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Article

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On shrinkage covariance estimators: how inefficient is 1/N strategy of covariance estimation for portfolio selection in foreign exchange market?

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ABSTRACT

We investigate portfolio selection performance as in Markowitz by evaluating variance matrix estimation criteria in the currency market. This study challenges theoretically rigorous shrinkage covariance estimators using multiple evaluation metrics: systematic loss function, risk profile of minimum variance portfolios, Herfindahl index, financial efficiency, and concentration level. We assess out-of-sample performance across conventional models, factor models, linear shrinkage estimators, and equally weighted portfolios by applying mean-variance criteria and minimum variance framework to the 10 most traded currencies. Our findings reveal that mean-variance optimal portfolios are concentrated, counterintuitive, and highly sensitive to optimizer input choices in currency markets. We discovered that shrinkage estimators do not provide additional benefits to investors and fund managers regarding systematic loss function and minimum variance portfolio risk profiles. The research highlights critical limitations in traditional portfolio construction approaches, demonstrating that portfolios built using mean-variance criteria are prone to significant input data sensitivity and tend to create overly concentrated investments. Consequently, the study suggests that investors and fund managers should exercise caution when selecting covariance estimators and consider exploring more diversified strategies to optimize portfolio performance in foreign exchange markets.

IMPACT STATEMENT

This study critically evaluates the efficacy of covariance estimators in optimizing portfolios within the foreign exchange market, highlighting limitations of traditional and sophisticated shrinkage methods. Through comprehensive analysis, it demonstrates that mean-variance optimal portfolios are prone to input sensitivity and concentration, offering little advantage to fund managers over simpler 1/N diversification strategies. With a comprehensive dataset spanning decades, the research provides actionable insights into minimizing estimation errors and improving portfolio stability. The findings challenge prevailing methodologies, emphasizing simplicity and diversification to enhance decision-making for investors, fund managers, and policymakers navigating the complexities of currency markets.

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1. Introduction

Asset allocation is an investment strategy in which investors distribute their wealth across numerous asset classes by considering the risks and rewards of portfolios. Factors such as globalization, services proliferation, financial innovation, and technology drastically alter the conventional way of portfolio formulation (Chao et al., 2019; Kou et al., 2019). Modern investment philosophy is grounded on the concept of diversification. In financial engineering literature, Markowitz (1952) mean-variance optimization gained wide acceptance for portfolio selection and assumes that an investor's utility function is solely

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driven by risk and return on investment (Liu & Li, 2017). The persuasiveness of Markowitz's (1952) paper was electronically surveyed by Kolm et al. (2014), and reported that it was cited 19,016 times in Google Scholar, hit 590,000 times in the Google search engine, and there are 531 videos available on YouTube. At the same time, it has been criticized by several studies such as Michaud (1989), Best & Grauer (1991), Scutellà & Recchia (2013), Yue et al. (2015), Kaucic et al. (2019) and Schmidt, (2019), Michaud (1989) terms it an 'estimation error maximizer', Disatnik & Benninga (2007) also raise many questions regarding the performance of mean-variance optimal portfolios, and Moura et al. (2020) argued that it yields questionable results.

A plethora of literature on modern investment philosophy can be characterized into two fundamental cuts: a theoretical approach and an implementation approach. Theories, underlying assumptions, and other notional aspects of mean-variance optimization are covered in the theoretical approach, whereas the implementation approach emphasizes the estimation of variance-covariance matrix and estimation of return vectors (Isaias et al., 2015; Ortobelli et al., 2019; Pandolfo et al., 2019). Our study targets the estimation of the variance-covariance matrix for portfolio selection and empirically investigates the concentration level and financial efficiency of mean-variance portfolios, minimum-variance portfolios, and naively diversified portfolios in the currency framework. Many studies such as DeMiguel et al. (2009a), and Elton & Gruber (1973) have documented the importance of covariances in portfolio selection. Simultaneously, covariance estimators are fundamental components in the range of econometrics and statistical applications such as portfolio optimization, hedging, value at risk, risk management, financial engineering, pricing of derivatives, and asset pricing (Moura et al., 2020; Kaucic et al., 2019; Lin et al., 2016; Wu et al., 2019; De Nard et al., 2021; Hounyo, 2017; DeMiguel et al., 2020; Münnix et al., 2014; Rahiminezhad Galankashi et al., 2020; Pakel et al., 2021; Ikeda & Kubokawa, 2016; Steland, 2020; Guo et al., 2020; Farnè & Montanari, 2020; Corazza, 2021).

The traditional methods for portfolio selection rely on sample covariance estimator, but they have been criticized by Michaud (1989), Jobson & Korkie (1980), and Pafka & Kondor (2004). The literature also supports using systematic risk factors to estimate the covariance matrix in portfolio selection (Sharpe, 1963; Vasicek, 1973; Blume, 1971; King, 1966). Moura & Noriller (2019) and Hautsch & Voigt (2019) came up with a way to model large realized covariance matrices. Overall mean approach for covariance estimation has been advocated by Elton & Gruber (1973), Elton et al. (2006), and Eun & Resnick (1992) while Engle et al. (2019) and Ardia et al. (2017) used the dynamic conditional correlation approach. De Nard et al. (2021) used a factor structure for the same purposes. The literature on covariance estimation is too extensive to survey, but it unanimously reveals that it is prone to errors. It may originate during the estimation of the portfolio or specification of parameters (DeMiguel et al., 2020). Therefore, DeMiguel et al. (2009a) empirically reveal that an equally weighted strategy *(1/N)* for asset allocation outperforms competitive 14 sophisticated strategies for asset allocation in seven different databases in the USA equity market. This outperformance is due to numerical instability and estimation errors during portfolio optimization.

The 21st-century financial literature attempts to optimize the specification error and estimation errors during portfolio selection in the equity market. Studies such as Ledoit & Wolf (2003, 2004) used the Bayesian shrinkage approach to shrink sample estimator to a systematic risk factor and overall mean base covariance estimators for the NYSE and AMEX stock markets. Their study minimized the quadratic loss function and reduced the sample estimator without initializing specification error. Later on, Disatnik & Benninga (2007) and Jagannathan & Ma (2003) criticized the work of Ledoit & Wolf (2003, 2004) on shrinkage estimation for the NYSE and AMEX stock markets and developed a simple solution for portfolio selection in the equity market. The existing literature provides no consensus on the estimation of the covariance matrix for portfolio selection.

The empirical financial engineering literature on the estimation of covariance matrices for portfolio selection is skewed toward the equity markets (Bengtsson & Holst, 2002; Ledoit & Wolf, 2003, 2004; Jagannathan & Ma, 2003; Disatnik & Benninga, 2007; DeMiguel et al., 2009a; Liu & Lin, 2010). However, factors such as globalization, services proliferation, financial innovation, and technology drastically alter the conventional way of portfolio formulation, and the global village nature of today's world provides opportunities for investors, and fund managers to go beyond the traditional way of portfolio selection. Therefore, the investors can equip their portfolio with an appropriate mix of asset classes from equities,

currencies, and commodities. Our study focuses on the currency market because daily in the currency market is 5.09 trillion US dollars. As per the trading volume, the foreign currency market is the largest in the world (Record, 2004). Therefore, fund managers, practitioners, and investors are seeking for optimal way to estimate the covariance matrices and develop optimal portfolios in the foreign currency market. However, the existing literature provides no real solution for the estimation of inputs to portfolio optimization and the standing of modern portfolio frameworks, even in the stock markets of developing and developing countries.

This study contributes to the existing literature in the following ways. First, we extend the debate on the use of optimal shrinkage covariance matrices as inputs to portfolio optimization and examine the complex, theoretically rigorous Ledoit & Wolf (2003,2004) covariance estimators in the currency market. The existing literature on covariance estimation is still confusing and offers no real consensus on the optimal framework for the estimation of the covariance matrix as an essential input to portfolio optimization (DeMiguel et al., 2020; Ledoit & Wolf, 2003, 2004; Engle et al., 2019; Disatnik & Benninga, 2007; Liu & Lin, 2010; Moura et al., 2020; DeMiguel et al., 2009a; Jagannathan & Ma, 2003; Bjerring, Ross & Weissensteiner, 2017; De Nard et al., 2021).

Second, we concentrate on the global currency market because it has been largely ignored in past studies on covariance estimates and portfolio selection. Furthermore, we conduct an empirical investigation of the mean-variance portfolio's performance, comparing its level of concentration and financial efficiency to that of the lowest variance portfolio in the forex market. It will advise investors, fund managers, policymakers, and researchers on the need of a modern portfolio framework in making dynamic asset allocation decisions in the currency market. In addition, we evaluate the performance of linear shrinkage estimators to classical models, factor models, and equally weighted portfolios of covariance estimators in the global currency market. Third, we used a set of evaluation criteria to make our findings more credible and reliable. It comprises a systematic loss function, often known as root mean square error, the risk profile of optimal portfolios, diversification measures, and efficiency measurements of optimal weights for portfolio selection in the forex market. Finally, we employ the most recent and longest data sets, with a sample period ranging from January 1985 to January 2021. The use of daily, weekly, fortnightly, and monthly datasets in our study lends credibility to our conclusions.

Based on a battery of portfolio weight characteristics, such as the Herfindahl index, total number of positive weights, total number of negative weights, weight standard deviation, and range of optimal weights, it was discovered that mean-variance optimal portfolios are concentrated, counterintuitive, and sensitive to the currency market's inputs to the optimizer. When we compare the performance of linear shrinkage covariance estimators to the 1/N strategy of covariance formulation, we discover that shrinkage estimators do not provide any additional benefit to investors or fund managers in terms of the systematic loss function and risk profile of minimum variance portfolios in the currency market. Our findings are robust throughout the study's daily, weekly, fortnightly, and monthly datasets. The remaining sections of this work are organized as follows. Section 2 examines the literature on portfolio optimization. Section 3 contains the data used in this study, as well as the complete research methodology. It also provides data on portfolio formulation in the sample currencies. Section 4 presents the empirical results. Section 5 presents our empirical findings, followed by a brief conclusion.

2. Review of related literature

Rabbi Issac Bar Aha first used the concept of diversification for asset allocation in the fourth century (DeMiguel et al., 2009a). After the lull in literature, Markowitz (1952) developed the modern portfolio theory, and numerous studies support the mean-variance criteria for portfolio selection (Chan et al., 1999). The modern portfolio framework requires two main inputs: a covariance matrix and the expected return vector. The estimation of covariance matrices is also frequently used in econometrics and statistical applications, including value-at-risk, asset pricing, risk management, and financial engineering (Hounyo, 2017; Bjerring et al., 2017; Ikeda & Kubokawa, 2016; De Nard et al., 2021; Lin et al., 2016; Pakel et al., 2021; Zhai & Bai, 2017; Sui et al., 2020; Zhou et al., 2020). Furthermore, Fabozzi et al. (2010)

provide a survey on the contribution of portfolio development strategies from finance and operation research to portfolio theory. The existing literature provides many options for estimating the covariance matrix, ranging from simple and straightforward covariance estimators to theoretical complex covariance estimators (Pakel et al., 2021; Moura et al., 2020; Disatnik & Benninga, 2007).

Sample covariance estimators are the traditional method for covariance estimation; however, they face heavy criticism from researchers such as Michaud (1989) and Jobson & Korkie (1980). It is less vulnerable to error if the total cross-section is larger than the sample size, which also dictates a low structure. Lin et al. (2016) argue that a traditional sample covariance matrix does not provide healthy estimates when investors increase their sample sizes and its dimensions. Sharpe (1963) proposed the concept of estimating covariance matrix by systematic risk factor (market factor). Furthermore, it was extended by Blume (1971), Vasicek (1973), and King (1966). Ardia et al. (2017) applied dynamic conditional correlation methods. Moura & Noriller (2019) developed ways to model the covariance matrices. Then, the overall mean-based covariance estimation approach was improved by Elton et al. (2006). Principal component analysis (PCA), a non-theoretical and statistical-based measure, is also used to estimate the covariance matrix. Furthermore, Rahiminezhad Galankashi et al. (2020) developed the fuzzy-ANP approach for asset allocation in Tehran. Researchers such as Steland (2020), Guo et al. (2020), Farnè & Montanari (2020), and Corazza (2021) also discuss variance-covariance matrices from various perspectives.

Ledoit & Wolf (2003) also developed the covariance estimators by using the Bayesian shrinkage approach. Further, Ledoit & Wolf (2004) shrank the sample covariance estimators without initiating specification errors. Different researchers have proposed algorithms and shrank the sample covariance toward numerous target matrices, identity, diagonal, single-index-based estimators, and overall mean methods. Linear shrinkage estimators were advocated by Schäfer & Strimmer (2005), Fisher & Sun (2011), Ledoit & Wolf (2004), and Touloumis (2015). A plethora of studies on the estimation of covariance matrices, such as Ledoit & Wolf (2003, 2004), Disatnik & Benninga (2007), Liu & Lin (2010), Bengtsson & Holst (2002), Jagannathan & Ma (2003) and conclude that every estimation technique is vulnerable to errors such as estimation errors or specification errors. These errors are the two ends of one stick, and investors can optimize these errors by applying the famous decision theory for optimal decision-making. Stein (1956) proposed a weighted average solution for this problem.

Non-linear shrinkage estimators are used by studies such as Fan et al. (2016), Cai & Zhou (2012), Cai & Liu (2011), Ledoit & Wolf (2015), and Rothman et al. (2009). Harris & Yilmaz (2010) proposed a hybrid multivariate estimation technique for exponentially weighted moving-average covariance estimators. Münnix et al. (2014) examined the correlation and covariance matrices from historical data, while Byström (2004) analyzed covariance estimators for Nordic stock indices. Ikeda & Kubokawa (2016) discuss different aspects of sample covariance estimators. Kremer et al. (2018) compared the performance of eight risk minimization strategies, including naive diversification and minimum risk portfolios in five regions.

However, DeMiguel et al. (2009a) recently documented that the equally weighted technique performs better than other estimation techniques. This is because of estimation errors during the portfolio selection process. A recent stream of studies further enhance the importance of optimal weights selection criterion while addressing optimal portfolios under uncertainty created because of various factors. Former researchers explicitly discussed optimal portfolio decision-making in markets where uncertainty is common and proposed a multiobjective credibility portfolio selection model for inaccurate data. Similarly, Huang & Ma (2023) propose an uncertain mean-chance model for portfolio selection, incorporating multiplicative background risk to better capture real-world uncertainties in investment environments. Likely, Wang et al. (2023) introduce a multi-criteria fuzzy portfolio selection model that incorporates three-way decisions and cumulative prospect theory to enhance investment decision-making under uncertainty. Similarly, González-Bueno (2019) proposed a multi-objective portfolio optimization. Keeping in view the importance of ESG factors, according to Giese et al. (2019), ESG factors are a useful tool for both investors and policymakers since they are not only ethically important but also financially significant.

Prior researchers have focused on the equity market, but our study attempts to analyze the phenomenon of portfolio selection in the currency market. Investment managers, economists, and finance specialists need a framework in which limited resources can be utilized for optimum returns in currency markets. Therefore, this study is an attempt to critically analyze the complex shrinkage estimators and

investigate the level of concentration and financial efficiency of modern and minimum variance portfolios in a currency environment.

3. Data and research Methodology

3.1. Data description

This study used the Bloomberg database to collect historical data for a sample of 10 currencies. The sample period spans from January 1985 to January 2021. To check the robustness of the results, we used daily, weekly, fortnightly, and monthly data series of selected currencies. The study selects the top 10 most traded currencies in the world, with the United States Dollar as the base currency in each currency pair. Our selection of the United States Dollar is based on its involvement in over 85% of world trade in the foreign exchange market, and 39% of the world debt is issued in United States Dollars. Additionally, our choice of the 10 most traded currencies covers a major portion of trade in the foreign exchange market. Table 1 lists the details of the selected currencies for this study.

3.1.2. List of selected currencies of study

Table 1 presents the details of the selected currencies, the exchange rate of United States dollars for each selected currency, and their respective shares in the foreign exchange market. Exchange rate quotes for each currency pair were collected from the Bloomberg database as of Tuesday, January 31, 2017. Data related to the proportion of trade for each currency pair were gathered from the Triennial Central Bank Survey on foreign exchange turnover in April 2016 by the Monetary and Economic Department of the Bank for International Settlements, Switzerland.

We divide the sample period into two sub-samples. The details of each of our study's subsamples are shown in Table 2. The variance-covariance matrices are calculated using the first subsample, and the ex-post correctness of our estimated variance-covariance matrices is assessed using the second subsample. Furthermore, in the first subsample, we estimate the weights of the mean-variance and minimum-variance portfolios. We then use these computed weights in the second subsample, which is the out-of-sample window, to estimate the risk-return characteristics of the ideal portfolio weights in currency markets. Our prime motive behind using the daily, weekly, fortnightly, and monthly data series of currencies' exchange rates for portfolio selection is to comment on the robustness of results.

3.2. Variance-covariance estimator

Variance-covariance (VC) is a square matrix consisting of variances and covariances. Variance shows the squared mean dispersion, whereas covariances show how the two currencies change together. A VC matrix can be written as follows:

Table 1. List of selected currencies of study.

Unit	Currency Name	Exchange Rate	Trade
Australia	Australian Dollar-AUD	AUD 1.32	6.90%
Canada	Canadian Dollar-CAD	CAD 1.3	5.10%
China	Renminbi-CNY	CNY 6.88	4.00%
European Union	Euro-EUR	EUR 0.93	31.40%
Japan	Japanese Yen-JPY	JPY 112.8	21.60%
Mexico	Mexican Peso-MXN	MXN 20.8	1.90%
New Zealand	New Zealand Dollar-NZD	NZD 1.37	2.10%
Sweden	Swedish Krona-SEK	SEK 8.74	2.20%
Switzerland	Swiss Franc-CHF	CHF 0.99	4.80%
UK	Pound Sterling-GBP	GBP 0.79	12.80%
USA	United States Dollar-USD	USD 1.00	87.60%

Table 1 presents the details of selected currencies, the exchange rate of United States Dollar with each selected currency and their respective share in foreign exchange market. Exchange rate quotes of each currency pair is collected from the Bloomberg database as on Tuesday, January 31, 2017. Data related to proportion of trade of each currency pair is gathered from the triennial central bank survey foreign exchange turnover in April 2016 by the monetary and economic department of Bank for International Settlements, Switzerland.

Table 2. Details of sample and subsamples from currency market.

Serial No	Subsample	Start Date	End Date	Frequency
1	1	1-Jan-85	15-Jan-03	Daily
	2	18-Jan-03	31-Jan-21	
2	1	7-Jan-85	13-Jan-03	Weekly
	2	20-Jan-03	25-Jan-21	
3	1	14-Jan-85	6-Jan-03	Fortnightly
	2	20-Jan-03	11-Jan-21	
4	1	30-Jan-85	31-Dec-03	Monthly
	2	29-Jan-03	30-Dec-21	

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sum z_1^2/k & \sum z_1 z_2/k & \cdots & \sum z_1 z_l/k \\ \sum z_2 z_1/k & \sum z_2^2/k & \cdots & \sum z_2 z_l/k \\ \cdots & \cdots & \cdots & \cdots \\ \sum z_l z_1/k & \sum z_l z_2/k & \cdots & \sum z_l^2/k \end{bmatrix}$$

Here Σ is the VC matrix of order $l*l$, k counts the observations, z_i denotes the mean deviation, term $\sum z_p^2/k$ depicts the variance of p^{th} currencies, and $\sum z_p z_q/k$ measures the covariances among the p^{th} and q^{th} currency of our study. We use the continuous compounded returns of each currency pair for portfolio optimization and compute them using the formula $R_{l,t} = \ln(a_t/a_{t-1})$. Here, $a_t \& a_{t-1}$ denotes the currency quotes for the current and previous periods of any currency, respectively. The following are details about the estimation of covariance estimators across four categories: traditional methods, index models, portfolios of estimators, and shrinkage estimators.

3.2.1. Diagonal covariance estimator

The diagonal covariance matrix assumes that all off-diagonal elements of the square matrix are zero, whereas diagonal entries are equal to the variance of the return series of each currency. Equation 1 presents a typical diagonal covariance matrix (VCE-1).

$$\boldsymbol{\Sigma} = VCE - 1 = \begin{bmatrix} \sum z_1^2/k & 0 & \cdots & 0 \\ 0 & \sum z_2^2/k & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sum z_j^2/k \end{bmatrix} \quad (1)$$

The diagonal covariance estimator (VCE-1) provides a basis for the weighted average portfolio of covariance estimators such as the optimal shrinkage covariance estimators.

3.2.2. Sample estimator

Suppose, 'l' is the total number of currency pairs, k counts the total observations, y_{it} is the continuous compounded return of i^{th} currency pairs at the time 't', and $\overline{y_i} = \sum_{t=1}^{K} \frac{y_{it}}{K}, i = 1, 2, 3, \ldots, l$ then the matrix of excess return ($\boldsymbol{\Gamma}$) can be written as follows:

$$\boldsymbol{\Gamma} = \begin{bmatrix} (y_{11} - \overline{y_1}) & (y_{21} - \overline{y_2}) & \cdots & (y_{l1} - \overline{y_l}) \\ (y_{12} - \overline{y_1}) & (y_{22} - \overline{y_2}) & \cdots & (y_{l2} - \overline{y_l}) \\ \cdots & \cdots & \cdots & \cdots \\ (y_{1K} - \overline{y_1}) & (y_{2K} - \overline{y_2}) & \cdots & (y_{lK} - \overline{y_l}) \end{bmatrix}$$

If $\boldsymbol{\Gamma}^{\mathbf{t}}$ is the response of the excess return matrix $\boldsymbol{\Gamma}$ then sample covariance estimator (VCE-2) can be as follows:

$$\boldsymbol{\Sigma} = \frac{\boldsymbol{\Gamma}^{\mathbf{t}} * \boldsymbol{\Gamma}}{K - 1} \quad (2)$$

We use Equation 2 to estimate the sample estimator for the currency market. Equation 2 can be used to estimate the covariance matrix in any direction.

3.2.3. Constant correlation base covariance estimator

According to Elton & Gruber (1973), the constant correlation coefficient is the sole driver of co-movement among securities. Consistent with Chan et al. (1999), we used the following formula scheme for covariance estimation (VCE-3):

$$\boldsymbol{\sigma_{lm}} = \begin{cases} \sigma_{ll} = \sigma_{l}^{2} & \text{when } l = m \\ \sigma_{lm} = \rho_{lm}\sigma_{l}\sigma_{m} & \text{when } l \neq m \end{cases}$$

The correlation coefficient and covariance are related by $\sigma_{lm} = \rho_{lm}\sigma_{l}\sigma_{m}$. Therefore, we first estimate the correlation matrix among all currency pairs, compute the average correlation coefficient, and finally utilize this overall mean correlation to estimate the variance-covariance matrix.

3.2.4. Single index base covariance estimator

Sharpe (1963) presents a novel approach for the estimation of covariances among securities. Additionally, compared to sample covariance estimators, this unique technique requires comparatively fewer parameters as shown below;

$$\pi_{mt} = \tau_{m} + \omega_{m}x_{t} + \psi_{t}$$

Here x_t, represents the return that is unassociated with the unexplained term ψ_t. Furthermore $E(\psi_i, \psi_j) = 0$ holds. The Association $(\boldsymbol{\sigma_{ij}})$ among any two pairs is written as follows:

$$\sigma_{ij} = \Lambda\sigma^{2}\Lambda^{t} + \Psi$$

where, $\boldsymbol{\Lambda}$ denotes the slope vector, σ^2 is the variance and $\boldsymbol{\Psi}$ is the disturbance term vector. The variance-covariance matrix (VCE-4) can be estimated by the following Equation (3):

$$\boldsymbol{\Sigma} = \boldsymbol{\lambda}\sigma^{2}\hat{\boldsymbol{\lambda}} + \boldsymbol{\zeta} \tag{3}$$

On the one hand, investors can minimize the estimation error by using Equation 3 but the sole factor nature of the single-index framework also introduces a specification error.

3.2.5. 1/N portfolio of covariance estimators

Ledoit & Wolf (2003, 2004) propose an optimal method for the computation of optimal linear shrinkage intensity and shrink the sample covariance estimator towards a target matrix. Furthermore, they adopted a complex procedure for determining the optimal shrinkage intensity factor. However, Jagannathan & Ma (2003) study criticizes their complex formulation of the shrinkage covariance estimator. It proposes a simple solution for estimating weighted covariance estimators for the New York Stock Exchange. Following the work of Husnain et al. (2016a) and Husnain et al. (2016b), we also shrunk the sample covariance estimators towards different targets. However, a key difference between our study and Ledoit & Wolf (2003, 2004) is the fact that we use an equal weight scheme (1/N strategy) for the computation of the following five covariance estimators (vce-5, vce-6, vce-7, vce-8, and vce-9) in the currency market. It is a simple and intuitive investment approach where the investors allocate an equal proportion of their wealth to each asset in the portfolio. Specifically, if there are N assets, the investor allocates 1/N of their total wealth to each asset.

- *Equally weighted of VCE-2 & VCE-1:* The sample covariance estimator and diagonal covariance estimator are equally weighted in our VCE-5 portfolio.

$$\text{VCE} - 5 = (\text{VCE} - 2 + \text{VCE} - 1)/\text{N} \tag{4}$$

- *Equally weighted of VCE-2 &VCE-4:* The sample covariance estimator and the single-index base covariance estimator are equally weighted in our VCE-6 portfolio.

$$\text{VCE} - 6 = (\textit{VCE} - 2 + \text{VCE} - 4)/\text{N} \tag{5}$$

- *Equally weighted of VCE-2 & VCE-3:* The sample covariance estimator and the constant correlation base covariance estimator are equally weighted in our VCE-7 portfolio.

$$\text{VCE} - 7 = (\text{VCE} - 2 + \text{VCE} - 3)/\text{N} \tag{6}$$

- *Equally weighted of VCE-2, VCE-4 & VCE-3:* The sample covariance estimator, single index base covariance estimator, and constant correlation base covariance estimator are equally weighted in VCE-8.

$$\text{VCE} - 8 = (VCE - 2 + VCE - 3 + VCE - 4)/\text{N} \tag{7}$$

- *Equally weighted of VCE-2, VCE-1, VCE-4, & VCE-3:* The sample covariance estimator, diagonal covariance estimator, single-index based covariance estimator, and constant correlation-based covariance estimator are all equally weighted in our VCE-9.

$$\text{VCE} - 9 = (VCE - 1 + VCE - 2 + VCE - 3 + VCE - 4)/\text{N} \tag{8}$$

3.2.6. Shrinkage covariance estimators

The sample covariance estimator is prone to an estimation error in portfolio selection, whereas the single-index base covariance estimator introduces a specification error in portfolio optimization. Therefore, our VCE-2 and VCE-4 are the two edges of one rod. Ledoit & Wolf (2003) criticizes the covariances through a sample estimator and single index base model, and documents that these traditional estimators give an estimation error and specification error, respectively.

If Ψ is the unrestricted parameters and Λ denotes the restricted parameters then corresponding elements denoted by Φ and λ. Then, a weighted average covariance estimator can be written;

$$\boldsymbol{\Sigma} = (\lambda)\delta + \Phi(1 - \delta) \tag{9}$$

Here, Φ is sample matrix, λ denotes a highly structured target matrix. The practical challenge is determining the value of the shrinkage constant. By choosing a delta that is only between 0 and 1, a compromise between the sample and the structured matrix would be reached. But it also gives you a lot of choices. It makes sense that there would be an 'optimal' shrinking constant. It is the one that reduces the anticipated difference between the shrinkage estimator and the actual covariance matrix. The shrinkage estimators are normally based on finite sample statistical decision theory and on the work of Frost & Savarino (1986). A quadratic measure of the difference between the true and estimated covariance matrices based on the Frobenius norm was proposed by Ledoit & Wolf (2003) which is a straightforward loss function. We have used this method for the estimation of optimal shrinkage intensity (δ^*). The value of the optimal shrinkage intensity (δ) ranged from 0 to 1 is $1 \geq \delta \geq 0$. When shrinkage is zero then, $\delta = 0$ and we will end up with $\boldsymbol{\Sigma} = \Phi$, whereas in the case of full shrinkage, $\delta = 1$ and we have $\boldsymbol{\Sigma} = \lambda$.

The value of the optimal shrinkage intensity δ can be estimated based on calculated intuition, or it may be reached by minimizing the following loss function;

$$R(\delta) = \boldsymbol{E}\left(\sum_{\boldsymbol{m}=1}^{\boldsymbol{r}} \left(\pi_m^* - \Psi_m\right)^2\right)$$

They first proposed a technique for optimal shrinkage intensity δ, which shrinks the sample estimator to the single index model. They develop a way to shrink the sample covariance estimator towards a constant correlation-based base estimator. There is a way to reach δ to guarantee the minimization of the mean square errors. Suppose the existence of the 1st & 2nd moments of Φ and λ, then the loss function can be written as:

$$R(\delta) = \sum_{\boldsymbol{m}=\boldsymbol{1}}^{\boldsymbol{r}} Var\left(\pi_m^*\right) + \left(E(\pi_m^*) - \Psi_m\right)^2) \tag{10}$$

After minimizing the Equation (10) w.r.t. δ;

$$\delta^* = \frac{\sum_{m=1}^{r} Var(\pi_m) - Cov(\pi_m, t_m)}{\sum_{m=1}^{r} E[(t_m - \pi_m)^2]} \tag{11}$$

Table 3. Summary of the variance-covariance estimators.

Category	Symbol	Covariance Estimators
Traditional estimators	VCE-1	Diagonal covariance estimator
	VCE-2	Sample estimator
	VCE-3	Constant-correlation base estimator
Factor model	VCE-4	Single-factor base estimator
Equally weighted Portfolio of estimators	VCE-5	*1/N* portfolio of VCE-2 & VCE-1
	VCE-6	*1/N* portfolio of VCE-2 & VCE-4
	VCE-7	*1/N* portfolio of VCE-2 & VCE-3
	VCE-8	*1/N* portfolio of VCE-2, VCE-4 & VCE-3
	VCE-9	*1/N* portfolio of VCE-2, VCE-4, VCE-3 & VCE-1
Optimal Shrinkage estimators	VCE-10	Shrinking the VCE-2 towards VCE-1
	VCE-11	Shrinking the VCE-2 towards VCE-4
	VCE-12	Shrinking theVCE-2 towards VCE-3

We have used the above Equation 11 for the estimation of the optimal shrinkage intensity (δ^*). For a full-length discussion on the optimal shrinkage intensity δ, we refer to the study of Ledoit & Wolf (2003, 2004). We also follow the study of Kwan (2011) and shrink the sample estimator towards a diagonal covariance estimator. We shrink the sample covariance estimator with three targets: a diagonal covariance estimator, a single-index base covariance estimator, and a constant correlation base covariance estimator in the currency market. We refer to the studies of Ledoit & Wolf (2003, 2004), Kwan (2011), Bengtsson & Holst (2002), and Husnain et al. (2016a) for a full-length discussion on portfolio selection. Table 3 provides the details of the variance-covariance estimators we used for our study across four categories: traditional methods, index models, portfolio of estimators, and shrinkage estimators in the currency market.

3.3. Mean-variance optimization

In addition to introducing a technique for calculating the expected risk and return of the portfolio, Markowitz (1952) offers a statistical method for developing optimal portfolios. If w_k is the weight then risk and return can be computed using the following formula:

$$E(r_p) = \sum_{k=1}^{q} w_k E(r)_k = \boldsymbol{w^t E(r)} \tag{12}$$

The portfolio return is the weighted return of individual currencies. In Equation (12), $E(r_p)$ denotes the return of the portfolio, W is the column vector of the weight of each currency, and $E(r)$ is the column vector of return of q currencies in the currency market such that;

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_q \end{bmatrix} \quad \& \quad \boldsymbol{E(r)} = \begin{bmatrix} E(r_1) \\ E(r_2) \\ E(r_3) \\ \vdots \\ E(R_q) \end{bmatrix}$$

Portfolio return is the weighted return of currencies, whereas portfolio risk is not solely the weighted risk but critically depends on the covariances among currencies. Markowitz (1952) proposed that the variance of asset classes is the optimal measure to capture their risk. The below formula is used to apply the risk of our currency portfolio[1]:

$$Var(r_p) = \begin{bmatrix} w_1 & w_2 & w_3 & \cdots & w_q \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \ldots & \sigma_{1q} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \ldots & \sigma_{2q} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \ldots & \sigma_{3q} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \sigma_{q1} & \sigma_{q2} & \sigma_{q3} & \ldots & \sigma_{qq} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_q \end{bmatrix}$$

$$\sigma_p{}^2 = \sum_{l=1}^{q} \sum_{k=l+1}^{q} w_l w_k \ \sigma_{lk} = \mathbf{w^t \Sigma w} \tag{13}$$

Equation (13) measures the risk of currency portfolios. In the above equation, Σ denotes the variance-covariance estimator, w_l, w_k is the weight of l^{th} and k^{th} currencies, and $\sigma_{lk} = Cov\ (R_l, R_k) = \rho_{lk}\ \sigma_l \sigma_k$. Markowitz's modern portfolio assumes that risk and return are the sole drivers of investor utility. The utility function can be written as $E[U(x)] = E(r_p - r_f) - \lambda \sigma_p^2$ s.t.$\mathbf{w^T} = 1$; λ is the coefficient of risk aversion. Therefore;

$$\text{Max}_w = \mathbf{w^t} \boldsymbol{E}(\boldsymbol{r}) - \lambda \mathbf{w_q^t \Sigma w_q} \ \text{such that } '\boldsymbol{w}\ \mathbf{1} = 1 \tag{14}$$

If we take the derivative of the above Equation (14) for w, and simplify using the equation, we obtain the following expression of weight for the modern portfolio framework;

$$\hat{\boldsymbol{w}}_{\boldsymbol{k}} = \frac{\hat{\Sigma}_{\mathbf{k}}^{-1} \mathbf{E(r)}}{S[\hat{\Sigma}_{\boldsymbol{k}}^{-1} \boldsymbol{E}(\boldsymbol{r})]} \tag{15}$$

We follow Equation (15) to obtain the optimal weights for portfolio selection.

3.4. Minimum-variance portfolio

Following the studies of DeMiguel et al. (2009b), Liu & Lin (2010), and Ardia et al. (2017), we utilize minimum variance portfolios for optimizing portfolios. Minimum variance portfolios are independent of return vectors and depend solely on the choice of covariance estimators. Portfolio managers aim to minimize the following investment problem:

$$\text{Min}\ \sigma_p{}^2 = \boldsymbol{w^t \Sigma w}\ \text{s.t}\ \boldsymbol{w^t 1} = 1$$

If '***1***' is a vector of ones, we utilize the following expression (16) for the computation of weights under minimum variance portfolios in the currency market:

$$\hat{w} = \frac{\hat{\Sigma}_i^{-1} 1}{1^t \hat{\Sigma}_i^{-1} 1} \tag{16}$$

The Herfindahl index is another tool we use to evaluate the degree of concentration of mean-variance optimal portfolios and minimal variance portfolios in the currency market and computed as $HI_l = w_{1l}^2 + w_{2l}^2 + w_{3l}^2 + ...w_{ql}^2$.

3.5. Evaluation of optimal portfolios

The two main evaluation criteria that are commonly supported in the body of literature on portfolio optimization are the systematic loss function, root mean square error (RMSE), and the risk profile of minimum variance portfolios. We follow the research of Jagannathan & Ma (2003), Byström (2004), Liu & Lin (2010), and Husnain et al. (2016a) for an in-depth understanding of RMSE and minimum variance portfolios. In Markowitz optimization or other mean-variance approaches, accurate estimation of the covariance matrix of asset returns is crucial. RMSE can be used to assess the errors in covariance matrix estimation. The RMSE would quantify how much the estimated covariance matrix deviates from the true (but often unobservable) covariance matrix, if available through simulation or theoretical models. Further, it helps quantify the accuracy of various models or estimation techniques used in constructing optimal portfolios. However, according to Markowitz's mean-variance optimization, asset returns are non-normal, therefore, supplementary assessment criteria will be used to assess and compare the performance of different models. Our first subsample frame serves as the basis for estimating covariance matrices, while estimators are evaluated for ex-post accuracy in the second subsample window. RMSE is computed as;

$$RMSE = \sqrt{\frac{H(H-1)}{2}\sum_{x=1}^{q}\sum_{y=1, x\neq y}^{q}\left(\hat{\sigma}_{xy} - \sigma_{xy}\right)^2} \tag{17}$$

If H counts the columns of the covariance estimator, then $\frac{H(H-1)}{2}$ denotes the pair-wise covariance estimators. Further, if the actual covariance between currencies x and y then σ_{xy} denotes the estimated covariance $\hat{\sigma}_{xy}$. The value of the RMSE is unit-free; therefore, it is easy to interpret. The covariance estimator with a lower RMSE results better in comparison to its counterpart.

Our next criterion is the performance profile of the optimal weights of the minimum variance portfolio (MVP) in the currency market. The performance profile of the optimal weights of the Minimum Variance Portfolio (MVP) provides insight into how the portfolio minimizes risk (variance) while not necessarily maximizing returns. The MVP is a key component of Markowitz Modern Portfolio Theory, and its performance is driven by the asset weights that minimize the overall portfolio variance, given the covariance structure of asset returns. The rationale behind selecting the MVP lies in its independence from the expected return vector; thus, the MVP relies solely on the covariance estimator. We opted for an unconditional MVP for several reasons, differing from the viewpoint of Jagannathan & Ma (2003). First, our primary concern is to analyze estimation errors that arise during the estimation process rather than assessing the performance of optimal portfolios. Second, our focus is on estimating covariances using buy-and-hold minimum variance portfolios rather than regularly rebalancing positions. Finally, the unconstrained MVP aligns with our second evaluation criterion, RMSE, ensuring consistency across both assessment criteria. Following Liu & Lin (2010), we first use each covariance estimator to construct the unconstrained MVP. We then evaluate the optimal portfolios' out-of-sample performance using these ideal MVP weights. Finally, we wrap up by looking at the MVP's risk and return characteristics in the currency market. While RMSE underscores the accuracy of covariance estimators, the MVP highlights the effectiveness of these estimators in achieving the MVP in the currency market. However, we assume that returns are non-normal based on the real-world behavior of assets.

Using performance indicators and concentration measurements, we assess the results of the mean-variance efficient portfolio and the lowest variance portfolio. In particular, we use 12 variance-covariance estimators in the currency market, including the standard deviation of portfolios, the Herfindahl index (HI), the number of positive weights (NPW), the number of negative weights (NNW), the highest value of weight (Max), and the minimum value of weight (Min).

4. Empirical findings

Table 4 provides details on the RMSE of 12 covariance estimators. It presents pairwise covariance estimations and their corresponding out-of-sample values. Additionally, the table includes RMSE values for covariance estimators computed from daily, weekly, fortnightly, and monthly data series of 10 traded currencies. A lower RMSE value for a covariance estimator indicates better performance compared to other estimators.

The results in Table 4 reveal that the sample estimator (VCE-2) performs worse than the constant correlation base estimator (VCE-3), but outperforms the single-factor base covariance estimator (VCE-4)

Table 4. Summary of RMSE of covariance estimators in currency market.

	Daily	Weekly	Fortnightly	Monthly
VCE-1	0.000000	0.000000	0.000000	0.000000
VCE-2	0.001043	0.004484	0.008495	0.027393
VCE-3	0.000630	0.004168	0.007249	0.015932
VCE-4	0.001468	0.007528	0.013003	0.026559
VCE-5	0.000522	0.002242	0.004247	0.013697
VCE-6	0.001062	0.004875	0.008801	0.022269
VCE-7	0.000680	0.003081	0.005688	0.015870
VCE-8	0.000874	0.004325	0.007691	0.018077
VCE-9	0.000655	0.003244	0.005769	0.013558
VCE-10	0.001043	0.004479	0.008479	0.027266
VCE-11	0.001043	0.004483	0.008485	0.026587
VCE-12	0.001038	0.004361	0.008158	0.024918

across all datasets except for monthly currency market data. Moreover, the overall mean method (VCE-3) yields a lower RMSE compared to the single-index base covariance estimator. Thus, the VCE-2 & VCE-3 exhibit closer RMSE values relative to the single-index base covariance estimator, with these findings holding robustly across four different data frequencies in our study.

We also compare the RMSE of traditional estimators with equally weighted portfolios of estimators, which are primarily proposed by Jagannathan & Ma (2003), across four different datasets in our study. It is evident from the table that VCE-7 outperforms the sample covariance estimator. Similarly, VCE-6 and VCE-8 outperform the single index-based covariance estimator in the currency market. Table 4 also reveals that VCE-7 results in a lower RMSE than the constant correlation-based covariance estimator under the weekly and fortnightly datasets, and VCE-7 remains competitive with VCE-3 in the monthly data series. Nevertheless, VCE-3 resulted in a relatively lower RMSE compared to VCE-6. Therefore, it is found that investors may achieve low RMSE by averaging two or more covariance estimators equally, rather than relying on any single traditional covariance estimator. Among the equally weighted portfolios of covariance estimators, VCE-7 outperforms VCE-6 and VCE-8; however, VCE-6 results in a relatively higher RMSE than VCE-8 in the currency market.

Next, we compare the RMSE of shrinkage covariance estimators with the traditional covariance estimators. The results show that VCE-12 outperforms the sample covariance estimator (VCE-2). Furthermore, VCE-11 results in a lower RMSE than VCE-4, except for in the monthly data. The sample covariance estimator produces competitive results with VCE-11 for daily datasets, whereas VCE-11 outperforms the sample covariance for the rest of the data series. Overall, the mean covariance estimator results in a lower RMSE than VCE-12 in the currency market.

The comparison between the comparatively basic equally weighted average scheme of covariance estimators, primarily introduced by Jagannathan & Ma (2003), and the more complex shrinkage covariance estimators proposed by Ledoit & Wolf (2003, 2004) is the most significant and critical. The shrinkage estimator VCE-10 performed worse than the identically weighted portfolio of estimator VCE-5. Similar to this, the complex shrinkage covariance estimator VCE-10 in the currency market has a higher RMSE than an equally weighted portfolio of the covariance estimator VCE-7. Additionally, in the daily time series, covariance estimators VCE-6 and VCE-11 reached competitive RMSE; however, VCE-11 outperformed the other estimators in the weekly and fortnightly data series. In currency market monthly datasets, VCE-6 performed better than VCE-11. In the currency market, VCE-12 did better than VCE-10 and VCE-11. Consequently, we conclude that investors are not further motivated to choose Ledoit & Wolf (2003, 2004) most sophisticated shrinkage covariance estimators rather than the currency market's equally weighted portfolio of simpler covariance estimators. Moreover, these results hold up well to the daily, weekly, fortnightly, and monthly quotes of the currency market in Paris.

The outcomes of our second comparison criterion, which looked at the risk profile of portfolios with the lowest variance under 12 variance-covariance estimators in the currency market, are shown in Table 5. Additionally, it shows the average standard deviation of the minimum variance portfolio weights for the 10 chosen currencies' daily, weekly, fortnightly, and monthly exchange rate quotes. We assessed the performance of the lowest variance portfolios built with different covariance estimation techniques; a covariance estimator performs better than another if it offers a comparatively less hazardous solution for portfolio optimization.

Table 5. Risk of GMVP under covariance estimators in currency market.

	Daily	Weekly	Fortnightly	Monthly
VCE-1	0.00458	0.01002	0.01374	0.02237
VCE-2	0.00478	0.01069	0.01495	0.02310
VCE-3	0.00495	0.01122	0.01558	0.02474
VCE-4	0.00491	0.01094	0.01524	0.02473
VCE-5	0.00472	0.01041	0.01446	0.02276
VCE-6	0.00489	0.01087	0.01515	0.02385
VCE-7	0.00492	0.01104	0.01535	0.02398
VCE-8	0.00493	0.01102	0.01533	0.02420
VCE-9	0.00485	0.01080	0.01501	0.02378
VCE-10	0.00478	0.01068	0.01495	0.02310
VCE-11	0.00478	0.01069	0.01495	0.02318
VCE-12	0.00478	0.01070	0.01496	0.02323

The Global Minimum Variance Portfolio (GMVP) standard deviation shows that the sample covariance estimate (VCE-2) performs better than the constant correlation-based covariance estimator (VCE-3), in contrast to RMSE, which assesses prediction accuracy. In particular, VCE-3 and VCE-4 continue to be competitive, but VCE-4 turns out to be marginally less hazardous than VCE-3. In the currency market, the sample covariance estimator yields a GMVP with a comparatively lower standard deviation than the single-index-based covariance estimate (VCE-4), which is in line with the RMSE findings.

The comparatively straightforward equally weighted average of covariance estimators VCE-6 and VCE-7, presented by Jagannathan & Ma (2003), outperforms the single index-based and constant correlation-based covariance estimators, respectively, which is another consistent result across both assessment criteria. Similar to the RMSE criterion, the sample covariance estimator leads to a GMVP with a relatively lower standard deviation compared to VCE-6 in our study.

Additional consistent results for both criteria are as follows: competitive covariance estimators are produced by VCE-11 outperforming VCE-4 and VCE-5 outperforming VCE-10, VCE-2, and VCE-11. Furthermore, VCE-12 outperforms VCE-2 based on the RMSE criterion; nonetheless, both exhibit competitive outcomes when compared to the GMVP risk profile in the currency market. Additionally, in contrast to RMSE, VCE-12 yields a GMVP that is less hazardous than VCE-7. Overall, we find that there are little differences in the relative performance of various covariance estimators under the daily, weekly, fortnightly, and monthly data series of currency exchange rates, as well as across our two assessment criteria.

Overall, we also observe that, given risk assessment criteria, the standard deviation of the least variance portfolio in the currency market shows that the relative performance of somewhat sophisticated estimators generally outperforms simpler estimators. These results are consistent with those of Jagannathan & Ma (2003), who indicate that even in cases where the limits appear arbitrary, limited portfolios typically yield significantly lower standard deviations.

For portfolio optimization in the currency market, we utilize the least variance portfolio formula and the mean-variance framework. We use performance evaluations, concentration levels in the portfolio, and diversification criteria to evaluate the efficacy of these computed optimal weights. We specifically calculate the following in the currency market: the highest weight value (Max), the minimum weight value (Min), the number of positive weights (NPW), the number of negative weights (NNW), the Herfindahl index (HI), and the portfolio standard deviation (P.Sd). Twelve variance-covariance estimators—classified as factor models, traditional estimators, equally weighted portfolios of estimators, and optimal shrinkage estimators—are reported using these assessment measures. We analyze and compare the characteristics of optimal weights computed from high-frequency (daily, weekly, fortnightly, and monthly) data of ten exchange rate quotes from the foreign exchange market. Table 6 and Table 7 provide comprehensive details of these computed weights for daily, and weekly datasets, respectively.

When it comes to the standard deviation of optimal weights, the mean-variance portfolios with the highest risk are those with monthly datasets, while higher-frequency exchange rate data series have the lowest standard deviation of ideal weights. Hence, the riskiness of portfolio weights varies significantly depending on the data frequency chosen. Similarly, portfolio concentration levels increase notably under monthly datasets compared to daily datasets. Furthermore, the number of positive weights and short positions appear largely independent of the data frequency choice. Similarly, Table 8, and Table 9 provide comprehensive details of these computed weights for fortnightly, and monthly datasets, respectively.

Additionally, these output characteristics are computed across 12 variance-covariance estimators. It is evident that mean-variance portfolios tend to exhibit concentration and often include short positions in the currency market. Notably, the minimum variance portfolio remains independent of the expected return vector chosen, indicating that the choice of covariance estimator affects the concentration level. Specifically, the constant correlation-based covariance estimator (VCE-3) and the equally weighted covariance estimator combining the sample estimator and constant correlation base (VCE-7) result in higher Herfindahl index values. Similarly, equally weighted portfolios (VCE-7 and VCE-8) tend to include relatively more short positions compared to VCE-10, VCE-11, and VCE-12 in minimum-variance portfolios.

Table 6. Characteristics of portfolios under alternative covariance estimators in currency market (Daily data).

	Weights under minimum variance portfolios						Efficient portfolio weights under mean-variance criteria					
Vcm	P.Sd	HI	NPW	NNW	Max	Min	P.Sd	HI	NPW	NNW	Max	Min
VCE-1	0.00094	0.6881	10	0	0.83	0.01	0.00135	1.1130	6	4	1.05	−0.09
VCE-2	0.00101	0.9002	7	3	0.95	−0.01	0.00157	1.2678	5	5	1.10	−0.13
VCE-3	0.00099	1.1408	1	9	1.07	−0.01	0.00131	1.3747	5	5	1.16	−0.12
VCE-4	0.00102	0.9221	7	3	0.96	−0.01	0.00154	1.2346	6	4	1.09	−0.12
VCE-5	0.00100	0.8613	8	2	0.93	0.00	0.00143	1.1830	5	5	1.08	−0.11
VCE-6	0.00101	0.9121	7	3	0.95	−0.01	0.00155	1.2397	5	5	1.09	−0.13
VCE-7	0.00103	1.0104	5	5	1.01	−0.01	0.00146	1.3045	5	5	1.13	−0.13
VCE-8	0.00103	0.9775	6	4	0.99	−0.01	0.00149	1.2746	5	5	1.11	−0.13
VCE-9	0.00103	0.9495	7	3	0.97	−0.01	0.00146	1.2422	5	5	1.10	−0.12
VCE-10	0.00101	0.9002	7	3	0.95	−0.01	0.00157	1.2677	5	5	1.10	−0.13
VCE-11	0.00101	0.9004	7	3	0.95	−0.01	0.00157	1.2672	5	5	1.10	−0.13
VCE-12	0.00101	0.9020	7	3	0.95	−0.01	0.00157	1.2680	5	5	1.10	−0.13

Table 6 presents various evaluation criteria for minimum variance portfolios and mean-variance portfolios in the currency market. These criteria encompass descriptive characteristics of optimal weights, performance metrics, diversification measures, and concentration measures. Specifically, Table 6 includes the portfolio standard deviation (P.Sd), Herfindahl index (HI), number of positive weights (NPW), number of negative weights (NNW), maximum weight value (Max), and minimum weight value (Min) in the currency market. It provides these evaluation metrics across all 12 considered variance-covariance estimators based on daily data series of 10 exchange rate quotes from the foreign exchange market. Equations 1 and 2 are used to compute the weights for the modern portfolio framework and minimum variance portfolios in the currency market, respectively; $\hat{\boldsymbol{w}}_{\boldsymbol{k}} = \hat{\Sigma}_{\boldsymbol{k}}^{-1}\mathbf{E}(\mathbf{r})/S[\hat{\Sigma}_{\boldsymbol{k}}^{-1}\boldsymbol{E}(\boldsymbol{r})]...(1)$ $\hat{w} = \hat{\Sigma}_i^{-1}\mathbf{1}/\mathbf{1}^t\hat{\Sigma}_i^{-1}\mathbf{1}...(2)$. For the value of Herfindahl index we uses the formula (3) and formula (4) is used for the computation of variance (Risk of portfolio) of portfolio in currency market; $HI_l = w_{1l}^2 + w_{2l}^2 + w_{3l}^2 + ...w_{ql}^2...(3)$, $\sigma_p{}^2 = \sum_{l=1}^{q}\sum_{k=l+1}^{q} w_l w_k \sigma_{lk}...(4)$.

From the results, it is evident that VCE-3 and VCE-7 yield relatively high values of HI (Herfindahl index), specifically 1.1408 and 1.0104, respectively, under minimum variance portfolios. Similarly, equally weighted portfolios (VCE-7 and VCE-8) result in a higher number of short positions compared to their counterparts VCE-10, VCE-11, and VCE-12 under minimum variance portfolios. The constant correlation-based covariance estimator (VCE-3) exhibits the second lowest standard deviation of portfolio weights, following VCE-1, under minimum variance portfolios. The results from the Herfindahl index indicate that mean-variance efficient portfolios tend to be concentrated.

When comparing the number of negative weights and positive weights between mean-variance portfolios and minimum variance portfolios, it becomes clear that mean-variance criteria yield counterintuitive results. Additionally, mean-variance criteria often lead to concentrated negative positions (minimum values of weights) in the currency market under daily datasets.

Table 7. Characteristics of different portfolios under battery of covariance estimators in currency market (Weekly data).

	Weights under minimum variance portfolios						Efficient portfolio weights under mean-variance criteria					
Vcm	PSd	HI	NPW	NNW	Max	Min	PSd	HI	NPW	NNW	Max	Min
VCE-1	0.00207	0.6735	10	0	0.82	0.01	0.00303	1.1324	6	4	1.06	−0.10
VCE-2	0.00225	0.9376	7	3	0.97	−0.02	0.00359	1.3944	4	6	1.14	−0.15
VCE-3	0.00219	1.1550	1	9	1.07	−0.01	0.00292	1.3996	5	5	1.17	−0.13
VCE-4	0.00226	0.9524	6	4	0.98	−0.01	0.00346	1.2904	5	5	1.11	−0.13
VCE-5	0.00224	0.8818	7	3	0.94	0.00	0.00327	1.2534	5	5	1.11	−0.12
VCE-6	0.00226	0.9455	6	4	0.97	−0.01	0.00348	1.3184	5	5	1.12	−0.14
VCE-7	0.00227	1.0436	3	7	1.02	−0.01	0.00324	1.3672	5	5	1.15	−0.14
VCE-8	0.00227	1.0104	5	5	1.00	−0.01	0.00332	1.3360	5	5	1.14	−0.14
VCE-9	0.00228	0.9757	6	4	0.99	−0.01	0.00326	1.2987	5	5	1.13	−0.13
VCE-10	0.00225	0.9376	7	3	0.97	−0.02	0.00359	1.3937	4	6	1.14	−0.15
VCE-11	0.00225	0.9382	7	3	0.97	−0.02	0.00358	1.3860	4	6	1.14	−0.15
VCE-12	0.00226	0.9472	7	3	0.97	−0.02	0.00355	1.3856	4	6	1.14	−0.15

Table 7 encompasses characteristics of optimal weights, performance metrics, diversification measures, and concentration measures. It includes the portfolio standard deviation (P.Sd), Herfindahl index (HI), number of positive weights (NPW), number of negative weights (NNW), maximum weight value (Max), and minimum weight value (Min) in the currency market. This table reports these evaluation metrics across all 12 variance-covariance estimators considered on the weekly data series of 10 exchange rate quotes from the foreign exchange market.

Equations 1 and 2 were utilized to compute the weights for the modern portfolio framework and minimum variance portfolios in the currency market, respectively; $\hat{\boldsymbol{w}}_{\boldsymbol{k}} = \hat{\Sigma}_{\boldsymbol{k}}^{-1}\mathbf{E}(\mathbf{r})/S[\hat{\Sigma}_{\boldsymbol{k}}^{-1}\boldsymbol{E}(\boldsymbol{r})]...(1)$ $\hat{w} = \hat{\Sigma}_i^{-1}\mathbf{1}/\mathbf{1}^t\hat{\Sigma}_i^{-1}\mathbf{1}...(2)$. For the value of Herfindahl index we uses the formula (3) and formula (4) is used for the computation of variance (Risk of portfolio) of portfolio in currency market; $HI_l = w_{1l}^2 + w_{2l}^2 + w_{3l}^2 + ...w_{ql}^2...(3)$, $\sigma_p{}^2 = \sum_{l=1}^{q}\sum_{k=l+1}^{q} w_l w_k \sigma_{lk}...(4)$.

Consistent with the daily data output, Table 7 shows that VCE-3 and VCE-7 result in relatively high HI values, 1.1550 and 1.0436 respectively, under minimum variance portfolios. However, the concentration level of the minimum variance portfolio increases in the weekly data compared to the daily exchange rate quotes. Similarly, equally weighted portfolios (VCE-7 and VCE-8) result in a relatively higher number of short positions than their counterparts VCE-10, VCE-11, and VCE-12 under minimum variance portfolios. The constant correlation base covariance estimator (VCE-3) results in the second lowest standard deviation of portfolio weights, just after VCE-1, under minimum variance portfolios. From the results of the Herfindahl index, it is clear that mean-variance efficient portfolios are concentrated. When we compare the number of negative weights and positive weights of mean-variance portfolios with minimum variance portfolios, it is clear that mean-variance criteria provide counterintuitive results. In addition, the mean variance criteria also end up with a concentrated negative position (minimum value of weights) in the currency market under the weekly datasets.

Table 8. Characteristics of different portfolios under series of covariance estimators in currency market (Fortnightly data).

	Weights under minimum variance portfolios						Efficient portfolio weights under mean-variance criteria					
vcm	PSd	HI	NPW	NNW	Max	Min	PSd	HI	NPW	NNW	Max	Min
VCE-1	0.00285	0.6526	10	0	0.81	0.01	0.00481	1.2837	5	5	1.12	−0.13
VCE-2	0.00311	0.9456	6	4	0.97	−0.03	0.00586	1.6769	6	4	1.23	−0.25
VCE-3	0.00303	1.1656	1	9	1.08	−0.01	0.00437	1.4885	5	5	1.20	−0.15
VCE-4	0.00312	0.9548	6	4	0.98	−0.02	0.00546	1.4524	5	5	1.17	−0.18
VCE-5	0.00311	0.8808	7	3	0.94	0.00	0.00518	1.4140	5	5	1.17	−0.17
VCE-6	0.00313	0.9499	5	5	0.97	−0.02	0.00563	1.5247	5	5	1.19	−0.20
VCE-7	0.00313	1.0527	3	7	1.03	−0.01	0.00501	1.5137	5	5	1.20	−0.19
VCE-8	0.00314	1.0174	5	5	1.01	−0.01	0.00518	1.4877	5	5	1.19	−0.19
VCE-9	0.00316	0.9794	5	5	0.99	−0.01	0.00507	1.4329	5	5	1.17	−0.17
VCE-10	0.00311	0.9454	6	4	0.97	−0.03	0.00586	1.6748	6	4	1.23	−0.25
VCE-11	0.00312	0.9458	6	4	0.97	−0.03	0.00584	1.6560	5	5	1.23	−0.25
VCE-12	0.00313	0.9602	6	4	0.98	−0.03	0.00573	1.6402	5	5	1.23	−0.24

Table 8 reports different evaluation criteria of minimum variance portfolios and mean-variance portfolios in currency market. These evaluation measures can be club into descriptive characteristics of optimal weights, performance measures, diversification measures, and concentration measures. Table 8 also includes the standard deviation of portfolios (P.Sd), Herfindahl index (HI), number of positive weights (NPW), number of negative weights (NNW), maximum value of weight (Max), and minimum value of weight (Min) in currency market. It also reports these evaluation dimensions across all considered 12 variance covariance estimators on the fortnightly data series of 10 exchange rate quotes from foreign exchange market. Specifically, we computed the following Equations 1 and 2 for the computation of weights for modern portfolio framework and weights under minimum variance portfolios in currency market, respectively; $\hat{\boldsymbol{w}}_{\boldsymbol{k}} = \hat{\Sigma}_{\boldsymbol{k}}^{-1}\mathbf{E}(\mathbf{r})/S[\hat{\Sigma}_{\boldsymbol{k}}^{-1}\boldsymbol{E}(\boldsymbol{r})]\ldots(1)$ $\hat{w} = \hat{\Sigma}_i^{-1}\mathbf{1}/\mathbf{1}^t\hat{\Sigma}_i^{-1}\mathbf{1}\ldots(2)$. For the value of Herfindahl index we uses the formula (3) and formula (4) is used for the computation of variance (Risk of portfolio) of portfolio in currency market; $HI_I = w_{1I}^2 + w_{2I}^2 + w_{3I}^2 + \ldots w_{qI}^2\ldots(3)$, $\sigma_p{}^2 = \sum_{I=1}^{q}\sum_{k=I+1}^{q} w_I w_k \sigma_{Ik}\ldots(4)$.
Consistent with the daily and weekly data output, Table 7 shows that VCE-3 and VCE-7 result in relatively high HI values, 1.1656 and 1.0527 respectively, under minimum variance portfolios. However, the concentration level of the minimum variance portfolio increases in the fortnightly data compared to the daily and weekly exchange rate quotes. Similarly, equally weighted portfolios (VCE-7 and VCE-8) result in a relatively higher number of short positions than their counterparts VCE-10, VCE-11, and VCE-12 under minimum variance portfolios. The constant correlation base covariance estimator (VCE-3) results in the second lowest standard deviation of portfolio weights, just after VCE-1, under minimum variance portfolios. From the results of the Herfindahl index, it is clear that mean-variance efficient portfolios are concentrated. When we compare the diversification measures and performance measures of mean variance portfolios with minimum variance portfolios, it is clear that mean-variance criteria provide counterintuitive results. In addition, the mean variance criteria also end up with a concentrated negative position (minimum value of weights) in the currency market under the fortnightly datasets.

Table 9. Characteristics of different portfolios under series of covariance estimators in currency market (Monthly data).

	Weights under minimum variance portfolios						Efficient portfolio weights under mean-variance criteria					
Vcm	PSd	HI	NPW	NNW	Max	Min	PSd	HI	NPW	NNW	Max	Min
VCE-1	0.00486	0.5949	10	0	0.77	0.02	0.00905	1.3867	5	5	1.16	−0.18
VCE-2	0.00539	0.9383	6	4	0.97	−0.03	0.01051	2.0086	5	5	1.31	−0.35
VCE-3	0.00520	1.2041	1	9	1.10	−0.02	0.00797	1.6306	5	5	1.25	−0.21
VCE-4	0.00540	0.9208	7	3	0.96	−0.03	0.01115	1.7236	5	5	1.24	−0.28
VCE-5	0.00540	0.8579	8	2	0.93	0.00	0.00951	1.5424	5	5	1.21	−0.23
VCE-6	0.00543	0.9254	5	5	0.96	−0.02	0.01072	1.7898	5	5	1.26	−0.30
VCE-7	0.00545	1.0634	4	6	1.03	−0.02	0.00910	1.6907	5	5	1.25	−0.26
VCE-8	0.00547	1.0116	4	6	1.01	−0.01	0.00974	1.6823	5	5	1.24	−0.27
VCE-9	0.00550	0.9686	4	6	0.98	−0.01	0.00951	1.5839	5	5	1.22	−0.24
VCE-10	0.00539	0.9379	6	4	0.97	−0.03	0.01049	1.9946	5	5	1.30	−0.34
VCE-11	0.00540	0.9346	6	4	0.96	−0.03	0.01052	1.9370	5	5	1.29	−0.33
VCE-12	0.00543	0.9727	5	5	0.98	−0.02	0.01002	1.8599	5	5	1.28	−0.32

Table 9 reports different evaluation criteria of minimum variance portfolios and mean-variance portfolios in currency market. These evaluation measures can be club into descriptive characteristics of optimal weights, performance measures, diversification measures, and concentration measures. Table 8 also includes the standard deviation of portfolios (P.Sd), Herfindahl index (HI), number of positive weights (NPW), number of negative weights (NNW), maximum value of weight (Max), and minimum value of weight (Min) in currency market. It also reports these evaluation dimensions across all considered 12 variance covariance estimators on the monthly data series of 10 exchange rate quotes from foreign exchange market. Specifically, we computed the following Equations 1 and 2 for the computation of weights for modern portfolio framework and weights under minimum variance portfolios in currency market, respectively; $\hat{\boldsymbol{w}}_{\boldsymbol{k}} = \hat{\Sigma}_{\boldsymbol{k}}^{-1}\mathbf{E}(\mathbf{r})/S[\hat{\Sigma}_{\boldsymbol{k}}^{-1}\boldsymbol{E}(\boldsymbol{r})]\ldots(1)$ $\hat{w} = \hat{\Sigma}_i^{-1}\mathbf{1}/\mathbf{1}^t\hat{\Sigma}_i^{-1}\mathbf{1}\ldots(2)$. For the value of Herfindahl index we uses the formula (3) and formula (4) is used for the computation of variance (Risk of portfolio) of portfolio in currency market; $HI_I = w_{1I}^2 + w_{2I}^2 + w_{3I}^2 + \ldots w_{qI}^2\ldots(3)$, $\sigma_p{}^2 = \sum_{I=1}^{q}\sum_{k=I+1}^{q} w_I w_k \sigma_{Ik}\ldots(4)$.
Consistent with the daily and weekly data output, Table 7 shows that VCE-3, VCE-7, and VCE-8 result in relatively high values of the Herfindahl Index (HI), i.e. 1.2041, 1.0634, and 1.0116, respectively, under minimum variance portfolios. In some cases, the concentration level of the minimum variance portfolio increases in monthly data compared to the daily, weekly, and fortnightly exchange rate quotes. Similarly, equally weighted portfolios (VCE-7 and VCE-8) result in a relatively higher number of short positions than their counterparts VCE-10, VCE-11, and VCE-12 under minimum-variance portfolios. The constant correlation-based covariance estimator (VCE-3) results in the second lowest standard deviation of portfolio weights, just after VCE-1, under minimum variance portfolios. From the results of the Herfindahl Index, it is clear that mean-variance-efficient portfolios are concentrated. When we compare the diversification measures and performance measures of mean-variance portfolios with minimum variance portfolios, it is clear that mean-variance criteria provide counterintuitive results. In addition, the mean-variance criteria also end up with a concentrated negative position (minimum value of weights) in the currency market under the monthly datasets.

Moreover, the minimal variance portfolio's risk profile implies that the frequency of the dataset has an impact on how risky optimal weights are. These results hold up well when compared to low-frequency datasets of the currencies we picked for our analysis.

5. Discussion

Fund managers, investors, and practitioners in the financial industry are seeking optimal ways to estimate covariance matrices and develop optimal portfolios. Covariance estimators are fundamental components in a range of econometrics and statistical applications, such as portfolio optimization, hedging, value at risk, risk management, financial engineering, pricing of derivatives, and asset pricing (Lin et al., 2016; Bjerring et al., 2017; Hounyo, 2017; Münnix et al., 2014; Harris & Yilmaz, 2010; Ikeda & Kubokawa, 2016). However, existing literature does not provide a real solution for the estimation of inputs to portfolio optimization within the modern portfolio framework. Our findings in the currency market are consistent with the studies of Liu & Lin (2010), Disatnik & Benninga (2007), and Moura et al. (2020), indicating that investors cannot earn additional benefits from complex shrinkage estimators of covariances. Similarly, our results align with the study of DeMiguel et al. (2009a) that sophisticated strategies result in estimation errors. Therefore, we suggest that investors should rely on a 1/N strategy for portfolio selection in the currency market. Although Ledoit & Wolf (2003, 2004) use an optimal method to estimate the optimal shrinkage intensity factor, our findings show that their shrinkage estimators cannot consistently outperform the 1/N strategy. A possible reason for this poor performance is the errors that arise during the process of portfolio optimization in the currency market. Further, we recommend that investors and fund managers consider the 1/N strategy and mean-variance optimal portfolio as good starting points for portfolio selection in the currency market.

We believe that different strategies for portfolio selection encounter estimation errors in different directions, depending on the underlying set of assumptions. Therefore, rather than focusing on a single estimation strategy, investors should utilize multiple estimation techniques to diversify errors and develop optimal portfolios in the currency market. This idea of diversifying errors is rooted in the study of Jagannathan & Ma (2003) on the NYSE in the USA market. Similarly, Ledoit & Wolf (2003, 2004) also focused on the diversification of errors through complex shrinkage covariance estimators. However, we believe that rather than concentrating on optimal shrinkage intensity, investors should also consider the 1/N strategy for developing covariance estimators in the currency market. Therefore, we recommend that investors, fund managers, and practitioners acknowledge that theoretically rigorous shrinkage covariance estimators are elegant but not necessarily useful for portfolio selection in the currency market. Similarly, the renewed mean-variance criteria often suggest concentrated and meaningless portfolios in the currency market.

Investors and fund managers require a robust framework for portfolio selection that considers the risks and rewards of portfolios. This study adds insights into the portfolio optimization literature in the currency market by suggesting an optimal method for portfolio selection. It provides guidelines for econometrics and various statistical applications, including portfolio optimization, financial engineering, and the pricing of derivatives in equities, currencies, and commodities. Estimating covariances among asset classes is as important as estimating the variance for a single asset class. The outcomes of this study provide useful guidelines for financial institutions: economists can use it for portfolio maximization, while risk managers can calculate value at risk based on the variance-covariance matrix. It is equally important for risk identification, risk mitigation, risk analysis, and risk management through hedging, options, derivative securities, and financial engineering.

6. Conclusion

The global nature of today's world provides opportunities for investors and fund managers to go beyond traditional portfolio selection methods and equip their portfolios with an appropriate mix of asset classes. We investigate the phenomenon of currency market portfolio selection. The empirical literature on the estimation of covariance matrices for portfolio selection is heavily skewed toward equity markets and provides no real consensus on the optimal method for portfolio selection. In the

investment basket of the ten most traded currencies, our paper empirically examines the concentration level and financial efficiency of mean-variance portfolios, minimum-variance portfolios, and naively diversified portfolios. Our goal is to estimate the variance-covariance matrix for portfolio selection. We test the out-of-sample performance of factor models, conventional models, linear shrinkage estimators, and equally weighted portfolios of estimator portfolios, as well as the theoretically rigorous shrinkage covariance estimators. We employ daily, weekly, fortnightly, and monthly data series of exchange rate quotes covering the time frame from January 1985 to January 2021 to ensure robustness.

The evaluation criteria used in this study include the systematic loss function (also known as the root mean square error), the risk profile of minimum variance portfolios, the Herfindahl index, the standard deviation of optimal portfolios, the number of positive weights, the total number of short positions, the maximum and minimum values of weights, and the degree of diversification. It was revealed that mean-variance optimal portfolios in the currency market are sensitive to the optimizer's input selection, concentrated, and counterintuitive. When we compare the performance of linear shrinkage covariance estimators with the 1/N strategy of covariance formulation, we find that shrinkage estimators do not provide any additional benefit to investors and fund managers in terms of the systematic loss function and risk profile of the minimum variance portfolios in the currency market. Our findings are robust across the daily, weekly, fortnightly, and monthly datasets.

The findings can help practitioners avoid unnecessary complexity and focus on practical, efficient portfolio-building strategies. Furthermore, the emphasis on dispersing estimating errors highlights the need for a more roboust and sophisticated approach to portfolio management. Combining several estimation approaches allows practitioners to reduce the effects of potential biases while also improving overall portfolio performance. Finally, this study provides practitioners with a practical and fact-based approach to portfolio management, allowing them to make more informed and profitable investment decisions.

Although we employ the most sophisticated comparison criteria, future researchers can develop better comparison criteria than the risk profile of the minimum variance portfolio and root mean square error in the currency market. This is because the root mean square error primarily overlooks the matrix's general structure and concentrates mainly on pairwise differences, and the least variance portfolio is just one portfolio on the frontier. Furthermore, behavioral aspects can be applied in future studies, which should concentrate on behavioral portfolio optimization for portfolio selection.

This study recommends the followings: firstly, simplicity trumps complexity: our findings show that when designing portfolios, investors and fund managers should choose simplicity over complexity. The 1/N technique is a trustworthy and effective method, especially in the currency market. Secondly, diversification of estimate errors: investors should consider employing multiple tactics to spread estimation errors rather than relying just on one. This method can improve overall portfolio performance while mitigating the consequences of potential biases. Thirdly, investors should focus on building robust portfolios that can endure market volatility and forecasting errors. This is performed by taking into account factors such as investment timeframe, risk tolerance, and specific investment objectives. Fourthly, continual portfolio monitoring and rebalancing are required to maintain optimal performance and respond to changing market conditions. By following these guidelines, practitioners and portfolio managers can improve the overall performance of their portfolios and make more informed investment decisions.

Note

1. We used the following formula for computation of inputs to portfolio optimization in global currency framework; *Variance of Currency* $= \sigma_k^2 = \frac{1}{m} \sum_{t=1}^{m} [(R_{kt} - k)]^2$, Correlation Coefficient;

$$\rho_{i,j} = \frac{\sum_{t=1}^{m} [R_{kt} - E(r_k)][R_{kt} - E(r_k)]}{\sqrt{\sum_{t=1}^{m} [r_{kt} - E(r_k)]^2 \sum_{t=1}^{m} [R_{kt} - E(r_k)]^2}} \quad Covariance = Cov\ (r_k, r_j) = \frac{1}{m} \sum_{t=1}^{m} [r_{k,t} - E(r_k)]\left[r_{j,t} - E(r_j)\right].$$

Covariance and correlation are interrelated with the expression $\rho_{kj} = \frac{Cov(r_k, r_j)}{\sigma_k \sigma_j}$.

Authors' contributions

Conceptualization, Muhammad Husnain; Methodology, Muhammad Husnain, Shamrez Ali; Software, Muhammad Husnain, Qaiser Munir; Formal analysis, Shamrez Ali, Qaiser Munir; Writing—original draft preparation, Muhammad Husnain, Qaiser Munir; writing—review and editing, Ammar Jreisat. All authors have read and agreed to the published version of the manuscript.

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Data availability statement

The dataset used in this study was the secondary data mentioned in the data description section of this research paper. Data can be provided upon a reasonable request.

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