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## The rise of the 1% and the fall of the labor share: An automation-driven doom loop?

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# Working Paper Research

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The rise of the 1% and the fall of the labor share:  
an automation-driven doom loop?

by Arthur Jacobs



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## Abstract

I evaluate the link between automation and the rise in top income concentration when inequality matters for macro. The novel mechanism is that automation redistributes income towards high-wealth households who save more, which lowers the interest rate and incites firms to automate more. To operationalize this, I build a tractable heterogeneous-agent model (1) with wealth in the utility function as a luxury good, and (2) a firm-side choice on automation. I find that introducing realistic savings rate heterogeneity largely eliminates the need for ad hoc technology shifts. Rather, automation is the outcome of increased top income concentration, not just its driver.

Keywords: automation, wealth inequality, capitalist spirit, task-based production, heterogeneous-agent.

JEL-Codes: E25, J23, O33.

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## Non-technical summary

The study puts forward a novel explanation for the joint decline of the labor share of income and the rise of top wealth and income concentration in the United States. The explanation puts automation centre stage, and thus interprets the decreasing labor share as the displacement of labor from tasks in the production process by capital. Instead of imposing automation as an ad-hoc, purely technological process without economic antecedents, I propose to view it as the result of greater top income concentration. Drawing on old and new empirical evidence, I build a quantitative heterogeneous-agent model where wealthy, high-income households save at a higher pace. The result is a framework in which inequality matters for macro: greater permanent income inequality is not neutral, but raises aggregate savings, lowers the interest rate and makes capital more cost-effective vis-à-vis labor thus inciting firms to expand automation. Since automation favours high-wealth households by raising the return to wealth, a feedback loop between automation and top income concentration readily emerges.

To operationalize this mechanism, I insert non-homotheticity in household preferences through a capitalist-spirit motive. Wealth enters the utility function directly as a luxury good, ensuring that rich households accumulate at a higher pace for status reasons. Households are hit by stochastic, downward wealth mobility shocks through which their assets dissipate in full. This ensures that the asset supply is bounded, and that the model's wealth distribution is stationary and nondegenerate. The firm side of the model features task-based production: firms execute a continuum of tasks using either capital or labor. Since the relative capital-labor productivity schedule is continuous over the range of tasks, a lower capital factor cost expands the set of tasks which can cost-effectively be executed by capital. This contraction in the labor-executed set of tasks is how automation is modelled.

Calibrating the model for the United States, I find that it matches the existing evidence on savings rate heterogeneity well. I show that non-homothetic savings behaviour helps to explain the US labor share decline without relying on exogenous technology shocks. In its absence, the slowdown in (population) growth and the rise in wage inequality can only account for around a quarter of the decline in the headline labor share since 1980, but – for the same production technology – this improves to up to 70% with non-homothetic savings. Vice versa, automation helps to keep the return to wealth high even in the face of low growth because firms can tap into new uses for capital as it becomes abundant. As a result, the increase in top wealth concentration is better explained when automation is considered. As opposed to the technology shock story, automation induced endogenously by slower growth is compatible with the observed decline in the aggregate net investment rate and the fall of the interest rate. Depending on the technical scope for future automation, further slowdowns in (population) growth can cause explosive changes in top wealth concentration and the labor share because of the non-linearity generated by the mutual feedback effects between automation and top income concentration.

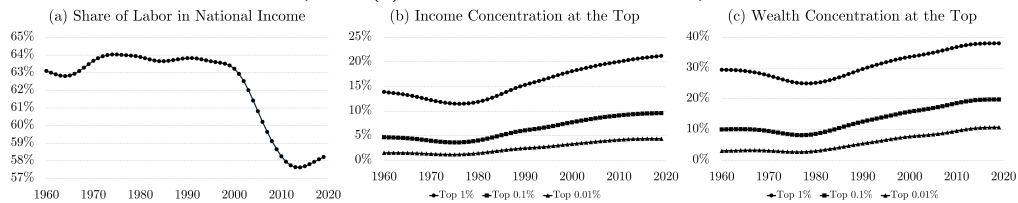
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## 1 Introduction

During the last four decades, the United States has witnessed strong shifts in both (1) the functional income distribution (between capital and labor), and (2) the personal income and wealth distribution (between households). Figure 1 depicts these trends. By now, it is an established fact that labor's share in the national income is considerably below its pre-1980 stable level.<sup>1</sup> Moreover, both income and wealth concentration at the top of the distribution have risen starkly.<sup>2</sup> Faced with these two distributional shifts, some macro studies have turned to the adoption of labor-replacing technology in the form of automation for their joint explanation. This involves two steps. First, the fall in the labor share of income is interpreted as automation. There is convincing empirical evidence that the displacement of labor by capital due to the adoption of new technologies indeed represents an important part of the decline of the labor share (Acemoglu, Lelarge, and Restrepo, 2020; Acemoglu and Restrepo, 2020; Bergholt et al., 2022). The second step consists out of the construction of a theoretical model where automation becomes the *cause* of the rise of top income and wealth concentration. Recently, Moll et al. (2022) show that automation can help explain the rise in top income and wealth concentration in the US via a rise in the return to wealth.

Figure 1: Evolution of selected variables in the US for 1960-2020: (1) the labor share of income, (2) income concentration at the top, and (3) wealth concentration at the top



Data sources. First panel: US Bureau of Labor Statistics (labor share statistic for the nonfinancial corporate sector). Second panel: World Inequality Database on Pre-Tax National Income by Tax Units (Income shares of top 1%, top 0.1% and top 0.01%). Third Panel: World Inequality Database on Net Wealth (Wealth shares of top 1%, top 0.1% and top 0.01%). All data are HP-filtered with  $\lambda$  of 100.

The striking question is then where this automation came from. Why is technology now labor-replacing after decades of neutrality? Why do firms now execute more tasks using capital, and less with labor? In short, why is this time different? In this paper, I propose to see the rise of top income concentration as the most important explanation. In this account, income inequality is not only the *consequence* of automation, but also its *cause*. The novel mechanism I introduce is that wealthy people save more, such that permanently greater top income concentration lowers the interest rate and incites firms to automate more. The possibility of a feedback loop readily emerges: automation redistributes income from poor to wealthy households, and this redistribution increases aggregate savings, which leads to more automation.

<sup>1</sup>Karabarbounis and Neiman (2014) were first to remark that the immobility of the factor shares had ceased to be a 'stylized fact of economic growth'. The 'labor share decline' narrative is not without contestation, however: the capitalization of intellectual property (Koh et al., 2020), the rise of 'pass-through' business (Smith et al., 2022) and, the treatment of self-employment income (Elsby et al., 2013) have all been invoked to (partially) invalidate the notion of a fundamental break with constancy. However, Grossman and Oberfield (2022) conclude in their review article that "there exists a fair amount of consensus that the labor share today however measured is well below its level of the 1970s and 1980s, at least in the US." (p. 99).

<sup>2</sup>Income is more concentrated at the very top of the US income distribution than forty years ago. Both individual tax returns data (Piketty and Saez, 2003) and survey data (CPS and SCF resp., Bricker et al., 2016; Burkhauser et al., 2012) point to important increases in top income shares.

To the best of my knowledge, I am the first to study the link between automation and top income concentration in a context where inequality matters for macro.<sup>3</sup> To operationalize this feedback loop, I build a stylized heterogeneous-agent model with two important features. First, wealth is present in the utility function as a luxury good (Carroll, 1998), which breaks the constancy of the savings rate over the permanent income distribution (Straub, 2019) that is inherent to workhorse macro models (e.g., Aiyagari, 1994). The model produces empirically realistic savings rate heterogeneity over wealth (Fagereng et al., 2019) and permanent income (Dynan et al., 2004). Second, production is task-based and firms respond to cheaper capital by automating more (Acemoglu, Manera, and Restrepo, 2020; Jacobs, 2023a; Nakamura and Nakamura, 2008).

Since both automation and top income concentration are endogenous model outcomes, I still require an exogenous shifter of either to explain their joint increase. For this, I rely on the exogenous slowdown in productivity growth and population growth which we observed over the last four decades. Following Piketty (2014), top income and wealth concentration is strongly linked with the return-growth differential  $r - g$ : slower growth contributes to greater concentration. I find that the growth slowdown (together with growing wage dispersion) can account for more than half of the fall in the labor share, and almost the entire increase in the top 1% wealth share. Crucially, however, this is only the case if there is a two-way feedback loop between automation and top income concentration as in my model. Under standard savings behavior, the observed growth slowdown can explain much less than half of the actual decline in the labor share.<sup>4</sup> This changes under realistic savings rate heterogeneity. The inequality effect of the growth slowdown then drives capital accumulation by the wealthiest households, which induces automation. The introduction of the feedback loop thus raises the explanatory power of the growth slowdown dramatically. Vice versa, the observed rise in top income and wealth concentration can only be matched in case the model incorporates automation: the rate of return to capital falls less as growth slows down if firms can respond to capital deepening by automating. The return-growth differential  $r - g$  increases by more if one takes into account automation, and wealthy households thus gain more.

The core story of this paper lies close to that of Piketty (2014) with its claim that the growth slowdown is an important cause of both the fall in the labor share and the rise in income concentration. However, it takes two of the most major critiques of his theoretical mechanism seriously. First, echoing the “second fundamental law of capitalism”, falling growth lies at the source of a rising capital-output ratio in my model. Krusell and Smith (2015) argue that, in assuming a constant net aggregate savings rate, Piketty (2014) may overestimate the impact of the growth slowdown on the capital-output ratio. My model acknowledges this: savings behavior is micro-founded and net aggregate savings and investment rates fall as growth slows down.<sup>5</sup> This is in line with the data and in contrast to a technology-based account of automation. Second, to convert a rising capital-output ratio into a declining labor share, Piketty (2014) relies on a neoclassical production function with an elasticity of substitution greater than one. The single most frequent critique of Piketty’s theoretical mechanism is that his elasticity of substitution

<sup>3</sup>This is with the exception of the work of Jacobs (2023). That study considers automation in a simple two-class framework with capitalists and workers, where capitalists save at a higher rate than workers.

<sup>4</sup>Even without inequality mattering for macro, the growth slowdown is a driver of automation because of the ‘relative scarcity of labor’ it induces and to which firms reply by automating more. With productivity growth originating from labor-augmenting technical progress, a slowdown of productivity growth means that the actual labor provided by the workforce grows less quickly. Lower population growth does the same.

<sup>5</sup>We find, however, that the decline in net saving and investment is less strong if one adds realistic savings rate heterogeneity.



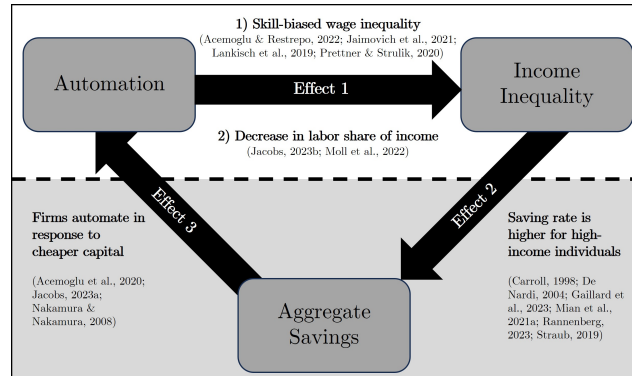
is greater than what is empirically warranted. In my model, the macro elasticity of substitution is the result of the technology adoption decision of firms, however. By treating automation as an endogenous reaction to changes in factor costs, a value for the elasticity of substitution above one becomes justifiable (Karabarbounis, 2023).

The remainder of this paper is structured as follows. In the second section, I discuss in more detail the existing literature on the interaction between automation and inequality. In the third section, I set out the quantitative model and explain my calibration strategy. The fourth section covers the results regarding (1) the model’s explanation of past stylized facts, and (2) the future distributional effects of a further slowdown of growth. Section 5 concludes.

## 2 Related literature

In this section, I briefly discuss the literature related to each of the three effects underpinning the interaction between automation and income inequality in my theoretical framework. This is summarized in Figure 2.

Figure 2: Schematic Overview of the Causal Chain Between Automation and Income Inequality



### 2.1 Effect 1: Automation Affects Income Inequality

First, let us consider the labor market consequences of automation in the recent macro literature (Effect 1 in Figure 2). Since Acemoglu and Restrepo (2018b) used the task-based production model of Zeira (1998) to define automation as “the displacement of workers from tasks by machines”, automation has frequently been invoked to explain both (1) the rise in the skill premium and (2) the fall in the labor share. Echoing the literature on skill-biased technical change, automation is often modelled as a process whose net labor demand effects are more advantageous to the higher-skilled workforce (e.g., Acemoglu and Restrepo, 2022; Jacobs and Heylen, 2024; Jaimovich et al., 2021; Lankisch et al., 2019; Prettnier and Strulik, 2020). This seems to be in line with existing empirical evidence regarding the labor market effects of robotics (e.g., Acemoglu and Restrepo, 2020; Dauth et al., 2017). The second, entirely distinct channel through which automation can generate rising income inequality is by lowering the labor share thus benefiting capital owners vis-à-vis workers. In the task-based framework, automation lowers the labor share by definition (Acemoglu and Restrepo, 2018a). In models with realistic inequality in capital

ownership, this fall in the labor share results in more overall income inequality (e.g., Jacobs, 2023b; Moll et al., 2022). Empirically, there is credible evidence indicating that technological factors like automation have been a quantitatively important driver of the lowering of the labor share in advanced economies (e.g., Acemoglu, Lelarge, and Restrepo, 2020; Acemoglu and Restrepo, 2020; Bergholt et al., 2022).

Regarding the effect of automation on inequality (Effect 1), I opt for a task-based production setting where automation has persistent effects on the labor share. I do not consider skill-biased wage effects related to automation, and focus instead on the right tail of the wealth and income distribution.

## 2.2 Effect 2: Income Inequality Affects Aggregate Savings

First, it is important to realize that in standard macro models — even those which allow for the fine-grained study of income inequality (e.g., Aiyagari, 1994) —, a shift in the permanent income distribution typically has no macro effects. This is because savings rates are constant across the permanent income distribution. This is generated by the homothecity of the household's maximization problem which generates policy functions which are linear in permanent income (Straub, 2019).<sup>6</sup> In standard models, automation may very well generate greater inequality (Moll et al., 2022), but this greater inequality is in itself no driver of automation. For my purposes, it is important that there is empirical evidence contradicting the simple model: both the savings rate and the marginal propensity to save out of income appear to increase over the permanent income distribution (Dynan et al., 2004; Kumhof et al., 2015; Straub, 2019) and the wealth distribution (Albers et al., 2022; Fagereng et al., 2019; Gaillard et al., 2023; Mian et al., 2021b). From a dynastic perspective, the finding that intentional bequests act as a luxury good (e.g., De Nardi, 2004) contributes to higher savings rates for wealthy dynasties. Finally, the fact that consumption inequality has not increased proportionally with income inequality (Heathcote et al., 2023) suggests concavity of savings with respect to permanent income, given that the rise in US income inequality seems mostly generated by greater permanent rather than transitory income variance (De Backer et al., 2013; Kopczuk et al., 2010).

To align households' behavior with these findings, I will introduce non-homothecity in households' utility function by using a capitalist-spirit motive, as in Carroll (1998). This means that wealth is inserted in the flow utility function as a luxury good. This allows the rise in inequality to act as a driver of increased aggregate savings. This is effect 2 in Figure 2. This modelling choice follows a nascent literature on the modelling of the right-skewed wealth distribution which uses the capitalist-spirit motive as a micro-foundation for heterogeneous savings rates (Gaillard et al., 2023; Mian et al., 2021a; Michau et al., 2023; Rannenberg, 2023) instead of relying on stochastic discount factor heterogeneity (e.g., Hubmer et al., 2021; Krusell and Smith, 1998).<sup>7</sup> With non-homothetic savings behavior, inequality can increase aggregate savings. As such, it becomes a driver behind the long-run fall in the interest rate (e.g., Heylen et al., 2024; Mian et al., 2021b; Rachel and Summers, 2019; Rannenberg, 2023).

<sup>6</sup>Note that I consider *permanent* income here. In the Aiyagari (1994) model, there is some non-linearity of the savings function in current income. However, this is only the case close to the borrowing constraint. Because of this, there is "approximate aggregation" à la Krusell and Smith (1998): often only the first moment of the distribution matters for the forecast of future prices. Hence, inequality does not matter, exactly because of the near-linearity of policy functions.

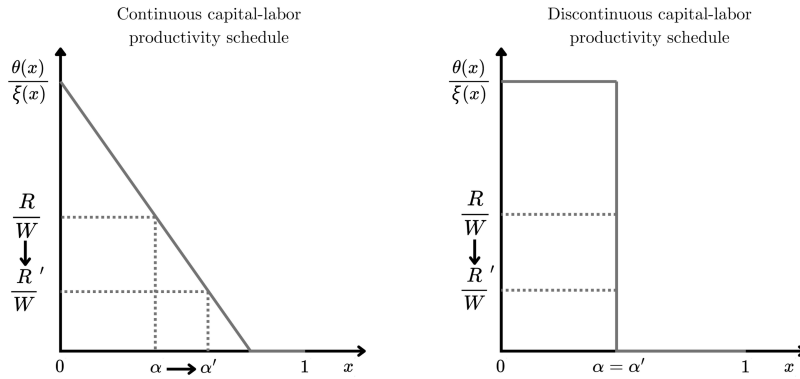
<sup>7</sup>In a recent contribution, Lee (2024) shows that savings rates that are increasing in wealth can also emerge in a model with homothetic preferences and multiplicative income risk.

### 2.3 Effect 3: Aggregate Savings Affect Automation

Finally, for the feedback mechanism between automation and inequality in Figure 2 to hold, the increase in aggregate savings must translate into increasing automation. This requires two elements. First, savings are lent out to firms in the form of capital. This is a cornerstone of all canonical neoclassical growth models and many heterogeneous-agent models which feature a production economy, as in Aiyagari (1994). Second, cheaper capital should incite firms to automate. This is true for task-based models under very general conditions, as shown next.

Let us consider an economy where the production of a single final good relies on the execution of different tasks, denoted by  $x$ , which can be executed using either capital or labor. Following Acemoglu, Manera, and Restrepo (2020), rational firms execute a task with capital if this is more cost-effective than labor. If the respective productivities of capital and labor as a function of a task  $x$  are given by  $\theta(x)$  and  $\xi(x)$ , and the respective costs of capital and labor are  $R$  and  $w$ , then a task  $x$  is executed by capital if  $\frac{\theta(x)}{\xi(x)} > \frac{R}{w}$ . When ordering tasks by declining productivity of capital vis-à-vis labor  $\frac{\theta(x)}{\xi(x)}$ , the automation choice of firms can be displayed in Figure 3.

Figure 3: Illustration of the two possibilities for the relative productivity paths of capital vis-à-vis labor. This decides whether factor costs matter.



In terms of notation,  $x$  represents the task index ranging from a continuous range from 0 to 1.  $R$  and  $W$  denote the factor costs related to capital and labor, respectively.  $\theta(x)$  and  $\xi(x)$  denote the productivity of capital and labor at the execution of the task  $x$ , respectively. The marginally automated task is denoted by  $\alpha$ : all tasks with an index lower than  $\alpha$  are executed by capital (=automated).

For any continuous relative productivity schedule, a fall in  $\frac{R}{w}$  then leads to an expansion in the set of tasks executed by capital and a contraction in the set of tasks executed by labor. This is shown in the left-hand side panel of Figure 3. In short, firms react to cheaper capital by executing more tasks with capital. This shift in production technology is what we call automation. This is in contrast to the case where the relative productivity schedule features a discontinuity at the current level of automation (right-hand side panel of Figure 3). In such a model, factor cost changes have no effect on automation take-up, and automation is fully determined by exogenous technological evolutions in factor productivities. By opting for a continuous productivity path, the optimal level of automation becomes endogenous (an outcome, not an exogenous driver of inequality as in Moll et al. (2022)). In the spirit of Acemoglu, Manera, and Restrepo (2020), Jacobs (2023a), and León-Ledesma and Satchi (2019), my production

function will feature a difference in the capital-labor substitutability in the short run (when automation is fixed) and in the long run (when factor cost changes affect technology adoption). Karabarbounis (2023) argues that this mechanism can help bridge the gap between micro-level estimates of the elasticity of substitution below one (e.g., Chirinko, 2008) and macro-level estimates above one (e.g., Hubmer, 2023; Karabarbounis and Neiman, 2014), as technology adoption may happen outside the bounds of the firm or even sector. As a result, the high elasticity of substitution required for the explanation of the decline in the labor share is not imposed from the outset (Piketty, 2014), but results from the endogenization of technology adoption.

## 2.4 Conclusion of related literature

In the text above, I have tried to show that the three theoretical mechanisms underpinning the causality between inequality and automation are (1) established features of state-of-the-art macro models, and (2) plausible in light of recent empirical findings. The novelty of this work lies only in the combination of all three mechanisms. In particular, this is, to the best of my knowledge, the first time that the effects of automation on top income concentration are studied when inequality matters for macro.<sup>8</sup>

The work of Moll et al. (2022) is most closely related to this paper. Compared to their work, I innovate in two crucial dimensions. First, I invoke the growth slowdown (and not technology shocks) as the underlying cause behind automation and the fall in the labor share. Automation is not an exogenous object here, but the result of firms' cost-minimization. Second, savings behavior is the result of non-homothetic preferences in our framework, such that inequality matters for macro. Mutual feedback effects between automation and inequality thus become possible.

Following the work of Rachel and Summers (2019), there is also a recent set of papers that invoke rising inequality as an explanation for the decline in the safe rate of return to capital  $r^*$  by introducing non-homothetic preferences in the savings-consumption decision (Aladangady et al., 2021; Mian et al., 2021b; Platzer and Peruffo, 2022; Rannenberg, 2023; Straub, 2019). My model is similar to their work in the sense that the rise of permanent income inequality drives capital accumulation. In my model, however, the story does not stop there: the fall in the cost of capital elicits firms to automate and this shifts the functional income distribution more towards capital. This further benefits high-wealth households who rely more on capital income, and less on labor income.<sup>9</sup>

<sup>8</sup>Moll et al. (2022) highlight that this approach could be interesting (own emphases). Moll et al. (2022) state that "the net effect of technology on top-tail income and wealth inequality depends on whether the higher demand for capital brought by automation translates into a higher rate of wealth accumulation for rich households in equilibrium. *Exploring this question in models where "inequality matters for macro" is another interesting area for future research.*" (p. 2680).

<sup>9</sup>Somewhat similarly, Straub (2019) briefly considers (in his section 6.6) the possibility of a two-way interaction between inequality and capital accumulation. The focus is very different, however. I focus on the labor share effects of capital accumulation (through induced automation), while Straub (2019) considers the skill-biased wage effects of capital accumulation. As a result, my model produces a feedback loop between income concentration among high-wealth households and automation, while Straub (2019) focuses on an interaction between *labor income* inequality and skill-biased capital accumulation.

### 3 The model

#### 3.1 Production side

Output is produced by a perfectly competitive representative firm. Equation (1) explains its production technology.

$$Y_t = H \prod_0^1 T_t(x)^{dx} \quad (1)$$

Equation (1) states that production is the result of the execution of a continuum of tasks  $T_t(x)$  by a representative firm. Here,  $T_t(x)$  denotes the execution of the task with index  $x$  at historical time  $t$ . The task continuum is, without loss of generality, normalized to the interval between 0 and 1. Tasks are combined via a Cobb-Douglas aggregator, hence the product integral. The factor  $H$  denotes Hicks-neutral technology.

$$T_t(x) = \theta_t(x)k_t(x) + \xi_t(x)l_t(x) \quad (2)$$

Equation (2) states that the execution of each task  $T_t(x)$  can be done by capital or labor that is assigned to that task, denoted by  $k_t(x)$  and  $l_t(x)$ , respectively.  $\theta_t(x)$  and  $\xi_t(x)$  denote their respective productivities at the execution of task  $x$ . At the execution of a task, there is perfect substitutability between factors.

We proceed by imposing more structure on the productivity schedule of capital and labor over the range of tasks in equation (3).

$$\theta_t(x) = b^{1+\gamma}(B-x)^\gamma \quad \text{and} \quad \xi_t(x) = A_t(1-b)^{1+\gamma}x^\gamma \quad \text{with} \quad 0 < b < 1, \quad \gamma > 0 \quad \text{and} \quad B \geq 1 \quad (3)$$

Given the monotonically decreasing relative productivity of capital over labor  $\frac{\theta_t(x)}{\xi_t(x)}$ , capital (labor) has a competitive advantage in lower-indexed (higher-indexed) tasks. The parameter  $b$  determines the overall productivity of capital vis-à-vis labor at time  $t$ . The parameter  $\gamma$  indicates how steeply the productivity of capital (labor) decreases (increases) over the range of tasks. Technical progress in the model economy is labor-augmenting technical at rate  $g$ :  $A_t = A_0 e^{gt}$ . The parameter  $B$  determines the capital productivity in the task least suitable for automation — the task with index 1. In the default case where  $B = 1$  (considered by Nakamura and Nakamura, 2008), the productivity of capital is zero for that task. If  $0 < B < 1$ ,  $B$  denotes the highest task for which capital still has non-negative productivity. If  $B > 1$ , the productivity of capital is strictly positive for the whole range of tasks.

As explained in Proof A1 in Appendix A, the perfect substitutability between capital and labor leads firms to execute tasks using *either* capital or labor, but never a combination of both. Let us denote the set of tasks which are executed using capital by  $\mathfrak{K}_t \subset [0, 1]$  and the subset of tasks which are executed using labor by  $\mathfrak{L}_t = [0, 1] \setminus \mathfrak{K}_t$ . Regarding terminology, I also call  $\mathfrak{K}_t$  ( $\mathfrak{L}_t$ ) the set of automated (non-automated) tasks. We now make a distinction between the short-run and long-run behavior of firms.

##### 3.1.1 Short-run production function

In the short run, firms cannot change the allocation of production factors to tasks. In other words,  $\mathfrak{K}_t$  and  $\mathfrak{L}_t$  are given. We denote the given level of automation by  $\alpha_t$ :  $\mathfrak{K}_t = [0, \alpha_t]$  and  $\mathfrak{L}_t = ]\alpha_t, 1]$ . As a

result, the short-run production function is equation (4).

$$Y_t = H \prod_0^{\alpha_t} (b^{1+\gamma} (B - x)^\gamma k_t)^{dx} \prod_{\alpha_t}^1 (A_t (1 - b)^{1+\gamma} x^\gamma l_t)^{dx} \quad (4)$$

Note that, because of the Cobb-Douglas aggregator at the level of each task, each task that is automated (not automated) uses the same amount of capital  $k_t$  (labor  $l_t$ ) (see Appendix A).

Given the definition of a product integral, it can be shown that the short-run production function simplifies to the expression in equation (5). This is shown in Proof A2 of Appendix A.

$$Y_t = \tilde{H} (B - \alpha_t)^{-\gamma(B-\alpha_t)} (1 - \alpha_t)^{\gamma(1-\alpha_t)} \left( \left( \frac{b}{\alpha_t} \right)^{1+\gamma} K_t \right)^{\alpha_t} \left( A_t \left( \frac{1-b}{1-\alpha_t} \right)^{1+\gamma} L_t \right)^{1-\alpha_t} \quad (5)$$

The Hicks-neutral technology term  $H$  is redefined as  $\tilde{H} = H e^{-\gamma B \gamma B}$ . Here,  $K_t$  ( $L_t$ ) denotes the aggregate stock of capital (supply of labor) across the range of tasks:  $K_t = \int_0^{\alpha_t} k_t dx = \alpha_t k_t$  and  $L_t = \int_{\alpha_t}^1 l_t dx = (1 - \alpha_t) l_t$ .

One can see that in the short-run, when firms are bound to their automation decision of the past ( $\alpha_t$  is a constant), the production function is of the Cobb-Douglas type. We assume perfectly competitive markets such that marginal products equal factor costs. This results in equations (6) and (7).

$$\frac{\partial Y_t}{\partial K_t} = \alpha_t \frac{Y_t}{K_t} = R_t \quad (6)$$

$$\frac{\partial Y_t}{\partial L_t} = (1 - \alpha_t) \frac{Y_t}{L_t} = W_t \quad (7)$$

### 3.1.2 Long-run production function

In the long run, firms choose to which extent they automate based on cost-effectiveness. Given that the relative productivity of capital vis-à-vis labor  $\frac{\theta_t(x)}{\xi_t(x)}$  in equation (3) is monotonically diminishing over the range of tasks, firms automate up to the marginally automated task  $\alpha_t$  where capital and labor are equally cost-effective  $\frac{\theta_t(\alpha_t)}{\xi_t(\alpha_t)} = \frac{R_t}{W_t}$ . This ensures that capital is strictly more cost-effective in all tasks with index  $x$  lower than  $\alpha_t$ , and that labor is strictly more cost-effective in all tasks with index  $x$  greater than  $\alpha_t$ .

Combining the condition for cost-effective automation with the imposed capital-labor productivity schedule of equation (3), we find the optimal level of automation in equation (8).

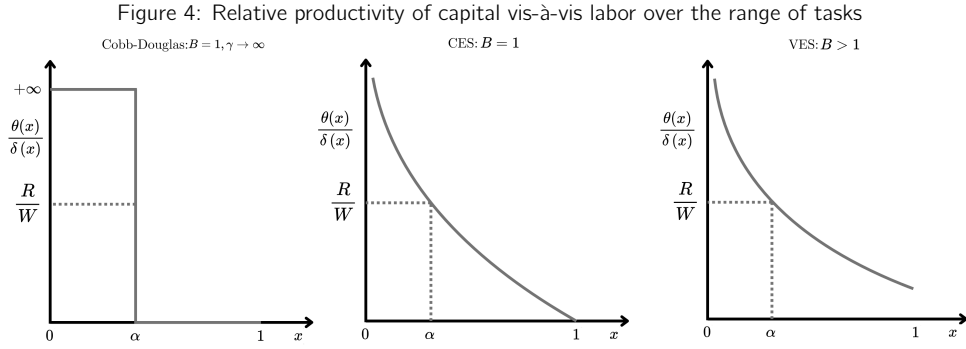
$$\frac{\theta_t(\alpha_t)}{\xi_t(\alpha_t)} = \frac{R_t}{W_t} \iff \alpha_t = \min \left( \frac{B}{1 + \left( \frac{A_t R_t}{W_t} \right)^{\frac{1}{\gamma}} \left( \frac{1-b}{b} \right)^{\frac{1+\gamma}{\gamma}}}, 1 \right) \quad (8)$$

In a perfectly competitive setting, the factor costs of capital  $R_t$  and labor  $W_t$  are the marginal product of capital and labor in equations (6) and (7). Inserting this in the expression for the optimal level of automation (8) results in an expression for the ratio of capital to effective labor  $\frac{K_t}{A_t L_t}$  in terms of the level

of automation  $\alpha_t$  given by equation (9).

$$\frac{K_t}{A_t L_t} = \left( \frac{1-b}{b} \right)^{1+\gamma} \frac{\alpha_t^{1+\gamma}}{(1-\alpha_t)(B-\alpha_t)^\gamma} \quad \text{with } 0 < \alpha_t < \min(B, 1) \quad (9)$$

The combination of equations 5 and 9 pins down the long-run production function, in which producers optimally choose the level of automation as a function of factor costs. I now proceed by showing what the aggregate production function looks like under different conditions on the capital-labor productivity schedule in (3). More specifically, I consider three cases: (1) the case where  $B = 1$  and  $\gamma \rightarrow \infty$ , (2) the case where  $B = 1$  and  $\gamma$  has a strictly positive, finite value, and (3) the case where  $B > 1$  and  $\gamma$  has a strictly positive, finite value. These three functional forms for the relative productivity of capital vis-à-vis labor respectively correspond to (1) a Cobb-Douglas production function, (2) a constant-elasticity-of-substitution (CES) production function, and (3) a variable-elasticity-of-substitution (VES) production function. Figure 4 shows the capital-labor productivity schedule in these three cases.



In terms of notation,  $x$  represents the task index ranging from 0 to 1.  $R$  and  $W$  denote the factor costs related to capital and labor, respectively.  $\theta(x)$  and  $\xi(x)$  denote the productivity of capital and labor at the execution of the task  $x$ , respectively. The marginally automated task is denoted by  $\alpha$ : all tasks with an index lower than  $\alpha$  are executed by capital (=automated).

The relative capital-labor productivity schedule  $\frac{\theta(x)}{\xi(x)}$  is given by  $\frac{1}{A_t} \left( \frac{b}{1-b} \right)^{1+\gamma} \left( \frac{B-x}{x} \right)^\gamma$ . The three parameters  $b$ ,  $\gamma$  and  $B$  play a distinct role. A rise in  $b$  shifts the entire productivity schedule in favor of capital without affecting the slope. A greater value for  $\gamma$ , on the other hand, implies a steeper decline in  $x$ . The parameter  $B$  determines what the productivity schedule looks like for the least automatable tasks (those closest to 1): for values of  $B > 1$ , capital has a non-zero relative productivity in those and full automation ( $\alpha = 1$ ) becomes technically feasible.

First, consider the case where  $B = 1$ . As explained before, this is the case where capital has zero productivity in the least automatable task (with index 1), but strictly positive productivity for all other tasks. In that case, we can rework equation (9) to find an expression for the optimal level of automation

$\alpha_t$  in terms of the capital-labor ratio in equation (10).

$$\alpha_t = \frac{b \left( \frac{K_t}{A_t L_t} \right)^{\frac{1}{1+\gamma}}}{b \left( \frac{K_t}{A_t L_t} \right)^{\frac{1}{1+\gamma}} + 1 - b} \quad (10)$$

Re-inserting this in equation (5) results in the following long-run production function in equation (11).

$$Y_t = \tilde{H} \left( b K_t^{\frac{1}{1+\gamma}} + (1-b) (A_t L_t)^{\frac{1}{1+\gamma}} \right)^{1+\gamma} \quad (11)$$

Clearly, this production function is of the CES type with a constant elasticity of substitution equal to  $1 + \frac{1}{\gamma}$ . This finding is also reported by Nakamura and Nakamura (2008).

Allowing firms to respond to changes in factor prices by expanding or contracting the set of automated tasks, opens up an additional margin of adjustment. In the short run, firms can only respond to changes in factor prices by substituting *between* tasks (e.g., executing relatively more automated tasks, and less non-automated tasks). In the long run, firms can rewire their production setting and opt for a different, more cost-effective allocation of capital and labor to tasks. This is represented by the additional term  $\frac{1}{\gamma}$ . The strength of this additional substitutability depends on the steepness of the relative capital-labor productivity schedule. A higher value for  $\gamma$  represents both (1) a faster decline in capital productivity over the range of tasks, and (2) a faster increase in labor productivity. Intuitively, a steeper relative productivity schedule  $\frac{\theta_t(x)}{\xi_t(x)}$  means that the same change in factor prices elicits less of a response in terms of automation  $\alpha_t$ .

For the limit case of  $\gamma \rightarrow +\infty$ , the capital-labor productivity schedule features a discontinuity at  $\alpha = b$ : the allocation of production factors to tasks ( $\mathfrak{K}_t$  and  $\mathfrak{L}_t$ ) is then entirely fixed and firms cannot respond to changes in factor prices in terms of automation. The result is a long-run production function that is of the Cobb-Douglas type. In this case, firms' hands are tied by technology and they cannot adjust the level of automation  $\alpha$  in response to factor cost changes as under equation (8).

**Lemma 1.**

Given that

1. the respective productivities of capital and labor over the range of tasks are  $\theta_t(x) = b^{1+\gamma}(B-x)^\gamma$  and  $\xi_t(x) = A_t(1-b)^{1+\gamma}x^\gamma$  with  $0 < b < 1$  and  $\gamma > 0$  where  $x$  denotes the task index and the range of tasks is the interval  $[0, 1]$

2. the tasks  $T_t(x)$  are combined through a Cobb-Douglas aggregator:  $Y_t = H \prod_0^1 T_t(x)^{dx}$

3. there is perfect substitutability between capital  $k_t(x)$  and labor  $l_t(x)$  for the execution of each task  $T_t(x) = \theta_t(x)k_t(x) + \xi_t(x)l_t(x)$

imposing that  $B = 1$  and  $\gamma \rightarrow +\infty$  results in an aggregate production function of the Cobb-Douglas type:  $Y_t = H e^{-\gamma} K_t^b (A_t L_t)^{1-b}$ .



imposing that  $B = 1$  with a general strictly positive value for  $\gamma$  results in an aggregate production function of the CES type:  $Y_t = \tilde{H} \left( bK_t^{\frac{1}{1+\gamma}} + (1-b)(A_t L_t)^{\frac{1}{1+\gamma}} \right)^{1+\gamma}$  with elasticity of substitution  $1 + \frac{1}{\gamma}$  ■

Given a general value for  $B$ , the elasticity of substitution between capital and labor is no longer equal to a constant  $1 + \frac{1}{\gamma}$  as it is under  $B = 1$ . Based on equations (6) and (7), it is easy to show that  $\frac{K_t}{L_t} = \frac{\alpha_t}{1-\alpha_t} \frac{W_t}{R_t}$ . Replacing  $\alpha_t$  by its expression in equation (8) and deriving with respect to  $\frac{W_t}{R_t}$  leads to an expression for the elasticity of substitution between  $K_t$  and  $L_t$  that depends on the share of automated tasks  $\alpha_t$  in equation (12).<sup>10</sup>

$$\frac{\partial \frac{K_t}{L_t}}{\partial \frac{W_t}{R_t}} \frac{\frac{W_t}{R_t}}{\frac{K_t}{L_t}} = \frac{1}{B} \left( 1 + \frac{1}{\gamma} \frac{B - \alpha_t}{1 - \alpha_t} \right) \quad (12)$$

For  $B = 1$ , the elasticity of substitution is again constant to  $1 + \frac{1}{\gamma}$ . For  $B > 1$ , the substitution elasticity increases in the share of automated tasks  $\alpha_t$ . For  $K_t \rightarrow \infty$  (which corresponds to  $\alpha_t \rightarrow 1$  under equation (9)), the elasticity of substitution goes to  $\infty$ . When  $B > 1$  the long-run production function becomes a VES. There is some evidence for a rise in the US elasticity of substitution over recent decades (cfr. Knoblich and Stöckl, 2020 and the references therein; Ialenti and Piali, 2024). This provides support for the view of a VES production technology where  $\sigma$  increases with capital intensity.

**Lemma 2.**

Given that

1. the respective productivities of capital and labor over the range of tasks are  $\theta_t(x) = b^{1+\gamma}(B-x)^\gamma$  and  $\xi_t(x) = A_t(1-b)^{1+\gamma}x^\gamma$  with  $0 < b < 1$  and  $\gamma > 0$  where  $x$  denotes the task index and the range of tasks is the interval  $[0, 1]$

2. the tasks  $T_t(x)$  are combined through a Cobb-Douglas aggregator:  $Y_t = H \prod_0^1 T_t(x)^{dx}$

3. there is perfect substitutability between capital  $k_t(x)$  and labor  $l_t(x)$  for the execution of each task  $T_t(x) = \theta_t(x)k_t(x) + \xi_t(x)l_t(x)$

imposing that  $B > 1$  with a strictly positive general value for  $\gamma$  results in an aggregate production function of the type VES with elasticity of substitution increasing in the capital share ■

Both the CES and VES technologies do not satisfy the Inada conditions, since  $\lim_{K \rightarrow \infty} F_K(L, K) > 0$ . However, both satisfy the conditions required for Uzawa's theorem under all circumstances since (1) technological progress is purely of the Harrod-neutral form, and (2) returns to scale are constant. Hence, the standard conditions for a balanced-growth path are met.

### 3.2 Household side

There exists a unit range of infinitely-lived households who differ in their fixed labor endowment  $l_i$ . The share of households that has a labor endowment  $l_i$  is denoted by  $n_i$ . Each member of a household of

<sup>10</sup>Here I assume that  $B \geq 1$ , since this results in the case that is most interesting for my purposes (namely a VES production function where the elasticity of substitution increases with capital deepening).

type  $i$  inelastically provides  $l_i$  units of effective labor. Each household has a population mass of  $N_t = e^{nt}$  members, where  $n$  represents the rate of population growth over time. The number of households is thus fixed to 1, but population growth continuously takes place within the household. The aggregate labor supply is given by  $L_t = e^{nt} \sum_i n_i l_i$ .

Each household maximizes the discounted flow of utility from time  $t = 0$  onwards. The household maximization problem is cast in continuous time. The objective function is given by equation (13).

$$\max_{c_{i,t}, a_{i,t}} E_0 \int_0^\infty e^{-\rho t} N_t u(A_t c_{i,t}, a_{i,t}) dt = \max_{c_{i,t}, a_{i,t}} E_0 \int_0^\infty e^{-(\rho-n)t} u(A_t c_{i,t}, a_{i,t}) dt \quad (13)$$

The flow utility function is given by  $u(A_t c_{i,t}, a_{i,t})$ , where  $c_{i,t}$  and  $a_{i,t}$  respectively denote household consumption per efficiency unit of labor and household wealth per efficiency unit of labor ( $\frac{W_t}{A_t N_t}$ ). The pure rate of time preference is  $\rho$ , but the effective rate of time preference (after considering population growth) is given by  $\rho - n$ . There will be a restriction imposed on  $\rho - n$  to ensure that the utility integral is bounded.

The utility maximization problem of equation (13) is subject to the following constraints.

$$da_{i,t} = (w_i + (R - \delta - g - n)a_{i,t} - c_{i,t}) dt - a_{i,t} dM_t \quad (14)$$

First, equation (14) characterizes the budget constraint in the form of the law of motion for household assets per efficiency unit of labor  $a_{i,t}$ . In this equation,  $w_i = \frac{l_i W_t}{A_t}$  denotes the detrended wage rate for households with labor endowment  $l_i$ , and  $R$  denotes the return to capital. We focus on the steady state, where (detrended) factor prices are constant. Note that there is no labor income risk in the model. Moreover,  $\delta$  denotes the depreciation rate,  $g$  denotes the rate of labor-augmenting technical change, and  $n$  denotes the rate of population growth. In Proof B1 of Appendix B, I show that this first part of the budget constraint holds. The second term of equation (14) is stochastic.  $M_t$  represents a Poisson process with arrival rate  $p$ . This Poisson process represents an iid shock to which all households are subject. During 'normal times',  $dM_t$  is zero and the asset stock per efficiency unit of labor drifts according to the standard budget constraint:  $\dot{a}_{i,t} = w_i + (R - \delta - g - n)a_{i,t} - c_{i,t}$ . In case of a 'dissipation event',  $dM_t$  is one and the asset stock jumps discontinuously to zero. This is what Moll et al. (2022) refer to as a 'dissipation shock': households hit by the shock lose all assets. Without such shocks, sufficiently wealthy households would accumulate unboundedly, capital supply would explode and the distribution of wealth would be non-stationary. More substantially, this process captures, albeit in a very reduced-form fashion, capital income risk. The presence of dissipation shocks ensures that households with the same labor endowment  $l_i$  are ex-post heterogeneous.

$$a_{i,t} \geq 0 \quad (15)$$

Equation (15) characterizes the ad-hoc borrowing limit of zero for all households.

The flow utility function  $u(A_t c_{i,t}, a_{i,t})$  is non-standard since individuals do not only value consumption per capita  $A_t c_{i,t}$ , but also their wealth level per efficiency unit of labor  $a_{i,t}$ . The direct presence of wealth in the utility function is called the "capitalist spirit" motive for asset holdings (Carroll, 1998). Under standard preferences, individuals hold assets only with the aim to (potentially) consume out of them in the future. Here, in contrast, assets offer *direct* utility to households because of the services they

provide when not consumed (e.g., prestige, influence ...). Since the social status and power likely arise from the comparison of asset levels to the economy-wide standard of living, we scale the direct services rendered by assets per capita by the technology level  $A_t$ . This follows the work of Michau et al. (2023). Intuitively, if GDP per capita grows tenfold, one's asset level must increase tenfold as well to command the same respect as before.

Equation (16) specifies the functional form for the flow utility function  $u(A_t c_{i,t}, a_{i,t})$ .

$$u(A_t c_{i,t}, a_{i,t}) = \ln(A_t c_{i,t}) + \frac{(a_{i,t} + S)^{1-\zeta}}{1-\zeta} \quad \text{with } 0 < \zeta < 1 \text{ and } S \geq 0 \quad (16)$$

It imposes additive separability between the utility coming from consumption, and asset holdings. In imposing that  $\zeta < 1$ , I ensure that preferences are non-homothetic: asset holdings  $a_{i,t}$  become a luxury good because the marginal utility of asset holdings declines less quickly than the marginal utility of consumption. The parameter  $S$  functions as a Stone-Geary parameter, ensuring that the capitalist spirit motive plays no role for households with low consumption. The role of both parameters will be discussed in more detail when deriving the Euler equation for consumption.

Before the Euler equation can be derived, it is important to note that the household optimization problem can be cast in the analytically more convenient form of equation (17).

$$\begin{aligned} \max_{c_{i,t}, a_{i,t}} E_0 \int_0^\infty e^{-(\rho' - n)t} \left( \ln(A_t c_{i,t}) + \frac{(a_{i,t} + S)^{1-\zeta}}{1-\zeta} \right) dt \\ \text{s. t. } \dot{a}_{i,t} = w_i + (R - \delta - g - n)a_{i,t} - c_{i,t} \\ a_{i,t} \geq 0 \end{aligned} \quad (17)$$

In Proof B2 of Appendix B, I argue based on the work of Moll et al. (2022) that the presence of dissipation shocks implies that households use the discount rate  $\rho' = \rho + p$ , but otherwise disregard the presence of these shocks. In other words, households behave as if faced with the deterministic utility maximization problem, but with a stronger time preference. In equation (17), we introduce  $t$ . It denotes the time since the household was last hit by a dissipation shock, and is thus different from historical time  $t$ . At  $t = 0$ , households are just now hit by a dissipation shock:  $a_{i,0} = 0$ . To ensure boundedness of lifetime utility, it has to hold that  $\rho' - n > 0$ .

Based on the utility maximization problem of equation (17), it is straightforward to come up with the Euler equation for consumption (18) when the individual is not at the borrowing constraint (see Proof B3 in Appendix B).

$$\frac{\dot{c}_{i,t}}{c_{i,t}} = R - \delta - \rho' - g + \frac{(a_{i,t} + S)^{-\zeta}}{c_{i,t}^{-1}} \quad (18)$$

Compared to the standard continuous-time Euler equation, we have the additional term  $\frac{(a_{i,t} + S)^{-\zeta}}{c_{i,t}^{-1}}$  which represents the ratio of the marginal utility of wealth holdings to that of consumption  $\frac{\partial u}{\partial a_{i,t}} / \frac{\partial u}{\partial c_{i,t}}$ . Preferences are only homothetic if  $\zeta = 1$  and  $S = 0$  as scaling assets and consumption by the same factor then leaves the ratio  $\frac{\partial u}{\partial a_{i,t}} / \frac{\partial u}{\partial c_{i,t}}$  unaffected and the growth rate of consumption  $\frac{\dot{c}_{i,t}}{c_{i,t}}$  stays constant. For values of  $\zeta$  smaller than one, this no longer holds:  $\frac{\dot{c}_{i,t}}{c_{i,t}}$  then increases if  $a_{i,t}$  and  $c_{i,t}$  are scaled by the same positive factor. The presence of the Stone-Geary parameter  $S$  ensures that the term  $\frac{\partial u}{\partial a_{i,t}} / \frac{\partial u}{\partial c_{i,t}}$  converges

to zero for very low consumption levels, such that low-income households behave as standard savers.

I follow Michau et al. (2023) in tracing the consumption-saving dynamics by considering the nullclines  $\dot{a}_{i,t} = 0$  and  $\dot{c}_{i,t}$  in a phase diagram. The nullclines are given by equations (19) and (20).

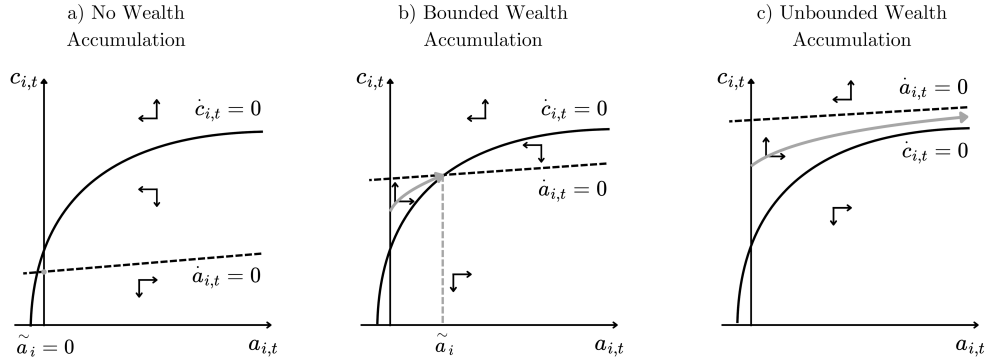
$$\dot{c}_{i,t} = 0 \iff c_{i,t} = (\delta + \rho' + g - R)(a_{i,t} + S)^\zeta \quad (19)$$

$$\dot{a}_{i,t} = 0 \iff c_{i,t} = w_i + (R - \delta - g - n)a_{i,t} \quad (20)$$

We restrict our attention to the case where  $R - \delta - g < \rho'$ . Intuitively, households would then not save if not for the capitalist-spirit motive. In other words, we restrict our attention to the case where individuals would not save under standard, homothetic preferences, but the presence of the capitalist-spirit motive incites some to still do so. The permanent heterogeneity in wage rate  $w_i$  (driven by heterogeneity in their productivity / labor endowment  $l_i$ ) is important here. It is discussed below.

Figure 5 indicates the three possible scenarios for the accumulation rule for assets per efficiency unit of labor  $a_{i,t}$ . The solid and dotted curves display  $\dot{c}_{i,t} = 0$ , and  $\dot{a}_{i,t} = 0$ , respectively. The actual trajectories for  $a_{i,t}$  and  $c_{i,t}$  (if not hit by a dissipation shock) are shown by the gray arrows.

Figure 5: Phase diagram for assets  $a_{i,t}$  and consumption  $c_{i,t}$  per efficiency unit of labor: three cases depending on the wage rate  $w_i$



The dotted line denotes the nullcline for assets per efficiency unit of labor. The solid curve denotes the nullcline for consumption per efficiency unit of labor. The three panels represent individuals with a different level of permanent labor income  $w_i$  (shift of the  $\dot{a}_{i,t}$  nullcline). The actual trajectories for  $a_{i,t}$  and  $c_{i,t}$  are shown by the gray arrows.

Let us consider the situation of a household that is just hit by a dissipation shock ( $t = 0$ ). This household does not have any assets ( $a_{i,0} = 0$ ). The first possibility (panel a) is that the wage rate  $w_i$  is too low for individuals to start accumulating assets  $a_{i,t}$ . This happens when  $w_i \leq (\delta + \rho' + g - R)S^\zeta$ . These individuals consume all of their labor income ( $c_{i,t} = w_i$ ) when at the borrowing constraint ( $a_{i,t} = 0$ ). The intuition is that their consumption is so low that that they prefer consumption to asset holdings at the margin: their permanent income is simply not high enough to make it worthwhile to accumulate assets for capitalist-spirit reasons. Hence, their optimal asset level is zero:  $\lim_{t \rightarrow \infty} a_{i,t} = a_i^* = 0$ . If the wage rate is sufficiently high ( $w_i > (\delta + \rho' + g - R)S^\zeta$ ), households do accumulate assets. The second possibility (panel b) is that households build up their wealth up to a positive, finite level  $\lim_{t \rightarrow \infty} a_{i,t} = a_i^* > 0$ . That means that, even if the household were never hit by a shock, their assets would not diverge to infinity.

The third possibility (panel c) is that individuals accumulate assets unboundedly when not hit by a dissipation shock:  $\lim_{t \rightarrow \infty} a_{i,t} = +\infty$ . This happens when  $w_i > (R - \delta - g - n)S + \frac{1-\zeta}{\zeta} \left( \frac{\zeta(\delta + \rho + g - R)}{(R - \delta - g - n)^\zeta} \right)^{\frac{1}{1-\zeta}}$ . Following Francis (2009) the share of income that goes to saving  $\left( \frac{\dot{a}_{i,t}}{w_i + (R - \delta - g - n)a_{i,t}} \right)$  approaches one for  $a_{i,t} \rightarrow +\infty$  for households in panel c.<sup>11</sup> In other words, as households become infinitely rich, the share of income that flows to the luxury good (the build-up of assets) goes to one, and the share that goes to the basic good (consumption) to zero.

An interesting interpretation of this three-way typology of household types is that it mirrors the qualitative division in society between (1) the poor, (2) the middle class, and (3) the wealthy. The poor consume the entirety of their income. Their consumption needs are insufficiently satisfied to begin accumulating assets for status reasons. The middle class accumulates assets for status considerations up to some degree (that depends positively on their permanent income level  $w_i$ ). Finally, the wealthy are on a path towards infinite wealth accumulation. If not for adverse shocks to their fortune, they would not cease to build up their asset stock. In the calibrated version of model for the US in 2017, the respective shares of the poor, middle class and wealthy are 8%, 77% and 15%.

Note that microfounding savings rate heterogeneity with the capitalist-spirit motive for wealth implies that preference heterogeneity is ex post. It is only because of wage heterogeneity and the random materialization of dissipation shocks that some households end up thriftier than others. This is in stark contrast to modeling ex ante heterogeneity in the discount factor (e.g., Krusell and Smith, 1998). In Krusell and Smith (1998) thrifty people become rich, while here rich people become thrifty.

### 3.3 Market clearing

#### 3.3.1 Capital supply

Solving the consumer maximization problem of equation (17), results in the policy function for asset holdings per efficiency unit of labor  $a_{i,t}$  where  $t$  does not denote the historical time, but the time since the household was last hit by a dissipation shock. The share of households that have not been hit by a dissipation shock for exactly a period of time  $t$  is given by  $pe^{-pt}$ .<sup>12</sup> The total capital supply per efficiency unit of labor  $\frac{K}{AL}$  is therefore given by equation (21).

$$\frac{K}{AL} = \sum_{i=1}^Z n_i \int_0^\infty a_{i,t} p e^{-pt} dt \quad (21)$$

The pace at which wealthy households accumulate assets per efficiency unit of labor is determined by the rate of return to capital  $R - \delta$  minus the growth rate of output  $g + n$ . This  $R - \delta - g - n$  is called the return gap by Moll et al. (2022). Individuals in the situation of panel c of Figure 5 accumulate assets unboundedly. Individuals at the very top of the wealth distribution will, in the limit  $a_{i,t} \rightarrow +\infty$ , allocate their entire income flow to saving:  $\lim_{a_{i,t} \rightarrow \infty} \frac{\dot{a}_{i,t}}{w_i + (R - \delta - g - n)a_{i,t}} = 1$ . Disregarding labor income (which becomes unimportant for  $a_{i,t} \rightarrow \infty$ ), the law of motion for assets per efficiency unit of labor for these high-wealth individuals is given by  $\dot{a}_{i,t} = (R - \delta - g - n)a_{i,t}$ . The return gap  $R - \delta - g - n$  is bounded from above by the dissipation rate  $p$ . Otherwise, the capital supply per efficiency unit of labor  $\frac{K}{AL}$

in equation (21)

<sup>11</sup> Carroll (1998) shows this analytically for a simple two-period model with wealth as a luxury good in the utility function.

<sup>12</sup> The fraction of the total population that has just been hit is  $p$ . This mass of individuals declines at rate  $p$  over time. It can be easily verified that  $\int_0^\infty p e^{-pt} dt = 1$

explodes to  $+\infty$  as the rate at which the wealthiest accumulate then exceeds the rate at which they 'die'.

A rising return gap  $R - \delta - g - n$  is tied together with rising wealth concentration at the top. This is straightforward to see, as wealthy households accumulate assets at rate  $R - \delta$  while the total stock of assets  $K$  grows at rate  $g + n$  along the BGP. A rise in the return gap then means that the wealthiest jump further ahead. More technically, this exponential growth of assets, coupled with the dissipation rate of  $p$ , implies that the very top of the wealth distribution approximates a Pareto distribution with tail parameter of  $\frac{p}{R - \delta - g - n}$  (e.g., Jones, 2015).<sup>13</sup> Again, it follows that  $R - \delta - g - n < p$  is required to ensure the boundedness of capital supply. If not, the tail parameter becomes smaller than one and, since the Pareto distribution only has finite moments of an order smaller than the tail parameter, total asset supply of wealthy households then converges to infinity.

Figure 6 gives a schematic overview of the market for loanable funds. Upward shifts along the capital supply curve generate greater wealth concentration as the return gap nears the dissipation rate. Small changes in the return gap  $R - \delta - g - n$  can have large implications for capital supply, due to the explosive accumulation by the wealthy. This is key to understand why the growth slowdown can have strong implications for automation if wealth is a luxury good: the explosive accumulation by the wealthiest then generates abundant and cheap capital, which firms use to automate.<sup>14</sup>

### 3.3.2 Capital demand

Under competitive factor markets, capital demand follows from the equality of the cost of capital  $R$  and the marginal product of capital. The lower bound on the marginal product of capital is of particular interest and depends on the functional form of the capital-labor productivity schedule. Using equation (6) and replacing  $Y$  and  $K$  by their respective expressions in equations (5) and (9), we find equation (22) for  $R$ .

$$R = \tilde{H}\alpha^{-\gamma_i}b^{1+\gamma_i}(B - \alpha)^{-\gamma_i(B-1)} \quad (22)$$

Given equation (9), we know that  $\frac{K}{AL} \rightarrow \infty$  corresponds to  $\alpha \rightarrow 1$ : firms fully automate the economy if the ratio of capital to effective labor goes to infinity. Hence, the lower bound for the marginal product of capital for  $\frac{K}{AL} \rightarrow \infty$  is  $\tilde{H}b^{1+\gamma_i}(B - 1)^{-\gamma_i(B-1)}$ . Henceforth, we denote this lower bound by  $\mu$ . For the Cobb-Douglas case ( $B = 1$  and  $\gamma \rightarrow +\infty$ ), the lower bound on the marginal product of capital is zero. For the CES and VES case ( $B = 1$  and  $B > 1$  respectively), the lower bound on the marginal product of capital is strictly positive. Under appropriate normalizations based on the initial labor share of income, capital-output ratio and elasticity of substitution, it holds that the VES case results in a strictly greater lower bound on the marginal product than the CES case (cfr. infra). Intuitively, production without human labor becomes more feasible in the VES case where relative productivity of capital vis-à-vis labor is higher in not yet automated tasks (cfr. Figure 5).

<sup>13</sup>This is the general finding of Wold and Whittle (1957) that the inverse of the tail parameter is given by the  $\frac{\text{growth rate of wealth}}{\text{mortality rate}}$  for a random growth process. Solving the differential equation for top wealth accumulation of the wealthy  $\dot{a}_{i,t} = (R - \delta - g - n)a_{i,t}$  results in  $a_{i,t} = A_0 e^{(R - \delta - g - n)t}$ . The Poisson process for dissipation shocks implies that  $Pr(t > T) = e^{-pT}$ . Combined, they imply that  $Pr(a > A) = \left(\frac{A}{A_0}\right)^{\frac{p}{R - \delta - g - n}}$  which is the complementary distribution function of the Pareto distribution with tail parameter  $\frac{p}{R - \delta - g - n}$ .

<sup>14</sup>Impullitti and Rendahl (2025) also propose an explanation for rising wealth concentration that is based on slower growth with heterogeneity in the savings response by wealth. Their study focuses on rising market power instead of automation and the labor share, however.

In Figure 6, the role of endogenous automation in explaining top wealth inequality can be elucidated by looking at the market for loanable funds. The decline in productivity growth  $g$  and population growth  $n$  mean that “labor grows less quickly” and thus represents a positive shift in the supply of capital *per effective unit of labor*  $\frac{K}{AL}$ . Hence the capital supply curve shifts downwards. The degree to which this increases wealth inequality depends on the induced change in the return gap  $R - \delta - g - n$ . This is where automation comes in. In the absence of automation, the aggregate production function is Cobb-Douglas and the rate of return  $R - \delta$  drops strongly in response to a decline in growth  $g + n$  because of the limited substitutability of capital and labor. Under strong endogenous automation, however, (e.g. the VES case with  $B > 1$ ), firms respond to greater capital abundance by automating. The return to capital  $R - \delta$  then falls by less, since firms keep up capital productivity by expanding the share of automated tasks and finding new purposes for capital. As a result, the return gap  $R - \delta - g - n$  rises more in response to a fall in  $g + n$  if firms can respond to capital abundance in terms of automation. This generates a stronger rise in top wealth and income concentration.

### 3.3.3 Market for loanable funds

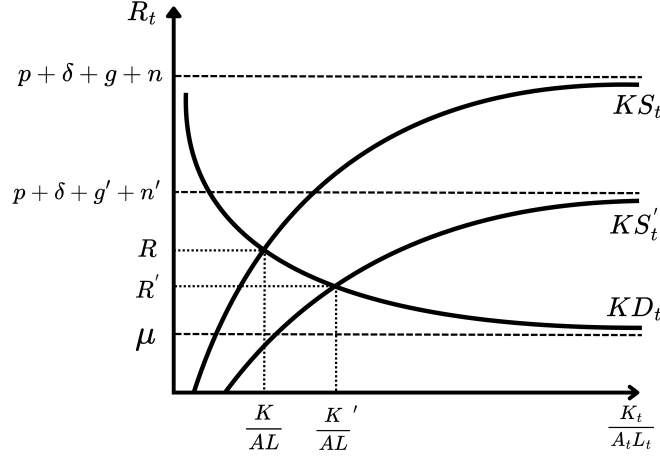
Compared to the standard Ramsey growth model, our loanable funds market is different both regarding capital supply and capital demand. The capital supply is upward-sloping, and bounded from above by  $p + \delta + g + n$ . Wealth concentration in the right tail is pinned down by the ratio  $\frac{p}{R - \delta - g - n}$  such that upward shifts along the capital supply curve generate greater wealth concentration. The capital demand curve has its typical negative slope, but its lower bound may be strictly greater than zero. Downward shifts along the capital demand curve generate greater levels of automation and thus a lower labor share. Graphically, it becomes clear how the historical decline in US population  $n$  and GDP per capita growth  $g$  over the last four decades could be an engine of greater wealth concentration and a fall in the labor share. The fall in  $g + n$  results in a clockwise rotation of the capital supply curve. The result is a decline in  $R$ , but less than one-to-one such that the tail parameter of the wealth distribution  $\frac{p}{R - \delta - g - n}$  falls. Hence, wealth concentration increases. The greater capital abundance  $\frac{K}{AL}$  lowers the cost of capital  $R$ , which sparks more automation and a lower labor share.

Sustained drops in  $g$  and  $n$  create the potential for an automation-inequality feedback loop. If  $g$  and  $n$  fall sufficiently for  $p + \delta + g + n < \mu$ , full automation and perfect wealth inequality ensue. The rate at which the wealthiest accumulate capital per efficiency unit of labor  $\mu - \delta - g - n$  then becomes greater than the rate at which wealthy households vanish  $p$ . The production technology then becomes of the AK type, and permanent growth at rate  $\mu - \delta - p$  emerges. This is the culmination of the ‘doom loop’ between automation and top income concentration, and a sufficient slowdown in population (growth) can elicit it. Lemma 3 summarizes this crucial result.

#### Lemma 3.

Given that (1) the depreciation rate of capital is given by  $\delta$ , (2) wealth “dissipates” at a rate  $p$  (meaning that at a rate  $p$ , (3) households are hit by a shock that makes them consume their entire asset stock), (4) the rate of labor-augmenting technical progress is  $g$  and the rate of population growth is  $n$ , and (5) the production technology is such that the lower bound on the marginal product of capital for  $\frac{K}{AL} \rightarrow \infty$  is given by  $\mu$

Figure 6: Schematic overview of the market for loanable funds, and the effects of a fall in growth from  $g + n$  to  $n' + g'$ .



The horizontal (vertical) axis denotes the capital stock per efficiency unit of labor  $\frac{K}{AL}$  (the return to capital  $R$ ). KS and KD resp. represent the supply and the demand for capital per efficiency unit of labor. The dissipation rate is given by  $p$ , and the depreciation rate by  $\delta$ .  $g$  and  $n$  resp. denote the growth rate of labor augmenting technology and population. The lower bound on the marginal product of capital is given by  $\mu$ . The effects of a fall in growth from  $g + n$  to  $n' + g'$  are shown: a positive shift in the supply of capital *per efficiency unit of labor* from KS to KS'.

imposing that wealth enters directly into the flow utility function as a luxury good implies that  $p + \delta + g + n > \mu$  must hold for a steady state to exist. If  $p + \delta + g + n \leq \mu$  the capital stock diverges to positive infinity and top wealth concentration tends to perfect concentration. For  $\mu > 0$ , the labor share then tends to zero.

### 3.3.4 Competitive equilibrium

The competitive equilibrium that defines the balanced growth path is the collection of quantities  $\{K, Y\}$  growing at a rate  $g + n$ , factor prices  $\{R, W_t\}$ , policy rules for the level of assets per efficiency unit of labor  $a_{i,t}$  (where  $t$  denotes the time since the household was last hit by a dissipation shock) and distributions  $g_i$  over the asset space such that

- (i) individuals maximize their utility taking prices as given  $\{R, w_i\}$ :  $a_{i,t}$  is a solution to the utility maximization problem described in equation (17).
- (ii) final goods producers maximize profits taking as given the price of the final good and factor costs  $\{R, W_t\}$ .
- (iii) the market for loanable funds clears:  $\frac{K}{AL} = \sum_{i=1}^Z n_i \int_0^\infty a_{i,t} p e^{-\rho t} dt$  with  $t$  denoting the time since the household was last hit by a dissipation shock.
- (iv) labor supply per capita  $l_i$  is exogenously given by the labor endowment such that the household's type-dependent, detrended wage rate per capita is given by a constant  $w_i = \frac{l_i W_t}{A_t}$ . Total labor supply grows exogenously at the rate of population growth:  $L_t = e^{nt} \sum_i n_i l_i$ .
- (v) the distribution over the asset space  $g_i$  is stationary for each household type  $i$ .



Since the households' accumulation rule  $a_{i,t}$  cannot be solved analytically, I rely on numerical methods to find the competitive equilibrium. I solve for the balanced growth path equilibrium numerically in Matlab in continuous time using the mean-field game approach of Achdou et al. (2022). More details can be found in Appendix C. While the uniqueness and stability of the equilibrium cannot be proved, I find that the numerical results are robust to the use of different initial conditions for the interest rate.

### 3.4 Calibration

#### 3.4.1 Calibration of production side

I calibrate the model to match key moments for the US economy in 2017. The economy is assumed to be on a balanced-growth path in that year. Table 1 shows the parameters of the model, their values and the relevant calibration targets.

Table 1: Parameters and their calibration for the US in 2017.

Parameter	Value	Calibration target	Value of target	Source for target
<i>Technology</i>				
Labor endowment by household type $l_i$	vector	Relative hourly wages by percentile*	-	US Census 2017
Long-run elasticity of substitution $\sigma$	1.35	Directly from literature	1.35	Hubmer (2023)
Capital share parameter $b$	Depends on production function	Share of labor in national income 2017	58.6%	BLS
Hicks-neutral technology factor $\bar{H}$	Depends on production function	Normalization of $y_0$	1	Arbitrary normalization
Depreciation rate $\delta$	5%	Directly from literature	5%	Standard calibration
<i>Household Preferences</i>				
Discount rate $\rho$	33.4%	Wealth to income ratio	$K/Y = 4$	World Inequality Database
Stone-Geary parameter for CS motive $S$	0.363	Share of wealth owned by top 1%**	32.5%	DINA (Piketty et al., 2018)
Luxury good parameter for CS motive $\zeta$	0.512	Share of wealth owned by top 0.01%**	9.1%	DINA (Piketty et al., 2018)
<i>Miscellaneous</i>				
Dissipation rate $p$	4.5%	Wealth persistence at the top over 16 years	49%	Gomez (2023) and Zheng (2020)
Rate of labor-augmenting technical change $g$	1.08%	Average annual growth rate of GDP per working-age person 2005-2015	1.08%	Real GDP (BEA) and Working-age population OECD
Rate of population growth $n$	0.72%	Average annual working-age population growth 2005-2015	0.72%	Working-age population (15-64) OECD

\*To make the data on the labor income distribution comparable to the model, the sample is limited to full-time employed individuals.

\*\*To make the data on top wealth concentration comparable to the model, two adjustments are made. First, only households with labor income above the minimum wage are withheld. Second, negative net wealth is treated as zero.

Following Klump et al. (2012), we normalize the production side such that we can compare the CD, CES and VES cases. As a result,  $\tilde{H}$ ,  $b$  and  $\gamma$  are set to match values for the capital-output ratio  $\frac{k_0}{y_0}$ , the capital share  $\alpha_0$  and the elasticity of substitution  $\sigma_0$ . The derivation of this normalization is discussed in Appendix D for the three cases (CD, CES and VES).

The capital-output ratio  $\frac{k_0}{y_0}$  is set to 4. This is based on the reported net national wealth to net national income ratio in labor endowmentality database (WID) for 2017. A value of 5 is reported. Based on a standard depreciation rate of 5%, this implies a capital-output ratio of 4 when equating capital with wealth. For the value of the capital share  $\alpha_0$ , we impose 41.4%. That is based on the reported value of the labor share of 58.6% of the BLS for 2017 for the nonfinancial corporate sector.

For the macro elasticity of substitution between capital and labor  $\sigma_0$ , we target a value of 1.35 in 2017. This corresponds to the middle of the range in the empirical work of Hubmer (2023) and Karabarbounis and Neiman (2014). Acemoglu, Manera, and Restrepo (2020) also consider values within this range. As indicated before, we follow the reasoning of Karabarbounis (2023) that distinguishing between a production function without and with automation can help to bridge the gap between micro-level estimates of the elasticity of substitution below one (e.g., Chirinko, 2008) and macro-level estimates above one (to which we calibrate the long-run production function). Our short-run production function (without automation) features an elasticity of substitution of one. Only after considering the endogenous adoption of automation technology in response to factor prices — which is a process that does not necessarily take place within the same firm, or even the same sector — does the elasticity of substitution exceed one.

The value of  $B$  is inherently speculative since the parameter captures the readiness of tasks for future automation. Figure 4 shows how the capital-labor productivity schedule depends on  $B$ . To speculate in an informed way, it is worth considering what the relative productivity schedule looks like for different values of  $B$  given our normalization in Appendix D. To assess this, we report (1) the mean relative productivity of capital vis-à-vis labor in not yet automated tasks as a percentage of the relative productivity in the marginally automated task, and (2) the relative productivity in the least automatable task (that is the task with index 1) as a percentage of the relative productivity in the marginally automated task. Additionally, we are interested in the lower bound on the marginal product of capital  $\mu$  for each calibration. Table 2 summarizes this information for some values of  $B$ .<sup>15</sup> Instead of selecting one “baseline” choice, I proceed by considering the results under these different scenarios. When referring to the VES case, we use  $B = 6$  from now on.

Additionally, the labor endowment by household type  $l_i$  is set to match the real hourly wage distribution in the US in 2017. We distinguish 100 different household types  $i$  and each type corresponds to a percentile of the wage distribution. Hence, the population share of each type  $i$  is 1%:  $n_i = 0.01$ . The data on real hourly wages comes from the US Census for 2017, and is limited to full-time employed individuals.

### 3.4.2 Household preferences

The rate of pure time preference  $\rho$  is set to generate the capital-output ratio of 4.

<sup>15</sup>For  $B \rightarrow \infty$ , the lower bound  $\mu$  does not change much compared to when  $B = 6$ . The limit value for  $\mu$  when  $B \rightarrow \infty$  equals 7.1% under the normalization with  $\frac{k_0}{y_0}$ ,  $\alpha_0$  and  $\sigma_0$ .

Table 2: Capital-labor productivity schedule and the implied value for the lower bound on MPK for different aggregate production functions.

	Cobb-Douglas	CES (B=1)	VES with B = 1.5	VES with B = 6
Mean relative productivity nonautomated tasks as % of relative productivity marginally automated task	0%	14.5%	30.6%	50.8%
Relative productivity least automatable task as % of relative productivity marginally automated task	0%	0%	5.0%	26.4%
Lower limit on marginal product of capital $\mu$	0%	0.8%	4.2%	6.7%

These figures follow from the normalization in Appendix D with  $\frac{k_0}{y_0} = 4$  and  $\alpha_0 = 41.4\%$ . Given the calibration, the marginal product of capital (MPK) in the steady state is 10.35%. For more background info on the task-based origins of the production function, see section 3.1 and Figure 4.

Regarding the capitalist-spirit preferences (the Stone-Geary parameter  $S$  and the curvature parameter  $\zeta$ ), they are set to match, as best as possible, the wealth share of the top 1% and the top 0.01% wealthiest American households for 2017. Data for the calibration targets come from the Distributional National Accounts (DINA) (Piketty et al., 2018). To ensure that the data is model-conform, I restrict the sample to households with a yearly labor income above the minimum wage for one full-time employee. Additionally, I treat negative net wealth values as zeroes given the model's borrowing constraint. The result is a target wealth share of 32.5% for the top 1%, and of 9.1% for the top 0.01%.

The rate at which wealth dissipates  $p$  is calibrated to match the degree of mobility at the very top of the wealth distribution. Gomez (2023) assembled data on the composition of the Forbes 400 (a list of the wealthiest 400 Americans) and the family relations between members. Based on his data, I compute that 49% of the 2017 Forbes 400 list has family ties to members of the 2001 Forbes list. Our model equivalent for this fraction is the probability that a dynasty is not hit by a dissipation shock for 16 years:  $e^{-16p}$ . This results in a value for  $p$  of 4.5%.<sup>16</sup> This is identical to the value of Moll et al. (2022), even though their calibration strategy for  $p$  is not based on top wealth mobility. In a similar vein, Zheng (2020) reports that 20% of those on the 1982 list have family members who appear on the 2018 list. This is also in accordance with a dissipation rate of 4.5%.

Finally, (1) the growth rate of labor-augmenting technology  $g$  and (2) the population growth rate  $n$  are set to match the average growth rate of (1) potential GDP per working-age person, and (2) the working-age population in 2015-2015.

## 4 Results

### 4.1 Validation of household side

Before checking whether our model can explain recent macro and inequality evolutions, we look at the static distributional implications of the model. Note that the household distribution is entirely independent of the choice of the production side (CD, CES or VES).

<sup>16</sup>We focus on long-run wealth mobility, since year-on-year evolutions in Forbes 400 membership are likely to capture volatility in business valuations rather than serious downward mobility (e.g., business owners repeatedly leapfrogging each other). For instance, Zheng (2020) reports a much higher yearly exit rate of Forbes 400 members of around 10%.

Table 3 compares the distribution of net wealth and total income in the model economy with that in the data. Actual data comes from the distributional national accounts (DINA) for 2017. The non-homothetic model (with wealth as a luxury good) is calibrated to match the top 1% and top 0.01% of the wealth distribution as well as possible. Nevertheless, we consider its distributional fit to be encouraging given the relatively stylized nature of the model. The combination of stochastic asset growth (dissipation) and non-homothetic savings behavior (capitalist-spirit) seems sufficient to match the wealth and total income distribution reasonably well, although the fit is not perfect.<sup>17</sup> As in Mian et al. (2021a), I also consider a homothetic version of the model where households still face a direct (capitalist-spirit) preference for wealth, but no longer as a luxury good:  $\zeta$  and  $S$  are set to 1 and 0 respectively. This results in constant savings rates over the permanent income distribution (cfr. *infra*). Clearly, the homothetic version of the model results in too little concentration of both income and wealth at the top of the distribution.

Table 3: Personal Income and Wealth distribution in 2017 for the US						
	Bottom 50%	51 to 90%	91 to 99%	Top 1%	Top 0.1%	Top 0.01%
Total Income						
Actual	17.8%	39.0%	24.7%	18.5%	8.5%	3.8%
Model Homothetic	24.9%	44.7%	23.2%	7.2%	0.9%	0.1%
Model Non-Homothetic	20.0%	40.3%	25.7%	14.1%	4.8%	2.0%
Net Wealth						
Actual	3.9%	31.1%	32.5%	32.5%	17.1%	9.1%
Model Homothetic	17.9%	46.9%	26.5%	8.7%	1.3%	0.1%
Model Non-Homothetic	4.0%	26.4%	33.1%	36.5%	16.2%	7.4%

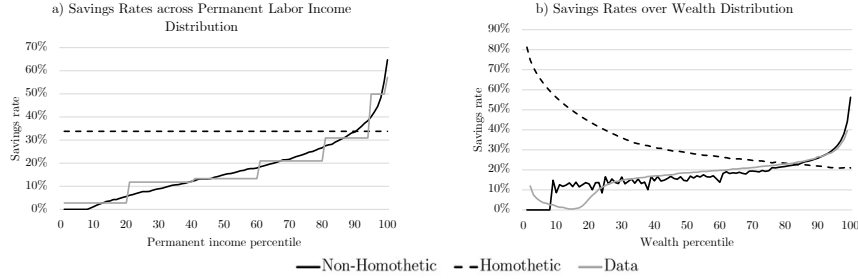
The non-homothetic model is the baseline model with the parameters of Table 1. The homothetic model follows the same calibration of Table 1, but  $\zeta$  and  $S$  are set to 1 and 0 respectively and the value of  $\rho$  is re-calibrated to ensure  $\frac{K}{Y}$  equals 4. Data for net wealth and total income comes from DINA for 2017. Two adjustments are made to the data. First, only households with labor income above the minimum wage are withheld. Second, negative net wealth is treated as zero.

The model's implications regarding the distribution of savings rates across the permanent labor income distribution and wealth distribution are shown in Figure 7. Here, we report *net* savings rates, meaning that capital income is net of depreciation. As indicated, the non-homothetic model produces rising savings rates across both the income and wealth distribution because wealth is treated as a luxury good. The extent of savings rate heterogeneity matches nicely with the influential empirical results of Dynan et al. (2004) and Fagereng et al. (2019). Dynan et al. (2004) use SCF data and find that savings rates increase over the permanent income distribution (which is proxied by instrumenting current income by lagged income). Fagereng et al. (2019) use Norwegian administrative data to find savings rates over the wealth distribution. The comparison suggests that the modelled non-linearity in savings behavior is not overly strong. Note that 8% of households do not accumulate any assets whatsoever. In terms of the categorization of 5, we find a division of the total population between the poor, middle class and wealthy of 8%, 77% and 15%, respectively. For comparison, 12.7% of households with a yearly labor income above the minimum wage have non-positive net wealth according to the 2017 DINA.

In any case, it seems to capture the extent of heterogeneity in savings rates much better than the

<sup>17</sup>The non-homothetic model does not produce a Pareto type I distribution for total income and net wealth, but a Pareto type II distribution. This means that the shape parameter of the Pareto distribution is non-constant over the distribution (as opposed to what Moll et al. (2022) obtain). Because of the increasing savings rates of wealthy households, income and wealth concentration further increase as one nears the very top. This echoes recent results that point to increasing Pareto inequality as one nears the very top for both income (Blanchet et al., 2022) and wealth (Wildauer et al., 2023) in the US.

Figure 7: Net savings rates over the permanent income and wealth distribution.



The model savings rates are net, hence capital depreciation is not included in savings. They reflect household savings when not hit by a dissipation shock. The non-homothetic is the baseline model with the parameters of Table 1. The homothetic model follows the same calibration of Table 1, but  $\zeta$  and  $S$  are set to 1 and 0 respectively and the value of  $\rho$  is re-calibrated to ensure  $\frac{K}{Y}$  equals 4. The model savings rates across the permanent labor income distribution are compared with the empirical findings of Dynan et al. (2004) for the US using the 1983–89 data of the Survey of Consumer Finances (their Table 5, column 1). The model savings rates across the wealth distribution are compared with the empirical findings of Fagereng et al. (2019) for Norway using 2005–2015 administrative data (their Figure 1). The empirical savings rates are re-scaled by a constant factor to ensure that their unweighted mean is that of the non-homothetic model.

model with homothetic preferences. With homothetic preferences, savings rates are perfectly constant across the permanent labor income distribution (Straub, 2019). Across wealth, savings rates decline in our homothetic case. The intuition is that households do not unboundedly accumulate a risky asset if the marginal utility of wealth declines as quickly as that of consumption.<sup>18</sup>

#### 4.2 Explanation of historical shifts in labor share and wealth distribution

We now consider whether the model can jointly explain the shifts in both (1) the functional income distribution (between capital and labor), and (2) the personal income and wealth distribution. We compare two balanced-growth paths: one is the calibrated balanced growth path for the US in 2017, the other represents the 1980 balanced growth path. Our preferred explanation focuses on the higher rate of labor-augmenting technical change  $g$  and population growth  $n$  in the 1980 growth path. Table 4 summarizes extent of growth slowdown from 1980 to 2017. Additionally, the rise in wage dispersion from 1980 to 2017 (cfr. Figure 8) is considered as a driver of automation and top wealth concentration by exogenously varying the labor endowment by household type  $l_i$  in the model.<sup>19</sup>

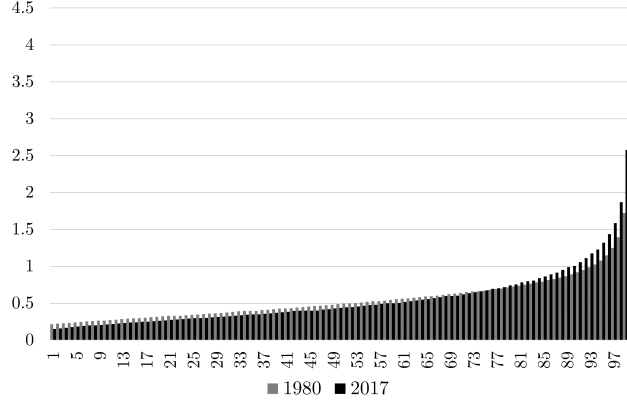
Table 4: Growth Rate of Selected Variables			
	1970-1990	2005-2015	Change
Real GDP per Person of Working Age $g$	1.54%	1.08%	-0.46%
Working-Age Population (15-64) $n$	1.56%	0.72%	-0.84%

Real GDP data is from the BEA. Working-Age Population data is from OECD. The variables consider GDP divided by Working-Age Population, and Working-Age Population.

<sup>18</sup>Just like in the Aiyagari (1994) model, homothetic households which are not hit by a negative (dissipation) shock will in the limit accumulate up to some “buffer stock” level. In the homothetic model, this happens if the effective discount rate  $\rho' - n$  exceeds twice the return gap  $2(R - \delta - g - n)$ . The nullclines in equations (19) and (20) then lead to a maximum wealth level of  $\frac{w_i}{(\rho' - n) - 2(R - \delta - g - n)}$ .

<sup>19</sup>This interpretation interprets the rise of labor income inequality as a rise in *permanent* labor income dispersion, rather than increased labor income risk. This is corroborated by empirical findings De Backer et al. (2013) and Kopczuk et al. (2010).

Figure 8: US Census Data on Real Hourly Wages by Percentile in 1980 and 2017.



The hourly wages are normalized such that their mean is one. The sample is limited to full-time employed individuals. This is imposed in the model by exogenously varying the labor endowment by household type  $l_i$ .

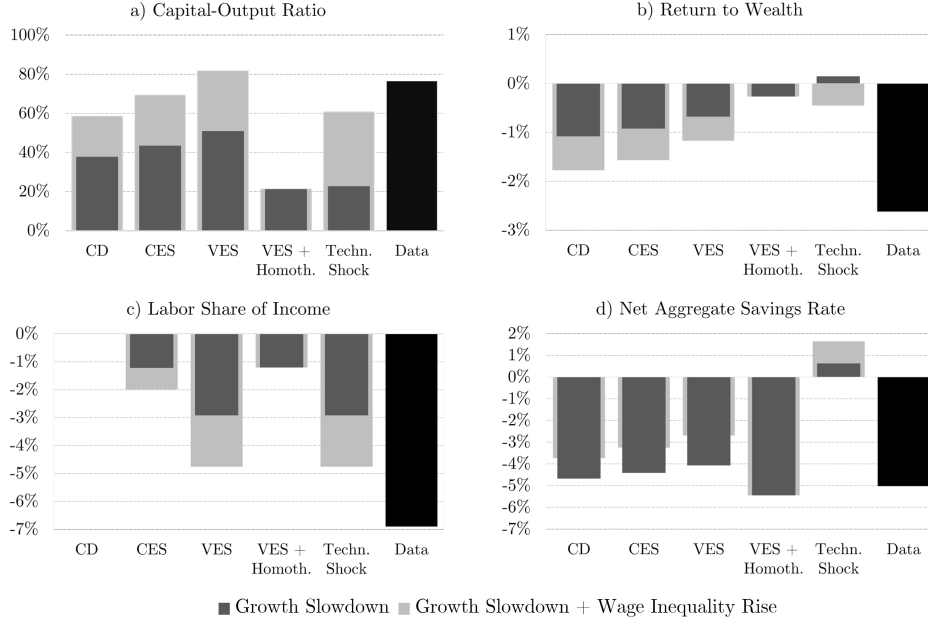
So what are the macro and distributional effects of the growth slowdown? This is summarized in Figures 9 and 10, respectively. In both figures, five different scenarios are considered and compared to the data. The first three columns show the effects of the growth slowdown (and exogenous rise in wage inequality) for the baseline model with non-homothetic savings behavior and an aggregate production function that is (1) Cobb-Douglas, (2) CES or (3) VES. The fourth column considers the same growth slowdown in the VES case, but now with homothetic preferences ( $\zeta = 1$  and  $S = 0$ ). The fifth column replaces the growth slowdown with an exogenous technology shock in  $b$  that generates the same labor share decline as the growth slowdown in the VES case; the model also features non-homothetic preferences.

The macro story is as follows. The decline in labor-augmenting technology growth  $g$  and population growth  $n$  implies that effective labor  $AL$  grows less quickly. The result is a relative abundance of capital supply per efficiency unit of labor  $\frac{K}{AL}$ : the capital-output ratio  $\frac{K}{Y}$  rises in panel (a). In the short run, this does not elicit automation as the allocation of production factors to tasks is fixed. Hence, the return to wealth  $R$  drops proportionally to the rise in the capital-output ratio  $\frac{K}{Y}$ .<sup>20</sup> As a result, the labor share, as the product of  $\frac{K}{Y}$  and  $R$ , remains constant in the absence of automation. In the long run, however, firms may respond to this drop in the cost of capital  $R$  by expanding the set of automated tasks: changing factor costs induce a shift in production technology and a rise in capital demand. The induced automation raises the return to capital  $R$  (though not as high as the initial level) in panel b and elicits further capital deepening  $\frac{K}{Y}$ , and the result is a fall in the labor share in panel c. The labor share falls more so if the capital-labor productivity schedule is less steep such that automation is a more potent margin of adjustment (eg. VES instead of CES). In the case of non-homothetic preferences, the rise in wage dispersion generates an additional impulse to automation as households with high permanent labor income save at a higher rate.

Compared to the technology-oriented explanation of Moll et al. (2022), my growth-oriented explanation has the advantage that it can reconcile automation with the observed decline in the aggregate savings and investment rate in panel d. The technology-oriented explanation basically states that there

<sup>20</sup>In equation (8),  $R$  is inversely proportional to  $\frac{K}{Y}$  for a given share of automated tasks  $\alpha$

Figure 9: Changes in macro variables from 1980 to 2017 in percentage points. Model effects of the growth slowdown under different scenarios vs. data



Five different scenarios are considered and compared to the data. The first three columns show the effects of the growth slowdown (and exogenous rise in wage inequality) for the baseline model with non-homothetic savings behavior and an aggregate production function that is (1) Cobb-Douglas, (2) CES or (3) VES. The fourth column considers the same growth slowdown in the VES case, but now with homothetic preferences ( $\zeta = 1$  and  $S = 0$ ). The fifth column replaces the growth slowdown with an exogenous technology shock in  $b$  that generates the same labor share decline as the growth slowdown under the VES case (column 3). The dark gray bars correspond to the case where  $g$  drops from 1.54% to 1.08%, and  $n$  drops from 1.56% to 0.72%. The light gray bars correspond to the same fall in  $g$  and  $n$ , and additionally the wage distribution changes as measured in the Census Data (1980-2017) (cfr. Figure 8). Capital-output ratio data is net national wealth to GDP from the World Inequality Database (400.5% in 2017). Interest rate data comes from Holston et al. (2017) estimates of  $r^*$  (0.92% in 2017). Data on the labor share comes from the BLS for the nonfinancial corporate sector. Net aggregate savings rate data come from own computations based on BEA data (3.33% in 2017).

has been an upward shift in the capital demand curve in Figure 6 compared to 2017. This results in a greater capital-output ratio  $\frac{K}{Y}$  with constant growth  $g+n$ . Following Piketty's second fundamental law of capitalism, it holds that the steady-state capital-to-net-output ratio is determined by the ratio of the net aggregate savings rate  $s$  and GDP growth  $g+n$ :  $\frac{K}{Y-\delta K} = \frac{s}{g+n}$ . Technology-driven automation generates an increase in  $\frac{K}{Y}$  through a rise in the net savings rate  $s$ .<sup>21</sup> With a decline in growth  $g+n$ , however, there is scope for the net savings (or investment) rate  $s$  to decline with rising capital-output ratio  $\frac{K}{Y}$ . Even as a smaller share of net output flows to the expansion of the capital stock (net investment  $s$ ), the capital-output ratio  $\frac{K}{Y}$  can converge to a greater level if GDP growth  $g+n$  is systematically lower. This decline in  $s$  in response to falling  $g+n$  is precisely what we find: as suggested by Krusell and Smith (2015), lower growth rates go hand in hand with slower savings. The reason is that the rise of  $\frac{K}{Y}$  results in a lower

<sup>21</sup>In the balanced growth equilibrium, the capital-output ratio  $\frac{K}{Y}$  and the capital-to-net-output ratio  $\frac{K}{Y-\delta K}$  move in the same direction. It holds that  $\frac{K}{Y-\delta K} = \frac{\frac{K}{Y}}{1-\delta \frac{K}{Y}}$ .

return to wealth, despite the induced automation, and the lower  $R$  dissuades the accumulation of assets.<sup>22</sup>

The story for top income and wealth concentration is more straightforward. The drop in the sum of population and productivity growth  $g + n$  generates a rise in the return gap  $R - \delta - g - n$ . This is because the rate of return to capital does not fall proportionally to the decline in growth  $g + n$ . That would only be the case under vertical capital supply in Figure 6 (which would require a Leontief production function). This rise in the difference between the rate of return to wealth  $R - \delta$  and GDP growth  $g + n$  implies that wealthy households jump further ahead compared to the rest of the economy. Intuitively, the non-homothetic model explains a greater rise in top wealth and income shares as the savings behavior of the wealthy is more explosive in that case. For the explanation of the observed fall in the income and wealth shares of the bottom 50%, the return gap is of less importance and the rise in wage dispersion is crucial. When considering the classification of Figure 5, our results also corroborate the so-called “middle class squeeze”. In the model with a VES, the rise in labor income inequality and the increase in the return gap cause the population share of the poor (who do not accumulate assets) to increase from 0% to 8%. Simultaneously, the share of the wealthy who accumulate unboundedly grew from 10% to 15%. Jointly, the model thus generates a decline in the middle class population share from 90% to 77% from 1980 to 2017.

The interaction between automation and non-homothetic savings behavior is of crucial importance for the strength of the mechanisms. First, the explanation of the fall in the labor share is far stronger in the non-homothetic case. The upward redistribution implied by the fall in the labor share triggers an additional increase in aggregate savings because rich people save more than poor. In a model without this savings heterogeneity (the homothetic model), this rise in inequality has no such capital deepening implications and less automation ensues. If inequality matters for macro in a realistic way, we find that the growth slowdown and rise in wage dispersion can account for up to 70% of the labor share decline. In model where inequality does not matter for macro, only 25% of the labor share decline is explained, for the same technology and same growth slowdown.

Second, the strength of the automation reaction of firms also matters for top wealth and income concentration. This is because, if firms can react more strongly to factor costs in terms of automation, the return to wealth  $R - \delta$  declines less in response to a fall in growth  $g + n$ . Firms can then make more effective use of capital abundance by expanding the set of automated tasks, hence capital productivity drops less. The result is a stronger increase in the return gap  $R - \delta - g - n$  and more top wealth and income concentration. In the model with VES production, the rise in the top 1% wealth share is almost fully explained. In a model without automation, less than half is explained.

There are two reasons for which I am not concerned with even the most techno-optimist model undershooting the full decline in the labor share in the data. First, other explanations might play a role for the observed decline in the labor share. Most notably, the increase of firms’ market power has been put forward (Barkai, 2020; De Loecker et al., 2020).<sup>23</sup> While there is various evidence that points to

<sup>22</sup>This can formally be shown by transforming the Euler equation for detrended consumption in equation (18) to actual consumption. A lower  $R$  then implies a less steep path for consumption growth.

<sup>23</sup>However, both explanations may operate at the same time and need not be independent: Hubmer and Restrepo (2021) shows how, with the presence of a fixed cost to automation, the decline of the labor share can be driven by large firms automating and gaining market share. Acemoglu et al. (2022) finds indeed that automation adoption is concentrated amongst large firms. My model could generate an alternative link between both explanations. To the extent that the increase in of the price mark-up shifts the income distribution in favor of wealthier households, this can drive additional



the importance of technology in the explanation of the declining labor share (Acemoglu, Lelarge, and Restrepo, 2020; Acemoglu and Restrepo, 2020; Bergholt et al., 2022), imperfect markets may very well be an important part of the explanation. Second, there are several studies indicating that the headline figures overstate the actual labor share decline because of changes in the treatment of intellectual property as investment (Koh et al., 2020), self-employment and housing (Gutiérrez and Piton, 2020) and the rise of the pass-through sector (Smith et al., 2022). Regarding top income concentration, the findings of DINA (Piketty et al., 2018) are not without any criticism either: Auten and Splinter (2019) report a more muted top income concentration increase since 1980.

### 4.3 Model predictions for the future

A nice feature of considering the growth slowdown as the core driver behind automation instead of technology is that it leaves scope for meaningful predictions regarding the future. Of course, this exercise first requires establishing reasonable predictions for future productivity and population growth. For population growth, I assume a future rate of 0%. This corresponds to the Census baseline projection for the growth rate of the total US population in 2080 (after which total population growth turns negative).<sup>24</sup> In the case of a low immigration scenario, population growth would halt much quicker. The future long-run growth rate of GDP per capita is surrounded by much more uncertainty. For this reason, we consider the implications of the model for different scenarios of the GDP per capita growth rate. In the context of the climate crisis and the call by some for degrowth, even negative values are considered. For the future, all other technological and household parameters assumed to remain constant. Again, we focus on statics here: all information refers to the new balanced growth equilibrium given these changes in  $n$  and  $g$ .

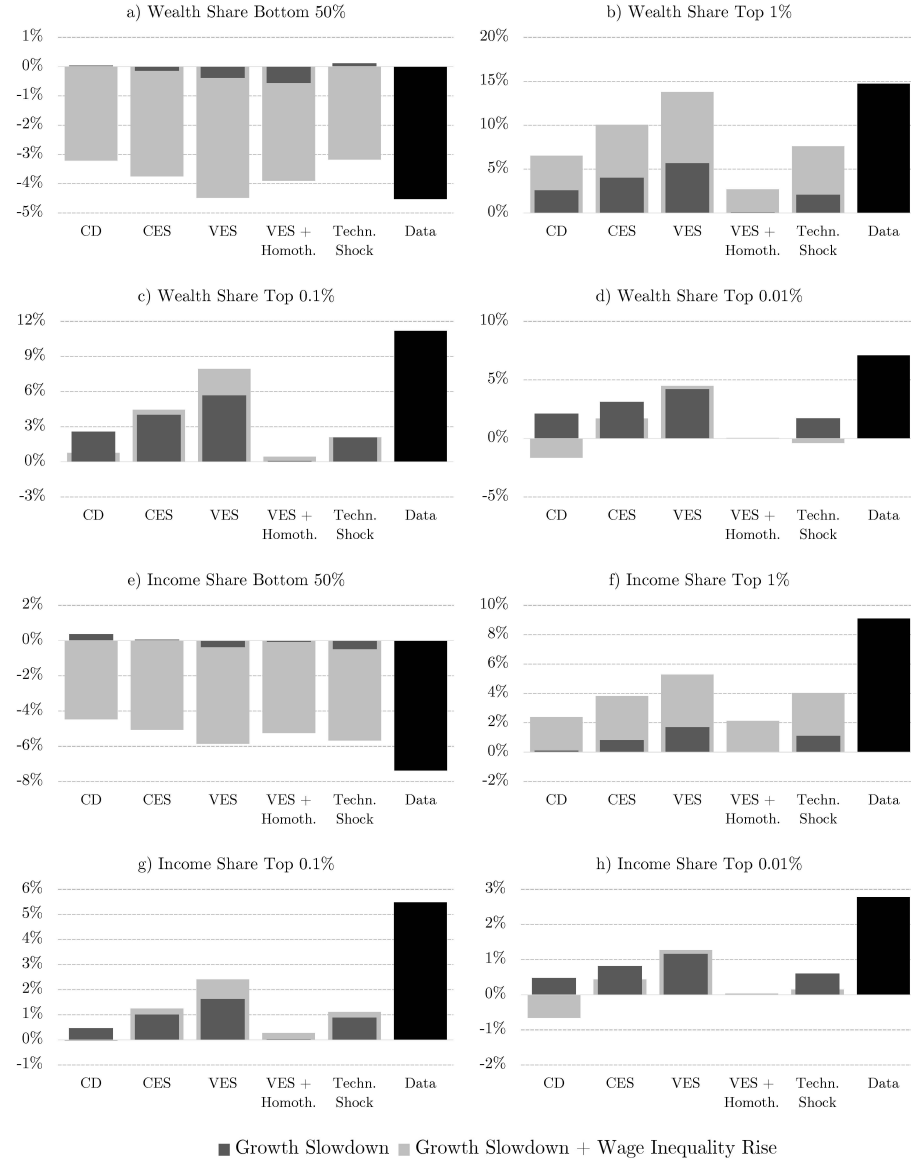
Before turning to numerical analysis, it is worth considering asymptotics. As argued in section 3.3, the automation-inequality doom loop brings about total automation and perfect wealth and income inequality if the rate at which the wealthiest accumulate capital per efficiency unit of labor  $\mu - \delta - g - n$  becomes greater than the rate at which households lose their wealth due to dissipation shocks  $p$ . Even in the most techno-optimistic scenario, the lower bound on the marginal product of capital  $\mu$  is 'only' 6.7% when normalizing the production function appropriately. Given our parameterization of the dissipation rate  $p$  at 4.5% and the depreciation rate  $\delta$  at 5%, this implies that the sum of population growth and labor-augmenting technology growth  $g + n$  has to fall below -2.8%. In the case of CES technology with constant elasticity of substitution of 1.35, this is even -8.7%. Even under the most pessimistic growth projections and most optimistic evaluations of the potential of automation technology, full automation and perfect inequality as in Lemma 3 thus do not occur. Nevertheless, both automation and inequality can still increase considerably in the future as a result of the growth slowdown, depending on (1) the functional form of the capital-labor productivity schedule, and (2) the strength of non-homothecity in the savings behavior. To evaluate this, we turn to our numerical exercise.

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investment and make more automation cost-effective. But this is beyond the scope of this work.

<sup>24</sup>The growth rate of the population aged 18 to 64 is projected to turn negative earlier, from 2050 onwards.

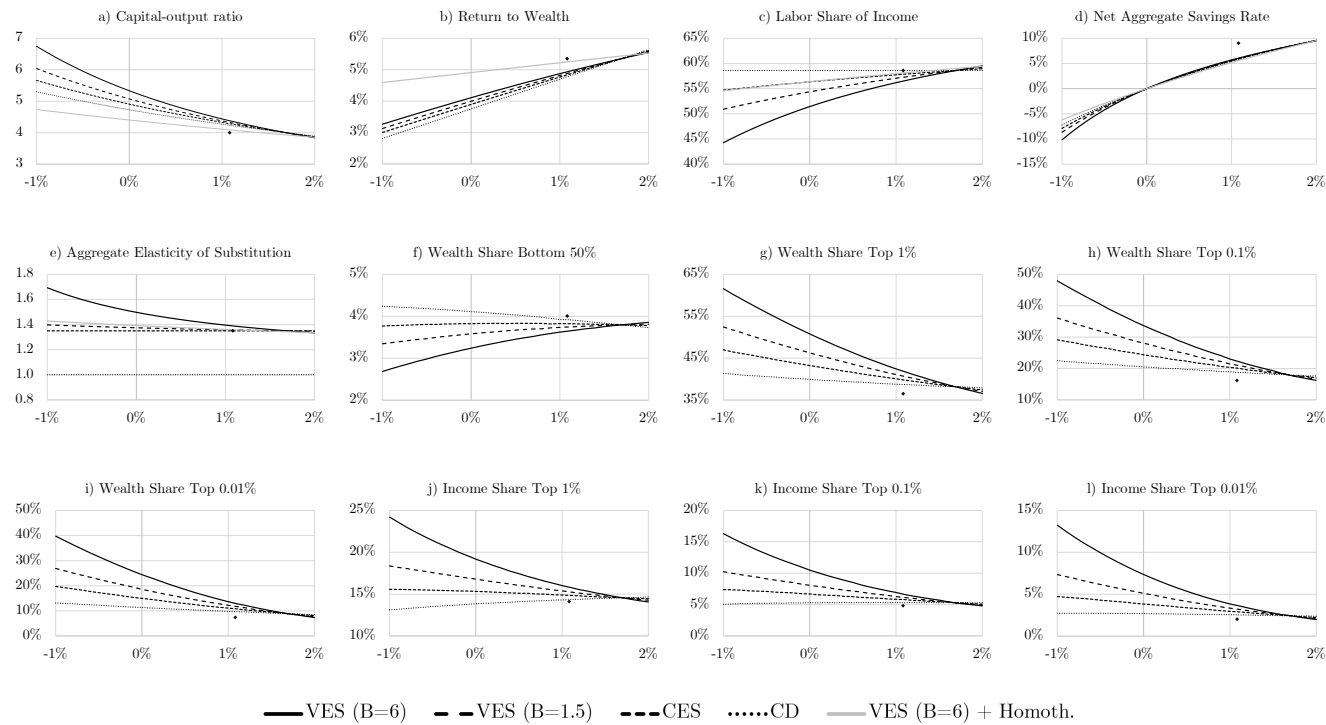
Figure 10: Increase in wealth and income inequality in the US from 1980 to 2017. Model effects of the growth slowdown under different scenarios vs. data



Five different scenarios are considered and compared to the data. The first three show the effects of the growth slowdown (and exogenous rise in wage inequality) for the baseline model with non-homothetic savings behavior and an aggregate production function that is (1) Cobb-Douglas, (2) CES or (3) VES. The fourth considers the same growth slowdown in the VES case, but now with homothetic preferences ( $\zeta = 1$  and  $S = 0$ ). The fifth replaces the growth slowdown with an exogenous technology shock in  $b$  that generates the same labor share decline as the growth slowdown in the VES case. The dark gray bars correspond to the case where  $g$  drops from 1.54% to 1.08%, and  $n$  drops from 1.56% to 0.72%. The light gray bars correspond to the same fall in  $g$  and  $n$ , and the wage distribution changes as measured in the Census Data (1980-2017).

Data on wealth and income inequality comes from DINA and concerns net wealth and pre-tax income. To make the distributional data model-consistent, (1) we consider negative net wealth as zero, and (2) consider only households with pre-tax income larger than the minimum wage.

Figure 11: Evolution of selected macro-economic variables and inequality under different growth scenarios.



The value on the vertical axis is that of the variable in question in the balanced-growth path that is characterized by the rate of labor-augmenting technical change  $g$  indicated on the horizontal axis. All scenarios have a zero population growth rate ( $n = 0$ ). The black diamond indicates the value of this variable for the 2017 BGP. The considered scenarios are (1) the VES case with  $B = 6$  (full line), (2) the VES case with  $B = 1.5$  (dashed line), (3) the CES case (finely dashed line), and (4) the Cobb-Douglas case (dotted line). These all have the baseline non-homothetic preferences. Additionally, scenario (5) (with grey line) is the VES case where preferences are homothetic ( $\zeta = 1$  and  $S = 0$ ). The homothetic results for wealth and income inequality are not shown since it results in far too little concentration at the top (cfr. Table 3).

Figure 11 summarizes the results of this numerical exercise. It shows what would happen, according to the model, if population growth goes to zero and the long-run growth rate of labor-augmenting technical progress is equal to the value on the horizontal axis. The overall prediction for a “low growth” future is that it would come with a lower labor share, and stronger wealth and income concentration. The size of the changes depends massively on (1) the extent to which firms can respond to changing factor costs in terms of automation, and (2) the presence of heterogeneous savings behavior.

Clearly, the ease of future automation opportunities matters *a lot* for the amount of wealth concentration induced by the growth slowdown. In the Cobb-Douglas case — where no automation is induced by the fall in  $g + n$  —, wealth and income concentration increase only modestly. In the VES case — where a lot of automation is induced by the fall in  $g + n$  — wealth and income concentration increase spectacularly. The intuition is analogous to that of the previous section: the growth slowdown and the corresponding rise in the capital-output ratio elicit a far smaller decline in the return to wealth if firms can respond to cheaper capital by automating more. Hence, the rate at which the wealthiest accumulate capital per efficiency unit of labor  $R - \delta - g - n$  increases more since  $R$  barely declines if  $g + n$  declines. As a result, the wealthiest jump further ahead compared to the rest. A complete halt in productivity and population growth would imply a 3 pp rise in the top 1% wealth share in the case of Cobb-Douglas technology but a spectacular 14 pp increase in the VES case. This is the first important point: the rise in top inequality generated by the growth slowdown is far larger if firms can respond to cheaper capital by automating more. If we do not anticipate that cheaper capital will make some unused automation opportunities profitable in the future, we thus risk underestimating the rise in top income concentration. This is very different from the findings of Hubmer et al. (2021) (in their Table 5) where the fall in the labor share even generates a slight reduction in top wealth concentration. In contrast to their model, the non-homothetic savings behavior in our model implies that saving of the poor is far less elastic with respect to the interest rate than the saving of the rich.

Likewise, the (non)linearity of savings behavior matters a lot for the macro implications of the growth slowdown. If population and productivity growth were to stop completely, the non-homothetic model predicts a 7 pp fall in the labor share as a result of the growth slowdown, while under homothetic savings behavior this is less than half. The reason is that the elasticity of capital supply with regard to the rate of return is far lower, i.e. the capital supply curve is far steeper under homothecity than under non-homotheticity. The intuition is that, if rich people in the limit save their entire income flow, there is an explosive increase in capital accumulation by the wealthy as the return gap  $R - \delta - g - n$  approaches the dissipation rate  $p$ . This is the second point: the rise in automation generated by the growth slowdown is far larger if savings behavior is non-homothetic. Imposing linear savings functions thus risks underestimating the fall in the labor share.

Faced with the unpalatable implications of rising top wealth and income concentration and the erosion of the labor income tax base through the decline of the labor share, policymakers may want to correct both. While we stop short of doing an actual policy analysis, it is worthwhile to mention that the introduction of a uniform yearly wealth tax comes close to perfectly compensating for the distributional effects of the growth slowdown. Both regarding the labor share and top inequality, reacting to the drop in  $g + n$  by raising the wealth tax one-to-one more or less offsets the inequality effects of the growth slowdown. For changes in the growth rate of productivity  $g$ , the offset is actually perfect. The reason is

that the annual wealth tax can be seen as an increase in the depreciation rate  $\delta$ , and  $\delta$  and  $g$  enter into the household optimization problem in the exact same way. For population growth  $n$ , this offset is not perfect since population growth also affects the rate of households' time preference (cfr. equation (17)).

## 5 Conclusion

In this work, I construct a tractable heterogeneous-agent model to study the interaction between automation and top income concentration when inequality matters for macro. Using non-homothetic household preferences, we match the empirical finding that high-wealth households save at a higher pace. As automation redistributes income towards high-wealth households, this then lowers the interest rate and incites firms to automate more. Automation can thus generate the capital investment that fuels further automation. In short, the rise of the 1% is not just the result of automation (as found in the literature), but also its cause. I find that the inequality driver of automation is quantitatively important.

The core findings are as follows. First, the observed fall of productivity and population growth and the rise of wage inequality since 1980 can explain up to 70% of the fall in the labor share and almost the entire increase in the top 1% wealth share. Second, the strength of the labor share effect depends heavily on the presence of non-homothetic savings behavior. Without it, only 25% of the fall in the labor share can be explained. Third, the strength of the inequality effect depends heavily on the responsiveness of automation to changes in factor costs. If firms can react to the growth slowdown by automating, the rate of return declines less, the interest-growth differential increases more and, hence, wealth inequality worsens more. In the absence of further automation opportunities, a total halt of population and productivity growth leads to a 3 pp rise in the top 1% wealth share; if automation in response to cheaper capital is possible, this can increase to 14 pp in the most techno-optimist scenario. Fourth, the model results in a clear analytical threshold for when the automation-inequality doom loop materializes. The intuition is that the presence of endogenous automation implies that the lower bound on the marginal product of capital becomes strictly greater than zero, and that the capitalist-spirit motive for wealth implies that wealthy households save the entire return on their wealth in the limit. Hence, if the sum of population growth and productivity growth drops below the lower bound on the net rate of return to wealth, the production process becomes fully automated and inequality becomes perfect.

My theory predicts a strong effect of past and future slowdowns in (population) growth on the functional and personal income distribution of the US. A doom loop with positive feedback between rising automation and rising wealth accumulation by the wealthiest may lead to perfect inequality and full automation. Such a scenario does not seem imminent, however, even when considering the most pessimistic forecasts regarding productivity growth. Nevertheless, serious further inequality increases are to be expected in the face of a further slowdown in (population) growth. A clear limitation of this model is its stylized nature, and lack of serious labor market frictions (full employment assumption). Nevertheless, our model is sophisticated enough to consider the robustness to different assumptions on both production and household side. In particular, I have considered different scenarios regarding (1) the ease of future automatability (and thus the evolution in the elasticity between capital and labor), and (2) the extent to which inequality matters for macro. A notable omission from the work is the role of heterogeneous wage effects of automation, which would further strengthen the feedback between automation and inequality if future automation raises the skill premium.

## Appendix A

### Proof A1

Given equations (1) and (2) of the main text, the price of a certain task  $x$  in terms of the final good is given by (23).

$$p_{x,t} = \frac{\partial Y_t}{\partial T_t(x)} = dx \frac{Y_t}{T_t(x)} \quad (23)$$

Because of the perfect substitutability between capital and labor within a task  $x$ , firms choose to execute that task using *either* capital or labor depending on which one is more cost-effective. The intuition is that the isoquant for the execution of a task  $x$  is linear because the marginal rate of technical substitution is constant:  $\frac{\frac{\partial Y_t}{\partial k_t(x)}}{\frac{\partial Y_t}{\partial l_t(x)}} = \frac{\theta_t(x)}{\xi_t(x)}$ . Two sets of tasks can be defined.  $\mathfrak{R}_t$  denotes the subset of tasks for which capital is more cost-effective than labor, and  $\mathfrak{L}_t$  denotes the subset of tasks for which labor is more cost-effective. This is formally set out in equations (24) and (25). Regarding terminology, we call  $\mathfrak{R}_t$  the set of automated tasks and  $\mathfrak{L}_t$  the set of non-automated tasks.

$$\mathfrak{R}_t = \left\{ x : \frac{\frac{\partial Y_t}{\partial k_t(x)}}{\frac{\partial Y_t}{\partial l_t(x)}} \geq \frac{R_t}{W_t} \wedge 0 \leq x \leq 1 \right\} \quad \text{and} \quad \mathfrak{L}_t = \left\{ x : \frac{\frac{\partial Y_t}{\partial k_t(x)}}{\frac{\partial Y_t}{\partial l_t(x)}} < \frac{R_t}{W_t} \wedge 0 \leq x \leq 1 \right\} = [0, 1] \setminus \mathfrak{R}_t \quad (24)$$

$$\forall x \in \mathfrak{R}_t : T_t(x) = \theta_t(x)k_t(x) \quad \text{and} \quad \forall x \in \mathfrak{L}_t : T_t(x) = \xi_t(x)l_t(x) \quad (25)$$

A cost-minimizing firm allocates capital to the execution of an automated task  $x \in \mathfrak{R}_t$  up to the point that its marginal product (priced in terms of final output) is equal to the cost of capital  $R_t$  (eq. (26)).

$$\forall x \in \mathfrak{R}_t : \frac{\partial Y_t}{\partial k_t(x)} = p_{x,t} \frac{\partial T_t(x)}{\partial k_t(x)} = p_{x,t} \theta_t(x) = R_t \quad (26)$$

Likewise, a cost-minimizing firm allocates labor to the execution of a non-automated task  $x \in \mathfrak{L}_t$  up to the point that its marginal product (priced in terms of final output) is equal to the cost of labor  $W_t$  (eq. (27)).

$$\forall x \in \mathfrak{L}_t : \frac{\partial Y_t}{\partial l_t(x)} = p_{x,t} \frac{\partial T_t(x)}{\partial l_t(x)} = p_{x,t} \xi_t(x) = W_t \quad (27)$$

I can now solve equations (26) and (27) for  $k_t(x)$  and  $l_t(x)$  respectively (by plugging in the results of equations (23) and (25)). This results in equations (28) and (29). It is clear that the amount of capital (or labor) that is assigned to a task does not vary over the the set of automated (non-automated) tasks.

$$\forall x \in \mathfrak{R}_t : k_t(x) = \frac{Y_t dx}{R_t} \wedge l_t(x) = 0 \quad (28)$$

$$\forall x \in \mathfrak{L}_t : k_t(x) = 0 \wedge l_t(x) = \frac{Y_t dx}{W_t} \quad (29)$$

### Proof A2

Given the productivity schedule of equation (3) of the main text and given the sets of automated and non-automated tasks  $\alpha_t$ :  $\mathfrak{R}_t = [0, \alpha_t]$  and  $\mathfrak{L}_t = ]\alpha_t, 1]$ , we can show that equation (5) of the main text holds.

Inserting the task-productivities in equation (1) of the main text results in equation (30) which is also

equation (4) of the main text.

$$Y_t = H \prod_0^{\alpha_t} (b^{1+\gamma}(B-x)^\gamma k_t) dx \prod_{\alpha_t}^1 (A_t(1-b)^{1+\gamma} x^\gamma l_t) dx \quad (30)$$

The definition of a product integral is  $\prod_a^b f(x) dx = \exp\left(\int_a^b \ln f(x) dx\right)$ . As a result, we can rework the production function. It results in equation (31).

$$Y_t = H \exp\left(\int_0^{\alpha_t} \ln(b^{1+\gamma}(B-x)^\gamma k_t) dx\right) \exp\left(\int_{\alpha_t}^1 \ln(A_t(1-b)^{1+\gamma} x^\gamma l_t) dx\right)$$

$$Y_t = H \exp\left(\alpha_t(1+\gamma) \ln(b) + \alpha_t \ln(k_t) + \gamma \int_0^{\alpha_t} \ln(B-x) dx\right) \exp\left((1-\alpha_t) \ln(A_t) + (1-\alpha_t)(1+\gamma) \ln(1-b) + (1-\alpha_t) \ln(l_t) + \gamma \int_{\alpha_t}^1 \ln(x) dx\right)$$

$$Y_t = H \exp\left(\alpha_t(1+\gamma) \ln(b) + \alpha_t \ln(k_t) + \gamma(B \ln(B) - (B-\alpha_t) \ln(B-\alpha_t) - \alpha_t)\right) \exp\left((1-\alpha_t) \ln(A_t) + (1-\alpha_t)(1+\gamma) \ln(1-b) + (1-\alpha_t) \ln(l_t) - \gamma(1-\alpha_t + \alpha_t \ln(\alpha_t))\right)$$

$$Y_t = H b^{\alpha_t(1+\gamma)} k_t^{\alpha_t} B^{\gamma B} (B-\alpha_t)^{-\gamma(B-\alpha_t)} \exp(-\gamma \alpha_t) A_t^{1-\alpha_t} (1-b)^{(1-\alpha_t)(1+\gamma)} l_t^{1-\alpha_t} \exp(-\gamma(1-\alpha_t)) \alpha_t^{-\gamma \alpha_t}$$

$$Y_t = H \exp(-\gamma) B^{\gamma B} (B-\alpha_t)^{-\gamma(B-\alpha_t)} (1-\alpha_t)^{\gamma(1-\alpha_t)} (\alpha_t^{-\gamma} b^{1+\gamma} k_t)^{\alpha_t} \left((1-\alpha_t)^{-\gamma} A_t (1-b)^{1+\gamma} l_t\right)^{1-\alpha_t} \quad (31)$$

We now invoke the condition for factor market clearing. Total supply of capital (labor) has to be equal to the sum of all allocation of capital (labor) by firms:  $K_t = \int_0^{\alpha_t} k_t = \alpha_t k_t$  and  $L_t = \int_{\alpha_t}^1 l_t = (1-\alpha_t) l_t$ . Inserting this into equation (31) results in equation (32) which is also equation (5) of the main text.

$$Y_t = H \exp(-\gamma) B^{\gamma B} (B-\alpha_t)^{-\gamma(B-\alpha_t)} (1-\alpha_t)^{\gamma(1-\alpha_t)} \left(\left(\frac{b}{\alpha_t}\right)^{1+\gamma} K_t\right)^{\alpha_t} \left(A_t \left(\frac{1-b}{1-\alpha_t}\right)^{1+\gamma} L_t\right)^{1-\alpha_t} \quad (32)$$

## Appendix B

### Proof B1

To figure out the law of motion for the asset level per efficiency unit of labor  $a_{i,t}$  in the absence of dissipation shocks, we start from the law of motion for total household assets  $A_{i,t}$  in equation (33).<sup>25</sup> Here,  $W_{i,t}$  denotes the wage rate,  $N_{i,t}$  denotes the total supply of labor of the household at time  $t$ ,  $R_t$  denotes the cost of capital, and  $C_{i,t}$  denotes total consumption of the household.

$$\dot{A}_{i,t} = W_{i,t}N_{i,t} + (R_t - \delta)A_{i,t} - C_{i,t} \quad (33)$$

We can find an expression for  $\frac{\dot{A}_{i,t}}{A_t N_t}$  by dividing everything by the level of technical progress  $A_t$  and the population size  $N_t$ . In equation (34), small letters denote variables per efficiency unit of labor.

$$\frac{\dot{A}_{i,t}}{A_t N_t} = w_{i,t} + (R_t - \delta)a_{i,t} - c_{i,t} \quad (34)$$

Since  $a_{i,t} = \frac{A_{i,t}}{A_t G_t}$ , taking logs and deriving with respect to time implies that  $\frac{\dot{a}_{i,t}}{a_{i,t}} = \frac{\dot{A}_{i,t}}{A_{i,t}} - g - n$ . When rearranged, this gives  $\frac{\dot{A}_{i,t}}{A_t N_t} = \dot{a}_{i,t} + g a_{i,t} + n a_{i,t}$ . When this is inserted in equation (34), this results in (35) which corresponds to the budget constraint of equation (14) of the main text, in the absence of dissipation shocks.

$$\dot{a}_{i,t} = w_{i,t} + (R_t - \delta - g - n)a_{i,t} - c_{i,t} \quad (35)$$

### Proof B2

Here, I argue that the presence of dissipation shocks changes the households' utility optimization problem only by making them discount the future at a higher rate. In effect, the argument here is very similar to that of Moll et al. (2022).

As Moll et al. (2022), I assume that the dissipation shock follows a Poisson process with arrival rate  $p$ . Upon hit by the shock, a household becomes infinitely impatient and only value their consumption in the following  $T$  periods with  $T \rightarrow 0$ . Additionally, the household loses their capitalist spirit motive for holding assets while in the 'impatience state'.

The value of being in the impatience state  $V^I$  can then be characterized by equation (36).

$$\begin{aligned} V^I &= \lim_{T \rightarrow 0} \max_{c_{i,t}, a_{i,t}} E_0 \int_0^T e^{-(\rho' - n)t} \ln(A_t C_{i,t}) dt \\ \text{s. t. } \dot{a}_{i,t} &= w_i + (R - \delta - g - n)a_{i,t} - c_{i,t} \\ a_{i,t} &\geq 0 \end{aligned} \quad (36)$$

Intuitively, the best thing a household can do when faced with the problem of equation (36), is to consume all available funds (and thus run down the entire asset stock) in the infinitesimally short period  $[0, T]$  with  $T \rightarrow 0$ . This forms the micro-foundation for the Poisson process of capital risk in equation (14) in the

<sup>25</sup>The level of household assets  $A_{i,t}$  should not be confused with the unrelated level of labor-augmenting technical progress  $A_t$ .



main text. Under this consumption schedule,  $V_t^I \rightarrow 0$ . The intuition is given by equation (37). Here,  $a_0$  denotes the asset level with which the household has entered the impatience state.

$$V^I \approx \lim_{T \rightarrow 0} \int_0^T \ln\left(\frac{a_0}{T}\right) dt = \lim_{T \rightarrow 0} T \ln\left(\frac{a_0}{T}\right) = \lim_{T \rightarrow 0} \frac{\ln\left(\frac{a_0}{T}\right)}{1/T} \stackrel{\text{L'Hôp.}}{=} \lim_{T \rightarrow 0} \frac{T(-1/T^2)}{-1/T^2} = \lim_{T \rightarrow 0} T = 0 \quad (37)$$

This implies that households outside of the impatience state behave as if faced with the utility maximization problem described by equation (17).

For a more rigorous, fully fledged proof of the workings of dissipation shocks, I refer to the online appendix C of Moll et al. (2022).

### Proof B3

The household maximization problem described in equation (17) of the main text can be represented by the current-value Hamiltonian in equation (38).

$$H_t(a_{i,t}, c_{i,t}) = \ln(A_t c_{i,t}) + \frac{(a_{i,t} + S)^{1-\zeta}}{1-\zeta} + \lambda_{i,t} [w_i + (R - \delta - g - n)a_{i,t} - c_{i,t}] + \kappa_{i,t} a_{i,t} \quad (38)$$

The first-order conditions are given by equations (39) and (40).

$$\begin{aligned} \frac{\partial H_t}{\partial c_{i,t}} &= 0 \\ \iff \frac{1}{c_{i,t}} &= \lambda_{i,t} \end{aligned} \quad (39)$$

$$\begin{aligned} \frac{\partial H_t}{\partial a_{i,t}} &= -\dot{\lambda}_{i,t} + (\rho' - n)\lambda_{i,t} \\ \iff (a_{i,t} + S)^{-\zeta} + \lambda_{i,t}(R - \delta - g - n) + \kappa_{i,t} &= -\dot{\lambda}_{i,t} + (\rho' - n)\lambda_{i,t} \end{aligned} \quad (40)$$

Taking logs of equation (39) and deriving with respect to time results in equation (41).

$$\frac{\dot{\lambda}_{i,t}}{\lambda_{i,t}} = -\frac{\dot{c}_{i,t}}{c_{i,t}} \quad (41)$$

Inserting equations (39) and (41) in equation (40) results in the Euler for  $a_{i,t} > 0$  equation of the main text in equation (18).

$$\frac{\dot{c}_{i,t}}{c_{i,t}} \geq R - \delta - \rho' - g + \frac{(a_{i,t} + S)^{-\zeta}}{c_{i,t}^{-1}} \quad (42)$$

The complementary slackness condition related to equation (17) is given by equation (43).

$$\kappa_{i,t} a_{i,t} = 0, a_{i,t} \geq 0, \kappa_{i,t} \geq 0 \quad (43)$$

The transversality condition related to equation (17) is given by equation (44).

$$\lim_{t \rightarrow \infty} e^{-(\rho' - n)t} \frac{1}{c_{i,t}} a_{i,t} = 0 \quad (44)$$

## Appendix C

In this appendix, I briefly set out how the household's optimization problem is solved numerically. The household's optimization problem is defined by equations (13), (14), (15) and (16) of the main text.

I use the methodology proposed by Achdou et al. (2022) to frame the heterogeneous-agent optimization problem as a mean-field game in continuous time.

First, this requires defining the Hamilton-Jacobi-Bellman (HJB) equation that is related to this optimization problem. This is given by equation (45).

$$(\rho' - n)v_i(a_i) = \ln(c_i) + \frac{(a_i + S)^{1-\zeta}}{1-\zeta} + v'_i(a_i)(w_i + (R - \delta - g - n)a_i - c_i) \quad (45)$$

The task here is then to solve the value function  $v(a_i)$  on a discrete grid of asset values to back out the savings function. While I stop short of giving a fully fledged explanation of the entire process, the conceptual steps required to solve for the balanced-growth equilibrium are the following. The interested reader is referred to the work of Achdou et al. (2022), where all technical details can be found.

*Step 1.* Start out with an initial guess for the interest rate  $r = R - \delta$ .

*Step 2.* Solve the HJB equation by using the finite difference method to approximate the derivative of the value function ( $v'_i(a_i) \approx \frac{v_i(a_i + \Delta a) - v_i(a_i)}{\Delta a}$ ) and iterating on the value function using an implicit scheme based on equation (45) until convergence. This is done for each household type  $i$ .

*Step 3.* Solve the Kolmogorov-Forward equation to find the stationary distribution of individuals over assets. Sum over the asset distribution and each household type  $i$  to find the capital supply.

*Step 4.* Compare the capital supply with the capital demand for a given value for  $r$  and update the value of  $r$  using the bisection method. If the difference between capital supply and demand is sufficiently close to zero, the equilibrium is found. Otherwise, return to step 2.

In practice, I use an unevenly-spaced grid with 10 000 grid points to discretize the asset space. I always verify that the share of total asset supply coming from households with are at the maximum grid point is negligible.

## Appendix D: Normalization of the production side

Regarding the production technology, we have a Cobb-Douglas, CES or VES technology depending on the restrictions imposed on the capital-labor productivity schedule (cf. *supra*). There are only two parameters in the Cobb-Douglas case: Hicks neutral technology  $\tilde{H}$  and the share parameter  $b$ . In the CES case, there is also the steepness of the capital-labor productivity schedule  $\gamma$  which determines the long-run elasticity of substitution between capital and labor. In the VES case, we must also pin down  $B$  which reflects the availability of future automation opportunities and determines how the elasticity of substitution varies as capital deepens. We follow Klump et al. (2012) by normalizing the production functions to make meaningful comparisons between situations with different elasticities of substitution possible.

### Normalization of the Cobb-Douglas case

From Lemma 1, it follows that the Cobb-Douglas production technology is given by equation (46), where  $y_t$  and  $k_t$  denote variables per efficiency unit of labor  $\frac{\cdot}{A_t L_t}$ .

$$y_t = \tilde{H} k_t^b \quad (46)$$

We normalize the Cobb-Douglas production function by imposing initial values for the output per efficiency unit of labor  $y_0$ , the capital stock per efficiency unit of labor  $k_0$  and the capital share of income  $\alpha_0$ .

$$\begin{cases} y_0 = \tilde{H} k_0^b \\ \alpha_0 = b \end{cases} \iff \begin{cases} \tilde{H} = \frac{y_0}{k_0^b} \\ b = \alpha_0 \end{cases} \quad (47)$$

### Normalization of the CES case

From Lemma 1, it follows that the CES production technology is given by equation (48), where  $y_t$  and  $k_t$  denote variables per efficiency unit of labor  $\frac{\cdot}{A_t L_t}$ .

$$y_t = \tilde{H} \left( b k_t^{\frac{1}{1+\gamma}} + (1-b) \right)^{1+\gamma} \quad (48)$$

Again, we normalize the CES production function by imposing initial values for the output per efficiency unit of labor  $y_0$ , the capital stock per efficiency unit of labor  $k_0$  and the capital share of income  $\alpha_0$ . The steepness of the capital-labor productivity schedule  $\gamma$  is chosen to match the constant elasticity of substitution  $\sigma$ :  $\gamma = \frac{1}{\sigma-1}$ .

$$\begin{cases} y_0 = \tilde{H} \left( b k_0^{\frac{1}{1+\gamma}} + (1-b) \right)^{1+\gamma} \\ \alpha_0 = \frac{b k_0^{\frac{1}{1+\gamma}}}{b k_0^{\frac{1}{1+\gamma}} + (1-b)} \end{cases} \iff \begin{cases} \tilde{H} = \frac{y_0}{\left( \alpha_0 k_0^{\frac{1}{1+\gamma}} + (1-\alpha_0) \right)^{1+\gamma}} \\ b = \frac{\alpha_0}{\alpha_0 + (1-\alpha_0) k_0^{\frac{1}{1+\gamma}}} \end{cases} \quad (49)$$

The power of the normalization routine above becomes apparent when verifying the value for the lower bound of the marginal product of capital  $\mu = \tilde{H} b^{1+\gamma}$  according to equation (10). It would be problematic if the value for the lower bound were to change in function of the arbitrarily chosen values of  $y_0$  or  $k_0$ . This is not the case. We find that  $\mu = \frac{y_0}{k_0} \alpha_0^{1+\gamma}$ . The lower bound for the marginal product of capital thus only depends on (1) the initial capital-output ratio, (2) the initial value for the capital share, and

(3) the elasticity of substitution. Scaling  $y_0$  and  $k_0$  by the same constant does not affect  $\mu$ .

### Normalization of the VES case

The VES production technology that follows from a general  $B \geq 1$  is characterized by equations (5), (9) and (12). By imposing initial values for the output per efficiency unit of labor  $y_0$ , the capital stock per efficiency unit of labor  $k_0$ , the capital share of income  $\alpha_0$  and the elasticity of substitution  $\sigma_0$ ,  $\gamma$ ,  $\tilde{H}$  and  $b$  receive a value in equation (50).

$$\begin{cases} y_0 = \tilde{H} (B - \alpha_0)^{-\gamma(B-\alpha_0)} (1 - \alpha_0)^{\gamma(1-\alpha_0)} \left( \left( \frac{b}{\alpha_0} \right)^{1+\gamma} k_0 \right)^{\alpha_0} \left( \left( \frac{1-b}{1-\alpha_0} \right)^{1+\gamma} \right)^{1-\alpha_0} \\ k_0 = \left( \frac{1-b}{b} \right)^{1+\gamma} \frac{\alpha_0^{1+\gamma}}{(1-\alpha_0)(B-\alpha_0)^\gamma} \\ \sigma_0 = \frac{1}{B} \left( 1 + \frac{1}{\gamma} \frac{B-\alpha_0}{1-\alpha_0} \right) \end{cases} \iff \begin{cases} \gamma = \frac{B-\alpha_0}{(1-\alpha_0)(B\sigma_0-1)} \\ b = \frac{\alpha_0}{\alpha_0 + k_0^{\frac{1}{1+\gamma}} (1-\alpha_0)^{\frac{1}{1+\gamma}} (B-\alpha_0)^{\frac{\gamma}{1+\gamma}}} \\ \tilde{H} = \frac{y_0}{k_0} (B - \alpha_0)^{\gamma(B-1)} \left( \alpha_0 + k_0^{\frac{1}{1+\gamma}} (1-\alpha_0)^{\frac{1}{1+\gamma}} (B-\alpha_0)^{\frac{\gamma}{1+\gamma}} \right)^{1+\gamma} \end{cases} \quad (50)$$

Again, the resulting normalization implies that the lower bound  $\mu = \tilde{H} b^{1+\gamma} (B-1)^{-\gamma(B-1)}$  is unaffected by arbitrary changes in  $y_0$  and  $k_0$  that keep  $\frac{k_0}{y_0}$  constant. We find that  $\mu = \left( \frac{B-\alpha_0}{B-1} \right)^{\frac{(B-\alpha_0)(B-1)}{(1-\alpha_0)(B\sigma_0-1)}} \frac{y_0}{k_0} \alpha_0^{1+\frac{B-\alpha_0}{(1-\alpha_0)(B\sigma_0-1)}}$ . Again, the lower bound for the marginal product of capital depends only on (1) the initial capital-output ratio, (2) the initial value for the capital share, and (3) the initial elasticity of substitution. Additionally, the parameter  $B$  matters for the lower bound of the marginal product of capital.<sup>26</sup>

Lemma D1 summarizes the relationship between the capital-labor productivity schedule and the lower bound on the marginal product of capital  $\mu$ .

#### Lemma D1.

Given that

1. the respective productivities of capital and labor over the range of tasks are  $\theta_t(x) = b^{1+\gamma}(B-x)^\gamma$  and  $\xi_t(x) = A_t(1-b)^{1+\gamma}x^\gamma$  with  $0 < b < 1$  and  $\gamma > 0$  where  $x$  denotes the task index and the range of tasks is the interval  $[0, 1]$

2. the tasks  $T_t(x)$  are combined through a Cobb-Douglas aggregator:  $Y_t = H \prod_0^1 T_t(x)^{dx}$

3. there is perfect substitutability between capital  $k_t(x)$  and labor  $l_t(x)$  for the execution of each task  $T_t(x) = \theta_t(x)k_t(x) + \xi_t(x)l_t(x)$

imposing that  $B = 1$  with a general strictly positive value for  $\gamma$  results in an aggregate production function of the CES type. After appropriate normalization, the lower bound on the marginal product of capital  $\mu = \lim_{K_t \rightarrow \infty} \frac{\partial Y_t}{\partial K_t}$  depends only on (1) the initial capital-output

<sup>26</sup>One can also compute the value for the lower bound  $\mu$  for  $B \rightarrow \infty$  by taking the log of  $\mu$ , calculating its limit for  $B \rightarrow \infty$  and exponentiating the result. The result is that  $\lim_{B \rightarrow \infty} \mu = e^{\frac{1}{\sigma_0} \frac{y_0}{k_0} \alpha_0^{1+\frac{1}{\sigma_0(1-\alpha_0)}}}$

ratio  $\frac{K_0}{Y_0}$ , (2) the initial value for the capital share  $\alpha_0$ , and (3) the elasticity of substitution  $1 + \frac{1}{\gamma}$ . It holds that  $\mu = \frac{Y_0}{K_0} \alpha_0^{1+\gamma}$ .

imposing that  $B > 1$  with a general strictly positive value for  $\gamma$  results in an aggregate production function of the VES type. After appropriate normalization, the lower bound on the marginal product of capital  $\mu = \lim_{K_t \rightarrow \infty} \frac{\partial Y_t}{\partial K_t}$  depends only on (1) the initial capital-output ratio  $\frac{K_0}{Y_0}$ , (2) the initial value for the capital share  $\alpha_0$ , and (3) the initial elasticity of substitution  $\sigma_0$ . It holds that  $\mu = \left( \frac{B-\alpha_0}{B-1} \right)^{\frac{(B-\alpha_0)(B-1)}{(1-\alpha_0)(B\sigma_0-1)}} \frac{Y_0}{K_0} \alpha_0^{1+\frac{B-\alpha_0}{(1-\alpha_0)(B\sigma_0-1)}}$ .

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