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Working Paper

Stochastic Discount Factor Approach to International Risk-Sharing: Evidence from Fixed Exchange Rate Episodes

Discussion Papers Series, No. 07-33

Provided in Cooperation with:

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Suggested Citation: Hadzi-Vaskov, M.; Kool, Clemens J. M. (2007) : Stochastic Discount Factor Approach to International Risk-Sharing: Evidence from Fixed Exchange Rate Episodes, Discussion Papers Series, No. 07-33, Utrecht University, Utrecht School of Economics, Tjalling C. Koopmans Research Institute, Utrecht

This Version is available at:

<https://hdl.handle.net/10419/322745>

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Stochastic Discount Factor Approach to International Risk-Sharing: Evidence from Fixed Exchange Rate Episodes

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March 2008

Abstract

This paper presents evidence of the stochastic discount factor approach to international risk-sharing applied to fixed exchange rate regimes. We calculate risk-sharing indices for two episodes of fixed or very rigid exchange rates: the Eurozone before and after the introduction of the Euro, and several emerging economies in the period 1993-2005. This approach suggests almost perfect bilateral risk-sharing among all countries from the Eurozone. Moreover, it implies that emerging markets with fixed/rigid nominal exchange rates against the US dollar in the period achieved almost perfect risk-sharing with the US. We conclude that risk-sharing measures crucially depend on the behavior of the nominal exchange rate, implying almost perfect risk-sharing among countries with fixed/rigid nominal exchange rates. Second, a counterintuitive ranking of the risk-sharing levels under different nominal exchange rate regimes suggests a limited use of this approach for cross-country risk-sharing comparisons. Real exchange rates might be very smooth, but risk-sharing across countries is not necessarily perfect.

Keywords: International Risk-Sharing, Stochastic Discount Factor, Fixed Exchange Rates, Exchange Rate Regimes

JEL Classification: F31, F33, G12, G15

1 Introduction

The stochastic discount factor model suggests that countries share a very high portion of all macroeconomic risks they face. Starting with the underlying assumption that real exchange rate changes equal the difference between domestic and foreign marginal utility growth (or discount factors), Brandt, Cochrane, and Santa-Clara (2006) argue that only a small portion of total risk is left unshared across countries. In fact, they calculate domestic and foreign marginal utility growth rates through stochastic discount factors derived from asset markets data (using the excess returns of the stock market index above the risk-free rate) and compare the volatility of these stochastic discount factors with the volatility of the real exchange rate. The main finding in their study is that the real exchange-rates (difference between marginal utility growth rates) are much less volatile than what the stochastic discount factors (proxies for marginal utility growth) of the corresponding countries would imply. Therefore, they conclude that marginal utility growth rates must be very highly correlated across countries, i.e. a large portion of macroeconomic risk is shared internationally.

The results from the approach employed in this study crucially depend on the behavior of the real exchange rate, which comprises two components: the ratio of price levels in the two countries, and the nominal exchange rate. Therefore, fluctuations in the real exchange rate can come from two sources: the behavior of the inflation rate differential, and the behavior of the nominal exchange rate changes. Hence, it might be interesting to see whether by fixing the (bilateral) nominal exchange rate, two countries “*automatically (or mechanically)*” achieve perfect levels of international risk-sharing. Does the stochastic discount factor approach imply perfect risk-sharing between countries when their nominal exchange rate is fixed?

We present some new evidence about the stochastic discount factor approach to measuring international risk-sharing when applied to episodes of fixed/rigid nominal exchange rates. We calculate risk-sharing indices for two such episodes and arrive at two main results: first, when applied to the period after the introduction of the Euro, this asset markets-based approach implies almost perfect risk-sharing for each Euro-zone country vis-à-vis the Euro-zone as a whole. This comes from the fact that the major part of fluctuations in the real exchange-rate (i.e. the difference in marginal utility growth rates across countries according to the asset pricing model) is eliminated after the introduction of the common currency. Second, when applied to emerging markets with fixed nominal exchange rates against the US dollar in the period 1993-2005, the approach again implies almost perfect risk-sharing for each country against the USA. In fact, these countries achieved much more risk-sharing vis-a-vis the US compared to developed countries like the UK or Japan.

Two main conclusions can be drawn from our findings: first, the risk-sharing measures crucially depend on the behavior of the nominal exchange rate, implying almost perfect risk-sharing among countries with fixed/rigid nominal exchange rates. Second, a counterintuitive ranking of risk-sharing levels under different nominal exchange rate regimes suggests a limited use of this approach for cross-country risk-sharing comparisons.

The rest of this paper is organized as follows: section 2 develops the theoretical framework and presents the calculations of the stochastic discount factors and the risk-sharing index. Section 3 investigate the importance of fixed nominal exchange rate regimes. We discuss the relevance of our findings in section 4. Section 5 concludes the paper.

2 Theoretical Framework

2.1 Pricing Kernels

In this section we derive the theoretical framework linking the change in the real exchange-rate with the domestic and foreign marginal utility growth rates (stochastic discount factors). Following the approach taken in Backus, Foresi, and Telmer (1996) and Backus, Foresi, and Telmer (2001), we model asset prices with pricing kernels, i.e. stochastic processes that govern the prices of state-contingent securities¹.

Let v_t represent the domestic currency value at time t of an uncertain, stochastic cash flow of d_{t+1} domestic currency units one period in the future. Then, the basic asset pricing relation relates v_t and d_{t+1} in the following way:

$$v_t = E_t(m_{t+1}d_{t+1}) \quad (1)$$

by dividing both sides of equation 1 by the initial investment v_t at time t , i.e. the value of the uncertain cash flow at time t , we get an expression in terms of returns:

$$1 = E_t(m_{t+1}R_{t+1}) \quad (2)$$

where $R_{t+1} = d_{t+1}/v_t$ is the gross return on this asset/investment between time t and $t + 1$, and m_{t+1} is the domestic currency pricing kernel. The kernel m_{t+1} occupies a central place since it gives the “gross rate” at which economic agents discount the uncertain payment d_{t+1} one period in

¹Several conditions should be satisfied in order to derive a relationship between the (real) exchange rate and the stochastic discount factors in the two currencies. First, there should be free trade in assets denominated in each currency as well as free trade in each of the corresponding currencies. Second, no pure (zero initial investment) arbitrage opportunities should exist on any of the markets.

the future, i.e. it represents the (nominal) intertemporal marginal rate of substitution between time t and $t + 1$ for all assets traded in the domestic economy².

Similar relations should hold for assets denominated in foreign currency and traded in the foreign economy. In fact, there are two equivalent ways to show these relations for foreign assets. First, through substitution of all domestic variables from equations 1 and 2 with their foreign counterparts we get the following equations for foreign assets:

$$v_t^* = E_t(m_{t+1}^* d_{t+1}^*) \quad (3)$$

and, in terms of gross returns:

$$1 = E_t(m_{t+1}^* R_{t+1}^*) \quad (4)$$

Second, the cash flows (or gross returns) received in foreign currency can be converted into domestic currency units at the expected future spot exchange rate, and then discounted using the domestic pricing kernel or domestic discount factor, just as in the case of domestic assets. According to this approach, we get the following relations:

$$v_t^* = E_t \left[m_{t+1} (S_{t+1}/S_t) d_{t+1}^* \right] \quad (5)$$

and, in terms of gross returns:

$$1 = E_t \left[m_{t+1} (S_{t+1}/S_t) R_{t+1}^* \right] \quad (6)$$

² m_{t+1} will be a unique solution of equations 1 and 2 only if the domestic economy has a complete set of state-contingent securities that can be freely traded. Otherwise, there are multiple solutions for m_{t+1} .

where S_t stands for the current spot nominal exchange rate (the price of foreign currency in domestic currency units) at time t , and S_{t+1}/S_t represents its gross rate of change between time t and $t + 1$.

Because these two approaches must give equivalent results, we can equate 3 with 5:

$$E_t(m_{t+1}^* d_{t+1}^*) = E_t[m_{t+1}(S_{t+1}/S_t) d_{t+1}^*] \quad (7)$$

or 4 with 6, respectively:

$$E_t(m_{t+1}^* R_{t+1}^*) = E_t[m_{t+1}(S_{t+1}/S_t) R_{t+1}^*] \quad (8)$$

If no pure arbitrage opportunities exist and markets in both countries are complete, then the following should hold³:

$$m_{t+1}^* = m_{t+1}(S_{t+1}/S_t) \quad (9)$$

which, in turn, gives the relation between the change of the exchange rate and the nominal discount factors in the two countries. Hence, the (nominal) exchange rate should move (depreciate/appreciate) exactly by the difference between the discount factors in the respective countries. More specifically, equation 9 implies that domestic currency depreciates when the domestic nominal discount factor is lower than the foreign nominal discount factor in the corresponding period.

Although the discussion in this section focused on *nominal* variables, a similar condition can be stated in terms of *real* variables. Thus, taking

³This relation holds in the case of complete markets in both countries (for currencies and risky assets). In incomplete markets, m_{t+1}^* and m_{t+1} will not be uniquely determined - combinations of the discount factors with some random disturbances ϵ_{t+1}^* and ϵ_{t+1} that are orthogonal to the underlying shocks will also price all assets.

the logarithm of both sides of equation 9 and changing all nominal variables (exchange rates, gross returns, discount factors) into their real counterparts, we arrive at a condition that equates the real exchange rate to the difference between changes in foreign and domestic intertemporal marginal rates of substitution between time t and $t + 1$:

$$\ln \frac{e_{t+1}}{e_t} = \ln \frac{\lambda_{t+1}^*}{\lambda_{t+1}} = \ln \lambda_{t+1}^* - \ln \lambda_{t+1} \quad (10)$$

where e_t is the real exchange rate - the relative price of foreign in terms of domestic goods⁴, λ_{t+1} is the gross rate of change in domestic marginal utility between time t and $t + 1$, λ_{t+1}^* is the gross rate of change in foreign marginal utility between time t and $t + 1$ (both measured in units of real, consumption goods)⁵. Rearranged in real terms, this condition states that in equilibrium the change in the relative price of foreign in terms of domestic goods (given by gross rate of change in the real exchange rate) should equal the ratio between foreign and domestic marginal utility changes (stochastic discount factors or pricing kernels). Derived through this simple asset pricing framework, equation 10 is of central importance for the stochastic discount factor approach to measuring international risk-sharing, elaborated in this study⁶.

⁴The real exchange rate is defined as the price of foreign goods over the price of domestic goods. Therefore, an increase in the real exchange rate implies a real appreciation (depreciation) of foreign (domestic) goods.

⁵The stochastic discount factors λ_{t+1} and λ_{t+1}^* represent gross real returns in the corresponding markets. They can be defined through in traditional consumption-based models as $\lambda_{t+1} = \beta(u'(c_{t+1})/u'(c_t))$, where β is the reciprocal of the gross rate of time preference and $(u'(c_{t+1})/u'(c_t))$ is the gross rate of change in marginal utility growth between time t and $t + 1$. Therefore, the values for the discount factors will be always positive in this framework, typically in the vicinity of 1.

⁶For more extensive discussion on the application of this equation see Backus et al. (2001) and Brandt and Santa-Clara (2002) for example.

2.2 Risk-Sharing Index

The perfect international risk-sharing hypothesis implies complete equalization of marginal utility growth rates across countries. In our framework, given by equation 10, it means equality between λ_{t+1} and λ_{t+1}^* at any point in time. Thus, if this asset pricing condition holds *and* all country-specific risks are shared internationally, then the left-hand side of this equation should always be zero. Put differently, the departures from this perfect situation can be measured by the deviations on the left-hand side, i.e. the fluctuations of the real exchange rate.

Brandt et al. (2006) use this intuition to propose a measure of international risk-sharing based on asset markets. First, they take variances of both sides of equation 10:

$$\begin{aligned}\sigma^2\left(\ln \frac{e_{t+1}}{e_t}\right) &= \sigma^2\left(\ln \lambda_{t+1}^* - \ln \lambda_{t+1}\right) = \\ &= \sigma^2\left(\ln \lambda_{t+1}^*\right) + \sigma^2\left(\ln \lambda_{t+1}\right) - 2\rho\sigma\left(\ln \lambda_{t+1}^*\right)\sigma\left(\ln \lambda_{t+1}\right)\end{aligned}\quad (11)$$

where σ^2 symbolizes a variance, σ a standard deviation, and ρ is the coefficient of correlation between the two discount factors λ_{t+1} and λ_{t+1}^* . Therefore, if the following two conditions hold: i) assets and currencies are priced according to equation 10 at any point in time; and ii) all risks are shared internationally, then: $\rho = 1$, $\lambda_{t+1} = \lambda_{t+1}^*$ and $\sigma^2\left(\ln \frac{e_{t+1}}{e_t}\right) = 0$. In general, the correlation between marginal utility growth rates will be given by:

$$\rho = \frac{\left[\sigma^2\left(\ln \lambda_{t+1}^*\right) + \sigma^2\left(\ln \lambda_{t+1}\right) - \sigma^2\left(\ln \frac{e_{t+1}}{e_t}\right)\right]}{2\sigma\left(\ln \lambda_{t+1}^*\right)\sigma\left(\ln \lambda_{t+1}\right)}\quad (12)$$

indicating that risk-sharing across countries decreases in the variability

of the real exchange rate. Based on this idea, Brandt et al. (2006) construct the following risk-sharing index

$$RSI = 1 - \frac{\sigma^2\left(\ln \frac{e_{t+1}}{e_t}\right)}{\sigma^2\left(\ln \lambda_{t+1}^*\right) + \sigma^2\left(\ln \lambda_{t+1}\right)} \quad (13)$$

where the numerator of the second term captures the variability in the real exchange rate (which, according to the argumentation above, measures the deviations from perfect risk-sharing), and the denominator is the sum of the variabilities in marginal utility growth in the two countries (the total risk that exists and can be shared across countries). Hence, this term gives a ratio between risk still not shared and total risk that can be shared between the two countries. Brandt et al. (2006) indicate that this index gives the portion of total (diversifiable) risk that is already shared by the two countries⁷.

2.3 Basic Calculations

In order to calculate the risk-sharing index given in the previous section, first we have to recover the log discount factors (or marginal utility growth rates) from asset markets data in the corresponding countries⁸. For this purpose, we closely follow the exposition given in Brandt et al. (2006). We start by assuming that the following assets are traded in a two-country setting:

$$\frac{dB^d}{B^d} = r^d dt \quad (14)$$

⁷In this way, the framework presented by Brandt et al. (2006) can be viewed as an extension of the Hansen-Jagannathan (1991) volatility bounds to the international setting.

⁸For ease of exposition and manipulation in the further calculations (translating between levels and logarithms), the demonstration here uses continuous time formulation. Empirically, all variables are calculated using the corresponding discrete time approximations, see the section on data issues.

$$\frac{dS^d}{S^d} = \theta^d dt + dz^d \quad (15)$$

$$\frac{de}{e} = \theta^e dt + dz^e \quad (16)$$

$$\frac{dB^f}{B^f} = r^f dt \quad (17)$$

$$\frac{dS^f}{S^f} = \theta^f dt + dz^f \quad (18)$$

where B^d is the domestic risk-free bond (with expected return r^d), S^d is the domestic risky asset (expected return θ^d), e is the real exchange rate, i.e. the relative price of foreign in terms of domestic goods (expected return θ^e), B^f is the foreign risk-free bond, and S^f is the foreign risky asset (expected return θ^f). There are three sources of uncertainty in this setting, related to the domestic asset, the real exchange rate, and the foreign asset. These shocks can be collected into a vector of shocks dz :

$$dz = \begin{bmatrix} dz^d \\ dz^e \\ dz^f \end{bmatrix}$$

with a corresponding variance-covariance matrix given by⁹:

$$\Sigma = \frac{1}{dt} E(dz dz') = \begin{bmatrix} \Sigma^{dd'} & \Sigma^{de} & \Sigma^{df'} \\ \Sigma^{ed'} & \Sigma^{ee} & \Sigma^{ef'} \\ \Sigma^{fd'} & \Sigma^{fe} & \Sigma^{ff'} \end{bmatrix}$$

Furthermore, the calculation of the discount factor(s) from asset markets depends primarily on the variability of the excess returns on risky assets,

⁹This variance-covariance matrix is the same for domestic and foreign investors because they face the same vector of shocks in this symmetric, bilateral setting.

driven by the shocks in vector dz^{10} . We derive all excess return equations in the appendix, and here present only their expected values. Thus, the domestic investor faces the following set of expected excess returns:

$$\mu^d = \begin{bmatrix} \theta^d - r^d \\ \theta^e + r^f - r^d \\ \theta^f - r^f + \Sigma^{ef} \end{bmatrix}$$

The first term in this vector gives the excess return that a domestic resident expects to get by investing on the domestic stock market. It equals the difference between the average real return on the domestic stock market index (θ^d) and the average real risk-free rate in the domestic economy (r^d) during the entire investment period. The expected excess return on the foreign exchange market is given by the second term in vector μ^d . It represents the average deviation from (uncovered) interest parity, calculated as borrowing in the domestic currency, converting the borrowed amount into the foreign currency, lending at the ongoing one-month foreign interest rate, and converting the proceeds back into domestic currency after one month. The last term in vector μ^d gives the expected excess return that a domestic investor expects to get by investing in the foreign stock market. Therefore, it represents a difference between the average return on the foreign stock market and the domestic one-month risk-free interest rate. The last part of this term Σ^{ef} results from the continuous-time formulation and gives the (average) co-movement between the returns on the foreign stock market and the exchange rate. Therefore, by correcting for the movements of the nominal exchange rate, this term facilitates the translation of excess returns obtained on the foreign market¹¹.

¹⁰Since we work with (expected) excess returns in this analysis, we do not make a real/nominal returns distinction.

¹¹For example, Σ^{ef} is added to the excess return on the foreign market for the domestic

A similar vector of expected excess returns applies to the foreign investor:

$$\mu^f = \begin{bmatrix} \theta^d - r^d - \Sigma^{ed} \\ -(\theta^e + r^f - r^d - \Sigma^{ee}) \\ \theta^f - r^f \end{bmatrix}$$

The interpretation of the terms is analogous to that given for the domestic investor. The expected excess return on the foreign exchange market is exactly the opposite of the one for the domestic investor (corrected for the continuous-time term Σ^{ee}).

Then, the following discount factors price all assets according to the basic pricing conditions¹²:

$$\frac{d\Lambda^i}{\Lambda^i} = -r^i dt - \mu^{i'} \Sigma^{-1} dz, i = d, f \quad (19)$$

where $\frac{d\Lambda^i}{\Lambda^i}$ is the growth rate of the discount factor, r^i is the risk-free return, and μ^i is the vector of excess returns for risky assets in country i . In order to calculate the change in the log discount factor $\ln \lambda^i$ required in equation 10, we use Ito's lemma and get the following expression:

$$d \ln \Lambda^i = \frac{d\Lambda^i}{\Lambda^i} - \frac{1}{2} \frac{d\Lambda^{i2}}{\Lambda^{i2}} = -\left(r^i + \frac{1}{2} \mu^{i'} \Sigma^{-1} \mu^i\right) dt - \mu^{i'} \Sigma^{-1} dz \quad (20)$$

and for its standard deviation:

$$\frac{1}{dt} \sigma^2(d \ln \Lambda^i) = \mu^{i'} \Sigma^{-1} \mu^i, i = d, f \quad (21)$$

The change in the log discount factor $d \ln \Lambda$ corresponds to $\ln \lambda_{t+1}$ in the basic asset pricing condition 10. Therefore, the risk-sharing index given investor, suggesting that foreign expected excess returns are amplified when associated with appreciation of the foreign currency.

¹²For more details on finding the discount factor in this setting see Brandt et al. (2006, p.675-677) or Chapter 4 in Cochrane (2004).

by 13 can be calculated directly from the second moments according to the following expression:

$$RSI = 1 - \frac{\sigma^2(d \ln \Lambda^d - d \ln \Lambda^f)}{\sigma^2(d \ln \Lambda^d) + \sigma^2(d \ln \Lambda^f)} = 1 - \frac{\Sigma^{ee}}{\mu^d \Sigma^{-1} \mu^d + \mu^f \Sigma^{-1} \mu^f} \quad (22)$$

In order to show the symmetric structure of our framework, we relate the shocks facing the domestic with those facing the foreign investor. The expected excess returns vectors μ^d and μ^f differ only by the exchange rate changes¹³:

$$\mu^d - \mu^f = \begin{bmatrix} \theta^d - r^d \\ \theta^e + r^f - r^d \\ \theta^f - r^f + \Sigma^{ef} \end{bmatrix} - \begin{bmatrix} \theta^d - r^d - \Sigma^{ed} \\ \theta^e + r^f - r^d - \Sigma^{ee} \\ \theta^f - r^f \end{bmatrix} = \begin{bmatrix} \Sigma^{ed} \\ \Sigma^{ee} \\ \Sigma^{ef} \end{bmatrix} \quad (23)$$

From these formulae, it is clear that the expected excess return vectors differ exactly by the middle column of the common variance covariance matrix Σ^e :

$$\mu^d - \mu^f = \begin{bmatrix} \Sigma^{ed} \\ \Sigma^{ee} \\ \Sigma^{ef} \end{bmatrix} = \Sigma^e \quad (24)$$

In turn, we can derive a relationship between the domestic and foreign discount factor loadings (given by the last term of equation 20):

$$\mu^d \Sigma^{-1} = (\mu^f + \Sigma^e) \Sigma^{-1} = \mu^f \Sigma^{-1} + \Sigma^e \Sigma^{-1} = \mu^f \Sigma^{-1} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (25)$$

¹³In order to derive this relation, we disregard the change in sign before the foreign exchange excess returns when moving from domestic to foreign investor perspective.

Equation 25 shows that domestic and foreign discount factors load equally on domestic and foreign stock market shocks, while their loadings on the foreign exchange shocks differ by exactly 1. Therefore, this implies that the only difference between the two discount factors comes from fluctuations in the real exchange rate.

3 Fixed Nominal Exchange Rates

The international risk-sharing index represents a comparison between real exchange rate volatility and the sum of the discount factor volatilities in the corresponding countries. The idea behind this set-up is that real exchange rates fluctuate by the difference between marginal utility growth rates in the corresponding countries, and thereby reflect hurdles for complete cross-country risk-sharing. Therefore, the results from the approach employed in this study crucially depend on the behavior of the real exchange rate. In turn, the real exchange rate is composed of two components: the ratio of price levels in the two countries, and the nominal exchange rate. Therefore, fluctuations in the real exchange rate can come from two sources: the behavior of the inflation rate differential, and the behavior of the nominal exchange rate changes.

Beginning with Mussa (1986), a large body of literature demonstrates that real exchange rate behavior crucially depends on the underlying nominal exchange rate regime. In fact, Mussa (1986) shows that real exchange rates are more volatile under floating compared to fixed nominal exchange rate regimes. Moreover, he finds that this extra variability comes primarily from the nominal exchange rate rather than the inflation differential. Ever since, a number of empirical and theoretical studies have found support for this argument (Baxter and Stockman, 1989; Dixon, 1999; Flood and Rose,

1999; Lothian and McCarthy, 2002; MacDonald, 1999). This is not surprising because under a floating regime, the nominal exchange rate represents an asset that is traded daily on financial markets. On the other hand, the inflation rate differential is one of the most important overall macroeconomic indicators, depending on numerous economy-wide factors, and does not show asset characteristics. Therefore, we are interested in the following question: to what extent does the value of the risk-sharing index depend on the nominal exchange rate regime? Does the stochastic discount factor approach imply perfect risk-sharing between countries when their nominal exchange rate is fixed?

The main issue we want to investigate is whether by fixing the (bilateral) nominal exchange rate, two countries “*automatically (or mechanically)*” achieve perfect levels of international risk-sharing according to the approach developed in this study. In order to answer this question(s), we calculate risk-sharing indices for two episodes with fixed nominal exchange rate regimes. The first one refers to the experience of the countries in the EMU after the introduction of the common currency - the Euro - on 1 January 1999. The second example refers to a group of emerging economies that had their nominal exchange rates (almost) fixed against the US dollar over extended periods of time in recent past.

3.1 Eurozone

3.1.1 Data Sources

We focus on 12 countries from the Eurozone in the period January 1993-December 2005. We collect monthly data on widely-used stock market indices for each country and for the Eurozone as a whole. For the Eurozone we include two indices: Eurozone FTSE Local made up of major Eurozone-

Table 1: Stock Market Indices

Country	Index
AUS	Austrian Traded ATX
BEL	BEL 20
FIN	OMX Helsinki 25
FRA	CAC 40
GER	DAX 30
GRE	ASE 20
IRE	ISEQ Irish Overall
ITA	MIB 30
LUX	LUXX
NL	AEX
POR	Portugal PSI General
SPA	IBEX 35

Note: This table contains a list of stock market indices for the Eurozone countries. When available, we include the leading stock market indices for each of the 12 countries. Some of the missing observations in the leading indices for Greece, Italy, and Luxembourg have been supplemented by alternative stock market indices: Athex Composite for ASE 20 (Greece), MIB Storico General for MIB 30 (Italy), and LUXX Datastream-Calculated for LUXX (Luxembourg).

based corporations that receive more than 70 percent of their total revenues domestically, i.e. from within the Eurozone; and Dow Jones Euro Stoxx 50 - the most widely-used index of 50 major corporations based in the Eurozone. A list with stock market indices per country can be found in Table 1¹⁴.

Moreover, we collect data series on interest rates for one-month Eurocur-

¹⁴The leading stock market indices for Greece, Italy and Luxembourg miss several observations for the period January 1993-December 2005. Therefore, they have been supplemented by alternative stock market indices: Athex Composite for ASE 20 (Greece), MIB Storico General for MIB 30 (Italy), and LUXX Datastream-Calculated for LUXX (Luxembourg).

rency deposits, consumer price indices, and nominal exchange rates for the pre-Euro period. Eurocurrency deposit rates for Greece and for the Eurozone were not available for the entire time period. Therefore, we supplemented the series for Greece for the period January 1993-January 2001 with interest rates series on bank deposits in domestic currency (Greek drachmas). For the Eurozone we use the German interest rates on one-month Eurocurrency deposits in the period January 1993-December 1998. After the introduction of the Euro, we use the Eurozone rate on one-month Eurocurrency deposits¹⁵. CPI indices refer to mid-month values of the consumer price indices. Therefore, we calculate inflation rates as monthly changes in the CPI. For each country in the pre-Euro period we use nominal spot exchange rates (in terms of domestic currency units per European Currency Unit (ECU)) at the beginning of the month. Moreover, for Greece we use the same nominal spot exchange rate until June 2000, when the drachma was fixed against the Euro.

In order to capture the importance of the introduction of the Euro, i.e. the fixation of all bilateral nominal exchange rates, we separate the entire time period into two subperiods of approximately equal length: one before the introduction of the Euro, from January 1993 till December 1998; and the other after the introduction of the Euro, from January 1999 till December 2005. In this way, we separate the dataset symmetrically with respect to 1999 - the year of the introduction of the Euro¹⁶. Subsequently, we do the calculations for each subperiod separately.

¹⁵Therefore, due to the convergence of interest rates after the introduction of the Euro in January 1999, one may argue that the Eurozone interest rate equals the German rate throughout the entire time period.

¹⁶Therefore, we control for the possibility that the number of observations might have an influence on the results from the calculations.

3.1.2 Summary Statistics

The summary statistics for the countries from the Eurozone are presented in Tables 2, 3, and 4. Table 2 shows annualized means and standard deviations for the real excess stock market returns for two sub-periods: before the introduction of the Euro (1993-1998), and after (1999-2005). The data suggests large differences between mean excess returns in the two subperiods: stock market returns in the pre-Euro period were much higher compared to the Euro period. To a large extent, this behavior is influenced by the “dot-com bubble”, which was characterized by a prolonged upward trend in the first, and abrupt downward correction in the second subperiod. However, while the mean excess returns were clearly influenced by this bubble, their volatility was not: standard deviations are comparable across the two subperiods.

In fact, this joint development - a decrease in mean excess returns without a significant change in their variability, implies lower Sharpe ratios. Indeed, the corresponding columns in Table 2 indicate that the Sharpe ratio decreased for all but one Eurozone country (Austria). As the Sharpe ratio measures investor’s compensation for risk taken, its declining trend suggests that markets lowered the reward given to risk-bearers in the second (Euro-period) compared to the first (pre-Euro) period.

Table 3 gives similar type of information for real foreign exchange excess returns. It shows the annualized means and standard deviations for real excess returns earned on the foreign exchange market¹⁷. Two important points deserve attention in this figure. First, mean real excess returns are typically very small in absolute value and of comparable magnitudes across

¹⁷ECU is the benchmark currency in the pre-Euro period, and the Euro is the benchmark currency in the calculations for the Euro-period.

Table 2: Summary Statistics: Stock Market Returns (Annualized)

	1993-1998			1999-2005		
country	mean	std dev	Sharpe ratio	mean	std dev	Sharpe ratio
AUS	8.72	23.04	0.38	18.25	16.37	1.11
BEL	13.93	16.19	0.86	-0.57	17.03	-0.03
FIN	29.65	24.76	1.19	10.94	28.82	0.38
FRA	8.91	21.02	0.42	2.25	20.06	0.11
GER	16.29	19.42	0.84	1.42	24.74	0.06
GRE	5.86	38.60	0.15	2.08	28.24	0.07
IRE	24.05	17.85	1.35	7.56	17.79	0.43
ITA	13.89	26.23	0.53	0.29	20.15	0.01
LUX	19.29	14.89	1.29	9.91	25.12	0.39
NL	19.51	18.37	1.06	-2.02	22.63	-0.09
POR	19.06	24.45	0.78	-1.28	16.39	-0.08
SPA	18.36	23.83	0.77	0.95	20.48	0.05
EURO	15.62	17.99	0.87	1.09	20.89	0.05

Note: This table contains summary statistics for stock market excess returns for each of the 12 countries from the Eurozone. The first three columns contain statistics for the pre-Euro period (January 1993-December 1998), while the last three columns contain statistics for the Euro-period (January 1999-December 2005). Excess returns are calculated over one-month Eurocurrency deposit rates for the corresponding country. The excess returns for the Eurozone are calculated over one-month Eurocurrency deposit rates for Germany (in the pre-Euro period) and for the Eurozone (in the Euro-period). All data-series are retrieved from Datastream and all summary statistics are annualized and expressed in percentage terms (rounded to two decimal places).

time subperiods. Second, the real foreign exchange excess returns show much lower volatility in the second (after the introduction of the Euro) compared to the first time subperiod (before the Euro). The latter observation is in line with the common intuition that (excess returns on) real exchange rates become much less volatile under fixed nominal exchange rate regimes.

Finally, Table 4 presents correlations between returns on stock market indices in the countries from the Eurozone and the overall Eurozone index Euro Stoxx 50 for the two time subperiods. We also used an alternative overall index for the Eurozone - Eurozone FTSE Local - but the correlations were very similar and we do not present them here¹⁸. The general impression is that the correlations for most national indices with the Eurozone index were very high. Moreover, they increased after the introduction of the common currency for most countries, the exceptions being Austria, Belgium, and Portugal, where these correlations (slightly) decreased.

3.1.3 Results for the Eurozone

The results for the risk-sharing index are presented in Table 5. All calculations are done using bilateral setting between each of the 12 countries and the Eurozone¹⁹. The first column refers to the results before, and the second column to the results after the introduction of the common currency.

¹⁸Calculations based on this alternative index for the Eurozone differ only marginally. All these calculations are available from the authors upon request.

¹⁹Strictly speaking, data on the Eurozone should exclude the country concerned. In our case, since interest rates across the countries in the Eurozone are virtually the same and nominal exchange rates are fixed, this point can only be valid for the Eurozone inflation rate and Eurozone stock market returns. However, due to computational difficulties, we do not exclude the contribution of each country's inflation towards the Eurozone inflation rate nor the fact that some of the companies included in the Euro Stoxx 50 index are based in the corresponding countries.

Table 3: Summary Statistics: Foreign Exchange Returns (Annualized)

	1993-1998		1999-2005	
country	mean	std dev	mean	std dev
AUS	4.81	3.03	2.94	0.38
BEL	-0.35	2.42	0.04	0.17
FIN	4.96	6.15	2.59	0.57
FRA	-0.97	2.47	-0.31	0.13
GER	0.20	2.89	-0.56	0.15
GRE	-7.54	5.22	-1.63	1.35
IRE	3.73	5.56	4.60	0.63
ITA	-3.81	6.89	0.35	0.16
LUX	-0.31	2.45	0.24	0.17
NL	0.40	2.77	0.56	0.34
POR	-4.88	3.16	1.04	0.30
SPA	-4.49	5.01	1.12	0.16

Note: This table contains summary statistics for excess returns on the foreign exchange market for each of the 12 countries from the Eurozone. The first two columns contain statistics for the pre-Euro period (January 1993-December 1998), while the last two columns contain statistics for the Euro-period (January 1999-December 2005). Excess returns on the foreign exchange market are calculated as (real) deviations from uncovered interest rate parity ($\theta^e + r^f - r^d$). Eurozone is used as the benchmark (domestic) country in all calculations. ECU is the benchmark currency in the pre-Euro period, and the Euro is the benchmark currency in the Euro-period. All data-series are retrieved from Datastream and all summary statistics are annualized and expressed in percentage terms (rounded to two decimal places).

Table 4: Stock Market Correlations with Euro Stoxx 50

Country	pre-Euro	Euro
AUS	0.793	0.476
BEL	0.845	0.684
FIN	0.671	0.825
FRA	0.929	0.980
GER	0.914	0.944
GRE	0.358	0.531
IRE	0.637	0.699
ITA	0.720	0.867
LUX	0.558	0.717
NL	0.903	0.934
POR	0.781	0.638
SPA	0.823	0.833

Note: This table presents figures for stock market correlations for all 12 countries from the Eurozone. All correlations are measured by the coefficient of correlation between monthly returns on each stock market index and monthly returns on Euro Stoxx 50 - the overall index for the Eurozone. The first column contains statistics for the pre-Euro period (January 1993-December 1998), while the second column contains statistics for the Euro-period (January 1999-December 2005). All figures are rounded up to three decimal places.

Two important observations deserve attention in this table. First, the risk-sharing index gets very high values for the first subperiod, typically about 0.999. Second, the risk-sharing index in the second subperiod - after the introduction of the Euro - gets extreme values, even much higher than in the first subperiod. In fact, the risk-sharing index asymptotically approaches the value of 1 for each country in the Eurozone. Using the terminology in Brandt et al. (2006), these values suggest *perfect risk-sharing* between each of these 12 countries and the Eurozone, and thereby, between each individual pair of countries.

Table 5: Risk-Sharing Index

Country	pre-Euro	Euro
AUS	0.999893	0.999999
BEL	0.999656	0.999975
FIN	0.999011	0.999999
FRA	0.999842	0.999999
GER	0.999502	0.999999
GRE	0.999635	0.999941
IRE	0.999448	0.999999
ITA	0.998154	0.999999
LUX	0.999842	0.999999
NL	0.999741	0.999998
POR	0.999841	0.999999
SPA	0.999276	0.999999

Note: This table presents figures for the bilateral risk-sharing index between each of the 12 Eurozone countries and the Eurozone as a whole. The risk-sharing index is calculated according to the following formula:

$RSI = 1 - \frac{\Sigma^{ee}}{\mu^d \Sigma^{-1} \mu^d + \mu^f \Sigma^{-1} \mu^f}$. The first column contains statistics for the pre-Euro period (January 1993-December 1998), while the second column contains statistics for the Euro-period (January 1999-December 2005). All figures are rounded up to six decimal places.

3.1.4 Understanding the Results

Having observed these extreme values for the international risk-sharing index, we now attempt to understand the underlying reasons. Therefore, we look at the behavior of the two index components: real exchange rate volatility, and discount factor volatility.

Figure 1 shows monthly changes in the nominal exchange rate (domestic currency per unit of ECU or Euro) for 11 countries that adopted the common currency on 1 January 1999. The vertical axis measures monthly percentage changes in the exchange rate, and the vertical line in the middle of the figure

denotes the separation of the complete time period into pre-Euro and Euro subperiods. The figure shows a continuous decrease in the nominal exchange rate variability for each country in the pre-Euro period. Moreover, it shows zero-variability in all nominal exchange rates starting in January 1999 with the adoption of the Euro. Figure 2 includes Greece as well, showing its late adoption of the Euro in January 2001²⁰.

In order to see the link between nominal and real exchange rate volatility, we present similar graphs for the real exchange rates in Figures 3 and 4. Figure 3 shows that the behavior of the real exchange rate was very similar to the behavior of the nominal exchange rate. In fact, the time-lines in Figure 3 closely resemble those for the nominal exchange rate in Figure 1. Therefore, the adoption of the Euro does not only eliminate nominal, but also the major (bulk) portion of real exchange rate volatility. There are only minor fluctuations left after January 1999, coming exclusively from the inflation rate differential. Finally, Figure 4 shows a similar picture for all 12 countries, including Greece. As can be seen from the graph, the real exchange rate for Greece stabilizes only with a two-year lag, due almost entirely to its late adoption of the Euro.

²⁰The Greek drachma was fixed against the Euro on 19 June 2000, and officially replaced by the common European currency on 1 January 2001.

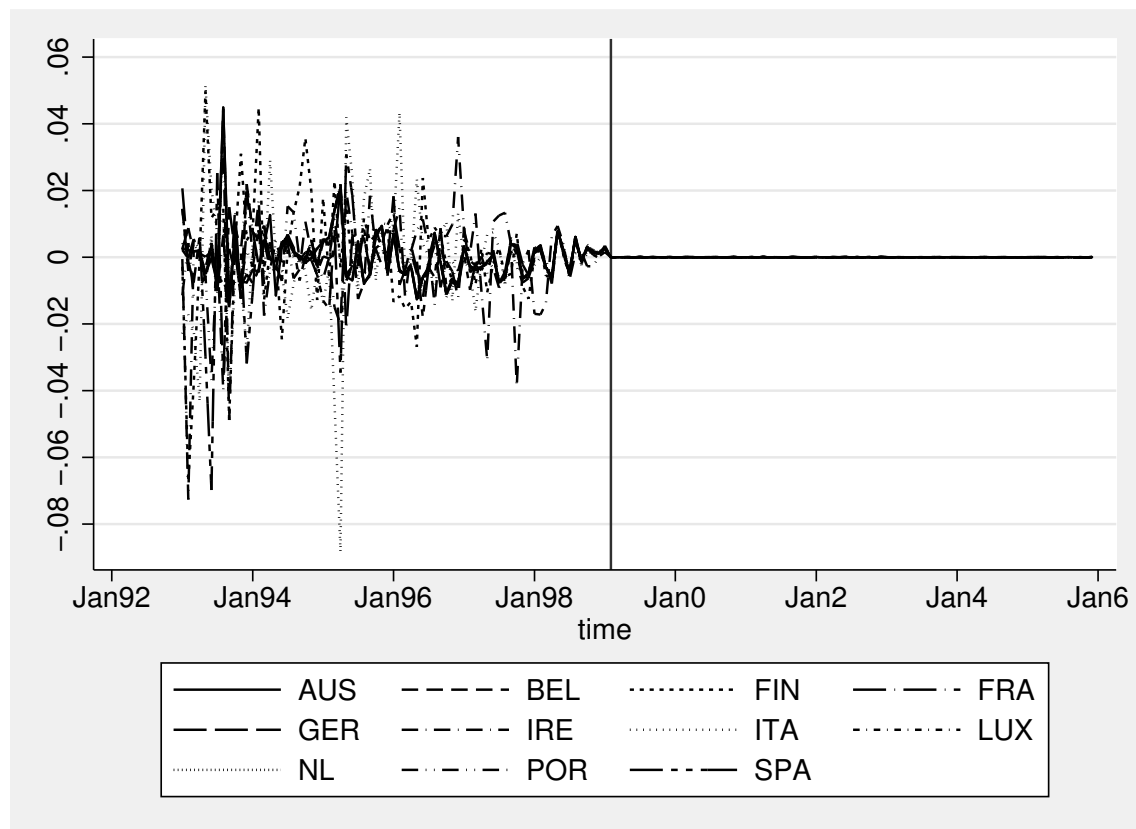


Figure 1: Nominal Exchange Rates (1993-2005)

Note: This figure presents monthly changes in the nominal exchange rates for 11 countries in the Eurozone (excluding Greece) over the period January 1993-December 2005. The ECU is used as benchmark currency in the pre-Euro period, and the Euro afterwards. Positive changes in the nominal exchange rate indicate depreciation of the benchmark currency (appreciation of the currency for the corresponding country). The vertical line marks the introduction of the Euro (January 1999).

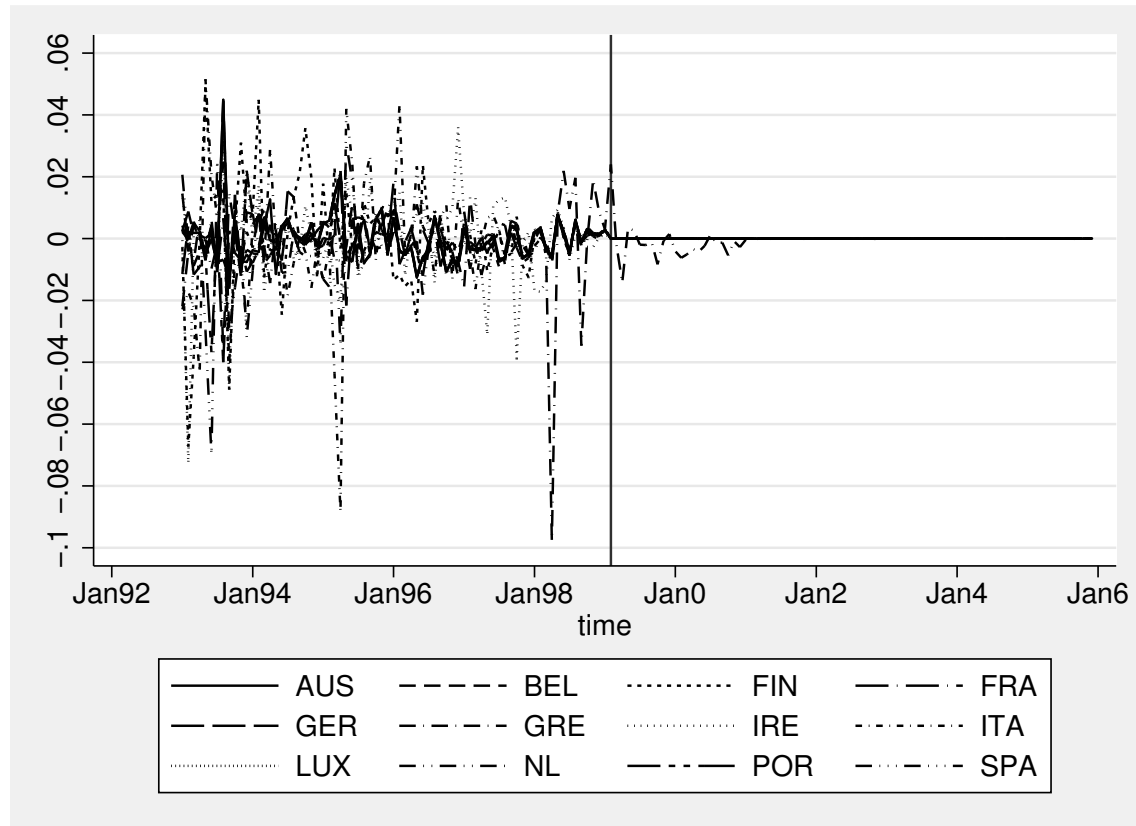


Figure 2: Nominal Exchange Rates (1993-2005)

Note: This figure presents monthly changes in the nominal exchange rates for all 12 countries in the Eurozone over the period January 1993-December 2005. The ECU is used as benchmark currency in the pre-Euro period, and the Euro afterwards. Positive changes in the nominal exchange rate indicate depreciation of the benchmark currency (appreciation of the currency for the corresponding country). The vertical line marks the introduction of the Euro (January 1999).

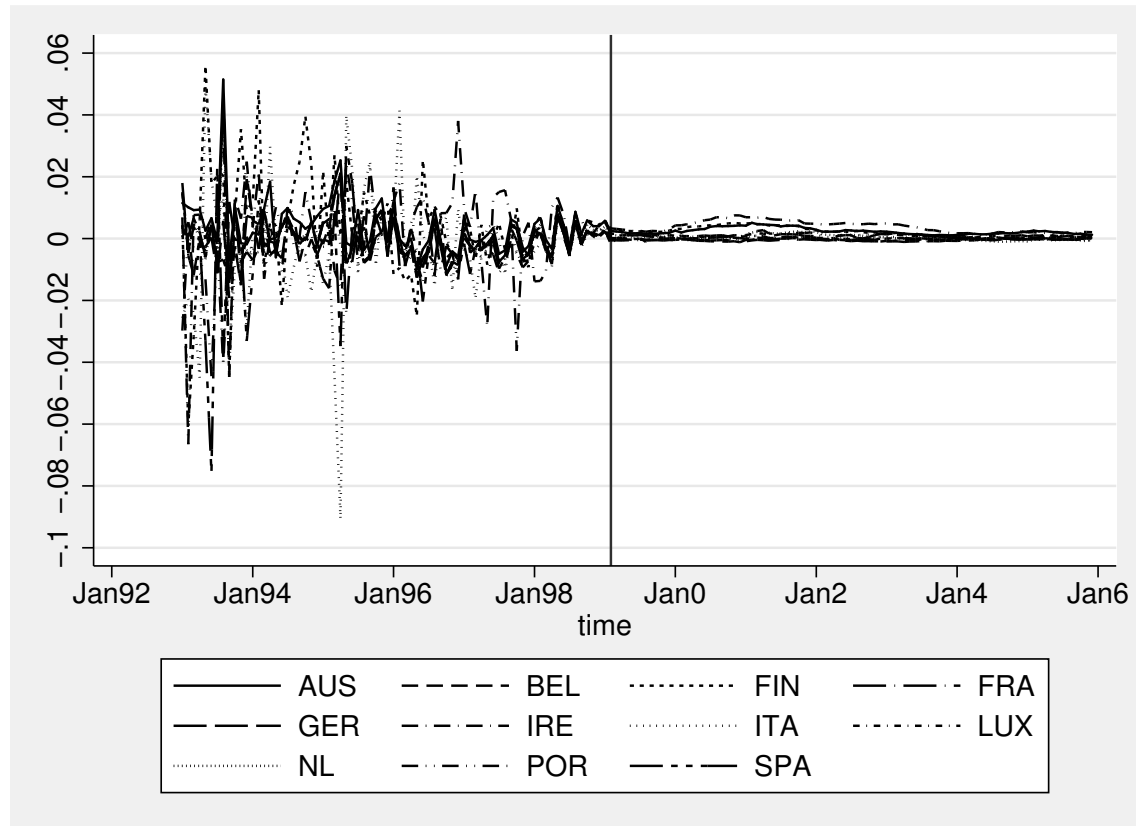


Figure 3: Real Exchange Rates (1993-2005)

Note: This figure presents monthly changes in the real exchange rates for 11 countries in the Eurozone (excluding Greece) over the period January 1993-December 2005. The ECU is used as benchmark currency in the pre-Euro period, and the Euro afterwards. Real exchange rate changes are calculated as nominal exchange rate changes corrected for the inflation differential. Positive changes in the real exchange rate indicate real depreciation of the benchmark currency (real appreciation of the currency for the corresponding country). The vertical line marks the introduction of the Euro (January 1999).

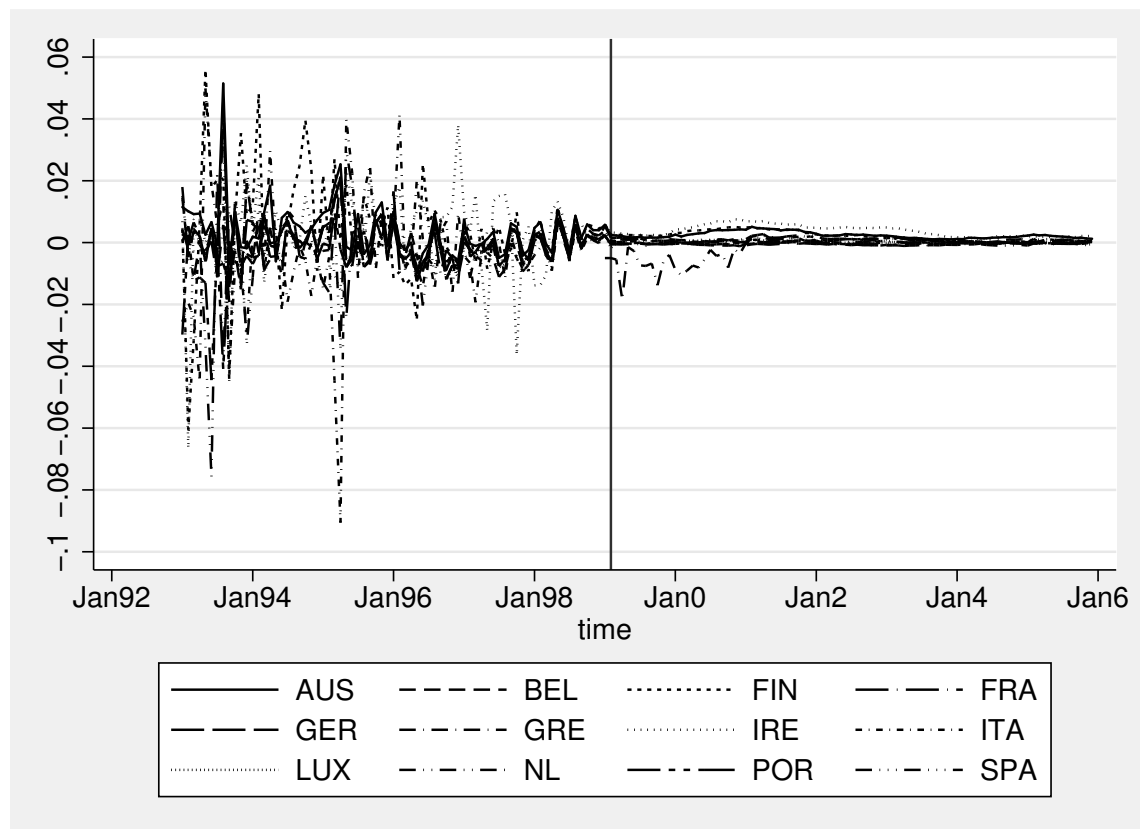


Figure 4: Real Exchange Rates (1993-2005)

Note: This figure presents monthly changes in the real exchange rates for all 12 countries in the Eurozone over the period January 1993-December 2005. The ECU is used as benchmark currency in the pre-Euro period, and the Euro afterwards. Real exchange rate changes are calculated as nominal exchange rate changes corrected for the inflation differential. Positive changes in the real exchange rate indicate real depreciation of the benchmark currency (real appreciation of the currency for the corresponding country). The vertical line marks the introduction of the Euro (January 1999).

Table 6: Real Exchange-Rate Volatility (Annualized Values)

Country	pre-Euro	Euro
AUS	3.01	0.38
BEL	2.40	0.17
FIN	6.11	0.57
FRA	2.45	0.13
GER	2.87	0.15
GRE	5.19	1.34
IRE	5.52	0.63
ITA	6.84	0.16
LUX	2.43	0.17
NL	2.75	0.34
POR	3.14	0.30
SPA	4.97	0.16

Note: This table presents figures for real exchange rate volatility for each of the 12 Eurozone countries. Volatility is measured by the annualized standard deviation calculated from monthly changes in the real exchange rate. The first column contains statistics for the pre-Euro period (January 1993-December 1998), while the second column contains statistics for the Euro-period (January 1999-December 2005). All figures are expressed in percentages terms, rounded up to two decimal places.

In Tables 6, 7, and 8 we go one step further in understanding the results for the risk-sharing index in the Eurozone. Tables 6 and 7 show statistics for real exchange rate and discount factor volatility in each country from the Eurozone. They refer to the annualized values of the two components used in the calculation of the risk-sharing index. Table 6 gives additional evidence about the rapid drop in the real exchange rate volatility after the adoption of the Euro. In fact, all figures in the second column (Euro period) are on average about 10 times lower than the corresponding entries in the first column (pre-Euro period).

Table 7 shows annualized values for the discount factor volatility in each country. The first columns in Table 7 give values for the Eurozone discount factor volatility and the last two columns give the corresponding values for the other countries used in the bilateral calculations. Moreover, for each country, the first column refers to the pre-Euro, while the second for the Euro period. Hence, we can see that the discount factors display similar volatility levels for each Eurozone-country pair (compare column 1 with column 3, or column 2 with column 4). Furthermore, for most countries, discount factors became more variable in the later period, although these changes are not as strong as in the case of the real exchange rate. In sum, the changes in both components of the risk-sharing index work in the same direction: they unambiguously lead to inflation of the index. In order to see the relative importance of each component, we proceed with a comparison of the changes in their volatility levels between the two subperiods.

Table 8 presents ratios between the volatility level in the pre-Euro period over the volatility level in the Euro period for each index component. Values above 1 indicate a decrease in the variability of the corresponding variable after the introduction of the Euro. We present ratios for the following five variables: real exchange rate, Eurozone discount factor, country discount factor, sum of both discount factors, and for the ratio of the real exchange rate over the sum of the discount factors. The first column in Table 8 displays ratios much higher than 1, indicating huge drop in the real exchange rate volatility for each country in the Eurozone. Moreover, these ratios differ widely across different countries. Thus, the real exchange rate was about 8 times more volatile in the pre-Euro period for Austria (ratio of 7.87) and the Netherlands (8.05), but 31 times for Spain and 43 times for Italy. The smallest drop comes for Greece (ratio of 3.86), but this is largely due to

Table 7: Discount Factors Volatility (Annualized Values)

	Eurozone DF	Eurozone DF	Country DF	Country DF
Country	pre-Euro	Euro	pre-Euro	Euro
AUS	206.69	811.29	204.38	810.93
BEL	91.33	24.48	91.74	24.34
FIN	139.02	479.16	135.54	478.63
FRA	137.41	250.85	138.13	250.98
GER	90.94	375.39	90.76	375.54
GRE	189.96	123.65	193.96	124.96
IRE	167.38	754.88	165.23	754.27
ITA	110.80	223.01	114.39	222.86
LUX	136.44	150.46	136.69	150.31
NL	120.92	173.56	120.622	173.24
POR	173.93	359.45	176.74	359.16
SPA	128.83	713.94	132.37	713.79

Note: This table presents figures for discount factor volatility for each of the 12 Eurozone countries. Volatility is calculated by the annualized standard deviation of the discount factor according to equation 21: $\frac{1}{dt} \sigma^2(d \ln \Lambda^i) = \mu^{i'} \Sigma^{-1} \mu^i$. The first column contains statistics for the pre-Euro period (January 1993-December 1998), while the second column contains statistics for the Euro-period (January 1999-December 2005). All figures are expressed in percentage terms, rounded up to two decimal places.

the fact that in the period January 1999-June 2000 its nominal exchange rate was not fixed yet. The next three columns generally show values below 1, indicating an increase in the volatility of the discount factors (for the Eurozone, for the corresponding country, and for the sum of the two)²¹. Finally, the last column presents changes in the overall ratio of real exchange rate volatility over the sum of discount factor volatilities. Therefore, this

²¹The exceptions being Belgium and Greece, where stock market returns imply less volatile discount factors after the introduction of the Euro.

ratio shows the mirror image of the risk-sharing index (see equation 13). Values higher than 1 indicate that this ratio was several times higher before the introduction of the Euro for each country in the dataset. Moreover, the decrease of this ratio (and the corresponding increase in the risk-sharing index) was not identical for all countries that adopted the Euro: it was just a little bit higher for Greece (2.50) and Belgium (3.73), while much higher for Italy (86.39) or Spain (172.78) in the pre-Euro period²². In sum, the relative volatility levels just strengthen the evidence from Tables 6 and 7: a strong decrease in real exchange rate volatility and a slight increase in (the sum of) discount factors volatility yielding an immense decrease in their ratio.

Unambiguously, the adoption of the Euro influenced both components of the risk-sharing index: directly, it led to a fall in real exchange rate volatility across Eurozone countries, and indirectly, it contributed to an increase in discount factor(s) volatility. While we elaborated broadly on the first effect above, here we pay more attention to the latter effect. Therefore, we present the discount factor loadings (calculated as $\mu^d \Sigma^{-1}$ and $\mu^f \Sigma^{-1}$) in Table 9. This table contains three rows for each country: one for each source of shocks in the model. The first row refers to the discount factor loading on the Eurozone stock market shocks (domestic stock market shock), the second to the real exchange rate shock (between the Eurozone and the corresponding country), and the third to the stock market shock of the corresponding (foreign) country. In line with equation 25, domestic and foreign discount factors load equally on each of the stock market shocks, and domestic discount factor loads on the exchange rate shocks by one more than the foreign discount factor. The last point implies that the difference

²²The main reason for the very small change in the case of Greece is its late adoption of the Euro. For Belgium this is primarily due to its much calmer stock market index BEL 20 in the Euro period.

Table 8: Relative Volatility Levels (pre-Euro period/Euro period)

Country	X-Rate	Eurozone DF	Country DF	Sum of DFs	X-Rate/DFs
AUS	7.88	0.25	0.25	0.25	31.09
BEL	13.98	3.73	3.77	3.75	3.73
FIN	10.81	0.29	0.28	0.29	37.71
FRA	19.56	0.55	0.55	0.55	35.63
GER	18.78	0.24	0.24	0.24	77.62
GRE	3.86	1.54	1.55	1.54	2.50
IRE	8.78	0.22	0.22	0.22	39.82
ITA	43.63	0.49	0.51	0.51	86.39
LUX	14.61	0.90	0.91	0.91	16.08
NL	8.05	0.69	0.69	0.69	11.56
POR	10.63	0.48	0.49	0.49	21.79
SPA	31.61	0.18	0.19	0.18	172.78

Note: This table compares the volatility of the risk-sharing index components before and after the introduction of the Euro. It presents ratios for the volatility level in the pre-Euro period over the volatility level in the Euro period for each country in the Eurozone. Values above 1 indicate a decrease in the variability of the corresponding variable after the introduction of the Euro. The first three columns contain figures for real exchange rate volatility (Σ^{ee}), and discount factor volatilities ($\mu^d \Sigma^{-1} \mu^d$ and $\mu^f \Sigma^{-1} \mu^f$). The last two columns contain figures for the sum of discount factor volatilities and for the ratio between unshared risk and total risk in the two countries. All figures are calculated as ratios of the annualized standard deviations for the corresponding variables.

between the two discount factors at each point in time equals the exchange rate. Moreover, two findings in Table 9 deserve particular attention.

First, most loadings (especially those on exchange rate shocks) are higher (in absolute value terms) compared to those given in Brandt et al. (2006). Second, the loadings on the exchange rate shocks attain *extremely* high values in the second period (1999-2005). This latter finding means that both discount factors load much more on the exchange rate shocks relative to the

stock market shocks. In fact, both of these findings come as a result of the extremely low volatility of the real exchange rate. Since the discount factors are calculated by the following formula $\mu^d \Sigma^{-1}$, low values for the denominator (variability of the real exchange rate) imply high values for whole term (discount factor loading). Thus, limited volatility of the real exchange rate explain why all discount factor loadings in the period preceding the introduction of the Euro are higher than in Brandt et al. (2006)²³. In turn, the extremely low real exchange rate variability is the main reason for the extremely high values for the exchange rate shock loadings after the introduction of the Euro. Finally, some of the discount factor loadings in Table 9 change sign. For example, some stock markets with relatively low Sharpe ratio are associated with negative discount factor loadings. This reflects the fact that certain markets are strictly dominated by (a combination of) other assets present in the system. In fact, investors that assets in some of these “dominated” markets forego better risk-return profiles (investment opportunities) available in other markets. Therefore, this sub-optimal (anomalous) behavior is reflected in the anomalous sign for the discount factor loadings on these assets (markets)²⁴.

3.2 Emerging Economies

In this section, we show the limitations of the stochastic discount factor approach to international risk-sharing under fixed nominal exchange rates with

²³In fact, low variability for the real exchange rate implies low values for most terms in the variance-covariance matrix used in the calculation of all discount factors.

²⁴This sub-optimal situation is not allowed in the equilibrium asset pricing model. In fact, the model suggests that shocks on all stock markets will be associated with positive loadings in equilibrium: favorable (negative) shock leads to a decrease (increase) in marginal utility growth. Clearly, this framework does not allow for dominated assets in equilibrium.

Table 9: Discount Factor Loadings: Eurozone Countries

1993-1998			1999-2005		
Country		Eurozone	Country	Eurozone	Country
AUS	dz^d	11.23	11.23	1.73	1.73
	dz^e	60.92	59.92	2148.07	2147.07
	dz^f	-4.72	-4.72	14.76	14.76
BEL	dz^d	3.02	3.02	0.67	0.67
	dz^e	5.92	4.92	132.01	131.01
	dz^f	2.77	2.77	-1.10	-1.10
FIN	dz^d	2.03	2.03	-5.56	-5.56
	dz^e	11.51	10.51	856.79	855.79
	dz^f	3.49	3.49	7.77	7.77
FRA	dz^d	19.44	19.44	2.50	2.50
	dz^e	-4.45	-5.45	-2015.42	-2016.42
	dz^f	-13.56	-13.56	-1.14	-1.14
GER	dz^d	3.22	3.22	11.03	11.03
	dz^e	8.35	7.35	-2518.02	-2519.02
	dz^f	1.90	1.90	-7.26	-7.26
GRE	dz^d	7.48	7.48	-1.41	-1.41
	dz^e	-33.55	-34.55	-93.99	-94.99
	dz^f	-1.64	-1.64	0.80	0.80
IRE	dz^d	-1.44	-1.44	10.86	10.86
	dz^e	18.88	17.88	1240.52	1239.52
	dz^f	9.74	9.74	-2.69	-2.69
ITA	dz^d	4.26	4.26	0.02	0.02
	dz^e	-10.78	-11.78	1428.25	1427.25
	dz^f	1.04	1.04	-0.99	-0.99
LUX	dz^d	1.53	1.53	-1.96	-1.96
	dz^e	17.16	16.16	850.10	849.10
	dz^f	8.74	8.74	2.62	2.62
NL	dz^d	-4.41	-4.41	7.78	7.78
	dz^e	21.49	20.49	500.25	499.25
	dz^f	10.66	10.66	-6.49	-6.49
POR	dz^d	4.87	4.87	-0.72	-0.72
	dz^e	-48.29	-49.23	1243.58	1242.58
	dz^f	-0.50	-0.50	4.57	4.57
SPA	dz^d	4.57	4.57	-1.26	-1.26
	dz^e	-18.95	-19.95	4546.58	4545.58
	dz^f	0.51	0.51	-0.42	-0.42

Note: This table presents figures for the discount factor loadings in the bilateral setting. The loadings for the Eurozone (domestic) discount factor are given by $\mu^{d'}\Sigma^{-1}$ and the corresponding loadings for the Eurozone countries (foreign) discount factors are given by $\mu^{f'}\Sigma^{-1}$. The row marked dz^d contains figures for discount factor loadings on the Eurozone (domestic) stock market shocks, row dz^e refers to discount factor loadings on the foreign exchange market shocks, and row dz^f refers to discount factor loadings on the stock market shocks for the corresponding country in the Eurozone. The first two columns contain statistics for the pre-Euro period (January 1993-December 1998), while the last two columns contain statistics for the Euro-period (January 1999-December 2005).

a second example. In the past two decades many developing or emerging countries adopted regimes of fixed or very rigid exchange rates against some of the major world currencies, mainly against the dollar. The motivations for these moves differed from country to country, ranging from macroeconomic price stabilization, a desire to attract foreign capital, to promotion of more international trade. Common for all these cases is that both nominal and real exchange rates became much less variable against the major world currencies. Here we investigate whether these (quasi) fixed nominal exchange rate episodes had a significant impact on the stochastic discount factor measures of international risk-sharing.

3.2.1 Data Sources

We collect monthly data on 6 emerging economies: Argentina, Chile, South Korea, Malaysia, Mexico, and Thailand in the period 1993-2005. Additionally, we collect data on Japan and UK for comparison. We use USA as the anchor country in this framework. Moreover, we separate the time period into two subperiods (1993-1998 and 1999-2005), exactly as in the case of the Eurozone. For the calculation of excess stock market returns, we take widely used stock market indices for the corresponding countries. A complete list of all stock market indices can be found in Table 10²⁵.

Data on one-month Eurocurrency deposit rates for these emerging markets was not available in Datastream. Therefore, we include interest rates series on one-month domestic bank deposits to approximate for the risk-free rate. Nominal exchange rate are measured as domestic currency units per US dollar. Finally, inflation data comes from the CPIs of the corresponding

²⁵In case we cannot collect comparable data on the most widely used index for certain country, we include an alternative index. For example, we include BURCAP instead of Merval for Argentina.

Table 10: Stock Market Indices

Country	Index
ARG	BURCAP
CHI	IPSA (Indice de Precios Selectivo de Acciones)
KOR	KOSPI (Korea Composite)
MAL	KLCI (Kuala Lumpur Composite Index)
MEX	IPC 35 (Indice de Precios y Cotizaciones)
THA	SET 50 (Stock Exchange of Thailand)
UK	FTSE All
JAP	NIKKEI 225
USA	S&P 500

Note: This table contains a list of leading stock market indices used in the analysis for the emerging economies. The upper panel lists the stock market indices for the emerging economies, while the lower panel lists the stock market indices for UK, Japan, and USA.

countries measured at mid-month. For Japan, UK and USA we use the same data sources as in Brandt et al. (2006).

3.2.2 Summary Statistics

The summary statistics are presented in Tables 11 and 12. The figures in Table 11 demonstrate that stock market returns in emerging economies are much more volatile than in industrial countries. In fact, the annualized standard deviations for emerging economies (columns 2 and 4) are typically double the figures for industrial economies²⁶. Moreover, columns 1 and 3 make clear that several stock market downfalls happened in this period. The effect of the financial crises in the second half of the 1990s is evident from

²⁶The only exception being Chile, where the figures are comparable to those in the industrial countries.

Table 11: Summary Statistics: Stock Market Returns (Annualized)

country	1993-1998			1999-2005		
	mean	std dev	Sharpe ratio	mean	std dev	Sharpe ratio
ARG	9.06	34.08	0.27	18.44	44.80	0.41
CHI	0.49	26.49	0.02	9.26	17.37	0.53
KOR	-10.95	33.98	-0.32	12.81	35.22	0.36
MAL	-2.62	39.71	-0.07	6.94	26.78	0.26
MEX	-2.84	32.79	-0.09	19.20	27.58	0.69
THA	-15.76	44.11	-0.36	12.19	32.61	0.37
UK	5.49	12.67	0.43	-2.47	16.61	-0.15
JAP	-1.55	21.99	-0.07	2.09	21.70	0.10
USA	12.60	12.17	1.03	-1.19	17.01	-0.07

Note: This table contains summary statistics for stock market excess returns for emerging economies and industrial countries. The upper panel refers to the group of emerging economies (Argentina, Chile, Korea, Malaysia, Mexico, and Thailand), while the lower panel refers to the group of industrial countries (UK, Japan, and USA). The first three columns contain statistics for period January 1993-December 1998, while the last three columns contain statistics for period January 1999-December 2005. Excess returns are calculated over one-month domestic bank deposit rates for the emerging economies, and over one-month Eurocurrency deposit rates for the industrial countries. All data-series are retrieved from Datastream and all summary statistics are annualized and expressed in percentage terms (rounded to two decimal places).

this table. In fact, most emerging economies display negative values for real excess stock market returns (columns 1 and 3) in the first subperiod. A similar conclusion applies to the industrial countries in the second period, reflecting the burst of the dotcom bubble in 2000-2001. Finally, the Sharpe ratios closely follow the trend in mean excess returns, i.e. they increase for all emerging markets and Japan in the second period and decrease for the UK and the USA²⁷.

²⁷Hence, the changes in the Sharpe ratios largely reflect the two stock market downturns in the corresponding periods: the emerging market crises in the first period and the dotcom-bubble in the second period.

Table 12 displays summary statistics for excess returns on the foreign exchange market. The negative values for most emerging economies in columns 1 and 3 indicate a net loss on investments in foreign exchange in these countries²⁸. Moreover, excess returns volatility levels differ widely across countries and time subperiods. In general, foreign exchange excess returns vary much more in emerging economies compared to Japan or the UK. However, there are subperiods for certain emerging economies when exactly the opposite is true. For example, Argentina and Chile in the first subperiod, and Malaysia in the second subperiod display extremely low volatility in foreign exchange returns.

Finally, Table 13 presents stock market return correlations with the USA (S&P 500). The correlation coefficients for emerging economies are comparable to those for industrial countries. In fact, most stock markets correlations in emerging economies fall inbetween the values for Japan and UK.

3.2.3 Results for Emerging Economies

The results for the (bilateral) risk-sharing index between the corresponding countries and the USA are displayed in Table 14. Several observations from this table deserve attention. First, all values are very high, suggesting almost perfect levels of international risk-sharing of all countries with the USA. More precisely, there are three episodes of literally perfect risk-sharing: Argentina and Chile in the first period and Malaysia in the second period. All three episodes reflect the fact that nominal exchange rates were fixed against the US dollar in the corresponding periods. Second, emerging economies achieve similar or even higher levels of risk-sharing with the USA than Japan or the UK. In fact, the risk-sharing index for Argentina and

²⁸In fact, this partly reflects some peso-problem related issues.

Table 12: Summary Statistics: Foreign Exchange Returns (Annualized)

	1993-1998		1999-2005	
country	mean	std dev	mean	std dev
ARG	-3.30	0.84	-17.43	27.15
CHI	-8.28	5.61	-2.37	9.79
KOR	-6.99	19.25	0.95	7.98
MAL	-5.90	15.57	-0.65	0.52
MEX	-15.68	19.36	1.46	8.85
THA	-6.45	18.47	-1.19	7.45
UK	0.31	7.35	-0.71	6.87
JAP	3.05	14.04	0.90	10.67

Note: This table contains summary statistics for excess returns on the foreign exchange market for the group of emerging economies and industrial countries. USA is used as the benchmark (domestic) country, and the US dollar is the benchmark currency in all calculations. Excess returns on the foreign exchange market are calculated as (real) deviations from uncovered interest rate parity ($\theta^e + r^f - r^d$): borrowing at the US interest rate, converting to the foreign currency, investing on the foreign interest rate, and converting the proceeds back to US dollars. The first two columns contain statistics for the period January 1993-December 1998, while the last two columns contain statistics for the period January 1999-December 2005. All data-series are retrieved from Datastream and all summary statistics are annualized and expressed in percentage terms (rounded to two decimal places).

Table 13: Stock Market Correlations with S&P 500

Country	1993-1998	1999-2005
ARG	0.549	0.272
CHI	0.480	0.488
KOR	0.238	0.677
MAL	0.371	0.299
MEX	0.481	0.666
THA	0.371	0.469
UK	0.682	0.823
JAP	0.404	0.501

Note: This table presents stock market correlations for the group of emerging and industrial countries. All correlations are measured by the coefficient of correlation between monthly returns on each stock market index and monthly returns on S&P 500. The first column contains statistics for the period January 1993-December 1998, while the second column contains statistics for the period January 1999-December 2005. All figures are rounded up to three decimal places.

Chile is higher, while for the other emerging economies it is very similar to the index for Japan or the UK in the first subperiod. In the second subperiod, the risk-sharing index for almost every emerging economy is higher than the index for Japan or the UK²⁹. Hence, the stochastic discount factor approach suggests that emerging markets achieved more international risk-sharing with the USA, than the much more globally integrated economies like Japan and the UK. In sum, the results from this analysis imply that the nominal exchange rate regime has a more profound impact on the risk-sharing measure than do all other economic characteristics of the corresponding country-pairs.

²⁹The only exception is Argentina whose risk-sharing index is slightly lower than that for the UK.

Table 14: Risk-Sharing Index

Country	1993-1998	1999-2005
ARG	0.999997	0.968468
CHI	0.999588	0.992272
KOR	0.988865	0.993513
MAL	0.991606	0.999966
MEX	0.992549	0.997553
THA	0.990011	0.993846
UK	0.997864	0.973277
JAP	0.992833	0.884481

Note: This table presents figures for the bilateral risk-sharing index between each country and USA. The risk-sharing index is calculated according to the following formula: $RSI = 1 - \frac{\Sigma^{ee}}{\mu^d/\Sigma - 1\mu^d + \mu^f/\Sigma - 1\mu^f}$. USA is the benchmark (domestic) country in all calculations. The first column contains statistics for the period January 1993-December 1998, while the second column contains statistics for the period January 1999-December 2005. All figures are rounded up to six decimal places.

3.2.4 Understanding the Results

The results for the risk-sharing index in Table 14 convey a very counterintuitive message. Despite the widely documented observation that Japan and the UK are more financially and economically integrated with the USA than the group of emerging economies, the stochastic discount factor approach suggests the opposite. In fact, we consider this result to be a major limitation of the framework presented in Brandt et al. (2006) and in this paper. In order to understand the reasons underlying this result, we analyze the behavior of the two risk-sharing index components in the rest of this section.

Figures 5 and 6 show the behavior of the nominal and real exchange rates for the 6 emerging economies, respectively. The vertical axis measures monthly percentage changes in the exchange rate, while the vertical line in

the middle of the figure denotes a separation of the complete time period into two subperiods. In general, these economies enjoyed periods of relatively stable currencies, characterized by very small fluctuations in their nominal exchange rates against the US dollar, which were interrupted by several crises. Three main crisis episodes, accompanied by sharp exchange rate fluctuations, can be seen in this figure: the Mexican peso crisis in 1994-1995, the East Asian financial crisis in the second half of 1997 (affected Thailand, Korea, and Malaysia), and the Argentine financial crisis in late 2001 - early 2002. As can be seen in the figure, each of these crises was characterized by large falls in the value of the domestic currency (sometimes followed by smaller reversals) against the US dollar. Furthermore, Figure 6 shows the behavior of the real exchange rate, measured as monthly changes in nominal exchange rates corrected for the inflation differential. As already documented in the example with the Eurozone, Figure 6 shows that nominal and real exchange rates are very closely related. In fact, the evidence in these figures only strengthens the argument that the variability of the real exchange rate crucially depends on the behavior of the nominal exchange rate.

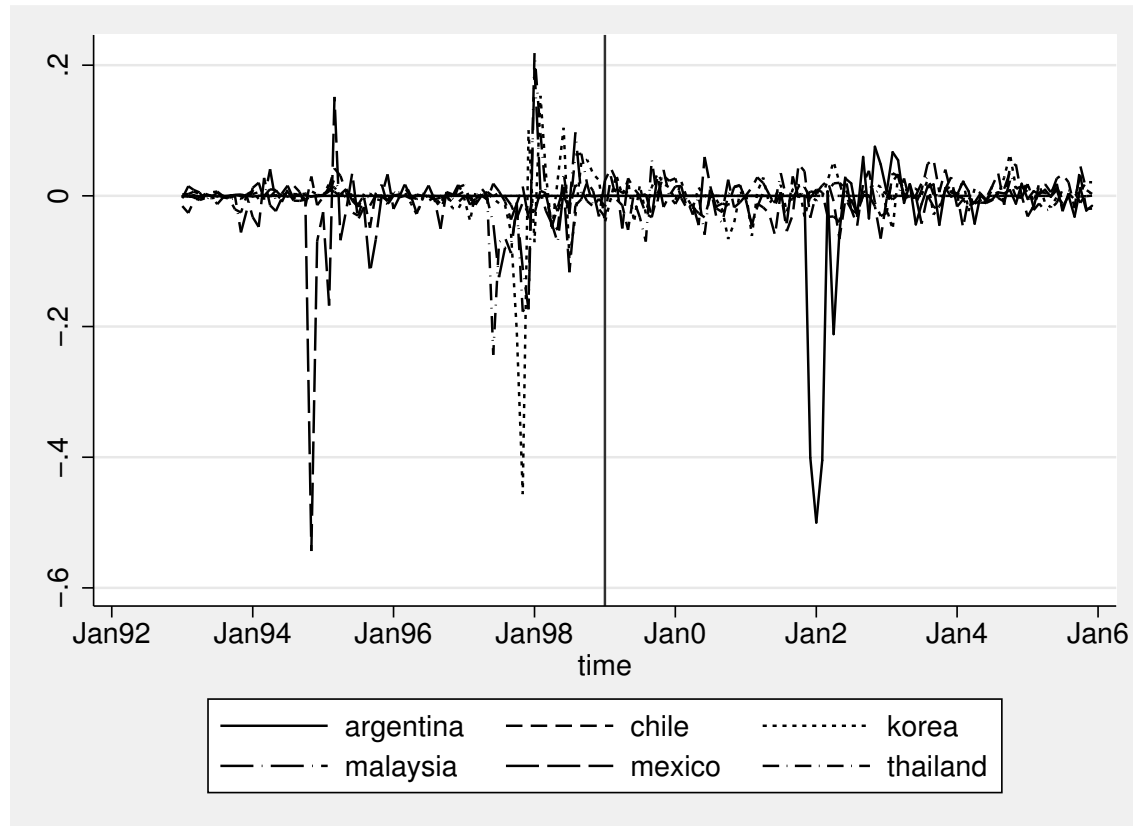


Figure 5: Nominal Exchange Rates (1993-2005)

Note: This figure presents monthly changes in the nominal exchange rates for the group of emerging and industrial countries over the period January 1993-December 2005. The US dollar is used as benchmark currency. Positive changes in the nominal exchange rate indicate depreciation of the US dollar (appreciation of the currency for the corresponding country). The separation into two time subperiods is marked by a vertical line (January 1999).

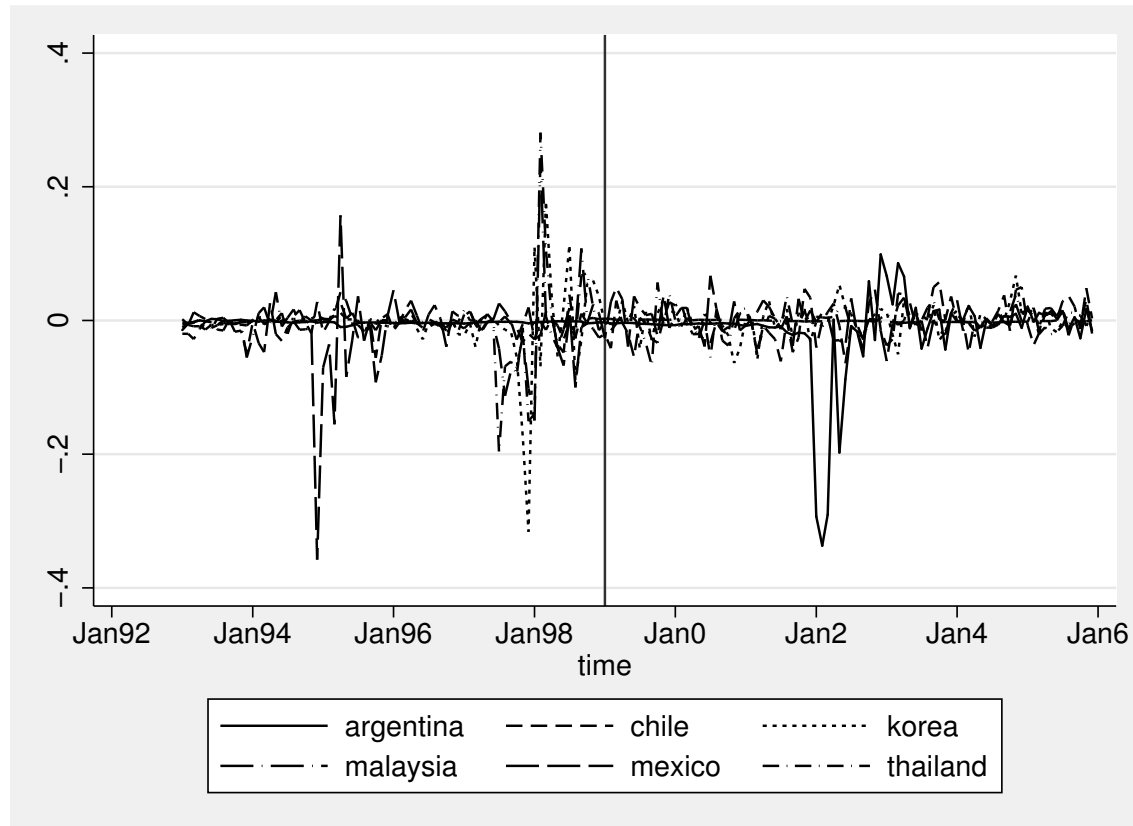


Figure 6: Real Exchange Rates (1993-2005)

Note: This figure presents monthly changes in the real exchange rates for the group of emerging and industrial countries over the period January 1993-December 2005. The US dollar is used as benchmark currency. Real exchange rate changes are calculated as nominal exchange rate changes corrected for the inflation differential. Positive changes in the real exchange rate indicate real depreciation of the US dollar (real appreciation of the currency for the corresponding country). The separation into two time subperiods is marked by a vertical line (January 1999).

Having visualized the relation between nominal and real exchange rate volatility, we turn to an in-depth analysis of the two components of the risk-sharing index here. Table 15 shows the real exchange rate volatility figures for each country in the dataset. All figures refer to annualized standard deviations measured in percentage terms over the corresponding subperiod.

In general, real exchange rate volatility decreased for most emerging economies in the second compared to the first time subperiod, reaching even lower levels than for Japan or the UK. The only exceptions are Argentina and Chile. In fact, the former experienced substantially higher real exchange in the second subperiod mainly due to the currency crisis in 2001-2002 and the subsequent abandonment of the fixed one-to-one Argentine peso - US dollar parity. Moreover, two extremely low figures deserve special attention: Argentina in the first and Malaysia in the second subperiod. Both of these figures refer to fixed exchange rate episodes: the Argentine peso was pegged to the US dollar from 1991 till December 2001, and the Malaysian ringgit was fixed against the US dollar in the aftermath of the East Asian financial crisis in 1998. In turn, exactly these fixed exchange rate episodes correspond with the perfect risk-sharing scores in Table 14.

In Table 16 we present figures for the second component of the risk-sharing index - discount factor volatility. All results are based on bilateral calculations, where the first two columns refer to the anchor (domestic) country (USA for each country pair in this case), and the last two columns to the other country. As can be seen from the comparison of the results in columns 2 and 4 relative to columns 1 and 3, discount factor volatility decreased for most countries in the second compared to the first subperiod.

Finally, equation 13 indicates that the risk-sharing index changes if either the real exchange rate volatility or the discount factor volatility changes. In

Table 15: Real Exchange-Rate Volatility (Annualized Values)

Country	1993-1998	1999-2005
ARG	0.84	22.68
CHI	5.57	9.86
KOR	19.16	7.57
MAL	15.46	0.78
MEX	19.22	7.87
THA	18.37	6.60
UK	7.30	7.76
JAP	13.94	10.29

Note: This table presents figures for real exchange rate volatility for the group of emerging and industrial countries. Volatility is measured by the annualized standard deviation calculated from monthly changes in the real exchange rate against the US dollar (US dollar is used as benchmark currency). The first column contains statistics for the period January 1993-December 1998, while the second column contains statistics for the period January 1999-December 2005. All figures are expressed in percentages terms, rounded up to two decimal places.

order to see the relative contribution of these two components for the risk-sharing index, we present comparisons of their values for the two subperiods in Table 17. Table 17 presents ratios of volatility in the first subperiod over volatility in the second subperiod. Hence, values below 1 indicate volatility increase in the second compared to the first subperiod. As there was no systematic change comparable to the adoption of the Euro in the previous example, the values in this figure do not suggest any clear trend in real exchange rate volatility. In fact, it increased for two, decreased for the other four emerging markets, and remained stable for Japan and especially for the UK. However, two extreme figures deserve attention: first, there was a sharp increase in Argentine real exchange rate volatility, and second, Malaysian real exchange rate was 20 times higher in the first compared to the second period. Finally, the three middle columns show a clear trend in

Table 16: Discount Factor Volatility (Annualized Values)

	USA DF	USA DF	Country DF	Country DF
Country	1993-1998	1999-2005	1993-1998	1999-2005
ARG	399.98	78.45	400.81	100.77
CHI	192.15	77.46	196.49	81.06
KOR	124.60	66.96	131.49	65.96
MAL	116.31	94.45	122.26	95.14
MEX	151.79	112.95	162.93	111.92
THA	126.63	58.32	132.89	60.69
UK	111.70	32.04	111.66	35.04
JAP	117.34	22.26	115.56	20.52

Note: This table presents figures for discount factor volatility for each of the group of emerging and industrial countries. Volatility is calculated by the annualized standard deviation of the discount factor according to equation 21: $\frac{1}{dt} \sigma^2(d \ln \Lambda^i) = \mu^{i'} \Sigma^{-1} \mu^i$. USA is used as benchmark (domestic) country in all calculations. The first column contains statistics for the period January 1993-December 1998, while the second column contains statistics for the period January 1999-December 2005. All figures are expressed in percentage terms, rounded up to two decimal places.

discount factor volatility: they became less variable for each country pair in the second subperiod. This reflects the fall in the highest Sharpe ratio shock documented in Table 11³⁰.

We close the discussion of the results from the risk-sharing index with a short note on the discount factor loadings in Table 18. Similar as in the case with the Eurozone, this table contains three rows for each country: one for each source of shocks in the model. The first row refers to the discount factor loading on the domestic stock market shock (USA for all countries

³⁰Discount factor volatility crucially depends on the excess return shock with the highest Sharpe ratio. Although the Sharpe ratios for all emerging markets increased in the second period, discount factor volatility decreased because of the dramatic fall in the highest Sharpe ratio from the first period - the US stock market shock (see Table 11).

Table 17: Relative Volatility Levels (1993-1998 period/1999-2005 period)

Country	X-Rate	USA DF	Country DF	Sum of DFs	X-Rate/DFs
ARG	0.037	5.10	3.98	4.47	0.01
CHI	0.56	2.48	2.42	2.45	0.23
KOR	2.53	1.86	1.99	1.93	1.31
MAL	19.93	1.23	1.28	1.26	15.84
MEX	2.44	1.34	1.46	1.40	1.75
THA	2.78	2.17	2.19	2.18	1.27
UK	0.94	3.49	3.19	3.33	0.28
JAP	1.35	5.27	5.63	5.44	0.25

Note: This table compares the volatility of the risk-sharing index components between the first and the second subperiod. It presents ratios for the volatility level in the period 1993-1998 over the volatility level in the period 1999-2005. Values above 1 indicate a decrease in the variability of the corresponding variable in the latter period. The first three columns contain figures for real exchange rate volatility (Σ^{ee}), and discount factor volatilities ($\mu^{d'}\Sigma^{-1}\mu^d$ and $\mu^{f'}\Sigma^{-1}\mu^f$). The last two columns contain figures for the sum of discount factor volatilities and for the ratio between unshared risk and total risk in the two countries. USA is used as benchmark (domestic) country in all calculations. All figures are calculated as ratios of the annualized standard deviations for the corresponding variables.

here), the second to the real exchange rate shock (between the USA and the corresponding country), and the third to the (foreign) stock market shock of the corresponding country. In line with equation 25 and the evidence for the Eurozone, domestic and foreign discount factors load equally on each of the stock market shocks, and domestic discount factor loads on the exchange rate shocks by one more than the foreign discount factor.

Finally, two observations deserve particular attention. First, the discount factor loadings on the US stock market change sign in most cases³¹: from positive in the first they turn negative in the second period. This

³¹The loading on the US stock market stays positive only in the calculations against the UK because the UK stock market has even more negative Sharpe ratio for the second period (see Table 11).

Table 18: Discount Factor Loadings: Emerging Economies

		1993-1998		1999-2005	
Country		USA	Country	USA	Country
ARG	dz^d	6.27	6.27	-0.73	-0.73
	dz^e	-466.85	-467.85	-3.13	-4.13
	dz^f	-2.01	-2.01	0.40	0.40
CHI	dz^d	11.40	11.40	-2.82	-2.82
	dz^e	-27.40	-28.40	-3.11	-4.11
	dz^f	-2.30	-2.30	5.15	5.15
KOR	dz^d	10.10	10.10	-4.84	-4.84
	dz^e	-2.08	-3.08	2.39	1.39
	dz^f	-1.45	-1.45	2.83	2.83
MAL	dz^d	9.68	9.68	-1.91	-1.91
	dz^e	-1.81	-2.81	-113.80	-114.80
	dz^f	-1.12	-1.12	1.89	1.89
MEX	dz^d	11.76	11.76	-7.05	-7.05
	dz^e	-5.19	-6.19	1.23	0.23
	dz^f	-1.10	-1.10	6.08	6.08
THA	dz^d	10.90	10.90	-2.45	-2.45
	dz^e	-0.12	-1.12	-3.68	-4.68
	dz^f	-1.74	-1.74	2.14	2.14
UK	dz^d	11.77	11.77	2.65	2.65
	dz^e	-0.52	-1.52	-3.13	-4.13
	dz^f	-4.48	-4.48	-4.03	-4.03
JAP	dz^d	10.48	10.48	-1.40	-1.40
	dz^e	0.50	-0.50	1.22	0.22
	dz^f	-2.65	-2.65	1.09	1.09

Note: This table presents figures for the discount factor loadings in the bilateral setting. The loadings for the USA (domestic) discount factor are given by $\mu^{d'}\Sigma^{-1}$ and the corresponding loadings for the other (foreign) discount factors are given by $\mu^{f'}\Sigma^{-1}$. The row marked dz^d contains figures for discount factor loadings on the USA (domestic) stock market shocks, row dz^e refers to discount factor loadings on the foreign exchange market shocks, and row dz^f refers to discount factor loadings on the stock market shocks for the corresponding (emerging or industrial) country. The first two columns contain statistics for the period January 1993-December 1998, while the last two columns contain statistics for the period January 1999-December 2005.

reflects the dramatic fall in the Sharpe ratio on the US stock market (the negative excess returns earned on the US stock market in the second period), which becomes “dominated” by the emerging markets’ Sharpe ratios in second period. Second, the exchange rate loadings for Argentina and Chile in the first, and for Malaysia in the second subperiod obtain extremely high (absolute) values. In line with the discussion for the Eurozone, these loadings reflect rigidity in the corresponding exchange rates.

4 Discussion

In this study, we showed that the risk-sharing index leads to absurdly high results when applied to countries with fixed nominal exchange rate regimes. The main point to show here was that a large “disconnect” between the asset side (denominator) and the macroeconomic side (numerator) drives the results of the risk-sharing index. In fact, we showed that just by elimination of the asset component in the real exchange rate, the stochastic discount factor “automatically” leads to absurdly high levels of risk-sharing. First, we demonstrated that the stochastic discount factor model implies literally perfect risk-sharing across all countries from the Eurozone after the introduction of the Euro. This result reflected the nominal exchange rate regime and not other economic similarities or the high level of economic integration among this set of countries: the comparison with the pre-Euro period demonstrated that the complete elimination of the nominal exchange rates had a profound effect for countries that already had very high risk-sharing indices. Second, we further strengthened this evidence with the analysis for the group of emerging markets. Irrespective of many underlying differences between the countries considered, fixed nominal exchange rates “automatically” implied perfect levels of international risk-sharing.

5 Concluding Remarks

In this study we present some new evidence of the stochastic discount factor approach to international risk-sharing. At the beginning, we present the theoretical framework that links the minimum-variance discount factors in two countries with the corresponding real exchange rate. We elaborate on the calculation of the discount factors and the construction of the risk-

sharing index. Subsequently, we investigate the importance of the nominal exchange rate regime for the risk-sharing measures.

We arrive at two main conclusions: first, the international risk-sharing index calculated in this way crucially depends on the behavior of the nominal exchange rate. In fact, we demonstrate that this approach “mechanically” leads to (almost) perfect international risk-sharing under fixed nominal exchange rate regimes. This is true for all countries in the Eurozone, as well as the emerging economies like Argentina and Malaysia that had (almost) fixed exchange rates against the US dollar. Second, this approach suggests more integration in terms of better risk-sharing between several emerging economies and the USA than between the UK and the USA. Having demonstrated that this counterintuitive result is almost entirely driven by the rigidity in the nominal exchange rate, we consider it as a major limitation. In fact, the over-dependence of this approach on the nominal exchange rate regime restricts its application in cross-country risk-sharing comparisons.

In sum, the evidence in this study demonstrates that fixing the nominal exchange rate directly implies (almost) fixed real exchange rates, and, according to the stochastic discount factor model, perfect international risk-sharing as well. We conclude that real exchange rates might be very smooth (as in this case), but this does not necessarily imply perfect risk-sharing across countries.

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A Excess Returns in Bilateral Setting

This section presents formulae for excess returns in the bilateral framework. First, we present a general derivation for excess return formulae for each asset. Second, we derive vectors of expected excess returns for each country. A general distinction is made between formulae for domestic country (with superscript d) and foreign country (with superscript f) assets. USA is the domestic country in the first two country-pairs, and UK is the domestic country in the last country-pair.

A.1 Excess Return Processes for Domestic Investor

The investors in the domestic country face the following three types of excess return shocks: domestic stock, foreign bond, and foreign stock.

The excess returns on domestic stock are calculated difference between returns on the domestic stock market and the risk-free rate on domestic bond:

$$\frac{dS^d}{S^d} - \frac{dB^d}{B^d} = (\theta^d - r^d)dt + dz^d \quad (26)$$

The corresponding excess return on the foreign bond for the domestic investor is given as the difference between foreign bond return expressed in domestic currency and domestic bond return. Hence, although foreign bond is risk-free for the foreign investor, it is risky asset from the perspective of the domestic investor due to the currency risk it contains.

$$\frac{d(eB^f)}{eB^f} - \frac{dB^d}{B^d} = \frac{de}{e} + r^f dt - r^d dt = (\theta^e + r^f - r^d)dt + dz^e \quad (27)$$

Finally, excess returns on foreign stock for the domestic investor are calculated as the difference between returns on foreign stock and returns on

foreign bonds when both are expressed in domestic currency units. Excess returns on the foreign stock for the domestic investor is calculated as follows:

$$\begin{aligned}
\frac{d(eS^f)}{eS^f} - \frac{d(eB^f)}{eB^f} &= \frac{dS^f}{S^f} + \frac{de}{e} \frac{dS^f}{S^f} - \frac{dB^f}{B^f} - \frac{de}{e} \frac{dB^f}{B^f} \\
&= \left(1 + \frac{de}{e}\right) \left(\frac{dS^f}{S^f} - \frac{dB^f}{B^f}\right) \\
&= (1 + \theta^e dt + dz^e)(\theta^f dt + dz^f - r^f dt) \\
&= \theta^f dt + dz^f - r^f dt \\
&\quad + \theta^e dt \theta^f dt + \theta^e dt dz^f - \theta^e dt r^f dt + dz^e \theta^f dt + dz^e dz^f - dz^e r^f dt \\
&= (\theta^f - r^f) dt + dz^e dz^f + dz^f \\
&= (\theta^f - r^f + \Sigma^{ef}) dt + dz^f
\end{aligned} \tag{28}$$

A.2 Excess Return Processes for Foreign Investor

The investors in the foreign country face the following three types of excess return shocks: domestic bond, domestic stock, and foreign stock.

The excess return on the domestic bond for the foreign investor is given as the difference between domestic bond return expressed in domestic currency and foreign bond return. Hence, although domestic bond is risk-free for the domestic investor, it is a risky asset from the perspective of the foreign investor due to the currency risk it contains. These excess returns are given as follows:

$$\begin{aligned}
\frac{d\left(\frac{B^d}{e}\right)}{\left(\frac{B^d}{e}\right)} - \frac{dB^f}{B_f} &= \left(\frac{dB^d}{B^d} - \frac{de}{e} + \frac{de_1^2}{e_1^2} - \frac{de}{e} \frac{dB^d}{B^d}\right) - \frac{dB^f}{B_f} \\
&= r^d dt - \theta^e dt - dz^e + \Sigma^{ee} dt - \theta^d dt r^d dt - r^f dt \\
&= (r^d - r^f - \theta^e + \Sigma^{ee}) dt - dz^e \\
&= -[(\theta^e + r^f - r^d - \Sigma^{ee}) dt + dz^e]
\end{aligned} \tag{29}$$

The excess returns on domestic stock from the perspective of foreign investor are calculated as difference between domestic stock and domestic bond returns, both translated into foreign currency:

$$\begin{aligned}
\frac{d\left(\frac{S^d}{e}\right)}{\frac{S^d}{e}} - \frac{d\left(\frac{B^d}{e}\right)}{\frac{B^d}{e}} &= \left(\frac{dS^d}{S^d} - \frac{de}{e} + \frac{de_1^2}{e_1^2} - \frac{de}{e} \frac{dS^d}{S^d}\right) \\
&\quad - \left(\frac{dB^d}{B^d} - \frac{de}{e} + \frac{de_1^2}{e_1^2} - \frac{de}{e} \frac{dB^d}{B^d}\right) \\
&= \frac{dS^d}{S^d} - \frac{dB^d}{B^d} - \frac{de}{e} \left(\frac{dS^d}{S^d} - \frac{dB^d}{B^d}\right) \\
&= \left(1 - \frac{de}{e}\right) \left(\frac{dS^d}{S^d} - \frac{dB^d}{B^d}\right) \\
&= (1 - \theta^e dt - dz^e)(\theta^d dt + dz^d - r^d dt) \\
&= \theta^d dt + dz^d - r^d dt - \theta^e dt \theta^d dt - \theta^e dt dz^d \\
&\quad + \theta^e dt r^d dt - dz^e \theta^d dt + dz^e dz^d - dz^e r^d dt \\
&= (\theta^d - r^d - \Sigma^{ed}) dt + dz^d
\end{aligned} \tag{30}$$

Finally, the excess returns that foreign investors get by investing on the foreign stock market are given as the difference between returns on foreign stock market and returns on foreign bond. Since the latter is a risk-free asset from the perspective of foreign investors. Hence, the foreign stock market excess returns are given by the following equation:

$$\frac{dS^f}{S^f} - \frac{dB^f}{B^f} = (\theta^f - r^f) dt + dz^f \tag{31}$$

A.3 Expected Excess Returns

This section presents the expected values for the excess return processes calculated in the previous two sections. The term in front of the dt term refers to the expected values in the continuous-time formulation employed

here. Therefore, domestic investor faces the following set of expected excess returns:

$$\mu^d = \begin{bmatrix} \theta^d - r^d \\ \theta^e + r^f - f^d \\ \theta^f - r^f + \Sigma^{ef} \end{bmatrix}$$

This vector stacks the expected values of the expected return processes given by equations 26 (domestic stock), 27 (foreign bond), and 31 (foreign stock).

The foreign investor faces a similar set of expected excess returns. The following vector stack the expected values of the expected return processes given by equations 29 (domestic bond), 30 (domestic stock), and 31 (foreign stock):

$$\mu^f = \begin{bmatrix} \theta^d - r^d - \Sigma^{ed} \\ -(\theta^e + r^f - r^d - \Sigma^{ee}) \\ \theta^f - r^f \end{bmatrix}$$