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## Firm ownership and the macroeconomics of incentive leakages

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### Firm ownership and the macroeconomics of incentive leakages

No 3033

## Abstract

Questions about market power have become salient in macroeconomics. We consider the role of institutional structures in addressing these within a dynamic general equilibrium framework. Standard models account for monopoly profits as a lump-sum transfer to the representative agent. We label this an “incentive leakage,” and show this to be a general characteristic of firm-optimal arrangements. We show that shareholder-operated or worker-operated firms that eliminate leakage can generate within-firm incentives that effectively reduce monopoly distortion in equilibrium. When all firms operate similarly, an additional general equilibrium effect arises through internalization of an aggregate demand externality. We characterize steady-state welfare across structures, and show how zero-leakage institutions lead to improvements towards the Golden Rule benchmark. Overall, our paper takes the first step towards an analysis of the macroeconomics of institutions without incentive leakage.

**Keywords:** Monopolistic competition, incentive leakage, aggregate demand externality, Golden Rule, patience gap, monopoly gap.

**JEL classifications:** E10, E22, E24, E25

## Non-technical Summary

In recent decades, a salient feature of capitalist economies has been a decline in business dynamism characterized by increased market concentration, the rise of superstar firms, a persistent increase in mark-ups. At the macroeconomic level these aggregate into decreasing share of labour and, with some qualifications, increasing share of profits in gross value added implying an increase in factor market inefficiencies. While policymakers often aim to induce firms to take social welfare into account, they struggle to deliver effective regulation to address market-power distortions. With this in mind, we look for alternative solutions based on institutional incentives.

In standard real business cycle (RBC) models with monopolistic competition, all firms are owned by the representative household and firms can be thought of as being run by a profit-maximising manager on behalf of households. The firm exploits monopoly power in the factor markets, so that labour and capital are underpaid relative to their marginal product. The resultant monopoly profits are transferred by the manager as a lump-sum payment to households. These residual payments do not have any substitution effect on factor provision, at best having an income effect. We define this as an “incentive leakage.” Our contribution is to embed alternative structures in an otherwise standard dynamic general equilibrium macroeconomic model with monopolistic competition, characterise novel outcomes and demonstrate that institutional arrangements can be a potent channel for mitigating distortions arising from market power. Specifically, we consider the efficacy of zero-incentive-leakage structures.

Our approach preserves monopoly but eliminates leakage through alternative ownership and remuneration structures. Specifically, zero leakage is achieved by disbursing the entire firm revenue net of any non-tied factor costs as payment to tied-factor providers. The latter are remunerated in proportion to contribution, and the firm aggregates tied-factor provision in a decentralised manner. The proportional remuneration scheme generates extra competition among agents to supply the tied factor(s). This results in a within-firm negative externality so that tied factors are oversupplied and therefore output is higher relative to the joint optimum of firm owners. The outcome is as if the firm under-exploits its monopoly power. Firm-level overproduction results in a positive externality on other firms. Thus, our decentralised zero-leakage schemes effectively internalise an aggregate demand externality. The ownership and remuneration structures that engender such efficacious properties are stable and individual actions are incentive compatible. Our results show that harnessing monopoly profits as part of such a remuneration structure generates extra incentives not available under perfect

competition. We propose decentralised zero-leakage institutional forms with either capital or labour as the tied factor (*Entrepreneurial Shareholders* (ES) and *Entrepreneurial Workers* (EW), respectively). We also consider the more general arrangement with both factors tied (*Workers Enterprises*).

Beyond specific structures, the broader idea is to show the role of remuneration structure in changing firm behaviour, complementing the literature on changing firm objectives. To consider welfare implications of the proposed zero-leakage arrangements, we rely on comparing steady-state outcomes, with the benchmark given by the Golden Rule. We show that relative to the perfect competition RBC setting, the Golden Rule requires the discount factor to approach unity. This creates two steady-state welfare gaps: the first is between the Golden Rule and perfect competition RBC (the patience gap) and the second is the standard gap between the latter and RBC under monopolistic competition (the monopoly gap). We show that EW closes part of the monopoly gap, while ES and WE not only close the monopoly gap, but also close part of the patience gap, thus outperforming even perfect competition in the steady state. We show formally that the welfare ranking is related to capital accumulation incentives across structures. Importantly, ES and WE structures rely on the monopoly gap to generate extra incentives to save compared to perfect competition, allowing improvements beyond the latter in the steady state.

ES, EW and perfect competition RBC (RBC-PC) models all feature zero incentive leakage. For RBC-PC, this arises through the absence of monopoly power, while ES and EW keep monopoly power intact but set incentive leakage to zero through revenue allocation mechanisms. The resulting incentives deliver higher steady state welfare for the ES firm as compared to the RBC-PC and lower steady state welfare for the EW firm as compared to the RBC-PC. A further interesting observation is that the Workers Enterprises behave similarly to the ES firms.

The results also serve to highlight the contrast between managerial and decentralised outcomes and indicate that eliminating even part of the incentive leakage by moving partially towards decentralised decision making can have a welfare-enhancing impact on the firm-level outcomes. They also suggest that using tax policies to target firm structure, for example through preferential tax treatment of firms that link profits to factor provision incentives, might have a more desirable long-run impact compared to competition policies that seek to limit monopoly power.

# 1 Introduction

In recent decades, a salient feature of capitalist economies has been a decline in business dynamism (Akçigit and Ates, 2021, De Loecker, Eeckhout, and Mongey, 2021). It is characterized by increased market concentration in most industries, the rise of superstar firms, a persistent increase in mark-ups as well as an increase in monopsony power in labor markets.<sup>1</sup> At the macroeconomic level these aggregate into decreasing share of labor and, with some qualifications, increasing share of profits in gross value added.<sup>2</sup> These transformations imply growing factor market inefficiencies. Using a multi-sector monopolistic competition model, Behrens, Mion, Murata, and Suedekum (2020) compute welfare distortions associated with factor market inefficiencies of the order of 6% to 10% of the UK and French GDP.

With the rise in market power, questions about ownership structure as well as objective of firms have become salient across social sciences.<sup>3</sup> The usual practice for firms, especially in the US and the UK, is to act on behalf of its shareholders.<sup>4</sup> This typically implies exploitation of market power to maximise profits, leading to distortions in factor markets and macroeconomic inefficiencies. Since the pioneering work of Ward (1958), a large literature has addressed questions about implications of firm structure, specifically the efficacy of worker-operated firms.<sup>5</sup> Important work by Sertel (1982, 1987, 1991) has shown that introducing partnership markets can resolve several concerns raised in the literature about incentives in such firms, though the question posed here is whether alternative ownership structures can perform as well as a standard profit-maximising firm. A further strand of literature seeks

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<sup>1</sup>For example, Philippon (2019), Eeckhout (2021) note the increased concentration, Autor, Dorn, Katz, Patterson, and Van Reenen (2020) show the rise of superstar firms, De Loecker, Eeckhout, and Unger (2020) and Gutiérrez and Philippon (2017) note the rise in mark-ups, Manning (2003) and Benmelech, Bergman, and Kim (2022) provide evidence for the rise in monopsony power. Certain ways of accounting for return on capital suggest a decline even in the capital share in US gross value added (Barkai, 2020) and at the global scale (Esfahani, Fernald, and Hobijn, 2024).

<sup>2</sup>See, for example, Elsby, Hobijn, and Sahin (2013) and Eeckhout (2021).

<sup>3</sup>See, for example, Ferreras, Battilana, and Méda (2022), a part of the Democratizing Work global initiative.

<sup>4</sup>For example, in the US, Delaware law pertaining to firm management gives primacy to shareholders rights.

<sup>5</sup>A substantial body of empirical work considers factor choice and allocational implications of alternative ownership structures. The evidence on structure suggests that compared to conventional firms, workers' enterprises or cooperatives are at least as productive, not smaller or less capitalized (e.g. Fakhfakh, Pérotin, and Gago, 2012, Burdin, 2014, Mirabel, 2022).

to alleviate distortions arising from profit maximisation by introducing greater pro-sociality in the objective of firm itself (Magill, Quinzii, and Rochet, 2015, Fleurbaey and Ponthière, 2023). The approach in these papers is to assume consumers, workers and shareholders as separate stakeholder entities, and explore firm behaviour if it were to maximise stakeholder rights. We discuss these further later, but here it suffices to note that our approach aligns with these broad areas of enquiry. In particular, we show that certain ownership and factor remuneration structures can endogenously alter behaviour of firms, which leads to lower monopoly welfare distortions and in turn effectively internalises an aggregate positive externality. Moreover, the proposed structures are stable in the sense that resulting behaviour is incentive compatible.

We consider a dynamic general equilibrium setting. To model firms with market power, we use the monopolistic competition framework à la Dixit and Stiglitz (1977).<sup>6</sup> In a general equilibrium model, there is no separation between stakeholders such as capital owners, workers and consumers/savers (i.e. households). Thus household utility remains the sole welfare criterion. To maximise profit, firms with monopoly power reduce demand for factors to limit output, so that a wedge arises between the marginal product of a factor and its real price. In this setting, if firms were to exploit their monopoly power less, aggregate outcomes would obviously improve. The basic question we pose in this paper is how this could be achieved in an incentive compatible manner.

While policymakers often aim to induce firms to take social welfare into account, they struggle to deliver effective regulation to address market-power distortions. With this in mind, we look for alternative solutions based on institutional incentives. Specifically, we take both imperfect competition and standard self-interested rational behaviour as given and focus on the positive role monopoly profits can play in incentivising factor provision through changes in the ownership and remuneration structures at the firm level. We show how these lead to a lower effective exploitation of market power in equilibrium and, how, in turn, this gives rise to an additional general equilibrium effect that enhances macroeconomic outcomes.

In standard real business cycle (RBC) models with monopolistic competition, all firms are

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<sup>6</sup>This ensures analytical tractability and allows comparison with outcomes of standard real business cycle models. As we explain in the literature review, our main insight would carry over to an oligopoly setting as well. Originally, the Dixit-Stiglitz equations capture a demand system with preference for variety and monopolistic competition, and a large literature exploits such features (Dhingra and Morrow, 2019, Behrens et al., 2020). We, however, simply use these equations to capture the production of a final good using intermediate goods, where the latter sector features monopolistic competition among firms.



owned by the representative household and firms can be thought of as being run by a profit-maximising manager on behalf of households. The firm exploits monopoly power in the factor markets, so that labour and capital are underpaid relative to their marginal product. The resultant monopoly profits are transferred by the manager as a lump-sum payment to households. These residual payments do not have any substitution effect on factor provision, at best having an income effect<sup>7</sup>. We define this as an “incentive leakage.” The standard model is useful in that it captures factor market distortions, but it leaves open the normative question of the best use of monopoly profits. In this paper we consider firm structures that eliminate this leakage, and systematically work out the impact on individual firms and on the general equilibrium outcome, and characterize the significant welfare implications that emerge from these.

To proceed, we need to understand whether lump-sum profit distribution is just a model-closing assumption or if incentive leakage is a more general phenomenon. To approach this question, we assign a separate mass of households to each firm, clarifying who owns the firm. We then solve the problem of a firm-level planner who maximises the utility of owners, taking market power as given. We show that the optimal solution features an incentive leakage. Moreover, if the solution is implemented using internal factor prices, the leakage coincides with the lump-sum transfer under profit maximisation. Thus, changing the objective of the firm from maximising profit to utility maximisation does not change the nature of the solution, establishing incentive leakage as a deeper phenomenon and allowing us to obtain a generalised definition that does not depend on market prices.<sup>8</sup>

Next, we propose firm-level institutional structures that can eliminate the incentive leakage by using the entire firm revenue towards factor payments. Note that the perfect competition RBC model does not feature any incentive leakage, while also eliminating market power. Our structures, on the other hand, eliminate the leakage while keeping market power unchanged. Our analysis therefore leads to a comparison across zero-leakage systems, either without market power (perfect competition) or with unchanged market power, but with specific ownership and remuneration structures. That the former improves on monopolistic

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<sup>7</sup>Hall (1989) and Karabarbounis and Neiman (2018) interpret these residual payments as pure profits or factorless income, reflecting the difference between gross revenues and the sum of measured payments to relevant production factors. Kalecki (1938, 1954) provides a clear definition and separation of workers’ and capitalists’ revenue streams in national income and their direct association with consumption and investment.

<sup>8</sup>Hart and Zingales (2017) note that firm manager should maximise shareholder utility rather than profit. Our result shows that the two objectives produce identical outcomes.



competition is obvious. More interestingly, we show that zero-leakage institutional structures we suggest without removing market power also improves on monopolistic competition, to different degrees depending on specific features. Essentially, our paper takes the first step towards analysing the macroeconomics of firms without incentive leakage and clarifying the significant implications of this feature for aggregate welfare. This is the main contribution of our paper.

Let us elaborate on the idea of zero leakage under monopolistic firms. In the firm-level planner's exercise mentioned above, owners supply both capital and labour to the firm. However, other intermediate arrangements are possible. For example, if households who own a firm supply capital exclusively to the firm, while labour is hired in an economy-wide market, we say that capital is the tied factor, while labour is not tied to any specific firm. In this case, the firm's surplus is the revenue net of payments to labour. We show that the firm would optimally exploit its market power in hiring labour in the same way as a profit-maximising firm, introducing the same wedge between the real wage and the marginal product of labour. The net surplus after paying labour is then paid to the tied factor, capital in this example. Households are paid a share of the surplus in proportion to their contribution to total capital. The remuneration scheme implies that the more capital a household supplies, the greater the fraction of future surplus they capture. In other words, the mass of households assigned to a firm compete internally for shares of the surplus.

The firm's decision making is decentralised: capital employed by the firm is simply the total capital supplied by its owners. To put it differently, once we assign ownership of a firm to a certain mass of households, we assume that they supply tied factors based on individual incentives, and the total firm demand for the tied factor simply aggregates across individual decisions; there is no centralised manager to decide factor provision. Our results therefore also serve to highlight the contrast between managerial and decentralised outcomes, and indicate that eliminating even part of the leakage by moving partially towards decentralised decision making can have an impact on firm-level outcomes.

We denote the arrangement above with capital as the tied factor as “Entrepreneurial Shareholders (ES).” Similarly, we consider an “Entrepreneurial Workers (EW)” arrangement where labour is the tied factor. We show that the utility maximising solution in these cases is equivalent to maximising surplus after paying the non-tied factor. That is, the information requirement for these cases is minimal.

The basic intuition for the aggregate impact of eliminating incentive leakage is as follows. The

remuneration schemes promotes internal competition among tied-factor providers. When capital is the tied factor, provision of extra capital by any household imposes a negative externality on others since it reduces the share of total capital provision of others. This impact is not internalised, leading to over-provision of capital relative to the joint optimum of owners and consequent higher output, which, in turn, has a positive externality on other firms. Thus within-firm negative externalities lead to a positive aggregate demand externality. In other words, the internal structure of the firm can, by itself, effectively reduce the exploitation of market power in equilibrium, in turn internalising an aggregate demand externality.

To understand the efficiency impact of the aggregate externality, let us first clarify our welfare notion. Policy is typically evaluated using a dynamic welfare criterion: change in lifetime utility starting at the initial period (the period when a policy is introduced). While it serves to understand the impact over business-cycle frequencies, this may not be best suited to evaluate a permanent change in institutional structure, which instead calls for measuring long-term impact. Since our dynamic general equilibrium model can be thought of as a growth model with zero growth, we can characterize the corresponding steady-state outcome to measure long-term impact. Crucially, the steady-state welfare benchmark is not the same as the dynamic welfare benchmark. The latter is addressed by the first welfare theorem, which implies that starting at the initial period, lifetime utility is highest under perfect competition RBC (RBC-PC), in turn implying that any change in institutional structure or nature of competition would produce lower dynamic welfare. On the other hand, the steady-state benchmark in growth models is given by the Golden Rule, which is achieved under perfect competition RBC in the limit as the household discount factor approaches unity.

The upshot is that given any discounting of future utility, we have a gap between the steady-state Golden Rule benchmark and welfare under perfect competition RBC (patience gap). Of course we also have the usual welfare gap between perfect and monopolistic competition RBC (monopoly gap).<sup>9</sup> Figure 1 shows the gaps. We present a general possibility result to show that, at the steady state, zero-leakage systems can not only close the monopoly gap, but can outperform perfect competition RBC by closing part of the patience gap. We further explain the crucial role played by capital provision incentives in this process.

Our results show that over a standard range of parameter values, the institutional form with

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<sup>9</sup>This gap obtains both dynamically and in the steady state. We consider the latter here.

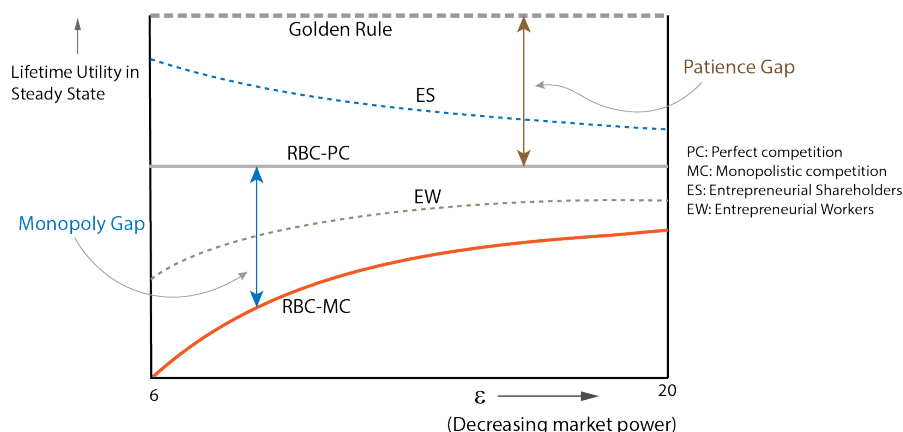


Figure 1: Twin welfare gaps in the steady state as functions of market power, which is decreasing in the parameter  $\epsilon$ . The schematic is based on simulations presented later, featuring values of  $\epsilon$  between 6 and 20. We show that perfect competition RBC achieves the Golden Rule if and only if the discount factor  $\beta \rightarrow 1$ . For any given  $\beta < 1$ , there is a steady-state “patience gap.” Further, the standard distortion under RBC with monopolistic competition gives rise to the “monopoly gap.” Greater capital accumulation incentives under ES imply that it closes not only the monopoly gap, but part of the patience gap, thus outperforming perfect competition RBC in the steady state.

labour as the tied factor (Entrepreneurial Workers, or EW) produces welfare that is higher than RBC with monopolistic competition, but lower than perfect competition RBC. That is, EW eliminates part of the monopoly gap. On the other hand, the institutional form with capital as the tied factor (Entrepreneurial Shareholders, or ES) not only closes the monopoly gap fully, but eliminates even part of the patience gap (see Figure 1). The results demonstrate that profits arising from imperfect competition can be harnessed to produce powerful long-term incentives.

The fact that steady-state welfare levels from ES and EW fit neatly on two sides of the level from RBC-PC (see Figure 1) is not a coincidence. As mentioned previously, all three are zero-leakage institutions. RBC-PC eliminates leakage by removing monopoly power, while ES and EW keep monopoly power intact but set leakage to zero through revenue allocation mechanisms. As we clarify in Section 6.2, the resulting differences in capital-provision incentives push ES and EW in different directions relative to RBC-PC.

Finally, we analyse the case of a Workers’ Enterprise, with both factors tied to the firm. This is a firm owned by its workers. Under a zero-leakage scheme, the firm optimally allocates shares of revenue to either factor, before remunerating the factor-providers. Each owner is then remunerated for either factor in proportion to their contribution to total supply

of that factor. This is a more general arrangement compared to ES and EW, as the firm can set the shares to imitate either case. While the analysis is technically challenging, the results do not add new insights about the aggregate impact of firm-level incentives. Steady-state simulations show that for a standard range of parameter values, the optimal structure of the WE is close to ES. This raises a different question. Since firms do not set out to consider aggregate externalities in choosing internal structure, why do they optimally choose a structure close to the one that generates high aggregate steady-state welfare? We clarify the intuition in Section 7.

Taken together, our results show that whether greater market power leads to greater inefficiency depends on the institutional structure. While competition policy typically aims to curb market power, our work shows the scope for alternative schemes to raise welfare through institutional arrangements that encourage factor provision.

## Related literature

Our paper is related to several strands of literature. In a well-known work featuring monopolistic competition and labour as the factor of production, [Blanchard and Kiyotaki \(1987\)](#) note that if all firms could coordinate a price reduction, the resulting aggregate demand externality would be Pareto improving. While such price reduction is not incentive compatible by itself, the authors show that small menu costs that prevent firms from adjusting prices fully after a money-balance shock can be welfare improving through a similar effect. In this paper we suggest schemes that generate aggregate externalities, but from a different source. Specifically, we suggest ownership and remuneration structures that generate intra-firm incentives that lead firm output to exceed the joint optimum of owners. This effectively reduces the exploitation of monopoly power in equilibrium, in turn generating welfare-improving aggregate demand externalities. Further, our dynamic analysis includes capital as well as labour, leading to novel implications about inter-temporal incentives and their connection to aggregate welfare.

[Blanchard and Giavazzi \(2003\)](#) consider the distribution of monopolistic competition rents to workers via wage bargaining that produces higher employment in the long-run. The EW setting here can be seen as a version of this idea, with added intensive labour margin and a specific remuneration structure, leading to long-run aggregate welfare improvements.

De Loecker et al. (2021) consider a continuum of markets, each modeled as a Cournot oligopoly. With purely labour as input, they quantify welfare implications of technology and market structure. While we use monopolistic competition with a continuum of firms, it is worth noting that our intuition for welfare improvement through firm structure would carry over to a Cournot oligopoly model. The negative externality from the tied-factor remuneration structure is internal to the firm and therefore independent of the number of firms, and the positive aggregate externality would arise with a finite number of firms in the same qualitative manner.

Macroeconomic analysis of the implications of firm ownership is relatively scant. An antecedent is recent work by Brzustowski and Caselli (2021), who compare an economy with competitive firms with one comprising cooperatives. In an overlapping-generations framework, a cooperative firm sets aside a fixed fraction of revenue today to pay capital in the next period, and divides the rest equally across agents. The set-aside revenue is equally divided in the next period. The authors show that worker cooperatives are inefficiently small (static inefficiency), however under certain conditions, cooperative economies can be dynamically efficient. Our approach differs in several ways. We consider an RBC model with monopolistic competition, and focus on institutional structures that eliminate the incentive leakage and determine the share of revenue accruing to any tied-factor provider endogenously. Thus our approach gives primacy to the equilibrium impact of internal competition, and resulting within-firm negative externalities that serve to internalise positive aggregate externalities.

Turning to firm objectives, works by Magill et al. (2015) and Fleurbaey and Ponthière (2023) mentioned previously model consumers, workers and shareholders as separate stakeholder groups. Magill et al. (2015) note that investment decisions by a large firm impacts all stakeholder groups, so that maximising just shareholder value implies underinvestment. In a single-firm setting, they introduce membership markets to resolve the problem. Fleurbaey and Ponthière (2023) develop optimal management rules to operationalise the idea of stakeholder value maximisation. They show that profit-maximisation under rules that suggest price-taking behaviour effectively serves to maximise stakeholder value. In contrast, we consider a dynamic general equilibrium model without delineation across stakeholder groups so that consumer welfare remains the sole efficiency criterion. We take monopoly power as given, and focus on internal structures that effectively reduce usage of that power in equilibrium, generating an aggregate improvement.

A substantial body of empirical work considers factor choice and allocational implications of

alternative ownership structures. The evidence on firm structure and remuneration practices suggests that compared to conventional firms, workers' enterprises or cooperatives are at least as productive, not smaller or less capitalized, and less likely to default (Craig and Pencavel (1995), Fakhfakh et al. (2012), Mirabel (2022), Burdin (2014)). Recently, Nimier-David, Sraer, and Thesmar (2023) show that the main implication of the French mandated profit-sharing law that has passed in 1967 led to redistribution of excess profits to lower-skill workers in a firm without generating significant distortions or productivity effects. All in all, existent evidence suggests that firms featuring different forms of ownership and remuneration structure can thrive, and underlines the importance of exploring the incentive structures for such cases. In this paper, we take a first step at this task for firms that implement a zero-incentive-leakage structure, and explore its aggregate impact.

The rest of the paper is organised as follows. Section 2 presents the standard RBC model with monopolistic competition. Section 3 introduces the concepts of incentive leakage and zero-leakage decentralised organisations. Sections 4 and 5 study the *Entrepreneurial Shareholders* and the *Entrepreneurial Workers* cases, respectively. Section 6 clarifies the steady-state welfare notion and ranks outcomes across arrangements. Section 7 studies the *Workers' Enterprise* case. Section 8 concludes.

## 2 Real Business Cycle Model with Monopolistic Competition and Additive Profits

We start by presenting the standard RBC model with monopolistic competition (RBC-MC), which adapts the framework of Dixit and Stiglitz (1977). Firms are owned by representative households and run by a profit-maximising manager who transfers profits back to households in the form of lump-sum payments. Under these assumptions, we obtain standard results, where the representative firm's monopoly power reduces its demand for labour and capital by introducing a wedge between the marginal product of the production inputs and their corresponding marginal costs. In the aggregate, the monopoly friction implies investment and labour fall below their perfect-competition (RBC-PC) levels, causing a loss in welfare.

## 2.1 Households

The representative household maximises utility while subject to a budget constraint to set consumption  $C_t$ , labour supply  $L_t$  and savings in capital  $K_t$

$$\max_{L_t, K_t, C_t} \mathbb{E}_t \sum_{s \geq t}^{\infty} \beta^{s-t} U(C_s, L_s) \quad (2.1)$$

$$\text{s.t. } P_s C_s + P_s K_s \leq W_s L_s + \Pi_s + (1 - \delta) P_s K_{s-1} + R_s K_s, \quad (2.2)$$

where  $P_t$  is the price of the consumption good,  $W_t$  the market wage and  $\Pi_t$  the lump sum profits distributed by the firms to households. By definition  $P_t$  is the price of the consumed good,  $R_t$  the rental price of capital,  $W_t$  wages and  $\Pi_t$  the profits generated by firms owned by households. The optimisation problem yields the standard intertemporal Euler condition

$$1 = \mathbb{E}_t D_{t,t+1} \left[ \left( 1 + \frac{R_{t+1}}{P_{t+1}} - \delta \right) \right], \quad (2.3)$$

where  $D_{t,t+1} := \beta \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}$  is the stochastic discount factor. Labour is determined at a level where households are indifferent between leisure and consumption

$$\frac{W_t}{P_t} = - \frac{U_L(C_t, L_t)}{U_C(C_t, L_t)}. \quad (2.4)$$

## 2.2 Final Producer

We assume the existence of a unit measure of intermediate producers, indexed  $i$ , who produce intermediate inputs  $Y_{i,t}$ . These are used to produce a final good  $Y_t$  subject to the production technology given below  $Y_t := \left[ \int_i Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$ , where  $\epsilon$  represents the degree of substitutability of intermediate inputs. The final producer maximises profits  $P_t \left[ \int_i Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} - \int_i P_{i,t} Y_{i,t} di$ , where  $P_t$  is the price of the final good,  $P_{i,t}$  is the price of intermediate good  $i$ . The final production optimisation problem yields the standard demand function for intermediate good  $i$

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-1/\epsilon} Y_t. \quad (2.5)$$

Combining the demand for intermediate inputs from Equation 2.5 with the final production technology function yields the price of the final good  $P_t = \left( \int_i P_{i,t}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ .



### 2.3 Profit-Maximising Firm

We start with the standard firm-ownership structure assumed in the macroeconomic literature. Firm  $i$  produces an intermediate good and is run by a manager who maximises the firm's profits  $\Pi_{i,t} = P_{i,t}Y_{i,t} - R_tK_{i,t} - W_tL_{i,t}$ , where  $W_t$  is the market wage and  $R_t$  the rental price of capital.

Formally, the manager maximises

$$\max_{P_{i,t}, K_{i,t}, L_{i,t}} P_{i,t}Y_{i,t} - R_tK_{i,t} - W_tL_{i,t}$$

subject to the potentially uncertain production technology  $F_t$  with constant returns to scale, the budget constraint and the monopolistic competition constraint below

$$Y_{i,t} \leq F_t(K_{i,t}, L_{i,t}),$$

$$R_tK_{i,t} + W_tL_{i,t} \leq P_{i,t}Y_{i,t},$$

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t.$$

Furthermore, we assume that the production function satisfies the conditions  $F_t(K_{i,t}, 0) = F_t(0, L_{i,t}) = 0$  to ensure interior solutions. The intermediate production optimisation problem yields

$$\frac{W_t}{P_{i,t}} = \left( 1 - \frac{1}{\epsilon} \right) F_{L,i,t}, \quad (2.6)$$

$$\frac{R_t}{P_{i,t}} = \left( 1 - \frac{1}{\epsilon} \right) F_{K,i,t}, \quad (2.7)$$

where  $F_{L,i,t} := \frac{\partial F_t}{\partial L_{i,t}}$  and  $F_{K,i,t} := \frac{\partial F_t}{\partial K_{i,t}}$  are the marginal products of labour and capital, respectively. Equations 2.6 and 2.7 set, respectively, the profit-maximising firm's demand for labour and capital. Monopolistic competition introduces a wedge between the price of both inputs and their respective marginal products. As a shorthand, we introduce the notation for input wedges under symmetry as being

$$\omega^L = \frac{F_{L,t}}{\frac{W_t}{P_{i,t}}} - 1 \quad (2.8)$$

in the case of labour and

$$\omega^K = \frac{F_{K,t}}{\frac{R_t}{P_{i,t}}} - 1 \quad (2.9)$$

in the case of capital inputs. We denote resulting factor wedges as  $\omega^{RBC-MC,L} = \omega^{RBC-MC,K} = \omega^{RBC-MC} = \frac{1}{\epsilon-1}$  where RBC-MC superscript represents the monopolistic competition RBC model. As  $\epsilon \rightarrow \infty$ ,  $\omega^{RBC-MC} \rightarrow 0$ , so that we obtain the perfect competition RBC result in the limit.

## 2.4 General Equilibrium

In the absence of any monetary effects, the model requires a closing assumption to determine the aggregate price index  $P_t$ . We assume that the price of the final good is normalised to one, i.e.  $P_t = 1$ .

The economy's equilibrium is then determined by a vector of prices  $(W_t, R_t, P_t, P_{1,t}, P_{2,t}, \dots)$  and a vector of quantities  $(C_t, L_t, K_t, Y_t, C_{1,t}, C_{2,t}, \dots, Y_{1,t}, Y_{2,t}, \dots)$  such that the agents' equilibrium conditions are satisfied and the economy's markets clear:

- The market for capital clears so that the firms' aggregate demand for capital equals the aggregate capital accumulated by the households in the previous period  $K_{t-1} = \int_i K_{i,t}$ .
- The labour market clears so that the firms' demand for labour equals the households' supply  $L_t = \int_i L_{i,t}$ .
- The intermediate goods markets clear  $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ .
- The final goods market clears.

Since we consider different firm structures and market arrangement later, it is useful to express the conditions setting aggregate outcomes without referring to market prices.

**Proposition 1. (RBC-MC Equilibrium)** *Under the profit-maximising firm arrangement aggregate labour and capital satisfy the conditions below*

$$\frac{U_{L,t}}{U_{C,t}} + \left(1 - \frac{1}{\epsilon}\right) F_{L,t} = 0, \quad (2.10)$$

$$1 = \mathbf{E}_t D_{t,t+1} \left[ 1 - \delta + \left(1 - \frac{1}{\epsilon}\right) F_{K,t+1} \right]. \quad (2.11)$$

*In addition, under a constant returns to scale technology, the firm's profit  $\Pi_t := P_{i,t} Y_{i,t} - R_t K_{i,t} - W_t L_{i,t}$  is given by*

$$\Pi_t = \frac{1}{\epsilon} Y_t. \quad (2.12)$$

*Proof.* Follows immediately from the first-order conditions by dropping the index  $i$  due to firms' symmetry. ■

Proposition 1 restates a standard result in the literature. The presence of monopoly power introduces a wedge between the marginal product of factors and their corresponding prices.<sup>10</sup>

### 3 Profit versus Utility Maximising Firms and Incentive Leakages

So far, our results show that a supply sector composed of profit-maximising firms engaged in a monopolistic competition sets demand for production inputs (labour and capital) such that the price of each input is lower than its marginal product. In this section we show that these factor distortions arise because of the lump-sum nature of profit distribution. We first show that distortions remain irrespective of whether firms engage in profit or utility maximisation.

In order to preserve a monopolistically competitive economy while changing the firm objective to utility maximisation, we consider a firm-level planner and assume that each and every household/worker/capital-owner triplet is assigned to a specific firm. For simplicity, we assume that both labour and capital are internal to the firm with no market for either factor.<sup>11</sup> In this setting, we study the problem of a firm-based planner who maximises utility of the households assigned to the firm. We show that even though the firm maximises (representative) owner/worker utility, the equilibrium outcome is the same as in the case of the profit-maximising firm.

We show later (Proposition 4) that a firm with a constant return to scale technology that fully uses its revenues to reward production inputs (no lump-sum profits) cannot set its demand of both factors lower than the perfect competition RBC first-best levels. Taken together, these two results show that while a change in the nature of the firms' ownership does not necessarily improve on the RBC-MC outcomes, absence of lump-sum payments might have an impact on the monopoly distortion.

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<sup>10</sup>Similar inefficiencies are present in any standard New Keynesian model (e.g., Galí (2015)).

<sup>11</sup>At least tying one production factor is necessary for a meaningful maximisation of firm-level household utility. However, we can assume that one of the two factors is hired in an economy-wide market without changing the results.

### 3.1 Utility Maximisation: Firm-level Planner's Problem

Formally, the firm-based planner maximises the utility of firm  $i$ 's owner/worker subject to monopolistic competition and a budget constraint by setting the variety sale price  $P_{i,t}$ , production factors  $L_{i,t}$  and  $K_{i,t}$  and household consumption  $C_{i,t}$

$$\max_{P_{i,t}, C_{i,t}, L_{i,t}, K_{i,t}} \sum_{s \geq t} \mathbf{E}_t \beta^{s-t} U(C_{i,s}, L_{i,s}) \quad (3.1)$$

$$Y_{i,s} \leq \left( \frac{P_{i,s}}{P_s} \right)^{-\epsilon} Y_s \text{ for } s = t, t+1, \dots \quad (3.2)$$

$$P_s C_{i,s} + P_s K_{i,s} \leq P_{i,s} Y_{i,s} + (1 - \delta) P_s K_{i,s-1} \text{ for } s = t, t+1, \dots, \quad (3.3)$$

The proposition below show that the conditions for optimally setting labour and capital in the economy coincide with those under RBC-MC.

**Proposition 2. (Equivalence of Utility vs Profit Maximisation)** *Under the firm-based planner arrangement aggregate labour and capital satisfy the same conditions as under the RBC-MC model (Proposition 1)*

$$\frac{U_{L,t}}{U_{C,t}} + \left( 1 - \frac{1}{\epsilon} \right) F_{L,t} = 0, \quad (3.4)$$

$$1 = \mathbf{E}_t D_{t,t+1} \left[ 1 - \delta + \left( 1 - \frac{1}{\epsilon} \right) F_{K,t+1} \right]. \quad (3.5)$$

*Proof.* See Appendix B.1. ■

Proposition 2 shows that the firm-level planner chooses the same production factors as the profit-maximising firm.

### 3.2 Implementing the Firm-level Planner's Solution

We now turn to the issue of incentive leakages associated with lump-sum distribution of profits. However, in the absence of factor prices, we cannot refer to a notion of profit in the firm-based planner arrangement. The proposition below shows that the firm-level planner can implement their preferred allocation using factor markets internal to the firm. These are within-firm competitive markets where factor supply is freely decided by the firm's workers/owners who maximise their own utility and where the factor demand is set by the firm level planner.

**Proposition 3. (Implementation using Internal Prices)** *The firm-based planner can replicate profit-maximising manager's behaviour leading to the outcome described in Proposition 2 by implementing firm-level internal labour and capital markets and setting the demand for factors such that*

$$\frac{W_{i,t}}{P_{i,t}} = \left(1 - \frac{1}{\epsilon}\right) F_{i,L,t}, \quad (3.6)$$

$$\frac{R_{i,t}}{P_{i,t}} = \left(1 - \frac{1}{\epsilon}\right) F_{i,K,t}, \quad (3.7)$$

where  $W_{i,t}$  and  $R_{i,t}$  denote the firm-specific wage and gross return on capital respectively. Assuming constant returns to scale production, the planner would then distribute a fraction  $1/\epsilon$  of the firm's revenues in the form of a lump-sum payment to workers/owners  $\Pi_{i,t} = \frac{1}{\epsilon} Y_{i,t}$ .

*Proof.* Derives immediately from Proposition 2. ■

Note that the demand for factors by the firm-based planner introduces the same wedges as in the case of the profit-maximising firm studied in Section 2. More importantly, the proposition shows that the firm-based planner of Proposition 2 behaves as if they are making the same lump-sum distribution made by the profit-maximising firm. This indicates the presence of an incentive leakage even in the absence of factor prices.

### 3.3 Incentive Leakage

Incentive leakage is a more general concept compared to lump-sum profits as it arises even in the absence of factor markets. Here, we introduce a definition of an economy-wide incentive leakage as the expected share of lump-sum profits in aggregate output (share of factorless income) in the presence of factor markets and under firm symmetry,

$$\Lambda_t = \frac{\mathbf{E}_t D_{t,t+1} \left( Y_{t+1} - \frac{W_{t+1}}{P_{t+1}} L_{t+1} - \frac{R_{t+1}}{P_{t+1}} K_t \right)}{\mathbf{E}_t D_{t,t+1} Y_{t+1}}. \quad (3.8)$$

Next, we assume the absence of factor markets and introduce factor shadow prices by exploiting the households' equilibrium conditions introduced in Section 2.1. The shadow real wages is given by the intratemporal consumption-leisure condition  $W_{t+1}/P_{t+1} = -U_{L,t+1}/U_{C,t+1}$ . and the shadow real return on capital is replaced for using the intertemporal consumption Euler condition  $\mathbf{E}_t D_{t,t+1} \frac{R_{t+1}}{P_{t+1}} = 1 - D_{t,t+1}(1 - \delta)$ , leading the following price-independent definition of dynamic one-period ahead incentive leakage.

**Definition 1.** In an economy where households have the utility  $U$ , incentive leakage  $\Lambda_t$  is defined as

$$\Lambda_t := \frac{\mathbf{E}_t D_{t,t+1} \left( Y_{t+1} + \frac{U_{L,t+1}}{U_{C,t+1}} L_{t+1} \right) - (1 - D_{t,t+1}(1 - \delta)) K_t}{\mathbf{E}_t D_{t,t+1} Y_{t+1}},$$

where  $D$  is the stochastic discount factor,  $Y$  the final good output,  $K$  capital,  $L$  labour. The parameter  $\delta$  represents the depreciation rate of capital and the parameter  $\beta$  the household discount factor.

It is an immediate corollary of Proposition 1 that  $\Lambda_t = \frac{1}{\epsilon}$  when the economy is composed of profit-maximising firms in a monopolistically competitive economy. Similarly, from Proposition 2 identical incentive leakage obtains when the profit-maximising firm is replaced by a firm-level planner, i.e.,  $\Lambda_t = \frac{1}{\epsilon}$ . This also means that in the absence of monopoly power ( $\epsilon = \infty$ ), there is no incentive leakage,  $\Lambda_t = 0$ .<sup>12</sup>

To highlight the importance of incentive leakages, we explore the consequences of closing the leakage on factor demand. To this end, we abstract from the ownership structure of the firm and assume that the supply side is made of firms, indexed  $i$ , that use labour and capital to produce, while being constrained by the production function  $Y_{i,t} = F_{i,t}(K_{i,t}, L_{i,t})$  and the budget constraint

$$P_{i,t} Y_{i,t} = W_{i,t} L_{i,t} + R_{i,t} K_{i,t}, \quad (3.9)$$

The constraint 3.9 states that the firms revenues are fully used to compensate labour and capital. Note that the prices of labour and capital might depend on the firm. The firm can operate internal factor markets as in the case of the firm-based planner discussed above or can hire labour and capital from economy-wide markets as is the case for the profit-maximising firm. Under constant returns to scale, the identity  $Y_{i,t} = F_{L,i,t} L_{i,t} + F_{K,i,t} K_{i,t}$  and the budget constraint 3.9 imply zero leakage as defined in Definition 1.

The proposition below stands without making any assumption on the firm's optimisation problem beyond the constant returns to scale technology and the budget constraint 3.9.

**Proposition 4. (Characterising Zero Incentive Leakage)** *A firm  $i$  that is subject to the zero-leakage budget constraint 3.9 and a constant returns to scale technology  $F_{i,t}$ , sets its*

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<sup>12</sup>In the steady state, incentive leakage is  $\Lambda := \frac{Y + \frac{U_L}{U_C} L - (\frac{1}{\beta} - 1 + \delta) K}{Y}$ . This clarifies the role of patience, represented by  $\beta$ , in the determination of incentive leakage. The welfare implications of households' patience will be discussed further below.

labour and capital demand such that

$$\left\{ F_{L,i,t} - \frac{W_{i,t}}{P_{i,t}} \right\} L_{i,t} + \left\{ F_{K,i,t} - \frac{R_{i,t}}{P_{i,t}} \right\} K_{i,t} = 0. \quad (3.10)$$

*Proof.* The constant return to scale production implies the identity  $Y_{i,t} = F_{L,i,t}L_{i,t} + F_{K,i,t}K_{i,t}$ . Combined with the budget constraint 3.9, this identity yields Equation 3.10. ■

When production displays constant return to scale and revenues are fully used to reward input providers, the absence of a lump-sum payment means that we cannot have a situation where both factors are underpaid relative to their respective marginal product.

To summarise, a radical change from a profit-maximising to a utility-maximising firm maintains the monopoly wedges affecting demand for production inputs, leading to identical welfare distortions in aggregate. However, closing the incentive leakage changes outcomes by avoiding situations where both production inputs are paid below their marginal product.

### 3.4 Decentralised Organisation of a Firm and Incentive Leakage

In the remainder of the paper, we propose specific organisational forms that eliminate the leakage, and show the impact on aggregate output, capital formation and welfare. Here, we clarify salient features of the structures we propose.

Suppose each agent supplies either capital or labour or both factors to a specific firm. Let us denote the factors through which an agent is tied to a firm as tied factors. Let us now define a decentralised organisation.

**Definition 2. Decentralised Organisation** *A firm is said to have a decentralised organisation if it has the following properties:*

- *There is a positive measure of agents tied through one or more factors to firm  $i$ . Each agent takes the behavior of other agents as given and makes an optimal decision to supply the tied factor given the remuneration structure.*
- *The firm's demand for the tied factor is equal to the aggregate equilibrium supply across agents tied to the firm.*



Note that under a decentralised organisation, the remuneration structure of a firm cannot have negative leakage. A negative leakage with one or more tied factors implies that total factor payments exceed total firm revenue. For leakage to be negative, the deficit must be financed without affecting factor prices, implying that it is financed through a lump-sum charge on agents. However, in a decentralised organisation, each agent would optimally choose not to pay the lump-sum charge (irrespective of whether others pay). Therefore a negative leakage cannot be part of a Nash equilibrium in a decentralised organisation. In other words, without outside intervention (e.g. a government imposing a lump-sum tax and redistributing as factor subsidy) the leakage is either zero or strictly positive. We assume the absence of such intervention since our focus is on the aggregate impact of internal incentive structure of firms.

Now, as we have shown above, the leakage is strictly positive under the firm-based planner's solution. This implies that if the factor demand is fully decided by a firm-level decision maker who maximises profit or utility (of tied agents), the optimal solution involves strictly positive leakage. We depart from this through decentralised organisation structures that have zero leakage.

**Definition 3. Decentralised Organisation with Zero Incentive Leakage** *A decentralised organisation with zero leakage satisfies the properties in Definition 2 and additionally satisfies the following:*

- *If there is an untied factor, it is paid a market price, and the demand for this factor is determined by a firm-level decision maker. The surplus (total firm revenue minus untied factor payment) is paid to the tied factor. A tied agent receives a fraction of the surplus commensurate with the fraction of tied factor the agent supplies.*
- *If both factors are tied, the firm decides shares of total revenue that goes to each factor. Each agent then gets remunerated in the same way as above.*

Note that under the above structure, no lump-sums are paid to any agent. All surplus (after paying any untied factor) is used to provide incentives to agents supplying tied factors.

### 3.5 Negative Internal Externalities and Positive Aggregate Externalities

In a decentralised organisation with zero leakage as defined above, each agent tied to a firm confers a negative externality on other agents tied to the firm. This is because greater supply

of the tied factor(s) by an agent reduces the share of surplus that others receive. However this externality is not internalised by the agent, implying that the total supply of the tied factor(s) is higher than the joint optimum of owners (providers of tied factor(s)). In other words, if there were a firm-level decision maker such as an owner or manager, they would decide to optimally limit the demand for each factor to exploit market power. However, a decentralised organisation oversupplies the tied factor(s) relative to such a joint optimum level.

The internal negative externality and consequent oversupply of tied factor(s) by all firms pushes up aggregate demand. In other words, the negative-externality-induced oversupply of tied factor(s) by each firm generates a positive externality for other firms. This results in an aggregate demand externality that raises final output as well as the level of tied factor provision. We explore the welfare implications of these effects in Section 6.

## 4 Entrepreneurial Shareholders

In Section 2, we analysed the benchmark case of monopolistic competition with incentive leakages. We now analyse the first of our decentralised zero-leakage arrangements: a firm owned by its shareholders, who hire labour externally. We assume that each and every household-shareholder is assigned to one specific firm, so that the ownership of the firm is well-defined. Here capital is the tied factor, and labour is rented in an economy-wide competitive market. As noted in the previous section, zero incentive leakage is achieved by paying the entire surplus (revenue net of labour costs) to capital in the following manner. Each shareholder receives a share of the surplus in proportion to their contribution to total capital. The firm's total capital is then an aggregation of decentralised investments decided at the shareholder-level. We label this firm *Entrepreneurial Shareholders* (ES).

### 4.1 Households

A unit measure of households assigned to firm  $i$  decide their consumption  $C_{i,t}$ , labour  $L_{i,t}$ , and capital supply  $K_{i,t}$  by maximising lifetime utility.

$$\max_{C_{i,s}, L_{i,s}, K_{i,s}} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_{i,s}, L_{i,s}) \quad (4.1)$$

$$\text{s.t. } P_s C_{i,s} + P_s \{K_{i,s} - (1 - \delta)K_{i,s-1}\} \leq W_s L_{i,s} + R_{i,s} K_{i,s-1}, \quad (4.2)$$

where  $P_t$  is the price of consumption goods,  $W_t$  is the market wage,  $R_{i,t}$  the firm-specific return on capital and  $\delta$  the depreciation rate. Note that the wage is external to the firm as it is determined by the economy-wide labour market. The return on investment  $R_{i,t}$ , however, is specific to the firm  $i$  that each household partly owns. First-order conditions for labour supply and saving yield

$$U_L(C_{i,s}, L_{i,s}) + \frac{W_t}{P_t} U_C(C_{i,s}, L_{i,s}) = 0, \quad (4.3)$$

$$\mathbf{E}_t D_{i,t,t+1} \left( 1 - \delta + \frac{R_{i,t+1}}{P_{t+1}} \right) = 1, \quad (4.4)$$

where  $D_{i,t,t+1} := \beta \frac{U_{i,C,t+1}}{U_{i,C,t}}$  is the firm  $i$  owners' stochastic discount factor. To further clarify ideas, consider the household's indirect utility function

$$V_{i,t} = V(K_{i,t-1}; W_t, W_{t+1}, \dots; R_{i,t}, R_{i,t+1}, \dots) := \mathbb{E}_t \sum_{u=t}^{\infty} \beta^{u-t} U(C_{i,u}, L_{i,u}).$$

The household's indirect utility is a function of last period accumulated capital stock  $K_{i,t-1}$ , the market wages  $W_t$  and the firm-specific rate of return on capital  $R_{i,t}$ . While wages are determined in the economy-wide labour market and are external to the firm, the firm can affect its owners' indirect utility through the return on capital  $R_{i,t}$ , as the households' capital is tied to the firm it co-owns.

## 4.2 The ES Firm

The monopolistically competitive firm is owned and run by entrepreneurial shareholder households. The capital supply decision  $K_{i,t}$  is taken by households internalising that future surplus  $S_{i,t+1} := P_{i,t+1} Y_{i,t+1} - W_{t+1} L_{i,t+1}$  is distributed proportionally to their capital contribution. Given the households' capital decision from the previous period, the firm sets demand for labour ( $L_{i,t}$ ) and the intermediate input price ( $P_{i,t}$ ) to maximize the households' indirect utility subject to the monopolistic competition, production technology and budget constraints. Formally, the optimisation problem is given by

$$\max_{L_{i,t}, P_{i,t}} V_{i,t}(K_{i,t-1}; W_t, W_{t+1}, \dots; R_{i,t}, R_{i,t+1}, \dots) \quad (4.5)$$

$$R_{i,t} K_{i,t-1} = P_{i,t} F_t(K_{i,t-1}, L_{i,t}) - W_t L_{i,t}, \quad (4.6)$$

$$Y_{i,t} \leq \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t \quad (4.7)$$

Note that firms are identical, yielding  $P_{i,t} = P_t$ . Given the symmetry of firms and households, we can now drop the subscript  $i$ .

Proposition 5 below presents the firm's labour demand and the household's capital and labour supply behaviour. Because the firm employs market labour, its labour demand displays the same wedge between marginal product of labour and real wages as the profit-maximising firm. Consistent with Proposition 4, in the absence of incentive leakage, the monopoly profits extracted by paying labour less than its marginal product are used to incentivise investments.

**Proposition 5. (ES Equilibrium)** *The firm with entrepreneurial shareholders (ES) sets its labour demand such that the labour wedge is positive  $\frac{W_t}{P_t} = (1 - \frac{1}{\epsilon}) F_{L,t}$ . In addition, the firm with entrepreneurial shareholder sets its capital demand such that the capital wedge, that is the difference between the marginal product of capital and real returns, is negative*

$$\frac{R_t}{P_t} = F_{K,t} + \frac{1}{\epsilon} F_{L,t} \frac{L_t}{K_{t-1}} > F_{K,t}. \quad (4.8)$$

*This implies the equilibrium conditions for labour and capital*

$$\frac{U_{L,t}}{U_{C,t}} + \left(1 - \frac{1}{\epsilon}\right) F_{L,t} = 0, \quad (4.9)$$

$$1 = \mathbf{E}_t D_{t,t+1} \left\{ 1 - \delta + F_{K,t+1} + \underbrace{\epsilon^{-1} F_{L,t+1} L_{t+1} / K_t}_{\text{extra returns on investments}} \right\}. \quad (4.10)$$

*Proof.* See appendix B.2. ■

The labour wedge is the same as in the case of the RBC-MC model given by  $\omega^{ES,L} = \frac{1}{\epsilon-1}$  but from Equation 4.10 the capital wedge is now negative as the firm pays labour less than its marginal product and uses the proceeds to incentivise investment  $\omega^{ES,K} = -\frac{1}{\epsilon \frac{F_{K,t} K_{t-1}}{F_{L,t} L_t} + 1}$ .

### 4.3 Characterising the ES outcome

**Proposition 6. (Higher Capital Formation)** *The equilibrium capital formation under the Entrepreneurial Shareholder firm (ES) is higher than under the RBC model with monopolistic competition.*

*Proof.* See Appendix B.3. ■

The intuition for higher capital formation by the firm lies in the design of remuneration and the fact that the firm aggregates individual household decisions. As explained in Section 3.5, the remuneration structure induces competition for shares of surplus among tied-factor providers, so that each agent exerts a negative externality on others, in turn leading to an oversupply of the tied factor (capital in this case). Further, higher capital provision by each firm generates a positive aggregate externality, leading to higher levels of investment and output at the aggregate level. Higher capital levels also raise demand for labour and wages rise. As the aggregate income also increases, all firms face a less restrictive competitive monopoly condition (Equation 4.7).

While we have assumed the firm maximises utility of owners, the next result shows that the information requirement is minimal under ES. In particular the firm can operate without knowing agent utilities. This arises from the static nature of the ES firm's optimisation problem, which in turn implies that results equivalent to Proposition 5 hold when the firm maximises any strictly increasing function of the indirect utility, given the same constraints. The proposition below results from the fact that indirect utility is strictly increasing in surplus.<sup>13</sup>

**Proposition 7. (Informational Robustness)** *The outcome in Proposition 5 obtains when to the ES firm maximises surplus  $S_{i,t} := P_{i,t}Y_{i,t} - W_tL_{i,t}$ , subject to the competitive monopoly constraint 4.7 and the zero-leakage condition 4.6.*

*Proof.* See Appendix B.4. ■

## 5 Entrepreneurial Workers

In the previous section, we considered the ES structure where capital is tied to the firm and labour is rented in an economy-wide competitive market. We now consider the opposite case: a decentralised firm with zero leakage owned by workers. Labour is now tied and capital is hired in an economy-wide competitive market. The surplus after paying capital costs goes to remunerate labour. Each worker receives a share of the surplus in proportion to their contribution to total labour. We label this firm *Entrepreneurial Workers* (EW).

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<sup>13</sup>Even if we were to assume heterogeneous household preferences, maximising surplus would maximise the utility of households inheriting non-zero capital from the previous period. Households with no capital would be indifferent to the firm's behaviour.

## 5.1 Households

Households behave in the same way as in Section 4, except that wages are firm-specific and the return on capital is not. Formally, households maximise the expression in 4.1 subject to the budget constraint Equation 4.2, with  $W_s$  replaced by  $W_{i,s}$  and  $R_{i,s}$  replaced by  $R_s$ . The factor supply conditions are also the same as Equations 4.3 and 4.4, with the same replacement.

## 5.2 The EW Firm

The firm sets demand for rented capital ( $K_{i,t}^R$ ) and the intermediate input price ( $P_{i,t}$ ) to maximize its representative worker indirect utility subject to the monopoly and budget constraints. The labour supply decision is taken by households internalising that surplus  $S_{i,t} := P_{i,t}Y_{i,t} - R_t K_{i,t}^R$  is distributed proportionally to their labour contribution. Internalising the households' labour supply, the firm solves the following optimisation problem subject to the monopoly, the production technology and the budget constraints, i.e.

$$\max_{K_{i,t}^R, P_{i,t}} V_{i,t}(K_{i,t-1}; W_{i,t}, W_{i,t+1}, \dots; R_t, R_{t+1}, \dots)$$

subject to

$$W_{i,t}L_{i,t} = F_t(K_{i,t}^R, L_{i,t}) - R_t K_{i,t}^R, \quad (5.1)$$

as well the competitive monopoly constraint given by 4.7.

Note that in the current set-up, we distinguish between the capital rented by the firm in the competitive economy-wide market  $K_{i,t}^R$  and the capital previously accumulated by the firm's owners/workers  $K_{i,t-1}$ . Also, note that the firm takes (internal) labour supply by member households as given, reflecting our decentralised setting. As in the entrepreneurial shareholder case, the symmetry of firms and households implies that we can drop the subscript  $i$ . Moreover, the symmetry arguments combined with the capital market clearing condition enable us to write  $K_t^R = K_{t-1}$ .

Proposition 8 below presents the firm's labour demand and investment behaviour.

**Proposition 8. (EW Equilibrium)** *The firm with entrepreneurial workers (EW) sets its capital demand such that the capital wedge is positive*

$$\frac{R_t}{P_t} = \left(1 - \frac{1}{\epsilon}\right) F_{K,t}.$$

In addition, the entrepreneurial workers firm sets its labour demand such that the real wage is higher than the marginal product of labour

$$\frac{W_t}{P_t} = F_{L,t} + \frac{1}{\epsilon} F_{K,t} \frac{K_{t-1}}{L_t} > F_{L,t}. \quad (5.2)$$

This implies the equilibrium conditions for labour and capital

$$\frac{U_{L,t}}{U_{C,t}} + F_{L,t} + \underbrace{\epsilon^{-1} F_{K,t} K_{t-1} / L_t}_{\text{extra return on labour}} = 0, \quad (5.3)$$

$$1 = \mathbf{E}_t D_{t,t+1} \left\{ 1 - \delta + \left( 1 - \frac{1}{\epsilon} \right) F_{K,t+1} \right\}. \quad (5.4)$$

*Proof.* See appendix B.5. ■

In the entrepreneurial workers set-up, we obtain the diametrically opposite result to the entrepreneurial shareholder case (Proposition 5). The capital wedge is the same as in the RBC-MC case  $\omega^{EW,K} = \frac{1}{\epsilon-1}$  and the labour wedge is negative  $\omega^{EW,L} = - \left\{ \epsilon \frac{F_{L,t} L_t}{F_{K,t} K_{t-1}} + 1 \right\}^{-1}$ . Unsurprisingly, given the additional incentive to labour supply, labour employment is higher in equilibrium under the EW firm than under RBC-MC, as per the proposition below.

We can characterize the EW outcome in the same way as the ES outcome. Just as ES leads to higher use of capital compared to RBC-MC, EW leads to higher use of labour. The result is similar to Proposition 6. We omit formal details. Further, as we show below, the informational robustness property of ES (Proposition 7) holds for EW as well.

Specifically, as with the ES firm, the nature of the EW firm's optimisation problem is static. Further, so long as firm owners' labour supply elasticity is not too negative, indirect utility is strictly increasing in surplus. These imply that the EW firm can target surplus instead of utility, implying it can operate without knowing the utility of agents. Proposition 9 below presents the result formally.<sup>14</sup>

**Proposition 9. (Informational Robustness)** *Assume that the wage elasticity of the firm-level labour supply is higher than -1  $\left( \frac{W_{i,t}}{L_{i,t}} \frac{\partial L_{i,t}}{\partial W_{i,t}} > -1 \right)$ , then the outcome in Proposition 8 obtains when the EW firm maximises surplus  $S_{i,t} := P_{i,t} Y_{i,t} - R_t K_{i,t}^R$ , subject to the competitive monopoly constraint 4.7 and the zero-leakage condition 5.1.*

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<sup>14</sup>This further implies that the behaviour of the EW firm remains unaffected when workers/owners have heterogeneous utilities, provided some of them supply labour and that the wage elasticity of the firm-level labour supply is not too negative.



*Proof.* See Appendix B.6. ■

## 6 Welfare Results and Numerical Evaluation

### 6.1 Dynamic versus Steady-State Welfare Benchmarks

The standard dynamic benchmark for welfare in macroeconomic models is the perfect competition RBC (RBC-PC) model. Under a positive level of monopoly power ( $\epsilon > 1$  and finite), firms distort factor payments in order to maximise profit. This makes the outcome inefficient compared to the RBC-PC model across periods and makes for a straightforward dynamic welfare comparison.

However, for the arrangements we propose in the paper, welfare comparison is more complicated. Starting from the RBC-MC set-up, a change to any of the institutional arrangement we propose changes the path of consumption and leisure. Obviously, starting from the initial period, welfare from any alternative arrangement must be below the RBC-PC level. This follows from the First Welfare Theorem, and is the standard dynamic notion of welfare comparison. But this “only-initial-period” view fails to capture important aspects of welfare change over time, as the rankings may be altered after a few periods. In the absence of an unambiguous welfare ranking that applies to all periods, one way to compare welfare is to consider the steady-state outcome. Indeed, such a long-term comparison might be the natural way to rank welfare effects of an institutional change. But in this case, the RBC-PC model is no longer the relevant benchmark. Rather, the steady-state benchmark is given by the Golden Rule.

In standard growth models, the Golden Rule sets the rate of capital formation that maximises steady state consumption (Phelps, 1961, 1965, Romer, 2018). Our model can be considered to be a zero-growth model, for which we can define an appropriate Golden Rule. Unlike standard growth models, our setting incorporates decisions on the intensive margin of labour (in standard growth models labour is inelastically supplied, as the focus is on population growth).<sup>15</sup> This implies that the relevant Golden Rule in our zero-growth model is the rate of capital formation that maximises *steady-state utility* which includes leisure in addition to consumption. We formally characterise the Golden Rule for our setting.

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<sup>15</sup>The choice of zero growth enables us to consider steady-state growth in an RBC set-up (King, Plosser, and Rebelo, 1988).

## 6.2 Characterising Welfare Gaps: The Monopoly Gap versus the Patience Gap

It is important to note that the neoclassical growth model with capital accumulation and positive time discounting does not implement the Golden Rule.<sup>16</sup> In this instance, the Golden Rule represents the limiting case of the neoclassical growth model as the discount factor  $\beta$  approaches 1.<sup>17</sup> We demonstrate formally (Proposition 10) that this result extends to the case of intensive labour margin as well.<sup>18</sup> Note that since RBC-PC with any given  $\beta < 1$  does not implement the Golden Rule, there is scope for steady-state welfare to be improved beyond the RBC-PC level.

The discussion above points to two welfare gaps. First, the welfare gap between the Golden Rule allocation and that of RBC-PC noted above. Since the former can be achieved by the latter if the discount factor approaches 1, this gap arises purely through impatience. We call this the **patience gap**. Second, the usual welfare gap between RBC-PC and RBC-MC, which we call the **monopoly gap**. This gap is present both in a dynamic analysis as well as in the steady-state. We focus on the monopoly gap at the steady-state. Figure 1 in the introduction illustrates the gaps.

Our numerical results show that over a standard range of parameter values, the institutional form with labour as the tied factor (EW) produces higher welfare compared to RBC-MC, but remains below RBC-PC. In other words, EW closes part of the monopoly gap. On the other hand, the institutional form with capital as tied factor (ES) not only eliminates the monopoly gap fully, but closes even part of the patience gap, thus outperforming even perfect competition RBC in the steady state.

This gives rise to another interesting question: why do ES and EW welfare levels fall neatly on two sides of RBC-PC? The answer lies in the fact that all three are zero-leakage institutions. RBC-PC achieves this by eliminating market power, and the other forms achieve this by keeping market power intact while changing the revenue-allocation mechanism. Thus as market power rises from zero, but zero leakage is preserved, the outcomes differ relative to RBC-PC. Looking at zero leakage as the main characteristic thus gives us a natural comparison across RBC-PC and our institutional forms. To clarify the nature of comparison,

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<sup>16</sup>See for example the Ramsey-Cass-Koopmans model (Romer, 2018).

<sup>17</sup>Equivalently, the Golden Rule corresponds to the rate of time preference approaching 0.

<sup>18</sup>Essentially, as the discount factor rises, steady-state capital rises, driving a rise in steady-state consumption. But for standard utility specifications, steady-state labour does not rise so much that the consequent fall in leisure would offset the utility improvement from higher consumption.

we develop a formal result (Proposition 11) that shows that under zero leakage, steady-state welfare rises in the steady-state capital-output ratio. Compared to RBC-PC, EW has more incentive for labour provision. This leads to a lower capital-output ratio as the saving incentive is dampened relative to RBC-PC. On the other hand, the stronger inter-temporal incentives for capital provision under ES implies a higher steady-state capital-output ratio compared to that under perfect competition. Therefore ES can raise steady-state welfare beyond the latter and move closer to the Golden Rule.

### 6.3 Characterising the Golden Rule

The standard Golden Rule aims at maximising balanced growth path welfare when utility is function of consumption alone. We propose a leisure-augmented Golden Rule aimed at maximising steady-state welfare  $\sum_{s \geq t} \beta^{s-t} U(C, L) = \frac{U(C, L)}{1-\beta}$  subject to the usual resource constraint  $C \leq F(K, L) - \delta K$ . The steady-state allocation of capital and labour is given by the first-order conditions  $F_K(K, L) = \delta$ , and  $U_C F_L + U_L = 0$ .

Here we formally demonstrate that the steady-state welfare under perfect competition rises in  $\beta$  under a standard utility specification used in macroeconomics. This confirms that the standard result that the Golden Rule requires  $\beta \rightarrow 1$  extends to our model with an intensive margin for labour.

**Proposition 10. (Steady-state Welfare under Perfect Competition rises in  $\beta$ )** *Assume a utility function of the form*

$$U(C_t, L_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{\chi}{1+\eta} L^{1+\eta}, \quad (6.1)$$

where  $\sigma$ ,  $\eta$  and  $\chi$  are positive constants. Under the perfect-competition RBC model, steady-state welfare is increasing in  $\beta$ .

*Proof.* See appendix B.7. ■

As noted previously, RBC-PC under  $\beta < 1$  cannot implement the Golden Rule, leaving scope for improvements. What institutional structures might be able to exploit this scope? The proposition below establishes a general possibility result showing that zero-leakage institutions can indeed implement such improvements. It further shows that the zero-leakage allocation with the highest steady-state utility corresponds to a larger capital-output ratio

than the RBC-PC outcome but a lower ratio than under the Golden Rule. The next result then derives capital-output ratios under the various arrangements.

**Proposition 11. (General Possibility Result for Zero-Leakage Institutions)** *Assume a utility function of the form given by 6.1. There exists a zero-incentive-leakage steady-state allocation that improves steady-state welfare relative to the RBC-PC outcome. The zero-incentive-leakage allocation with the highest steady-state welfare implies a capital-output ratio that is higher than the RBC-PC outcome but lower than that under the Golden Rule. Further, steady-state welfare is increasing in the capital-output ratio for all values below the optimum level.*

*Proof.* See appendix B.8. ■

We now assume a Cobb-Douglas technology and derive the steady-state capital-output ratios for all the cases we have considered so far.

**Proposition 12. (Steady-State Capital-Output Ratios)** *Assume that for each firm  $i$  production is constrained by the Cobb-Douglas technology  $Y_{i,t} = F_t(K_{i,t}, L_{i,t}) = Z_t K_{i,t}^\alpha L_{i,t}^{1-\alpha}$ , where  $Z_t$  is TFP and  $\alpha$  the share of capital in output. The capital-output ratio  $\frac{K}{Y}$  corresponding to the different cases are as follows:  $\frac{\alpha}{\delta}$  (Golden Rule),  $\frac{\alpha}{\delta} \frac{1}{1+(1/\beta-1)/\delta}$  (RBC-PC),  $\frac{\alpha}{\delta} \frac{1-1/\epsilon}{1+(1/\beta-1)/\delta}$  (RBC-MC),  $\frac{\alpha}{\delta} \frac{1+(1/\alpha-1)/\epsilon}{1+(1/\beta-1)/\delta}$  (ES), and  $\frac{\alpha}{\delta} \frac{1-1/\epsilon}{1+(1/\beta-1)/\delta}$  (EW).*

*Proof.* The capital-output ratio under RBC-MC can be derived from Proposition 1. The limit as  $\epsilon \rightarrow \infty$  then gives us the ratio under RBC-PC, and the limit of the ratio under RBC-PC as  $\beta \rightarrow 1$  gives us that under the Golden Rule. Finally, the ratios under ES and EW are derived from Propositions 5 and 8, respectively. ■

Note that  $\left(\frac{K}{Y}\right)^{EW} = \left(\frac{K}{Y}\right)^{RBC-MC} < \left(\frac{K}{Y}\right)^{RBC-PC} < \left(\frac{K}{Y}\right)^{GoldenRule}$  and  $\left(\frac{K}{Y}\right)^{RBC-PC} < \left(\frac{K}{Y}\right)^{ES}$  for all feasible model parameters. The second inequality shows that when monopoly profits are harnessed to incentivise investments, as under ES, the capital-output ratio overshoots its value under perfect competition. Finally, the investment ratio under ES can even surpass the Golden Rule ratio when the firm holds enough monopoly power:  $\epsilon < \delta \frac{1/\alpha-1}{1/\beta-1}$ . Given the calibration in Table 1 below, this condition implies  $\epsilon < 4.4$ , which corresponds to higher monopoly power than typically assumed in the literature.

Combining the results of Propositions 11 and 12, we can further conclude that the zero-leakage allocation with highest steady-state welfare corresponds to a capital-output ratio strictly between  $\frac{\alpha}{1/\beta-1+\delta}$  and  $\frac{\alpha}{\delta}$ .

Note that Proposition 12 can be reformulated using the marginal rate of technical substitution (MRTS). Under a Cobb-Douglas technology, the MRTS is increasing in the capital-output ratio in the steady state:  $mrts := \frac{\partial F/\partial L}{\partial F/\partial K} = \frac{1-\alpha}{\alpha} \left(\frac{1}{Z} \frac{K}{Y}\right)^{\frac{1}{1-\alpha}}$ , where  $Z$  designates the steady-state TFP. As the discussion in Section 3.5 indicates, RBC-MC delivers the optimal firm-level MRTS under monopolistic competition. While, both the ES and EW arrangements remove incentive leakage, EW maintains the same MRTS as the RBC-MC arrangement while ES implies a higher MRTS (equivalently a higher capital-output ratio) by using monopoly profits to incentivise capital supply. Although this higher investment is suboptimal at the level of the firm (negative externality), it leads to higher aggregate welfare (positive aggregate demand externality).

## 6.4 Steady State Numerical Results under ES and EW

We now provide steady-state numerical results for the equilibrium outcomes under the two firm arrangements discussed above (ES and EW) and compare these with the outcomes under monopolistic competition, perfect competition as well as the Golden Rule as characterised above. We assume the following utility function for households:

$$U(C_t, L_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{\chi}{1+\eta} L_t^{1+\eta}.$$

Further, we assume that the production function is Cobb-Douglas with  $\alpha$  denoting the share of capital and  $Z_t$  denoting aggregate total factor productivity (TFP) with  $Z = 1$  in the steady state:

$$F_t(K_{t-1}, L_t) = Z_t K_{t-1}^\alpha L_t^{1-\alpha}.$$

Log TFP follows an AR(1) process  $\ln(Z_t) = \rho^Z \ln(Z_{t-1}) + \sigma^Z \epsilon$  where  $\epsilon \sim N(0, 1)$  are normal *i.i.d.* shocks. Assuming the RBC-MC set-up, we calibrate the relative importance of the disutility of work  $\chi$  to match steady-state labour hours around a third of total time endowment and following Galí (2015) we assume  $\epsilon = 9$ . We borrow the remaining parameters from the literature (see Table 1). Steady states are computed for all arrangements for different levels of the parameter  $\epsilon$ . Note that monopoly power is captured by  $1/\epsilon$ . The power is therefore decreasing along the horizontal axis in the figure below.

The steady-state results for output, capital, labour and consumption are presented in Figure 2. Note that levels of these variables are higher than the monopolistic competition levels for all arrangements over the relevant range of market power. The more interesting comparison is with respect to perfect competition RBC. Essentially, the same idea as discussed in

| Parameter   | Value |
|---|-------|
| Discount factor $\beta$                           | 0.99  |
| Intertemporal elasticity of substitution $\sigma$ | 1     |
| Inverse of Frisch elasticity $\eta$               | 1     |
| Labour disutility scale parameter $\chi$          | 8     |
| Share of capital $\alpha$                         | 0.36  |
| Volatility of TFP $\sigma^Z$                      | 1%    |
| TFP persistence parameter $\rho^Z$                | 0.9   |
| Capital depreciation rate $\delta$                | 2.5%  |
| Monopoly power parameter $\epsilon$               | 9     |

Table 1: Calibrated model parameters.

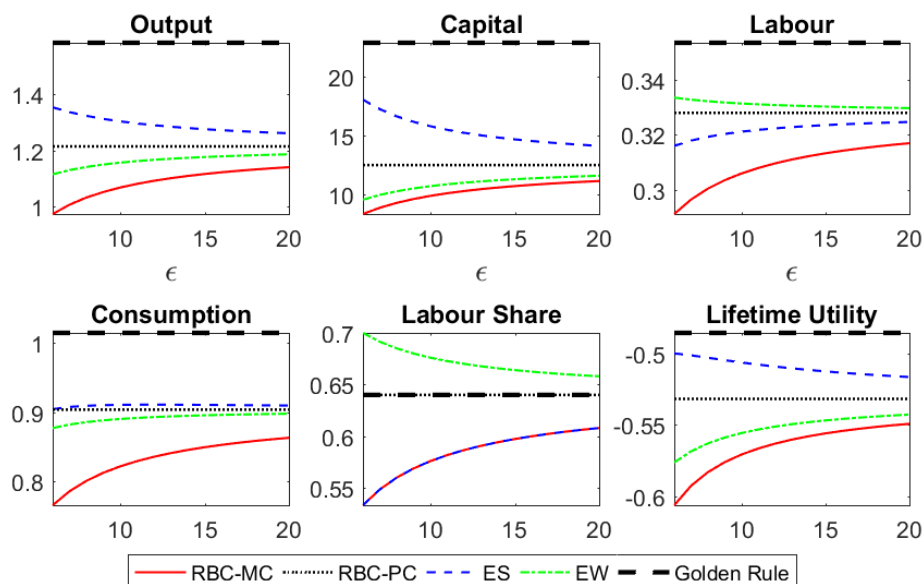


Figure 2: The steady-state output, capital, labour, consumption, labour share and lifetime utility for RBC-MC (red, continuous line), ES (blue, dashed line), EW (green, continuous-dashed line), RBC-PC (black, dotted line) and the Golden Rule (black, thick dashed line) for various values of the parameter  $\epsilon$ . A higher value of  $\epsilon$  corresponds to lower monopoly power. Note that the welfare gap between RBC-PC and RBC-MC is the monopoly gap, while the gap between the Golden Rule and RBC-PC is the patience gap.

Section 6.2 (which is about steady-state welfare) applies here: ES, EW and RBC-PC are all zero-leakage systems, with different in-built incentives. RBC-PC features firms with no market power, so the factor payments align with marginal products. But ES and EW preserve

market power, so the elimination of leakage necessarily leads to more-than-marginal-product payment for the tied factor, biasing the incentive structure relative to RBC-PC. Further, as expected from theory, when monopoly power fades ( $\epsilon \rightarrow \infty$ ), steady-state variables converge to their respective RBC-PC levels under all firm arrangements.

Since the steady-state level of capital is higher under ES compared to RBC-PC, ES output also surpasses that under RBC-PC. The high levels of output under ES arrangement leads to higher consumption than under RBC-MC for all values of  $\epsilon$ , and is also a little above the RBC-PC level for most values of  $\epsilon$ .

EW, on the other hand, results in higher labour in the steady state than under RBC-MC or RBC-PC. Output and capital are also higher than under RBC-MC leading to higher consumption in the steady state. However, as noted previously, given that the direct incentives are on labour, capital is lower compared to RBC-PC, which leads to lower output and consumption relative to RBC-PC.

Figure 2 also presents the shares of both production inputs under different firm arrangements. Under RBC-MC, both factors are remunerated less than under RBC-PC, as some revenues are leaked in the form of additive profits. As expected by theory, in each of the alternative arrangements, the factor tied to the firm receives a higher share than under the RBC-PC outcome, while the factor not tied receives the same share as under RBC-MC.

Finally, Figure 2 shows the steady-state welfare under all firm arrangement for different values of  $\epsilon$ . This figure is the basis for the schematic in Figure 1, discussed in the introduction. Counterintuitively, note that more monopoly power (lower values of  $\epsilon$ ) leads to better allocation of resources through aggregate externalities under the ES firm, such that the welfare gap between ES and the Golden Rule is further reduced.

## 6.5 Convergence Dynamics: Firm-Level Institutional Change

In this section, we study the dynamic evolution of the macroeconomic variables and welfare under different firm-level institutional arrangements. In a standard business cycle analysis, the institutional structure is given and the dynamic analysis is based on how the economy responds to TFP shocks. Our problem is different as we analyse the response of the economy to a change in the institutional structure itself. To do so, we assume that the economy is initially at the steady state of the RBC-MC model and that the institutional form of the firm changes in the initial period to each of the alternative arrangements discussed so far. We



then let the system converge to the new steady state under each alternative arrangement. The results with the parameter values in Table 1 are presented in Figure 3.

The figure also displays the effects of removing all monopoly power amounting to the RBC-PC case. As expected from the First Welfare Theorem, removing monopoly power provides the highest welfare gains initially relative to all alternative arrangements. However, for the reasons explained above, the long-run steady state ranking is different. ES and EW improve on RBC-MC. The more interesting comparison, as above, is among the three zero-leakage arrangements, with ES and EW outcomes on two sides of RBC-PC.

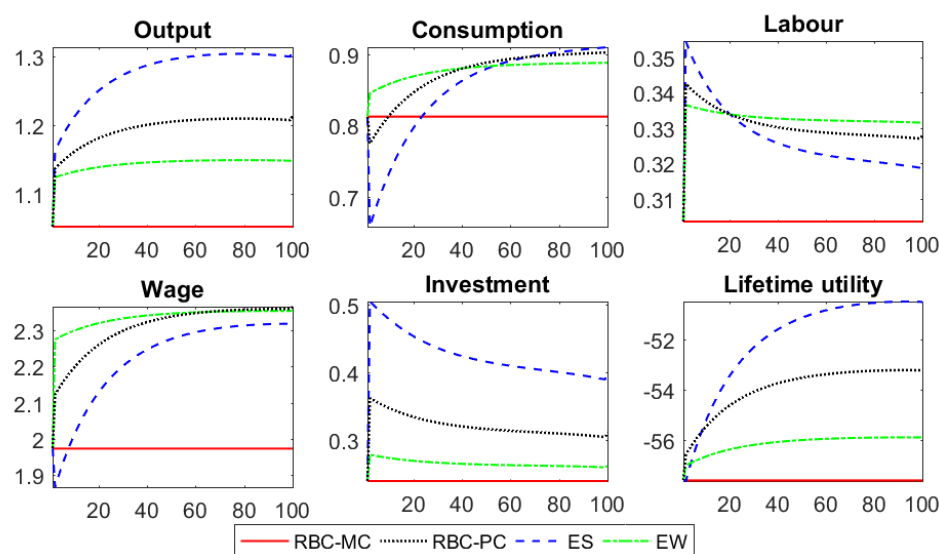


Figure 3: Convergence to new steady-states under different institutional arrangement. The horizontal axis represents time in quarters. Starting from the steady-state under RBC-MC, the economy is assumed to adopt the ES arrangement (blue, dashed line), the EW arrangement (green, dashed-continuous line) and RBC-PC arrangement (black, dotted line). The initial steady state under RBC-MC is represented using the red, continuous line. All variables are in levels.

## 6.6 Impulse Response under Firm-Level Institutional Change

The steady state macroeconomic outcomes and welfare differ substantially across firm-level institutional arrangements. Do they also differ in terms of their dynamic response to aggregate shocks starting from the respective steady states? The impulse responses following an unexpected one standard deviation shock to TFP are presented in Figure 4 assuming the parameters in Table 1 in the paper. The figure shows that the reaction of output, labour

and wages to unpredicted TFP shocks is similar under all firm arrangements. Steady-state welfare improvement when switching to the proposed arrangements (ES or EW) does not lead to higher aggregate volatility.

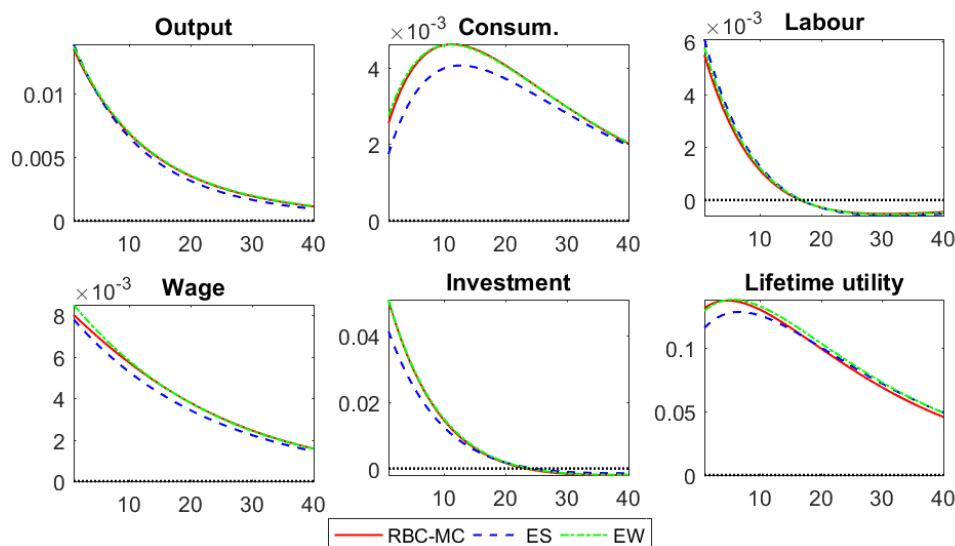


Figure 4: Impulse responses w.r.t. one standard deviation TFP shock. The horizontal axis represents time in quarters. The figure shows the response under RBC-MC (red continuous line), ES (blue, dashed line) and EW (green, dashed-continuous line), starting from the respective steady states. All variables are in log form and are expressed as deviation from the models' respective steady states, except for lifetime utility where no log transformation is applied.

## 7 Workers' Enterprise

In this section we consider the behaviour of an economy composed of Workers' Enterprises (WE) engaged in a competitive monopoly. We define the WE following [Sertel \(1982, 1987, 1996\)](#) as being an enterprise whose shareholders (partners) and workers are by statute identical. In other words, in contrast with previous arrangements, both factors are now tied to the firm. This adds another layer of decision making. Since both factors are tied, and there is no incentive leakage, a WE must decide the share of next period's revenue that is awarded to each factor. Promising shares to production factors ensures that the firm complies with the decentralised arrangement (Definition 2) by guaranteeing that no external source of funds is

required to compensate for potential fluctuations in revenues.<sup>19</sup>

Note that the WE presents a more general case compared to ES or EW, as either can be obtained as a special case of WE by setting share appropriately. However, the additional optimisation problem of deciding factor revenue shares one period ahead makes the analysis technically challenging. Our steady-state simulations show that for a standard range of parameter values, the optimal structure of the WE is close to ES. This shows that WE firms, optimising individually, end up adopting a structure that generates higher aggregate welfare. Indeed, the extra flexibility in the structure of WE implies it generates a level of welfare even higher than that under ES, but simulations show that the difference is small. This is surprising, since firms do not set out to consider aggregate externalities in choosing internal structure, yet optimally choose a structure that results in a steady-state welfare level close to ES.

We can gain a rough intuition by considering dynamic incentives. We know from Section 3 that the firm-level social planner with unrestricted choice of structure would optimally choose one with an incentive leakage achieved through a lump-sum payment. Let us argue that within zero-leakage systems, the ES structure comes close to replicating a lump-sum effect, making it an attractive choice for a WE. As explained previously, the ES structure provides strong incentives to save. In period  $t + 1$ , savings of  $t$  are available as a given stock and the return on this acts as a lump-sum receipt at  $t + 1$ . In other words, arriving in period  $t + 1$  with greater capital (higher savings from previous periods) serves a utility-boosting function similar to having a lump-sum. Mainly incentivising labour, on the other hand, takes away more from leisure today, and weaker incentive on savings imply a weaker version of the lump-sum impact in the future. In choosing a structure that maximises steady-state utility, WE firms therefore lean towards ES.

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<sup>19</sup>That is, the revenue share is decided at the time the capital investment is made. Additional to the considerations raised in the model, this provides the following robustness feature. In an overlapping generation (OLG) setting, future generations of workers/owner might hold less capital than incumbents, inducing them to allocate a larger share of revenue to compensate labour. Such loss of future share would nullify, partly or fully, investment incentives at the present. Deciding shares in advance mitigates the problem. The WE structure is therefore robust to a change in the model to an OLG setting.

## 7.1 Households

We assume the existence of a unit measure of worker/owner households and that each household is tied to a single WE, denoted  $i$ , with respect to both factors. The households' supply of labour and capital depend on the WE internal wage and internal return on capital. The worker-owner households maximise utility to set consumption  $C_{i,t}$ , labour supply  $L_{i,t}$  and the capital  $K_{i,t}$ . Formally,

$$\max_{C_{i,s}, L_{i,s}, K_{i,s}} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_{i,s}, L_{i,s}) \quad (7.1)$$

$$\text{s.t. } P_s C_{i,s} + P_{i,s} \{K_{i,s} - (1 - \delta)K_{i,s-1}\} \leq W_{i,s} L_{i,s} + R_{i,s} K_{i,s-1}. \quad (7.2)$$

The household's problem first-order equations yield the labour and capital supply conditions

$$U_L(C_{i,t}, L_{i,t}) + \frac{W_{i,t}}{P_t} U_C(C_{i,t}, L_{i,t}) = 0,$$

$$1 = \mathbf{E}_t D_{i,t,t+1} (1 - \delta + R_{i,t+1}/P_{t+1}),$$

where  $D_{i,t,t+1}$  is the firm-specific stochastic discount factor. The indirect utility of the household is a function of the WE internal factor prices and savings

$$V_{i,t} = V(K_{i,t-1}; W_{i,t}, W_{i,t+1}, \dots; R_{i,t}, R_{i,t+1}, \dots).$$

## 7.2 Factor Shares

The WE, indexed  $i$ , decides at time  $t$  the shares of time  $t + 1$  revenues dedicated to capital  $\xi_{i,t}^K$  and labour  $1 - \xi_{i,t}^K$ . The current shares are predetermined at time  $t - 1$

$$R_{i,t} K_{i,t-1} = \xi_{i,t-1}^K P_{i,t} Y_{i,t}, \quad (7.3)$$

$$W_{i,t} L_{i,t} = (1 - \xi_{i,t-1}^K) P_{i,t} Y_{i,t}. \quad (7.4)$$

Moreover, incentives of the agents tied to the WE are summarised by the labour ( $L^S$ ) and the capital ( $K^S$ ) supply functions

$$L_{i,t} = L^S(K_{i,t-1}; W_{i,t}, W_{i,t+1}, \dots; R_{i,t}, R_{i,t+1}, \dots), \quad (7.5)$$

$$K_{i,t} = K^S(K_{i,t-1}; W_{i,t}, W_{i,t+1}, \dots; R_{i,t}, R_{i,t+1}, \dots). \quad (7.6)$$

In addition, the firm is subject to the competitive monopoly constraint

$$Y_{i,t} \leq \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t. \quad (7.7)$$

The worker/owners determine factor shares to maximise their welfare. Workers are constrained by the previously set factor shares 7.3 and 7.4, labour and capital supply functions 7.5 and 7.6, the monopoly constraint 7.7 and the production technology. Formally, workers set the future shares of capital  $\xi_{i,t}^K, \xi_{i,t+1}^K, \dots$ , the sale prices  $P_{i,t}, P_{i,t+1}, \dots$ , the wages  $W_{i,t}, W_{i,t+1}, \dots$  and the returns on capital  $R_{i,t}, R_{i,t+1}, \dots$  to maximise their indirect utility

$$\begin{aligned}
& \max_{\xi_{i,s}^K, P_{i,s}, W_{i,s}, R_{i,s} \text{ for } s=t, t+1, \dots} V_{i,t}(K_{i,t-1}; W_{i,t}, W_{i,t+1}, \dots; R_{i,t}, R_{i,t+1}, \dots) \\
& \text{s.t. } 0 < \xi_{i,t}^K < 1, \\
& W_{i,s} L_{i,s} = (1 - \xi_{i,s-1}^K) P_{i,s} Y_{i,s} \text{ for } s = t, t+1, \dots \\
& R_{i,s} K_{i,s-1} = \xi_{i,s-1}^K P_{i,s} Y_{i,s} \text{ for } s = t, t+1, \dots \\
& L_{i,s} = L^S(K_{i,s-1}; W_{i,s}, W_{i,s+1}, \dots; R_{i,s}, R_{i,s+1}, \dots) \text{ for } s = t, t+1, \dots \\
& K_{i,s} = K^S(K_{i,s-1}; W_{i,s}, W_{i,s+1}, \dots; R_{i,s}, R_{i,s+1}, \dots) \text{ for } s = t, t+1, \dots \\
& Y_{i,s} \leq \left( \frac{P_{i,s}}{P_s} \right)^{-\epsilon} Y_s \text{ for } s = t, t+1, \dots \\
& Y_{i,s} = F_s(K_{i,s-1}, L_{i,s}) \text{ for } s = t, t+1, \dots
\end{aligned}$$

### 7.3 Workers' Enterprise First-Order Conditions

To simplify notation, we suppose implicitly that both production inputs are set by the households' supply  $L_{i,s} = L^S(K_{i,s-1}, W_{i,s}, R_{i,s})$  and  $K_{i,s} = K^S(K_{i,s-1}, W_{i,s}, R_{i,s})$ , and that the output is set by the production function  $Y_{i,s} = F_s(K_{i,s-1}, L_{i,s})$ . We can, thus, write the problem's Lagrangian as follows

$$\begin{aligned}
\mathcal{L}_{i,t} = & V_{i,t} + \mathbf{E}_t \sum_{u \geq t} D_{t,u} \lambda_{i,u}^F \left\{ \left( \frac{P_{i,u}}{P_u} \right)^{-\epsilon} Y_u - Y_{i,u} \right\} \\
& + D_{t,u} \mu_{i,u}^W \left\{ (1 - \xi_{i,u-1}^K) P_{i,u} Y_{i,u} - W_{i,u} L_{i,u} \right\} + D_{t,u} \mu_{i,u}^K \left\{ \xi_{i,u-1}^K P_{i,u} Y_{i,u} - R_{i,u} K_{i,u-1} \right\}.
\end{aligned}$$

The first-order conditions with regard to the price  $P_{i,t}$  and the share of capital  $\xi_{i,t}^K$  yield respectively

$$\begin{aligned}
\lambda_{i,t}^F \frac{\epsilon}{P_{i,t}} &= \mu_{i,t}^W (1 - \xi_{i,t-1}^K) + \mu_{i,t}^K \xi_{i,t-1}^K, \\
\mathbf{E}_t D_{t,t+1} P_{i,t+1} Y_{i,t+1} \mu_{i,t+1}^W &= \mathbf{E}_t D_{t,t+1} P_{i,t+1} Y_{i,t+1} \mu_{i,t+1}^K.
\end{aligned} \tag{7.8}$$

Using the envelope theorem on the household's problem yields the expression  $\frac{\partial V_{i,t}}{\partial W_{i,t+1}} = \mathbf{E}_t D_{t,t+1} \frac{U_{C,t}}{P_t} L_{i,t+1}$ . Defining  $\mu_{i,t} := \mu_{i,t}^W (1 - \xi_{i,t-1}^K) + \mu_{i,t}^K \xi_{i,t-1}^K$  and noting that  $\lambda_{i,t}^F = \frac{1}{\epsilon} P_{i,t} \mu_{i,t}$ ,

the first-order condition with regard to the wage  $W_{i,t+1}$  yields

$$\begin{aligned}
& \mu_{i,t} P_{i,t} \frac{\partial L_{i,t}}{\partial W_{i,t+1}} \tilde{\omega}_t^L + \mathbf{E}_t D_{t,t+1} \mu_{i,t+1} P_{i,t+1} \frac{\partial L_{i,t+1}}{\partial W_{i,t+1}} \tilde{\omega}_{t+1}^L \\
& + D_{t,t+1} \frac{\partial K_{i,t}}{\partial W_{i,t+1}} \mu_{i,t+1} P_{i,t+1} \left[ \tilde{\omega}_{t+1}^K + \frac{\partial L_{i,t+1}}{\partial K_{i,t}} \tilde{\omega}_{t+1}^L \right] \\
& + D_{t,t+2} \frac{\partial K_{i,t+1}}{\partial W_{i,t+1}} \mu_{i,t+2} P_{i,t+2} \left[ \tilde{\omega}_{t+2}^K + \frac{\partial L_{i,t+2}}{\partial K_{i,t+1}} \tilde{\omega}_{t+2}^L \right] \\
& = \mathbf{E}_t D_{t,t+1} \left\{ \mu_{i,t+1}^W - \frac{U_{C,t}}{P_t} \right\} L_{i,t+1}, \quad (7.9)
\end{aligned}$$

where  $\tilde{\omega}_t^L = (1 - \frac{1}{\epsilon}) F_{L,t} - \frac{\mu_{i,t}^W W_{i,t}}{\mu_{i,t} P_{i,t}}$  is the labour wedge and  $\tilde{\omega}_t^K = (1 - \frac{1}{\epsilon}) F_{K,t} - \frac{\mu_{i,t}^K R_{i,t}}{\mu_{i,t} P_{i,t}}$  is the capital wedge.

Using the expression  $\frac{\partial V_{i,t}}{\partial R_{i,t+1}} = \mathbf{E}_t D_{t,t+1} \frac{U_{C,t}}{P_t} K_{i,t}$ , the first-order conditions with regard to the return on capital  $R_{i,t+1}$  similarly yields

$$\begin{aligned}
& \mu_{i,t} P_{i,t} \frac{\partial L_{i,t}}{\partial R_{i,t+1}} \tilde{\omega}_t^L + \mathbf{E}_t D_{t,t+1} \mu_{i,t+1} P_{i,t+1} \frac{\partial L_{i,t+1}}{\partial R_{i,t+1}} \tilde{\omega}_{t+1}^L \\
& + D_{t,t+1} \frac{\partial K_{i,t}}{\partial R_{i,t+1}} \mu_{i,t+1} P_{i,t+1} \left[ \tilde{\omega}_{t+1}^K + \frac{\partial L_{i,t+1}}{\partial K_{i,t}} \tilde{\omega}_{t+1}^L \right] + \\
& D_{t,t+2} \frac{\partial K_{i,t+1}}{\partial R_{i,t+1}} \mu_{i,t+2} P_{i,t+2} \left[ \tilde{\omega}_{t+2}^K + \frac{\partial L_{i,t+2}}{\partial K_{i,t+1}} \tilde{\omega}_{t+2}^L \right] \\
& = \mathbf{E}_t D_{t,t+1} \left\{ \mu_{i,t+1}^K - \frac{U_{C,t}}{P_t} \right\} K_{i,t+1}. \quad (7.10)
\end{aligned}$$

The first two terms of the left-hand side equation 7.9 expresses the contemporaneous impact of changes the wage  $W_{i,t+1}$ . Changing the next period's wage the workers receive impacts their labour supply at times  $t$  and  $t + 1$ , thus impacting the overall wage payment the workers' enterprise has to make as well as the firm's output in these two periods. Because the increase in the firm output decreases its monopoly power, the overall impact on output is decreased by a factor of  $\frac{1}{\epsilon}$ . The third and fourth term of the left-hand side of equation 7.9 reflects the impact of changing  $W_{i,t+1}$  on the workers' saving decision. If changing  $W_{i,t+1}$  changes the capital accumulated at times  $t$  and  $t + 1$ , it would also change the periods  $t + 1$  and  $t + 2$  capital payments made by the firm as well as the these periods' production (again moderated by a factor  $\frac{1}{\epsilon}$  reflecting the firm's limited monopoly power). In addition, the workers' saving decision can impacts their future supply of labour. This has an impact on the wages payment by the firm and output in the periods  $t + 1$  and  $t + 2$ . The right-hand

side of equation 7.9 represents the effect of the wage on the workers' indirect utility and part of the impact of changing  $W_{i,t}$  on the current wage payment.

Similarly to the analysis presented above for the wage first-order condition, the right-hand side of equation 7.10 represents the impact of changing the capital payment rate  $R_{i,t+1}$  on the workers' indirect utility and part of the impact on the capital payments due to members. The first two term of the left-hand side of equation 7.10 reflects the impact of changes to the rate of capital payments operating through changes in the labour supplied by workers while the last two term represents the impact through changes in the workers' saving decisions. We present the steady-state characterization of the optimal solution below.

## 7.4 Steady State Results with Workers' Enterprise

Define the following factor wedges relative to the RBC-MC case.

**Definition 4. (Factor wedges relative to RBC-MC and relative wedge ratio  $A$ )** *Let  $\tilde{\omega}^L$  and  $\tilde{\omega}^K$  denote the wedges relative to the RBC-MC case. These are defined as follows*

- $\tilde{\omega}^L := (1 - \frac{1}{\epsilon}) F_L - \frac{W}{P}$  *the steady state marginal product of labour net of labour cost and corrected for the loss in sales resulting from the monopolistic competition effect*
- $\tilde{\omega}^K = (1 - \frac{1}{\epsilon}) F_K - \frac{R}{P}$  *the steady state marginal product of capital net of capital cost and corrected for the loss in sales resulting from the monopolistic competition effect*

*We define the steady state relative wedge ratio in factor markets  $A := \tilde{\omega}^K / \tilde{\omega}^L$ .*

The proposition below provides steady state results for labour and capital wedges when the economy is populated with WEs described in this section.

**Proposition 13. (WE Equilibrium)** *At the steady state, the workers' enterprise demand for factor shares is given by*

$$\begin{aligned} F_L - \frac{W}{P} &= \frac{1}{\epsilon} \frac{A F_L - F_K}{A + L/K}, \\ F_K - \frac{R}{P} &= \frac{1}{\epsilon} \frac{A^{-1} F_K - F_L}{A^{-1} + K/L}. \end{aligned}$$

Moreover, using the WE's first-order conditions at the steady state,  $A$  takes the expression below

$$A = -\frac{\partial L}{\partial K_{-1}} + \frac{\left\{ \frac{1}{L} \frac{\partial L}{\partial W_{+1}} - \frac{1}{K} \frac{\partial L}{\partial R_{+1}} \right\} + \beta \left\{ \frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R} \right\}}{\beta \left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\} + \beta^2 \left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}} \quad (7.11)$$

where  $\frac{\partial L}{\partial W}$  ( $\frac{\partial L}{\partial R}$ ) is the steady-state sensitivity of labour supply with respect to current wages (return on capital),  $\frac{\partial L}{\partial W_{+1}}$  ( $\frac{\partial L}{\partial R_{+1}}$ ) is the steady-state sensitivity of labour supply with respect to next period's wages (return on capital);  $\frac{\partial L}{\partial K_{-1}}$  is the steady-state sensitivity of labour supply with respect to previous period's capital. Capital sensitivity terms are defined similarly.

The variable  $A$  summarises the steady state equilibrium effects of factor supply sensitivities. We clarify this term further below.

## 7.5 Clarifying the term $A$

From Proposition 13, we know that

$$A = -\frac{\partial L}{\partial K_{-1}} + \frac{\left\{ \frac{1}{L} \frac{\partial L}{\partial W_{+1}} - \frac{1}{K} \frac{\partial L}{\partial R_{+1}} \right\} + \beta \left\{ \frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R} \right\}}{\beta \left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\} + \beta^2 \left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}} \quad (7.12)$$

We now clarify this expression further. We can rewrite the equation above as

$$\begin{aligned} & \frac{PY}{L} \left\{ \frac{\partial L}{\partial W_{+1}} \tilde{\omega}^L + \beta \frac{\partial L}{\partial W} \tilde{\omega}^L + \beta \frac{\partial K}{\partial W_{+1}} \left[ \tilde{\omega}^K + \frac{\partial L}{\partial K_{-1}} \tilde{\omega}^L \right] + \beta^2 \frac{\partial K}{\partial W} \left[ \tilde{\omega}^K + \frac{\partial L}{\partial K_{-1}} \tilde{\omega}^L \right] \right\} \\ &= \frac{PY}{K} \left\{ \frac{\partial L}{\partial R_{+1}} \tilde{\omega}^L + \beta \frac{\partial L}{\partial R} \tilde{\omega}^L + \beta \frac{\partial K}{\partial R_{+1}} \left[ \tilde{\omega}^K + \frac{\partial L}{\partial K_{-1}} \tilde{\omega}^L \right] + \beta^2 \frac{\partial K}{\partial R} \left[ \tilde{\omega}^K + \frac{\partial L}{\partial K_{-1}} \tilde{\omega}^L \right] \right\}. \end{aligned} \quad (7.13)$$

Equation 7.13 describes the equilibrium trade-off the workers' enterprise is facing when it decides to increase the labour share against increasing the capital share. In equilibrium, the firm's marginal gain from increasing the share of capital  $\xi^K$  (RHS) equals the marginal cost to the firm from decreasing the corresponding labour share  $1 - \xi^K$  (LHS). The term  $\frac{PY}{L}$  represents the increase of next period's wages when reducing the share of capital  $\xi^K$ .<sup>20</sup> The LHS term between braces represents change the firm's (real) resources from increasing next

<sup>20</sup>Remember that  $W_{t+1} = (1 - \xi_t^K) \frac{P_{t+1} Y_{t+1}}{L_{t+1}}$ .



period's wages:  $\frac{\partial L}{\partial W_{+1}}\tilde{\omega}^L$  represents the marginal impact of increasing next period's wages on current firm resources through the reaction of current labour supply;  $\beta\frac{\partial L}{\partial W}\tilde{\omega}^L$  represents the marginal impact of increasing next period's wages on next period's firm resources through the reaction of next period's labour supply; the term  $\beta\frac{\partial K}{\partial W_{+1}}\left[\tilde{\omega}^K + \frac{\partial L}{\partial K_{-1}}\tilde{\omega}^L\right]$  represents the impact on next period's firm resources through current savings by households as savings affect the capital available for production in the next period; finally the term  $\beta^2\frac{\partial K}{\partial W}\left[\tilde{\omega}^K + \frac{\partial L}{\partial K_{-1}}\tilde{\omega}^L\right]$  represents the marginal impact of the next period wage through savings in the next period. Similarly, the RHS of 7.13 represents the marginal product on the firm resources of changing the capital share. The term  $\frac{PY}{K}$  in the LHS of 7.13 corresponds to the change of next period's returns on capital  $R_{+1}$  when changing the share of capital  $\xi^K$  and the term between braces is the marginal change in the firm's resources when changing the next period's return on capital.

## 7.6 Deriving $A$ when Utility is Additive

Proposition 13 underlies that the steady state variable  $A$  is key in determining the steady state demand for production factors. The following proposition provides an  $A$  expression for additively separable utility functions.

**Proposition 14.** *Assume that the utility function takes the form*

$$U(C_t, L_t) = u(C_t) - v(L_t),$$

with  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$  and  $v'' > 0$ , then

$$A = \frac{-u''(C)}{\beta\{v''(L) - W^2u''(C)\}} \left[ -\beta W + \frac{\frac{L}{K}\{v''(L) - W^2u''(C)\} + (1 + \beta)Wu''(C)}{v''(L)} \right].$$

In Proposition 14,  $A$  is expressed as a function of the households' utility and the wage. However, using the labour supply first-order condition, one can substitute for the wage using the marginal utility of consumption and the marginal disutility of labour ( $W = v'(L)/u'(C)$ ), thus making  $A$  a function of the workers' utility and its derivatives at the equilibrium consumption and labour.

## 7.7 Steady-State Numerical Results with Workers' Enterprise

We present the steady state results when the firm arrangement corresponds to the workers' enterprise and contrast it against other arrangements studied in Sections 4 and 5 in the

paper. The model's parameters are otherwise the same as presented in Table 1 in the paper.

Figure 5 shows the steady state capital, output, labour, consumption, capital share and welfare under the WE arrangement for different levels of the parameter  $\epsilon$  and contrasts these steady state variables against their values under the RBC-MC and the ES arrangements. The figure shows that the steady state equilibria under the WE arrangement are almost identical to equilibria under the ES firm. With extra flexibility in labour payment, the WE outperforms the ES in welfare terms, but only slightly. Essentially, the WE chooses to behave as if labour was outside the organisation, thus exploiting labour (to an even greater extent than under ES), while pushing the share of capital higher than in the under the RBC model. We discussed the intuition in the initial part of the section.

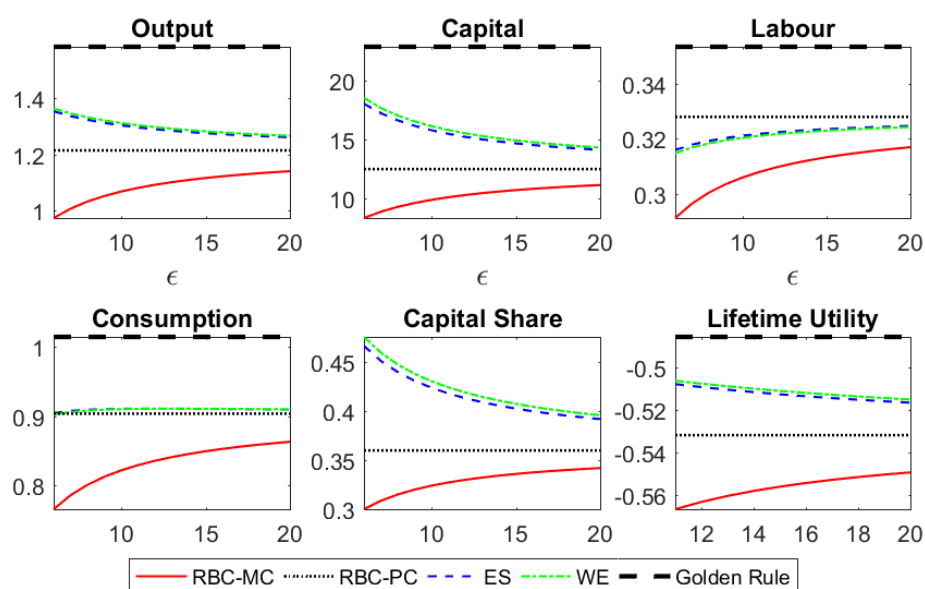


Figure 5: The steady state output, capital, labour, consumption, capital share and welfare under RBC-MC (red, continuous line), ES (blue, dashed line) and WE arrangements (green, continuous-dashed line) are presented for various values of the parameter  $\epsilon$ . The RBC-PC (black, dotted line) and the Golden Rule (black, thick dashed line) outcomes are also shown as reference.

## 8 Conclusion

In a backdrop of significant market power for firms and consequent welfare distortions, questions about the institutional structure of the firm and its objectives have assumed salience. Our contribution is to embed alternative institutional structures in an otherwise standard dynamic general equilibrium macroeconomic model with monopolistic competition, characterise novel outcomes and demonstrate that institutional arrangements can be a potent channel for mitigating distortions arising from market power.

Specifically, we consider the efficacy of zero-incentive-leakage structures. The perfect competition RBC model is an example of such a structure, given the absence of market power. Our approach preserves monopoly, but eliminates leakage through alternative ownership and remuneration structures. Specifically, zero leakage is achieved by disbursing the entire firm revenue net of any non-tied factor costs as payment to tied-factor providers. The latter are remunerated in proportion to contribution, and the firm aggregates tied-factor provision in a decentralised manner.

The proportional remuneration scheme generates extra competition among agents to supply the tied factor(s). This results in a within-firm negative externality so that tied factors are oversupplied and therefore output is higher relative to the joint optimum of firm owners. The outcome is as if the firm under-exploits its monopoly power. Firm-level overproduction results in a positive externality on other firms. Thus our decentralised zero-leakage schemes effectively internalise an aggregate demand externality. The ownership and remuneration structures that engender such efficacious properties are stable as individual actions are incentive compatible.

Our results show that harnessing monopoly profits as part of such a remuneration structure generates extra incentives not available under perfect competition. We propose decentralised zero-leakage institutional forms with either capital or labour as the tied factor (ES and EW, respectively). Overall, our paper takes the first step towards analysing the macroeconomics of firms without incentive leakage and clarifying the significant implications of this feature for aggregate welfare. Beyond specific structures, the broader idea is to show the role of remuneration structure in changing firm behaviour, complementing the literature on changing firm objectives discussed in the introduction.

To consider welfare implications of the proposed zero-leakage arrangements, we rely on comparing steady-state outcomes, with the benchmark given by the Golden Rule. We show that

EW closes part of the welfare gap between RBC-PC and RBC-MC arising from monopoly, while ES not only eliminates this gap, but also eliminates part of the patience-based welfare gap between RBC-PC and the Golden Rule, thus outperforming even perfect competition in the steady state. We show formally that the welfare ranking is related to capital accumulation incentives across structures. Importantly, the ES structure relies on the monopoly gap to generate extra incentives to save compared to perfect competition. In other words, it is the presence of monopoly power that allows setting up incentives that improve welfare beyond perfect competition in the steady state.

The results also serve to highlight the contrast between managerial and decentralised outcomes, and indicate that eliminating even part of the incentive leakage by moving partially towards decentralised decision making can enhance welfare. They also suggest that using tax policies to target firm structure, for example through preferential tax treatment of firms that link profits to factor provision incentives, might have a more desirable long-run impact compared to competition policies seeking to limit monopoly power.

We conclude by noting some limitations of our model. We assign agents to firms to define ownership, where owners are residual claimants after paying non-tied factors. However, the assignment is detail-free: we do not spell out any precise mechanism for firms to hire members. We also abstract from the use of external finance. This precludes questions about the role of a capital market in diversifying firm-specific risk, or the role of a labour market in mitigating productivity misallocation. Such questions would be especially relevant in a model with some degree of heterogeneity or with factor market frictions. Here we take a first step towards analysing the macroeconomics of zero-leakage firms. We hope to address a more complete set of issues under heterogeneous agents in future work.

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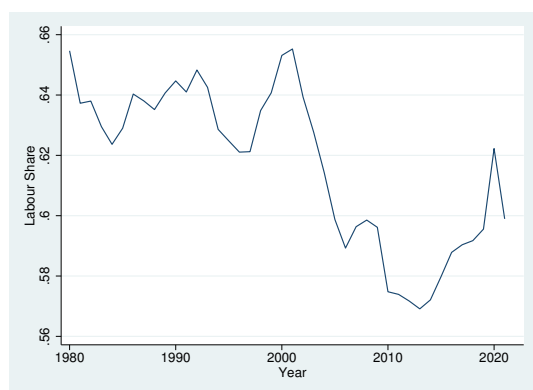
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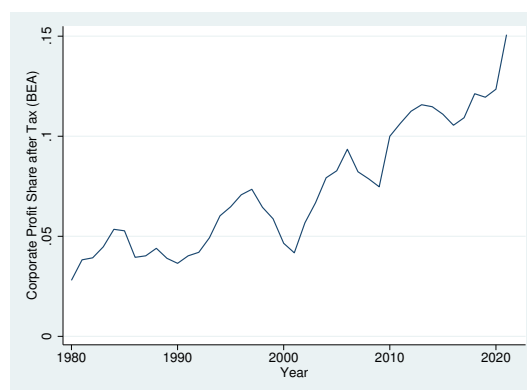
## A Rising Profits and Distorted Factor Shares: Data and Computations

### A.1 Empirical Evidence

In Figure 6a, we plot the corporate sector labour share in GVA as measured by the employee compensation and in Figure 6b the share of corporate profits net of taxes and subsidies with inventory valuation adjustment (IVA) and capital consumption adjustment (CCAdj) as a fraction of gross value added (GVA). The profit share in GVA, shows an increase from around 3% in 1980 to almost 13% by 2020 and labour share in GVA has declined from around 66% to roughly 60%. Computation of capital share in GVA is more complex as profits are attributed to capital share in NIPA Table 1.14 (net operating surplus). However, an implicit measure for capital share can be developed that accounts for the cost of capital including the opportunity cost associated with the equity. Recent work by Barkai (2020) indeed suggests that the conceptual capital share in GVA was also declining in recent decades in conjunction with a rise in the measure of pure profit share as computed by the difference between nominal gross value added against the employee, capital compensation and indirect taxes.



(a) Labour Share in GVA  
Source: Bureau of Economic Analysis

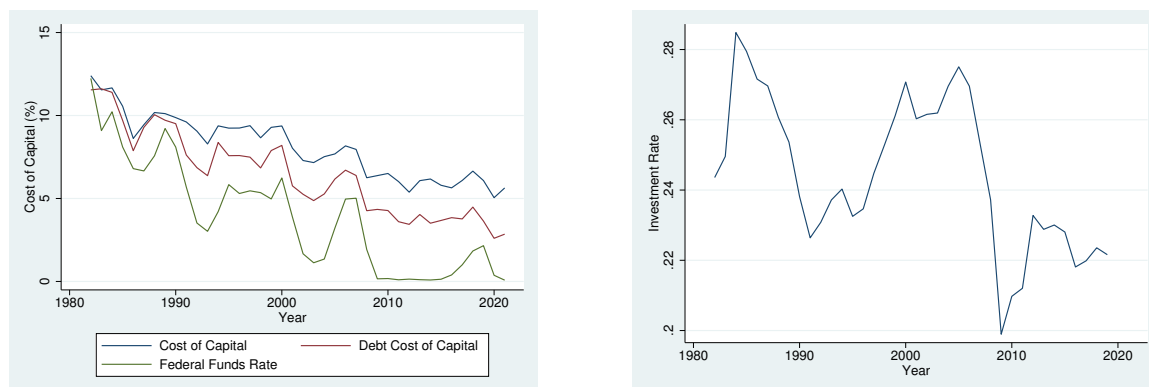


(b) Corporate Profit Share (with IVA and CCAdj) in GVA

Next, we plot, in Figure 7a and , our computation of the cost of capital that closely follows the definitions provided by Barkai (2020) for required rate of return and its components, debt and equity cost of capital together with the evolution of the Federal Funds rate (FFR). It is clear that in line with the riskless rates the debt costs to finance capital investment has steadily declined over the course of four decades from around 13% to about 3%. the natural question is whether this decline in investment financing costs led to an increase in the investment share. We plot in 7b the share of gross capital formation at PPP as reported



by the Penn World Tables. We note the marked decline in investment rate from around 27% pre Global Financial Crisis to roughly 23% post Global Financial Crisis despite increased profit shares, declining costs of investment finance and increasing capital wedge. Favourable profitability and financing conditions do not translate into capital investment.



(a) Required Rate of Return and Debt Cost of Capital (b) Share of Gross Capital Formation at PPP  
a). Cost of capital computations follow Barkai (2020). Representative debt cost of capital is computed as the weighted average of costs of financing via commercial paper, corporate bonds and loans. Equity cost of capital is the yield on 10 years government bonds plus 5%. Cost of capital is the average of debt and equity cost of capital treating for the corporate income tax rate. b). Share of Gross Capital Formation at PPP (Penn World Tables)

## A.2 Data

### • National Accounts

- Gross value added of nonfinancial corporate business: from line 17 in NIPA Table 1.14.
- Compensation of employees: from line 20 in NIPA Table 1.14.
- Taxes on production and imports less subsidies: from line 23 in NIPA Table 1.14.
- Profit per unit of real gross value added of nonfinancial corporate business: U.S. Bureau of Economic Analysis, Profit per unit of real gross value added of nonfinancial corporate business: Corporate profits after tax with IVA and CCAdj (unit profits from current production) [A466RD3Q052SBEA], retrieved from FRED on September 5, 2022.<sup>21</sup>

<sup>21</sup>Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/A466RD3Q052SBEA>

- Corporate income tax rate and capital allowances: Corporate tax rate from the Organisation for Economic Cooperation and Development (OECD) Tax Database and capital allowance from the Tax Foundation.
- Cost of Capital
  - Corporate bond rate: Moody’s Seasoned Aaa Corporate Bond Yield [AAA], retrieved from FRED September 4, 2022.<sup>22</sup>
  - Corporate commercial paper rate: 90-Day AA Nonfinancial Commercial Paper Interest Rate [DCPN3M], retrieved from FRED on September 4, 2022.<sup>23</sup>
  - Corporate loans rate: Bank Prime Loan Rate [MPRIME], retrieved from FRED on September 4, 2022.<sup>24</sup>
  - Open market paper, municipal securities, corporate bonds, loans, corporate equities are all from Bureau of Economic Analysis, Integrated Macroeconomic Accounts Table S5a, Nonfinancial Corporate Business
  - Yield on the 10-year U.S. Treasury: Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis [DGS10], retrieved from FRED on September 4, 2022.<sup>25</sup>
  - Expected inflation ( $px - md$ ): Median expected price change next 12 months from University of Michigan, Surveys of Consumers

Cost of capital computations follow [Barkai \(2020\)](#). Representative debt cost of capital is computed as the weighted average of costs of financing via commercial paper, corporate bonds and loans. Equity cost of capital is the yield on 10 years government bonds plus 5%. Cost of capital is the average of debt and equity cost of capital treating for the corporate income tax rate. b). Capital wedge is computed as the S&P 500 real 10 years annualized stock returns (from [Shiller \(2015\)](#)) net of real debt costs of capital.

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<sup>22</sup>Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/AAA>

<sup>23</sup>Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/DCPN3M>

<sup>24</sup>Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/MPRIME>

<sup>25</sup>Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/DGS10>

## B Proofs

### B.1 Proof of Proposition 2

The Lagrangian of the firm-level planner problem is

$$\begin{aligned}\mathcal{L}_{i,t} = & \sum_{s \geq t} \mathbf{E}_t \beta^{s-t} \left[ U(C_{i,s}, L_{i,s}) \right. \\ & \left. + \lambda_{i,s} (P_{i,s} Y_{i,s} + (1 - \delta) P_t K_{i,s-1} - P_s C_{i,s} - P_s K_{i,s}) + \mu_{i,s} \left( \left( \frac{P_{i,s}}{P_s} \right)^{-\epsilon} Y_s - Y_{i,s} \right) \right]\end{aligned}$$

The first-order condition with regard to the sale price  $P_{i,t}$ , consumption  $C_{i,t}$ , labour  $L_{i,t}$  and capital  $K_{i,t+1}$  yield, respectively

$$\begin{aligned}\lambda_{i,t} P_{i,t} &= \epsilon \mu_{i,t}, \\ U_{C,i,t} &= P_t \lambda_{i,t}, \\ U_{L,i,t} + \lambda_{i,t} P_{i,t} F_{L,i,t} - \mu_{i,t} F_{L,i,t} &= 0, \\ \lambda_{i,t} P_t &= \beta \mathbf{E}_t [\lambda_{i,t+1} P_{i,t+1} F_{K,i,t+1} + \lambda_{i,t+1} P_{t+1} (1 - \delta) - \mu_{i,t} F_{K,i,t+1}].\end{aligned}$$

Noting that all firms are identical, and by using the model closing assumption, we can now drop the firm index and replace for all sale prices  $P_{i,t} = 1$ . Combination of first-order conditions yields firm-level planner outcomes described in 3.4 and 3.5.

### B.2 Proof of Proposition 5

Consider the firm's problem is  $\max_{P_{i,s}, L_{i,s}} \mathcal{L}_{i,t}^F$ , with

$$\mathcal{L}_{i,t}^F = V_{i,t} + \mathbf{E}_t \sum_{s \geq t} \beta^{s-t} \mu_{i,s}^F \left\{ \left( \frac{P_{i,s}}{P_s} \right)^{-\epsilon} Y_s - Y_{i,s} \right\}.$$

where the return on capital is replaced in  $V_{i,t}$  using  $R_{i,t} = (P_{i,t} Y_{i,t} - W_t L_{i,t}) / K_{i,t-1}$ , implying  $\frac{\partial R_{i,t}}{\partial P_{i,t}} = \frac{Y_{i,t}}{K_{i,t-1}}$  and  $\frac{\partial R_{i,t}}{\partial L_{i,t}} = \frac{1}{K_{i,t-1}} (P_{i,t} F_{L,i,t} - W_t)$ .

The first-order conditions with regard to  $P_{i,t}$  and  $L_{i,t}$  are respectively

$$\begin{aligned}\frac{\epsilon \mu_{i,t}^F}{P_{i,s}} &= \frac{1}{K_{i,t-1}} \frac{\partial V_{i,t}}{\partial R_{i,t}}, \\ \mu_{i,t}^F F_{L,i,t} &= \frac{1}{K_{i,t-1}} \frac{\partial V_{i,t}}{\partial R_{i,t}} (P_{i,t} F_{L,i,t} - W_t).\end{aligned}$$

Combining the price and labour demand first-order conditions yields  $\frac{W_t}{P_t} = (1 - \frac{1}{\epsilon}) F_{L,t}$ , where we dropped the index  $i$  because all firms are symmetrical. Given the zero-leakage binding budget constraint and the symmetry of all firms, the return on capital is  $R_t = \frac{P_t}{K_{t-1}} \left\{ Y_t - \frac{W_t}{P_t} L_t \right\}$ . Using the labour first-order condition,

$$\frac{R_t}{P_t} = \frac{1}{K_{t-1}} \left\{ Y_t - \left(1 - \frac{1}{\epsilon}\right) F_{L,t} L_t \right\}.$$

Given that the technology displays constant return to scale, which implies  $Y_t = F_{L,t} L_t + F_{K,t} K_{t-1}$ , one can rewrite the binding budget constraint as follows

$$\frac{R_t}{P_t} = F_{K,t} + \frac{1}{\epsilon} F_{L,t} \frac{L_t}{K_{t-1}} > F_{K,t},$$

The remaining results of Proposition 5 follow by using the expression above combined with the household's first order conditions.

### B.3 Proof of Proposition 6

Suppose, on the contrary, that  $K_t^{ES} \leq K_t^{RBC-MC}$ . Now,  $K_t^{RBC-MC}$  is given implicitly by the solution to 2.11, while  $K_t^{ES}$  satisfies 4.10. Since  $K_t^{ES} \leq K_t^{RBC-MC}$ ,  $F_{K,t+1}(K = K_t^{ES}) \geq F_{K,t+1}(K = K_t^{RBC-MC})$ . This implies

$$\begin{aligned} & \mathbf{E}_t D_{t,t+1} \left\{ 1 - \delta + F_{K,t+1} + \epsilon^{-1} F_{L,t+1} L_{t+1} / K_t \right\} \quad \text{with } K = K_t^{ES} \\ & > \mathbf{E}_t D_{t,t+1} \left\{ 1 - \delta + F_{K,t+1} \right\} \quad \text{with } K = K_t^{ES} \\ & > \mathbf{E}_t D_{t,t+1} \left\{ 1 - \delta + F_{K,t+1} \right\} \quad \text{with } K = K_t^{RBC-MC} \\ & = 1 \end{aligned}$$

Note that the first inequality above follows from the fact that  $\epsilon^{-1} F_{L,t+1} L_{t+1} / K_t > 0$ . In other words, the inequality holds irrespective of the level of  $L_t$ . The above implies Equation 4.10 is violated as the right hand side is shown above to strictly exceed 1. Hence we have a contradiction. This proves that  $K_t^{ES} > K_t^{RBC-MC}$ .

### B.4 Proof of Proposition 7

The Lagrangian for the household's problem is given by

$$\mathcal{L}_{i,t}^H = \mathbf{E}_t \sum_{s \geq t} \beta^{u-t} \left[ U(C_{i,s}, L_{i,s}) + \mu_{i,t}^H (W_s L_{i,s} + R_{i,s} K_{i,s-1} - P_s C_{i,s} - P_s \{K_{i,s} - (1 - \delta) K_{i,s-1}\}) \right]$$

The first-order condition with regard to consumption yields

$$\mu_{i,t}^H = \frac{1}{P_t} \frac{\partial U}{\partial C_{i,t}}. \quad (\text{B.1})$$

The envelope theorem applied to the household's maximisation problem yields  $\frac{\partial V_{i,t}}{\partial R_{i,t}} = \mu_{i,t}^H K_{i,t-1}$ . Combining with B.1,  $\frac{\partial V_{i,t}}{\partial R_{i,t}} = \frac{K_{i,t-1}}{P_t} \frac{\partial U}{\partial C_{i,t}} > 0$ . The household's share of the surplus is proportional to their share of capital  $S_{i,t} = R_{i,t} K_{i,t-1}$ . This implies  $\frac{\partial V_{i,t}}{\partial S_{i,t}} = \frac{1}{K_{i,t-1}} \frac{\partial V_{i,t}}{\partial R_{i,t}}$ . Thus

$$\frac{\partial V_{i,t}}{\partial S_{i,t}} = \frac{U_C(C_{i,t}, L_{i,t})}{P_t}, \quad (\text{B.2})$$

which implies that  $\frac{\partial V_{i,t}}{\partial S_{i,t}} > 0$  as long as  $K_{i,t-1} > 0$ . The static nature of the ES firm's problem implies that maximising the indirect utility is equivalent to maximising surplus.

## B.5 Proof of Proposition 8

Consider the firm's problem is  $\max_{K_{i,s}^R, P_{i,s}} \mathcal{L}_{i,t}^F$ , with

$$\mathcal{L}_{i,t}^F = V_{i,t} + \mathbf{E}_t \sum_{s \geq t} \beta^{s-t} \mu_{i,s}^F \left\{ \left( \frac{P_{i,s}}{P_s} \right)^{-\epsilon} Y_s - Y_{i,s} \right\}.$$

where the wage is replaced in  $V_{i,t}$  using  $W_{i,t} = (P_{i,t} Y_{i,t} - R_t K_{i,t}^R) / L_{i,t}$ , implying  $\frac{\partial W_{i,t}}{\partial P_{i,t}} = \frac{Y_{i,t}}{L_{i,t}}$  and  $\frac{\partial W_{i,t}}{\partial K_{i,t}^R} = \frac{1}{L_{i,t}} (P_{i,t} F_{K,i,t} - R_t)$ . The first-order conditions with regard to  $P_{i,t}$  and  $K_{i,s}^R$  are respectively

$$\begin{aligned} \frac{\epsilon \mu_{i,t}^F}{P_{i,s}} &= \frac{1}{L_{i,t}} \frac{\partial V_{i,t}}{\partial W_{i,t}}, \\ \mu_{i,t}^F F_{K,i,t} &= \frac{1}{L_{i,t}} \frac{\partial V_{i,t}}{\partial W_{i,t}} (P_{i,t} F_{K,i,t} - R_t). \end{aligned}$$

Combining the price and capital demand first-order conditions and exploiting symmetry to drop the index  $i$  yields  $\frac{R_t}{P_t} = (1 - \frac{1}{\epsilon}) F_{K,t}$ . Replacing for  $R_t$  in the firm's binding budget constraint using the capital first-order condition, we obtain an expression for the real wage

$$\frac{W_t}{P_t} = \frac{1}{L_t} \left\{ Y_t - \left( 1 - \frac{1}{\epsilon} \right) F_{K,t} K_{t-1} \right\}.$$

Given that the technology displays constant return to scale, which implies  $Y_t = F_{L,t} L_t + F_{K,t} K_t$ , one can rewrite the binding budget constraint as follows

$$\frac{W_t}{P_t} = F_{L,t} + \frac{1}{\epsilon} F_{K,t} \frac{K_{t-1}}{L_t} > F_{L,t},$$

The remaining results of Proposition 8 follow by using the expression above combined with the household's first order conditions.

## B.6 Proof of Proposition 9

The Lagrangian for the household's problem is given by

$$\mathcal{L}_{i,t}^H = \mathbf{E}_t \sum_{s \geq t} \beta^{u-t} [U(C_{i,s}, L_{i,s}) + \mu_{i,t}^H (W_{i,s} L_{i,s} + R_s K_{i,s-1} - P_s C_{i,s} - P_s \{K_{i,s} - (1 - \delta) K_{i,s-1}\})]$$

The first-order condition with regard to consumption yields

$$\mu_{i,t}^H = \frac{1}{P_t} \frac{\partial U}{\partial C_{i,t}}. \quad (\text{B.3})$$

The envelope theorem applied to the household's maximisation problem yields  $\frac{\partial V_{i,t}}{\partial W_{i,t}} = \mu_{i,t}^H L_{i,t}$ . Combining with B.3, we get  $\frac{\partial V_{i,t}}{\partial W_{i,t}} = \frac{L_{i,t}}{P_t} \frac{\partial U}{\partial C_{i,t}} > 0$ . The household's share of the surplus is proportional to their labour supply  $S_{i,t} = W_{i,t} L_{i,t}$ . This implies

$$\frac{\partial S_{i,t}}{\partial W_{i,t}} = \frac{\partial (W_{i,t} L_{i,t})}{\partial W_{i,t}} = L_{i,t} + W_{i,t} \frac{\partial L_{i,t}}{\partial W_{i,t}}.$$

The assumption  $\frac{W_{i,t}}{L_{i,t}} \frac{\partial L_{i,t}}{\partial W_{i,t}} > -1$  yields

$$\frac{\partial V_{i,t}}{\partial S_{i,t}} = \frac{\partial V_{i,t}}{\partial W_{i,t}} \frac{\partial W_{i,t}}{\partial S_{i,t}} = \frac{U_C(C_{i,t}, L_{i,t})}{P_t} \frac{1}{1 + \frac{W_{i,t}}{L_{i,t}} \frac{\partial L_{i,t}}{\partial W_{i,t}}} > 0, \quad (\text{B.4})$$

The static nature of the ES firm's problem implies that maximising the indirect utility is equivalent to maximising surplus.

## B.7 Proof of Proposition 10

From the model closing assumption  $P = 1$ . The Euler equation yields, in the steady state (SS),  $r = 1/\beta + \delta - 1$ . The technology in SS is  $Y = K^a L^{1-a}$ . The first order condition for capital yields

$$L = \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta - 1 + \delta}{a} \right]^{\frac{1}{1-a}} K, \quad (\text{B.5})$$

or

$$Y = \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta - 1 + \delta}{a} K. \quad (\text{B.6})$$

This and the firm's labour demand FOC pin down the SS wages

$$W = (1 - a) \left(1 - \frac{1}{\epsilon}\right)^{\frac{1}{1-a}} \left(\frac{1/\beta - 1 + \delta}{a}\right)^{\frac{-a}{1-a}}. \quad (\text{B.7})$$

From the final good clearing equation

$$C = Y - \delta K = \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta - 1 + \delta}{a} - \delta \right] K \quad (\text{B.8})$$

Given the utility function, the marginal utilities are  $U_C(C_t, L_t) = C_t^{-\sigma}$  and  $U_L(C_t, L_t) = -\chi L_t^\eta$ . The labour supply equation becomes  $\chi L_t^\eta C_t^\sigma = W_t$ . In the steady-state  $\chi L^\eta C^\sigma = W$ . Plugging the last equations expressing SS labour, SS consumption and SS wages into the SS labour provision equation yields SS capital as a function of the model's parameters and rearranging

$$K = \left( \frac{(1-a) \left(1 - \frac{1}{\epsilon}\right)^{\frac{1+\eta}{1-a}} \left(\frac{1/\beta - 1 + \delta}{a}\right)^{\frac{-a-\eta}{1-a}}}{\chi \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta - 1 + \delta}{a} - \delta \right]^\sigma} \right)^{\frac{1}{\eta+\sigma}}.$$

This shows that the steady-state capital increases with  $\beta$  for  $0 < a < 1$  and  $\eta > 0$ . From [B.6](#), the steady-state output is

$$Y = \left( \frac{(1-a) \left(1 - \frac{1}{\epsilon}\right)^{\frac{a+\eta}{1-a}} \left(\frac{1/\beta - 1 + \delta}{a}\right)^{-\eta \frac{1+a}{1-a}}}{\chi \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} - \delta \left(\frac{1/\beta - 1 + \delta}{a}\right)^{-1} \right]^\sigma} \right)^{\frac{1}{\eta+\sigma}}.$$

The last expression shows that the steady-state output is increasing in  $\beta$ . From expression [B.8](#), the steady-state consumption is

$$C = \left( \frac{1-a}{\chi} \left(1 - \frac{1}{\epsilon}\right)^{\frac{1+\eta}{1-a}} \left(\frac{1/\beta - 1 + \delta}{a}\right)^{\frac{-a-\eta}{1-a}} \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta - 1 + \delta}{a} - \delta \right]^\eta \right)^{\frac{1}{\eta+\sigma}}.$$

Note  $X := \frac{1/\beta - 1 + \delta}{a}$  and  $h$  the function  $h(X) := X^{\frac{-a-\eta}{1-a}} \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} X - \delta \right]^\eta$ . It is immediate that the steady-state consumption is increasing in  $\beta$  if and only if  $h$  is decreasing in  $X$ . In addition,  $h'(X) < 0$  iff  $\delta \frac{a+\eta}{1-a} - \left(1 - \frac{1}{\epsilon}\right)^{-1} \left(\frac{a+\eta}{1-a} - \eta\right) X < 0$ , which is verified as  $X > \delta/a$ . This means that steady-state consumption is increasing in  $\beta$ .

Finally, from expression [B.5](#), the steady-state labour is

$$L = \left( \frac{(1-a) \left(1 - \frac{1}{\epsilon}\right)^{\frac{1+\eta}{1-a}} \left(\frac{1/\beta - 1 + \delta}{a}\right)^{\frac{\sigma-a}{1-a}}}{\chi \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta - 1 + \delta}{a} - \delta \right]^\sigma} \right)^{\frac{1}{\eta+\sigma}}.$$

Steady-state labour is not monotonic in  $\beta$  for all values of the other parameters. Note  $g$  the function  $g(X) := X^{\frac{\sigma-a}{1-a}} \left[ \left(1 - \frac{1}{\epsilon}\right)^{-1} X - \delta \right]^{-\sigma}$ . From the labour supply condition

$$L^\eta = C^{-\sigma} W = C^{-\sigma} a \frac{Y}{L} = C^{1-\sigma} a \frac{Y}{CL}.$$

Rearrange

$$\frac{L^{1+\eta}}{1+\eta} = \frac{C^{1-\sigma}}{1-\sigma} a \frac{1-\sigma}{1+\eta} \frac{Y}{C} = \frac{C^{1-\sigma}}{1-\sigma} a \frac{1-\sigma}{1+\eta} \frac{\left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta-1+\delta}{a}}{\left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta-1+\delta}{a} - \delta}.$$

This leads to the utility expression

$$U = \frac{C^{1-\sigma}}{1-\sigma} \left[ 1 - a \frac{1-\sigma}{1+\eta} \frac{\left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta-1+\delta}{a}}{\left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta-1+\delta}{a} - \delta} \right].$$

This time note  $X := \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{1/\beta-1+\delta}{a}$ . Steady-state utility is increasing in  $\beta$  iff the function  $g(X) := X^{\frac{-a-\eta}{1-a} \frac{1-\sigma}{\eta+\sigma}} (X - \delta)^{\frac{\eta(1-\sigma)}{\eta+\sigma}} \left[ 1 - a \frac{1-\sigma}{1+\eta} \frac{X}{X-\delta} \right]$  is decreasing in  $X$ . Noting  $\gamma := \frac{a+\eta}{1-a} > a, \eta$  and  $\psi := \frac{1-\sigma}{1+\eta} < 1, 1/\eta$  enables us to simplify the expression of  $g$

$$g(X) = X^{-\gamma\psi} (X - \delta)^{\eta\psi} \left[ 1 - a\psi \frac{X}{X-\delta} \right] = X^{-\gamma\psi} (X - \delta)^{\eta\psi} - a\psi X^{-\gamma\psi+1} (X - \delta)^{\eta\psi-1}.$$

First note that the term  $X^{-\gamma\psi} (X - \delta)^{\eta\psi}$  is decreasing in  $X$  (because  $C$  is increasing in  $\beta$ ). Then note that when  $\sigma \geq 1$ , the constant  $\psi$  is non-positive and the term  $\left[ 1 - a\psi \frac{X}{X-\delta} \right]$  is (weakly) decreasing in  $X$ , which means that  $g$  is also decreasing in  $X$ . We now need to consider the case  $\sigma < 1$  (or  $\psi > 0$ ), which we assume going forward.

$$\frac{g'(X)}{\psi X^{-\gamma\psi-1} (X - \delta)^{\eta\psi-2}} = -\gamma(X - \delta)^2 + \eta X(X - \delta) - a(-\gamma\psi + 1)X(X - \delta) - a(\eta\psi - 1)X^2.$$

We now need to study the sign of the function quadratic  $h(X) := \frac{g'(X)}{\psi X^{-\gamma\psi-1} (X - \delta)^{\eta\psi-2}}$ . Through some algebra we show that  $h(0) = -\gamma\delta^2 < 0$ ,  $h(\delta) = a(1 - \eta\psi)\delta^2 > 0$  and  $h(\delta/a) = -a[1 - a + a(1 - \psi)](\delta/a)^2 < 0$ . These inequalities and the quadratic nature of  $h$  imply that  $h$  is negative for all  $X > \delta/a$ .  $X$  is greater than  $\delta/a$  for all admissible values of  $\beta$ . This concludes the proof that steady-state utility is increasing in  $\beta$  for the more general case under competitive monopoly.

## B.8 Proof of Proposition 11

We define the steady-state capital-output ration  $K/Y \equiv \zeta$ , and propose to show that, for zero-leakage arrangements, steady-state utility is maximised for some  $\zeta$  between  $\frac{\alpha}{1/\beta-1+\delta}$  and  $\frac{\alpha}{\delta}$ .

Replacing for  $Y$  using the Cobb-Douglas technology  $Y = K^\alpha L^{1-\alpha}$

$$L = \left( \frac{1}{\zeta} \right)^{\frac{1}{1-\alpha}} K. \quad (\text{B.9})$$



From the zero-incentive leakage condition and the definition of  $\zeta$

$$WL = Y - RK = \left[ \frac{1}{\zeta} - R \right] K, \quad (\text{B.10})$$

where the notation  $R := 1/\beta - 1 + \delta$  is adopted for simplicity, despite the potential absence of markets for capital in the economy under study in this proposition. Clearly for positive consumption, we must have  $1/\zeta > R > \delta$ .

Plugging [B.9](#) in the above,

$$W = \left[ \frac{1}{\zeta} - R \right] \zeta^{\frac{1}{1-\alpha}}. \quad (\text{B.11})$$

From the resource constraint condition

$$C = Y - \delta K = \left[ \frac{1}{\zeta} - \delta \right] K. \quad (\text{B.12})$$

The labour supply condition is  $\chi C^\sigma L^\eta = W$ . Replace for  $W$  using [B.11](#), for  $L$  using [B.9](#) and for  $C$  using [B.12](#)

$$K^{\eta+\sigma} = \frac{1}{\chi} \left[ \frac{1}{\zeta} - \delta \right]^{-\sigma} \left[ \frac{1}{\zeta} - R \right] \zeta^{\frac{1+\eta}{1-\alpha}}.$$

Using [B.12](#), we retrieve the steady-state consumption

$$C^{\eta+\sigma} = \frac{1}{\chi} \left[ \frac{1}{\zeta} - \delta \right]^\eta \left[ \frac{1}{\zeta} - R \right] \zeta^{\frac{1+\eta}{1-\alpha}}. \quad (\text{B.13})$$

Now, note that from the labour supply condition  $\chi L^{1+\eta} = \frac{WL}{C} C^{1-\sigma}$ . Replace for  $WL$  using [B.10](#) and for  $C$  using [B.12](#)  $\chi L^{1+\eta} = \frac{\frac{1}{\zeta} - R}{\frac{1}{\zeta} - \delta} C^{1-\sigma}$ . The latter expression enables us to rewrite the steady-state utility as a function of consumption

$$U = \left[ \frac{1}{1-\sigma} - \frac{1}{1+\eta} \frac{\frac{1}{\zeta} - R}{\frac{1}{\zeta} - \delta} \right] C^{1-\sigma}.$$

Finally, an expression for steady-state utility obtains by replacing for  $C$  using [B.13](#)

$$U = \left( \frac{1}{\chi} \right)^{\frac{1-\sigma}{\eta+\sigma}} \left[ \frac{1}{1-\sigma} - \frac{1}{1+\eta} \frac{\frac{1}{\zeta} - R}{\frac{1}{\zeta} - \delta} \right] \left[ \frac{1}{\zeta} - \delta \right]^{\eta \frac{1-\sigma}{\eta+\sigma}} \left[ \frac{1}{\zeta} - R \right]^{\frac{1-\sigma}{\eta+\sigma}} \zeta^{\frac{1+\eta}{1-\alpha} \frac{1-\sigma}{\eta+\sigma}},$$

where we assume  $\sigma \neq 1$  (the case  $\sigma = 1$  will be studied separately below). We need to study the monotonicity of the function

$$g(X) := \left[ \frac{1}{1-\sigma} - \frac{1}{1+\eta} \frac{X-R}{X-\delta} \right] [X-\delta]^{\eta \frac{1-\sigma}{\eta+\sigma}} [X-R]^{\frac{1-\sigma}{\eta+\sigma}} X^{-\frac{1+\eta}{1-\alpha} \frac{1-\sigma}{\eta+\sigma}},$$

where the variable  $X$  replaces  $1/\zeta$ , with  $X > R > \delta$

$$\begin{aligned} \frac{g'(X)}{g(X)} = & -\frac{1}{1+\eta} \frac{R-\delta}{(X-\delta)^2} \left[ \frac{1}{1-\sigma} - \frac{1}{1+\eta} \frac{X-R}{X-\delta} \right]^{-1} \\ & + \eta \frac{1-\sigma}{\eta+\sigma} [X-\delta]^{-1} + \frac{1-\sigma}{\eta+\sigma} [X-R]^{-1} - \frac{1+\eta}{1-\alpha} \frac{1-\sigma}{\eta+\sigma} X^{-1}. \end{aligned}$$

Rearrange

$$\frac{\eta+\sigma}{(1-\sigma)g(X)} g'(X) = -\frac{\eta+\sigma}{1+\eta} \frac{R-\delta}{(X-\delta)^2} \left[ 1 - \frac{1-\sigma}{1+\eta} \frac{X-R}{X-\delta} \right]^{-1} + \eta [X-\delta]^{-1} + [X-R]^{-1} - \frac{1+\eta}{1-\alpha} X^{-1}.$$

Noting that the terms  $\frac{\eta+\sigma}{(1-\sigma)g(X)}$ ,  $\left[ 1 - \frac{1-\sigma}{1+\eta} \frac{X-R}{X-\delta} \right]$ ,  $[X-\delta]$ ,  $[X-R]$  and  $X$  are all positive, it suffices to study the sign of the function  $h$  defined below, the zeros of which are (local) extrema of the steady-state utility.

$$\begin{aligned} h(X) = & -\frac{\eta+\sigma}{1+\eta} (R-\delta)X(X-R) + \eta X(X-\delta)(X-R) \left[ 1 - \frac{1-\sigma}{1+\eta} \frac{X-R}{X-\delta} \right] \\ & + X(X-\delta)^2 \left[ 1 - \frac{1-\sigma}{1+\eta} \frac{X-R}{X-\delta} \right] - \frac{1+\eta}{1-\alpha} (X-\delta)^2(X-R) \left[ 1 - \frac{1-\sigma}{1+\eta} \frac{X-R}{X-\delta} \right]. \end{aligned}$$

$h$  is a third degree polynomial, with a negative leading coefficient  $-(\eta+\sigma)\frac{\alpha}{1-\alpha}$ . Through some algebra

$$h(R) = R(R-\delta)^2,$$

$$h(\delta/\alpha) = \delta(R-\delta)^2/\alpha,$$

$$h(R/\alpha) = -R(R-\delta)[\eta(R-\alpha\delta) + (1-\alpha)R\sigma]/\alpha^2.$$

**Case  $\beta = 1$ :** In this case,  $R = \delta$  and  $h(\delta) = h(\delta/\alpha) = 0$ . More generally.

$$h(X) = \frac{\eta+\sigma}{1-\alpha} (X-\delta)^2(\delta-\alpha X).$$

Thus,  $h$  is positive for  $\delta = R < X < R/\alpha = \delta/\alpha$  and negative for  $X > \delta/\alpha$ , with  $\delta$  and  $\delta/\alpha$  being the only zeros of  $h$ . In this case, optimal steady-state welfare is reached for  $\zeta = \alpha/\delta$ .

**Case  $R \neq \delta/\alpha$ ,  $\beta < 1$ :** In this case,  $0 < \min(R, \delta/\alpha) < \max(R, \delta/\alpha) < R/\alpha$ ,  $h(\min(R, \delta/\alpha)) < h(\max(R, \delta/\alpha))$  and  $h(R/\alpha) < 0$ . Given that  $h$  is a third degree polynomial with a negative lead, we conclude that  $h$  admits a single zero for all values of  $X$  greater than  $R$ ; this root of  $h$  is between  $\max(R, \delta/\alpha)$  and  $R/\alpha$ . Furthermore,  $h$  is positive between  $R$  and the said single root and negative for values of  $X$  higher than the root of interest. As a result, the optimal steady-state welfare is reached for a value  $\zeta$  strictly between  $\frac{\alpha}{R}$  and  $\frac{\alpha}{\delta}$ .

**Case  $R = \delta/\alpha$ ,  $\beta < 1$ :** Through continuity arguments, the same results of the previous case apply. This is because the zeros of  $h$  are a continuous function of  $R$  when the remaining polynomial parameters are fixed.

**Case  $\sigma = 1$ ,  $\beta < 1$ :** The steady-state utility takes the form

$$U = -\ln(\chi) + \eta \ln(1/\zeta - \delta) + \ln(1/\zeta - R) + \frac{1+\eta}{1-\alpha} \ln(\zeta) - \frac{1}{1+\eta} \frac{1/\zeta - R}{1/\zeta - \delta}.$$

Again, noting  $X = 1/\zeta$ ,

$$\frac{\partial U}{\partial X} = \frac{\eta}{X - \delta} + \frac{1}{X - R} - \frac{1+\eta}{1-\alpha} \frac{1}{X} - \frac{1}{1+\eta} \frac{R - \delta}{(X - \delta)^2}.$$

It suffices to study the sign of

$$\tilde{h}(X) = \eta X(X - \delta)(X - R) + X(X - \delta)^2 - \frac{1+\eta}{1-\alpha} (X - \delta)^2 (X - R) - \frac{1}{1+\eta} (R - \delta) X (X - R).$$

We have  $\delta < R$ ,  $h(\delta) = \frac{1}{1+\eta} \delta (R - \delta)^2 > 0$ ,  $h(R) = \frac{1}{1+\eta} R (R - \delta)^2 > 0$ ,  $\delta < R$ ,  $h(\delta) < h(R)$  and

$$h(R/\delta) = -R(R - \delta)((1 - \alpha)R + \eta(\eta + 1)(R - \alpha\delta))/(\alpha^2(\eta + 1)) > 0.$$

So the reasoning of the case where  $R \neq \delta/\alpha$  applies here too.

## B.9 Proof of Proposition 13

In the steady-state  $\mu^W = \mu^K = \mu$ . The firm's first-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial W_{+1}} \tilde{\omega}^L + \beta \frac{\partial L}{\partial W} \tilde{\omega}^L + \left\{ \beta \frac{\partial K}{\partial W_{+1}} + \beta^2 \frac{\partial K}{\partial W} \right\} \left[ \tilde{\omega}^K + \frac{\partial L_{+1}}{\partial K} \tilde{\omega}^L \right] &= \beta \left\{ 1 - \frac{U_C}{\mu} \right\} L, \\ \frac{\partial L}{\partial R_{+1}} \tilde{\omega}^L + \beta \frac{\partial L}{\partial R} \tilde{\omega}^L + \left\{ \beta \frac{\partial K}{\partial R_{+1}} + \beta^2 \frac{\partial K}{\partial R} \right\} \left[ \tilde{\omega}^K + \frac{\partial L_{+1}}{\partial K} \tilde{\omega}^L \right] &= \beta \left\{ 1 - \frac{U_C}{\mu} \right\} K. \end{aligned}$$

Combine the two first-order conditions

$$\begin{aligned} \left\{ \frac{1}{L} \frac{\partial L}{\partial W_{+1}} - \frac{1}{K} \frac{\partial L}{\partial R_{+1}} \right\} \tilde{\omega}^L + \beta \left\{ \frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R} \right\} \tilde{\omega}^L + \\ \left( \beta \left\{ \frac{1}{L} \frac{\partial K}{\partial W_{+1}} - \frac{1}{K} \frac{\partial K}{\partial R_{+1}} \right\} + \beta^2 \left\{ \frac{1}{L} \frac{\partial K}{\partial W} - \frac{1}{K} \frac{\partial K}{\partial R} \right\} \right) \left[ \tilde{\omega}^K + \frac{\partial L_{+1}}{\partial K} \tilde{\omega}^L \right] &= 0. \end{aligned} \tag{B.14}$$

Definition  $A := \tilde{\omega}^K / \tilde{\omega}^L$  and exploiting expression B.14, thus yields equation 7.12.

## B.10 Proof of Proposition 14

We provide several lemmas from which Proposition 14 is derived.

**Lemma 1.** *In the steady state,*

$$\frac{\partial K}{\partial K_{-1}} = 1.$$

*Proof.* Write the savings Euler equation

$$U_{C,t} = \mathbf{E}_t U_{C,t+1} (1 - \delta + R_{t+1}).$$

Derive with regard to  $K_{t-1}$

$$\frac{\partial U_{C,t}}{\partial K_{t-1}} = \frac{\partial K_t}{\partial K_{t-1}} \beta \mathbf{E}_t \frac{\partial U_{C,t+1}}{\partial K_t} (1 - \delta + R_{t+1}).$$

In the steady state  $1 = \beta(1 + r - \delta)$ , which yields the proposition's result. ■

**Lemma 2.** *Assume that the utility function takes the form*

$$U(C_t, L_t) = \mathcal{U}(u(C_t) - v(L_t)),$$

*with  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$ ,  $v'' > 0$  and  $\mathcal{U}' > 0$ . then in the steady state*

$$\begin{aligned} \frac{\partial L}{\partial K_{-1}} &= \frac{(1/\beta - 1)W u''(C)}{v''(L) - W^2 u''(C)}, \\ \frac{\partial C}{\partial K_{-1}} &= \frac{(1/\beta - 1)v''(L)}{v''(L) - W^2 u''(C)}. \end{aligned}$$

*Proof.* Given the assumed form of the utility function, we can write

$$U_{C,t} = u'(C_t) \mathcal{U}'(u(C_t) - v(L_t)),$$

$$U_{L,t} = -v'(L_t) \mathcal{U}'(u(C_t) - v(L_t)).$$

This enables us to write the labour supply condition as follows  $u'(C_t) \frac{W_t}{P_t} = v'(L_t)$ . Plug this in the households' budget constraint

$$u'(W_t L_t + (1 + R_t - \delta)K_{t-1} - K_t) \frac{W_t}{P_t} = v'(L_t). \quad (\text{B.15})$$

Derive with regard to  $K_{t-1}$

$$u''(C_t) \left[ W_t \frac{\partial L_t}{\partial K_{t-1}} + (1 + R_t - \delta) - \frac{\partial K_t}{\partial K_{t-1}} \right] \frac{W_t}{P_t} = v''(L_t) \frac{\partial L_t}{\partial K_{t-1}}.$$

In the steady state, exploiting the fact that  $\frac{\partial K}{\partial K_{-1}} = 1$  and rearranging yields the first result

$$\frac{\partial L}{\partial K_{-1}} = \frac{(1/\beta - 1)Wu''(C)}{v''(L) - W^2u''(C)}.$$

Deriving the budget constraint with regards to  $K_{t-1}$

$$\frac{\partial C_t}{\partial K_{t-1}} = W_t \frac{\partial L_t}{\partial K_{t-1}} + (1 + R_t - \delta) - \frac{\partial K_t}{\partial K_{t-1}}.$$

In the steady state, again exploiting the identity  $\frac{\partial K}{\partial K_{-1}} = 1$

$$\frac{\partial C}{\partial K_{-1}} = W \frac{\partial L}{\partial K_{-1}} + 1/\beta - 1.$$

Using the expression for  $\frac{\partial L}{\partial K_{-1}}$  and rearranging yields the proposition's second result

$$\frac{\partial C}{\partial K_{-1}} = \frac{(1/\beta - 1)v''(L)}{v''(L) - W^2u''(C)},$$

■

**Lemma 3.** *Assume that the utility function takes the form*

$$U(C_t, L_t) = u(C_t) - v(L_t),$$

*with  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$  and  $v'' > 0$ , then in the steady state*

$$\frac{1}{\beta K} \frac{\partial K}{\partial R} = 1, \tag{B.16}$$

$$\frac{1}{\beta L} \frac{\partial K}{\partial W} = 1 + \frac{v'(L)}{v''(L)L}, \tag{B.17}$$

$$\frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} = -\beta \frac{v'(L)}{v''(L)L},$$

$$\frac{1}{K} \frac{\partial K}{\partial R_+} - \frac{1}{L} \frac{\partial K}{\partial W_+} = \frac{-\beta u'(C)}{Ku''(C)v''(L)} \{v''(L) - W^2u''(C)\} - \beta \frac{v'(L)}{v''(L)L}.$$

*Proof.* Plugging the budget constraint into the household's labour supply condition yields an expression linking the new capital  $K_t$  and the labour supply  $L_t$

$$W_t u'((1 - \delta + R_t)K_{t-1} + W_t L_t - K_t) = v'(L_t). \tag{B.18}$$

The right hand side of the equation above is increasing in  $L_t$  and the left hand side is decreasing in  $L_t$  (note that  $v'' > 0$  and  $u'' < 0$ ). Equation B.18 therefore has a unique solution that we note  $\mathcal{L}_t^K$ . That is

$$L_t = \mathcal{L}_t^K(W_t, R_t, K_{t-1}; K_t), \quad (\text{B.19})$$

where  $\mathcal{L}_t^K\left(\frac{W_t}{P_t}, R_t, K_{t-1}; K_t\right)$  is the level of labour supply for a given level of new capital  $K_t$ . As the *rhs* of B.18 is increasing in  $K_t$ ,  $\mathcal{L}_t^K$  is itself increasing in  $K_t$ . To obtain the derivative of  $\mathcal{L}_t^K$  with regard to  $K_t$ , derive B.18 with regard to  $K_t$

$$W_t u''(C_t) \left\{ W_t \frac{\partial \mathcal{L}_t^K}{\partial K_t} - 1 \right\} = v''(L_t) \frac{\partial \mathcal{L}_t^K}{\partial K_t}.$$

Rearrange to get

$$\frac{\partial \mathcal{L}_t^K}{\partial K_t} = \frac{-W_t u''(C_t)}{v''(C_t) - W_t^2 u''(C_t)}.$$

Similarly derive with regard to  $W_t$  and  $R_t$  to get

$$\frac{\partial \mathcal{L}_t^K}{\partial W_t} = \frac{u'(C_t) + W_t L_t u''(C_t)}{v''(L_t) - W_t^2 u''(C_t)},$$

$$\frac{\partial \mathcal{L}_t^K}{\partial R_t} = \frac{W_t K_{t-1} u''(C_t)}{v''(L_t) - W_t^2 u''(C_t)}.$$

Expression B.19 enables to write  $C_t$  as a function of  $K_t$ , using the household's budget constraint  $C_t = W_t \mathcal{L}_t^K(K_t) + (1 - \delta + R_t)K_{t-1} - K_t$ . Plug the latest expression of  $C_t$  into the Euler savings condition

$$u'(W_t \mathcal{L}_t^K + (1 - \delta + R_t)K_{t-1} - K_t) = \beta \mathbf{E}_t(1 - \delta + R_{t+1})u'(C_{t+1}). \quad (\text{B.20})$$

Derive both sides of the equation above with regard to  $R_t$

$$\begin{aligned} & \left\{ \frac{\partial K_t}{\partial R_t} \left[ W_t \frac{\partial \mathcal{L}_t^K}{\partial K_t} - 1 \right] + W_t \frac{\partial \mathcal{L}_t^K}{\partial R_t} + K_{t-1} \right\} u''(C_t) = \\ & \beta \mathbf{E}_t(1 - \delta + R_{t+1}) \frac{\partial K_t}{\partial R_t} \frac{\partial C_{t+1}}{\partial K_t} u''(C_{t+1}). \end{aligned}$$

In the steady-state, this can be written as

$$\frac{\partial K}{\partial R} \left[ \frac{\partial C}{\partial K_{-1}} - W \frac{\partial \mathcal{L}^K}{\partial K} + 1 \right] = K + W \frac{\partial \mathcal{L}^K}{\partial R}.$$

Replace using the steady-state expressions of  $\frac{\partial \mathcal{L}^K}{\partial R}$ ,  $\frac{\partial \mathcal{L}^K}{\partial K}$  and  $\frac{\partial C}{\partial K_{-1}}$

$$\frac{1}{\beta K} \frac{\partial K}{\partial R} = 1.$$

Derive the savings Euler equation with regard to wages  $W_t$

$$\left\{ \frac{\partial K_t}{\partial W_t} \left[ W_t \frac{\partial \mathcal{L}_t^K}{\partial K_t} - 1 \right] + W_t \frac{\partial \mathcal{L}_t^K}{\partial W_t} + L_t \right\} u''(C_t) = \beta \mathbf{E}_t(1 - \delta + R_{t+1}) \frac{\partial K_t}{\partial W_t} \frac{\partial C_{t+1}}{\partial K_t} u''(C_{t+1}).$$

In the steady state

$$\frac{\partial K}{\partial W} \left[ W \frac{\partial \mathcal{L}^K}{\partial K} - 1 \right] + W \frac{\partial \mathcal{L}^K}{\partial W} + L = \frac{\partial K}{\partial W} \frac{\partial C}{\partial K_{-1}},$$

Rearrange

$$\frac{\partial K}{\partial W} \left[ \frac{\partial C}{\partial K_{-1}} - W \frac{\partial \mathcal{L}^K}{\partial K} + 1 \right] = W \frac{\partial \mathcal{L}^K}{\partial W} + L.$$

Replace using the steady-state expressions of  $\frac{\partial \mathcal{L}^K}{\partial W}$ ,  $\frac{\partial \mathcal{L}^K}{\partial K}$  and  $\frac{\partial C}{\partial K_{-1}}$

$$\frac{\partial K}{\partial W} \left[ \frac{(1/\beta - 1)v''(L)}{v''(L) - W^2 u''(C)} - W \frac{-W_t u''(C)}{v''(L) - W_t^2 u''(C)} + 1 \right] = W \frac{u'(C) + W_t L_t u''(C)}{v''(L) - W^2 u''(C)} + L.$$

Rearrange

$$\frac{1}{\beta L} \frac{\partial K}{\partial W} = 1 + \frac{W u'(C)}{v''(L) L}.$$

Use the steady-state version of the labour supply condition ( $W u'(C) = v'(L)$ )

$$\frac{1}{\beta L} \frac{\partial K}{\partial W} = 1 + \frac{v'(L)}{v''(L) L}. \quad (\text{B.21})$$

Combine [B.16](#) and [B.21](#)

$$\frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} = -\beta \frac{v'(L)}{v''(L) L}.$$

Derive [B.20](#) with regard to  $R_{t+1}$  in the steady-state

$$u''(C) \frac{\partial K}{\partial R_+} \left\{ W \frac{\partial \mathcal{L}^K}{\partial K} - 1 \right\} = \beta u'(C) + u''(C) \frac{\partial K}{\partial R} \left\{ W \frac{\partial \mathcal{L}^K}{\partial K} - 1 \right\} + u''(C) \left\{ K + W \frac{\partial \mathcal{L}^K}{\partial R} \right\}.$$

Rearrange

$$\left\{ W \frac{\partial \mathcal{L}^K}{\partial K} - 1 \right\} \left\{ \frac{\partial K}{\partial R_+} - \frac{\partial K}{\partial R} \right\} = \beta \frac{u'(C)}{u''(C)} + \left\{ K + W \frac{\partial \mathcal{L}^K}{\partial R} \right\}$$

Replace for  $\frac{\partial \mathcal{L}^K}{\partial K}$ ,  $\frac{\partial \mathcal{L}^K}{\partial R}$  and  $\frac{\partial K}{\partial R}$

$$\frac{1}{K} \frac{\partial K}{\partial R_+} = -(1 - \beta) - \frac{\beta u'(C)}{u''(C) K v''(L)} \{v''(L) - W^2 u''(C)\}.$$

Derive B.20 with regard to  $W_{t+1}$  in the steady-state

$$\frac{\partial K}{\partial W_+} \left\{ W \frac{\partial \mathcal{L}^K}{\partial K} - 1 \right\} = \frac{\partial K}{\partial W} \left\{ W \frac{\partial \mathcal{L}^K}{\partial K} - 1 \right\} + \left\{ L + W \frac{\partial \mathcal{L}^K}{\partial W} \right\}.$$

Replace for  $\frac{\partial \mathcal{L}^K}{\partial K}$ ,  $\frac{\partial \mathcal{L}^K}{\partial W}$  and  $\frac{\partial K}{\partial W}$

$$\frac{1}{L} \frac{\partial K}{\partial W_+} = -(1 - \beta) + \beta \frac{v'(L)}{v''(L)L}.$$

We, now, can write

$$\frac{1}{K} \frac{\partial K}{\partial R_+} - \frac{1}{L} \frac{\partial K}{\partial W_+} = \frac{\beta W^2 u'(C)}{K v''(L)} - \frac{\beta u'(C)}{K u''(C)} - \beta \frac{v'(L)}{v''(L)L}.$$

■

**Lemma 4.** Assume that the utility function takes the form

$$U(C_t, L_t) = u(C_t) - v(L_t),$$

with  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$  and  $v'' > 0$ , then in the steady state

$$\begin{aligned} [v''(L_t) - W_t^2 u''(C_t)] \left\{ \frac{1}{L_t} \frac{\partial L_t}{\partial W_t} - \frac{1}{K_t} \frac{\partial L_t}{\partial R_t} \right\} &= u'(C_t)/L_t + W_t u''(C_t) \left( 1 - \frac{K_{t-1}}{K_t} \right) \\ &+ W_t u''(C_t) \left\{ \frac{1}{K_t} \frac{\partial K}{\partial R_t} - \frac{1}{L_t} \frac{\partial K_t}{\partial W_t} \right\}, \\ [v''(L_t) - W_t^2 u''(C_t)] \left\{ \frac{1}{L_t} \frac{\partial L_t}{\partial W_{t+1}} - \frac{1}{K_t} \frac{\partial L_t}{\partial R_{t+1}} \right\} &= W_t u''(C_t) \left\{ \frac{1}{K_t} \frac{\partial K_t}{\partial R_{t+1}} - \frac{1}{L_t} \frac{\partial K_t}{\partial W_{t+1}} \right\}. \\ [v''(L_t) - W_t^2 u''(C_t)] \frac{\left\{ \frac{1}{L} \frac{\partial L}{\partial W_{+1}} - \frac{1}{K} \frac{\partial L}{\partial R_{+1}} \right\} + \beta \left\{ \frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R} \right\}}{\left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\} + \beta \left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}} &= \\ + \frac{\beta u'(C)/L}{\left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\} + \beta \left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}} &+ W u''(C). \end{aligned}$$

*Proof.* Coming back to identity B.18

$$W_t u' (W_t L_t + (1 - \delta + R_t) K_{t-1} - K_t) = v'(L_t).$$

Derive with regard to  $W_t$

$$u'(C_t) + W_t u''(C_t) \left[ W_t \frac{\partial L_t}{\partial W_t} + L_t - \frac{\partial K_t}{\partial W_t} \right] = v''(L_t) \frac{\partial L_t}{\partial W_t}.$$



Rearrange

$$[v''(L_t) - W_t^2 u''(C_t)] \frac{1}{L_t} \frac{\partial L_t}{\partial W_t} = u'(C_t)/L_t + W_t u''(C_t) - W_t u''(C_t) \frac{1}{L_t} \frac{\partial K_t}{\partial W_t}. \quad (\text{B.22})$$

Derive [B.18](#) with regard to  $R_t$

$$W_t u''(C_t) \left[ W_t \frac{\partial L_t}{\partial R_t} + K_{t-1} - \frac{\partial K_t}{\partial R_t} \right] = v''(L_t) \frac{\partial L_t}{\partial R_t}.$$

Rearrange

$$[v''(L_t) - W_t^2 u''(C_t)] \frac{1}{K_t} \frac{\partial L_t}{\partial R_t} = W_t \frac{K_{t-1}}{K_t} u''(C_t) - W_t u''(C_t) \frac{1}{K_t} \frac{\partial K_t}{\partial R_t}. \quad (\text{B.23})$$

Subtract [B.23](#) from [B.22](#) and rearrange

$$[v''(L) - W^2 u''(C)] \frac{\frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R}}{\frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W}} = \frac{u'(C)/L}{\frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W}} + W u''(C).$$

replace for  $\frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W}$  using proposition

$$[v''(L) - W^2 u''(C)] \frac{\frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R}}{\frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W}} = \frac{u'(C)/L}{-\beta \frac{W u'(C)}{v''(L)L}} + W u''(C) = W u''(C) - \frac{v''(L)}{\beta W}.$$

Derive the identity [B.18](#) with regard to  $W_{t+1}$

$$u''(C_t) \left[ W_t^2 \frac{\partial L_t}{\partial W_{t+1}} - W_t \frac{\partial K_t}{\partial W_{t+1}} \right] = v''(L_t) \frac{\partial L_t}{\partial W_{t+1}}.$$

Rearrange

$$[v''(L_t) - W_t^2 u''(C_t)] \frac{1}{L_t} \frac{\partial L_t}{\partial W_{t+1}} = -W_t u''(C_t) \frac{1}{L_t} \frac{\partial K_t}{\partial W_{t+1}}.$$

Similarly, derive the identity [B.18](#) with regard to  $R_{t+1}$  and rearrange

$$[v''(L_t) - W_t^2 u''(C_t)] \frac{1}{K_t} \frac{\partial L_t}{\partial R_{t+1}} = -W_t u''(C_t) \frac{1}{K_t} \frac{\partial K_t}{\partial R_{t+1}}.$$

Subtract the last equation from the one above

$$[v''(L_t) - W_t^2 u''(C_t)] \left\{ \frac{1}{L_t} \frac{\partial L_t}{\partial W_{t+1}} - \frac{1}{K_t} \frac{\partial L_t}{\partial R_{t+1}} \right\} = W_t u''(C_t) \left\{ \frac{1}{K_t} \frac{\partial K_t}{\partial R_{t+1}} - \frac{1}{L_t} \frac{\partial K_t}{\partial W_{t+1}} \right\}.$$

■

From the definition of  $A$ ,

$$\begin{aligned}
A &= - \frac{\frac{\partial L_{+1}}{\partial K}}{\frac{(1/\beta-1)Wu''(C)}{v''(L)-W^2u''(C)}} + \frac{1}{\beta} \frac{\left\{ \frac{1}{L} \frac{\partial L}{\partial W_{+1}} - \frac{1}{K} \frac{\partial L}{\partial R_{+1}} \right\} + \beta \left\{ \frac{1}{L} \frac{\partial L}{\partial W} - \frac{1}{K} \frac{\partial L}{\partial R} \right\}}{\left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\} + \beta \left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}} \\
&\quad \frac{1}{v''(L)-W^2u''(C)} \left[ \frac{\beta u'(C)/L}{\left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\} + \beta \left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}} + Wu''(C) \right] \\
&= \frac{1}{\beta} \frac{1}{v''(L) - W^2u''(C)} \left[ \beta Wu''(C) + \frac{\beta u'(C)/L}{\underbrace{\left\{ \frac{1}{K} \frac{\partial K}{\partial R_{+1}} - \frac{1}{L} \frac{\partial K}{\partial W_{+1}} \right\}}_{\frac{-\beta u'(C)}{Ku''(C)v''(L)} \{v''(L) - W^2u''(C)\} - \beta \frac{v'(L)}{v''(L)L}}} + \beta \underbrace{\left\{ \frac{1}{K} \frac{\partial K}{\partial R} - \frac{1}{L} \frac{\partial K}{\partial W} \right\}}_{-\beta \frac{v'(L)}{v''(L)L}} \right] \\
&= \frac{1}{\beta} \frac{1}{v''(L) - W^2u''(C)} \left[ \beta Wu''(C) + \frac{u'(C)/L}{\underbrace{\frac{-u'(C)}{Ku''(C)v''(L)} \{v''(L) - W^2u''(C)\} - (1 + \beta) \frac{v'(L)}{v''(L)L}}_{\frac{Wu'(C)}{v''(L)L}}} \right] \\
&= \frac{-u''(C)}{\beta \{v''(L) - W^2u''(C)\}} \left[ -\beta W + \frac{v''(L)}{\frac{L}{K} \{v''(L) - W^2u''(C)\} + (1 + \beta) Wu''(C)} \right].
\end{aligned}$$

## C External Stability

We have shown that agents act optimally given the actions of others in a decentralised firm with no-leakage remuneration. In other words, agents' actions form a Nash equilibrium, so no agent has an incentive to unilaterally deviate. We can think of this notion as internal stability.

Let us now consider whether the arrangements we introduce are also externally stable, in the sense that the suggested internal firm structures emerge as an equilibrium structure even under coalitional deviations that allow agents to collectively change the organisational structure of the firm (or set up another firm with a different structure). To this end, we consider a stability notion based on coalition-proof Nash equilibrium (Bernheim, Peleg, and Whinston, 1987). This is a natural concept of stability in an environment where agents can

communicate with each other but cannot make binding commitments. Any arrangement, including any possible deviation, is then required to be self-enforcing. In other words, any initial deviation by a coalition that is subject to further deviations by some sub-coalition of the deviating coalition is not self-enforcing. The notion of coalition-proof Nash equilibrium allows only deviations that are self-enforcing. Any arrangement that is not subject to any self-enforcing deviation is coalition proof.

The problem in our case is dynamic and any deviations to consider involve changes in the remuneration structure. Motivated by the discussion above, we construct the definition of external stability below.

First, note that in this paper we have proposed organisational forms in which the entire profit goes to factor payments (either capital only or to both factors), making factor prices internal. No part of profits is a payment independent of factor contribution of an agent, that is, the remuneration structure features no incentive leakage (NIL). We denote firms with such remuneration structure as NIL firms.

Next, we consider a possible deviation by any non-empty subset of agents (including the full set of agents) to set-up another firm that includes a lump-sum element in the payoff that represents an incentive leakage (an IL firm). We call this an IL deviation. However, we require that any candidate IL deviation must satisfy the consistency condition that it is itself self-enforcing, that is, immune to further deviations by any agent or subset of agents at some future date to return to an NIL structure (either by joining an existing NIL firm or setting up a new NIL firm). An NIL structure is then defined to be externally stable if there does not exist any such self-enforcing IL deviation.

More formally, let  $V_{t,T}(\mathbf{w}, \mathbf{r})$  denote the lifetime discounted (indirect) utility of an agent at time  $t$ , starting period  $T \geq t$  in an NIL firm with payment vectors  $\mathbf{w}, \mathbf{r}$  to labor and capital respectively. That is, these are vectors of payments over time for the relevant period.

Let  $\hat{V}_{t,T}(\hat{\mathbf{w}}, \hat{\mathbf{r}}, \mathbf{G})$  denote the lifetime discounted (indirect) utility of an agent at time  $t$ , starting period  $T \geq t$ , in an IL firm with payment vectors  $\hat{\mathbf{w}}, \hat{\mathbf{r}}$  to labor and capital respectively, as well as a lump-sum payment vector  $\mathbf{G}$  which is all or part of profit. Here,  $\mathbf{G} \geq 0$  and not equal to the zero vector.

An IL deviation at  $t$  is **self-enforcing** if for all  $T \geq t$ ,

$$\hat{V}_{t,T}(\hat{\mathbf{w}}, \hat{\mathbf{r}}, \mathbf{G}) \geq V_{t,T}(\mathbf{w}, \mathbf{r}) \quad (\text{C.1})$$

**Definition 5.** (External Stability) *An NIL structure is externally stable if there is no self-*

*enforcing IL deviation.*

**Model perturbation** Consider the following perturbation of the model. Suppose there is a positive measure of agents who are unproductive - that is, their labour supply does not generate any extra output. That is, under the perturbed labor force structure, there are two types of workers: one with zero productivity and another with the usual positive level of productivity (the argument also works with low but positive productivity, but assuming zero productivity is simpler). Once an agent is employed by a firm, their productivity can be observed.

We further assume there is turnover of a fraction  $\eta \in (0, 1)$  of agents each period. The firm has imperfect screening technology, so if the firm remuneration structure attracts entry applications from unproductive agents, the firm hires a fraction  $m_u \eta$  of such agents,  $0 < m_u \leq 1$ .

**Stability result under the perturbation** In an NIL firm, if labour is not tied, the wage payment for a zero-productivity agent would be zero. If labour is tied, zero-productivity labour supply does not add to the productive labour supply. Thus the labour remuneration received by zero-productivity agents would be zero in any NIL firm. Since there is no income, there is also no saving, implying these agents do not contribute to capital formation either. In other words, lifetime utility for an unproductive agent is 0 in an NIL firm starting any period, and therefore a zero-productivity agent would have no reason to enter an NIL firm. Since each period a fraction  $\eta$  of normal productivity types are hired, the agent population in an NIL firm remains the same every period. It follows that our analysis of NIL firms remain unchanged under the perturbation considered.

Let us now consider an IL firm. The lifetime utility of a zero-productivity agent under this arrangement is

$$\widehat{V}(\mathbf{0}, \mathbf{0}, \mathbf{G}) \equiv \widehat{V}_0(\mathbf{G})$$

Note that  $\widehat{V}_0(\mathbf{G}) > 0$ . This implies that unproductive agents would apply to enter any IL firm.

Now consider the scenario we explore in the paper: all firms follow an NIL structure. Suppose all agents in one firm deviate and start a new firm with an IL structure. Since this firm can implement the firm-level planner's solution (which features an incentive leakage), the firm

owners would have higher utility. This implies that ignoring turnover considerations, the deviation is beneficial. However, let us consider the impact of turnover.

Let  $U_{t,T}$  denote the fraction of unproductive types in the firm in period  $T$  with entry starting period  $t$ .

**Lemma 5.**  $U_{t,T}$  is increasing in  $T$ .

**Proof:** We have  $U_{t,t+1} = \eta m_u$ ,  $U_{t,t+2} = 2\eta m_u - \eta^2 \mu$ ,  $U_{t,t+3} = 3\eta m_u - 2\eta^2 m_u + \eta^3 \mu$  and so on.

In any period  $t + T$ , the fraction of unproductive agents is  $T\eta m_u - O(\eta^2)$ , where  $O(\eta^2)$  denotes terms of order  $\eta^2$  and higher orders. It is clear that the fraction of unproductive agents rises in  $T$ .||

**Proposition 15.** *Under the perturbation introduced above, any IL deviation is not self-enforcing.*

**Proof:** Note that starting at  $t$ , in any period  $T \geq t$ , the lump-sum payoff is

$$G_{t,T} \leq (1 - U_{t,T})\pi_T$$

Let us write  $\mathbf{G}$  as  $\mathbf{G}(U_{t,T})$  to make the dependence on the unproductive fraction explicit. Note that for  $T' > T$ ,  $U_{t,T'} > U_{t,T}$  (from Lemma 5). Therefore  $G(U_{t,T'}) \leq G(U_{t,T})$ , and for high enough  $T'$ , strict inequality holds.

We know that if  $\mathbf{G}(U_{t,T}) = \mathbf{0}$  for all  $T \geq t$  (that the lump sum is withheld from the agents), then

$$\hat{V}_{t,T}(\hat{\mathbf{w}}, \hat{\mathbf{r}}, \mathbf{0}) < V_{t,T}(\mathbf{w}, \mathbf{r})$$

for all  $T \geq t$ . Further, we know that if  $U_{t,T} = 0$  for all  $T \geq t$ , and  $\mathbf{G}(0) = \mathbf{\Pi}$ , where  $\mathbf{\Pi}$  is the vector of profits,

$$\hat{V}_{t,T}(\hat{\mathbf{w}}, \hat{\mathbf{r}}, \mathbf{\Pi}) > V_{t,T}(\mathbf{w}, \mathbf{r})$$

for all  $T \geq t$ . Since  $G(U_{t,T})$  must eventually decrease in  $T$ , starting from any  $t$ , there is a  $T^* \geq t$  such that

$$\hat{V}_{t,T^*}(\hat{\mathbf{w}}, \hat{\mathbf{r}}, \mathbf{G}(U_{t,T^*})) = V_{t,T^*}(\mathbf{w}, \mathbf{r})$$

Then for any  $T > T^*$ , equation (C.1) is violated, implying that the deviation to an IL firm is not self-enforcing.||

**Proposition 16.** *Under the perturbation introduced above, an NIL structure is externally stable.*

*Proof.* Proposition 15 shows that any IL deviation is not self-enforcing. The result follows immediately from Definition 5. ■

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