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Advertising, Consumption and Economic Growth: An Empirical Investigation

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Advertising, Consumption and Economic Growth: An Empirical Investigation*

- Preliminary Version -

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Abstract
It is sometimes argued that more advertising raises consumption which in turn stimulates output and so economic growth. We test this hypothesis using annual German data expressed in terms of GDP for the period 1950-2000. We find that advertising does not Granger-cause growth but Granger-causes consumption. Consumption, in turn, Granger-causes GDP growth. The data imply that the immediate impact of more advertising on consumption is positive. However, the long-run effect is negative. Furthermore, the immediate impact of higher consumption on growth is negative. But the long-run effect is positive. These results raise interesting questions for standard theory, political debates and advertising practitioners.

Keywords: Advertising, Consumption, Economic Growth

JEL Classification: O4, M3, E2

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1 Introduction

In this paper we investigate the link between advertising, consumption and economic growth. This link deserves attention because it is often argued that domestic demand deficiencies may be the cause of low growth.

For the short run it has become a regular finding by business cycle research that output co-varies pro-cyclically with private consumption and investment. These two aggregates are clearly important in national income accounting and for growth research. It is also well known that consumption in most advanced countries makes up around 60 percent of national income compared to around 15 percent that is made up by private investment. Thus, changes in private consumption should have a relatively important impact on aggregate demand and output.

It is, thus, argued that more consumption would raise output. One way to stimulate the former might be an increase in advertising. Clearly, advertising, at least in the consumer goods sector, is targeted directly at potential buyers of (final) goods. If the effect of advertising is to persuade customers to buy a good and they indeed do that, then advertising should have a positive effect on measured consumption. Hence, it is often hypothesized that more advertising raises consumption which in turn stimulates output and so economic growth. Although the hypothesis has intuitive appeal, theory leads one to be cautious about the validity of this chain of reasoning.

To our knowledge the direct link between advertising and economic growth has not been analyzed in detail so far. This is a bit unfortunate since advertising is a pervasive phenomenon that has been around for a long time. For example, the

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1For the United States see, for example, Blanchard and Fischer (1989), p. 16. For Germany see, for example, Tichy (1994), p. 78.
2A similar logic holds for advertising in the investment goods sector. If advertising there leads to more purchases of investment goods, this is expected to raise investment, bearing positively on output and personal incomes. Higher incomes then often also lead to more consumption.
3For an earlier analysis see, for example, Galbraith (1972), chs. 18-20. He argues that advertising serves to "manage" demand. The continuous upholding of high levels of the latter is argued to increase or keep output at a higher level than it would otherwise be. Thus, he basically advocates a hypothesis that higher advertising implies higher demand (consumption) and - thinking in Keynesian terms - that that would stimulate output and its growth. For an earlier formal analysis on these links see, for example, Frey (1969). The ensuing debates from that hypothesis were not really settled, however, and have not been taken up in more recent times.
history of brand creation can at least be dated back to the times of the Industrial Revolution. One such brand is "Made in Country X" which was created around that time.

Clearly, many business people consider advertising an important economic factor nowadays as the budgets spent on it testify. Thus, some theory on the relationship between advertising and growth would appear to be desirable.

On the other parts of the "causal" links embedded in the hypothesis theory would suggest the following: In accumulation-driven growth models in the tradition of Solow (1956) higher consumption cannot really lead to higher growth. Growth depends on investment and savings. Thus, in a simple Solow growth model a higher consumption share (aggregate private consumption in terms of GDP) would lower the savings share and would usually imply lower growth for some time, that is, in the transition from the old to a new steady state with higher consumption.\(^4\) Also, innovation-driven growth models like Romer (1986), Romer (1990), or Aghion and Howitt (1992) suggest the same. Hence, no clear and, if at all, rather a negative relationship between consumption shares and economic growth is usually predicted by theory.\(^5\)

The issue of the interaction between advertising and consumption has primarily been analyzed by economists in the industrial economics literature.\(^6\) (See e.g. Tirole (1988), ch. 7, or Martin (2002), ch. 9.) The focus here has usually been on the decision of firms to invest in advertising in order to get a competitive edge in a non-cooperative world. Illuminating for our purpose is the seminal paper by Dixit and Norman (1978). They model advertising as having a direct positive effect on utility. ("Don’t we all feel happy when we see those smiling people on billboards or tv?") But firms in an oligopolistic market will compete in advertising investment which will raise the price of goods and might be bad

\(^4\)Of course, in a simple Solow growth model the long-run steady state growth rate would not be affected by these changes. In that sense Solow (1956) would still imply that the long-run response to an increase in the consumption share would be zero and thus neutral. The same would usually apply to other accumulation-driven growth models. See e.g. Barro and Sala-i-Martin (1995).

\(^5\)In the 1990s many commentators argued that the increase in growth rates in the U.S. was consumption-driven. However, no clear academic research agenda in economics emanated from this, perhaps debatable, claim for long-run phenomena.

\(^6\)Clearly, the business administration literature has devoted a bulk of literature on the subject. However, for lack of space we confine ourselves to the debates featuring in the economics literature in this paper. For an early economics study of the advertising industry in the United Kingdom see, for example, Kaldor (1950).
for demand of the product. They show that the combined effect of more advertising is negative for welfare. Thus, demand might actually fall due to increases in advertising.

Summarizing, we find that the theoretical predictions on the relationship between advertising, consumption and growth is not clear. Thus, in this paper we test the hypothesis of the link from advertising to consumption to growth empirically. Our focus is on the long run because we want to link up to current growth research, and growth is ultimately a long-run phenomenon.

For our test we use annual German data expressed in terms of GDP for the period 1950-2000. The reason for focusing on Germany is threefold. First Germany is a big market. In terms of GDP it is the third largest economy in the world and the largest one in the European Union. Thus, consumption and advertising in that country should have an impact on its trading partners in a globalized world. Second, we have obtained reliable data for German advertising expenditure from sources that we are more familiar with. This is because knowledge of everyday advertising impacts on consumer behaviour appears to be required as “off-the-record-information” to relate to arguments by advertising practitioners in any country. In our case this is Germany. Third, we compare our findings to those obtained by earlier studies focusing on the United States as the world’s largest economy in terms of GDP. Hence, we compare findings for the largest economy in the world to the largest European one.

The data in this paper are taken from the German Statistical Office (Statistisches Bundesamt) and the German Central Association of Advertising Business (Zentralverband der Werbewirtschaft, ZAW). Our key variables are the consumption share, that is, the ratio of private consumption to GDP, the advertising share, that is, the ratio of total advertising expenditure GDP, and the growth rate of GDP.

We use standard methods to analyze our variables. For the data it turns out that all three variables are stationary. Next, we conduct pairwise comparisons of the time series of our data to check for Granger-causality.\footnote{Granger-causality establishes whether one series helps to predict another series. This concept is explained in more detail below.} We find that advertising does not Granger-cause growth but Granger-causes consumption. Consumption, in turn, Granger-causes GDP growth. For all other pairs we cannot reject the null hypothesis that no causality is present.
These results are interesting. For instance, Ashley, Granger, and Schmalensee (1980) find for U.S. data that aggregate consumption Granger-causes aggregate advertising expenditure using quarterly data.\textsuperscript{8} Their results would suggest that consumption predicts advertising, whereas we find a one way ”causality” according to which advertising predicts consumption in Germany. Of course, their data are of higher frequency so that the results are not necessarily incompatible with each other. As our focus is on the long run, our findings do, however, suggest a different prediction pattern - at least for Germany.

Furthermore, the exercise casts doubt on the maintained link. Granger causality is \textit{not} transitive. Thus, the direct Granger-causal effect of advertising on growth is not present in our data. Also, one cannot conclude from the other relationships that there is a link from advertising through consumption on growth. Thus, the causality analysis alone casts doubt on the hypothesis.

Building on these causality results we use standard, well established statistical arguments to build empirical models that best describe the time series of our data.

These models yield some surprising results. We find that the immediate impact of more advertising on consumption is positive. However, the long-run effect is \textit{negative}. Furthermore, the immediate impact of higher consumption on growth is negative. But the long-run effect of higher consumption on growth is \textit{positive}.

These results raise interesting questions for standard theory. We offer these suggestive explanations of our findings for the long run:

The negative effect of more advertising on consumption may indeed lend support to arguments in line with Dixit and Norman (1978). However, their conclusion focuses on welfare whereas we find that more advertising may ultimately be bad for more consumption at the macro level. Of course, our results only hold relative to GDP. An alternative viewpoint would be that more advertising does not foster more consumption - at least not in Germany. Furthermore, the result has suggestive implications for consumer sovereignty that are discussed in the main text.

The consumption-growth result is peculiar. It would cast doubt on standard growth models - at least for Germany. It might also be taken to imply for accumulation-driven growth models that investment may actually be driven by

\textsuperscript{8}In this context, Jung and Seldon (1995) find that consumption and advertising are characterized by a feedback relationship in the U.S. according to which advertising ”causes” consumption which in turn ”causes” advertising.
consumption. This would suggest that some form of accelerator in the spirit of e.g. Samuelson (1939) is at work determining investment and economic growth - at least in Germany.\footnote{For a recent publication that models growth as positively depending on the consumption share see, for example, Commendatore, D’Acunto, Panico, and Pinto (2003).}

The paper is organized as follows. Section 2 describes the data and presents some descriptive statistics. In section 3 we analyze stationarity and Granger-causality. Section 4 derives models capturing the times series properties of the data. Section 5 concludes.

\section{The Hypothesis and the Data}

It is convenient to restate succinctly the main claim we wish to test.

\textbf{Hypothesis} Advertising is good for consumption. Consumption is good for output and output growth. Hence, advertising is good for economic growth.

We test this claim using German, yearly data for the period 1950-2000. As our focus is on economic growth and, thus, on the long run, 50 years of observations serve our purposes well.\footnote{Similar times spans and so number of observations have been used for empirical arguments in the growth literature. See, for example, Jones (1995). For a multivariate time series analysis with a relatively smaller sample of twenty nine observations see, for instance, Teräsvirta (2004).} The data sources and precise definitions of our variables are presented below.

\subsection{The growth rate}

The yearly growth rate of \textit{real}, aggregate GDP is defined in a standard way as

\[ g_t = \frac{(Y^r_t - Y^r_{t-1})}{Y^r_t} \]

where $Y^r_t$ denotes real, aggregate GDP in period $t$. Data for $g_t$ are taken from publications of the Federal Statistical Office, Germany, (Statistisches Bundesamt).\footnote{In particular, we have used Table 23.2 of Statistisches Bundesamt (2004b), and Statistisches Bundesamt (2004c), Table 1.3.2.} For this paper we use the yearly growth rates of real GDP in constant prices.
Data are available for the period 1950-1970 in constant prices of 1991 and for the period 1971-2000 in constant prices of 1995.\(^{12}\)

Note we do not use the growth rate of GDP per capita as is often done in growth empirics. As GDP growth is a prerequisite for growth of GDP per capita and since the population in Germany has hardly grown, except for the one-time population shock due to unification, the results are (perhaps heroically) taken to apply to both concepts. Even if this does not apply, do we believe that concentrating on the growth rate of GDP is interesting in its own right and relevant for growth research.

### 2.2 The consumption share

Various sources provide data on the consumption share. We have chosen to construct our measure of the consumption share as much as possible from German data sources. First, we define the consumption share as

\[ c_t = \frac{C_t^N}{Y_t^N}, \]

where \( C_t^N \) and \( Y_t^N \) denote the nominal aggregate consumption expenditures of the private sector and the nominal aggregate GDP at time \( t \) in billions of Euros.\(^{13}\) \( c_t \) is taken to proxy for the same variable when consumption and output would have been measured in real terms. This procedure is often applied in the growth empirics literature and is followed here.

### 2.3 The advertising share

Investment in advertising comprises such expenditures as those on salaries, media and the production of means of publicity. The German Central Association of

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\(^{12}\)Thus, no growth rate is provided for 1970. To circumvent the problem of missing values in software packages that we used for our analysis we choose to specify a value of the growth rate for 1970. All checks revealed that the growth rate of 1970 was not an outlier in the evolution of growth rates over the immediate time period leading to and starting from 1970. It was around 5 percent. Thus, we have chosen to set \( g_{70} \) at the average value of \( g_{69} \) and \( g_{71} \) which was 5.4 percent. This also seems a reasonable choice when checking for the growth rate of German GDP in 1970 on the internet.

\(^{13}\)It is important to note that we concentrate on private sector consumption. Thus, our consumption share does not include government consumption. See the appendix for more details.
Advertising Business (Zentralverband der deutschen Werbewirtschaft, ZAW) in Bonn publishes data on these expenditures for the whole industry and for various years.\footnote{See ZAW (1950-2000), Table ”Investitionen in Werbung” (Investment in Advertising). The data were courteously assembled for and made available to us by ZAW.}

The data refer to gross expenditures in advertising and are expressed in billions of Euros and are, thus, in nominal terms. In order to relate to the growth rate we constructed the variable

\[ ad_t = \frac{\text{(Advertising Expenditure)}_t}{Y_t} \]

so that \( ad_t \) denotes the advertising share in nominal GDP at time \( t \). We again assume that \( ad_t \) is a proxy for the same variable when investment in advertising and output would have been measured in real terms.

\subsection*{2.4 Descriptive Statistics}

The growth rate of German GDP has fluctuated quite a lot over the last fifty years as Figure 1 demonstrates. Around 1950 the growth rate was 10 percent and raises to a maximum of 12 percent in 1955. From the figure one clearly identifies the recessions of 1967, 1974 (first oil price shock), 1981 (second oil price shock) and 1993 when the growth rate fell to zero and approximately minus one percent, respectively. Since 1970 the German growth rate fluctuates between -1 percent and +5 percent. Furthermore, for the period after 1990 it looks as if there is a downward trend in the growth rate, which may be interpreted as being a consequence of German reunification.
In comparison to the growth rate the (private) consumption share has fluctuated far less. The consumption share starts at a high value of 65 percent in 1950 and seems to fall to lower levels subsequently. Since the 1960s the consumption shares have been somewhere between 54 percent and 58 percent. Interestingly, they are quite high during, or right after the recessions of 1967 and 1981-1984. On the other hand, preceding the recession of 1974 the consumption shares for the years 1970 to 1974 are relatively low. Since 1990 the consumption share for all of Germany does not seem to follow a particular trend.
Figure 3 depicts the evolution of the advertising share. Advertising investment has increased from 0.64 percent of GDP in 1950 to a maximum of 1.72 percent in 1967. The advertising share then falls to a level of 1.2 percent in 1974 and then increases again. There does not seem to be a clear trend in the series after 1985, even after reunification. Since 1995, however, the advertising share appears to increase.

The following table summarizes the main descriptive properties of the series:

<table>
<thead>
<tr>
<th>Series</th>
<th>Period</th>
<th>Obs.</th>
<th>Mean</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_t$</td>
<td>1951-2000</td>
<td>49</td>
<td>3.930</td>
<td>12.10</td>
<td>-1.10</td>
<td>3.02</td>
</tr>
<tr>
<td>$c_t$</td>
<td>1950-2000</td>
<td>50</td>
<td>0.570</td>
<td>0.65</td>
<td>0.53</td>
<td>0.02</td>
</tr>
<tr>
<td>$ad_t$</td>
<td>1950-2000</td>
<td>50</td>
<td>0.014</td>
<td>0.017</td>
<td>0.006</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Over the sample period the mean growth rate of GDP was 3.93 percent, the mean consumption share was 57 percent, and that for the advertising share amounted to 1.4 percent. We note that the means of the series are all positive.
3 Stationarity and Granger-causality

3.1 Stationarity

We wish to relate the time series of three economic variables to each other. In order to apply standard econometric techniques the series must at least have the property of (weak) stationarity. The latter is given when (a) the mean of a series $x_t$ is independent of $t$, (b) the variance of $x_t$ is independent of $t$, and (c) the covariance between $x_t$ and $x_s$ depends only on the difference between $t$ and $s$ (and not on $t$ or $s$ separately).\footnote{See, for example, Harvey (1990), p. 23, Hamilton (1994), p. 45, or Greene (2003), p. 612.}

Looking at the graphs suggests that the series fluctuate around some positive value and that $ad_{t}$ appears to be trending upwards, whereas $g_{t}$ seems to have a negative trend. However, visual inspection may lead one astray, as statistical concepts often prove differently. This we will check next.

In order to test for (weak) stationarity one has to assume whether the series has a deterministic trend or is fluctuating around some particular value. If the latter is zero, the series is a random walk. As the descriptive statistics imply that the mean values of the three series are non-zero, the latter assumption is not checked any further.

For analyzing whether a series is (weakly) stationary it is common to employ the Augmented Dickey Fuller (ADF) test. If we do not know whether a series fluctuates around a constant term and/or has a trend, Hamilton (1994), p. 501, suggests that one tests whether the estimated coefficient $\rho$ in the regression with a constant term

$$\Delta x_t = \alpha + \rho x_{t-1} + \sum_{i=2}^{p} \beta_i \Delta x_{t-i+1} + \epsilon_t$$

where $\Delta x_t \equiv x_t - x_{t-1}$, or whether $\rho$ in the regression with a constant term and a trend

$$\Delta x_t = \alpha + \delta \cdot t + \rho x_{t-1} + \sum_{i=2}^{p} \beta_i \Delta x_{t-i+1} + \epsilon_t$$

is significantly different from zero.\footnote{Of course, we also assume that the error terms in these regressions have the standard prop-}

\[1\]
is particularly made for series that have non-zero means, as in our case. Here \( \alpha \) represents the presence of a constant term and \( \delta \) that of a time trend. In both expressions the term \( \sum_{i=2}^{p} \beta_i \Delta x_{t-i+1} \) is included in order to correct for possible serial correlation in the series and, in particular, the error terms. The unknown here is the order \( p \) of sum of the (first order) lags in \( \Delta x_{t-i+1} \).

To determine that lag structure we follow a "top down" approach for each series. We start with ten lags and, hence, with a regression of the form

\[
x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + \beta_{10} x_{t-10} + u_t
\]

where \( x_t = g_t, c_t, \) or \( ad_t \). In each case we found support for a simple AR(1) process

\[
x_t = \beta_0 + \beta_1 x_{t-1} + u_t.
\]

The (heuristic) criteria were t-statistics, \( R^2 \), the Durbin-Watson statistic and other criteria such as the Aikake and Schwartz information criterion as indicators of the quality of the models, especially of the presence of serial correlation in the error terms. (See appendix B.) The results imply that the terms \( \sum_{i=2}^{p} \beta_i \Delta x_{t-i+1} \) do not play any important role in the Augmented Dickey Fuller Tests. Thus, serial correlation does not seem to be a problem for the error terms so that we perform (simple) Dickey Fuller tests of the form

\[
M1: \quad \Delta x_t = \alpha + \rho x_{t-1} + \epsilon_t, \quad M2: \quad \Delta x_t = \alpha + \delta \cdot t + \epsilon_t.
\]

with \( \epsilon_t \sim N(0, \sigma^2) \). The results of these tests are presented below.

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17The null hypothesis, \( H_0 \), for these tests is \( \rho = 1 \). Thus, a stationary series would have \(-1 < \rho < 1\). Hence rejecting \( H_0 \) under the usual criteria of statistical significance implies that the alternative hypothesis of (weak) stationarity would then command higher probability and would be preferred.
The hypothesis of the series not being (weakly) stationary can broadly be rejected at the 95 percent level of statistical significance. This is because the ADF statistic of the sample is smaller than the critical values $ADF_{0.95}$ ($ADF_{0.90}$). According to that criterion the hypothesis that the series $g_t$ and $c_t$ are not (weakly) stationary at the 95% (and higher) level of statistical significance can be rejected, no matter whether we include a constant, or a constant and a trend for the test. For the advertising share we find that the series appears to be stationary if we only include a constant, $ADF = -3.249 < -2.920 = ADF_{0.95}$. However, if we also include a trend term the hypothesis of non-stationarity cannot be rejected at the 90% level. Although the value of −2.832 is not too far from the critical value of $-3.179$, the probability of rejecting the ”$H_0$: Non-stationarity” is lower than 90 percent. However, it is still higher (approximately 81 percent) than a probability that would lead one to definitely reject the alternative hypothesis of stationarity, i.e. ”$H_a$: Stationarity”\textsuperscript{18}

\textsuperscript{18}Model 2 (M 2) in equation (3) may look more general than M 1 since the test is about both a constant and a trend. One way to get stationarity would be to take first differences of $ad_t$. Tests then reveal that the series $\Delta ad_t$ seems to be stationary. However, first differences entail an information loss. Striking a balance here we opt to attach more disadvantage of losing information than to the loss in probability when rejecting the hypothesis of non-stationarity. On this see, for example, Goldrian (1995).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{Variable} & \textbf{M} & \textbf{ADF} & \textbf{p-val.} & \textbf{ADF}^{**}_{0.95} & \textbf{ADF}^{*}_{0.90} & \textbf{\textit{\alpha}} & \textbf{\textit{\delta}} & \textbf{\textit{\textit{R^2}}} & \textbf{\textit{\textit{DW}}} & \textbf{\textit{\textit{Obs.}}} \\
\hline
$g_t$ & 1 & -3.393 & 0.011 & -2.933 & -2.601 & 0.013 & (2.44) & 0.196 & 2.052 & 49 \\
$g_t$ & 2 & -4.461 & 0.002 & -3.504 & -3.182 & 0.042 & (-2.68) & 0.305 & 1.865 & 49 \\
$c_t$ & 1 & -5.063 & 0.000 & -2.930 & -2.600 & 0.169 & (5.01) & 0.348 & 1.876 & 50 \\
$c_t$ & 2 & -4.456 & 0.002 & -3.500 & -3.180 & 0.194 & (-0.826) & 0.357 & 1.824 & 50 \\
$ad_t$ & 1 & -3.249 & 0.017 & -2.930 & -2.598 & 0.002 & (3.55) & 0.180 & 1.845 & 50 \\
$ad_t$ & 2 & -2.832 & 0.185 & -3.500 & -3.180 & 0.002 & (0.80) & 0.191 & 1.809 & 50 \\
\hline
\end{tabular}
\caption{Tests for Stationarity}

\textsuperscript{18} T-statistics in parentheses. M refers to which model is contemplated. $ADF^{**}_{0.95}$ ($ADF^{*}_{0.90}$) denotes the critical value at the 95 % (90 %) of statistical significance for the rejection of the hypothesis of a unit root, $\rho = 1$. The critical values are from MacKinnon (1991) and used by EVIEWS 4.0 (2000). The column ”p-value” presents the probability of a unit root.

\textsuperscript{18}
Thus, in the following analysis we will proceed under the statistically backed hypothesis that the series for the growth rate \( g_t \), the consumption share \( c_t \) and the advertising share \( a_d_t \) are (weakly) stationary.

### 3.2 Causality

In this section we check by pairwise comparisons if one series helps to predict another one. This is the concept introduced by Granger (1969) and has become known as "Granger-causality". It should be borne in mind that this concept should not be taken as a philosophical concept in the sense of "cause and effect". Instead, "Granger-causality" is a statistical property of time series and rather refers to the possibility of temporal predictability of a series \( x_t \) by another series \( y_t \). See, for example, Harvey (1990), p. 304.

The starting point is to look at regressions of the form

\[
x_t = \alpha_0^1 + \alpha_1^1 x_{t-1} + \ldots + \alpha_l^1 x_{t-1} + \beta_1^1 y_{t-1} + \ldots \beta_l^1 y_{t-1} + \epsilon_t \tag{4}
\]

\[
y_t = \alpha_0^2 + \alpha_1^2 y_{t-1} + \ldots + \alpha_l^2 y_{t-1} + \beta_1^2 x_{t-1} + \ldots \beta_l^2 x_{t-1} + u_t \tag{5}
\]

where by assumption the error terms are normally distributed with \( \epsilon_t \sim N(0, \sigma_\epsilon^2) \) and \( u_t \sim N(0, \sigma_u^2) \).

The test then implies performing an F-test on the null hypothesis

\[ H_0 : \beta_1^j = \beta_2^j = \ldots = \beta_l^j = 0 \]

where \( j = 1, 2 \) depending on the direction of "Granger-causality" that is being maintained. In this context it is useful to use the notation below.
Table 3: Patterns of Causality

<table>
<thead>
<tr>
<th>Description</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $x$ and $y$ are unrelated</td>
<td>$(x, y)$</td>
</tr>
<tr>
<td>(2) Instantaneous causality only</td>
<td>$(x \rightarrow y)$</td>
</tr>
<tr>
<td>(3) $y$ causes $x$ only and not instantly</td>
<td>$(x \Rightarrow y)$</td>
</tr>
<tr>
<td>(4) $x$ causes $y$ only and instantly</td>
<td>$(x \rightarrow y)$</td>
</tr>
<tr>
<td>(5) $y$ causes $x$ only and not instantly</td>
<td>$(x \leftarrow y)$</td>
</tr>
<tr>
<td>(6) $y$ causes $x$ only and instantly</td>
<td>$(x \leftarrow y)$</td>
</tr>
<tr>
<td>(7) Feedback, not instantly</td>
<td>$(x \leftrightarrow y)$</td>
</tr>
<tr>
<td>(7) Feedback and instantaneous causality</td>
<td>$(x \equiv y)$</td>
</tr>
</tbody>
</table>

Furthermore, we use the symbol "¬" for "not". Thus, $\neg(x \rightarrow y)$ would denote that "$x$ does not only and not instantly, Granger-cause $y". The meaning of "instantaneous" in this context is explained farther below.

It is standard to test for relationships of the form $\neg(x \rightarrow y)$ first. Given our definitions the results for the simple "Granger-causality" tests of our data for hypotheses of the form $\neg(x \rightarrow y)$ are summarized below.

Table 4: Granger-Causality Tests I

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>Lag 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\neg(c_t \rightarrow a_t)$</td>
<td>50</td>
<td>1.914</td>
<td>0.173</td>
<td>49</td>
<td>1.026</td>
<td>0.367</td>
</tr>
<tr>
<td>$\neg(a_t \rightarrow c_t)$</td>
<td>50</td>
<td>4.090</td>
<td>0.049</td>
<td>49</td>
<td>4.322</td>
<td>0.019</td>
</tr>
<tr>
<td>$\neg(g_t \rightarrow a_t)$</td>
<td>49</td>
<td>0.791</td>
<td>0.378</td>
<td>48</td>
<td>0.746</td>
<td>0.480</td>
</tr>
<tr>
<td>$\neg(a_t \rightarrow g_t)$</td>
<td>49</td>
<td>2.306</td>
<td>0.136</td>
<td>48</td>
<td>1.742</td>
<td>0.187</td>
</tr>
<tr>
<td>$\neg(g_t \rightarrow c_t)$</td>
<td>49</td>
<td>0.235</td>
<td>0.630</td>
<td>48</td>
<td>1.146</td>
<td>0.327</td>
</tr>
<tr>
<td>$\neg(c_t \rightarrow g_t)$</td>
<td>49</td>
<td>10.468</td>
<td>0.000</td>
<td>48</td>
<td>5.542</td>
<td>0.007</td>
</tr>
</tbody>
</table>

F denotes the F-statistic.

We have chosen a maximum lag length of four, because we do not think that a change in advertising investment impacts on consumption after more than four years.\(^{19}\) The results then suggest the following when concentrating on a conventional 5 percent (or even 10 percent) level of statistical significance.

\(^{19}\)We have checked longer lags, but the qualitative features of the results do not change. In particular, the most significant results all seem to be captured by looking at 4 lags. The results for longer lags are available on request.
In most cases there is no Granger-causal relationship. In particular, there seems to be no "causal" relationship between advertising and growth. We find Granger-causality between the consumption share and the growth rate for lags 1, 2, and 3, and between the advertising and the consumption share for all considered lags. This leads us to the hypothesis that bivariate relationships exist between \( ad_t \) and \( c_t \), and between \( c_t \) and \( g_t \). The first case may justify regressing \( c_t \) on \( ad_t \), and the second one provides an argument for regressing \( g_t \) on \( c_t \). Furthermore, the results imply that such regressions might command the highest probability when regressing \( c_t \) on \( ad_t \) when introducing a lag of 2. This is because the hypothesis of "Non-Granger-causality" in this case is lower (1.9 percent probability) than for the other lags. For a regression of \( g_t \) on \( c_t \) a lag structure of order one seems to be a reasonable choice, given the lowest probability (zero percent) of rejecting "Non-Granger-causality" in that case.

Although we will use similar arguments below, we cannot apply them here since we also have to check for instantaneous Granger-causality.

### 3.3 Instantaneous Granger-causality

Instantaneous Granger-causality is given when we also allow for a contemporaneous effect of a variable \( y_t \) on \( x_t \). More precisely, we check in a regression with the standard assumptions on the error term

\[
x_t = \alpha_0 + \alpha_1 x_{t-1} + \ldots + \alpha_l x_{t-l} + \beta_0 y_t + \beta_1 y_{t-1} + \ldots + \beta_l y_{t-l} + \epsilon_t
\]

whether the coefficient \( \beta_0 \) is different from zero. The difference between equations (4) and (6) is that we now also allow (contemporaneous) \( y_t \), and possible lags thereof, to have an effect on \( x_t \).

This concept is important in our context, because we use annual data and the association between our variables may change relatively quickly within a year. However, our focus remains the long run. Thus, even if intra-annual changes matter, they will have an impact on the long run.

To test for instantaneous causality, \((y - x)\), the null hypothesis is \( H_0 : \beta_0 = 0 \), no matter how many other lags of \( y_t \) are included in the regression. A simple F-test for one variable or a conventional t-test for the statistical significance of \( y \) (in
the regression for $x$) can then be employed. Notice that this test is symmetric, that is, if $(y - x)$, then also $(x - y)$ holds true. Thus, the concept does not establish a "causal" direction.

To test for "$y$ causes $x$ only and instantaneously", i.e. $(y \Rightarrow x)$, the null hypothesis is $H_0 : \beta_0 = \beta_1 = \cdots = \beta_l = 0$. Again this can be tested using an F-test on the joint significance of $y_t, y_{t-1}, \cdots, y_{t-L}$ having a bearing on $x_t$.

Table 5: Granger-Causality Tests II

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>Lag 1</th>
<th></th>
<th>Lag 2</th>
<th></th>
<th>Lag 3</th>
<th></th>
<th>Lag 4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\neg(c_t - ad_t)$</td>
<td>50</td>
<td>3.774</td>
<td>0.058</td>
<td>49</td>
<td>3.972</td>
<td>0.053</td>
<td>48</td>
<td>3.320</td>
</tr>
<tr>
<td>$\neg(c_t \Rightarrow ad_t)$</td>
<td>50</td>
<td>2.901</td>
<td>0.065</td>
<td>49</td>
<td>2.654</td>
<td>0.120</td>
<td>48</td>
<td>0.984</td>
</tr>
<tr>
<td>$\neg(ad_t \Rightarrow c_t)$</td>
<td>50</td>
<td>4.053</td>
<td>0.024</td>
<td>49</td>
<td>4.400</td>
<td>0.009</td>
<td>48</td>
<td>3.527</td>
</tr>
<tr>
<td>$\neg(g_t - ad_t)$</td>
<td>49</td>
<td>0.848</td>
<td>0.362</td>
<td>48</td>
<td>1.205</td>
<td>0.279</td>
<td>47</td>
<td>0.995</td>
</tr>
<tr>
<td>$\neg(g_t \Rightarrow ad_t)$</td>
<td>49</td>
<td>0.818</td>
<td>0.448</td>
<td>48</td>
<td>0.902</td>
<td>0.448</td>
<td>47</td>
<td>0.824</td>
</tr>
<tr>
<td>$\neg(ad_t \Rightarrow g_t)$</td>
<td>49</td>
<td>1.573</td>
<td>0.219</td>
<td>48</td>
<td>1.568</td>
<td>0.211</td>
<td>47</td>
<td>0.645</td>
</tr>
<tr>
<td>$\neg(g_t - c_t)$</td>
<td>49</td>
<td>7.870</td>
<td>0.007</td>
<td>48</td>
<td>9.037</td>
<td>0.004</td>
<td>47</td>
<td>10.970</td>
</tr>
<tr>
<td>$\neg(g_t \Rightarrow c_t)$</td>
<td>49</td>
<td>4.070</td>
<td>0.023</td>
<td>48</td>
<td>3.920</td>
<td>0.015</td>
<td>47</td>
<td>3.190</td>
</tr>
<tr>
<td>$\neg(c_t \Rightarrow g_t)$</td>
<td>49</td>
<td>9.951</td>
<td>0.000</td>
<td>48</td>
<td>7.397</td>
<td>0.000</td>
<td>47</td>
<td>4.965</td>
</tr>
</tbody>
</table>

F denotes the F-statistic.

From table (5) the following can be inferred when taking a 10 percent level of statistical significance as our benchmark:

There seems to be a feedback relationship between $g_t$ and $c_t$. Thus, we find $(g_t \Leftrightarrow c_t)$ for all four lags. There is instantaneous causality between $ad_t$ and $c_t$, that is, $(ad_t - c_t)$, for all lags. Furthermore, for all lags we find "$ad_t$ causes $c_t$ only and instantaneously", $(ad_t \Rightarrow c_t)$. For lag 1 there appears to be a feedback relationship $(ad_t \Leftrightarrow c_t)$. It is a surprise to find that, as in the simple Granger-causality tests, there is no "causal" relationship between growth and the advertising share, $(ad_t, g_t)$, at a 10 percent level of statistical significance, even when checking for instantaneous causality. This surely casts doubt on an important part of the hypothesis that we wish to test.

20 Usually, t-statistics for $\beta_0$ are reported. But since a squared t-statistic is the F-statistic in a single variable case such as here, we report the F-statistic for convenience.

16
3.4 Summary

In terms of our hypothesis the most important finding is that there is no "causal" relationship between advertising and growth.

**Result 1** For the advertising share and the rate of economic growth in Germany for annual data from 1950-2000 we do not find any Granger-causal relationship. Thus, the statement that advertising "causes" growth may be problematic.

Recall that Granger-causality is not transitive. Given our evidence that no direct, "causal" relationship exists, we do not analyze this relationship any further. We think that the relationship is probably more complicated and may be non-linear. This is discussed further below.\(^{21}\)

As we find sufficient probability of the assumption of causal as well as feedback relationships for the pairs \(g_t\) and \(c_t\), as well as for \(ad_t\) and \(c_t\) we will analyze these pairs in more detail below.

4 Econometric Models

As a starting point to build models that capture the relationships between our variables best, we look at the cross-correlations of our data. This also reveals whether there are highly linear associations between our variables so that multicollinearity might be a problem.

It is well known that correlations between variables in excess of 0.8 indicate - as a rule of thumb - that multicollinearity might be a problem in any regression featuring highly collinear variables as independent variables.\(^{22}\)

\(^{21}\)The following argument is also possible. Suppose the relationship is non-linear. Finding insignificant results in a linear model may also be an indication that the relationship is at some optimum of an underlying non-linear relationship. Of course, the shape of that would have to be known. Optimality would then imply that we are at a (local) maximum or minimum. Given this interpretation, one may also argue that our result just captures that there is an inverted U-shaped relationship between \(ad_t\) and \(g_t\), and that the value of \(ad_t\) is optimal for \(g_t\). We plotted \(g_t\) (y-axis) against \(ad_t\) (x-axis) and fitted a quadratic curve to find that the relationship appears to be a U-shaped one, however. We will leave a more detailed analysis of this question for future research.

\(^{22}\)When multicollinearity is present then, this may affect the analysis in the following way: (a) Wide swings in the parameter estimates may be produced by small changes in the data. (b) The coefficients may have very high standard errors and low significance levels even though they are jointly significant. The \(R^2\) for the regression is quite high. (c) The coefficients may have the 'wrong' sign or implausible magnitudes. See Greene (2003), p. 57.
### 4.1 Multicollinearity and Cross-Correlations

Table 6: Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>$ad_t$</th>
<th>$ad_{t-1}$</th>
<th>$ad_{t-2}$</th>
<th>$c_t$</th>
<th>$c_{t-1}$</th>
<th>$c_{t-2}$</th>
<th>$g_t$</th>
<th>$g_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ad_t$</td>
<td>0.93**</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ad_{t-1}$</td>
<td>0.87**</td>
<td>0.94*</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_t$</td>
<td>-0.47</td>
<td>-0.60</td>
<td>-0.72</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{t-1}$</td>
<td>-0.43</td>
<td>-0.53</td>
<td>-0.65</td>
<td>0.83*</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{t-2}$</td>
<td>-0.45</td>
<td>-0.53</td>
<td>-0.62</td>
<td>0.73</td>
<td>0.84*</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_t$</td>
<td>-0.51</td>
<td>-0.48</td>
<td>-0.50</td>
<td>0.31</td>
<td>0.53</td>
<td>0.65</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$g_{t-1}$</td>
<td>-0.56</td>
<td>-0.55</td>
<td>-0.53</td>
<td>0.35</td>
<td>0.36</td>
<td>0.58</td>
<td>0.65</td>
<td>1.00</td>
</tr>
<tr>
<td>$g_{t-2}$</td>
<td>-0.60</td>
<td>-0.59</td>
<td>-0.59</td>
<td>0.44</td>
<td>0.40</td>
<td>0.43</td>
<td>0.44</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Values greater than 0.80 are marked by *. Number of observations: 48.

Thus, there seems to be a problem with $ad_t, ad_{t-1}$ and $ad_{t-2}$. There also is a problem with $c_t, c_{t-1}$ and $c_{t-2}$. A number of other interesting results emerge for the table: First, all the cross-correlations between the consumption and the advertising shares are negative. Furthermore, all the cross-correlations between advertising share and the growth rate are negative, and all those between the consumption share and the growth rate positive.

This suggests that one is likely to get a negative (instantaneous) relationship between advertising and consumption, as well as advertising and growth. But we might expect a positive relationship between growth and consumption.

Given this information on correlations that do not reveal anything about causality per se, we now turn to a multivariate analysis, starting with a concept that does not presuppose any causality.

### 4.2 Vector Autoregressive Regression (VAR) Analysis

In our context, VARs have the advantage that they do not assume any a priori causality. Thus, they serve our purposes well since we have found that feedback relationships may exist between our variables of interest.

#### 4.2.1 Theory

Consider the following vector autoregressive process of the order $p$. 
\[
\begin{bmatrix}
  x_t \\
  y_t
\end{bmatrix}
= \begin{bmatrix}
  \mu_1 \\
  \mu_2
\end{bmatrix} + \sum_{p=1}^{P} \begin{bmatrix}
  a_{11}^p & a_{12}^p \\
  a_{21}^p & a_{22}^p
\end{bmatrix} \begin{bmatrix}
  x_{t-p} \\
  y_{t-p}
\end{bmatrix} + \begin{bmatrix}
  \epsilon_{1,t} \\
  \epsilon_{2,t}
\end{bmatrix}
\]

(7)

where \(a_{ij}^p\) and \(i, j = 1, 2\) denotes the \(p\)-lag coefficient in the coefficient matrix measuring the (linear) effect of the \(j\)-th variable at lag \(p\) on the \(i\)-th dependent variable at time \(t\) in the system. By assumption the errors terms \(\epsilon_{i,t}\) are assumed to be normally distributed, uncorrelated over time and across the equations, with a zero mean and constant variance each. The \(\mu_i\) denote the constants in the regression for the \(i\)-th dependent variable.

We are interested in short and long-run effects of our variables. In a VAR the estimated coefficients for \(a_{12}^1\) would capture the immediate impact of a shock in \(y_{t-1}\) (e.g. now) on \(x_t\) (e.g. next year). Likewise, \(a_{21}^1\) would capture the immediate effect of a change in \(x_{t-1}\) on \(y_t\).

For the long-run we have to ask the following: Once the system has been shocked and reacted to it, there will be an overall, worked-out effect on the variables in the long-run. The long-run response of a variable is obtained as follows: First, we re-express our system in matrix form. Let

\[
\begin{bmatrix}
  1 \\
  -L\end{bmatrix}A_1 + \cdots + \begin{bmatrix}
  1 \\
  -L^P\end{bmatrix}A_P = \begin{bmatrix}
  \mu_1 \\
  \mu_2
\end{bmatrix}, \quad \epsilon_t = \begin{bmatrix}
  \epsilon_{1,t} \\
  \epsilon_{2,t}
\end{bmatrix}
\]

Then equation (7) can be represented as

\[
z_t = \mu + A_1 z_{t-1} + \cdots + A_P z_{t-P} + \epsilon_t.
\]

We can then reformulate this process in its Wold moving average (MA) representation as follows: Using the concept of a lag operator, which is defined as \(L^i x_t = x_{t-i}\) for \(i = 1, \cdots, P\), we have

\[
[1 - L^1 A_1 + \cdots + L^P A_P] z_t = \mu + \epsilon_t
\]

\[
z_t = [1 - L A_1 + \cdots + L^P A_P]^{-1} \mu + [1 - L A_1 + \cdots + L^P A_P]^{-1} \epsilon_t.
\]

(8)

where \(1\) denotes the identity matrix. The first \(p \leq P\) elements of the inverse matrix represent the response of the system due to (unforeseen) innovations at
some date $t$ in the system captured by changes in $\epsilon_t$. These first $p \leq P$ elements with $p = 1, 2, \cdots, P$ of the matrix $\left[1 - LA_1 + \cdots + L^P A_P\right]^{-1}$ are usually called *dynamic multipliers* and capture how the system reacts at that date due to an innovation in a particular variable in the system. They are used when looking at the *impulse response* of the system due to an innovation in a variable. In particular, the *short-run* multiplier is given by $a_{ij}^1$ and captures the immediate effect on impact on the system.

In the long-run the system is stable, that is, stationary and settled at an equilibrium. This steady state situation is given when $z_t = \cdots = z_{t-p} = z^*$, where $z^*$ does not depend on time anymore. In a long-run equilibrium the expectations are met so that the error terms in $\epsilon_t$ should be zero. In that case

$$z_t^* = \left[1 - A_1 + \cdots + A_P\right]^{-1} \mu$$

which denotes the *steady state value* of $z_t$, and where the elements of the inverse matrix are called *long-run* dynamic multipliers. Let $\Gamma$ denote that matrix, i.e. $\Gamma \equiv \left[1 - A_1 + \cdots + A_P\right]^{-1}$.

For a growth analysis they are particularly intuitive. The elements of $\Gamma$ capture the long-run (underlying) relationships. Among other things the elements of $\Gamma$ tell one how a *steady state* element of $z^*$ changes, if, for instance, another element of that vector $z^*$ changes. Thus, the long-run multipliers capture how *steady state* variables are related in a multivariate, time series setting, if another *steady state* variable changes.

To see more clearly what this involves we consider a simple VAR of order one. If $P = 1$, then the impact multiplier is $a_{ij}^1$ of variable $j$ on variable $i$ at date 1. Recalling the definition of $A_1$ above, it is then easy to show that the *long-run multipliers* in a first order VAR are given by

$$\Gamma \equiv\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} = \left[1 - A_1\right]^{-1} = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix}\right]^{-1}$$

$$= \Delta \begin{bmatrix} 1 - a_{22}^1 & a_{12}^1 \\ a_{21}^1 & 1 - a_{11}^1 \end{bmatrix}$$ \hspace{1cm} (9)$$

\footnote{Of course, that change would have to come from the other element of $\mu$ really. For instance, we can ask how $z^*$ changes if $\mu_2$, associated with the equation for $y^*$, changes by some small amount. Later we see that $\Gamma$ yields a similar answer to the question of what happens to $z_t$ in the long run if there is an unexpected shock in $\epsilon_t$.}
where $\Delta = \frac{1}{1 - a_{11}^1 - a_{12}^1 - a_{21}^1 - a_{22}^1 + a_{11}^1 \cdot a_{22}^1}$. For instance, to get the long-run response of variable $x_t$, i.e. $x^*$ in steady state, due to a change in $y_t$, that is, $y^*$ in steady state, in a VAR(1) like in equation (7) would be to look at the value of $\gamma_{12} = \Delta \times a_{12}^1$. These long-run multipliers will play an important role below.

4.2.2 Estimation

In section 3 we found support of the hypothesis that the variables are stationary when including a constant in the regressions. We generally did not find enough support to include a time trend for the analysis. Thus, we consider VARs with a constant and no time trend.

Motivated by the ”causality” results we consider two models. One analyzes the relationship between $a_{dt}$ and $c_t$, and the other one the relationship between $c_t$ and $g_t$. For each of the two VAR specifications we first checked the optimal lag structure according to the Akaike, Final Prediction Error, Hannan-Quinn, and Schwarz Criteria. All of them suggest that the optimal lag length for each VAR is one period. Thus, we take the VAR lag length to be one in each model.\(^{24}\)

4.2.3 Model 1: $c_t$ and $a_{dt}$

The estimation results for the first model are presented below (with standard errors in parentheses).

$$
\begin{bmatrix}
c_t \\
ad_t
\end{bmatrix}
= 
\begin{bmatrix}
0.254 & 0.583 \\
-0.004 & 0.009
\end{bmatrix}
+ 
\begin{bmatrix}
0.004 & -1.373 \\
0.007 & 0.923
\end{bmatrix}
\begin{bmatrix}
c_{t-1} \\
ad_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
u_{1,t}^2 \\
u_{2,t}^2
\end{bmatrix}.
$$

The estimated coefficients for $a_{11}^1, a_{12}^1, a_{22}^1$ and $\mu_1$ are all statistically significant at the 5 percent level, whereas the estimate for $a_{21}^1$ is statistically insignificant with probability of 0.166. The latter measures the effect of consumption on advertising in the second row of the VAR. This effect may therefore be zero. Also $\mu_2$ may be zero with a probability of 0.361. For our analysis, we have decided to keep

\(^{24}\)For the estimation we used \textit{JMulTi} by Lütkepohl and Krätzig (2004) to obtain our results. The programme is available free of charge at \textit{www.jmulti.de}.
insignificant results in the VAR in order to keep as much, even if redundant, information as possible.

We have checked the goodness of the model by a battery of standard tests. (See appendix C.) As regards serial correlation of the residuals, we find that the plots of the residuals, their autocorrelations and cross-correlations do not show convincing signs of serial correlation. The portmanteau test rejects serial correlation with a probability of 89 percent. The LM test for autocorrelation (Breusch-Godfrey) with five lags has a probability of 0.16, with the null hypothesis of there being no correlation. This is not too high, but does not lead us to accept the alternative. The test for joint non-normality due to Doornik and Hansen (1994) rejects non-normality with a probability of 100 percent. The Jarque-Bera test for normality for each residual reveals no problems for the innovations in \( c_t \), i.e. \( u(c, t) \), but rejects normality in \( u(ad, t) \). Of course, this is of concern for some tests and t-statistics. We tested whether that is no autoregressive conditional heteroskedasticity (ARCH) in the residuals and found that that had a probability of 30 percent for \( u(c, t) \), and 99 percent for \( u(ad, t) \). The multivariate ARCH-LM test with five lags for no ARCH has a probability of 45 percent.

We also checked for structural breaks. The plots of the cumulative sum of the residuals and the squared residuals, CUSUM and CUSMSQ, do not suggest an important break point problem. The Chow tests for sample-split, break-point, or forecasts identify 1971 and 1991/1992 as problematic.\(^{25}\) However, never do all three tests identify breaks simultaneously. Given this, probably very strong, criterion we argue that structural breaks are not a severe problem for our analysis. We also present recursive parameter estimates in the appendix and do not find clear indications of parameter instability.

Thus, we think that the model is not performing too badly as regards the standard assumptions. As our focus is on the point estimates for the ensuing analysis, we will keep our model as capturing the time series association of our

\(^{25}\)For a more detailed explanation of the tests see, for example, Lütkepohl, H. (2004), p. 135. We used the bootstrapped Chow test statistics as provided by JMulTi, and were, thus, able to evaluate almost every year in the sample. We found that the forecast test never identifies a break. For the other tests we note the following: For the year 1971 we do not have a good guess why there should be a break in a system with \( ad \) and \( c \). Maybe it was the important change in the German government in 1969 that may have caused different behaviour. The other date is probably driven by German re-unification.
variables sufficiently well.

Next, we looked at the impulse responses from our model. We choose to represent them up to 20 periods, because we believe that such a time period is long enough to capture the economically important responses. Figures 4 and 5 depict the impulse responses of one of the variables due to an innovation in the other variable.\(^{26}\)

Figure 4: Impulse Response of \(c_t\) due to innovation in \(ad_t\)

Figure 5: Impulse Response of \(ad_t\) due to innovation in \(c_t\)

Thus, \(c_t\) reacts negatively to an innovation in \(ad_t\), but \(ad_t\) reacts positively to an innovation in \(c_t\). As regards the dynamic multipliers, equation (10) tells us that the immediate response of \(c_t\) due to a change in \(ad_t\) is negative and given by \(a_{12} = -1.373\). The impact multiplier for \(ad_t\) for a change in \(c_t\) is \(a_{21} = 0.009\) and, thus, positive. In turn the long-run multipliers are given by

\[
\Gamma = \begin{bmatrix}
1.732 & -30.878 \\
0.202 & 9.378
\end{bmatrix}
\]

(11)

where we again denote the elements of \(\Gamma\) by \(\gamma_{ij}\) and \(i\) denotes the row and \(j\) the column position in this matrix. Thus, the long-run multiplier of \(ad_t\) affecting \(c_t\) is negative and given by \(\gamma_{12} = -30.878\). That for \(c_t\) affecting \(ad_t\) is positive and

\(^{26}\)We have used forecast error impulse responses. See, for example, Breitung, Brüggemann, and Lütkepohl (2004), p. 166, for a detailed explanation. Roughly speaking, these impulse responses tell one how \(y_t\) changes if there is an innovation in the residuals \(u_{jt}\), where \(j = x, y\). In the graphs below we concentrate on \(u(y, t)\). The results for both models are qualitatively similar, when using the alternative of orthogonalized impulse responses, also explained by those authors. The results based on the latter concept are available upon request.
given by \( \gamma_{21} = 0.202 \).

To interpret the latter results we can invoke the concept of an elasticity. For instance, around the sample means of the series\(^{27}\)

\[
E_{(x,y)} = \gamma_{(x,y)} \frac{\bar{y}}{\bar{x}} \approx \frac{\partial x/\bar{x}}{\partial y/\bar{y}}
\]

can be interpreted as an elasticity. See, for example, Pindyck and Rubinfeld (1998), p. 99. For this we have used that \( \gamma_{(x,y)} \) is the coefficient of the regression of \( x \) on \( y \) and the bars over the variables denote variables at their sample means. Thus, \( E_{(x,y)} \) measures how a 1 percent increase in \( y \) induces an \( E_{(x,y)} \) percent change in \( x \). In our case \( \bar{c} = 0.57 \) and \( \bar{ad} = 0.014 \), and so

\[
E_{c,ad} = \gamma_{12} \cdot \frac{\bar{ad}}{\bar{c}} = -0.76, \quad E_{ad,c} = \gamma_{21} \cdot \frac{\bar{c}}{\bar{ad}} = 8.22.
\]

Thus, a one percent increase in \( ad \) lowers the consumption share by 0.76 percent. Around the means the new, long-run consumption share would then be 0.566 instead of 0.570. In turn, a one percent increase in \( c \) raises the advertising share by 8.22 percent. Then the new, long-run advertising share would be 0.015.

Finally, we again checked for Granger causality in the VAR. The following table summarizes the results.\(^{28}\)

<table>
<thead>
<tr>
<th></th>
<th>F(1,1,94)</th>
<th>Prob.-F</th>
<th>( \chi^2(1) )</th>
<th>Prob.-( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(- (ad_t \rightarrow c_t))</td>
<td>4.090</td>
<td>0.046</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(- (c_t \rightarrow ad_t))</td>
<td>0.046</td>
<td>0.170</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>((c_t - ad_t))</td>
<td>-</td>
<td>-</td>
<td>3.524</td>
<td>0.061</td>
</tr>
</tbody>
</table>

There is instantaneous causality between \( c_t \) and \( ad_t \) at the 6 percent level of

---

\(^{27}\)It is not difficult to calculate the steady state values \( x^* \) and \( y^* \) for the series. In the first model they turn out to be very close to the sample means of the series. The elasticity concept would, of course, also hold when evaluating at the steady state values \( x^* \) and \( y^* \). The effects might be different then, however. For model I we find \( ad^* = 0.014 \) and \( c^* = 0.56 \). Thus, they are very similar to the sample means.

\(^{28}\)We follow Lütkepohl, H. (2004), p. 149, and note that for a VAR "testing for instantaneous causality can be done by determining the absence of instantaneous residual correlation. Because of the asymptotic properties of the estimator of the residual covariance matrix of a VAR process are unaffected by the degree of integration and cointegration in the variables, a test statistic based on the usual Wald or likelihood ratio principles has an asymptotic \( \chi^2 \) - distribution under the standard assumption." He also points out that the concept does not specify a causal direction. The latter must be determined from other sources. Furthermore, there may be indirect links between the two variables that may have to be taken into account.
statistical significance. If one is strict at the 5 percent level of significance, one would not accept the hypothesis that there is instantaneous causality.

The probability of causality from $c_t$ to $ad_t$ is around 17 percent. Thus, this hypothesis can be rejected at a 10 percent level of statistical significance. The probability of causality from $ad_t$ to $c_t$ is around 4.6 percent. Thus, we accept this hypothesis at the 5 percent (10 percent) level of significance. Hence, the evidence suggests that the probability of $ad_t$ causing $c_t$ is higher.

But the following caveat applies: Because of instantaneous causality, we cannot unambiguously say that $ad_t$ always “causes” $c_t$. Contemporaneous effects may be important in this context, because both - advertising and consumption - may react fast to each other. To obtain the instantaneous relationship in our VAR, one has to look at the correlation matrix of residuals. The corresponding correlation coefficient is positive and given by 1.013. Thus, we cannot exclude the possibility that a contemporaneously higher consumption share leads to a higher contemporaneous advertising share and vice versa. However, if there are adjustment lags then we conclude that the bulk of evidence suggests that advertising causes consumption, and that it affects it negatively.

**Result 2** We find instantaneous causality between the advertising and the consumption shares, which is positive. If time adjustment is allowed, then there is causality from advertising to consumption. The impact and the long-run response of more advertising on consumption is found to be negative.

Our result is in contrast to what Ashley, Granger, and Schmalensee (1980) show for the United States. Using quarterly data they find that consumption "causes" advertising there. Their result is interesting for the following reason: Consumer sovereignty is preserved by their results.\(^{29}\) Loosely speaking: Americans are independent consumers - according to their result - and advertising responds to the consumers’ needs. (Consumption leads advertising.) Our result suggests to call (long-run) consumer sovereignty of Germans into question, because at the macro level consumption seems to follow advertising in that country over longer time horizons. Furthermore, the evidence suggests that, contrary to

\(^{29}\)The concept of consumer sovereignty is one of the corner stones of economic thinking. For a critique of it, see, for instance, Galbraith (1972). Notice that the results of Jung and Seldon (1995), mentioned above, suggest feedback relationships for the United States. Thus, it is not entirely clear whether there may be consumer sovereignty in the United States, either.

25
conventional wisdom and given no instantaneous adjustments, more advertising may not be good for consumption in Germany in the long run. On the other hand, it may be good in the very short run.

4.2.4 Model 2: $c_t$ and $g_t$

The estimation results for our second model are given by

$$
\begin{bmatrix}
  c_t \\
  g_t
\end{bmatrix}
= \begin{bmatrix}
  0.146 \\
  -0.325 \\
\end{bmatrix}
\begin{bmatrix}
  0.793 & 0.022 \\
  0.608 & 0.501 \\
\end{bmatrix}
\begin{bmatrix}
  c_{t-1} \\
  g_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
  u_{3,t}^1 \\
  u_{3,t}^2
\end{bmatrix}. \tag{12}
$$

The estimates for $a_{11}^1, a_{21}^1, a_{22}^1$ and $\mu_1, \mu_2$ are all statistically significant at the 5 percent level, whereas the estimate for $a_{12}^1$ is statistically insignificant with probability of 0.628. The latter measures the effect of growth on consumption in the second row of the VAR. This effect is likely to be zero. Again we keep insignificant results in the VAR.

Appendix C presents the details of the checks we conducted for this model. The plots of the residuals do not suggest signs of serial correlation. The portmanteau test rejects serial correlation with a probability of 96 percent. The LM test for autocorrelation (Breusch-Godfrey) with five lags has a probability of 0.56, with the null hypothesis of there being no correlation. Thus, we do not accept the alternative. The test for joint non-normality due to Doornik and Hansen (1994) rejects non-normality with a probability of 92 percent. The Jarque-Bera test for normality for each residual reveals no problems for both of them. We tested whether there is no autoregressive conditional heteroskedasticity (ARCH) in the residuals. We found a probability of 44 percent for $u(c, t)$, and 72 percent for $u(g, t)$. The multivariate ARCH-LM test with five lags for no ARCH has a probability of 38 percent.

We also checked for structural breaks. The plots of the cumulate sum of the CUSUM and CUSMSQ do not indicate an important break point problem. The Chow tests for sample-split, break-point, or forecasts identify 1977 as problematic. But all three tests never identify breaks simultaneously. Given our earlier

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30For the year 1977 we believe that a break in the system with $c$ and $g$ might be explained by the wave of terrorism that shook Germany at that time. In 1977 the employer representative Hans Martin Schleyer was abducted and later killed by the Red Army Faction (RAF). In the data we identify a decrease in $c_t$ in the following years which may reflect a concomitant drop in consumer confidence. The growth rate also dropped in that year.
criterion we argue that structural breaks are not a severe problem for our analysis of the second model. We also present recursive parameter estimates in the appendix and do not find clear indications of parameter instability.

Thus, we think the model is not performing too badly either. Hence, we will keep our second model for what is to follow.

For the impulse responses we again consider 20 periods. The results are presented in figures 6 and 7, where the definition of innovations is the same as in the last model.

Figure 6: Impulse Response of $g_t$ due to innovation in $c_t$

![Figure 6](image1)

Figure 7: Impulse Response of $c_t$ due to innovation in $g_t$

![Figure 7](image2)

The impact multiplier of $c_t$ for a change in $g_t$ is $a_{12} = 0.022$. The impact multiplier for $g_t$ for a change in $c_t$ is positive at $a_{21} = 0.608$ and implies that on impact a higher consumption share raises the growth rate. The impulse response function for this effect suggests that growth reacts very strongly to an increase in $c_t$. This huge effect may, therefore, just highlight how quantitatively important changes in the consumption share might be for economic growth.\(^{31}\)

The corresponding long-run multipliers are given by

$$
\Gamma = \begin{bmatrix}
5.500 & 0.245 \\
6.762 & 2.302
\end{bmatrix}
$$

(13)

Thus, the long-run multiplier for $c_t$, when affected by $g_t$, is positive, that is,\(^{31}\)

\(^{31}\)Of course, one should not take the exact value to literally. The model ignores too many other factor that are usually also thought to be (quantitatively) important for the growth rate of aggregate GDP. But then most models suffer from this problem of making abstractions.
\( \gamma_{12} = 0.245 \). That for \( g_t \), when affected by \( c_t \), is positive and given by \( \gamma_{21} = 6.762 \).

In our case \( \bar{c}_t = 0.57 \) and \( \bar{g}_t = 0.039 \), and so

\[
E_{c,g} = \gamma_{12} \cdot \frac{\bar{g}_t}{\bar{c}_t} = 0.245 \cdot \frac{0.039}{0.57} = 0.017, \quad E_{g,c} = \gamma_{21} \cdot \frac{\bar{c}_t}{\bar{g}_t} = 6.762 \cdot \frac{0.57}{0.039} = 98.830.
\]

Thus, a one percent increase in the value of \( g_t \) would raise the long-run consumption share by 0.017 percent around the sample means.\(^{32}\) Thus, this change would be small. In turn, a one percent increase in \( c_t \) raises the the growth rate by almost 99 percent. Thus, the new, long-run growth rate of aggregate GDP would almost double around the sample means.

Clearly, such a result seems drastic and leads one to be cautious about the result. But in terms of the direction of change it lends support to the consumption-driven growth hypothesis. That is the aspect we will focus on here.

We are reinforced in our focus when looking at Granger-causality for our second model.

<table>
<thead>
<tr>
<th></th>
<th>( F(1,1,94) )</th>
<th>Prob.-( F )</th>
<th>( \chi^2(1) )</th>
<th>Prob.-( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -c_t \to g_t )</td>
<td>10.468</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( -g_t \to c_t )</td>
<td>0.235</td>
<td>0.629</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (c_t - g_t) )</td>
<td></td>
<td></td>
<td>6.349</td>
<td>0.012</td>
</tr>
</tbody>
</table>

From these results we are led to reject the hypothesis that growth ”causes” consumption. However, we find support for the hypothesis that consumption ”causes” growth at the 5 percent level of significance. We also find evidence for instantaneous causality. From the cross-correlations in the residuals we infer that the instantaneous relationship is negative in both ways and given by \(-1.412\).

**Result 3** There is instantaneous causality between the consumption share and the growth rate. The association is negative. For the longer run with time adjustments we find support of the hypothesis that growth is consumption-driven in Germany. The quantitative effect of a higher consumption share on the long-run growth rate may be huge.

\(^{32}\)For the steady state we found \( c^* = 0.731 \) and \( g^* = 0.239 \), which seem implausibly high. We simply interpret this as suggesting that the long-run equilibrium combination of \( c_t \) and \( g_t \) is higher than what the sample means for our time horizon capture. This would also suggest that Germany was not in a steady state over the period and for the variables considered in our second model. Recall that the first model would approximately imply a steady state for the relation \( ad_t \) and \( c_t \).
5 Concluding Remarks

The paper focuses on the popular hypothesis that advertising is good for consumption and that higher consumption means higher output and higher economic growth.

Using standard techniques we analyze these claims empirically. Using German data for the period 1950 to 2000 we find that parts of the hypothesis do not stand up to our empirical checks, when focussing on the causality concept of Granger.

For instance, we find that there is no direct, "causal" relationship between advertising and growth. As Granger causality is not transitive, we conclude that it is very likely that the relationship between advertising and economic growth is more complicated as suggested by simple linear empirical models. We conjecture that the long-run relationship may be non-linear and that advertising may indeed work through consumption on long-run economic growth. But this would require a more elaborate (formal) theory.

For the other links we find the following: There seems to be evidence that advertising is contemporaneously positively related to consumption. However, in the longer run advertising "causes" consumption, but negatively so. Thus, the short and long-run effects are very different. Second, we find evidence that the current consumption share is negatively associated with the current growth rate. When allowing for adjustment lags we find that consumption "causes" growth and the long-run effect of a higher consumption share may be huge.

Of course, we must acknowledge the limitations of a study such as the present one. We wonder whether our results generalize to more economies. We have only considered bivariate relationships due to the intransitive nature of Granger causality. That is restrictive. We have focused on simple linear empirical models only. In our context, non-linearity may be the driving force of the hypothesized relationships. In defence of our analysis we may note, however, that linear empirical models can be interpreted as first order approximations to some (possibly) highly non-linear relationship.

Despite these reservations do we think that further work on the hypothesis analyzed in this paper would be valuable. The present analysis should be regarded as one contribution in that direction.
A The data and its sources

All the data used in this paper refer to the Federal Republic of Germany for the period 1950 to 2000 and treat the Federal Republic as a single entity in its current and its pre-1990 form.

\textbf{AD}_t: Investment in advertising including expenditures on salaries, media and the production of means of publicity in billions of Euros for the period 1950-2000. The data are taken from the various ZAW Jahrbücher and were collected and provided to us by courtesy of Volker Nickel of the Zentralverband der deutschen Werbewirtschaft (ZAW). See also http://www.interverband.com/u−img/184/zaw_home_18_01_05.htm.

\textbf{Y}^N_t and \textbf{C}^N_t: Nominal GDP is denoted by \textbf{Y}^N_t and nominal private consumption expenditure by \textbf{C}^N_t. Both are in current prices in billions of Euros. The data for 1950-1969 are from Table 8 (interne lange Reihe) of Statistisches Bundesamt (2004a) and were made available to us by courtesy of Statistisches Bundesamt, Wolfgang Garjonis, SB III A. We transformed the data into Euros using the official exchange rate of the former Deutsche Mark to the Euro. The data for the period 1970-2000 are taken from Table 1.3.1 of Statistisches Bundesamt (2004c). See http://www.destatis.de.

\textbf{g}_t: The annual growth rate of real GDP, where the data for the latter are taken from Statistisches Bundesamt (2004b), Table 23.2, and Statistisches Bundesamt (2004c), Table 1.3.2, and are for 1950-1970 in constant prices of 1991, for 1971-2000 in constant prices of 1995. No growth rate is (initially) calculated for the year 1970.

\textbf{c}_t and \textbf{ad}_t: These variables were generated as explained in the main text.
### Table 7: The Data

<table>
<thead>
<tr>
<th>Year</th>
<th>$AD_t$</th>
<th>$Y_t^{N}$</th>
<th>$a_t$</th>
<th>$C_t^{N}$</th>
<th>$c_t$</th>
<th>$g_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>0.32</td>
<td>49.70</td>
<td>0.006</td>
<td>32.32</td>
<td>0.65</td>
<td>n.a.</td>
</tr>
<tr>
<td>1951</td>
<td>0.47</td>
<td>61.00</td>
<td>0.008</td>
<td>37.46</td>
<td>0.614</td>
<td>0.097</td>
</tr>
<tr>
<td>1952</td>
<td>0.61</td>
<td>69.80</td>
<td>0.009</td>
<td>41.32</td>
<td>0.592</td>
<td>0.088</td>
</tr>
<tr>
<td>1953</td>
<td>0.72</td>
<td>74.90</td>
<td>0.01</td>
<td>45.16</td>
<td>0.598</td>
<td>0.078</td>
</tr>
<tr>
<td>1954</td>
<td>0.82</td>
<td>80.60</td>
<td>0.01</td>
<td>48.16</td>
<td>0.598</td>
<td>0.087</td>
</tr>
<tr>
<td>1955</td>
<td>0.92</td>
<td>91.90</td>
<td>0.01</td>
<td>53.83</td>
<td>0.586</td>
<td>0.121</td>
</tr>
<tr>
<td>1956</td>
<td>1.07</td>
<td>101.60</td>
<td>0.011</td>
<td>59.63</td>
<td>0.587</td>
<td>0.077</td>
</tr>
<tr>
<td>1957</td>
<td>1.38</td>
<td>110.70</td>
<td>0.012</td>
<td>64.79</td>
<td>0.585</td>
<td>0.061</td>
</tr>
<tr>
<td>1958</td>
<td>1.48</td>
<td>119.00</td>
<td>0.012</td>
<td>69.94</td>
<td>0.588</td>
<td>0.045</td>
</tr>
<tr>
<td>1959</td>
<td>1.69</td>
<td>130.30</td>
<td>0.013</td>
<td>74.79</td>
<td>0.574</td>
<td>0.079</td>
</tr>
<tr>
<td>1960</td>
<td>1.89</td>
<td>154.80</td>
<td>0.012</td>
<td>87.86</td>
<td>0.568</td>
<td>0.086</td>
</tr>
<tr>
<td>1961</td>
<td>2.30</td>
<td>169.60</td>
<td>0.014</td>
<td>96.29</td>
<td>0.568</td>
<td>0.046</td>
</tr>
<tr>
<td>1962</td>
<td>2.51</td>
<td>184.50</td>
<td>0.014</td>
<td>104.71</td>
<td>0.568</td>
<td>0.047</td>
</tr>
<tr>
<td>1963</td>
<td>2.76</td>
<td>195.50</td>
<td>0.012</td>
<td>110.84</td>
<td>0.567</td>
<td>0.028</td>
</tr>
<tr>
<td>1964</td>
<td>3.12</td>
<td>214.80</td>
<td>0.015</td>
<td>119.39</td>
<td>0.556</td>
<td>0.067</td>
</tr>
<tr>
<td>1965</td>
<td>3.63</td>
<td>234.80</td>
<td>0.015</td>
<td>131.72</td>
<td>0.561</td>
<td>0.054</td>
</tr>
<tr>
<td>1966</td>
<td>4.09</td>
<td>249.60</td>
<td>0.016</td>
<td>140.64</td>
<td>0.563</td>
<td>0.028</td>
</tr>
<tr>
<td>1967</td>
<td>4.35</td>
<td>252.80</td>
<td>0.017</td>
<td>144.51</td>
<td>0.572</td>
<td>-0.003</td>
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<tr>
<td>1968</td>
<td>4.35</td>
<td>272.70</td>
<td>0.016</td>
<td>153.77</td>
<td>0.564</td>
<td>0.055</td>
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<tr>
<td>1969</td>
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<td>305.20</td>
<td>0.016</td>
<td>169.19</td>
<td>0.554</td>
<td>0.075</td>
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<td>4.76</td>
<td>352.00</td>
<td>0.014</td>
<td>190.08</td>
<td>0.540</td>
<td>*0.054</td>
</tr>
<tr>
<td>1971</td>
<td>5.11</td>
<td>390.00</td>
<td>0.013</td>
<td>211.52</td>
<td>0.542</td>
<td>0.033</td>
</tr>
<tr>
<td>1972</td>
<td>5.68</td>
<td>427.50</td>
<td>0.013</td>
<td>234.17</td>
<td>0.548</td>
<td>0.041</td>
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<tr>
<td>1973</td>
<td>6.08</td>
<td>476.70</td>
<td>0.013</td>
<td>257.13</td>
<td>0.539</td>
<td>0.046</td>
</tr>
<tr>
<td>1974</td>
<td>6.08</td>
<td>513.60</td>
<td>0.012</td>
<td>276.67</td>
<td>0.539</td>
<td>0.005</td>
</tr>
<tr>
<td>1975</td>
<td>6.34</td>
<td>536.00</td>
<td>0.012</td>
<td>303.8</td>
<td>0.567</td>
<td>-0.010</td>
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<tr>
<td>1976</td>
<td>7.00</td>
<td>583.90</td>
<td>0.012</td>
<td>329.15</td>
<td>0.564</td>
<td>0.050</td>
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<tr>
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<td>0.013</td>
<td>354.32</td>
<td>0.568</td>
<td>0.030</td>
</tr>
<tr>
<td>1978</td>
<td>9.05</td>
<td>669.30</td>
<td>0.014</td>
<td>376.75</td>
<td>0.563</td>
<td>0.030</td>
</tr>
<tr>
<td>1979</td>
<td>10.17</td>
<td>722.50</td>
<td>0.014</td>
<td>406.14</td>
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<td>0.042</td>
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<td>1980</td>
<td>11.2</td>
<td>766.60</td>
<td>0.015</td>
<td>434.70</td>
<td>0.567</td>
<td>0.013</td>
</tr>
<tr>
<td>1981</td>
<td>11.61</td>
<td>800.20</td>
<td>0.015</td>
<td>459.35</td>
<td>0.574</td>
<td>0.001</td>
</tr>
<tr>
<td>1982</td>
<td>12.27</td>
<td>831.80</td>
<td>0.015</td>
<td>476.85</td>
<td>0.573</td>
<td>-0.008</td>
</tr>
<tr>
<td>1983</td>
<td>13.29</td>
<td>872.20</td>
<td>0.015</td>
<td>498.85</td>
<td>0.572</td>
<td>0.016</td>
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<td>1984</td>
<td>15.13</td>
<td>915.00</td>
<td>0.017</td>
<td>521.47</td>
<td>0.570</td>
<td>0.028</td>
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<tr>
<td>1985</td>
<td>15.90</td>
<td>955.30</td>
<td>0.017</td>
<td>541.05</td>
<td>0.566</td>
<td>0.022</td>
</tr>
<tr>
<td>1986</td>
<td>16.41</td>
<td>1 010.20</td>
<td>0.016</td>
<td>559.24</td>
<td>0.554</td>
<td>0.024</td>
</tr>
<tr>
<td>1987</td>
<td>17.08</td>
<td>1 043.30</td>
<td>0.016</td>
<td>582.76</td>
<td>0.559</td>
<td>0.015</td>
</tr>
<tr>
<td>1988</td>
<td>17.90</td>
<td>1 098.50</td>
<td>0.016</td>
<td>605.82</td>
<td>0.551</td>
<td>0.037</td>
</tr>
<tr>
<td>1989</td>
<td>18.92</td>
<td>1 168.30</td>
<td>0.016</td>
<td>642.40</td>
<td>0.550</td>
<td>0.039</td>
</tr>
<tr>
<td>1990</td>
<td>20.20</td>
<td>1 274.90</td>
<td>0.016</td>
<td>686.49</td>
<td>0.538</td>
<td>0.057</td>
</tr>
<tr>
<td>1991</td>
<td>22.29</td>
<td>1 387.10</td>
<td>0.016</td>
<td>745.62</td>
<td>0.538</td>
<td>0.051</td>
</tr>
<tr>
<td>1992</td>
<td>24.13</td>
<td>1 613.20</td>
<td>0.015</td>
<td>914.30</td>
<td>0.567</td>
<td>0.022</td>
</tr>
<tr>
<td>1993</td>
<td>24.75</td>
<td>1 654.20</td>
<td>0.015</td>
<td>950.66</td>
<td>0.553</td>
<td>-0.011</td>
</tr>
<tr>
<td>1994</td>
<td>25.97</td>
<td>1 735.50</td>
<td>0.015</td>
<td>985.75</td>
<td>0.548</td>
<td>0.023</td>
</tr>
<tr>
<td>1995</td>
<td>27.41</td>
<td>1 801.30</td>
<td>0.015</td>
<td>1 024.79</td>
<td>0.547</td>
<td>0.017</td>
</tr>
<tr>
<td>1996</td>
<td>28.07</td>
<td>1 833.70</td>
<td>0.015</td>
<td>1 052.26</td>
<td>0.559</td>
<td>0.008</td>
</tr>
<tr>
<td>1997</td>
<td>28.94</td>
<td>1 871.60</td>
<td>0.015</td>
<td>1 079.77</td>
<td>0.562</td>
<td>0.014</td>
</tr>
<tr>
<td>1998</td>
<td>30.17</td>
<td>1 929.40</td>
<td>0.016</td>
<td>1 111.18</td>
<td>0.560</td>
<td>0.020</td>
</tr>
<tr>
<td>1999</td>
<td>31.44</td>
<td>1 978.60</td>
<td>0.016</td>
<td>1 155.97</td>
<td>0.562</td>
<td>0.020</td>
</tr>
<tr>
<td>2000</td>
<td>33.21</td>
<td>2 030.00</td>
<td>0.016</td>
<td>1 196.79</td>
<td>0.569</td>
<td>0.029</td>
</tr>
</tbody>
</table>

$AD_t$, $Y_t$, and $C_t$ are in billions of Euros. The growth rate for 1970 is interpolated, as explained in the main text.
### B Lag Selection for the Stationarity Analysis

#### B.1 Selection of the lag for $g_t$

Dependent Variable: GR  
Method: Least Squares  
Sample(adjusted): 1961 2000  
Included observations: 40 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.012318</td>
<td>0.007725</td>
<td>1.594424</td>
<td>0.1217</td>
</tr>
<tr>
<td>GR(-1)</td>
<td>0.422177</td>
<td>0.182042</td>
<td>2.319120</td>
<td>0.0276</td>
</tr>
<tr>
<td>GR(-2)</td>
<td>-0.266384</td>
<td>0.166746</td>
<td>-1.535951</td>
<td>0.1662</td>
</tr>
<tr>
<td>GR(-3)</td>
<td>-0.005968</td>
<td>0.201967</td>
<td>-0.029552</td>
<td>0.9766</td>
</tr>
<tr>
<td>GR(-4)</td>
<td>0.129841</td>
<td>0.194792</td>
<td>0.654328</td>
<td>0.5196</td>
</tr>
<tr>
<td>GR(-5)</td>
<td>-0.073824</td>
<td>0.196188</td>
<td>-0.376292</td>
<td>0.7094</td>
</tr>
<tr>
<td>GR(-6)</td>
<td>-0.061711</td>
<td>0.193322</td>
<td>-0.325691</td>
<td>0.7397</td>
</tr>
<tr>
<td>GR(-7)</td>
<td>0.136203</td>
<td>0.174641</td>
<td>0.780347</td>
<td>0.4415</td>
</tr>
<tr>
<td>GR(-8)</td>
<td>-0.029162</td>
<td>0.183294</td>
<td>-0.159099</td>
<td>0.8747</td>
</tr>
<tr>
<td>GR(-9)</td>
<td>0.119486</td>
<td>0.182550</td>
<td>0.654540</td>
<td>0.5179</td>
</tr>
<tr>
<td>GR(-10)</td>
<td>0.082480</td>
<td>0.164516</td>
<td>0.501352</td>
<td>0.6199</td>
</tr>
</tbody>
</table>

R-squared    0.301888  
Adjusted R-squared 0.028875  
S.E. of regression 0.020360  
Durbin-Watson stat 1.992457

AFTER ELIMINATING INSIGNIFICANT (5 percent level of significance) REGRESSORS:

Dependent Variable: GR  
Method: Least Squares  
Sample(adjusted): 1952 2000  
Included observations: 49 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.012646</td>
<td>0.005176</td>
<td>2.443437</td>
<td>0.0184</td>
</tr>
<tr>
<td>GR(-1)</td>
<td>0.647716</td>
<td>0.103222</td>
<td>6.238196</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared    0.452948  
Adjusted R-squared 0.021013  
S.E. of regression 0.021776  
Durbin-Watson stat 2.052080

AFTER ELIMINATING INSIGNIFICANT (5 percent level of significance) REGRESSORS:

#### B.2 Selection of the lag for $c_t$

Dependent Variable: CS  
Method: Least Squares  
Sample(adjusted): 1961 2000  
Included observations: 41 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.216710</td>
<td>0.091198</td>
<td>2.376268</td>
<td>0.0241</td>
</tr>
<tr>
<td>CS(-1)</td>
<td>0.672277</td>
<td>0.141233</td>
<td>4.776064</td>
<td>0.0000</td>
</tr>
<tr>
<td>CS(-2)</td>
<td>-0.031694</td>
<td>0.219810</td>
<td>-0.140992</td>
<td>0.8847</td>
</tr>
<tr>
<td>CS(-3)</td>
<td>0.018904</td>
<td>0.217972</td>
<td>0.086726</td>
<td>0.9315</td>
</tr>
<tr>
<td>CS(-4)</td>
<td>0.151230</td>
<td>0.213670</td>
<td>0.707110</td>
<td>0.4850</td>
</tr>
<tr>
<td>CS(-5)</td>
<td>-0.009273</td>
<td>0.210330</td>
<td>-0.044086</td>
<td>0.9651</td>
</tr>
<tr>
<td>CS(-6)</td>
<td>-0.245758</td>
<td>0.212032</td>
<td>-1.159063</td>
<td>0.2565</td>
</tr>
<tr>
<td>CS(-7)</td>
<td>0.009395</td>
<td>0.215650</td>
<td>0.041574</td>
<td>0.9671</td>
</tr>
<tr>
<td>CS(-8)</td>
<td>0.075730</td>
<td>0.207068</td>
<td>0.365743</td>
<td>0.7171</td>
</tr>
<tr>
<td>CS(-9)</td>
<td>-0.009299</td>
<td>0.230142</td>
<td>-0.036060</td>
<td>0.9715</td>
</tr>
<tr>
<td>CS(-10)</td>
<td>-0.011769</td>
<td>0.153316</td>
<td>-0.075773</td>
<td>0.9401</td>
</tr>
</tbody>
</table>

R-squared    0.477878  
Adjusted R-squared 0.030837  
S.E. of regression 0.009999  
Durbin-Watson stat 2.745781

AFTER ELIMINATING INSIGNIFICANT (5 percent level of significance) REGRESSORS:

Dependent Variable: CS
### Method: Least Squares

Sample (adjusted): 1951 2000

Included observations: 50 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.169366</td>
<td>0.033795</td>
<td>5.011536</td>
<td>0.0000</td>
</tr>
<tr>
<td>CS(-1)</td>
<td>0.698094</td>
<td>0.059634</td>
<td>11.70637</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.740595

Adjusted R-squared: 0.735191

S.E. of regression: 0.008632

Akaike info criterion: -6.627397

Sum squared resid: 0.003577

Schwarz criterion: -6.550917

Log likelihood: 167.6849

F-statistic: 137.0390

Durbin-Watson stat: 1.875881

Prob(F-statistic): 0.000000

### B.3 Selection of the lag for $ad_t$

Dependent Variable: AD

Method: Least Squares

Sample (adjusted): 1960 2000

Included observations: 41 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.002850</td>
<td>0.001323</td>
<td>2.154388</td>
<td>0.0394</td>
</tr>
<tr>
<td>AD(-1)</td>
<td>0.916361</td>
<td>0.178104</td>
<td>5.156330</td>
<td>0.0000</td>
</tr>
<tr>
<td>AD(-2)</td>
<td>0.186764</td>
<td>0.242140</td>
<td>0.760321</td>
<td>0.2222</td>
</tr>
<tr>
<td>AD(-3)</td>
<td>0.121935</td>
<td>0.235531</td>
<td>0.518966</td>
<td>0.6047</td>
</tr>
<tr>
<td>AD(-4)</td>
<td>0.111359</td>
<td>0.229033</td>
<td>0.486258</td>
<td>0.6303</td>
</tr>
<tr>
<td>AD(-5)</td>
<td>-0.028075</td>
<td>0.238883</td>
<td>-0.127040</td>
<td>0.2222</td>
</tr>
<tr>
<td>AD(-6)</td>
<td>-0.046622</td>
<td>0.223896</td>
<td>-0.209629</td>
<td>0.0999</td>
</tr>
<tr>
<td>AD(-7)</td>
<td>-0.128295</td>
<td>0.228112</td>
<td>-0.578399</td>
<td>0.5673</td>
</tr>
<tr>
<td>AD(-8)</td>
<td>0.066899</td>
<td>0.220689</td>
<td>0.303873</td>
<td>0.7636</td>
</tr>
<tr>
<td>AD(-9)</td>
<td>-0.175768</td>
<td>0.222802</td>
<td>-0.788982</td>
<td>0.4364</td>
</tr>
<tr>
<td>AD(-10)</td>
<td>0.201698</td>
<td>0.152125</td>
<td>1.325872</td>
<td>0.1949</td>
</tr>
</tbody>
</table>

R-squared: 0.820212

Adjusted R-squared: 0.760286

S.E. of regression: 0.000730

Akaike info criterion: -11.38195

Sum squared resid: 1.60E-05

Schwarz criterion: -10.92201

Log likelihood: 244.3300

F-statistic: 13.68635

Durbin-Watson stat: 1.938562

Prob(F-statistic): 0.000000

AFTER ELIMINATING INSIGNIFICANT (5 percent level of significance) REGRESSORS:

Dependent Variable: AD

Method: Least Squares

Sample (adjusted): 1951 2000

Included observations: 50 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.002014</td>
<td>0.000568</td>
<td>3.548780</td>
<td>0.0009</td>
</tr>
<tr>
<td>AD(-1)</td>
<td>0.868619</td>
<td>0.040433</td>
<td>21.375086</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.905786

Adjusted R-squared: 0.903805

S.E. of regression: 0.000750

Akaike info criterion: -11.57666

Sum squared resid: 2.35E-05

Schwarz criterion: -11.45056

Log likelihood: 293.3261

F-statistic: 461.3842

Durbin-Watson stat: 1.849443

Prob(F-statistic): 0.000000
C Model 1: Diagnostic Tests

Table 8: Standardized Residuals 1951-2000

Table 9: Residual Autocorrelations and Cross-correlations
PORTMANTEAU TEST (H0:Rh=(r1,...,rh)=0)
tested order: 16
test statistic: 46.8134
p-value: 0.8931
adjusted test statistic: 56.0296
p-value: 0.6215
degrees of freedom: 60.0000

LM-TYPE TEST FOR AUTOCORRELATION with 5 lags
Reference: Doornik (1996), LM test and LMF test (with F-approximation)
LM statistic: 26.0993
p-value: 0.1626
df: 20.0000
LMF statistic: 1.3468
p-value: 0.1793
df1: 20.0000
df2: 72.0000

TESTS FOR NONNORMALITY
Reference: Doornik & Hansen (1994)
joint test statistic: 31.4260
p-value: 0.0000
degrees of freedom: 4.0000
skewness only: 13.0713
p-value: 0.0015
kurtosis only: 18.3547
p-value: 0.0001
joint test statistic: 29.4532
p-value: 0.0000
degrees of freedom: 4.0000
skewness only: 12.6286
p-value: 0.0018
kurtosis only: 16.8246
p-value: 0.0002

JARQUE-BERA TEST

table: variable teststat p-Value(Chi^2) skewness kurtosis
u1 0.8763 0.6452 0.3159 2.8539
u2 51.9653 0.0000 -1.3222 7.2368

ARCH-LM TEST with 16 lags

variable teststat p-Value(Chi^2) F stat p-Value(F)
u1 18.1786 0.3135 2.4416 0.0384
u2 3.2203 0.9997 0.2223 0.9979

MULTIVARIATE ARCH-LM TEST with 5 lags

VARCHLM test statistic: 44.7240
p-value(chi^2): 0.4836
degrees of freedom: 45.0000
C.1 Stability

Table 10: CUSUM and CUSUMSQ

<table>
<thead>
<tr>
<th>CUSUM: $c_t$ (top), $ad_t$ (bottom)</th>
<th>CUSUMSQ: $c_t$ (top), $ad_t$ (bottom)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="CUSUM plots" /></td>
<td><img src="image2" alt="CUSUMSQ plots" /></td>
</tr>
</tbody>
</table>

These plots reveal that the CUSUM and the CUSUMSQ are within the bands of a significance level of 1 percent. However, there could be breaks. Especially, from the CUSUMSQ for $ad_t$ one may infer that there might be a problem around 1970 and around 1990.

The plots of the recursive coefficients reveals that for the period after 1975 the estimates look stable. This also holds true for the coefficients for $\mu$ which are not presented here.

Table 11: Recursive Coefficients I

<table>
<thead>
<tr>
<th>Recursive residuals suggest that there might be a break problems around 1970 and 1990.</th>
<th>Recursive residuals suggest that there might be a break problems around 1970 and 1990.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Recursive residuals plots" /></td>
<td><img src="image4" alt="Recursive residuals plots" /></td>
</tr>
</tbody>
</table>

The recursive residuals suggest that there might be a break problems around 1970 and 1990.
C.1.1 Chow Tests

We searched for breaks over the whole sample period of model 1. A routine in JMulTi calculates the asymptotic $\chi^2$ probability values and bootstrapped probability values for the Chow sample-split, break-dates, and forecast tests. This identified 1971 and 1991 as possible dates. Then we conducted the three Chow test for these particular years. When specifying particular dates the test uses more observations. The results are presented below.

<table>
<thead>
<tr>
<th>tested break date:</th>
<th>1971</th>
<th>tested break date:</th>
<th>1991</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20 observations before break)</td>
<td>(40 observations before break)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>break point Chow test:</td>
<td>25.1596</td>
<td>break point Chow test:</td>
<td>36.8616</td>
</tr>
<tr>
<td>bootstrapped p-value:</td>
<td>0.0400</td>
<td>bootstrapped p-value:</td>
<td>0.0200</td>
</tr>
<tr>
<td>asymptotic chi^2 p-value:</td>
<td>0.0028</td>
<td>asymptotic chi^2 p-value:</td>
<td>0.0000</td>
</tr>
<tr>
<td>degrees of freedom:</td>
<td>9</td>
<td>degrees of freedom:</td>
<td>9</td>
</tr>
<tr>
<td>sample split Chow test:</td>
<td>7.5160</td>
<td>sample split Chow test:</td>
<td>19.4244</td>
</tr>
<tr>
<td>bootstrapped p-value:</td>
<td>0.4800</td>
<td>bootstrapped p-value:</td>
<td>0.0100</td>
</tr>
<tr>
<td>asymptotic chi^2 p-value:</td>
<td>0.2758</td>
<td>asymptotic chi^2 p-value:</td>
<td>0.0035</td>
</tr>
<tr>
<td>degrees of freedom:</td>
<td>6</td>
<td>degrees of freedom:</td>
<td>6</td>
</tr>
<tr>
<td>Chow forecast test:</td>
<td>0.6459</td>
<td>Chow forecast test:</td>
<td>1.0387</td>
</tr>
<tr>
<td>bootstrapped p-value:</td>
<td>0.9200</td>
<td>bootstrapped p-value:</td>
<td>0.3800</td>
</tr>
<tr>
<td>asymptotic F p-value:</td>
<td>0.9281</td>
<td>asymptotic F p-value:</td>
<td>0.4312</td>
</tr>
<tr>
<td>degrees of freedom:</td>
<td>60, 32</td>
<td>degrees of freedom:</td>
<td>20, 72</td>
</tr>
</tbody>
</table>

As one can see, the Chow forecast test never accepts breaks at the 5 percent level of significance. This is the argument we use in the main text.

---

33These results are available on request, but can easily be replicated by the reader.
D  Model 2: Diagnostic Tests

Table 13: Standardized Residuals 1951-2000

Table 14: Residual Autocorrelations and Cross-correlations
PORTMANTEAU TEST \( \text{H}_{0}: \text{Rh}=(r_1,\ldots,r_h)=0 \)
tested order: 16
test statistic: 42.0101
p-value: 0.9625
adjusted test statistic: 51.4698
p-value: 0.7756
degrees of freedom: 60.0000

LM-TYPE TEST FOR AUTOCORRELATION with 5 lags
Reference: Doornik (1996), LM test and LMF test (with F-approximation)
LM statistic: 18.4211
p-value: 0.5597
df: 20.0000
LMF statistic: 0.8334
p-value: 0.6663
df1: 20.0000
df2: 70.0000

TESTS FOR NONNORMALITY
Reference: Doornik & Hansen (1994)
joint test statistic: 8.3313
p-value: 0.0802
degrees of freedom: 4.0000
skewness only: 2.1721
p-value: 0.3375
kurtosis only: 6.1592
p-value: 0.0460
joint test statistic: 4.4863
p-value: 0.3442
degrees of freedom: 4.0000
skewness only: 2.9982
p-value: 0.2233
kurtosis only: 1.4881
p-value: 0.4752

JARQUE-BERA TEST

<table>
<thead>
<tr>
<th>variable</th>
<th>teststat</th>
<th>p-Value(Chi^2)</th>
<th>skewness</th>
<th>kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>u1</td>
<td>3.9848</td>
<td>0.1364</td>
<td>0.5875</td>
<td>3.7558</td>
</tr>
<tr>
<td>u2</td>
<td>0.8061</td>
<td>0.6683</td>
<td>0.2250</td>
<td>2.5614</td>
</tr>
</tbody>
</table>

ARCH-LM TEST with 16 lags

<table>
<thead>
<tr>
<th>variable</th>
<th>teststat</th>
<th>p-Value(Chi^2)</th>
<th>F stat</th>
<th>p-Value(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u1</td>
<td>16.1615</td>
<td>0.4417</td>
<td>1.9796</td>
<td>0.0914</td>
</tr>
<tr>
<td>u2</td>
<td>12.2305</td>
<td>0.7280</td>
<td>1.2145</td>
<td>0.3511</td>
</tr>
</tbody>
</table>

MULTIVARIATE ARCH-LM TEST with 5 lags

VARCHLM test statistic: 47.2603
p-value(chi^2): 0.3804
degrees of freedom: 45.0000
D.1 Stability

Table 15: CUSUM and CUSUMSQ

<table>
<thead>
<tr>
<th>CUSUM: $c_t$ (top), $g_t$ (bottom)</th>
<th>CUSUMSQ: $c^2_t$ (top), $g_t$ (bottom)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="CUSUM plot" /></td>
<td><img src="image2" alt="CUSUMSQ plot" /></td>
</tr>
</tbody>
</table>

These plots reveal that the CUSUM and the CUSUMSQ are both well within the bands of a significance level of 1 percent. From them we do not infer that there is a problem with stability.

The recursive estimates for $\mu$ look very stable and are not presented for lack of space. The recursive parameter estimates of the coefficients are given below.

Table 16: Recursive Coefficients II

<table>
<thead>
<tr>
<th><img src="image3" alt="Recursive Coefficients plot" /></th>
</tr>
</thead>
</table>

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Table 17: Recursive Residuals

D.1.1 Chow Tests

We searched for breaks over the sample period of model 2 as in section C.1.1. This identified 1977 as a possible break date. Again we then conducted the Chow test 1977, using more observations. The results are presented below.

CHOW TEST FOR STRUCTURAL BREAK


sample range: [1952, 2000], T = 49

tested break date: 1977 (25 observations before break)

break point Chow test: 20.6684
bootstrapped p-value: 0.1200
asymptotic chi^2 p-value: 0.0142
degrees of freedom: 9

sample split Chow test: 18.0809
bootstrapped p-value: 0.0600
asymptotic chi^2 p-value: 0.0060
degrees of freedom: 6

Chow forecast test: 0.8811
bootstrapped p-value: 0.7100
asymptotic F p-value: 0.6658
degrees of freedom: 48, 42

As one can see, the Chow forecast test does not accept the break in 1977 at the 5 percent level of significance. This is the argument we use in the main text.
References


