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Analyzing the effects of minimum wages: a microeconomic approach

Clemens Thielen¹ · Philipp Weinschenk²

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Abstract

We use a microeconomic approach to analyze the effects of minimum wages. Agents are allowed to have different productivities at different principals as well as different costs of working. We obtain several new and interesting effects. Minimum wages could influence the generated surplus when leaving employment unaffected, and destroy jobs that generate relatively high levels of surplus when affecting employment. Furthermore, minimum wages could harm agents even if these stay employed, while principals could benefit from them. We provide a complete characterization of the effects and show that these hold independently of the specific bargaining procedure and information structure.

Keywords Minimum wages · Principal-agent model · Costs of working · Welfare effects

JEL Classification C78 · D21 · J31 · J38 · K31

1 Introduction

The insight that workers' occupational decisions depend not only on monetary compensation, but also on nonmonetary job characteristics, is widely accepted.¹ Employers might take these nonmonetary characteristics into consideration when making wage

¹ This insight—already conceived by Smith (1937)—is, for example, the key assumption in the literature on compensating wage differentials and has recently gained much attention, cf. Kaplan and Schulhofer-Wohl (2018).

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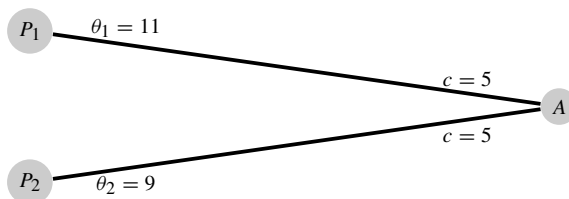
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offers.² Still, these aspects have not been fully considered when analyzing minimum wages, which is one of the most controversial, yet important topics in economics.³ We contribute to closing this gap by considering a simple microeconomic model where an agent (interpreted as a worker) can not only have different productivities at different principals (potential employers), but also different costs of working to which principals can adjust their wage offers. Agents are thus allowed to have preferences regarding their employment, and these preferences can be taken into account by principals when offering wages. This is important since job opportunities usually differ in their non-monetary characteristics, e.g., how satisfying, demanding, engaging, hazardous, and flexible they are, and what commuting costs they cause.⁴

Our model replicates the standard results we know from the existing literature in case each agent faces the same costs at all principals: Minimum wages redistribute income from principals to agents, such that principals suffer, while agents who stay employed benefit, and the efficiency is only affected if employment is decreased. Accordingly, the effects of minimum wages are a simple trade-off between efficiency (which is only affected if employment is lowered) and redistribution (directed from employers to workers).

This is illustrated in the following example.⁵ Consider a situation with two principals and one agent. Let the agent A have productivity $\theta_1 = 11$ when working for principal P_1 , and productivity $\theta_2 = 9$ when working for principal P_2 . The agent's costs c are 5 at both principals. Principals can make offers to the agent, who can



² Despite the fact that truck drivers earn well above the national average salary, the U.S. truck industry has problems finding drivers (CNN 2015; CNBC 2018), which indicates that relatively low-skilled workers also care for nonmonetary job characteristics. As a reaction, Walmart has increased truck driver salaries to \$87,500 a year on average (CNBC 2019).

³ According to the (International Labour Organization (ILO) 2017), 90% of ILO member states have minimum wages. In the U.S., for example, there is a controversial debate not only about the minimum wage at federal level, but also about the minimum wage in states and cities (New York Times 2021a, b, c). There is also a discussion in the UK about the potential rise of the minimum wage and living wage (Financial Times 2022).

⁴ Bonhomme and Jolivet (2009) empirically show that the type of work, the working conditions, the working times, the job security, and the distance to work are indeed important job characteristics for workers. Card et al. (2018) show that allowing workers to have idiosyncratic tastes for different workplaces is helpful to understand many well-documented empirical regularities on labor markets.

⁵ In this example as well as in the main part of the paper, we examine the single-agent case. As we show in Appendix C, our results carry over to the multi-agent case with an arbitrary number of agents if there are no (binding) capacity constraints for the principals, and the observed effects of minimum wages still arise when capacity constraints are considered.

then accept one of the offers or rejects all offers. For simplicity of exposition, we do not model the agent as a strategic player, but assume the agent to accept the utility-maximizing offer.⁶ In the Nash equilibrium without a minimum wage, the agent works for P_1 for a wage of $w = 9$, which results in utility $w - c = 4$ for the agent and a surplus of $\theta_1 - c = 6$. Every binding minimum wage \underline{w} that leaves the agent employed, i.e., $\underline{w} \in (9, 11]$, increases the agent's utility (to $\underline{w} - 5$) while leaving the surplus unaffected. Higher minimum wages $\underline{w} > 11$ cause the agent to become unemployed and lower the surplus.

The assumption that each agent faces the same costs at all principals—which might belong to different economic sectors or regions—is strong and generically violated.⁷ We, therefore, allow agents to have different costs and obtain new results that, in particular, invalidate the simple trade-off perspective on minimum wages. The main results are the following.

First, a minimum wage can adversely affect the generated surplus even if the employment level is unchanged. This effect arises in our model since a minimum wage could destroy job opportunities with relatively high levels of surplus while simultaneously maintaining opportunities with relatively low levels of surplus. These welfare losses may be hard to detect for outside observers (e.g., politicians or econometricians), since employment levels stay constant and wages increase. We can thus speak of “hidden costs” of minimum wages.

Second, an agent can also suffer from a minimum wage when remaining employed. There are two different reasons for this effect to emerge: (i) the minimum wage could force an agent to relocate to another principal, which we show to decrease his⁸ utility generically, or (ii) the minimum wage could allow an agent to stay at the same principal, but cause a reduction of the wage payment. This shows that minimum wages—which are usually intended to help agents—can actually harm agents also when they stay employed.

Some of the basic results are illustrated in the following example (see figure on the next page), which is identical to the previous example except that the agent's costs are different (e.g., due to different commuting costs or working conditions). Without a minimum wage, the agent works for P_2 for a wage of 7,⁹ which results in the utility $w - c_2 = 4$ and the surplus $\theta_2 - c_2 = 6$. Any minimum wage $\underline{w} \in (9, 11)$ causes the agent to stay employed (by relocating to P_1), but lowers the agent's utility to $\underline{w} - 7$ and the surplus to $\theta_1 - c_1 = 4$. Interestingly, not only agent's utility lowers, but also aggregate profits.

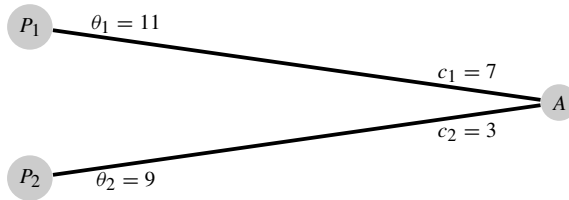
Relaxing the seemingly innocent assumption that each agent has identical costs at all principals thus drastically changes the effects of minimum wages. The results are robust and hold in a variety of different settings; see Sections 5 and 6 and the appendices for numerous extensions and robustness checks. Our analysis provides further robust effects.

⁶ In case of indifference, we let the agent behave according to natural tie-breaking rules that, as we show in Appendix A.1, exactly model the agent's behavior in any subgame-perfect Nash equilibrium.

⁷ Sorkin (2016, 2018), for instance, documents that nonpay characteristics differ substantially between economic sectors.

⁸ We follow the standard convention and talk about female principals and male agents.

⁹ P_1 will offer a wage of 11 so that P_2 has to offer at least 7 to attract the agent.



First, wages may overshoot. That is, equilibrium wages may increase strictly above the imposed minimum wage.

Second, principals may benefit from minimum wages. This effect arises when the minimum wage (i) forces an agent to relocate to another principal or (ii) shrinks the set of competing principals if no relocation occurs. Case (i) arises in the example above, where P_1 benefits from any minimum wage $\underline{w} \in (9, 11)$. Case (ii) arises in the example if the agent's cost at P_1 is changed to $c_1 = 4$. Here, the agent works for P_1 for a wage of 10 without a minimum wage, but any minimum wage $\underline{w} \in (9, 10)$ decreases the wage paid by P_1 to \underline{w} since P_2 will no longer compete.

Third, when causing unemployment, minimum wages may destroy jobs that generate relatively high levels of surplus. Accordingly, the jobs that are lost due to a minimum wage are not necessarily the ones that generate only marginal levels of surplus, and the effect of minimum wages on efficiency is not only of second order.

Fourth, minimum wages can cause an increase in equilibrium productivities by relocating agents from low to high productivity jobs. The productivity gains are, however, generically overcompensated by higher costs. Thus, the productivity gains caused by minimum wages are generically accompanied by efficiency losses.

We start our analysis with the basic bargaining procedure where the principals make offers. This simple procedure is standard in the principal-agent literature but quite specific. Therefore, we also consider alternative bargaining procedures as well as stable outcomes, which abstract from how parties bargain and what they know. We show that the set of stable outcomes is given by the convex combinations of the outcome obtained when principals make offers and the outcome when agents make offers. Interestingly, except for the boundary case corresponding to the situation where agents make offers, the effects of a minimum wage on stable outcomes are qualitatively exactly the same as in the case where the principals make offers.¹⁰ Therefore—and this is important—all effects of minimum wages we identify when principals make offers are the consequence of stability and not the consequence of the specific bargaining procedure or information structure.

1.1 Related literature

The empirical literature on minimum wages is extensive; see (Neumark and Wascher 2008; Manning 2021) for overviews. Early studies summarized by Brown et al. (1982)

¹⁰ For the boundary case where the stable outcome coincides with the case where agents make offers, the same effects arise as in case where the principals make offers, except that agents can never benefit from a minimum wage.

mainly found negative employment effects of minimum wages. This view was challenged by the famous study of Card and Krueger (1994), who find no indication for a reduction of employment. The debate on the employment effect of minimum wages is ongoing and there is still no consensus (Neumark 2019; Neumark et al. 2014b, a; Allegretto et al. 2011, 2017; Dube et al. 2010; Clemens and Wither 2019; Fang and Lin 2015; Addison et al. 2013; Liu et al. 2016; Thompson 2009; Muravyev and Oshchepkov 2016; Meer and West 2016; Wolfson and Belman 2019; Kreiner et al. 2020; Caliendo et al. 2018; Cengiz et al. 2019; Harasztosi and Lindner 2019; Aaronson et al. 2018). In the conclusion, we identify several factors that influence the employment effects of minimum wages, which might help us to understand the large variety of the empirical findings.

There is also a rich theoretical literature on minimum wages; see Neumark and Wascher (2008); Flinn (2010) for overviews. The traditional view is based on the supply-demand model, also referred to as the neoclassical competitive model (Mankiw 2017; Card and Krueger 1995). In its basic form, a unique equilibrium wage obtained at the intersection of the labor supply and demand curves is paid to all workers. A minimum wage is only effective if it is above this equilibrium wage. Such an effective minimum wage lowers employment (due to a decreased labor demand), causes unemployment (since labor demand falls short of labor supply), and lowers the generated surplus (by causing a deadweight loss). However, only the jobs that generate the lowest levels of surplus are destroyed. Since only marginal workers lose their jobs, a minimum wage has only a second-order effect on efficiency (Lee and Saez 2012, page 739). Overall, a minimum wage benefits workers who stay employed, but harms employers and workers who become unemployed.

Another class of models takes into account that employers could have market power. As explained by Robinson (1933) and Stigler (1946), a monopsonist optimally chooses an employment level below the competitive equilibrium in order to reduce its total wage payment. A minimum wage removes the monopsonist's incentive to keep employment artificially low and, thus, increases the wage (to the minimum wage) and the employment. This is beneficial for the workers and the generated surplus (since the deadweight loss is reduced) but harmful for the employer. In the related setting of monopsonistic competition studied by Bhaskar and To (1999), a minimum wage also increases firm-level employment, but may at the same time cause an exit of firms, thus leading to ambiguous effects on the aggregate employment level.

A third class of models is based on search theory. In these models, the wage offer distribution emerges as the equilibrium of a noncooperative wage search and wage posting game between workers and employers, and a minimum wage changes the game. See Flinn (2010) for a comprehensive overview. In a seminal paper, Burdett and Mortensen (1998) show that a minimum wage increases employment and shifts the equilibrium wage offer distribution to the right. Hence, the common result of the existing literature, according to which employed agents benefit from a binding minimum wage, also holds here. This type of model is typically referred to as “dynamic monopsony” since search-related frictions induce monopsony-like behavior (see Neumark and Wascher (2008, Chapter 3.2.2)).

Heterogeneous preferences over job characteristics are considered by Bhaskar et al. (2002); Bhaskar and To (1999, 2003) to analyze minimum wages. These authors

assume that firms are unable to make individual offers to workers that depend on the workers' specific preferences. Moreover, their work focuses more on the aggregate firm and industry employment effects of minimum wages, while our paper distinctly emphasizes the potential heterogeneity of wage, employment, and welfare outcomes. In our model, depending on the specific scenario, a (higher) minimum wage may lead to agents continuing to work for the same principals and accruing a higher or lower utility, relocating to different principals and experiencing a lower utility, or ending up out of work entirely—and these effects might be heterogeneous among agents. Thus, minimum wages can have a rich set of effects in our model, and we provide a complete characterization of them.

2 Model description

We now introduce our model for the case of a single agent, which is extended to multiple agents in Sect. 6. Consider an agent (interpreted as a worker) who can work for one of n principals (interpreted as potential employers). If the agent works for principal $i \in N = \{1, \dots, n\}$, his utility is

$$u_i = w_i - c_i,$$

where w_i is the wage paid by principal i and c_i is the agent's cost when working for i .¹¹ The cost depend on the job characteristics at principal i , e.g., commuting costs, type of work, working conditions, working times, and job security. If the agent does not work for any principal, we say the agent is *unemployed* and his utility is $u_0 = 0$.¹² The cost c_i is thus the agent's opportunity cost, i.e., the payment for which he is indifferent between working for principal i and not working. We analyze both the case of unrestricted wages and the case of restricted wages, where a minimum wage \underline{w} requires that $w_i \geq \underline{w}$. We underline all variables in case of restricted wages.

The profit of principal i if the agent works for her is

$$\pi_i = \theta_i - w_i,$$

where θ_i is the agent's productivity¹³ at principal i , while the profit is normalized to zero if the agent does not work for her. We consider only principals for which the

¹¹ We can also allow for non-linear utility functions $u(w_i, c_i) = h(w_i) - c_i$ (or some monotone transformation of it). This is equivalent to the case $u_i = w_i - c_i$ when transforming the productivities via h . Interestingly, if the utility function is concave in w_i , some of the possible negative effects of a minimum wage on the agent's utility and on the surplus might become even stronger.

¹² The case where the agent has a nonzero reservation utility is equivalent to the case with zero reservation utility and all costs increased by the initial reservation utility.

¹³ The productivity θ_i can be interpreted as principal i 's gross profit if the agent works for her. In case productivity is stochastic, θ_i is interpreted as expected gross profit. Similarly, if the agent's cost is stochastic, c_i can be interpreted as expected cost.

productivity exceeds the cost (i.e., $\theta_i > c_i$ for all i)¹⁴ and let $\theta_{\max} := \max_i \theta_i$ denote the maximum productivity.

The surplus generated when the agent works for principal i is

$$s_i = \pi_i + u_i = \theta_i - c_i.$$

If the agent does not work for any principal, the surplus is $s_0 = 0$.

We call the principal the agent works for the *winning principal* and let i^* denote her index. If the agent does not work for any principal, we set $i^* = 0$. The remaining principals $N \setminus \{i^*\}$ are referred to as the *losing principals*. The maximum surplus is $s_{\max} := \max_i s_i$ and $N_{\max} := \{i \in N : s_i = s_{\max}\}$ is the set of principals where this surplus can be generated.

In case of a minimum wage, we denote the principals who can afford the minimum wage without making a loss by $\underline{N} := \{i \in N : \theta_i \geq \underline{w}\}$. The maximum surplus among these principals is $\underline{s}_{\max} := \max\{0, s_i : i \in \underline{N}\}$. The set of principals who can afford the minimum wage and yield this surplus is $\underline{N}_{\max} := \{i \in \underline{N} : s_i = \underline{s}_{\max}\}$.

Recognize that different combinations of productivity and cost may not only stem from principals who are possibly located in different geographic regions or industrial sectors, but also from the possibility of choosing working hours, investments in working conditions, or efforts; see Appendix D.2. In all these scenarios, a higher productivity is naturally associated with a higher cost, but not necessarily with a higher surplus.

3 Basic bargaining procedure

We start by considering the basic bargaining procedure where the principals make take-it-or-leave-it offers. This procedure is predominant in the agency literature (cf. Laffont and Martimort (2002)) and allows for a simple and intuitive characterization. As we show later, the effects of minimum wages we obtain by using this procedure are robust.

The procedure is as follows: First, each principal i offers a wage $w_i \in \mathbb{R}$ to the agent or makes no offer. In case a minimum wage \underline{w} is imposed, the offered wages must satisfy $w_i \geq \underline{w}$. After receiving all offers, the agent either accepts one of the offers or rejects all offers, and the payoffs are realized.

We assume that the principals know the parameters of the model, which enables each principal to determine her best response to the other principals' offers. This assumption is not needed when interpreting the equilibrium of the basic bargaining procedure as the result of ascending offers made by principals (cf. Appendix A.5). It is not needed either when examining stable outcomes (cf. Appendix B.2), where one abstracts from how the parties bargain and what they know.

For simplicity of exposition, we do not model the agent as a strategic player that acts after the principals have made their offers. Instead, rationality of the agent is modeled by assuming that he always accepts an offer that maximizes his utility or no offer if all offers provide negative utility. In case of indifference, we let the agent

¹⁴ Principals i for which $\theta_i \leq c_i$ are redundant since employing the agent can never yield a positive profit/utility for one of the two parties without yielding a negative utility/profit for the other party.

behave according to the following tie-breaking rules that, as we show in Appendix A.1, exactly model the agent's strategic behavior in any subgame-perfect Nash equilibrium of the extensive form game. First, in case of indifference, the agent prefers to work, i.e., accept an offer instead of no offer. Second, if several offers maximize the agent's utility, he chooses one that maximizes the surplus among these offers. Third, if several offers maximize the agent's utility and the surplus, the agent chooses an offer according to an arbitrary deterministic tie-breaking rule. We concentrate on Nash equilibria in undominated strategies.¹⁵

For some results, we require the tie-breaking rule used in case that several offers simultaneously maximize the agent's utility and the surplus to satisfy the property of *independence of irrelevant alternatives*:

Definition 3.1 A tie-breaking rule satisfies *independence of irrelevant alternatives* if the following holds for any two subsets $N'' \subseteq N' \subseteq N$: If the tie-breaking rule selects principal j among the principals in N' , it also selects j among the principals in N'' whenever $j \in N''$.

We next characterize the pure-strategy Nash equilibria both for unrestricted and restricted wages and then analyze the effects of minimum wages.

3.1 Unrestricted wages

The following proposition establishes the existence of a Nash equilibrium. In the specified equilibrium, the principals where the maximum surplus can be generated offer wages equal to the second-highest surplus plus the agent's cost,¹⁶ while the remaining principals offer wages equal to the productivity.

Proposition 3.1 For unrestricted wages, the strategy profile

$$w_i = \begin{cases} \max\{0, \theta_j - c_j : j \in N \setminus \{i\}\} + c_i & \text{for } i \in N_{\max} \\ \theta_i & \text{for } i \notin N_{\max} \end{cases}$$

constitutes a Nash equilibrium yielding the maximum surplus s_{\max} .

Proof Observe that, for all $i \in N_{\max}$ and all $i' \notin N_{\max}$, the offers in Proposition 3.1 yield

$$u_i = w_i - c_i = \max\{0, \theta_j - c_j : j \in N \setminus \{i\}\} \geq \theta_{i'} - c_{i'} = w_{i'} - c_{i'} = u_{i'}.$$

Hence, $u_i \geq 0$ $i \in N_{\max}$ and the agent cannot do better than accepting an offer from a principal $i^* \in N_{\max}$ such that the maximum surplus s_{\max} is generated.

It remains to show that no principal can improve by changing her offer. Each principal $i \notin N_{\max}$ —i.e., each principal where the agent cannot generate the maximum

¹⁵ Weakly dominated strategies are discussed in Appendix A.2.

¹⁶ In case of $n \geq 2$ principals, an alternative interpretation is that each principal i where the maximum surplus can be generated offers a wage equal to the wage offered by a best competitor j (i.e., a principal with the second-highest surplus) plus the cost differential $c_i - c_j$.

surplus—currently has profit zero since the agent works for a principal in N_{\max} . No wage $w'_i < w_i = \theta_i$ can make the agent work for i , either. Any wage offer $w'_i > w_i = \theta_i$ yields a nonpositive profit and is, thus, weakly dominated by $w_i = \theta_i$.

For the principals in N_{\max} , first consider the case where $|N_{\max}| = 1$. Then, i^* 's current profit satisfies $\pi_{i^*} = \theta_{i^*} - w_{i^*} = \theta_{i^*} - c_{i^*} - \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} \geq 0$. Since all other principals offer wages of $w_i = \theta_i$, the current utility of the agent is

$$u_{i^*} = w_{i^*} - c_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} = \max\{0, u_j : j \in N \setminus \{i^*\}\}.$$

Hence, if principal i^* lowers her wage offer, the agent either works for another principal or for no principal at all. If i^* increases her wage offer, her profit reduces.

In case $|N_{\max}| \geq 2$, all principals in N_{\max} have profit zero since principal i^* 's current profit is $\pi_{i^*} = \theta_{i^*} - w_{i^*} = \theta_{i^*} - c_{i^*} - \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} = 0$. Moreover, we currently have $u_i = \max\{\theta_j - c_j : j \in N\}$ for all $i \in N_{\max}$. Hence, if a principal $i \in N_{\max}$ reduces the offered wage, the agent will accept the offer of another principal in N_{\max} , which cannot increase i 's profit. Increasing the offered wage to $w'_i > w_i$ leads the agent to accept i 's offer, but yields a negative profit for principal i . \square

The next proposition shows that the Nash equilibrium is essentially unique, in the sense that all Nash equilibria yield the same utility, profits, and surplus.

Proposition 3.2 *For unrestricted wages, every Nash equilibrium satisfies:*

- (I) *The agent accepts the offer of a principal $i^* \in N_{\max}$ and the surplus is $s_{i^*} = s_{\max}$.*
- (II) *The paid wage is $w_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$.*
- (III) *The agent's utility is $u_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}$, which is nonnegative and independent of the choice of i^* from N_{\max} .*
- (IV) *The profit of principal i^* is $\pi_{i^*} = \theta_{i^*} - c_{i^*} - \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}$, which is nonnegative, and equal to zero whenever $|N_{\max}| \geq 2$.*

Proof (I) Since not making an offer yields profit zero for any principal, the equilibrium profits of all principals must be nonnegative. Suppose contrary to our claim, that the agent works for principal $i^* \notin N_{\max}$. Then a surplus of only $s_{\max} - \epsilon$ for some $\epsilon > 0$ is generated. The agent's utility is then at most $s_{\max} - \epsilon$. Hence, any principal $i \in N_{\max}$ could make the agent work for her and increase her profit to $\frac{\epsilon}{2} > 0$ by offering a wage of $w_i = \theta_i - \frac{\epsilon}{2}$. This is a contradiction. Consequently, in any Nash equilibrium, the agent must work for a principal in N_{\max} and the surplus s_{\max} is generated.

- (II) Assume for the sake of a contradiction that $w_{i^*} \neq \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*} =: \tilde{w}$.

Case 1: $w_{i^*} > \tilde{w}$. If $|N_{\max}| = 1$, principal i^* could increase her profit by offering \tilde{w} , since no other principal i offers a wage larger than θ_i . If $|N_{\max}| \geq 2$, then $w_{i^*} > \tilde{w} = \theta_{i^*}$, so $\pi_{i^*} < 0$, which cannot hold in equilibrium.

Case 2: $w_{i^*} < \tilde{w}$. If $\tilde{w} = c_{i^*}$, then $w_{i^*} < c_{i^*}$, which cannot hold since the agent would then obtain a negative utility when working for i^* . If $\tilde{w} > c_{i^*}$,

then $\max\{\theta_j - c_j : j \in N \setminus \{i^*\}\} > 0$, and we let i denote a principal such that $\theta_i - c_i = \max\{\theta_j - c_j : j \in N \setminus \{i^*\}\} > 0$. The current utility of the agent is then $w_{i^*} - c_{i^*} < \tilde{w} - c_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} = \theta_i - c_i$. Hence, principal i could win and obtain a positive profit by offering a wage of $w_i = \theta_i - \epsilon$ for some $0 < \epsilon < (\theta_i - c_i) - (w_{i^*} - c_{i^*})$.

(III) & (IV) Directly from (I) and (II). \square

Proposition 3.2 shows that a surplus-maximizing allocation is implemented in any Nash equilibrium. This surplus is shared among the winning principal and the agent. The agent receives a share equal to the second-highest surplus (and thus realizing a positive utility) if there are at least two principals, while his share is zero if there is only one principal.

3.2 Restricted wages

Suppose wage setting is restricted by a minimum wage. We first establish that a Nash equilibrium still exists. In the specified equilibrium, principals who cannot afford the minimum wage make no offers; principals where the maximum surplus \underline{s}_{\max} can be generated offer wages equal to the maximum of the minimum wage and the sum of the second-highest surplus and the agent's cost; and the remaining principals offer wages equal to the productivity.

Proposition 3.3 *With a minimum wage \underline{w} , the strategy profile in which the principals in $N \setminus \underline{N}$ make no offers and the principals in \underline{N} offer*

$$\underline{w}_i = \begin{cases} \max\left\{\underline{w}, \max\{\theta_j - c_j : j \in \underline{N} \setminus \{i\}\} + c_i\right\} & \text{for } i \in \underline{N}_{\max} \\ \theta_i & \text{for } i \in \underline{N} \setminus \underline{N}_{\max} \end{cases}$$

constitutes a Nash equilibrium yielding surplus \underline{s}_{\max} .

Proof A principal $i \in N \setminus \underline{N}$ cannot increase her profit since making an offer requires to offer a wage $\underline{w}_i \geq \underline{w}$, which can never yield a positive profit for principal i . This proves the claim in the case where $\underline{N} = \emptyset$. In the case where $\underline{N} \neq \emptyset$, the rest of the argumentation is the same as in the proof of Proposition 3.1 except that the offered wages \underline{w}_i cannot fall short of \underline{w} . \square

We next show that the Nash equilibrium is also essentially unique for restricted wages. In order to avoid a tedious case distinction in the special case where $\underline{N} \neq \emptyset$ and $\theta_i = \underline{w}$ for all $i \in \underline{N}_{\max}$ (i.e., when all principals who can generate surplus \underline{s}_{\max} have productivities exactly equal to the minimum wage) we assume that at least one of the principals in \underline{N}_{\max} makes an offer.

Proposition 3.4 *With a minimum wage \underline{w} , every Nash equilibrium satisfies:*

- (I) *If $\underline{N} = \emptyset$, no principal makes an offer and the agent does not work for any principal, i.e., $i^* = 0$.*
- (II) *If $\underline{N} \neq \emptyset$:*

- (1) The agent accepts the offer of a principal $\underline{i}^* \in \underline{N}_{\max}$ and the surplus is $\underline{s}_{\underline{i}^*} = \underline{s}_{\max}$.
- (2) The paid wage is $\underline{w}_{\underline{i}^*} = \max \left\{ \underline{w}, \max \{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} + c_{\underline{i}^*} \right\}$.
- (3) The agent's utility is $\underline{u}_{\underline{i}^*} = \max \left\{ \underline{w} - c_{\underline{i}^*}, \max \{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} \right\}$, which is nonnegative and independent of the choice of \underline{i}^* from \underline{N}_{\max} .
- (4) The profit of principal \underline{i}^* is $\underline{\pi}_{\underline{i}^*} = \min \left\{ \theta_{\underline{i}^*} - \underline{w}, \theta_{\underline{i}^*} - c_{\underline{i}^*} - \max \{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} \right\}$, which is nonnegative, and equal to zero whenever $|\underline{N}_{\max}| \geq 2$.

Proof (I) If $\underline{N} = \emptyset$ and some principals offer wages, then the winning principal \underline{i}^* obtains a profit of $\underline{\pi}_{\underline{i}^*} = \theta_{\underline{i}^*} - \underline{w}_{\underline{i}^*} \leq \theta_{\underline{i}^*} - \underline{w} < 0$. Thus, this principal can increase her profit by not making an offer.

(II) For any principal $i \in \underline{N} \setminus \underline{N}$, making an offer (which would have to exceed the productivity θ_i as $\theta_i < \underline{w}$) is weakly dominated by not making an offer. Hence, no principal in $\underline{N} \setminus \underline{N}$ makes an offer and the game played by the principals in \underline{N} reduces to a game between the principals in \underline{N} . Claims (1) and (2) then follow exactly as (I) and (II) in the proof of Proposition 3.2 except that the paid wage $\underline{w}_{\underline{i}^*}$ cannot fall short of \underline{w} . Claims (3) and (4) then follow directly from (2) by plugging in the wage $\underline{w}_{\underline{i}^*}$. \square

4 Effects of minimum wages

We now analyze the effects of minimum wages by comparing the equilibria with restricted wages to those with unrestricted wages. Note that an increase of an existing minimum wage has the same effects as the introduction of a minimum wage, as we show in Appendix A.4.

The next corollary, which follows from Proposition 3.4, characterizes when the agent becomes unemployed due to a minimum wage.

Corollary 4.1 *The agent gets unemployed with a minimum wage \underline{w} (i.e., $\underline{i}^* = 0$) if and only if $\underline{N} = \emptyset$, which is equivalent to $\underline{w} > \theta_{\max}$. In this case, the agent's utility, the principals' profits, and the surplus drop to zero.*

Corollary 4.1 has two interesting implications. First, a minimum wage could cause agents who work in high-surplus jobs to become unemployed, while leaving agents with low-surplus jobs unaffected. This holds true since the relevant condition ($\underline{w} > \theta_{\max}$) is about productivities and not surpluses. Second, also minimum wages slightly above the initial equilibrium wage can cause substantial losses of surplus.¹⁷ Hence, the effect of minimum wages on efficiency is not only of second order.

We next study the effects of a minimum wage for the case where the agent stays employed, i.e., when $\underline{w} \leq \theta_{\max}$ holds.

¹⁷ The simplest example is the case of two identical principals, where any minimum wage above the initial equilibrium wage destroys all surplus.

4.1 Effects on the generated surplus

Theorem 4.1 characterizes the effects of a minimum wage on the generated surplus.

Theorem 4.1 *Suppose there is a minimum wage $\underline{w} \leq \theta_{\max}$.*

- (I) *If $\underline{w} \leq \max_{j \in N_{\max}} \theta_j$, the generated surplus remains unchanged: $s_{\underline{i}^*} = s_{i^*}$.*
- (II) *If $\underline{w} > \max_{j \in N_{\max}} \theta_j$, the generated surplus decreases: $s_{\underline{i}^*} < s_{i^*}$.*

Proof (I) If $\underline{w} \leq \max_{j \in N_{\max}} \theta_j$, at least one principal from N_{\max} is contained in \underline{N} , which directly implies that $\underline{N}_{\max} \subseteq N_{\max}$. Accordingly, by Propositions 3.2 and 3.4, the agent keeps working for a principal where the maximum surplus s_{\max} is generated.

(II) If $\underline{w} > \max_{j \in N_{\max}} \theta_j$, then $\underline{N} \cap N_{\max} = \emptyset$, so also $\underline{N}_{\max} \cap N_{\max} = \emptyset$, which implies that the agent cannot keep working for a principal where the maximum surplus s_{\max} is generated, i.e., $\underline{i}^* \notin N_{\max}$. Consequently, by Propositions 3.2 and 3.4, the generated surplus lowers, $s_{\underline{i}^*} = s_{\max} < s_{\max} = s_{i^*}$. \square

Theorem 4.1 shows that the minimum wage can lower the generated surplus even if the agent stays employed. The intuition is that, with heterogeneous costs, a minimum wage can destroy the employment opportunity with the highest surplus and thereby force the agent to work for another principal where only a relatively low level of surplus is generated. See the second illustrative example in the introduction.

If the tie-breaking rule satisfies independence of irrelevant alternatives or there are no ties requiring such a tie-breaking rule (which holds generically), we obtain another relevant insight: The minimum wage increases the agent's productivity and cost whenever he relocates to another principal.¹⁸ By Theorem 4.1, however, the productivity increase always falls short of the cost increase. The productivity gains caused by minimum wages are thus generically accompanied by efficiency losses.

4.2 Effects on the paid wage

We next characterize how a minimum wage affects the paid wage.

Theorem 4.2 *Suppose there is a minimum wage $\underline{w} \leq \theta_{\max}$.*

- (I) *If $\underline{w} > w_{i^*}$, the paid wage increases, $\underline{w}_{\underline{i}^*} > w_{i^*}$.*
- (II) *If $\underline{w} = w_{i^*}$, the paid wage increases weakly, $\underline{w}_{\underline{i}^*} \geq w_{i^*}$. If the tie-breaking rule satisfies independence of irrelevant alternatives, $\underline{w}_{\underline{i}^*} = w_{i^*}$.*
- (III) *If $\underline{w} < w_{i^*}$, the effect on the paid wage is ambiguous, i.e., $\underline{w}_{\underline{i}^*} \gtrless w_{i^*}$. If the tie-breaking rule satisfies independence of irrelevant alternatives, $\underline{w}_{\underline{i}^*} \leq w_{i^*}$.*

Proof (I) Follows since $\underline{w} > w_{i^*}$ and the minimum wage requires that $\underline{w}_{\underline{i}^*} \geq \underline{w}$.

(II)&(III) The relation $\underline{w}_{\underline{i}^*} \geq w_{i^*}$ in (II) follows from $\underline{w} = w_{i^*}$ and $\underline{w}_{\underline{i}^*} \geq \underline{w}$. The ambiguity in (III) follows since $\underline{w} \leq w_{i^*}$ implies that $i^* \in \underline{N}_{\max}$ and $\underline{N}_{\max} \subseteq$

¹⁸ The agent relocates to another principal if and only if $\underline{N}_{\max} \cap N_{\max} = \emptyset$. Accordingly, $\theta_{i^*} < \underline{w} \leq \theta_{\underline{i}^*}$, which implies $c_{i^*} < c_{\underline{i}^*}$ since $s_{\underline{i}^*} = s_{\max} < s_{\max} = s_{i^*}$.

N_{\max} , so any combination of $i^*, \underline{i}^* \in \underline{N}_{\max}$ is possible if the tie-breaking rule does not satisfy independence of irrelevant alternatives, which might lead to different wages with and without the minimum wage if $|\underline{N}_{\max}| \geq 2$. Hence, (II) and (III) follow if we show that $\underline{w} \leq w_{i^*}$ and a tie-breaking rule satisfying independence of irrelevant alternatives imply $\underline{w}_{\underline{i}^*} \leq w_{i^*}$. Suppose contrary to this claim that $\underline{w}_{\underline{i}^*} > w_{i^*}$. Observe that $\underline{w} \leq w_{i^*}$ implies $i^* \in \underline{N}_{\max}$ and $\underline{N}_{\max} \subseteq N_{\max}$ as seen above, which, by independence of irrelevant alternatives, implies that $\underline{i}^* = i^*$. Using $\underline{w} \leq w_{i^*} < \underline{w}_{\underline{i}^*}$ and Propositions 3.2 and 3.4 yields that

$$\begin{aligned} \underline{w}_{\underline{i}^*} &= \max \left\{ \underline{w}, \max \{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} + c_{\underline{i}^*} \right\} \\ &\leq \max \{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*} \\ &= \max \{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*} = w_{i^*} \end{aligned} \quad \square$$

A minimum wage above the initial equilibrium wage thus either leads the agent to become unemployed or to an increase of the paid wage. Interestingly, the paid wage might increase strictly above the minimum wage.

Example 4.1 Consider a situation with three principals, where $\theta_1 = 13$, $\theta_2 = 12$, $\theta_3 = 9$ and $c_1 = 7$, $c_2 = 8$, $c_3 = 2$. Without a minimum wage, we have $i^* = 3$ and $w_{i^*} = 8$. With a minimum wage of $\underline{w} = 10 > w_{i^*}$, however, we have $\underline{i}^* = 1$ and $\underline{w}_{\underline{i}^*} = 11 > \underline{w}$.¹⁹

This phenomenon, which we refer to as *wage overshooting*, is defined as follows.²⁰

Definition 4.1 A minimum wage $\underline{w} > w_{i^*}$ causes *wage overshooting* if the agent stays employed and the paid wage increases to $\underline{w}_{\underline{i}^*} > \underline{w}$.

Propositions 3.2 and 3.4 yield the following characterization of wage overshooting.

Corollary 4.2 A minimum wage \underline{w} causes wage overshooting if and only if $\underline{w} \leq \theta_{\max}$ and

$$\max \{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} + c_{\underline{i}^*} > \underline{w} > \max \{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}. \quad (\text{WOS})$$

Thus, a minimum wage leads to wage overshooting when it forces the agent to relocate to a principal where the cost $c_{\underline{i}^*}$ is rather large.²¹ Intuitively, the principal

¹⁹ Note that this wage overshooting effect and all other effects occurring in this example (which will be discussed in the following) are robust with regard to small changes of the values, in the sense that they also arise if all productivities and costs are independently perturbed by $\pm \epsilon$ for some $0 \leq \epsilon < 1/8$.

²⁰ Empirically, it has been documented that the introduction of a minimum wage changes the aggregate earnings distribution also above the minimum wage (Neumark and Wascher 2008), which is in line with the effects of wage overshooting.

²¹ van den Berg (2003) provides an alternative mechanism for wage overshooting, namely the existence of multiple equilibria and the selection effects caused by minimum wages.

must compensate the agent for a high cost by paying a relatively high wage. The necessary wage could lie above the minimum wage, i.e., cause wage overshooting.

While a minimum wage that exceeds the initial wage ($\underline{w} > w_{i^*}$) can cause wage overshooting, another interesting phenomenon can arise if the minimum wage falls short of the initial wage ($\underline{w} < w_{i^*}$). The minimum wage can then lead to a *decrease* of the paid wage. This holds even if the tie-breaking rule satisfies independence of irrelevant alternatives (or there are no ties requiring such a rule). We formally characterize the conditions for a wage decrease in the next corollary, which follows from Propositions 3.2 and 3.4.

Corollary 4.3 *A minimum wage leads to a decrease in the paid wage (i.e., $\underline{w}_{i^*} < w_{i^*}$) if and only if $\underline{w} < w_{i^*}$ and $\max\{0, \theta_j - c_j : j \in \underline{N} \setminus \{i^*\}\} + c_{i^*} < \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$. If the tie-breaking rule satisfies independence of irrelevant alternatives, then $\underline{i}^* = i^*$ and the second condition simplifies to*

$$\max\{0, \theta_j - c_j : j \in \underline{N} \setminus \{i^*\}\} < \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}. \quad (\text{EBC})$$

To gain the intuition, we concentrate on the case where the tie-breaking rule satisfies independence of irrelevant alternatives (or there are no ties requiring such a rule). From Corollary 4.3 we see that a minimum wage lowers the wage payment whenever the minimum wage falls short of the initial wage and the minimum wage eliminates the winning principal's best competitor(s). This is formally captured by the elimination of best competitor condition (EBC). This is intuitive: If a principal's best competitor is eliminated by the minimum wage since the competitor cannot afford it, the principal lowers her wage offer. See Example 4.2 below.²² Minimum wages, which are commonly intended for redistribution from principals to agents, can thus have the opposite effect, i.e., lead to a redistribution from agents to principals.

Example 4.2 Consider a situation with two principals, where $\theta_1 = 14$, $\theta_2 = 8$ and $c_1 = 9$, $c_2 = 5$. Without a minimum wage, we have $i^* = 1$ and $w_{i^*} = 12$. With a minimum wage of $\underline{w} = 10$, we still have $\underline{i}^* = 1$, but $\underline{w}_{i^*} = 10 < w_{i^*}$.

Figure 1 summarizes the effects of a minimum wage on the paid wage if the tie-breaking rule satisfies independence of irrelevant alternatives (or there are no ties requiring such a rule).

4.3 Effects on the agent's utility

We next determine the effects of a minimum wage on the agent's utility.

²² Note that this effect does not occur in the extreme scenario where the principals have no bargaining power and the agent makes offers to the principals since the equilibrium offer then guarantees that the agent receives the whole surplus. See Sect. 5 for a brief overview and Appendix B for a detailed analysis. Remarkably, in all intermediate cases (i.e., when both sides have some bargaining power), the effects are qualitatively identical to the case where the principals make the offers.

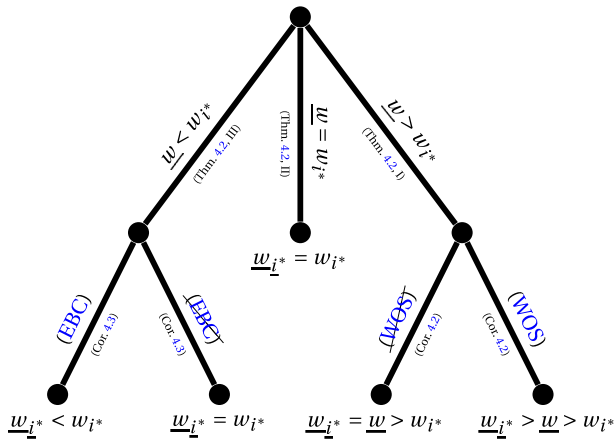


Fig. 1 Effects on the paid wage

Theorem 4.3 Suppose there is a minimum wage $\underline{w} \leq \theta_{\max}$.

(I) If $\underline{i}^* = i^*$ and

- (1) $\underline{w} > w_i^*$, the agent's utility increases, i.e., $\underline{u}_i^* > u_i^*$.
- (2) $\underline{w} = w_i^*$, the agent's utility remains unchanged, i.e., $\underline{u}_i^* = u_i^*$.
- (3) $\underline{w} < w_i^*$, the agent's utility decreases weakly, i.e., $\underline{u}_i^* \leq u_i^*$.

(II) If $\underline{i}^* \neq i^*$, the agent's utility decreases weakly, i.e., $\underline{u}_i^* \leq u_i^*$.

When the tie-breaking rule satisfies independence of irrelevant alternatives, Case (I) applies if and only if $\underline{w} \leq \theta_{i^*}$, and Case (II) applies otherwise.

Proof (I) (1) Follows since $\underline{w} > w_i^*$ and $\underline{w}_i^* \geq \underline{w}$ such that $\underline{w}_i^* > w_i^*$ as well as $\underline{i}^* = i^*$.

- (2) If $\underline{w} = w_i^*$, then $\underline{w} - c_{i^*} = w_i^* - c_{i^*}$. Since $\underline{i}^* = i^*$ and, by Proposition 3.2, $w_i^* - c_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}$, this yields

$$\underline{w} - c_{\underline{i}^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}.$$

Because the maximum is no smaller than $\max\{0, \theta_j - c_j : j \in N \setminus \{\underline{i}^*\}\}$,

$$\max\left\{\underline{w} - c_{\underline{i}^*}, \max\{0, \theta_j - c_j : j \in N \setminus \{\underline{i}^*\}\}\right\} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}.$$

By Propositions 3.2 and 3.4, this is equivalent to $\underline{u}_i^* = u_i^*$.

- (3) If $\underline{w} < w_i^*$, the same argumentation as in the proof of (2) shows that

$$\underline{w} - c_{\underline{i}^*} < \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}.$$

Since $\max\{0, \theta_j - c_j : j \in N \setminus \{\underline{i}^*\}\} \leq \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}$, Propositions 3.2 and 3.4 imply that $\underline{u}_i^* \leq u_i^*$.

(II) Proposition 3.2 and $\underline{i}^* \neq i^*$ yield

$$u_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} \geq \theta_{\underline{i}^*} - c_{\underline{i}^*} \geq \underline{w}_{\underline{i}^*} - c_{\underline{i}^*} = \underline{u}_{\underline{i}^*},$$

where the second inequality follows since the paid wage $\underline{w}_{\underline{i}^*}$ cannot exceed $\theta_{\underline{i}^*}$. The claim that, when the tie-breaking rule satisfies independence of irrelevant alternatives, Case (I) applies if and only if $\underline{w} \leq \theta_{i^*}$, while Case (II) applies otherwise, follows directly from the definition of independence of irrelevant alternatives and the fact that $i^* \in \underline{N}_{\max}$ if and only if $\underline{w} \leq \theta_{i^*}$. \square

Theorem 4.3 shows that a minimum wage increases the agent's utility when the minimum wage exceeds the initial wage and there is no relocation. Otherwise, the agent's utility decreases at least weakly. This also implies that the agent can never benefit from wage overshooting, i.e., the wage increase can never compensate the agent's cost increase. We show next that a minimum wage could cause a strict decrease of the agent's utility.

Corollary 4.4 *Suppose there is a minimum wage $\underline{w} \leq \theta_{\max}$.*

- (I) *If $\underline{i}^* = i^*$, the agent's utility decreases (i.e., $\underline{u}_{\underline{i}^*} < u_{i^*}$) if and only if $\underline{w} < w_{i^*}$ and (EBC) holds.*
- (II) *If $\underline{i}^* \neq i^*$, the agent's utility decreases (i.e., $\underline{u}_{\underline{i}^*} < u_{i^*}$) if and only if*

$$|\underline{N}_{\max}| = 1 \text{ or } \underline{i}^* \notin \operatorname{argmax}\{\theta_j - c_j : j \in N \setminus \{i^*\}\}. \quad (\text{UD})$$

Proof (I) Since $\underline{i}^* = i^*$, the agent's cost stay constant such that the agent's utility decreases if and only if the paid wage decreases. By Corollary 4.3, this happens if and only if $\underline{w} < w_{i^*}$ and $\max\{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} + c_{\underline{i}^*} < \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$, where the latter condition reduces to (EBC) since $\underline{i}^* = i^*$.

- (II) A decrease in the agent's utility occurs if and only if at least one of the two inequalities in Case (II) of the proof of Theorem 4.3 is strict, which happens exactly if $|\underline{N}_{\max}| = 1$ or $\underline{i}^* \notin \operatorname{argmax}\{\theta_j - c_j : j \in N \setminus \{i^*\}\}$. \square

A minimum wage can thus reduce the agent's utility for two reasons. First, when the minimum wage allows the winning principal to lower the wage payment, which arises whenever the minimum wage eliminates the principal's best competitor(s). See Example 4.2 for an illustration. Second, when the minimum wage causes the agent to relocate to another principal and condition (UD) holds, which is generically true. This is illustrated in Example 4.1, where the minimum wage lowers the agent's utility from 6 to 4.²³ Intuitively, the agent's utility generically suffers if he relocates because the higher cost overcompensates the higher wage payment.

Summarizing, even if the agent stays employed after the introduction or increase of a minimum wage, his utility may suffer—no matter whether he switches to another principal or not. If he switches, the agent's utility generically decreases, and this holds even if the minimum wage exceeds the previously paid wage. See Figure 2 for the overview.

²³ In this example, for all minimum wages between 9 and 12, condition (UD) is only violated in the non-generic case where $\theta_1 - c_1 = \theta_2 - c_2$.

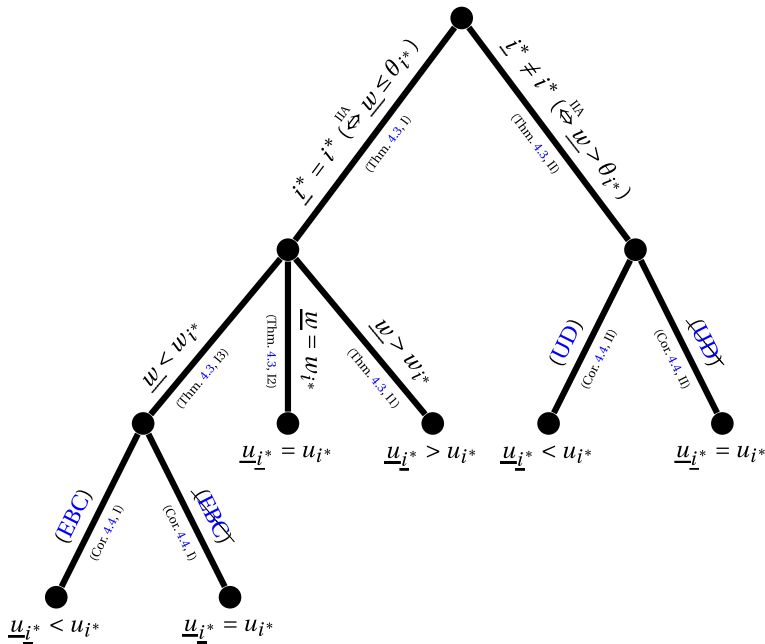


Fig. 2 Effects on the agent's utility

4.4 Effects on the principals' profits

By comparing Propositions 3.2 and 3.4, we directly obtain the effects on the profits.

Theorem 4.4 Suppose there is a minimum wage $\underline{w} \leq \theta_{\max}$.

- (I) Principal i 's profit decreases if and only if either (a) $i = i^* \neq \underline{i}^*$ and $|N_{\max}| = 1$, or (b) $i = i^* = \underline{i}^*$ and $\underline{w} > w_{i^*}$.
- (II) Principal i 's profit increases if and only if either (a) $i = \underline{i}^* \neq i^*$, $\underline{w} \neq \theta_{\underline{i}^*}$, and $|N_{\max}| = 1$, or (b) $i = i^* = \underline{i}^*$, $\underline{w} < w_{i^*}$, and (EBC) holds.

Theorem 4.4 shows that a principal can benefit from a minimum wage for two reasons. First, if the minimum wage forces an agent to relocate, the principal who is able to attract the agent now generically earns a positive profit. See Example 4.1. Second, if the minimum wage shrinks the set of competing principals when no relocation occurs and the minimum wage falls short of the initial wage. See Example 4.2. Interestingly, a minimum wage can simultaneously lower the agent's utility and the aggregate profits, as Corollary 4.4 and Theorem 4.4 reveal. See the second example in the introduction for an illustration.

5 Alternatives to the basic bargaining procedure

In Appendix B, we examine alternative bargaining procedures. First, we let the agent propose wages to the principals. Second, we abstract from how the parties bargain

and what they know by examining stable outcomes. We show that the set of stable outcomes is given by the convex combinations of the outcome obtained when the principals make offers and the outcome when the agent makes offers. Remarkably, for any fixed bargaining power of the agent except for the boundary case corresponding to the situation where the agent makes offers, the effects of minimum wages for stable outcomes are always *qualitatively identical* to the effects observed when the principals make offers. That is, if a minimum wage increases [decreases, does not change] the surplus, the wage, the agent's utility, or the principals' profits for stable outcomes, the same holds true when the principals make offers (and vice versa). This implies that all effects of minimum wages we have identified before for the procedure where the principals make offers are the consequence of the stability requirement, and not the consequence of the specific bargaining procedure or information structure.

6 Multiple agents

In Appendix C, we show that the model is readily generalized to the case where multiple agents interact with multiple principals. An important insight we obtain is that, in the situation where the principals have no (or no binding) capacity constraints that limit the numbers of agents they can employ, the principals compete for each of the m agents independently of the presence of the other agents. Accordingly, the situation decomposes into m single-agent problems, to which the analysis from the previous sections applies. In the absence of binding capacity constraints, all results shown for the single-agent case thus fully carry over to case with multiple agents. The same holds true if each principal has a capacity of one and the multi-agent model is generated by a duplication of a single-agent model. For the general case of capacity constraints, we prove the existence of stable outcomes via an algorithmic approach and show that the previously obtained effects of minimum wages still arise.

7 Conclusion

This paper uses a microeconomic approach to study the effects of minimum wages. We allow that agents can have different productivities at different principals as well as different costs of working. We identify a rich set of effects. We inter alia show that minimum wages may also lower efficiency when leaving employment unaffected, and destroy jobs that generate high levels of surplus when affecting employment. Minimum wages could harm agents even if these stay employed, while principals could benefit from minimum wages. Our analysis further reveals that the productivity gains caused by minimum wages are generically accompanied by efficiency losses.

We provide a complete characterization of the effects of minimum wages and show that all effects are robust. The effects are the consequence of stability, and not the consequence of specific bargaining procedures or information structures.

The results are policy-relevant. Suppose, for instance, that a government executes a minimum wage reform, and the unemployment rates stay unchanged. Our insights imply that we then cannot conclude that the reform necessarily helps workers and

leaves efficiency unchanged. Another important issue is that we have to distinguish between the productivity effects and the efficiency effects caused by minimum wages. If we observe that a minimum wage reform increases workers' productivities, these gains should not be taken as evidence of a positive efficiency effect.

The model also provides testable implications. First, an important mechanism in the model is that minimum wages may cause the relocation of workers to other employers where they suffer from higher costs. One could test whether minimum wages affect key components of workers' costs, e.g., increase commuting costs/distances.²⁴ Second, the model predicts that minimum wages reduce employment to a lesser extent (a) in environments where parties have more information,²⁵ (b) in more diverse economic environments,²⁶ and (c) when better transportation technologies are available.²⁷ Since the information and transportation technologies have improved substantially over the last decades, the model predicts that the employment effect of minimum wages should be weaker nowadays than in the past. This is precisely the pattern meta studies find.²⁸ Finally, the model might also help us to understand the variety of the empirical findings in given time periods, which is a notable puzzle of the empirical literature on minimum wages (cf. the brief overview in the introduction). Some of the variety of the empirical findings could be due to differences in the afore-described factors (a)–(c) among the different studies.

Appendices

A Discussion of the bargaining procedure and equilibrium concept

In this section, we discuss and substantiate the basic bargaining procedure and equilibrium concept used in Sects. 3 and 4. In Sect. A.1, we justify the tie-breaking rules used in this bargaining procedure. Sect. A.2 shows the effects of allowing weakly dominated strategies to be played by the principals, while Sect. A.3 considers the generalization of the equilibrium concept to mixed-strategy Nash equilibria. Section A.4 shows that the effects observed when introducing a minimum wage carry over to the case where an existing minimum wage is increased. Finally, Sect. A.5 analyzes an

²⁴ Dustmann et al. (2022) document that the introduction of a minimum wage in Germany has increased the commuting distances of low-wage workers by 1.5 km (or 8%) relative to high-wage workers. Moreover, as predicted by our model, the minimum wage has led to a relocation of low-wage workers to more productive firms. These observations can also be explained by the model of Bhaskar and To (1999), who also note that some workers might have to accept less preferred jobs after the introduction of a minimum wage.

²⁵ Formally, more information leads to more employment opportunities, which expands the set N and, thus, causes an agent to remain employed for a larger set of minimum wages.

²⁶ A more diverse economic environment can be captured by a mean-preserving spread of the productivities, which causes an agent to remain employed for a larger set of minimum wages.

²⁷ Lower transportation costs lead to an expansion of the set of principals for whom an agent can profitably work and, thus, to a larger set of minimum wages for which the agent stays employed.

²⁸ In their review, Brown et al. (1982) establish a much-cited consensus that the employment elasticity of minimum wages was between -0.3 and -0.1 . Wolfson and Belman's meta-analysis (Wolfson and Belman 2019) uses more recent data and establishes that the range has shifted to -0.13 and -0.07 .

alternative bidding procedure in which the principals make ascending offers to the agent.

A.1 Justification of the tie-breaking rules

We now discuss the tie-breaking rules used in the basic bargaining procedure by showing that, in the natural two-stage game where the principals first make their offers and the agent then either chooses one of these offers or rejects all offers, he always behaves according to these rules in any subgame-perfect Nash equilibrium. While we concentrate on the case of unrestricted wages in the following discussion, the arguments readily carry over to the case of restricted wages.

The first proposition shows that, in any subgame-perfect Nash equilibrium of the two-stage game, the agent always chooses an offer maximizing the surplus if several offers maximize his utility:

Proposition A.1 *Suppose that the principals' offers are such that $i', i'' \in \operatorname{argmax}_{i \in N} w_i - c_i$ with $s_{i'} > s_{i''}$. Then the agent choosing principal i'' does not constitute a subgame-perfect Nash equilibrium.*

Proof If $w_{i''} > \theta_{i''}$, then $\pi_{i''} = \theta_{i''} - w_{i''} < 0$, so principal i'' can increase her profit to zero by reducing her offer to $\theta_{i''}$. If $w_{i''} \leq \theta_{i''}$, then $u_{i''} = w_{i''} - c_{i''} \leq \theta_{i''} - c_{i''} = s_{i''} < s_{i'} = \theta_{i'} - c_{i'}$. Consequently, principal i' can increase her profit by offering $\theta_{i'} - \epsilon$ for some $0 < \epsilon < \theta_{i'} - c_{i'} - s_{i''}$ (which makes the agent accept the offer of i'). \square

The next proposition shows that, in any subgame-perfect Nash equilibrium, the agent always accepts an offer instead of no offer in case of indifference:

Proposition A.2 *Suppose that the principals' offers satisfy $\max_{i \in N} w_i - c_i = 0$. Then the agent choosing to reject all offers does not constitute a subgame-perfect Nash equilibrium.*

Proof If the agent rejects all offers, the assumption that $\theta_i > c_i$ for all i implies that any principal $i \in N$ can increase her profit by offering $w_i = c_i + \epsilon$ for some $0 < \epsilon < \theta_i - c_i$ (which makes the agent accept i 's offer). \square

Our last assumption on the agent's behavior states that he always chooses an offer according to an arbitrary deterministic tie-breaking rule (e.g., choosing the offer of the principal with the lowest index) in case that several offers simultaneously maximize both his utility and the surplus. While using a stochastic tie-breaking rule in this case would make the winning principal and (possibly) the paid wage stochastic, it would not influence the agent's utility, the generated surplus, or the principals' profits.

A.2 Allowing for weakly dominated strategies

In the analysis of the basic bargaining procedure, we concentrate on equilibrium strategies that are not weakly dominated. If weakly dominated strategies are allowed, Part (I)

of Proposition 3.2 and Parts (I) and (II), (1) of Proposition 3.4 still hold by the same argumentation as in the original proofs. Consequently, the winning principal and the equilibrium surplus remain unchanged and are still uniquely determined. However, the wage and, hence, the distribution of the surplus between the winning principal and the agent are, in general, not unique anymore even for a fixed winning principal $i^* \in N_{\max}$. We now demonstrate this for the case of unrestricted wages – the case of restricted wages is similar. Specifically, we now show that, for unrestricted wages, any wage $w_{i^*} \in [\max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}, \theta_{i^*}]$ can be obtained in a Nash equilibrium if weakly dominated strategies are allowed. This holds since the following strategy profile constitutes a Nash equilibrium: principal $i^* \in N_{\max}$ offers $w_{i^*} \in [\max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}, \theta_{i^*}]$, at least one other principal $j \in N \setminus \{i^*\}$ offers $w_j = w_{i^*} + c_j - c_{i^*}$, and each of the remaining principals $k \in N \setminus \{i^*, j\}$ offers a wage w_k weakly below $w_{i^*} + c_k - c_{i^*}$. Note that agent j 's wage offer w_j exceeds the productivity θ_j —and is, thus, weakly dominated—unless $j \in \arg\max\{\theta_j - c_j : j \in N \setminus \{i^*\}\}$ and $w_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$.

Two remarks are in order. First, an equilibrium in which weakly dominated strategies are played requires an implausibly high degree of coordination among players. At least one principal j who plays a weakly dominated strategy must exactly match the utility offered to the agent by the winning principal $i^* \in N_{\max}$, despite the fact that there is a continuum of possible utilities the agent could be offered by i^* . Simultaneously, principal j takes the chance of a loss if coordination fails, i.e., if she offers a slightly higher utility. This is why equilibria in weakly dominated strategies are usually ruled out. Second, it is interesting to recognize that, although Nash equilibria in weakly dominated strategies are implausible in the basic bargaining procedure where the principals design the offers, the set of possible equilibrium wages that can result from these equilibria is identical to the set of wages in stable outcomes; cf. the characterization of stable outcomes in Sect. B.2.

A.3 Mixed-strategy nash equilibria

We next show that, for both unrestricted and restricted wages, the generated surplus, the agent's utility, and the principals' profits in any (mixed-strategy) Nash equilibrium are as in every pure-strategy Nash equilibrium with probability 1. Consequently, the effects of a minimum wage on the surplus, utility, and profits do not depend on whether pure- or mixed-strategy equilibria are considered. With respect to the paid wage, this also holds if either we are in the generic case where $|N_{\max}| = |\underline{N}_{\max}| = 1$, or $|N_{\max}|, |\underline{N}_{\max}| \leq 2$ and a deterministic tie-breaking rule is used in case that several offers simultaneously maximize both the agent's utility and the surplus. If $|N_{\max}| \geq 3$ or $|\underline{N}_{\max}| \geq 3$, the equilibrium wage can be stochastic in some mixed-strategy equilibria.

Theorem A.1 *Suppose that the wages are unrestricted and $|N_{\max}| = 1$, i.e., there is a unique principal $i_{\max} \in N_{\max}$. Then every (mixed-strategy) Nash equilibrium satisfies:*

- (I) *With probability 1, principal i_{\max} offers a wage of $\check{w}_{i_{\max}} := \max\{0, \theta_j - c_j : j \in N \setminus \{i_{\max}\}\} + c_{i_{\max}}$.*
- (II) *With probability 1, the winning principal is $i^* = i_{\max}$ and $w_{i^*} = \check{w}_{i_{\max}}$.*

Consequently, with probability 1, the agent's utility, the principals' profits, and the generated surplus are as in every pure-strategy Nash equilibrium (cf. Proposition 3.2).

Proof (I) Since no principal i ever offers a wage $w_i > \theta_i$, principal i_{\max} wins with probability 1 when offering $w_{i_{\max}} = \check{w}_{i_{\max}}$, so offering a higher wage could only decrease her expected profit. Consequently, we must have $w_{i_{\max}} \leq \check{w}_{i_{\max}}$ with probability 1 and it only remains to show that also $w_{i_{\max}} \geq \check{w}_{i_{\max}}$ with probability 1. Suppose for the sake of a contradiction that principal i_{\max} offers a wage lower than $\check{w}_{i_{\max}}$ with positive probability. If $|N| = 1$, i.e., if i_{\max} is the only principal, we have $\check{w}_{i_{\max}} = c_{i_{\max}}$, meaning that i_{\max} offers a wage lower than the agent's cost with positive probability. This is a contradiction since offering a wage below the agent's cost yields profit zero for principal i_{\max} whereas she can obtain positive profit by offering $w_{i_{\max}} := \check{w}_{i_{\max}} = c_{i_{\max}} < \theta_{i_{\max}}$. For the case where $|N| \geq 2$, we let supp_i denote the support of i 's mixed strategy for $i \in N$. Then, since i_{\max} offers a wage below $\check{w}_{i_{\max}}$ with positive probability, we have $\inf(\text{supp}_{i_{\max}}) < \check{w}_{i_{\max}}$, and we claim that the offers of the other principals satisfy

$$\max\{w_i - c_i : i \in N \setminus \{i_{\max}\}\} > \inf(\text{supp}_{i_{\max}}) - c_{i_{\max}} \quad (\text{A.1})$$

with probability 1. Note that if this was not the case, we would in particular have $\inf(\text{supp}_{i'}) - c_{i'} \leq \inf(\text{supp}_{i_{\max}}) - c_{i_{\max}}$ for any principal $i' \in \arg\max_{i \neq i_{\max}} s_i$, so she has offers in her support that make her lose with probability 1 and, consequently, yield expected profit zero. For any $0 < \epsilon < \check{w}_{i_{\max}} - \inf(\text{supp}_{i_{\max}})$, however, an offer of $w_{i'} := \theta_{i'} - \epsilon$ would make principal i' win with positive probability and, thus, yield a positive expected profit, which is a contradiction.

Consequently, (A.1) holds with probability 1, which means that there exist offers in $\text{supp}_{i_{\max}}$ for which principal i_{\max} loses (and obtains profit zero) for sure. This yields a contradiction since principal i_{\max} can obtain positive profit by offering $w_{i_{\max}} := \check{w}_{i_{\max}}$.

(II) Since no principal i ever offers a wage $w_i > \theta_i$ and principal i_{\max} offers $\check{w}_{i_{\max}}$ with probability 1, it follows directly that $i^* = i_{\max}$ and $w_{i^*} = \check{w}_{i_{\max}}$ with probability 1. \square

Theorem A.2 Suppose that the wages are unrestricted and $|N_{\max}| \geq 2$. Then every (mixed-strategy) Nash equilibrium satisfies:

- (I) At least two principals $i \in N_{\max}$ offer wages of $w_i = \theta_i$ with probability 1.
- (II) With probability 1, the winning principal i^* belongs to N_{\max} and $w_{i^*} = \theta_{i^*}$.

Consequently, with probability 1, the agent's utility, the principals' profits, and the generated surplus are as in every pure-strategy Nash equilibrium (cf. Proposition 3.2).

Proof (I) We first show that at least one principal $i \in N_{\max}$ offers $w_i = \theta_i$ with probability 1. Suppose for the sake of a contradiction that no principal $i \in N_{\max}$ offers a wage of $w_i = \theta_i$ with probability 1. Then, since no principal $i \in N_{\max}$ ever offers a wage $w_i > \theta_i$ in equilibrium, we must have $\inf(\text{supp}_i) < \theta_i$ for all $i \in N_{\max}$, so $\min\{\theta_i - \inf(\text{supp}_i) : i \in N_{\max}\} =: \epsilon > 0$.

We now distinguish two cases:

Case 1: $\operatorname{argmax}\{\theta_i - \inf(\operatorname{supp}_i) : i \in N_{\max}\} \subsetneq N_{\max}$.

Consider any principal $i_0 \in \operatorname{argmax}\{\theta_i - \inf(\operatorname{supp}_i) : i \in N_{\max}\}$. Because $\operatorname{argmax}\{\theta_i - \inf(\operatorname{supp}_i) : i \in N_{\max}\} \subsetneq N_{\max}$, there exists a subset of $\operatorname{supp}_{i_0}$ with positive probability mass such that all offers in this set make principal i_0 lose with probability 1 and, thus, yield expected profit zero for i_0 . However, denoting the difference between the maximum surplus and the second-highest surplus by $\epsilon' := s_{\max} - \max\{0, s_i : i \in N \setminus N_{\max}\} > 0$, principal i_0 can win with positive probability and obtain a positive expected profit by offering $w_{i_0} = \theta_{i_0} - \frac{\min\{\epsilon, \epsilon'\}}{2}$, which is a contradiction.

Case 2: $\operatorname{argmax}\{\theta_i - \inf(\operatorname{supp}_i) : i \in N_{\max}\} = N_{\max}$. We distinguish two sub-cases:

Case 2.1: For some principal $i_1 \in N_{\max}$, the offer $w_{i_1} = \inf(\operatorname{supp}_{i_1})$ has probability 0 (i.e., there is no mass point at $\inf(\operatorname{supp}_{i_1})$).

Let $i_0 \in N_{\max} \setminus \{i_1\}$ be arbitrary. Then, as in Case 1, offering $w_{i_0} = \theta_{i_0} - \frac{\min\{\epsilon, \epsilon'\}}{2}$ yields a positive expected profit for principal i_0 , which we denote by $\tilde{\pi}_{i_0}$. Since the probability of principal i_1 offering $w_{i_1} = \inf(\operatorname{supp}_{i_1})$ is zero, however, principal i_0 's probability of winning tends to zero as w_{i_0} approaches $\inf(\operatorname{supp}_{i_0})$. Thus, since principal i_0 's profit obtained from any offer is upper bounded by s_{i_0} , also principal i_0 's expected profit tends to zero as her offer approaches $\inf(\operatorname{supp}_{i_0})$. Consequently, there exists a subset of $\operatorname{supp}_{i_0}$ with positive probability mass such that all offers in this subset yield an expected profit lower than $\tilde{\pi}_{i_0}$ for principal i_0 , which is a contradiction.

Case 2.2: Every principal $i \in N_{\max}$ offers $w_i = \inf(\operatorname{supp}_i)$ with positive probability (i.e., there is mass point at $\inf(\operatorname{supp}_i)$ for every $i \in N_{\max}$).

Then, with positive probability, *all* principals in N_{\max} offer $w_i = \inf(\operatorname{supp}_i)$ at the same time. If some principal $i_0 \in N_{\max}$ had expected profit of zero in this situation (and hence, also unconditional expected profit zero from the offer $w_{i_0} = \inf(\operatorname{supp}_{i_0})$), this principal could again obtain positive expected profit by offering $w_{i_0} = \theta_{i_0} - \frac{\min\{\epsilon, \epsilon'\}}{2}$ (where ϵ' is as in Case 1), which is a contradiction. Consequently, all principals in N_{\max} must have positive expected profit in this situation and, in particular, each one must have a positive probability of winning. Thus, each principal in N_{\max} must also have a positive probability of losing when all principals $i \in N_{\max}$ offer $w_i = \inf(\operatorname{supp}_i)$. This yields a contradiction since each principal in N_{\max} could then improve her (unconditional) expected profit by slightly raising her offer.

Hence, in all cases, at least one principal $i \in N_{\max}$ offers a wage of $w_i = \theta_i$ with probability 1. Now suppose that *only* one principal $i_0 \in N_{\max}$ offers a wage of $w_{i_0} = \theta_{i_0}$ with probability 1. Then, $\inf(\operatorname{supp}_i) < \theta_i$ for all $i \in N_{\max} \setminus \{i_0\}$ and, thus, $\bar{\epsilon} := \min\{\theta_i - \inf(\operatorname{supp}_i) : i \in N_{\max} \setminus \{i_0\}\} > 0$. Then, offering $w_{i_0} = \theta_{i_0} - \frac{\min\{\bar{\epsilon}, \epsilon'\}}{2}$ for ϵ' as in Case 1 above yields positive expected profit for principal i_0 whereas i_0 's current offer of $w_{i_0} = \theta_{i_0}$ yields expected profit zero, which is a contradiction. Consequently, at least two principals $i \in N_{\max}$ must offer wages of $w_i = \theta_i$ with probability 1 as claimed.

- (II) Since no principal i ever offers a wage $w_i > \theta_i$ and at least two principals $i \in N_{\max}$ offer wages of $w_i = \theta_i$ with probability 1, it follows directly that $i^* \in N_{\max}$ and $w_{i^*} = \theta_{i^*}$ with probability 1.

□

Theorem A.3 *When a minimum wage \underline{w} is imposed, every (mixed-strategy) Nash equilibrium satisfies:*

- (I) *If $\underline{N} = \emptyset$, then, with probability 1, no principal makes an offer.*
 (II) *If $\underline{N} \neq \emptyset$:*
- (1) *If $|\underline{N}_{\max}| = 1$, i.e., there is a unique principal $\underline{i}_{\max} \in \underline{N}_{\max}$, then, with probability 1, principal \underline{i}_{\max} offers a wage of $\underline{w}_{\underline{i}_{\max}} := \max \left\{ \underline{w}, \max\{0, \theta_j - c_j : j \in N \setminus \{\underline{i}_{\max}\}\} + c_{\underline{i}_{\max}} \right\}$ and $\underline{i}^* = \underline{i}_{\max}$.*
 - (2) *If $|\underline{N}_{\max}| \geq 2$, then, with probability 1, at least two principals $i \in \underline{N}_{\max}$ offer wages of $\underline{w}_i = \underline{w}$ and the winning principal i^* belongs to \underline{N}_{\max} .*

Consequently, with probability 1, the agent's utility, the principals' profits, and the generated surplus are as in every pure-strategy Nash equilibrium (cf. Proposition 3.4).

Proof (I) If $\underline{N} = \emptyset$ and some principals offer wages with positive probability, then at least one principal's expected profit will be negative, so she could improve by never making an offer.

- (II) For any principal $i \in N \setminus \underline{N}$, all mixed strategies in which i makes an offer with positive probability are weakly dominated by the strategy in which i never makes an offer. Hence, the principals in $N \setminus \underline{N}$ never make offers with positive probability and the game reduces to a game between the principals in \underline{N} . Then, Claim (1) follows since either $|\underline{N}| = 1$, in which case the only remaining principal \underline{i}_{\max} offers a wage of $\max\{\underline{w}, c_{\underline{i}_{\max}}\} = \underline{w}_{\underline{i}_{\max}}$, or $|\underline{N}| \geq 2$, in which case one can argue as in the proof of Theorem A.1 except that the offered wages cannot fall short of \underline{w} . Similarly, for Claim (2), one can argue as in the proof of Theorem A.2 (again taking into account that the offered wages cannot fall short of \underline{w}).

□

A.4 Increasing an existing minimum wage

Another natural question is whether the effects observed in our model when introducing a minimum wage carry over to the case where an existing minimum wage is increased. To this end, consider an instance in which an initial minimum wage $\underline{w}^{\text{initial}}$ already exists and is increased to $\underline{w}^{\text{new}} > \underline{w}^{\text{initial}}$. Let $\underline{N}^{\text{initial}} := \{i \in N : \theta_i \geq \underline{w}^{\text{initial}}\}$ and $\underline{N}^{\text{new}} := \{i \in N : \theta_i \geq \underline{w}^{\text{new}}\}$ denote the set of principals for which the productivity weakly exceeds the initial/new minimum wage, respectively. Moreover, let $\underline{N}_{\max}^{\text{initial}} := \arg\max_{i \in \underline{N}^{\text{initial}}} s_i$. We claim that the instance can be transformed into an equivalent instance in which no minimum wage exists initially.

For the construction of the transformed instance, we distinguish two cases:

Case 1: $|\underline{N}_{\max}^{\text{initial}}| \geq 2$, or $|\underline{N}_{\max}^{\text{initial}}| = 1$ and $c_{\underline{i}_{\max}} \geq \underline{w}^{\text{initial}}$ for $\underline{i}_{\max} \in \underline{N}_{\max}^{\text{initial}}$.

In this case, Proposition 3.4 shows that the minimum wage $\underline{w}^{\text{initial}}$ is not binding (i.e., it does not affect the equilibrium) in the bidding game between the principals in $\underline{N}^{\text{initial}}$ in the original instance. Hence, when defining the set of principals in the transformed instance as $N^{\text{trans}} := \underline{N}^{\text{initial}}$ and leaving all productivities and costs for the principals in N^{trans} as in the original instance, Propositions 3.2 and 3.4 show that the sets of possible winning principals and the paid wage (and, thus, also the surplus, utility, and profits) are the same in the original instance with minimum wage $\underline{w}^{\text{initial}}$ and in the transformed instance without any minimum wage. Consequently, increasing the minimum wage from $\underline{w}^{\text{initial}}$ to $\underline{w}^{\text{new}}$ in the original instance is equivalent to introducing a minimum wage of $\underline{w}^{\text{new}}$ in the transformed instance.

Case 2: $|\underline{N}_{\max}^{\text{initial}}| = 1$ and $c_{i_{\max}} < \underline{w}^{\text{initial}}$ for the unique principal $i_{\max} \in \underline{N}_{\max}^{\text{initial}}$.

In this case, the minimum wage $\underline{w}^{\text{initial}}$ can be binding in equilibrium in the bidding game between the principals in $\underline{N}^{\text{initial}}$ in the original instance. Therefore, we define the set of principals in the transformed instance as $N^{\text{trans}} := \underline{N}^{\text{initial}} \cup \{n+1\}$, where the productivity and cost of the new (dummy) principal $n+1$ are given by $\theta_{n+1} := \underline{w}^{\text{initial}}$ and $c_{n+1} := c_{i_{\max}}$. The remaining principals have the same productivities and costs as in the original instance. Intuitively, the dummy principal $n+1$ ensures that principal i_{\max} pays a wage of at least $\underline{w}^{\text{initial}}$ in equilibrium in the transformed instance even when no minimum wage is imposed. When a minimum wage of $\underline{w}^{\text{new}}$ is imposed, however, the dummy principal no longer influences the equilibrium since the productivity $\theta_{n+1} = \underline{w}^{\text{initial}}$ falls short of the minimum wage $\underline{w}^{\text{new}}$. Propositions 3.2 and 3.4, thus, imply that increasing the minimum wage from $\underline{w}^{\text{initial}}$ to $\underline{w}^{\text{new}}$ in the original instance is equivalent to introducing a minimum wage of $\underline{w}^{\text{new}}$ in the transformed instance.

We can, thus, conclude that the effects observed when introducing a minimum wage carry over to the case where an existing minimum wage is increased.

A.5 Alternative bidding process: ascending offers

An alternative bidding process consists of letting the principals provide ascending offers to the agent, who states which offer he prefers. For unrestricted wages, the bidding process then stops once a principal with maximum surplus offers the agent a utility equal to the second-highest surplus. For restricted wages and $\underline{N} \neq \emptyset$, the process stops once a principal in \underline{N} offers a utility equal to the maximum of this value and the difference of the minimum wage and the cost. Thus, this alternative bidding process results in exactly the same outcome as the originally considered process. When considering the alternative bidding process, however, the principals need not know all parameters of the model.

B Analysis of alternative bargaining procedures

In Appendix B.1, we let the agent propose wages to the principals. In Appendix B.2, we examine stable outcomes. Note that we do not study the (generalized) Nash bargaining solution due to the conceptual problems that would arise in our multi-player game.

B.1 Wage proposal by the agent

In the basic bargaining procedure considered so far, the principals offer wages to the agent, who can only choose among these offers, i.e., the agent has no bargaining power. Even though, historically, minimum wages were mainly created to compensate workers' lack of bargaining power (cf. Neumark and Wascher (2008)), we now examine the case where the agent has all bargaining power. Formally, this means that the agent can choose a principal to whom he proposes a wage, and the principal can then either accept or reject this proposal. By individual rationality, the principal accepts the proposed wage if and only if this leads to a nonnegative profit (in particular, each principal prefers to employ the agent at a wage equal to the agent's productivity instead of not employing the agent). The agent chooses the principal to propose to and the wage in order to maximize his utility. If a minimum wage \underline{w} is imposed, the agent must propose a wage of at least \underline{w} or make no proposal.

In the following analysis of this alternative bargaining procedure, we distinguish the relevant variables from the corresponding variables in the original bargaining procedure by using hats. The agent optimally proposes $\hat{w}_{\hat{i}^*} = \theta_{\hat{i}^*}$ to a principal $\hat{i}^* \in N_{\max}$, who then accepts, which yields a utility equal to the whole maximum surplus for the agent. Any other proposal would either be rejected or yield a lower utility.²⁹

Proposition B.1 *If the agent has all bargaining power and the wages are unrestricted, there exists a Nash equilibrium and every Nash equilibrium satisfies:*

- (I) *The agent proposes $\hat{w}_{\hat{i}^*} = \theta_{\hat{i}^*}$ to a principal $\hat{i}^* \in N_{\max}$, who accepts the offer, so that a surplus of s_{\max} is generated.*
- (II) *The paid wage is $\hat{w}_{\hat{i}^*} = \theta_{\hat{i}^*} = s_{\max} + c_{\hat{i}^*}$.*
- (III) *The agent's utility is $\hat{u}_{\hat{i}^*} = s_{\max}$, which is independent of the choice of \hat{i}^* from N_{\max} .*
- (IV) *The profit of principal \hat{i}^* is $\hat{\pi}_{\hat{i}^*} = 0$.*

In case a minimum wage \underline{w} is imposed, the agent optimally proposes $\hat{w}_{\hat{i}^*} = \theta_{\hat{i}^*}$ to a principal $\hat{i}^* \in \underline{N}_{\max}$ as long as $\underline{N} \neq \emptyset$, i.e., as long as there is some principal who can afford to pay the minimum wage. If $\underline{N} = \emptyset$, any proposal respecting the minimum wage will be rejected.

Proposition B.2 *If the agent has all bargaining power and a minimum wage \underline{w} is imposed, there exists a Nash equilibrium and every Nash equilibrium satisfies:*

- (I) *If $\underline{N} = \emptyset$, i.e., $\underline{w} > \theta_{\max}$, the agent does not work for any principal, i.e., $\hat{i}^* = 0$.*
- (II) *If $\underline{N} \neq \emptyset$, i.e., $\underline{w} \leq \theta_{\max}$:*
 - (1) *The agent proposes $\hat{w}_{\hat{i}^*} = \theta_{\hat{i}^*}$ to a principal $\hat{i}^* \in \underline{N}_{\max}$, who accepts the offer, so that a surplus of \underline{s}_{\max} is generated.*

²⁹ The Nash equilibrium is unique if some deterministic tie-breaking rule is applied if $|N_{\max}| \geq 2$.

- (2) The paid wage is $\hat{w}_{\underline{i}}^* = \theta_{\underline{i}}^* = s_{\max} + c_{\underline{i}}^*$.
- (3) The agent's utility is $\hat{u}_{\underline{i}}^* = s_{\max}$, independently of the choice of \underline{i}^* from N_{\max} .
- (4) The profit of principal \underline{i}^* is $\hat{\pi}_{\underline{i}}^* = 0$.

Proposition B.2 reveals that, as in the basic bargaining procedure where the principals make the offers, the agent becomes unemployed if and only if $\underline{w} > \theta_{\max}$. Since this is a condition on productivities and not surpluses, the insight that minimum wages may destroy jobs generating relatively high levels of surplus also holds in this alternative bargaining procedure.

We next concentrate on the case where the agent stays employed.

Theorem B.1 *Let the agent have all bargaining power and suppose there is a minimum wage $\underline{w} \leq \theta_{\max}$. Then:*

- (I.1) If $\underline{w} \leq \max_{j \in N_{\max}} \theta_j$, the agent proposes $\hat{w}_{\underline{i}}^* = \theta_{\underline{i}}^*$ to a principal $\underline{i}^* \in N_{\max} \subseteq N_{\max}$, who accepts the offer, and the generated surplus and the agent's utility remain unchanged, i.e., $s_{\underline{i}}^* = s_{\max} = s_{\hat{i}^*}$ and $\hat{u}_{\underline{i}}^* = \hat{u}_{\hat{i}^*}$.
- (I.2) If $\underline{w} > \max_{j \in N_{\max}} \theta_j$, the agent proposes $\hat{w}_{\underline{i}}^* = \theta_{\underline{i}}^*$ to a principal $\underline{i}^* \in N_{\max}$, who accepts the offer, where $N_{\max} \cap N_{\max} = \emptyset$, and the generated surplus and the agent's utility decrease, i.e., $s_{\underline{i}}^* = s_{\max} < s_{\hat{i}^*}$ and $\hat{u}_{\underline{i}}^* < \hat{u}_{\hat{i}^*}$.
- (II.1) If $\underline{w} > \hat{w}_{\hat{i}^*}$, the paid wage increases, i.e., $\hat{w}_{\underline{i}}^* > \hat{w}_{\hat{i}^*}$.
- (II.2) If $\underline{w} = \hat{w}_{\hat{i}^*}$, the paid wage increases weakly, i.e., $\hat{w}_{\underline{i}}^* \geq \hat{w}_{\hat{i}^*}$. If the tie-breaking rule satisfies independence of irrelevant alternatives, then $\hat{w}_{\underline{i}}^* = \hat{w}_{\hat{i}^*}$.
- (II.3) If $\underline{w} < \hat{w}_{\hat{i}^*}$ and the tie-breaking rule satisfies independence of irrelevant alternatives, the paid wage remains unchanged, i.e., $\hat{w}_{\underline{i}}^* = \hat{w}_{\hat{i}^*}$.
- (III) The winning principal's profit remains zero, i.e., $\hat{\pi}_{\underline{i}}^* = \hat{\pi}_{\hat{i}^*} = 0$.

Proof

- (I) As in the proof of Theorem 4.1.
- (II.1) Follows since $\underline{w} > \hat{w}_{\hat{i}^*}$ and $\hat{w}_{\underline{i}}^* \geq \underline{w}$.
- (II.2) & (II.3) That $\hat{w}_{\underline{i}}^* \geq \hat{w}_{\hat{i}^*}$ in II.2) is obvious. The rest of the statements follows since $\hat{i}^* = \underline{i}^*$ in both cases by independence of irrelevant alternatives, and $\hat{w}_{\hat{i}^*} = \theta_{\hat{i}^*}$ and $\hat{w}_{\underline{i}}^* = \theta_{\underline{i}^*}$ by Propositions B.1 and B.2.
- (III) Follows directly from Propositions B.1 and B.2.

□

By Theorem B.1, the insights that a minimum wage could adversely affect the generated surplus as well as the agent's utility even if the agent remains employed also hold when the agent has all bargaining power. To be precise, the theorem reveals that the generated surplus as well as the agent's utility decrease weakly due to a minimum wage. Both decreases are strict if and only if the minimum wage exceeds the agent's maximum productivity at the principals where the maximum surplus could be

generated. Furthermore, the minimum wage leaves the principals' profits unchanged at zero. A minimum wage can thus never be beneficial for any party, i.e., it may only cause a Pareto deterioration of utility and profits. Intuitively, the agent's utility decreases weakly while the principals' profits remain zero due to the fact that the agent already receives the whole maximum surplus without any minimum wage, and a minimum wage only imposes an additional restriction on the wages the agent can propose.

The following corollary, which follows directly from Propositions B.1 and B.2, shows that wage overshooting can also arise when the agent has all bargaining power.

Corollary B.1 *If the agent has all bargaining power, a minimum wage \underline{w} causes wage overshooting, i.e., $\hat{w}_{\hat{i}^*} > \underline{w} > \hat{w}_{\hat{i}^*}$, if and only if $\underline{w} \leq \theta_{\max}$ and $s_{\max} + c_{\hat{i}^*} > \underline{w} > s_{\max} + c_{\hat{i}^*}$.*

B.2 Stable outcomes

We now determine the set of stable outcomes of the model. Hence, we no longer model the bargain process and what parties know, but instead impose a natural a posteriori condition on the outcome. Formally, an outcome consists of a pair (i^*, w_{i^*}) (or $(\underline{i}^*, \underline{w}_{\underline{i}^*})$) of a winning principal's index $i^* \in N$ and a paid wage w_{i^*} . Stability of an outcome is defined as follows:

Definition B.1 For unrestricted wages, an outcome (i^*, w_{i^*}) is called *stable* if it is individually rational, i.e., $\pi_{i^*}, u_{i^*} \geq 0$, and $u_{i^*} \geq s_j$ holds for all principals $j \in N \setminus \{i^*\}$. Correspondingly, if a minimum wage \underline{w} is imposed, an outcome $(\underline{i}^*, \underline{w}_{\underline{i}^*})$ is called *stable* if it respects the minimum wage, i.e., $\underline{w}_{\underline{i}^*} \geq \underline{w}$, is individually rational, i.e., $\pi_{\underline{i}^*}, \underline{u}_{\underline{i}^*} \geq 0$, and $\underline{u}_{\underline{i}^*} \geq s_j$ holds for all principals $j \in \underline{N} \setminus \{\underline{i}^*\}$.

Hence, an outcome is stable if and only if (it respects the minimum wage and) both the winning principal's profit and the agent's utility are nonnegative and there exists no other principal $j \in N \setminus \{i^*\}$ (or $j \in \underline{N} \setminus \{\underline{i}^*\}$) who can offer a wage that increases the agent's utility, while at the same time yielding a nonnegative profit for principal j .³⁰

We now show that, as for the Nash equilibria arising for the bargaining procedures considered before, the winning principal in any stable outcome has to belong to the set N_{\max} in case of unrestricted wages, and to \underline{N}_{\max} in case of restricted wages. Interestingly, the equilibrium wages that arise in the situation where the agent has no/all bargaining power yield exactly the lower/upper boundary, respectively, of the interval of wages in stable outcomes.

B.2.1 Unrestricted wages

Proposition B.3 *For unrestricted wages, an outcome (i^*, w_{i^*}) is stable if and only if $i^* \in N_{\max}$ and $\max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*} \leq w_{i^*} \leq \theta_{i^*}$. In particular, every stable outcome (i^*, w_{i^*}) generates a surplus of s_{\max} .*

³⁰ Except for the special case in which $\theta_j = \underline{w}$, principal j can actually obtain a *strictly positive* profit in this case. Hence, except for this case, the common distinction between *weak* and *strong* stability (see, e.g., Irving (1994)) is meaningless in our model.

Proof If the outcome (i^*, w_{i^*}) is stable, then $u_{i^*} \geq s_j$ for all $j \in N \setminus \{i^*\}$ and $\pi_{i^*} \geq 0$. Adding these two inequalities yields that $s_{i^*} = u_{i^*} + \pi_{i^*} \geq s_j$ for all $j \in N \setminus \{i^*\}$, which shows that $i^* \in N_{\max}$.

It remains to show that, for $i^* \in N_{\max}$, the outcome (i^*, w_{i^*}) is stable if and only if $\max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*} \leq w_{i^*} \leq \theta_{i^*}$. To this end, note that, if $w_{i^*} > \theta_{i^*}$, then $\pi_{i^*} = \theta_{i^*} - w_{i^*} < 0$, so the outcome is not stable. Similarly, if $w_{i^*} < \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$, then either $w_{i^*} < c_{i^*}$, in which case $u_{i^*} = w_{i^*} - c_{i^*} < 0$, or $w_{i^*} < \theta_j - c_j + c_{i^*}$ for some $j \in N \setminus \{i^*\}$, in which case $u_{i^*} < \theta_j - c_j = s_j$ for this j . In both cases, the outcome (i^*, w_{i^*}) is not stable.

To show that (i^*, w_{i^*}) is stable for all wages between $\max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$ and θ_{i^*} if $i^* \in N_{\max}$, we first consider the two interval boundaries.

If $w_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$, then:

1. $\pi_{i^*} = \theta_{i^*} - w_{i^*} = \theta_{i^*} - \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*} = s_{i^*} - \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} \geq 0$ since $i^* \in N_{\max}$ and $s_{\max} \geq 0$.
2. $u_{i^*} = w_{i^*} - c_{i^*} \geq 0$ since $w_{i^*} \geq c_{i^*}$.
3. $u_{i^*} = w_{i^*} - c_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} \geq s_j$ for all $j \in N \setminus \{i^*\}$.

Thus, (i^*, w_{i^*}) is stable if $i^* \in N_{\max}$ and $w_{i^*} = \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$. If $w_{i^*} = \theta_{i^*}$, then:

1. $\pi_{i^*} = \theta_{i^*} - w_{i^*} = 0$.
2. $u_{i^*} = w_{i^*} - c_{i^*} = \theta_{i^*} - c_{i^*} = s_{i^*} = s_{\max} \geq \max\{0, s_j : j \in N \setminus \{i^*\}\}$ since $i^* \in N_{\max}$ and $s_{\max} \geq 0$.

Thus, (i^*, w_{i^*}) is stable if $i^* \in N_{\max}$ and $w_{i^*} = \theta_{i^*}$.

Since all requirements for stability are linear inequalities in the wage w_{i^*} , this implies that the outcome (i^*, w_{i^*}) with $i^* \in N_{\max}$ must also be stable for all wages between $\max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\} + c_{i^*}$ and θ_{i^*} . \square

Proposition B.3 shows that, in every stable outcome, the wage w_{i^*} must be at least as large as the wage arising for winning principal i^* in a Nash equilibrium when the agent has *no* bargaining power (which we denote by $w_{i^*}^{\text{NE}}$ in the following), but at most as large as the wage arising for winning principal i^* in a Nash equilibrium when the agent has *all* bargaining power (which we denote by $\hat{w}_{i^*}^{\text{NE}}$ in the following). This implies the following corollary.

Corollary B.2 *For unrestricted wages, an outcome (i^*, w_{i^*}) is stable if and only if $i^* \in N_{\max}$ and $w_{i^*} \in [w_{i^*}^{\text{NE}}, \hat{w}_{i^*}^{\text{NE}}]$. Hence, for each stable outcome (i^*, w_{i^*}) , there exists $\alpha \in [0, 1]$ such that*

$$w_{i^*} = \alpha \cdot \hat{w}_{i^*}^{\text{NE}} + (1 - \alpha) \cdot w_{i^*}^{\text{NE}} =: w_{i^*}^{\alpha}. \quad (\text{B.1})$$

In the case where $|N_{\max}| = 1$, there exists a unique winning principal $i^* \in N_{\max}$ and we have $w_{i^*}^{\text{NE}} < \hat{w}_{i^*}^{\text{NE}}$. Hence, given a stable outcome (i^*, w_{i^*}) , the weight α in Corollary B.2 is uniquely determined and can be interpreted as the bargaining power of the agent. The extreme cases $\alpha = 0/\alpha = 1$ correspond to the cases studied previously where the agent has no/all bargaining power, respectively. The agent's utility and the

winning principal's profit in a stable outcome can then also be expressed in terms of the bargaining power α :

$$\begin{aligned} u_{i^*} &= w_{i^*} - c_{i^*} \\ &= \left(\alpha \cdot \hat{w}_{i^*}^{\text{NE}} + (1 - \alpha) \cdot w_{i^*}^{\text{NE}} \right) - c_{i^*} \\ &= \alpha \cdot \left(\hat{w}_{i^*}^{\text{NE}} - c_{i^*} \right) + (1 - \alpha) \cdot \left(w_{i^*}^{\text{NE}} - c_{i^*} \right) \\ &= \alpha \cdot \hat{u}_{i^*}^{\text{NE}} + (1 - \alpha) \cdot u_{i^*}^{\text{NE}} =: u_{i^*}^\alpha, \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} \pi_{i^*} &= \theta_{i^*} - w_{i^*} \\ &= \theta_{i^*} - \left(\alpha \cdot \hat{w}_{i^*}^{\text{NE}} + (1 - \alpha) \cdot w_{i^*}^{\text{NE}} \right) \\ &= \alpha \cdot \left(\theta_{i^*} - \hat{w}_{i^*}^{\text{NE}} \right) + (1 - \alpha) \cdot \left(\theta_{i^*} - w_{i^*}^{\text{NE}} \right) \\ &= \alpha \cdot \hat{\pi}_{i^*}^{\text{NE}} + (1 - \alpha) \cdot \pi_{i^*}^{\text{NE}} =: \pi_{i^*}^\alpha. \end{aligned} \quad (\text{B.3})$$

Since $\alpha \in [0, 1]$, this shows that $u_{i^*} \in [u_{i^*}^{\text{NE}}, \hat{u}_{i^*}^{\text{NE}}] = [s_{\text{second}}, s_{\text{max}}]$ and $\pi_{i^*} \in [\hat{\pi}_{i^*}^{\text{NE}}, \pi_{i^*}^{\text{NE}}] = [0, s_{\text{max}} - s_{\text{second}}]$ for any stable outcome (i^*, w_{i^*}) , where $s_{\text{second}} := \max\{0, \theta_j - c_j : j \in N \setminus \{i^*\}\}$ denotes the second-highest surplus. The bargaining power α thus determines how the parties share the surplus $s_{\text{max}} - s_{\text{second}}$ created by their relationship in excess of the second-highest surplus s_{second} .

B.2.2 Restricted wages

With a minimum wage $\underline{w} > \theta_{\text{max}}$, there is no stable outcome since all wages satisfying the minimum wage cause negative profits. We thus focus on the case $\underline{w} \leq \theta_{\text{max}}$ in the following proposition, whose proof is analogous to that of Proposition B.3.

Proposition B.4 *If a minimum wage $\underline{w} \leq \theta_{\text{max}}$ is imposed, an outcome $(i^*, \underline{w}_{i^*})$ is stable if and only if $\underline{i}^* \in \underline{N}_{\text{max}}$ and $\max\left\{\underline{w}, \max\{0, \theta_j - c_j : j \in \underline{N} \setminus \{\underline{i}^*\}\} + c_{\underline{i}^*}\right\} \leq \underline{w}_{i^*} \leq \underline{\theta}_{i^*}$. In particular, every stable outcome $(\underline{i}^*, \underline{w}_{i^*})$ generates a surplus of $\underline{s}_{\text{max}}$.*

Similar to the case of unrestricted wages, Proposition B.4 shows that, when a minimum wage $\underline{w} \leq \theta_{\text{max}}$ is imposed, the wage in any stable outcome $(i^*, \underline{w}_{i^*})$ must be at least as large as the wage $\underline{w}_{i^*}^{\text{NE}}$ arising in a Nash equilibrium when the agent has no bargaining power, but at most as large as the wage $\hat{\underline{w}}_{i^*}^{\text{NE}}$ in a Nash equilibrium when the agent has all bargaining power. This yields the the following corollary.

Corollary B.3 *For restricted wages, an outcome $(i^*, \underline{w}_{i^*})$ is stable if and only if $\underline{i}^* \in \underline{N}_{\text{max}}$ and $\underline{w}_{i^*} \in [\underline{w}_{i^*}^{\text{NE}}, \hat{\underline{w}}_{i^*}^{\text{NE}}]$. Hence, for each stable outcome $(\underline{i}^*, \underline{w}_{i^*})$, there exists $\underline{\alpha} \in [0, 1]$ such that*

$$\underline{w}_{i^*} = \underline{\alpha} \cdot \hat{\underline{w}}_{i^*}^{\text{NE}} + (1 - \underline{\alpha}) \cdot \underline{w}_{i^*}^{\text{NE}} =: \underline{w}_{i^*}^{\underline{\alpha}}. \quad (\text{B.4})$$

If $|N_{\max}| = 1$, the winning principal is again unique and $\underline{w}_{i^*}^{\text{NE}} < \hat{w}_{i^*}^{\text{NE}}$. Consequently, the unique value $\underline{\alpha}$ can again be interpreted as the agent's bargaining power, and we can express the agent's utility and the winning principal's profit as

$$\underline{u}_{i^*} = \underline{\alpha} \cdot \hat{u}_{i^*}^{\text{NE}} + (1 - \underline{\alpha}) \cdot \underline{u}_{i^*}^{\text{NE}} =: \underline{u}_{i^*}^{\alpha} \quad \text{and} \quad (\text{B.5})$$

$$\underline{\pi}_{i^*} = \underline{\alpha} \cdot \hat{\pi}_{i^*}^{\text{NE}} + (1 - \underline{\alpha}) \cdot \underline{\pi}_{i^*}^{\text{NE}} =: \underline{\pi}_{i^*}^{\alpha}, \quad (\text{B.6})$$

which implies that $\underline{u}_{i^*} \in [\underline{u}_{i^*}^{\text{NE}}, \hat{u}_{i^*}^{\text{NE}}]$ and $\underline{\pi}_{i^*} \in [\hat{\pi}_{i^*}^{\text{NE}}, \underline{\pi}_{i^*}^{\text{NE}}]$ for any stable outcome. Similar to the case of unrestricted wages, the agent's bargaining power $\underline{\alpha}$ thus determines how the parties share the excess surplus created by their relationship. Formally, $\underline{u}_{i^*} \in [\underline{u}_{i^*}^{\text{NE}}, \hat{u}_{i^*}^{\text{NE}}] = [\sigma, s_{\max}]$ and $\underline{\pi}_{i^*} \in [\hat{\pi}_{i^*}^{\text{NE}}, \underline{\pi}_{i^*}^{\text{NE}}] = [0, s_{\max} - \sigma]$, where $\sigma := \max\{\underline{w} - c_{i^*}, s_{\text{second}}\}$ with $s_{\text{second}} := \max\{0, \theta_j - c_j : j \in \underline{N} \setminus \{i^*\}\}$ denoting the second-highest surplus among the principals in \underline{N} .

B.2.3 Effects of minimum wages

In order to explore the effects of a minimum wage, we consider the case where the agent does not get unemployed, i.e., where $\underline{w} \leq \theta_{\max}$. Moreover, in order to avoid tedious case distinctions, we concentrate on the generic case where $|N_{\max}| = 1$ and $|\underline{N}_{\max}| = 1$, in which case there exists a unique winning principal $i^* \in N_{\max}$ without a minimum wage and a unique winning principal $\underline{i}^* \in \underline{N}_{\max}$ with a minimum wage. We additionally assume that the bargaining power of the agent does not change when introducing the minimum wage, i.e., $\alpha = \underline{\alpha}$.³¹

As we can see from Equations (B.1) to (B.6), the effects of a minimum wage \underline{w} on the wage, utility, and winning principal's profit for any fixed value of $\alpha \in (0, 1)$ are given by the weighted effects arising in the extreme cases $\alpha = 0$ and $\alpha = 1$. The following theorem shows that, actually, the qualitative effects on each of these variables for any $\alpha \in (0, 1)$ are the same as for $\alpha = 0$.

Theorem B.2 *Suppose that $|N_{\max}| = |\underline{N}_{\max}| = 1$. The following holds for each of the variables generated surplus, paid wage, agent's utility, and winning principal's profit: If the minimum wage \underline{w} decreases the variable / leaves the variable unchanged / increases the variable for $\alpha = 0$, then the same applies for all $\alpha \in [0, 1)$.*

Proof We consider each of the variables separately.

For the generated surplus, we note that, by Propositions B.3 and B.4, without a minimum wage, every stable outcome generates a surplus of s_{\max} and, with a minimum wage \underline{w} , every stable outcome generates a surplus of \underline{s}_{\max} . Thus, the change in the generated surplus is identical for all $\alpha \in [0, 1]$.

³¹ Note that, as described before, the agent's bargaining power determines how the parties share the excess surplus $s_{\max} - s_{\text{second}}$ (or $s_{\max} - \sigma$ in case of restricted wages) created by their relationship. Hence, while both the generated surplus and the excess surplus—and, thus, the amount of surplus over which the parties bargain—may change when introducing a minimum wage, it is natural to assume that the agent's bargaining power remains unchanged.

For the paid wage, we note that, with $\alpha = \underline{\alpha}$ and the unique winning principals i^* and \underline{i}^* without and with a minimum wage, respectively, Eqs. (B.1) and (B.4) show that

$$\underline{w}_{\underline{i}^*}^{\alpha} - w_{i^*}^{\alpha} = \alpha \cdot \left(\underline{w}_{\underline{i}^*}^{\alpha=1} - w_{i^*}^{\alpha=1} \right) + (1 - \alpha) \cdot \left(\underline{w}_{\underline{i}^*}^{\alpha=0} - w_{i^*}^{\alpha=0} \right). \quad (\text{B.7})$$

If $\underline{w} \leq w_{i^*}^{\alpha=1}$, then $\underline{w}_{\underline{i}^*}^{\alpha=1} - w_{i^*}^{\alpha=1} = 0$ by Theorem B.1, so $\underline{w}_{\underline{i}^*}^{\alpha} - w_{i^*}^{\alpha} = (1 - \alpha) \cdot (\underline{w}_{\underline{i}^*}^{\alpha=0} - w_{i^*}^{\alpha=0})$, which shows that the change in the paid wage for any $\alpha \in (0, 1)$ is qualitatively the same as for $\alpha = 0$. If $\underline{w} > w_{i^*}^{\alpha=1}$, then also $\underline{w} > w_{i^*}^{\alpha=0}$ since $w_{i^*}^{\alpha=1} = \theta_{i^*} \geq w_{i^*}^{\alpha=0}$ by Proposition B.1. Consequently, the minimum wage causes all wages to increase since $\underline{w}_{\underline{i}^*}^{\alpha=1} - w_{i^*}^{\alpha=1} > 0$ and $\underline{w}_{\underline{i}^*}^{\alpha=0} - w_{i^*}^{\alpha=0} > 0$ in (B.7), so also $\underline{w}_{\underline{i}^*}^{\alpha} - w_{i^*}^{\alpha} > 0$. Hence, the change in the paid wage for any $\alpha \in (0, 1)$ is again qualitatively the same as for $\alpha = 0$.

For the agent's utility, we similarly have

$$\underline{u}_{\underline{i}^*}^{\alpha} - u_{i^*}^{\alpha} = \alpha \cdot \left(\underline{u}_{\underline{i}^*}^{\alpha=1} - u_{i^*}^{\alpha=1} \right) + (1 - \alpha) \cdot \left(\underline{u}_{\underline{i}^*}^{\alpha=0} - u_{i^*}^{\alpha=0} \right) \quad (\text{B.8})$$

by Equations (B.2) and (B.5). If $\underline{w} \leq \max_{j \in N_{\max}} \theta_j = \theta_{i^*}$, then $\underline{u}_{\underline{i}^*}^{\alpha=1} - u_{i^*}^{\alpha=1} = 0$ by Theorem B.1, so

$$\underline{u}_{\underline{i}^*}^{\alpha} - u_{i^*}^{\alpha} = (1 - \alpha) \cdot \left(\underline{u}_{\underline{i}^*}^{\alpha=0} - u_{i^*}^{\alpha=0} \right),$$

which shows that the change in the agent's utility for any $\alpha \in (0, 1)$ is qualitatively the same as for $\alpha = 0$. If $\underline{w} > \max_{j \in N_{\max}} \theta_j = \theta_{i^*}$, then $\underline{i}^* \neq i^*$ and the agent's utility decreases in all cases since $\underline{u}_{\underline{i}^*}^{\alpha=1} - u_{i^*}^{\alpha=1} < 0$ by Theorem B.1 and $\underline{u}_{\underline{i}^*}^{\alpha=0} - u_{i^*}^{\alpha=0} < 0$ by Corollary 4.4. Thus, the change for any $\alpha \in (0, 1)$ is again qualitatively the same as for $\alpha = 0$.

For the winning principal's profit, we have

$$\underline{\pi}_{\underline{i}^*}^{\alpha} - \pi_{i^*}^{\alpha} = \alpha \cdot \left(\underline{\pi}_{\underline{i}^*}^{\alpha=1} - \pi_{i^*}^{\alpha=1} \right) + (1 - \alpha) \cdot \left(\underline{\pi}_{\underline{i}^*}^{\alpha=0} - \pi_{i^*}^{\alpha=0} \right) \quad (\text{B.9})$$

by Equations (B.3) and (B.6). Exploiting that $\underline{\pi}_{\underline{i}^*}^{\alpha=1} = \pi_{i^*}^{\alpha=1} = 0$ by Theorem B.1, this shows the desired result. \square

Theorem B.2 reveals that the qualitative effects of minimum wages on each single variable of the model obtained for the basic bargaining procedure (where the principals have all bargaining power, i.e., $\alpha = 0$) extend to all cases where the principals have nonzero bargaining power (i.e., $\alpha < 1$).³²

³² Even though the theorem focuses on the generic case where $|N_{\max}| = |\underline{N}_{\max}| = 1$, an even stronger result holds in the nongeneric case where $|N_{\max}|, |\underline{N}_{\max}| \geq 2$. Then, we have $s_{i^*} = u_{i^*} = s_{\max}$, $\pi_{i^*} = 0$,

Concerning wage overshooting (which is formally defined for an arbitrary value of α in the following definition), however, the quantitative effect of the minimum wage on the paid wage is also important, as we now demonstrate.

Definition B.2 For a given bargaining power $\alpha \in [0, 1]$ of the agent, a minimum wage $\underline{w} > w_{i^*}^\alpha$ causes *wage overshooting* if the agent stays employed and the paid wage increases to $\underline{w}_{i^*}^\alpha > \underline{w}$.

The following theorem shows that, if wage overshooting occurs for a minimum wage \underline{w} in one of the extreme cases $\alpha = 0$ or $\alpha = 1$, then it also occurs for all intermediate cases.

Theorem B.3 Suppose that $|N_{\max}| = |\underline{N}_{\max}| = 1$. If a minimum wage \underline{w} causes wage overshooting for either $\alpha = 0$ or for $\alpha = 1$, then the minimum wage \underline{w} also causes wage overshooting for all $\alpha \in (0, 1)$.

Proof Note that $|N_{\max}| = 1$ implies that the agent stays employed in all cases, so this prerequisite for wage overshooting does not have to be considered in the rest of the proof.

First assume that \underline{w} causes wage overshooting for $\alpha = 0$. Fix some $\alpha \in (0, 1)$. Then by Definition 4.1 and Corollary 4.2:

$$\underline{w}_{i^*}^{\text{NE}} > \underline{w} > w_{i^*}^{\text{NE}}, \quad (\text{B.10})$$

$$\underline{i}^* \neq i^*. \quad (\text{B.11})$$

Together with (B.4) and $\hat{w}_{i^*}^{\text{NE}} \geq \underline{w}$, this yields

$$\underline{w}_{i^*}^\alpha = \alpha \cdot \underbrace{\hat{w}_{i^*}^{\text{NE}}}_{\geq \underline{w}} + (1 - \alpha) \cdot \underbrace{w_{i^*}^{\text{NE}}}_{> \underline{w}} > \underline{w}.$$

It remains to show that $w_{i^*}^\alpha < \underline{w}$. To this end, first note that $i^* \in \underline{N}$ would imply $i^* \in \underline{N}_{\max}$, so $i^* = \underline{i}^*$, which contradicts (B.11). Thus, we must have $i^* \notin \underline{N}$, which implies that $\theta_{i^*} < \underline{w}$ and also $\hat{w}_{i^*}^{\text{NE}} < \underline{w}$. Hence, by (B.1) and (B.10)

$$w_{i^*}^\alpha = \alpha \cdot \underbrace{\hat{w}_{i^*}^{\text{NE}}}_{< \underline{w}} + (1 - \alpha) \cdot \underbrace{w_{i^*}^{\text{NE}}}_{< \underline{w}} < \underline{w}. \quad (\text{B.12})$$

Now assume that \underline{w} causes wage overshooting for $\alpha = 1$. Again fix some $\alpha \in (0, 1)$. Then by the definition of wage overshooting given in Corollary B.1:

$$\hat{w}_{i^*}^{\text{NE}} > \underline{w} > \hat{w}_{i^*}^{\text{NE}}. \quad (\text{B.13})$$

and $w_{i^*} = \theta_{i^*}$ in any stable outcome without a minimum wage and, similarly, $s_{i^*} = u_{i^*} = s_{\max}$, $\pi_{i^*} = 0$, and $\underline{w}_{i^*} = \theta_{i^*}$ in any stable outcome with a minimum wage. Consequently, the qualitative and quantitative effects on the generated surplus, the agent's utility, and the winning principal's profit are completely independent of which stable outcomes are considered, while the effect on the paid wage only depends on the choice of the winning principal without and with the minimum wage from the sets N_{\max} and \underline{N}_{\max} , respectively.

Together with (B.4), this yields

$$\underline{w}_{i^*}^\alpha = \alpha \cdot \underbrace{\hat{w}_{i^*}^{\text{NE}}}_{> \underline{w}} + (1 - \alpha) \cdot \underbrace{w_{i^*}^{\text{NE}}}_{\geq \underline{w}} > \underline{w}.$$

In addition, since $\hat{w}_{i^*}^{\text{NE}} = \theta_{i^*}$ by Proposition B.1 and $w_{i^*}^{\text{NE}} \leq \theta_{i^*}$, the second inequality in (B.13) yields that also $w_{i^*}^{\text{NE}} < \underline{w}$. Hence, by (B.1), we obtain

$$w_{i^*}^\alpha = \alpha \cdot \underbrace{\hat{w}_{i^*}^{\text{NE}}}_{< \underline{w}} + (1 - \alpha) \cdot \underbrace{w_{i^*}^{\text{NE}}}_{< \underline{w}} < \underline{w}.$$

□

While wage overshooting in one of the extreme cases $\alpha = 0$ or $\alpha = 1$ is sufficient for wage overshooting in all intermediate cases $\alpha \in (0, 1)$, it is not necessary. Indeed, as the following example demonstrates, wage overshooting can occur for intermediate cases when it does not occur at the extremes:

Example B.1 Consider a situation with two principals, where $\theta_1 = 12$, $\theta_2 = 10$ and $c_1 = 2$, $c_2 = 2$. Without a minimum wage, we have $i^* = 1$, $w_{i^*} = 10$, and $\hat{w}_{i^*} = 12$. Now fix some $\alpha \in (0, 1)$ and consider a minimum wage \underline{w} with $10 + 2\alpha < \underline{w} < 12$. Then $N_{\max} = \underline{N}_{\max} = \{1\}$ and the Nash equilibrium wages at the winning principal $i^* = 1$ are $\underline{w}_{i^*}^{\text{NE}} = \underline{w}$ and $\hat{w}_{i^*}^{\text{NE}} = 12$. In particular, no wage overshooting occurs if the agent has no or all bargaining power. However, when the agent's bargaining power is α , wage overshooting *does* occur since we have

$$w_{i^*}^\alpha = \alpha \cdot 12 + (1 - \alpha) \cdot 10 = 10 + 2\alpha < \underline{w} \quad \text{and} \quad \underline{w}_{i^*}^\alpha = \alpha \cdot 12 + (1 - \alpha) \cdot \underline{w} > \underline{w}.$$

C Analysis of the multi-agent model

We previously examined how a minimum wage affects the interaction between an agent and a set of principals. In this section, we generalize the model to the case of $m \geq 2$ agents. We denote the set of agents by $M = \{1, \dots, m\}$, and let $c_{i,j}$ and $\theta_{i,j}$ denote agent j 's cost and productivity, respectively, when working for principal $i \in N$. Similarly, $u_{i,j} := w_{i,j} - c_{i,j}$ denotes the utility of agent j if he works for principal i at wage $w_{i,j}$, and $\pi_{i,j} := \theta_{i,j} - w_{i,j}$ denotes the profit resulting for principal i from this employment. Each agent can work for at most one principal, but a principal may employ several agents. The total profit π_i of principal i amounts to the sum of the profits resulting from all these employments, where $\pi_i := 0$ if she does not employ any agent. Similarly, we let u_j denote agent j 's utility at the principal he works for, where $u_j := 0$ if agent j does not work for any principal. The surplus generated when agent j works for principal i is denoted by $s_{i,j} := \theta_{i,j} - c_{i,j}$.

Note that, while we assumed (without loss of generality) that $\theta_i > c_i$ for all $i \in N$ in the single-agent case, we now allow that $\theta_{i,j} \leq c_{i,j}$ for some i, j (which means that

agent j can never be profitably employed by principal i). This allows us to consider situations where each agent is only productive (i.e., has productivity exceeding cost) at a subset of the principals.

In the following, it will be useful to summarize the costs and productivities in two $(n \times m)$ matrices $C = (c_{i,j})_{i,j}$ and $\Theta = (\theta_{i,j})_{i,j}$, respectively. Here, each row corresponds to a principal $i \in N$ and each column to an agent $j \in M$ and the corresponding entries $c_{i,j}$ of C and $\theta_{i,j}$ of Θ are the cost and productivity, respectively, resulting when principal i employs agent j . Similarly, the matrices $W = (w_{i,j})_{i,j}$, $U = (u_{i,j})_{i,j}$, and $\Pi = (\pi_{i,j})_{i,j}$ summarize the (offered) wages, utilities, and profits, respectively. We set $w_{i,j} := \text{NO}$ if principal i makes no offer to agent j . The assignment stating which principals employ which agents is summarized in the binary matrix $A = (a_{i,j})_{i,j}$, where $a_{i,j} = 1$ if principal i employs agent j and $a_{i,j} = 0$, otherwise. A pair (A, W) consisting of the assignment A and the wages W will be referred to as an *outcome*.

In the situation in which the principals have no capacity constraints limiting the numbers of agents they can employ, the principals compete for each agent independently of the presence of the other agents. Consequently, the situation decomposes into m single-agent problems, to which the analysis from the previous sections applies. Hence, in the absence of capacity constraints, all results shown for the single-agent case carry over to case of multiple agents.

In the situation where each principal has a capacity of one, i.e., a principal can only employ a single agent, it is easy to see that the results from the single-agent case carry over to the multi-agent case if the economy is simply duplicated a given number of times, i.e., there are k identical agents and k copies of each principal for some $k \geq 2$.

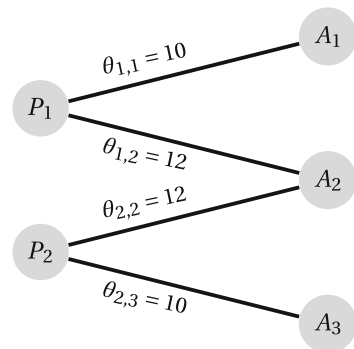
We now seek to explore the general multi-agent case with capacity constraints. Here, each principal $i \in N$ has a positive integer capacity $\kappa_i \in \mathbb{N}_{>0}$ that specifies the maximum number of agents she can employ and we summarize the capacities in a vector $\kappa = (\kappa_1, \dots, \kappa_n)$. We start by considering the canonical extension of our basic bargaining procedure in which each principal can offer wages to at most as many agents as her capacity allows her to employ. The following example, however, demonstrates that, in this case, pure-strategy Nash equilibria may fail to exist and mixed-strategy Nash equilibria may not be stable – even when no minimum wage is imposed:

Example C.1 Consider the situation with two principals with unit capacities $\kappa_1 = \kappa_2 = 1$ and three agents shown in Figure 3, where no minimum wage is imposed and all costs are zero. Obviously, there cannot be any pure-strategy Nash equilibrium in which either no principal makes an offer to agent 2 or both principals make their offers to agent 2. Hence, in a pure-strategy equilibrium, exactly one principal—say, principal 1—would have to make an offer to agent 2. Given that principal 2 does not make an offer to agent 2, principal 1's optimal offer to agent 2 is $w_{1,2} = 0$. But principal 2 could then increase her profit by offering $w_{2,2} = 1$ to agent 2.

There does, however, exist a (unique) symmetric, mixed-strategy Nash equilibrium in which each principal makes an offer to agent 2 with probability $1/6$, in which case she distributes the offered wage on $[0, 2]$, and offers wage zero to the agent who yields productivity 10 for her, otherwise. Then, however, agent 2 receives no offer with probability $(5/6)^2 = 25/36$, in which case each principal could improve her profit ex post by employing agent 2.

Fig. 3 Example C.1.

Connections that are omitted
yield negative surplus



We now show that stable outcomes do always exist in the setting with multiple agents and capacity constraints both with and without a minimum wage. Here, stability is formally defined as follows:

Definition C.1 If a minimum wage \underline{w} is imposed, an outcome (A, W) is called *stable* if it respects the capacities and the minimum wage, i.e., $\sum_{j \in M} a_{i,j} \leq \kappa_i$ for all $i \in N$ and $w_{i,j} \geq \underline{w}$ for all i, j with $a_{i,j} = 1$, is individually rational, i.e., $\pi_{i,j}, u_{i,j} \geq 0$ for all i, j with $a_{i,j} = 1$, and, for all i, j with $a_{i,j} = 0$, we have:

- (a) If $\sum_{j' \in M} a_{i,j'} < \kappa_i$, then $u_j \geq s_{i,j}$ or $\theta_{i,j} \leq \underline{w}$.
- (b) If $\sum_{j' \in M} a_{i,j'} = \kappa_i$, then $u_j + \min\{\pi_{i,j'} : a_{i,j'} = 1\} \geq s_{i,j}$ or $\theta_{i,j} \leq \underline{w} + \min\{\pi_{i,j'} : a_{i,j'} = 1\}$.

For unrestricted wages, stability is defined analogously, except that the conditions involving the minimum wage \underline{w} are omitted.

Note that, even though an outcome (A, W) contains wage offers $w_{i,j}$ for all pairs (i, j) , only those offers where $a_{i,j} = 1$ are relevant for the stability of the outcome. An outcome respecting both the capacities and the minimum wage is stable if and only if all employments in the current assignment yield nonnegative profit and utility for the corresponding principals and agents, respectively, and no principal i can offer a wage to an agent j currently not working for her in a way that both i and j would improve and the capacities as well as the minimum wage are still respected.

In Case (a), where principal i 's capacity κ_i is not exhausted in the current assignment, the condition that principal i cannot offer such a wage to agent j means that either j 's current utility u_j already weakly exceeds the surplus $s_{i,j}$ he would generate when working for i , or the minimum wage weakly exceeds j 's productivity $\theta_{i,j}$ at principal i . If both of these conditions are violated, there exist wage offers improving j 's utility, while at the same time respecting i 's capacity and the minimum wage and yielding a positive profit $\pi_{i,j}$ for principal i , for instance $w'_{i,j} := \theta_{i,j} - \frac{1}{2} \min\{s_{i,j} - u_j, \theta_{i,j} - \underline{w}\}$.

In Case (b), where principal i 's capacity κ_i is already exhausted in the current assignment, the conditions are similar except that principal i then has to get rid of one

(least profitable) agent she currently employs, which yields a profit loss of $\min\{\pi_{i,j'} : a_{i,j'} = 1\}$.³³

In the following, we refer to a wage offer of a principal i to an agent j that would increase both principal i 's profit and agent j 's utility by at least a given amount $\epsilon > 0$ as an ϵ -improving offer:

Definition C.2 Given an outcome (A, W) , a minimum wage \underline{w} , and $\epsilon > 0$, a wage offer $w'_{i,j} \geq \underline{w}$ of principal i to agent j is called an ϵ -improving offer if $w'_{i,j} - c_{i,j} \geq u_j + \epsilon$ and either $\sum_{j' \in M} a_{i,j'} < \kappa_i$ and $\theta_{i,j} - w'_{i,j} \geq \epsilon$, or $\sum_{j' \in M} a_{i,j'} = \kappa_i$ and $\theta_{i,j} - w'_{i,j} \geq \min\{\pi_{i,j'} : a_{i,j'} = 1\} + \epsilon$. For unrestricted wages, the definition is the same except that the condition $w'_{i,j} \geq \underline{w}$ is omitted. The outcome (A, W) is called ϵ -stable if it respects the capacities and the minimum wage if one exists, is individually rational, and no principal i can make an ϵ -improving offer to any agent j .

The following observation is obtained directly from the definition:

Observation C.1 For any outcome (A, W) , the following holds:

- (I) If principal i can make an ϵ -improving offer to agent j , then $a_{i,j} = 0$, i.e., j does not work for i in the current assignment.
- (II) If (A, W) is ϵ -stable for some $\epsilon > 0$, then it is also ϵ' -stable for all $\epsilon' > \epsilon$.

The next proposition provides the connection between stability and ϵ -stability:

Proposition C.1 An outcome (A, W) is stable if and only if it is ϵ -stable for all $\epsilon > 0$, i.e., if and only if there does not exist an ϵ -improving offer for any $\epsilon > 0$.

Proof If there exists an ϵ -improving offer of some principal i to some agent j for some $\epsilon > 0$, then $a_{i,j} = 0$ and the conditions in a) and b) of Definition C.1 are clearly violated for i, j .

Conversely, if the outcome (A, W) respects the capacities and the minimum wage if one exists but is *not* stable, then some principal i can make an offer to some agent j such that both i 's profit and j 's utility improve. This offer is then an ϵ -improving offer when choosing ϵ as the minimum of the two improvements. \square

We now algorithmically prove the existence of ϵ -stable outcomes for every $\epsilon > 0$, and then use Proposition C.1 in order to derive also the existence of a stable outcome. While Algorithm 1 is formulated for the case with a minimum wage, it straightforwardly applies also to the case without a minimum wage by setting $\check{w}_{i,j'} := c_{i,j'} + u_{j'} + \epsilon$ in line 3.

As long as there exists a principal who can make an ϵ -improving offer to some agent, Algorithm 1 chooses such a principal i who then makes an ϵ -improving offer to an agent j in a way that maximizes her profit. To do so, principal i could have to replace a least profitable agent l she has previously employed by agent j in case that her capacity κ_i is already exhausted. Such a replacement, however, never actually occurs in any iteration of the algorithm, as the proof of the following proposition shows.

³³ If both conditions in b) are violated, a possible wage offer improving j 's utility and i 's profit and respecting the minimum wage is given by $w'_{i,j} := \max\left\{\underline{w}, \theta_{i,j} - \min\{\pi_{i,j'} : a_{i,j'} = 1\} - \frac{\delta}{2}\right\}$, where $\delta := s_{i,j} - u_j - \min\{\pi_{i,j'} : a_{i,j'} = 1\}$.

Algorithm 1 ϵ -stable outcome

Input: Set N of principals, set M of agents, minimum wage \underline{w} , productivities Θ , costs C , capacities κ , and $\epsilon > 0$.

Output: An ϵ -stable outcome (A, W) .

- 1 Initialize $a_{i,j} := 0$, $w_{i,j} := \text{NO}$, $\pi_{i,j} := 0$, $u_j := 0$ for all $i \in N$, $j \in M$.
- 2 **while** there exists a principal i who can make an ϵ -improving offer to some agent **do**
- 3 Choose $j \in \operatorname{argmax}_{j' \in M} \{\theta_{i,j'} - \check{w}_{i,j'}\}$, where $\check{w}_{i,j'} := \max\{\underline{w}, c_{i,j'} + u_{j'} + \epsilon\}$.
- 4 **if** $\sum_{j' \in M} a_{i,j'} = \kappa_i$ **then**
- 5 Choose $l \in \operatorname{argmin}_{j' \in M: a_{i,j'} = 1} \{\pi_{i,j'}\}$.
- 6 Set $a_{i,l} := 0$, $\pi_{i,l} := 0$, and $u_l := 0$.
- 7 **end if**
- 8 Set $a_{i,j} := 1$ and $a_{i',j} := 0$ for all $i' \in N \setminus \{i\}$.
- 9 Set $w_{i,j} := \check{w}_{i,j}$ and $w_{i',j} := \text{NO}$ for all $i' \in N \setminus \{i\}$.
- 10 Set $\pi_{i,j} := \theta_{i,j} - \check{w}_{i,j}$ and $\pi_{i',j} := 0$ for all $i' \in N \setminus \{i\}$.
- 11 Set $u_j := \check{w}_{i,j} - c_{i,j}$.
- 12 **end while**
- 13 **return** (A, W)

Proposition C.2 *Algorithm 1 terminates after a finite number of iterations with an ϵ -stable outcome (A, W) .*

Proof Since the outcome returned by the algorithm at termination is clearly ϵ -stable, we only have to show that the algorithm terminates after a finite number of iterations of the while loop.

To this end, we now show that the utility u_j of each agent j is monotonously increasing during the while loop. Since the utility of some agent j increases by at least $\epsilon > 0$ in each iteration of the while loop and the sum of all utilities of the agents (which is zero before the first iteration) is bounded from above (e.g., by the sum of all $s_{i,j}$), this will directly imply that the number of iterations of the while loop is finite.

In order to show the desired monotonicity of the utilities, we first observe that an agent k 's utility u_k can never decrease as long as the agent stays employed (i.e., as long as $\sum_{i \in N} a_{i,k} = 1$ during the while loop). This holds since the agent then stays employed at the same principal for the same wage until he receives an ϵ -improving offer, which increases his utility by at least ϵ . Moreover, observe that the only point in the algorithm where a previously employed agent can become unemployed is in lines 4–6, where a principal i replaces an agent l by another agent j because her capacity κ_i is already exhausted when making an ϵ -improving offer to agent j . Consequently, the claim follows if we show that the condition of the if statement in line 4 is never satisfied, i.e., that no principal ever replaces an agent by another agent because her capacity is already exhausted.

Suppose that iteration t is the first of the while loop in which some principal i replaces an agent l by another agent j . Let $t' < t$ denote the latest previous iteration in which principal i offered a new wage $\check{w}_{i,l}(t')$ to agent l . Then, since no agent was ever replaced by another agent during the iterations $1, \dots, t-1$, agent j 's utilities $u_j(t)$ and $u_j(t')$ at the start of iterations t and t' must satisfy $u_j(t') \leq u_j(t)$. Thus, the

values $\check{w}_{i,j}(t)$ and $\check{w}_{i,j}(t')$ in iterations t and t' also satisfy $\check{w}_{i,j}(t') \leq \check{w}_{i,j}(t)$. Hence, denoting the values $\pi_{i,k}$ at the beginning of iteration t by $\pi_{i,k}(t)$, we have

$$\theta_{i,j} - \check{w}_{i,j}(t') \geq \theta_{i,j} - \check{w}_{i,j}(t) \geq \pi_{i,l}(t) + \epsilon = \theta_{i,l} - \check{w}_{i,l}(t') + \epsilon > \theta_{i,l} - \check{w}_{i,l}(t'),$$

where the second inequality follows since $l \in \operatorname{argmin}_{j' \in M: a_{i,j'}=1} \{\pi_{i,j'}(t)\}$ and $\check{w}_{i,j}(t)$ is an ϵ -improving offer in iteration t . This yields the desired contradiction since agent i should have made an ϵ -improving offer to agent j instead of agent l in iteration t' according to line 3. \square

In particular, Proposition C.2 yields the following result:

Corollary C.1 *For every $\epsilon > 0$, there exists an ϵ -stable outcome.*

We now use Proposition C.1 and Corollary C.1 to establish the existence of a stable outcome. To this end, for any fixed assignment A respecting the capacities, consider the following mixed integer linear program (MILP), which computes the smallest value $\epsilon' \geq 0$ such that the assignment A together with suitable wages W forms an outcome (A, W) that is ϵ -stable for all $\epsilon > \epsilon'$:

$$\begin{aligned} \min \quad & \epsilon' \text{ s.t.} & (C.1) \\ u_j = \sum_{i: a_{i,j}=1} (w_{i,j} - c_{i,j}) & \forall j \in M & (a) \\ \pi_{i,j} = \theta_{i,j} - w_{i,j} & \forall (i, j) \text{ with } a_{i,j} = 1 & (b) \\ \max\{\underline{w}, c_{i,j}\} \leq w_{i,j} \leq \theta_{i,j} & \forall (i, j) \text{ with } a_{i,j} = 1 & (c) \\ s_{i,j} \cdot x_{i,j} \leq u_j + 2\epsilon' & \forall (i, j) \text{ with } a_{i,j} = 0, \sum_{j' \in M} a_{i,j'} < \kappa_i & (d) \\ s_{i,j} \cdot x_{i,j} \leq u_j + \pi_{i,\tilde{j}} + 2\epsilon' & \forall (i, j) \text{ with } a_{i,j} = 0, \sum_{j' \in M} a_{i,j'} = \kappa_i, \forall \tilde{j} \text{ with } a_{i,\tilde{j}} = 1 & (e) \\ (\theta_{i,j} - \underline{w}) \cdot y_{i,j} \leq \epsilon' & \forall (i, j) \text{ with } a_{i,j} = 0, \sum_{j' \in M} a_{i,j'} < \kappa_i & (f) \\ (\theta_{i,j} - \underline{w}) \cdot y_{i,j} \leq \theta_{i,\tilde{j}} - w_{i,\tilde{j}} + \epsilon' & \forall (i, j) \text{ with } a_{i,j} = 0, \sum_{j' \in M} a_{i,j'} = \kappa_i, \forall \tilde{j} \text{ with } a_{i,\tilde{j}} = 1 & (g) \\ x_{i,j} + y_{i,j} \geq 1 & \forall (i, j) \text{ with } a_{i,j} = 0 & (h) \\ x_{i,j}, y_{i,j} \in \{0, 1\} & \forall (i, j) & (i) \\ \epsilon' \geq 0 & & (j) \end{aligned}$$

Note that the variable $w_{i,j}$ representing the wage paid by principal i to agent j only exists in case that $a_{i,j} = 1$, i.e., if agent j works for principal i . In the MILP, (a) and (b) link the utilities u_j and the profits $\pi_{i,j}$, respectively, to the paid wages $w_{i,j}$. The constraints (c) ensure individual rationality and that all paid wages adhere to

the minimum wage. The constraints (d)–(i) ensure that no principal i can make an ϵ -improving over to any agent j for any $\epsilon > \epsilon'$: Either $x_{i,j} = 1$ and the surplus $s_{i,j}$ allows a joint improvement of at most $2\epsilon'$ (by (d), or (e)) or $y_{i,j} = 1$ and principal i can improve by at most ϵ' due to the minimum wage (by (f) or (g)).

For each assignment A respecting the capacities, we denote the optimum objective value of MILP (C.1) by $\tilde{\epsilon}(A)$. Note that an optimal solution always exists as long as (C.1) is feasible (cf. Schrijver (1998)). If no feasible solution exists for a given assignment A , we let $\tilde{\epsilon}(A) := +\infty$. We then have $\tilde{\epsilon}(A) \geq 0$ for every assignment A due to constraint (j). Moreover, we let $\tilde{\epsilon} := \min_A \tilde{\epsilon}(A) \geq 0$ denote the minimum of the values $\tilde{\epsilon}(A)$ over all the (finitely many) assignments A . Note that $\tilde{\epsilon}$ is finite since (C.1) has a feasible solution for at least one assignment A respecting the capacities: If $a_{i,j} = 0$ for all i, j , then setting $x_{i,j} := 1$ and $y_{i,j}, \pi_{i,j}, u_j := 0$ for all i, j , and choosing $\epsilon' := \max_{i,j} \frac{s_{i,j}}{2}$ is clearly feasible.

We now use MILP (C.1) to establish the existence of stable outcomes:

Theorem C.1 *There exists a stable outcome (A, W) for any productivities Θ , costs C , capacities κ , and any minimum wage \underline{w} .*

Proof We show that $\tilde{\epsilon} = 0$ for any productivities Θ , costs C , capacities κ , and any minimum wage \underline{w} . This will prove the claim since a corresponding assignment A for which $\tilde{\epsilon}(A) = 0$ together with the wages $w_{i,j}$ from an optimal solution of (C.1) for this A constitute an outcome (A, W) that is ϵ -stable for all $\epsilon > 0$. Thus, this outcome (A, W) is stable by Proposition C.1.

Suppose for the sake of a contradiction that $\tilde{\epsilon} > 0$. Then, by Corollary C.1, there exists an outcome (A, W) that is $\frac{\tilde{\epsilon}}{2}$ -stable, and by Observation C.1, this outcome is also ϵ -stable for all $\epsilon > \frac{\tilde{\epsilon}}{2}$. Thus, the wages given by W induce a feasible solution of (C.1) with objective value at most $\frac{\tilde{\epsilon}}{2}$, so $\tilde{\epsilon}(A) \leq \frac{\tilde{\epsilon}}{2}$, which yields a contradiction since $\tilde{\epsilon}(A) \geq \tilde{\epsilon}$ by definition of $\tilde{\epsilon}$. \square

Theorem C.1 also holds in case no minimum wage is imposed, since a minimum wage $\underline{w} \leq \min_{i,j} c_{i,j}$ has no effect.³⁴ Further, recognize that the result in Theorem C.1 is constructive in the sense that, for any assignment A respecting the capacities, we can use MILP (C.1) in order to compute wages W such that the outcome (A, W) is stable (these wages are given by the variables $w_{i,j}$ in an optimal solution if the optimum objective value $\tilde{\epsilon}(A)$ is zero) or decide that no such wages exist for assignment A (which is the case if $\tilde{\epsilon}(A) > 0$).

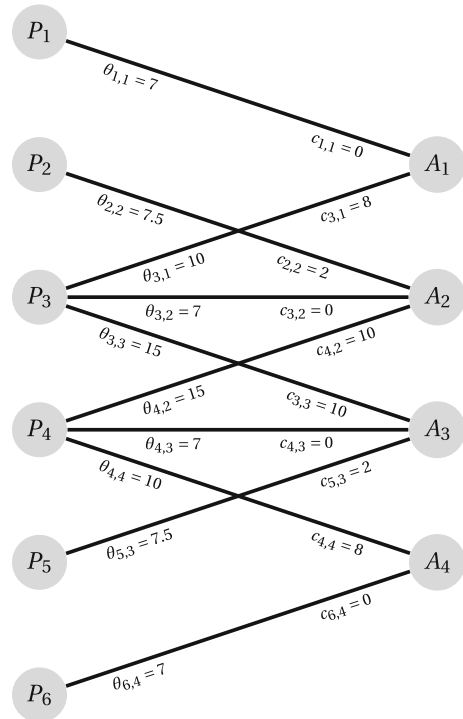
The following example demonstrates that the effects observed in the single-agent case still arise in the case of multiple agents:

Example C.2 Consider the situation with six principals with unit capacities $\kappa_i = 1$ for all i and four agents shown in Figure 4. When no minimum wage is imposed, the only possible assignment in a stable outcome is $a_{1,1} = a_{3,2} = a_{4,3} = a_{6,4} = 1$ and

³⁴ For the case where no minimum wage is imposed, the existence of a stable outcome can also be shown by using linear programming duality in order to calculate the utilities and profits in a stable outcome; see Shapley and Shubik (1971). This technique, however, does not easily generalize to the case in which a (nontrivial) minimum wage exists, since the minimum wage may prevent certain combinations of utilities and profits for each principal-agent pair.

Fig. 4 Example C.2.

Connections that are omitted
yield negative surplus



$a_{i,j} = 0$ otherwise. It can easily be shown that the wages W in a stable outcome must satisfy $5.5 \leq w_{3,2}, w_{4,3} \leq 7$, $w_{3,2} - 5 \leq w_{1,1} \leq 7$, $w_{4,3} - 5 \leq w_{6,4} \leq 7$, which yields $5.5 \leq u_2, u_3 \leq 7$ and $0 \leq \pi_3, \pi_4 \leq 1.5$.

With a minimum wage of $\underline{w} = 8$, however, the only possible assignment in a stable outcome is $a_{3,3} = a_{4,2} = 1$ and $a_{i,j} = 0$ otherwise. The wages W in a stable outcome must satisfy $10 \leq w_{3,3}, w_{4,2} \leq 13$, which yields $0 \leq u_2, u_3 \leq 3$ and $2 \leq \pi_3, \pi_4 \leq 5$. Consequently, for any possible combination of stable outcomes before and after the imposition of the minimum wage, the wages of agents 2 and 3 increase to a level above the minimum wage (i.e., wage overshooting occurs), while the utilities of these agents decrease strictly (so they do not benefit from wage overshooting). On the other hand, the profits of principals 3 and 4 increase due to the minimum wage (due to the elimination of competition). In addition, one can observe that the minimum wage destroys all employments that yield the highest surplus of 7. The total surplus (the sum of the surpluses generated by all employments) decreases from 28 to 10. Moreover, also the surplus generated only by the employments of the two agents 2 and 3 who stay employed after the introduction of the minimum wage reduces from 14 to 10.

Summarizing, the existence of multiple agents does not change anything if the agents have no capacity constraints, since the situation then decomposes into m single-agent problems. In the general multi-agent case with capacity constraints, stable outcomes still exist both with and without a minimum wage, and the obtained effects of minimum wages still arise.

D Extensions

D.1 Dynamic environments

We now show that our model and the obtained effects extend to two simple dynamic environments with finitely many time periods $t \in \{1, \dots, T\}$. This is not very surprising, since we showed that the identified effects are the consequence of stability (and not the consequence of specific bargaining procedures or information structures). The set of principals for which the agent can work might be different in each period, and we denote the corresponding set in period $t \in \{1, \dots, T\}$ by N^t . These sets might be deterministic or stochastic, so that the agent need not necessarily know which principals he can work for in future periods.

In the first dynamic environment, in each period t , the agent bargains with the principals in N^t and the contract that the agent may conclude with one of these principals is valid only for the current period. Consequently, since future periods do not play any role for the bargaining in period t , the situation decomposes into T static (single-period) problems. This holds for all of the considered bargaining procedures as well as for the setting of stable outcomes. Furthermore, this result also stays valid if, instead of concluding a single-period contract, the parties can conclude long-term contracts, but each side has the option to terminate the contract in each period.

In the second dynamic environment, the agent also bargains with the principals in N^t in each period t , but additionally has the option of continuing the contract of the previous period $t - 1$, i.e., the agent can choose to continue to work for the same principal as in period $t - 1$ at the same wage. This corresponds to the case where employment contracts are permanent, but can be terminated by the agent in each period. In order for the agent to be able to continue working at the same principal, we have to suppose that the set of principals does not shrink over time, i.e., that $N^t \subseteq N^{t+1}$ for all $t \in \{1, \dots, T - 1\}$.

Here, the situation in the final period T corresponds to a static (single-period) problem, in which the agent has a reservation utility $u_0 \geq 0$ given by his utility obtained from the existing contract of period $T - 1$ (where $u_0 = 0$ if no such contract exists). As argued before (see Footnote 12), this static problem with reservation utility u_0 corresponds to the static problem with reservation utility zero and all costs increased by u_0 , where the agent's final utility is readjusted by adding the initial reservation utility u_0 . Thus, all our results shown for the static problem with reservation utility zero apply to problem faced in the final period. In particular, since the agent's utility obtained in the static problem can never decrease by more than u_0 when increasing all costs by u_0 , the agents readjusted utility in period T is nondecreasing in u_0 . In other words, a higher reservation utility from the previous-period contract weakly increases the agent's final, readjusted utility in period T . Consequently, selecting a contract in period $T - 1$ that maximizes his utility in period $T - 1$ is not only optimal for the agent with respect to period $T - 1$, but at the same time optimal for his utility in period T since it maximizes the reservation utility in period T . Similarly, for the principals, a higher reservation utility from the contract of period $T - 1$ weakly decreases the winning principal's profit in period T , which implies that offering a contract that maximizes her

profit (and, thus, minimizes the agent's reservation utility) given the other principals' offers in period $T - 1$ is optimal for each principal with respect to both period $T - 1$ and period T . By induction, this shows that, in each period $t \in \{1, \dots, T\}$, the agent and the principals behave as if no future periods existed by maximizing their current utility and profits, respectively. Consequently, the situation again decomposes into T static (single-period) problems. As in the first dynamic extension, this holds for all of the considered bargaining procedures as well as for the setting of stable outcomes.

D.2 Working hours, endogenous working costs, and efforts

Beside remuneration, an important content of real-world employment contracts is the number of hours an agent has to work and minimum wages could potentially influence working hours. This has been empirically documented, e.g., in Neumark and Wascher (2008); Kreiner et al. (2020); Jardim et al. (2022).

We next incorporate working hours into our model by letting the principals choose between part-time and full-time employment when making an offer to the agent. If the agent works for principal i , we denote his productivity by θ_i^p in case of part-time employment and by θ_i^f in case of full-time employment. The agent's cost may also depend on the working hours, and we let c_i^p and c_i^f denote the cost of working part-time and full-time for principal i , respectively. It is reasonable that the cost and productivity are lower with part-time employment than with full-time employment, i.e., $\theta_i^p < \theta_i^f$ and $c_i^p < c_i^f$ for each $i \in N$.

The binary choice between part-time and full-time employment at each principal means that there are essentially two variants of each principal i : the part-time variant, where the productivity is θ_i^p and the cost is c_i^p , and the full-time variant, where the productivity is θ_i^f and the cost is c_i^f . Each principal then competes for the agent against all variants of the other principals.

In case of unrestricted wages, it is optimal for each principal i to choose the working hours that maximizes the surplus. That is, it is optimal for principal i to offer full-time employment if $\theta_i^f - c_i^f > \theta_i^p - c_i^p$, and to offer part-time employment if $\theta_i^f - c_i^f < \theta_i^p - c_i^p$ (if $\theta_i^f - c_i^f = \theta_i^p - c_i^p$, the principal is indifferent). The reason is straightforward: if principal i attracts the agent with an offer that does not maximize the surplus—i.e., with an offer that specifies part-time employment even though $\theta_i^f - c_i^f > \theta_i^p - c_i^p$ or full-time employment even though $\theta_i^f - c_i^f < \theta_i^p - c_i^p$ —then principal i can improve her profit by changing the working hours and simultaneously adjusting the wage in a way that keeps the agent's utility unchanged.³⁵

In case of restricted wages, each principal must fulfill a minimum wage requirement, which specifies that the offered wage at least weakly exceeds the hours worked times the (hourly) minimum wage \underline{w} . Denoting the working hours at principal i in case of

³⁵ There can also be equilibria where some principals (precisely those that cannot generate the highest or second-highest surplus) offer working hours that do not maximize the surplus since they are not able to attract the agent anyway and are, therefore, indifferent which working hours to offer. In all equilibria, however, at least the winning principal and the principal who can generate the second-highest surplus offer working hours that maximize the surplus.

part-time and full-time employment by h_i^p and h_i^f , respectively, principal i has to offer a wage of at least $h_i^p \cdot \underline{w}$ for part-time employment and of at least $h_i^f \cdot \underline{w}$ for full-time employment. It is then readily seen that a minimum wage may distort the principal's choice, in the sense that she may choose to offer full-time employment instead of part-time employment even though $\theta_i^f - c_i^f < \theta_i^p - c_i^p$, or part-time employment instead of full-time employment when $\theta_i^f - c_i^f > \theta_i^p - c_i^p$. The reason is that the minimum wage requirement could cause the principal's profit when choosing the surplus-maximizing working hours to be negative or rather small, so that she decides to select other working hours that do not maximize the surplus. A simple example is the case with one principal, $h_1^p = 20$, $h_1^f = 40$, $c_1^p = 50$, $c_1^f = 150$, $\theta_1^p = 250$, and $\theta_1^f = 400$, where the principal chooses full-time employment in case of no minimum wage and part-time employment in case of a minimum wage $\underline{w} = 10$.

Theorem D.1 *Suppose the principals specify the wage and the working hours (part-time or full-time) in their offers. For unrestricted wages, it is an equilibrium that each principal makes an offer that specifies the surplus-maximizing working hours. This need no longer be true with a minimum wage, where principals might offer working hours that do not maximize the surplus.*

With working hours as a choice variable, a minimum wage may, thus, decrease the generated surplus even if the agent stays employed at the same principal. Generically, this occurs whenever the minimum wage causes the winning principal to change the working hours.³⁶

Interestingly, with working hours as a choice variable, the agent's utility could suffer from a minimum wage even if he stays employed at the same principal.³⁷ To see this, we extend the example from before by a second principal with $h_2^p = 20$, $h_2^f = 40$, $c_2^p = 50$, $c_2^f = 150$, $\theta_2^p = 200$, and $\theta_2^f = 380$. Without a minimum wage, the agent works full-time for principal 1 for a total wage of 380 (9.5 per hour) and, thus, experiences a utility of 230. With a minimum wage of $\underline{w} = 10$ per hour, he stays at principal 1, but works part-time for a wage of 200 and, thus, experiences a utility of just 150.

An important alternative interpretation of this model is that each principal can invest in the working conditions or amenities agents face when working for her. Then, a principal can choose between different combinations of productivities net of the investment costs θ_i and working costs c_i . The previous results show that a minimum wage may cause a principal to reduce her investments in the agent's working conditions or amenities, which can lower the generated surplus and the agent's utility. A second alternative interpretation is that of efforts.³⁸ A principal may require an agent to invest a specific level of effort. A higher level of effort is then associated with a higher cost on the side of the agent as well as with a higher productivity on the side of the principal.

³⁶ Generically, $\theta_i^f - c_i^f > \theta_i^p - c_i^p$ or $\theta_i^f - c_i^f < \theta_i^p - c_i^p$ holds, such that either full-time employment or part-time employment maximizes the generated surplus and every shift of the working hours due to the minimum wage is associated with a strict decrease of the generated surplus.

³⁷ We thank an anonymous referee for mentioning this point.

³⁸ See (Rebitzer and Taylor 1995) for a study of minimum wages in an efficiency wage model. In their model, the workers' costs are the same at all firms.

Note that, for all possible interpretations (working hours, endogenous working conditions/amenities, or efforts), higher productivities are associated with higher costs, i.e., there is a monotone positive relationship, but higher productivities are not necessarily associated with higher levels of surplus. It is furthermore straightforward to extend the above model to more than two possible levels of working hours etc.

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