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Comparing the Effects of Subsidies on Target Goods

Markus Dertwinkel-Kalt* Christian Wey[†]

July 2025

Abstract

We analyze the equilibrium effects of different subsidies on target goods under both perfectly competitive and monopolistic market structures. We concentrate our analysis on three particularly common forms of subsidies: (i) a per-unit subsidy, (ii) an ad valorem subsidy, and (iii) an "inversely related" subsidy, which increases as the price of the target good decreases. To evaluate the price effects of the subsidies, we rely on two criteria, an "equal-relief" criterion—which relies on a pass-through analysis—and a cost-effectiveness criterion. Overall, the ad valorem subsidy always yields the strongest price-increasing effect, whereas an inversely related subsidy leads to the lowest price increase. Consequently, the ad valorem subsidy induces the largest output expansion under perfect competition, whereas the inversely related subsidy dominates the other subsidies in a monopoly under both criteria. Those findings are consistent with several empirical facts, such as observed price differences for green target goods across European countries.

JEL-Classification: D04, D40, H20.

Keywords: Subsidies, Target Goods, Equal-relief, Pass Through, Cost Effectiveness.

1 Introduction

Countries worldwide implement a variety of subsidy schemes to promote target goods, particularly for households in the context of their green economic transition, such as the adoption of heat pumps, the installation of solar panels, or the purchase of electric vehicles. In this context, understanding the extent to which each type of subsidy affects market equilibrium—particularly the prices and quantities of the target goods—is crucial.

In this paper, we compare the price and quantity effects of subsidies that are affine in the market price of the target good. We concentrate our analysis on three especially prevalent subsidies that are special cases of the affine subsidy: a *per-unit subsidy* (as implemented, e.g., recently in Germany for solar panel installations or electric vehicle purchases, or in France for heat pump adoptions), an *ad valorem subsidy* (as implemented, e.g., in Germany for heat pump adoptions), and an *inversely related subsidy* (as implemented, e.g., in Japan for solar panel installations). The inversely related subsidy is proportional to the difference between a reference price set by the

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regulator and the market price, meaning that the subsidy increases as the market price of the target good decreases. We conduct our analysis for different market structures, namely, for perfectly competitive supply as well as for a monopolistic supplier.

All three types of subsidies shift the inverse demand function upward; that is, they drive a wedge between the market price received by suppliers and the price actually paid by consumers. The per-unit subsidy increases the inverse demand for any quantity by the amount of the subsidy and thus represents a parallel shift of the inverse demand function. The ad valorem subsidy tilts the demand clockwise, and the inversely related subsidy tilts it counter-clockwise. It follows that the wedge between the subsidy-induced inverse demand and the original inverse demand is decreasing in quantity under the ad valorem subsidy, constant under the per-unit subsidy, and increasing under the inversely related subsidy.

We investigate the effects of the subsidies according to two criteria: the *equal-relief* criterion and the *cost-effectiveness* criterion. The former criterion assumes that the subsidies reduce the *initial* consumer price by the same amount. This captures the stated intent of many real-world interventions: policymakers frequently aim to relieve consumers by $\in X$ per unit. From an equal-relief perspective, a subsidy is preferred if it leads to a higher output level than all other considered alternatives. This reflects an inherent shortsightedness of the equal-relief approach, which focuses on the subsidy's output effects while neglecting its fiscal costs in equilibrium. Notably, in case of marginal changes of the subsidies, this approach leads to a *pass-through* analysis. The cost-effectiveness approach, in contrast, takes full account of the equilibrium adjustment induced by the subsidies. The question here is which of the subsidies achieves a certain output level with the lowest fiscal costs.

The comparison of subsidy types depends on the benchmark used for evaluation and the market structure. In a perfectly competitive market, existing results from the taxation literature imply that all three subsidy types—per-unit, ad valorem, and inversely-related—are equally cost effective. We confirm this result. However, when applying the equal-relief criterion, the ad valorem subsidy leads to the largest increase in the market price and, consequently, the greatest expansion in output under perfect competition. The intuition is as follows. If the market price were held fixed at the pre-subsidy level, then—under the equal-relief assumption—all subsidies would generate the same level of consumer demand. Given an upward-sloping supply function, the equilibrium price must rise relative to the pre-subsidy level, implying that equilibrium output will fall short of the level that would be achieved if the price remained unchanged. Now remember that under the ad valorem subsidy, the wedge between inverse demand with and without the subsidy increases as output falls; under the per-unit subsidy, it remains constant; and under the inversely-related subsidy, it declines. It follows that the ad valorem subsidy results in the greatest output increase, the inversely-related subsidy in the smallest, and the per-unit subsidy lies in between.

In the case of a monopoly, the optimal quantity is determined by equating marginal revenue and marginal cost. Marginal revenue, in turn, critically depends on the slope of the inverse demand function. A larger absolute value of this slope—*ceteris paribus*—implies a lower marginal revenue. The effects of the three subsidy types under monopoly can thus be analyzed by comparing how marginal changes in each subsidy are passed through to prices, under the

equal-relief assumption. A marginal increase in any of the three subsidies leads to an increase in monopoly output. However, the magnitude of this effect depends, again, on how the aforementioned wedge varies with output: under the inversely-related subsidy, the wedge increases with output; under the ad valorem subsidy, it decreases; and under the per-unit subsidy, it remains constant. As a result, the output effect is strongest for the inversely-related subsidy and weakest for the ad valorem subsidy, with the per-unit subsidy lying in between. Since this ordering holds not only relative to the no-subsidy benchmark, but also when a subsidy is already paid in the benchmark, the inversely-related subsidy dominates the other two—both in terms of output expansion under the equal-relief criterion and in terms of cost effectiveness. By contrast, the ad valorem subsidy is the least cost effective and leads to the highest monopoly price and the smallest output increase under the equal-relief benchmark.

Overall, our results highlight the price-increasing effect of the ad valorem subsidy. While this leads to the largest output expansion under perfect competition, the opposite is true in a monopoly setting, where the ad valorem subsidy results in the smallest output increase. In contrast, the inversely-related subsidy generates the smallest price increase and becomes strictly preferable when firms face downward-sloping demand curves—that is, when firms do not face perfectly elastic demand curves, as in monopolistic markets. When applying the cost-effectiveness criterion, the inversely-related subsidy is never outperformed by the other subsidy types. It is strictly preferred whenever firms possess some degree of market power.

Our findings are consistent with several empirical facts, such as observed price differences for green target goods across European countries, which originally motivated this paper. Our equal-relief approach helps explain why the German government—prioritizing a rapid, nation-wide transition to sustainable heating systems while giving only limited consideration to associated costs—has opted for ad valorem subsidies for heat pumps: in competitive markets, output expands more under an ad valorem subsidy than a unit subsidy. Standard cost-effectiveness arguments cannot account for this choice: under perfect competition, all subsidy types are equally cost effective, while in monopolistic markets, ad valorem subsidies are the least cost-effective option. Our framework also sheds light on the significantly higher prices for heat pumps, for instance, are significantly costlier in Germany compared to France, supporting the view that this disparity is largely due to differences in national subsidy schemes (Deutsche Umwelthilfe, 2025). Presumably, these high prices result from inelastic supply as the supply of heat pump installations is rather competitive than monopolistic.

Related Literature Our analysis builds on a foundational strand of the public finance literature that compares the incidence and efficiency of per-unit versus ad valorem taxation (Wicksell, 1896; Suits and Musgrave, 1953; Bishop, 1968; Hamilton, 1999; Anderson *et al.*, 2001; Auerbach and Hines, 2002).³ The main takeaway here is that an ad valorem tax is particularly effective in

 $^{^1\!}S\!ee\ https://www.tagesschau.de/wirtschaft/verbraucher/waermepumpen-kosten-foerderung-100.html.$

²For an overview of heat pump subsidies across Europe in 2023, see (European Heat Pump Association, 2023). See also https://www.euronews.com/green/2024/01/21/heat-pumps-government-subsidies-in-europe-are-making-green-tech-more-affordable for a more recent summary.

³To promote the adoption of green target goods, also very different types of subsidies have been implemented. For instance, in the case of solar panels, subsidies have sometimes targeted future electricity production rather than

environments of imperfect competition.⁴ While we provide a comprehensive analysis of subsidies commonly used to promote target goods, many of our findings concerning the per-unit subsidy and the ad valorem subsidy can be pieced out of this literature when acknowledging the "reverse" relation between taxes and subsidies. A central object of our analysis is the *inversely related subsidy* which has been studied by Kiso (2022) as a formalization of Japan's subsidy program for solar panel installations. Within an oligopolistic market model, he finds that the inversely related subsidy is more cost effective than a per-unit subsidy, which in turn outperforms an ad valorem subsidy. We show that the inversely related subsidy can be interpreted as a *combination* of a per-unit subsidy and an ad valorem tax, and here the respective literature has shown that such a combination is particularly effective in environments with imperfect competition (Delipalla and Keen, 1992; Dillén, 1995; Myles, 1995, 1996); thus echoing the results presented in Kiso (2022).⁵ We extend this literature by studying the inversely related subsidy in comparison with the per-unit subsidy and the ad valorem subsidy for the benchmark market structures of perfect competition and monopoly, and by examining the equal-relief and pass-through implications of the subsidies.

We proceed as follows. In Section 2, we introduce the subsidies, followed by Section 3, where we present the two criteria used to analyze them. Section 4 contains our analysis in competitive markets, while Section 5 extends the analysis to monopolistic markets. Section 6 concludes.

2 Consumer Demand and the Subsidies

We first establish our demand system (Section 2.1) before we introduce the different subsidy schemes that we analyze (Section 2.2) and demand under these subsidies (Section 2.3).

2.1 Consumer Demand

Consider a target good x for which the policymaker wishes to lower the market price p that consumers pay. The representative consumer has a quasilinear utility function, where the utility of the target good is given by u(x) with $u'(x) := \frac{\partial u(x)}{\partial x} > 0$ and $u''(x) := \frac{\partial^2 u(x)}{\partial x^2} < 0$ for all $x \ge 0$. Thus, consumer preferences for the target good are monotone, and marginal utility decreases with higher consumption levels. The consumer solves the utility maximization problem

$$\max_{x} u(x) - px.$$

This gives the first-order condition

$$u'(x) - p = 0, (1)$$

provided that there exists x such that marginal utility u'(x) is larger than the market price p. From (1) we get the demand function x(p). In the following, we mostly work with the inverse

providing upfront investment support (De Groote and Verboven, 2019). We do not analyze such subsidies in this paper.

⁴While we assume profit maximization, other contributions have compared per-unit and ad valorem taxes and subsidies under revenue maximization (Gaudin and White, 2014).

⁵Another key insight from this literature is that such a combination can offset monopoly distortions (via an ad valorem tax) and implement second-best (Ramsey-Boiteux) pricing in regulated markets (Myles, 1996).

demand function $p^D(x) := x^{-1}(p)$, which also follows from (1) and is given by

$$p^{D}(x) = u'(x), \tag{2}$$

with slope

$$\frac{\partial p^D(x)}{\partial x} = u''(x) < 0; \tag{3}$$

that is, demand is downward sloping.

2.2 The Subsidies

We consider subsidies for target goods, which are granted per unit of transaction. Quite generally, such a subsidy comes as a positive, affine transformation of the market price, p, of the target good and thus is of the form

$$t + sp \ge 0$$
, with $s, t \in \mathbb{R}$. (4)

By definition, a subsidy is never negative, and there exist prices p for which the subsidy is strictly positive. A subsidy (4) consists of a price-independent parameter t (the "per-unit" part) and a dimensionless parameter s, expressing the proportion of the market price that is subsidized (the "ad valorem" part). We assume $0 \le t < p$, so that t stands for a per-unit subsidy which is smaller than the market price. Likewise, it is sensible to restrict the ad valorem parameter to values below one; i.e., s < 1. Otherwise, consumer demand, x(p) is upward sloping in the market price p, meaning that the subsidy scheme could be milked indefinitely.⁶

With a subsidy (4) in place, the consumer price, \hat{p} , for a given market price p is obtained by subtracting the subsidy from the market price, which gives

$$\widehat{p} = p - (t + sp). \tag{5}$$

Given the pre-intervention market price p, we assume that the subsidy never reduces consumer demand x(p). This is guaranteed by assuming that the subsidized price consumers face, \widehat{p} , is strictly smaller than the pre-intervention market price p for any demand level x>x(p). Taken together, if p is the pre-intervention market price, then we restrict attention to subsidies that fulfill

$$t + sp > 0$$
 with $0 \le t < p$ and $s < 1$ so that $x > x(p)$. (6)

Importantly, a subsidy according to (6) drives a wedge between the *market price* p received by suppliers and the *consumer price* \hat{p} actually paid by consumers.

In practice, three parameter constellations of t and s are of particular relevance. First, for t > 0 and s = 0 we have a *per-unit subsidy*. Second, for t = 0 and $s \in (0,1)$ we get the case

 $^{^6}$ However, a subsidy with s>1 is not entirely unrealistic, as demonstrated by Italy's 'superbonus' program for home renovations, introduced in the aftermath of the COVID-19 pandemic. This initiative offered a highly generous tax credit—at times reaching 110%—for specific types of building improvements, particularly those aimed at enhancing energy efficiency and seismic resilience (Corsello and Ercolani, 2024). Unsurprisingly, the fiscal cost of the superbonus has proven to be substantial, but is mostly covered by the EUR 750 billion coronavirus recovery fund.

of an *ad valorem subsidy*. Notably, in both cases, the subsidy is globally strictly positive for all p>0. A third case is obtained for t>0 and s<1 in which case a per-unit subsidy is combined with either an ad valorem subsidy (for $s\in(0,1)$) or an *ad valorem tax* (for s<0).⁷ In the former case, the subsidy is globally positive for any p>0, which is not necessarily true in the latter case. Notably, the latter constellation is quite common, since most goods subsidized on a per-unit basis remain subject to the usual value-added tax. In principle, a combination of a per-unit subsidy and an ad valorem tax can lead to a net tax or a net subsidy depending on the value of the market price. However, our paper is concerned with subsidies, we require that such a combination satisfies (6) and thus yields a strictly positive relief for consumers. Now, observe that in this situation the subsidy (6) can be expressed as

$$(\overline{p} - p)r$$
, with $r := -s$, and $\overline{p} := -\frac{t}{s}$. (7)

This expression allows for a new interpretation of a regulatory regime in which a good is subsidized on a per-unit basis while simultaneously subject to an ad valorem tax. Specifically, such a combined intervention is equivalent to a subsidy defined by a fixed "reference price" \overline{p} being strictly larger than the market price p and a parameter $r \in (0,1)$, so that the subsidy is *inversely related* to the product's price. This subsidy scheme is interesting in its own right because ad valorem taxes are typically not set on an industry-specific basis. By setting the reference price $\overline{p}>0$ and the parameter $r\in (0,1)$, this subsidy scheme allows the government to combine a per-unit subsidy with an valorem tax on a specific targeted good.

To sum up, we consider the following three subsidies paid directly to consumers:

1. Per-unit subsidy ("t-subsidy"): The consumer receives a subsidy 0 < t < p, and the consumer price is

$$\widehat{p}_t := p - t. \tag{8}$$

2. Ad valorem subsidy ("s-subsidy"): The consumer receives a subsidy sp, with $s \in (0,1)$, and the consumer price is

$$\widehat{p}_s := p(1-s). \tag{9}$$

3. Inversely related subsidy ("r-subsidy"): The consumer receives a subsidy ($\overline{p} - p$)r, with $r \in (0,1)$ and $\overline{p} > p$, and the consumer price is

$$\widehat{p}_r := p - (\overline{p} - p)r. \tag{10}$$

Let $\sigma \in \Sigma := \{r, s, t\}$ indicate the type of subsidy in place. For any market price p, the amount paid by consumers decreases as the subsidy parameters t, s, or r increase, so that each subsidy provides financial relief to consumers purchasing the target good. A per-unit subsidy lowers the price of every unit by a fixed amount t; an ad valorem subsidy reduces the price by the fraction s of the market price; and an inversely related subsidy decreases the price by the proportion r of the gap between a reference price \overline{p} and the market price p.

 $^{^{7}}$ A fourth case could be a combination of a per-unit tax and an ad valorem subsidy. While this case is conceivable, it appears to be less practically relevant.

2.3 Induced Consumer Demand under the Subsidies

Replacing the market price p in (1) by the subsidized consumer price \hat{p}_{σ} , we get the *induced* inverse demand suppliers are facing when consumers pay $p^{D}(x)$ and receive the subsidy $\sigma \in \Sigma$. Those induced inverse demand functions are given by

$$p_t^D(x) := p^D(x) + t,$$
 (11)

$$p_s^D(x) := \frac{p^D(x)}{1-s}$$
, and (12)

$$p_r^D(x) := \frac{p^D(x) + \overline{p}r}{1+r}.$$
 (13)

Each subsidy creates a strictly positive wedge between the induced inverse demand and the original inverse demand, which is given by the difference $p_{\sigma}^D(x)-p^D(x)$ for the subsidies $\sigma\in\Sigma$. For any given quantity x, the wedge measures by how much a subsidy increases consumers' maximal willingness to pay per unit of the target good. In case of a t-subsidy, we get $p_t^D(x)-p^D(x)=t$, so that the wedge is a constant and independent of the consumed quantity. Thus, the induced inverse demand under a t-subsidy is obtained by shifting the original inverse demand upwards by the amount of t. For the s-subsidy, the wedge is given by $p_s^D(x)-p^D(x)=\frac{s}{1-s}p^D(x)$, so that the induced demand again lies above the original inverse demand, but because $\frac{\partial p^D(x)}{\partial x}<0$, the wedge is decreasing in the consumption level of the target good. This means that the subsidy per unit is higher for low consumption levels and lower for higher consumption levels of the target good. Finally, if the r-subsidy applies, then the wedge becomes $p_t^D(x)-p^D(x)=\frac{r}{1+r}(\overline{p}-p^D(x))$, so that the induced inverse demand lies above the original demand, but the wedge is now increasing in the consumed quantity of the target good.

3 Two Criteria to Compare the Subsidies

Although each subsidy *ceteris paribus* lowers the price consumers pay, any meaningful comparison of the subsidies must recognize that the policies also alter the equilibrium market price p. Below, we analyze the subsidies within the two benchmark market structures of perfect competition (indexed by "**") and monopoly (indexed by "M"). Let $\omega \in \Omega := \{*, M\}$ indicate the market structure. Given a market structure ω and a subsidy σ , let x_{σ}^{ω} , $p^{D}(x_{\sigma}^{\omega})$, and p_{σ}^{ω} denote the equilibrium values of the quantity, the consumer price, and the market price, respectively, where the market price follows from (11)-(13) with $p_{\sigma}^{\omega} := p_{\sigma}^{D}(x_{\sigma}^{\omega})$. Below we show that there exist such unique values under both market structures for all feasible parameter values of the subsidies.

It is helpful to define the set of all feasible triples θ^ω of the three subsidies that give rise to the same equilibrium output x^ω_σ under market structure ω by

$$\Theta^\omega:=\{(t,s,r)\in[0,p^\omega)\times[0,1)\times[0,1)|x_t^\omega=x_s^\omega=x_r^\omega\geq x^\omega\}$$
 ,

where x^{ω} and p^{ω} denote the pre-subsidy output level and the pre-subsidy market price, respectively.

We investigate the relative performance of the subsidies from two perspectives: the *equal-relief* criterion and the *cost-effectiveness* criterion. The former approach assumes that the subsidies reduce the *initial* consumer price by the same amount. Thus, the subsidies are "matched at the initial price" (Suits and Musgrave, 1953). The evaluation of the subsidies then depends on the induced quantity and price effects. Notably, in case of marginal changes of the subsidies, this approach leads to a standard *pass-through* analysis. The cost-effectiveness approach, in contrast, takes full account of the equilibrium adjustment induced by the subsidies. The question here is which of the subsidies achieves a certain output level with the lowest fiscal costs.

Equal Relief. Suppose the pre-subsidy equilibrium is given by (x^{ω}, p^{ω}) with $x^{\omega} > 0$. Now consider strictly positive values of the subsidies which reduce the pre-subsidy consumer price \hat{p}^{ω} (which is equal to the market price p^{ω}) by the same amount. This gives the *equal-relief conditions*

$$\widehat{p}_t = \widehat{p}_s = \widehat{p}_r \Leftrightarrow t = p^{\omega} s = (\overline{p} - p^{\omega})r. \tag{14}$$

Notably, this approach allows to compare the subsidies' quantity effects relative to the presubsidy market equilibrium (x^{ω}, p^{ω}) .

Definition 1 (Equal Relief Subsidies). Fix a market structure $\omega \in \Omega$. A subsidy triple (t, s, r) provides equal relief relative to the pre-subsidy market equilibrium (x^{ω}, p^{ω}) if and only if $t = p^{\omega}s = (\overline{p} - p^{\omega})r$ holds. From an equal-relief perspective, a subsidy is preferred if it leads to a higher output level than all other considered alternatives.

Thus, this approach assumes a fixed pre-intervention price, p^{ω} , which allows us to calculate the per-unit equivalents of the s- and r-subsidy, which are given by $p^{\omega}s$ and $(\overline{p}-p^{\omega})r$, respectively. The equal-relief criterion according to Definition (1) reflects the inherent shortsightedness of the equal-relief approach, which focuses solely on output while neglecting equilibrium price effects and thus the fiscal costs associated with the subsidies. Still, it has the advantage that it relies only on *observable* market variables—namely the prevailing price and the mandated transfer—whereas the standard cost-effectiveness criterion requires full knowledge of demand and supply schedules, an unrealistic requirement when short-run and long-run curves differ substantially and the relevant horizon is unclear.⁸

We now define the normalized subsidy values as

$$\gamma_t^{\omega} := t, \gamma_s^{\omega} := p^{\omega} s$$
, and $\gamma_r^{\omega} = (\widehat{p} - p^{\omega}) r$,

so that the equal-relief conditions (14) can be written as

$$\gamma_t^{\omega} = \gamma_s^{\omega} = \gamma_r^{\omega}$$
.

While so far, equal relief referred to the pre-subsidy equilibrium, we now generalize equal

⁸Interestingly, this approach can also be justified with behavioral-economic reasoning: both voters and policy-makers are often myopic and fail to anticipate equilibrium adjustments. There is ample evidence that citizens and policymakers struggle with contingent reasoning and tend to neglect equilibrium feedbacks, leading to systematic misperceptions of policy effectiveness—already noted by Smith (1776) and recently demonstrated in experimental work by Dal Bó *et al.* (2018) and Nunnari *et al.* (2024).

relief conditions to initial market equilibria with subsidies $\theta^\omega \in \Theta^\omega$ in place. In the corresponding initial market equilibrium $(x^\omega_\sigma, p^\omega_\sigma)$ there is a wedge between the consumer and the market price. The consumer price must be the same under all subsidies, whereas the market price could differ. Starting from such a situation, we refer to equal relief if

$$\Delta \gamma_t^{\omega} = \Delta \gamma_s^{\omega} = \Delta \gamma_r^{\omega}$$
, with $\Delta \gamma_t^{\omega} := \Delta t$, $\Delta \gamma_s^{\omega} := p_s^{\omega} \Delta s$, and $\Delta \gamma_r^{\omega} := (\widehat{p} - p_r^{\omega}) \Delta r$, (15)

holds for discrete increases Δt , Δs , and Δr . Likewise, we speak of equal relief for marginal subsidy increases if

$$d\gamma_t^\omega=d\gamma_s^\omega=d\gamma_r^\omega$$
, with $d\gamma_t^\omega:=dt$, $d\gamma_s^\omega:=p_s^\omega ds$, and $d\gamma_r^\omega:=(\widehat{p}-p_r^\omega)dr$,

holds for marginal increases dt, ds, and dr. We can now define the quantity effects and the pass-through rates of the prices that are induced by marginal changes of the normalized subsidy values.

Pass-through. The quantity effect of a marginal change of the normalized value of a subsidy σ is given by

$$\frac{dx_{\sigma}^{\omega}}{d\gamma_{\sigma}^{\omega}} \tag{16}$$

and the pass-through rates of the subsidies on the consumer and the market prices are given by

$$\frac{dp^D(x^\omega_\sigma)}{d\gamma^\omega_\sigma}$$
 and $\frac{dp^\omega_\sigma}{d\gamma^\omega_\sigma}$, respectively.

The normal reaction to an increase of a subsidy is that the market price goes up; however, in case of a monopoly it could be that the monopoly price p_{σ}^{M} is reduced when the subsidy is marginally increased. In accordance with the cost pass through literature, we refer to the case where the consumer price decreases by more than the subsidy increases (i.e., the market price decreases with a subsidy-increase) as "overshifting."

Relation between equal relief and pass-through. Now note that the quantity effect of a subsidy, $x_{\sigma}^{\omega} - x^{\omega}$, can be calculated as follows:

$$x_{\sigma}^{\omega} - x^{\omega} = \int_{0}^{\gamma_{\sigma}^{\omega}} \frac{dx_{\sigma}^{\omega}}{d\gamma_{\sigma}^{\omega}} d\gamma_{\sigma}^{\omega}.$$
 (17)

Likewise, the change in consumer prices from a subsidy follows from pass-through as follows:

$$p^{D}(x_{\sigma}^{\omega}) - p^{D}(x^{\omega}) = \int_{0}^{\gamma_{\sigma}^{\omega}} \frac{dp^{D}(x_{\sigma}^{\omega})}{d\gamma_{\sigma}^{\omega}} d\gamma_{\sigma}^{\omega}, \tag{18}$$

and also the change in market prices follows from pass-through:

$$p_{\sigma}^{\omega} - p^{\omega} = \int_{0}^{\gamma_{\sigma}^{\omega}} \frac{dp_{\sigma}^{\omega}}{d\gamma_{\sigma}^{\omega}} d\gamma_{\sigma}^{\omega}, \tag{19}$$

respectively.

Cost Effectiveness. Consider a triple of strictly positive subsidies, θ^{ω} , for a given market structure ω , so that the equilibrium quantity is identical across all three subsidies $\sigma \in \Sigma$ with $x_{\sigma}^{\omega} > x^{\omega}$. Denote by ϕ_{σ}^{ω} the equilibrium per-unit subsidy of the σ -subsidy that achieves this output level. Then,

$$\phi_t^{\omega} = t, \, \phi_s^{\omega} = p_s^{\omega} s, \, \text{and} \, \phi_r^{\omega} = (\overline{p} - p_r^{\omega})r;$$
 (20)

holds, where p_s^{ω} and p_r^{ω} denote the equilibrium market prices. The higher ϕ_{σ}^{ω} is, the less cost effective is σ .

Definition 2 (Cost Effectiveness). Fix a market structure $\omega \in \Omega$. Suppose θ^{ω} implements the same equilibrium quantity $x^{\omega}_{\sigma} > x^{\omega}$ for all $\sigma \in \Sigma$. Then, subsidy σ is said to be more, equally, or less cost effective than subsidy σ' if $\phi^{\omega}_{\sigma} < \phi^{\omega}_{\sigma'}$, $\phi^{\omega}_{\sigma} = \phi^{\omega}_{\sigma'}$, or $\phi^{\omega}_{\sigma} > \phi^{\omega}_{\sigma'}$, respectively, holds.

This "cost-effectiveness" approach is also examined in the commodity taxation literature, where it is referred to as the "equal-yields" approach (Suits and Musgrave, 1953).

4 Perfect Competition

We first present the model in Section 4.1, before we provide the equal-relief analysis of the subsidies (which also includes the pass-through analysis) in Section 4.2 and the analysis of the cost effectiveness criterion in Section 4.3.

4.1 Benchmark Model of Perfect Competition

Assume a standard partial equilibrium model of a perfectly competitive market for a target good x (see Mas-Colell et al., 1995, chap. 10C), where the inverse consumer demand is given by (2). On the supply side, we have a representative firm that maximizes profits, π , by choosing a supply quantity. The firm takes the market price p as given and solves

$$\max_{x} \pi = px - C(x),\tag{21}$$

where C(x) is the cost function with $C'(x) := \frac{\partial C(x)}{\partial x} > 0$ and $C''(x) := \frac{\partial^2 C(x)}{\partial x^2} \ge 0$. To ensure a strict gain of trade in the market for x > 0, we abstract from fixed production costs (i.e., we set C(0) = 0), and we assume that the choke price (which is the smallest price where demand is zero) is strictly larger than the marginal production cost of the first unit of the target good. The inverse supply function follows from the firm's first-order condition

$$p - C'(x) = 0,$$

and is given by

$$p^S(x) := C'(x). \tag{22}$$

In the absence of a subsidy, the price consumers pay and the price suppliers receive is the same in the unique (Walrasian) competitive equilibrium, (p^*, x^*) , which follows from the

market-clearing condition

$$p^{S}(x^{*}) = p^{D}(x^{*}) =: p^{*} \Leftrightarrow C'(x^{*}) = u'(x^{*}) = p^{*}, \text{ with } p^{*} > 0 \text{ and } x^{*} > 0,$$
 (23)

where the existence and uniqueness follows from (3) (i.e., the inverse demand is strictly downward sloping), $\frac{\partial p^S(x)}{\partial x} = C'' \geq 0$ (i.e., the inverse supply is non-decreasing), and from assuming a sufficiently large choke price.

4.2 Equal Relief and Pass-Through Analysis

With a subsidy $\sigma \in \Sigma$ in place, the competitive equilibrium follows from the intersection of the respective induced inverse demand function (see (11)-(13)) and the inverse supply function (22). Accordingly, the competitive equilibrium under subsidy σ is given by $(p_{\sigma}^*, x_{\sigma}^*)$ and follows from the market equilibrium condition

$$p^{S}(x_{\sigma}^{*}) = p_{\sigma}^{D}(x_{\sigma}^{*}) =: p_{\sigma}^{*} \Leftrightarrow C'(x_{\sigma}^{*}) = p_{\sigma}^{D}(x_{\sigma}^{*}) = p_{\sigma}^{*}, \text{ for all } \sigma \in \Sigma.$$
 (24)

Next, we analyze the effects of the subsidy programs under the equal-relief approach under three different supply conditions: i) horizontal supply, ii) normal supply, and iii) vertical supply.

Case i (Horizontal Supply). Suppose the market supply is perfectly competitive in the sense that the industry supply is perfectly elastic; that is, it is a horizontal line parallel to the x-axis in the market diagram. In this case, the market price is fixed by the industry's constant marginal costs C'(x) = c > 0, so that the equilibrium price is given by $p^* = c$ for all x > 0. We first analyze how the competitive equilibrium is affected by a marginal change in the subsidies' normalized values γ_{σ} . We get the following comparative statics results.

Proposition 1 (Pass-through under perfect competition with horizontal supply). Assume a horizontal inverse supply function. Suppose $\theta^* \in \Theta^*$. A marginal change of the normalized subsidy γ_{σ} , with $\sigma \in \Sigma$, affects the market equilibrium $(p_{\sigma}^*, x_{\sigma}^*)$ and the consumer price $p^D(x_{\sigma}^*)$ as follows:

i) The quantity effect is identical for all $\sigma \in \Sigma$ with

$$\frac{dx_{\sigma}^*}{d\gamma_{\sigma}} = -\frac{1}{\frac{\partial p^D(x_{\sigma}^*)}{\partial x}} > 0 \text{ for all } \sigma \in \Sigma.$$

ii) The pass-through rate of the consumer price is identical for all $\sigma \in \Sigma$ with

$$\frac{dp^D(x^*_\sigma)}{d\gamma_\sigma} = -1 \ \text{for all } \sigma \in \Sigma.$$

⁹The supply is perfectly elastic if the competitive industry is characterized by free entry, all firms have access to the same technology, and there are no resource constraints in input markets.

iii) The pass-through rate of the market price is identical for all $\sigma \in \Sigma$ with

$$\frac{dp_{\sigma}^*}{d\gamma_{\sigma}} = 0 \text{ for all } \sigma \in \Sigma.$$

Now, assume that the subsidies fulfill the equal-relief conditions (14). Using (17), (18), and (19), we get the total effects of the subsidies concerning their impacts on output levels, consumer prices, and market prices, respectively. Proposition (1) thus implies the following comparison of the three subsidies that fulfill the equal-relief conditions (14).

Corollary 1 (Equal relief with horizontal supply). Assume a horizontal inverse supply function. Suppose the strictly positive subsidies t, s, and r offer equal relief to consumers according to (14). Then,

$$p_t^* = p_s^* = p_r^* = p^*,$$

$$x_t^* = x_s^* = x_r^* > x^*,$$

and

$$p^{D}(x_{t}^{*}) = p^{D}(x_{s}^{*}) = p^{D}(x_{r}^{*}) < p^{*},$$

so that none of the subsidies is preferred from an equal-relief perspective.

This shows that all three subsidies affect the market equilibrium in exactly the same way if the supply side is perfectly price elastic and the level of financial relief is identical. Under such conditions of perfect competition, the policymaker only needs to determine the extent to which consumers should be relieved; the specific type of subsidy implemented is irrelevant.

Case ii (Normal Supply). Suppose the industry supply function is neither perfectly elastic (case i) nor perfectly inelastic (case iii), but upward sloping. Again, we first analyze how the competitive equilibrium is affected by a marginal change in the subsidies' normalized values γ_{σ} .

Proposition 2 (Pass-through under perfect competition with normal supply). Assume C''(x) > 0. Suppose $\theta^* = (t, s, r) \in \Theta^*$. A marginal change of the normalized subsidy γ_{σ} , with $\sigma \in \Sigma$, affects the market equilibrium $(p_{\sigma}^*, x_{\sigma}^*)$ and the consumer price $p^D(x_{\sigma}^*)$ as follows:

i) The quantity effects are

$$\frac{dx_t^*}{d\gamma_t} = \frac{-1}{\frac{\partial p^D(x_t^*)}{\partial x} - C''(x_t^*)},$$

$$\frac{dx_s^*}{d\gamma_s} = \frac{-1}{\frac{\partial p^D(x_s^*)}{\partial x} - (1-s)C''(x_s^*)},$$

$$\frac{dx_r^*}{d\gamma_r} = \frac{-1}{\frac{\partial p^D(x_r^*)}{\partial x} - (1+r)C''(x_r^*)},$$

and their ordering is

$$0 < \frac{dx_r^*}{d\gamma_r} < \frac{dx_t^*}{d\gamma_t} < \frac{dx_s^*}{d\gamma_s}.$$

ii) The pass-through rates of the consumer price are

$$\frac{dp^D(x^*_{\sigma})}{d\gamma_{\sigma}} = \frac{\partial p^D(x^*_{\sigma})}{\partial x} \frac{dx^*_{\sigma}}{d\gamma_{\sigma}} \text{ for all } \sigma \in \Sigma,$$

and their ordering is

$$\frac{dp^D(x_s^*)}{d\gamma_s} < \frac{dp^D(x_t^*)}{d\gamma_t} < \frac{dp^D(x_r^*)}{d\gamma_r} < 0.$$

iii) The pass-through rates of the market price are

$$\frac{dp_t^*}{d\gamma_t} = 1 + \frac{dp^D(x_t^*)}{d\gamma_t},$$

$$\frac{dp_s^*}{d\gamma_s} = \frac{1}{1-s} \left(1 + \frac{dp^D(x_s^*)}{d\gamma_s} \right),$$

$$\frac{dp_r^*}{d\gamma_r} = \frac{1}{1+r} \left(1 + \frac{dp^D(x_r^*)}{d\gamma_r} \right),$$

and their ordering is

$$0 < \frac{dp_r^*}{d\gamma_r} < \frac{dp_t^*}{d\gamma_t} < \frac{dp_s^*}{d\gamma_s},$$

with $\frac{dp_t^*}{d\gamma_t} < 1$.

Using (17), (18), and (19), Proposition (2) implies the following comparison of the three subsidies that fulfill the equal-relief conditions (14).

Corollary 2 (Equal relief with normal supply). Assume a normal supply with C'' > 0. Suppose the strictly positive subsidies t, s, and r offer equal relief to consumers according to (14). Then

$$p_s^* > p_t^* > p_r^* > p_r^*,$$

$$x_s^* > x_t^* > x_r^* > x^*,$$

and

$$p^{D}(x_{s}^{*}) < p^{D}(x_{t}^{*}) < p^{D}(x_{r}^{*}) < p^{*},$$

so that the s-subsidy results in the largest output effect and is thus preferred from an equal-relief perspective.

For a fixed relief per unit, Corollary (2) demonstrates that the ad valorem subsidy leads to the greatest output expansion among the three subsidy programs considered. Given the upward-sloping inverse supply function, achieving this higher output requires the equilibrium price to be highest under the ad valorem subsidy. These effects are illustrated in Figure 1, which shows how equal relief subsidies affect inverse demand and, consequently, shift the market equilibrium, with the r-subsidy leading to the lowest equilibrium output and the s-subsidy resulting in the highest.

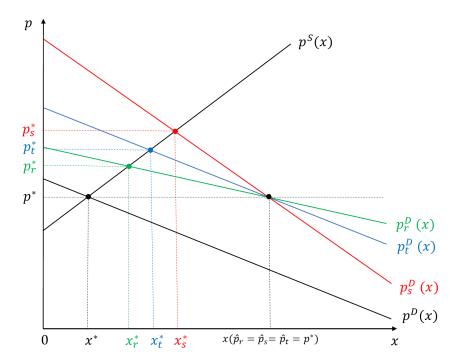


Figure 1: Equal-relief subsidies under perfect competition for linear demand

Notes: The subsidies are designed so that, for a fixed price, they provide equal relief—meaning the consumer pays the same fraction of the market price in all three cases. As a result, the shifted demand curves all pass through the same point $x(\hat{p}_r = \hat{p}_s = \hat{p}_t = p^*)$, located on a horizontal line through p^* . The t-subsidy shifts the original demand curve upward in parallel (blue line). The s-subsidy rotates the demand curve clockwise (red line), whereas the r-subsidy rotates it counterclockwise (green line). The equilibrium under each subsidy is determined by the intersection of the respective demand curve with the inverse supply curve. All subsidies increase both the market price and the equilibrium quantity, with the s-subsidy causing the largest increases and the r-subsidy the smallest.

Case iii (Vertical Supply). Suppose the supply function is perfectly inelastic at the pre-intervention competitive equilibrium (p^*, x^*) ; that is, it is represented by a vertical line (orthogonal to the x-axis) in the market diagram. The comparative statics results for marginal changes in the normalized subsidy values, γ_{σ} , are as follows.

Proposition 3 (Pass-through under perfect competition with vertical supply). Assume the supply quantity is fixed at $x^* > 0$ for all $p \geq 0$. Suppose $\theta^* = (t, s, r) \in \Theta^*$. A marginal change of the normalized subsidy γ_{σ} , with $\sigma \in \Sigma$, affects the market equilibrium $(p_{\sigma}^*, x_{\sigma}^*)$ and the consumer price $p^D(x_{\sigma}^*)$ as follows:

i) The quantity effects are all the same with

$$\frac{dx_{\sigma}^*}{d\gamma_{\sigma}} = 0 \text{ for all } \sigma \in \Sigma.$$

ii) The pass-through rates of the consumer price are all the same with

$$\frac{dp^D(x^*_{\sigma})}{d\gamma_{\sigma}} = 0 \text{ for all } \sigma \in \Sigma.$$

iii) The pass-through rates of the market price are

$$rac{dp_t^*}{d\gamma_t}=1$$
, $rac{dp_s^*}{d\gamma_s}=rac{1}{1-s}$, and $rac{dp_r^*}{d\gamma_r}=rac{1}{1+r}$,

and their ordering is

$$0 < \frac{dp_r^*}{d\gamma_r} < \frac{dp_t^*}{d\gamma_t} < \frac{dp_s^*}{d\gamma_s}.$$

Proposition (3) implies the following comparison of the three equal-relief subsidies.

Corollary 3 (Equal relief with vertical supply). Suppose a vertical supply function. Suppose the strictly positive subsidies t, s, and r offer equal relief to consumers according to (14). Then,

$$p_s^* > p_t^* > p_r^* > p_r^*,$$

$$x_t^* = x_s^* = x_r^* = x^*,$$

and

$$p^D(x^*_{\sigma}) = p^*,$$

so that none of the subsidies is preferred from an equal-relief perspective.

According to Corollary 3, if supply is vertical, the subsidy only increases prices, and the ad valorem subsidy has the strongest price-increasing effect.

4.3 Cost Effectiveness

How do the different subsidies compare in terms of cost effectiveness in expanding output? We obtain the following equivalence result under perfect competition, showing that the three subsidies are, in a perfectly competitive market, equally cost effective.

Proposition 4 (Cost effectiveness under perfect competition). Suppose strictly positive subsidies $(t, s, r) \in \Theta^*$, so that all subsidies implement the same equilibrium $(p^*_{\sigma}, x^*_{\sigma})$. Then, all three subsidies are equally cost effective with $\phi^*_{\sigma} = p^*_{\sigma} - u'(x^*_{\sigma})$ for all $\sigma \in \Sigma$.

The intuition is as follows. Fix some $x > x^*$. This constitutes an equilibrium if the induced inverse demand equals supply at this point. The per-unit subsidies are then given by the difference between the equilibrium price and the consumer price at this quantity. Thus, to implement x, the subsidies per unit are all the same (see Figure 2 for an illustration).

4.4 Summary

Altogether, under perfect competition, all types of subsidies are equally cost effective in achieving a desired output level of a target good. The cost-effectiveness criterion, therefore, does not allow for discriminating among the considered subsidies; this is different from an equal-relief perspective, where (in the most relevant case ii) we find that the ad valorem subsidy unfolds both the largest price-raising effect and the strongest output expansion effect. Thus, there is a

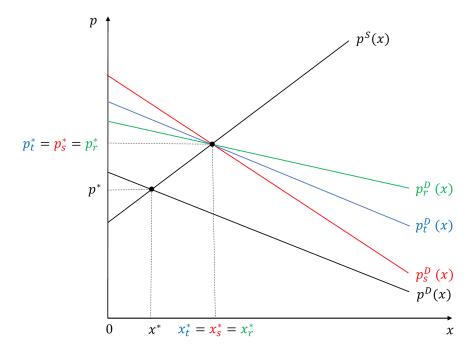


Figure 2: The cost-effectiveness criterion under perfect competition for linear demand *Notes*: The subsidies are designed so that they all achieve the same target quantity $x_t^* = x_s^* = x_r^*$. As a consequence, in this point the inverse demand functions must intersect the

inverse supply function. Thus, market prices and also subsidies per unit are the same.

case for using the ad valorem subsidy when the government applies the equal relief criterion and seeks to get the largest possible output effect from its subsidy policy. Notably, the different output effects of the subsidies under the equal-relief assumption cannot be detected from examining marginal changes of the subsidies in a small neighbourhood of the pre-subsidy market equilibrium, but only for discrete changes of the subsidies.¹⁰

Still, there remain some reservations about the ad valorem subsidy due to its implied fiscal costs, which could be entirely pocketed by suppliers when supply is inelastic, leaving consumers without any relief and resulting in no output expansion at all.

5 Monopoly

We first present the benchmark model with no subsidy in place (Section 5.1), before we present the comparison of the subsidies under the equal-relief assumption (which includes the pass-through analysis) in Section 5.2 and under the cost effectiveness criterion in Section 5.3.

5.1 Benchmark Monopoly Model

In the benchmark case without a subsidy in place, the monopolist maximizes (21) subject to $p = p^D(x)$. In the following, we assume that i) marginal costs are constant with C'(x) = c > 0, 11

The quantity effects and the pass-through rates of the prices are all the same at t = s = r = 0 (see Proposition (2).

<sup>(2).

11</sup> This assumption allows us to focus on the price-raising effects of monopoly power relative to the perfect competition benchmark, where the market price is never affected by any of the considered subsidies. Note also that we

and ii) marginal revenue, $M\!R(x) := p^D(x) + \frac{\partial p^D(x)}{\partial x} x$, is strictly downward sloping; that is,

$$2\frac{\partial p^D(x)}{\partial x} + \frac{\partial^2 p^D(x)}{\partial x^2}x < 0 \text{ for all } x > 0.$$

We define the *inverse demand curvature*, ρ , by

$$\rho := -\frac{\partial^2 p^D(x)}{\partial x^2} \frac{x}{\frac{\partial p^D(x)}{\partial x}}.$$
 (25)

Then, marginal revenue is strictly downward sloping if and only if

$$\frac{\partial p^D(x)}{\partial x} (2 - \rho) < 0 \Leftrightarrow \rho < 2; \tag{26}$$

that is, the inverse demand curve must not be too convex. The monopolist's problem is thus given by

$$\max_{x} \pi = \left(p^{D}(x) - c \right) x,\tag{27}$$

and the optimal monopoly solution (p^M, x^M) follows from the first-order condition

$$\frac{\partial \pi}{\partial x} = p^{D}(x) + \frac{\partial p^{D}(x)}{\partial x}x - c = 0, \text{ with } p^{M} := p^{D}(x^{M}). \tag{28}$$

The second-order condition holds because marginal revenue is strictly downward sloping and marginal costs are non-decreasing.

5.2 Equal Relief and Pass-Through Analysis

With a subsidy $\sigma \in \Sigma$ in place, the monopoly solution $(p_{\sigma}^M, x_{\sigma}^M)$ fulfills

$$x_{\sigma}^{M} := \operatorname*{argmax}_{x} \pi_{\sigma} := \left(p_{\sigma}^{D}(x) - c \right) x \text{ and } p_{\sigma}^{M} := p_{\sigma}^{D}(x_{\sigma}^{M}), \tag{29}$$

so that the monopoly price, that is p_{σ}^{M} , follows from evaluating the respective induced inverse demand function (see (11)-(13)) at the monopoly quantity, x_{σ}^{M} (which gives $p_{\sigma}^{M}=p_{\sigma}^{D}(x_{\sigma}^{M})$). The consumer price under subsidy σ follows from the original inverse demand (2) evaluated at the monopoly quantity x_{σ}^{M} , which gives $p^{D}(x_{\sigma}^{M})$.

We first analyze the comparative statics of the quantities, the consumer prices, and the monopoly prices under the three subsidies. As before, we consider marginal changes of the normalized subsidies γ_{σ} . We get the following results.

Proposition 5 (Pass-through in a monopoly). Suppose $\theta^M = (t, s, r) \in \Theta^M$. A marginal change of the normalized subsidy γ_{σ} , with $\sigma \in \Sigma$, affects the monopoly solution $(p_{\sigma}^M, x_{\sigma}^M)$ and the consumer price $p^D(x_{\sigma}^M)$ as follows:

exclude c=0 because in this case an ad valorem subsidy could not affect the monopoly solution.

i) The quantity effects are

$$\begin{split} \frac{dx_t^M}{d\gamma_t} &= -\frac{1}{\frac{\partial p^D(x_t^M)}{\partial x}(2-\rho)},\\ \frac{dx_s^M}{d\gamma_s} &= -\frac{c}{p_s^M} \frac{1}{\frac{\partial p^D(x_s^M)}{\partial x}(2-\rho)}, \end{split}$$

and

$$\frac{dx_r^M}{d\gamma_r} = -\frac{\overline{p} - c}{\overline{p} - p_r^M} \frac{1}{\frac{\partial p^D(x_r^M)}{\partial x} (2 - \rho)}'$$

and their ordering is

$$0 < \frac{dx_s^M}{d\gamma_s} < \frac{dx_t^M}{d\gamma_t} < \frac{dx_r^M}{d\gamma_r}.$$

ii) The pass-through rates of the consumer price are

$$\frac{dp^D(x_\sigma^M)}{d\gamma_\sigma} = \frac{\partial p^D(x_\sigma^M)}{\partial x} \frac{dx_\sigma^M}{d\gamma_\sigma} \text{ for } \sigma \in \Sigma,$$

and their ordering is

$$\frac{dp^D(x_r^M)}{d\gamma_r} < \frac{dp^D(x_t^M)}{d\gamma_t} < \frac{dp^D(x_s^M)}{d\gamma_s} < 0.$$

iii) The pass-through rates of the market price are

$$\begin{split} \frac{dp_t^M}{d\gamma_t} &= 1 - \frac{1}{2-\rho},\\ \frac{dp_s^M}{d\gamma_s} &= \frac{1}{1-s} \left(1 - \frac{c}{p_s^M} \frac{1}{2-\rho}\right),\\ \frac{dp_r^M}{d\gamma_r} &= \frac{1}{1+r} \left(1 - \frac{\overline{p}-c}{\overline{p}-p_r^M} \frac{1}{2-\rho}\right), \end{split}$$

and their ordering is

$$\frac{dp_r^M}{d\gamma_r} < \frac{dp_t^M}{d\gamma_t} < \frac{dp_s^M}{d\gamma_s},$$

with
$$\frac{dp_t^M}{d\gamma_t} \stackrel{\geq}{=} 0 \Leftrightarrow \rho \stackrel{\leq}{=} 1$$
.

Notably, part iii) of Proposition (5) highlights the fact that overshifting can occur for inverse demand curvatures of ρ < 1 under the r-subsidy but not under the other two subsidies. We next turn to the comparison of the three subsidies under the equal-relief assumption. Using (17), (18), and (19), Proposition (5) implies the following comparison of the three subsidies.

Corollary 4 (Equal relief in a monopoly). *Suppose the strictly positive subsidies* t, s, and r offer equal relief to consumers according to (14). Then,

$$p_s^M > p_t^M > p_r^M > p^M$$
 ,

$$x_r^M > x_t^M > x_s^M > x^M,$$

and

$$p^{M} > p^{D}(x_{s}^{M}) > p^{D}(x_{t}^{M}) > p^{D}(x_{r}^{M}),$$

so that the r-subsidy leads to the largest output effect and thus is preferred from an equal-relief perspective.

The preceding results show that the *s*-subsidy has especially unattractive consequences: it pushes the market price up more than any other subsidy while expanding output by less. These effects are illustrated in Figure 3, which shows how demand, then marginal revenue and then also the equilibrium shifts under the difference subsidies.

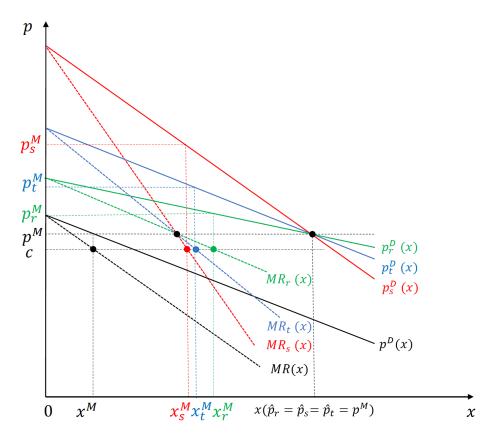


Figure 3: Equal-relief subsidies under monopoly for linear demand

Notes: The subsidies are set up in such a way that, for a fixed price, they provide equal relief—that is, the consumer pays the same fraction of the market price in all three cases. Consequently, the shifted demand curves all pass through the same point, which lies on a horizontal line through p^M . The t-subsidy results in a parallel upward shift of the original demand curve (blue line). The s-subsidy rotates the demand curve clockwise (red line), while the r-subsidy rotates it counterclockwise (green line). The intersections of the respective marginal revenue curves MR_r , MR_s , and MR_t with the marginal cost curve give the equilibria under each type of subsidy. All subsidies lead to an increase in the monopoly price and quantity, with the s-subsidy causing the largest price and smallest quantity increase and the r-subsidy causing the smallest price and the largest quantity increase.

5.3 Cost Effectiveness

A cost-effectiveness comparison shows that the r-subsidy is the most efficient, followed by the t-subsidy, with the s-subsidy being the least cost effective.

Proposition 6 (Cost effectiveness in a monopoly). Suppose strictly positive subsidies $(t, s, r) \in \Theta^M$, so that all subsidies implement the same monopoly quantity x_σ^M . Then, the monopoly solution (p_σ^M, x_σ^M) satisfies

$$0 < \phi_r^M < \phi_t^M < \phi_s^M$$

with $\phi_r^M = \frac{\overline{p} - p_r^M}{\overline{p} - c} \phi_t^M$ and $\phi_s^M = \frac{p_s^M}{c} \phi_t^M$; i.e., the r-subsidy is more cost effective than the t-subsidy, which in turn is more cost effective than the s-subsidy.

The intuition for this result follows directly from the quantity effects of the subsidies (see part i) of Proposition 5). For any quantity $x \ge x^M$, a marginal change of the normalized values of the subsidies leads to the largest output effect under the r-subsidy, implying that a certain target output level can be achieved with the lowest fiscal costs under the r-subsidy (see Figure 4 for an illustration).

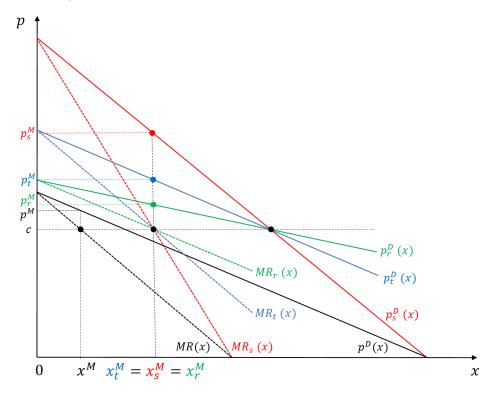


Figure 4: The cost-effectiveness criterion under monopoly for linear demand

Notes: The subsidies are designed such that all subsidies achieve the same quantity: $x_t^M = x_s^M = x_r^M$. At this quantity, each subsidy's marginal revenue curve must intersect the marginal cost curve. Given linear demand functions, the marginal revenue curves have twice the slope of the corresponding demand curves. Consequently, all demand curves must intersect the marginal cost curve at the same point to the right of the intersection point of the marginal revenue curves. The ranking of market prices for the different subsidies is then determined by the size of the wedge between the market price and the consumer price, which decreases in quantity for the s-subsidy, increases for the r-subsidy and stays constant for the t-subsidy. The higher the market price the less cost effective the subsidy.

5.4 Summary

We have shown that the core result from the previous section—that the ad valorem subsidy has a particularly strong upward pressure on market price under perfect competition—also holds if the supplier has market power. Here, this does not go along with a particularly high output expansion, though. Instead, the monopoly case highlights the attractiveness of the inversely related subsidy, which mirrors existing results of the taxation literature on the relative merits of an ad valorem tax. The inversely related subsidy implies that the wedge between the supply and the consumer price steadily grows as the subsidy per unit increases along the inverse demand curve. This feature tends to constrain the monopolist's margin, which in turn implies the strongest output expansion from an equal-relief perspective. Consequently, our monopoly analysis of the subsidies uncovers a strong case for the inversely related subsidy, suggesting that policymakers should consider this subsidy much more often than they have done so far.

6 Conclusion

In this paper, we provide a comprehensive analysis of subsidy-based market interventions and their price and quantity effects. While we have assumed that the subsidy is directly paid to the consumer, the analysis remains essentially unchanged when the subsidy is paid to the supplier. In that case, the resulting market prices coincide exactly with our consumer price results.

In line with the well-known welfare superiority of ad valorem taxation under monopolistic supply conditions (see, e.g., Auerbach and Hines, 2002 for an overview article or, e.g., Fischer *et al.*, 2024 for a recent empirical support), our analysis shows from different angles that ad valorem subsidies tend to have particularly strong price-increasing effects. Still, when supply is perfectly competitive, then the price-raising effect of an ad valorem subsidy also induces the largest output expansion under the equal-relief criterion; thus, creating a case for using the ad valorem subsidy when the policy goal is to achieve a maximal adoption level of the target good.

In contrast to the ad valorem subsidy, the inversely related subsidy exhibits particularly desirable properties independent of the market structure, from a cost-effectiveness perspective. It, therefore, enables an expansion of the output of the target good at relatively low fiscal cost. Our positive results on the inversely related subsidy can be deducted from existing results concerning the combination of a unit subsidy and an ad valorem tax, but the inversely related subsidy offers a new way of implementing such a combination.

Overall, the ad valorem subsidy has the strongest price-increasing effect, while the inversely related subsidy has the strongest price-dampening effect. Taking an equal-relief approach, the market structure then becomes critical for the subsidies' quantity effects. The price-increasing effect of the ad valorem subsidy leads to the highest output expansion under perfect competition. In contrast, it is the inversely related subsidy that induces the strongest output expansion under conditions of monopoly power.

Whether these theoretical results hold in practice remains an empirical question. A potential downside of the inversely related subsidy is that it may facilitate collusion if the reference price serves as a focal point for coordination. While such focal point collusion has been exper-

imentally documented in other contexts (e.g., Hunold and Werner, forthcoming), we have no empirical evidence supporting this concern.

In practice, ad valorem subsidies that—according to our analysis tend to drive up prices—are quite common—not only to support specific target goods but also, for example, when the federal government provides funds to municipalities to facilitate structural change (Bundesministerium für Wirtschaft und Klimaschutz, 2024). Such subsidies could help to overcome moral hazard on the recipient's side—recipients retain a direct stake in their decisions—but could yield the unintended consequence that suppliers raise their prices in response.

One further current example of an ad valorem subsidy being considered in a competitive market—where, according to our analysis, such subsidies have strong output-expanding effects—concerns the electricity market, with firms rather than households as recipients. In Germany, the so-called "industry electricity price" is under discussion¹², whereby firms would receive a subsidy covering half the wholesale electricity price for half of their consumption. This effectively amounts to a price subsidy of one quarter. Our analysis shows that, from an equal-relief perspective, this approach is particularly well-suited to counteracting deindustrialization—a major concern in Germany amid soaring energy prices—as it produces the largest output effect in competitive markets like electricity.

Ad valorem subsidies have also been implemented in variants not covered in this paper. One such variant includes a cap on the subsidized price. In this case, the subsidy functions as an ad valorem subsidy up to the threshold price, beyond which it effectively becomes a per-unit subsidy. Three types of equilibria can then arise: First, if the equilibrium market price is strictly below the cap, the subsidy operates as a pure ad valorem subsidy, and our corresponding analysis applies. Second, if the equilibrium price is strictly above the cap, the subsidy is equivalent to a per-unit subsidy, to which our earlier analysis also applies. Third, a boundary case may occur in which the equilibrium price is exactly at the cap. This outcome, which is anecdotally quite common, may stem from behavioral factors. This question is left for future research.

Appendix

Here we provide the missing proofs.

Proof of Propositions 1 and 2 (Pass-through rates under perfect competition with horizontal and normal supply). We first derive the pass-through results for each subsidy separately and then turn to the comparison of those results as stated in the two propositions.

Case 1 (t-subsidy). Assume a t-subsidy with $t \geq 0$. The equilibrium quantity x_t^* fulfills the equilibrium condition

$$p^{D}(x_{t}^{*}) + t - C'(x_{t}^{*}) = 0, (30)$$

where $p^D(x_t^*)$ is the consumer price and $p_t^* = p^D(x_t^*) + t$ is the market price suppliers receive. Equation (30) defines the implicit function $x_t^* = x_t^*(t)$. Using the implicit function theorem, we

 $^{^{12}} See \\ https://www.handelsblatt.com/politik/deutschland/bericht-konzept-fuer-industriestrompreis-bringt-vier-milliarden-euro-entlastung/100139718.html.$

get

$$\frac{dx_t^*}{dt} = -\frac{1}{\frac{\partial p^D(x_t^*)}{\partial x} - C''(x_t^*)} > 0.$$

In case of the t-subsidy a marginal change of the value of t is equal to a marginal change of the normalized subsidy γ_t ; i.e., we have $dt = d\gamma_t$. The quantity effect and the pass-through rates of the consumer price and the market price induced by a marginal change $d\gamma_t$, are the given by

$$\frac{dx_t^*}{d\gamma_t} = \frac{dx_t^*}{dt} = -\frac{1}{\frac{\partial p^D(x_t^*)}{\partial x} - C''(x_t^*)} > 0, \tag{31}$$

$$\frac{dp^D(x_t^*)}{d\gamma_t} = \frac{\partial p^D(x_t^*)}{\partial x} \frac{dx_t^*}{dt} = -\frac{\partial p^D(x_t^*)}{\partial x} \frac{1}{\frac{\partial p^D(x_t^*)}{\partial x} - C''(x_t^*)} \in [-1, 0), \tag{32}$$

$$\frac{dp_t^*}{d\gamma_t} = 1 + \frac{dp^D(x_t^*)}{dt} = 1 - \frac{\frac{\partial p^D(x_t^*)}{\partial x}}{\frac{\partial p^D(x_t^*)}{\partial x} - C''(x_t^*)} \in [0, 1).$$
(33)

Case 2 (s-subsidy). Assume an s-subsidy with $s \in [0, 1)$. The equilibrium quantity x_s^* fulfills the equilibrium condition

$$\frac{p^D(x_s^*)}{1-s} - C'(x_s^*) = 0, (34)$$

where $p^D(x_s^*)$ is the consumer price and $p_s^* = \frac{p^D(x_s^*)}{1-s}$ is the market price suppliers receive. Equation (34) defines the implicit function $x_s^* = x_s^*(s)$. Applying the implicit function theorem, we get

$$\frac{dx_s^*}{ds} = -\underbrace{\frac{p^D(x_s^*)}{1-s}}_{=p_s^*} \underbrace{\frac{1}{\partial p^D(x_s^*)} - (1-s)C''(x_s^*)}_{=p_s^*} > 0.$$

In this case, a marginal change of the value of s induces a marginal change of the normalized subsidy γ_s by $d\gamma_s=p_s^*ds$. The quantity effect and the pass-through rates of the consumer price and the market price induced by a marginal change of the normalized subsidy, $d\gamma_s$, are then given by

$$\frac{dx_s^*}{d\gamma_s} = \frac{1}{p_s^*} \frac{dx_s^*}{ds} = -\frac{1}{\frac{\partial p^D(x_s^*)}{\partial x} - (1-s)C''(x_s^*)} > 0, \tag{35}$$

$$\frac{dp^{D}(x_{s}^{*})}{d\gamma_{s}} = \frac{1}{p_{s}^{*}} \frac{dp^{D}(x_{s}^{*})}{ds} = -\frac{\frac{\partial p^{D}(x_{s}^{*})}{\partial x}}{\frac{\partial p^{D}(x_{s}^{*})}{\partial x} - (1 - s)C''(x_{s}^{*})} < 0, \text{ and}$$
(36)

$$\frac{dp_s^*}{d\gamma_s} = \frac{1}{p_s^*} \frac{dp_s^*}{ds} = \frac{1}{p_s^*} \left(\frac{p^D(x_s^*)}{(1-s)^2} + \frac{1}{1-s} \frac{\partial p^D(x_s^*)}{\partial x} \frac{dx_s^*}{ds} \right)$$

$$= \frac{1}{1-s} \left(1 - \frac{\frac{\partial p^{D}(x_{s}^{*})}{\partial x}}{\frac{\partial p^{D}(x_{s}^{*})}{\partial x} - (1-s)C''(x_{s}^{*})} \right) > 0.$$
 (37)

Case 3 (r-subsidy). Assume an r-subsidy with $r \in [0, 1)$. The equilibrium quantity x_r^* fulfills the equilibrium condition

$$\frac{p^D(x_r^*) + \overline{p}r}{1+r} - C'(x_r^*) = 0, (38)$$

where $p^D(x_r^*)$ is the consumer price and $p_r^* = \frac{p^D(x_r^*) + \bar{p}r}{1+r}$ is the market price suppliers receive. Equation (38) defines the implicit function $x_r^* = x_r^*(r)$. Applying the implicit function theorem, we get

$$\frac{dx_r^*}{dr} = -\underbrace{\frac{\overline{p} - p^D(x_r^*)}{1 + r}}_{=\overline{p} - p_r^*} \underbrace{\frac{1}{\frac{\partial p^D(x_r^*)}{\partial x} - (1 + r)C''(x_r^*)}}_{} > 0.$$

In this case, a marginal change of the value of r induces a marginal change of the normalized subsidy γ_r by $d\gamma_r = (\bar{p} - p_r^*)dr$. The quantity effect and the pass-through rates of the consumer price and the market price induced by a marginal change of the normalized subsidy, $d\gamma_r$, are the given by

$$\frac{dx_r^*}{d\gamma_r} = \frac{1}{\overline{p} - p_r^*} \frac{dx_r^*}{dr} = -\frac{1}{\frac{\partial p^D(x_r^*)}{\partial r} - (1+r)C''(x_r^*)} > 0, \tag{39}$$

$$\frac{dp^D(x_r^*)}{d\gamma_r} = \frac{1}{\overline{p} - p_r^*} \frac{dp^D(x_r^*)}{dr} = -\frac{\frac{\partial p^D(x_r^*)}{\partial x}}{\frac{\partial p^D(x_r^*)}{\partial x} - (1+r)C''(x_r^*)} < 0, \text{ and}$$
(40)

$$\frac{dp_r^*}{d\gamma_r} = \frac{1}{\overline{p} - p_r^*} \frac{dp^D(x_r^*)}{\partial r} = \frac{1}{\overline{p} - p_r^*} \left(-\frac{p^D(x_r^*) + \overline{p}r}{(1+r)^2} + \frac{\frac{dp^D(x_r^*)}{dr} + \overline{p}}{1+r} \right)
= \frac{1}{1+r} \left(1 - \frac{\frac{\partial p^D(x_r^*)}{\partial x}}{\frac{\partial p^D(x_r^*)}{\partial x} - (1-s)C''(x_r^*)} \right) \in [0, 1).$$
(41)

Comparison of results. Note first that the subsidies' quantity effects and their consumer price and market price pass-through rates are all equal for t=s=r=0; i.e., if $x_\sigma^*=x^*$ holds for all $\sigma\in\Sigma$. If, to the contrary $x_t^*=x_s^*=x_r^*>x^*$ holds (which requires t,r,s>0 to holds as well), we have to distinguish between the horizontal supply (Proposition 1) and the normal supply (Proposition 2) cases. In the former case, industry marginal costs are constant, so that C''(x)=0 holds for all $x\geq0$. Using this, we get that the subsidies' quantity effects (31), (35), and (39) are all the same as stated in part i) of Proposition 1. The subsidies' consumer price pass-through rates (32), (36), and (40) are also all the same with value -1. It then follows that the market price pass-through rates (33), (37), and (33) must be all zero.

In case of a normal supply, C'''(x) > 0 holds for all x. The subsidies' quantity effects, $\frac{dx_\sigma^*}{d\gamma_\sigma}$, are given by (31), (35), and (39) and their ordering as stated in part i) of Proposition 2 follows from r, s, t > 0. The subsidies' consumer price pass-through rates (see (32), (36), and (40)) fulfill $\frac{dp^D(x_\sigma^*)}{d\gamma_\sigma} = \frac{\partial p^D(x_\sigma^*)}{\partial \gamma_\sigma} \frac{dx_\sigma^*}{d\gamma_\sigma}$, so that their ordering —as stated in part ii) of Proposition 2—follows directly from the ordering of the quantity effects. The pass-through rates of the market prices are given by (33), (37), and (41), and their ordering as stated in part iii) of Proposition 2 follows from r, s, t > 0. Finally, the pass-through rates of the market prices under the t- and the t-subsidy can never be larger than 1, whereas the pass-through of the market price under the t-subsidy could be larger than one.

Proof of Proposition 3 (Pass-through under perfect competition with vertical supply). Suppose the supply quantity is fixed at $x^* > 0$ for all p. The demand function x(p) follows from (1).

In benchmark case without a subsidy, the market equilibrium condition can be written as

$$x(p^*) - x^* = 0,$$

which gives the equilibrium market price p^* . As the supply quantity is fixed at x^* , the subsidies' quantity effects must all be zero. For each subsidy, we next analyze the pass-through rates of the prices and then compare them.

Case 1 (t-subsidy). For $t \ge 0$, the equilibrium market price p_t^* follows from the market clearing condition

$$x(\underbrace{p_t^* - t}) - x^* = 0, \tag{42}$$

where $\hat{p}_t = p_t^* - t$ is the consumer price under the t-subsidy. Equation (42) defines the implicit function $p_t^*(t)$. Applying the implicit function theorem to (42), we get the pass-through rate of the market price

$$\frac{dp_t^*}{d\gamma_t} = \frac{dp_t^*}{dt} = 1,$$

which implies a pass-through rate of the consumer price, \hat{p}_t , of

$$\frac{d\widehat{p}_t}{d\gamma_t} = 0.$$

Case 2 (s-subsidy). For $s \in [0,1)$, the equilibrium market price p_s^* follows from the market clearing condition

$$x(\underbrace{p_s^*(1-s)}_{=\widehat{p}_s}) - x^* = 0, \tag{43}$$

where $\hat{p}_s = p_s^*(1-s)$ is the consumer price under the *s*-subsidy. Equation (43) defines the implicit function $p_s^*(s)$. Applying the implicit function theorem to (43), we get the pass-through rate of the market price

$$\frac{dp_s^*}{d\gamma_s} = \frac{1}{p_s^*} \frac{dp_s^*}{ds} = -\frac{1}{p_s^*} \frac{-\frac{\partial x}{\partial p} p_s^*}{\frac{\partial x}{\partial p} (1-s)} = \frac{1}{1-s},$$

which implies a pass-through rate of the consumer price, \hat{p}_s , of

$$\frac{d\widehat{p}_s}{d\gamma_s} = \frac{1}{p_s^*} \frac{d\widehat{p}_s}{ds} = \frac{1}{p_s^*} \left(\frac{dp_s^*}{ds} (1-s) - p_s^* \right) = \frac{1}{p_s^*} \left(\frac{-\frac{\partial x}{\partial p} p_s^*}{\frac{\partial x}{\partial p} (1-s)} (1-s) - p_s^* \right) = 0.$$

Case 2 (r-subsidy). For $r \in [0,1)$, the equilibrium market price p_r^* follows from the market clearing condition

$$x(\underbrace{p_r^* - (\overline{p} - p_r^*)r}_{=\widehat{p}_r}) - x^* = 0, \tag{44}$$

where $\hat{p}_r = p_r^* - (\bar{p} - p_r^*)r$ is the consumer price under the r-subsidy. Equation (44) defines the implicit function $p_r^*(r)$. Applying the implicit function theorem to (44), we get the pass-through

rate of the market price

$$\frac{dp_r^*}{d\gamma_r} = \frac{1}{\overline{p} - p_r^*} \frac{dp_r^*}{dr} = -\frac{1}{\overline{p} - p_r^*} \frac{-\frac{\partial x}{\partial p} (\overline{p} - p_r^*)}{\frac{\partial x}{\partial p} (1 + r)} = \frac{1}{1 + r},$$

which implies a pass-through rate of the consumer price, \hat{p}_r , of

$$\frac{d\widehat{p}_r}{d\gamma_r} = \frac{1}{\overline{p} - p_r^*} \frac{d\widehat{p}_r}{dr} = \frac{1}{\overline{p} - p_r^*} \left((1+r) \frac{dp_r^*}{dr} - (\overline{p} - p_r^*) \right) = 0.$$

The ordering of the subsidies' pass-through rates of the market prices—as stated in *part iii*) of the proposition—follows from r, s, t > 0.

Proof of Proposition 4 (Cost Effectiveness Under Perfect Competition). The values of t, s, and r which implement the same allocation $(p_{\sigma}^*, x_{\sigma}^*)$ follow from the equilibrium conditions (24) from which we get the following relations

$$u'(x_{\sigma}^*) + t = \frac{u'(x_{\sigma}^*)}{1-s} = \frac{u'(x_{\sigma}^*) + \overline{p}r}{1+r} = C'(x_{\sigma}^*) = p_{\sigma}^*.$$

We then get

$$t=p_\sigma^*-u'(x_\sigma^*)$$
, $s=rac{p_\sigma^*-u'(x_\sigma^*)}{p_\sigma^*}$, and $r=rac{p_\sigma^*-u'(x_\sigma^*)}{\overline{p}-p_\sigma^*}$,

and substituting this into (20), we get the result stated in the proposition.

Proof of Proposition 5 (**Pass-through in a monopoly**). We first derive the pass-through results for each subsidy separately and then turn to the comparison of those results as stated in the proposition.

Case 1 (t-subsidy). Assume a t-subsidy with $t \ge 0$. The monopolist solves

$$\max_{x} \pi = (\underbrace{p^{D}(x) + t}_{=p_{*}^{M}(x)} - c)x,$$

where $p_t^M(x) = p^D(x) + t$ is the monopoly price under the t-subsidy as a function of x. The optimal monopoly quantity x_t^M fulfills the first-order condition

$$\frac{\partial \pi}{\partial x} = p^{D}(x) + \frac{\partial p^{D}(x)}{\partial x}x + t - c = 0, \tag{45}$$

which defines the implicit function $x_t^M = x_t^M(t)$. Using the implicit function theorem, we get

$$\frac{dx_t^M}{dt} = -\frac{1}{\frac{\partial p^D(x_t^M)}{\partial x}(2-\rho)} > 0.$$

In case of the t-subsidy, a marginal change of the value of t is equal to a marginal change of the normalized subsidy γ_t ; i.e., we have $dt = d\gamma_t$. The quantity effect and the pass-through rates of

the consumer price and the market price induced by a marginal change $d\gamma_t$, are then given by

$$\frac{dx_t^M}{d\gamma_t} = \frac{dx_t^M}{dt} = -\frac{1}{\frac{\partial p^D(x_t^M)}{\partial x}(2-\rho)} > 0, \tag{46}$$

$$\frac{dp^D(x_t^M)}{d\gamma_t} = \frac{\partial p^D(x_t^M)}{\partial x} \frac{dx_t^S}{dt} = -\frac{1}{2-\rho} < 0, \text{ and}$$
 (47)

$$\frac{dp_t^M}{d\gamma_t} = 1 + \frac{\partial p^D(x_t^M)}{\partial x} \frac{dx_t^{\dot{i}}}{dt} = 1 - \frac{1}{2 - \rho}, \text{ with } \frac{dp_t^M}{d\gamma_t} \stackrel{\geq}{<} 0 \Leftrightarrow \rho \stackrel{\leq}{>} 1.$$
 (48)

Thus, there is overshifting under the t-subsidy with $\frac{dp_t^M}{d\gamma_t} < 0$ if $\rho > 1$. *Case 2 (s-subsidy)*. Assume an s-subsidy with $s \in [0,1)$. The monopolist solves

$$\max_{x} \pi = (\underbrace{\frac{p^{D}(x)}{1-s}}_{=p_{s}^{M}(x)} - c)x,$$

where $p_s^M(x) = \frac{p^D(x)}{1-s}$ is the monopoly price under the s-subsidy as a function of x. The optimal monopoly quantity x_s^M fulfills the first-order condition

$$\frac{\partial \pi}{\partial x} = \frac{1}{1 - s} \left(p^D(x) + \frac{\partial p^D(x)}{\partial x} x \right) - c = 0, \tag{49}$$

which defines the implicit function $x_s^M(s)$. Applying the implicit function theorem and using the first-order condition (49), we get

$$\frac{dx_s^M}{ds} = -\frac{c}{\frac{\partial p^D(x_s^M)}{\partial x}(2-\rho)} > 0.$$

In this case, a marginal change of the value of s induces a marginal change of the normalized subsidy γ_s by $d\gamma_s = p_s^M ds$. The quantity effect and the pass-through rates of the consumer price and the market price induced by a marginal change of the normalized subsidy, $d\gamma_s$, are then given by

$$\frac{dx_s^M}{d\gamma_s} = \frac{1}{p_s^M} \frac{dx_s^M}{ds} = -\underbrace{\frac{c}{p_s^M}}_{0} \underbrace{\frac{\partial p^D(x_s^M)}{\partial x}(2-\rho)}_{0} > 0, \tag{50}$$

$$\frac{dp^{D}(x_{s}^{M})}{d\gamma_{s}} = \frac{1}{p_{s}^{M}} \frac{dp^{D}(x_{s}^{M})}{ds} = -\frac{c}{p_{s}^{M}} \frac{1}{2 - \rho} < 0, \text{ and}$$
 (51)

$$\frac{dp_s^M}{d\gamma_s} = \frac{1}{p_s^M} \frac{dp_s^M}{ds} = \frac{1}{p_s^M} \left(\frac{p^D(x_s^M)}{(1-s)^2} + \frac{1}{1-s} \frac{\partial p^D(x_s^M)}{\partial x} \frac{dx_s^M}{ds} \right)
= \frac{1}{1-s} \left(1 - \frac{c}{p_s^M} \frac{1}{2-\rho} \right).$$
(52)

Case 3 (r-subsidy). Assume an r-subsidy with $r \in [0,1)$. The monopolist solves

$$\max_{x} \pi = (\underbrace{\frac{p^{D}(x) + \overline{p}r}{1 + r}}_{=p_{x}^{M}(x)} - c)x,$$

where $p_r^M(x) = \frac{p^D(x) + \overline{p}r}{1+r}$ is the monopoly price under the r-subsidy as a function of x. The optimal monopoly quantity x_r^M fulfills the first-order condition

$$\frac{\partial \pi}{\partial x} = \frac{1}{1+r} \left(p^D(x) + \frac{\partial p^D(x)}{\partial x} x + \overline{p}r \right) - c = 0.$$
 (53)

Equation (38) defines the implicit function $x_r^M = x_r^M(r)$. Applying the implicit function theorem and using the first-order condition (49), we get

$$\frac{dx_r^M}{dr} = -\frac{\overline{p} - c}{\frac{\partial p^D(x_r^M)}{\partial x}(2 - \rho)} > 0.$$

In this case, a marginal change of the value of r induces a marginal change of the normalized subsidy γ_r by $d\gamma_r = (\overline{p} - p_r^M)dr$. The quantity effect and the pass-through rates of the consumer price and the market price induced by a marginal change of the normalized subsidy, $d\gamma_r$, are then given by

$$\frac{dx_r^M}{d\gamma_r} = \frac{1}{\overline{p} - p_r^M} \frac{dx_r^M}{dr} = -\underbrace{\frac{\overline{p} - c}{\overline{p} - p_r^M}}_{>1} \frac{1}{\frac{\partial p^D(x_r^M)}{\partial x}(2 - \rho)} > 0, \tag{54}$$

$$\frac{dp^{D}(x_{r}^{M})}{d\gamma_{r}} = \frac{1}{\overline{p} - p_{r}^{M}} \frac{dp^{D}(x_{r}^{M})}{dr} = -\frac{\overline{p} - c}{\overline{p} - p_{r}^{M}} \frac{1}{2 - \rho} < 0, \text{ and}$$

$$\frac{dp_{r}^{M}}{d\gamma_{r}} = \frac{1}{\overline{p} - p_{r}^{M}} \frac{dp_{r}^{M}}{dr} = \frac{1}{\overline{p} - p_{r}^{M}} \left(-\frac{p^{D}(x_{r}^{M}) + \overline{p}r}{(1 + r)^{2}} + \frac{1}{1 + r} \left(\frac{\partial p^{D}(x_{r}^{M})}{\partial x} + \overline{p} \right) \right)$$

$$= \frac{1}{1 + r} \left(1 - \frac{\overline{p} - c}{\overline{p} - p_{r}^{M}} \frac{1}{2 - \rho} \right).$$
(55)

Comparison of results. The normalized subsidies' quantity effects, $\frac{dx_\sigma^M}{d\gamma_\sigma}$, are given by (46), (50), and (54) and their ordering as stated in part~i) of the proposition follows from r,s,t>0 and the fact that $\frac{c}{p_s^M}<1$ and $\frac{\bar{p}-c}{\bar{p}-p_r^M}>1$ must hold as well. The subsidies' consumer price pass-through rates (see (47), (51), and (55)) fulfill $\frac{dp^D(x_\sigma^M)}{d\gamma_\sigma}=\frac{\partial p^D(x_\sigma^M)}{\partial \gamma_\sigma}\frac{dx_\sigma^M}{d\gamma_\sigma}$, so that their ordering—as stated in part~ii) of Proposition 5—follows directly from the ordering of quantity effects. The pass-through rates of the market prices, $\frac{dp_\sigma^M}{d\gamma_\sigma}$, are given by (48), (52), and (54) and the ordering—as stated in part~iii) of Proposition 5 follows from r,s,t>0 and the fact that $\frac{c}{p_s^M}<1$ and $\frac{\bar{p}-c}{\bar{p}-p_r^M}>1$ must hold as well.

Proof of Proposition 6 (Cost effectiveness under monopoly).

Suppose a triple of subsidies $\theta^M > (0,0,0)$ so that the equilibrium outputs are all the same with $x_t^M = x_s^M = x_r^M > x^M$. For the per-unit subsidy, the value of t—and hence, the value of

 ϕ_t^M —follows from the first-order condition (45); this gives

$$\phi_t^M = -\left(p^D(x_t^M) + \frac{\partial p^D(x_t^M)}{\partial x} x_t^M - c\right). \tag{57}$$

Under the ad valorem subsidy, the value of s follows from the first-order condition (49) and is given by

$$s = -\frac{1}{c} \left(p^D(x_s^M) + \frac{\partial p^D(x_s^M)}{\partial x} x_s^M - c \right). \tag{58}$$

In this case, the per-unit subsidy is $\phi_s^M = p_s^M s$ (see (20)) and substituting (58), the per-unit subsidy is then given by

$$\phi_s^M = -\frac{p_s^M}{c} \left(p^D(x_s^M) + \frac{\partial p^D(x_s^M)}{\partial x} x_s^M - c \right). \tag{59}$$

Under the inversely-related subsidy, the value of r follows from the first-order condition (53) and is given by

$$r = -\frac{1}{\overline{p} - c} \left(p^D(x_r^M) + \frac{\partial p^D(x_r^M)}{\partial x} x_r^M - c \right). \tag{60}$$

In this case, the per-unit subsidy is $\phi_r^M=(\overline{p}-p_r^M)r$ (see (20)) and substituting (60), the per-unit subsidy is then given by

$$\phi_r^M = -\frac{\overline{p} - p_r^M}{\overline{p} - c} \left(p^D(x_r^M) + \frac{\partial p^D(x_r^M)}{\partial x} x_r^M - c \right). \tag{61}$$

Substituting (57) into (59) and (61), we get

$$\phi^M_s = \underbrace{\frac{p^M_s}{c}}_{>1} \phi^M_t$$
 and $\phi^M_r = \underbrace{\overline{p} - p^M_r}_{<1} \phi^M_t$, respectively,

so that the ordering $\phi^M_s > \phi^M_t > \phi^M_r$ follows from noticing that $p^M_s > c$ and $p^M_r > c$ must obviously hold in the respective monopoly solutions.

References

Anderson, S. P., De Palma, A. and Kreider, B. (2001). Tax incidence in differentiated product oligopoly. *Journal of Public Economics*, **81** (2), 173–192.

Auerbach, A. J. and Hines, J., James R. (2002). Taxation and economic efficiency. In A. J. Auerbach and M. Feldstein (eds.), *Handbook of Public Economics*, vol. 3, Amsterdam: Elsevier, pp. 1347–1421.

BISHOP, R. L. (1968). The effects of specific and ad valorem taxes. *The Quarterly Journal of Economics*, **82** (2), 198–218.

- Bundesministerium für Wirtschaft und Klimaschutz (2024). Modellvorhaben "unternehmen revier". https://www.foerderdatenbank.de/FDB/Content/DE/Foerderprogramm/Bund/BMWi/modellvorhaben-unternehmen-revier.html, Förderprogramm zur Unterstützung des Strukturwandels in deutschen Braunkohleregionen.
- Corsello, F. and Ercolani, V. (2024). The Role of the Superbonus in the Growth of Italian Construction Costs. *Bank of Italy Occasional Papers*, **903**.
- Dal Bó, E., Dal Bó, P. and Eyster, E. (2018). The demand for bad policy when voters underappreciate equilibrium effects. *The Review of Economic Studies*, **85** (2), 964–998.
- DE GROOTE, O. and VERBOVEN, F. (2019). Subsidies and time discounting in new technology adoption: Evidence from solar photovoltaic systems. *American Economic Review*, **109** (6), 2137–2172.
- Delipalla, S. and Keen, M. (1992). The comparison between ad valorem and specific taxation under imperfect competition. *Journal of Public Economics*, **49** (3), 351–367.
- Deutsche Umwelthilfe (2025). Forderungspapier für einen Wärmepumpenhochlauf. Policy Paper, Deutsche Umwelthilfe, Berlin, last accessed: April 3, 2025.
- DILLÉN, M. (1995). Corrective tax and subsidy policies in economies with Bertrand competition. *Journal of Public Economics*, **58** (2), 267–282.
- European Heat Pump Association (2023). Subsidies for Residential Heat Pumps in Europe. Report, European Heat Pump Association, Brussels, last accessed: April 3, 2025.
- FISCHER, K., MARTIN, S. and Schmidt-Dengler, P. (2024). *Indirect Taxation in Consumer Search Markets: The Case of Retail Fuel*. CEPR Discussion Paper 19095, Centre for Economic Policy Research.
- Gaudin, G. and White, A. (2014). Unit vs. ad valorem taxes under revenue maximization. *SSRN Working Paper*.
- Hamilton, S. F. (1999). Tax incidence under oligopoly: a comparison of policy approaches. *Journal of Public Economics*, **71** (2), 233–245.
- Hunold, M. and Werner, T. (forthcoming). Algorithmic price recommendations and collusion: Experimental evidence. *Experimental Economics*, forthcoming.
- Kiso, T. (2022). A subsidy that is inversely related to the product price. *The Economic Journal*, **132** (646), 2173–2206.
- Mas-Colell, A., Whinston, M. D. and Green, J. R. (1995). *Microeconomic Theory*. New York: Oxford University Press, 1st edn.
- Myles, G. D. (1995). Public Economics. Cambridge University Press.

- (1996). Imperfect competition and the optimal combination of ad valorem and specific taxation. *International Tax and Public Finance*, **3**, 29–44.
- Nunnari, S., Proto, E. and Rustichini, A. (2024). *Cognitive Abilities and the Demand for Bad Policy*. Cesifo working paper, CESifo.
- Sмітн, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations*. London: W. Strahan and T. Cadell.
- Suits, D. B. and Musgrave, R. A. (1953). Ad valorem and unit taxes compared. *The Quarterly Journal of Economics*, **67** (4), 598–604.
- Wicksell, K. (1896). Finanztheoretische Untersuchungen: Nebst Darstellung und Kritik des Steuerwesens Schwedens. Jena, Germany: G. Fischer.

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