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When no news is good news: Multidimensional heterogeneous beliefs in financial markets

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When No News is Good News: Multidimensional Heterogeneous Beliefs in Financial Markets *

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When No News is Good News: Multidimensional Heterogeneous Beliefs in Financial Markets

Abstract

We demonstrate the asset pricing implications of investors' belief heterogeneity in the frequency of news arrival and its joint impact with heterogeneous beliefs about news content. Investors trade volatility derivatives against each other to speculate on the rate of news arrival: greater disagreement of this kind gives rise to more extreme derivative positions. When disagreement about news arrival frequency is low, volatility exhibits mean reversion because extreme optimists and pessimists incur substantial wealth losses amid intense market swings. In contrast, high disagreement about the news arrival rate leads to volatility persistence. When news is absent in such environments, volatility sellers dominate, and extreme payoffs are underweighted in the formation of market expectations, resulting in lower implied volatility. In this context, "no news" effectively becomes good news for risky asset valuations.

Key words: *News arrival, heterogeneous beliefs, derivatives, volatility*

JEL codes: *G11, G12, D83, D84*

1 Introduction

News plays a central role in financial markets by triggering price adjustments, redistributing wealth, and shaping volatility through the arrival of new information. Yet what matters is not only what the news says, but also how often it arrives. This distinction introduces a crucial dichotomy between the content of news (e.g., whether it signals favorable or unfavorable fundamentals) and the frequency of news arrivals (i.e., how often informative events occur). While much of the literature focused on disagreement over content—capturing the classic divide between optimists and pessimists—less attention has been paid to heterogeneity in beliefs about the tempo of information flow. In practice, investors may broadly agree on the relative likelihood of good versus bad news, yet differ in their expectations about how often such news will materialize. This second dimension of belief is particularly relevant in markets where volatility itself becomes the object of speculation, such as in options and volatility derivatives. Understanding how these two dimensions—news content and arrival rate—interact is thus essential for explaining price dynamics and the persistence of volatility, even in the absence of major surprises.

This paper fills the gap by developing a tractable asset pricing framework that incorporates heterogeneity in both the content and frequency dimensions of investor beliefs. These beliefs translate into positions in both the underlying asset and derivative markets, allowing us to examine how wealth is redistributed across agents with divergent expectations. Those that are proven correct ex post accumulate more wealth, thereby exerting greater influence on asset prices. Bullish and bearish investors benefit, respectively, from favorable and unfavorable news. Simultaneously, volatility sellers profit when news is infrequent and market volatility is low, whereas frequent news arrivals generate persistent volatility that favors volatility buyers.

Realized frequency of news arrival exerts a subtle yet powerful influence on investor behavior and market outcomes. More frequent news arrivals reduce wealth-weighted disagreement over content, penalizing investors with incorrect beliefs, while simultaneously increasing the wealth-weighted average belief about the arrival rate. Crucially, the impact of news arrival frequency on asset prices and volatility depends on the structure of investor disagreement. When investors broadly agree on the rate of news arrival but disagree about its content, more frequent news is associated with higher asset prices and lower implied volatility. In contrast, when disagreement about the news arrival rate itself is substantial, increased frequency of news lowers asset prices and raises implied volatility. These patterns highlight the importance of modeling belief heterogeneity along multiple dimensions to understand complex dynamics in underlying and derivatives markets.

To formalize these insights, we develop a discrete-time model in which the state of the world evolves along a trinomial tree representing three possible outcomes "good news," "no news," and "bad news," that affect the terminal payoff of risky assets. Sometimes, even scheduled news announcements may correspond to no material change in fundamentals. In such cases, although these announcements are inherently news events, they are classified as "no news" within this framework. Accordingly, the level of fundamentals is reflected by the difference between the counts of good and bad news, while their sum captures the realized frequency of news arrivals.

Agents in this economy hold heterogeneous beliefs, characterized by two parameters: the perceived probability of news arrival, v, and the conditional probability of "good news" given arrival, u. These parameters capture disagreement along both the frequency and content dimensions of news. Initial wealth is distributed according to a flexible and analytically tractable two-dimensional beta distribution.

Investors trade two risky assets: a market asset that reflects fundamental values, and a one-period, delta-neutral derivative, similar to a straddle, that provides pure exposure to gamma. Portfolio allocations embody investors' subjective beliefs: positions in the market are shaped by their optimism or pessimism about fundamentals, while positions in the derivative are primarily driven by their beliefs about the frequency of informative events. Consequently, derivative holdings serve as an observable proxy for disagreement over news frequency, enabling us to distinguish between environments characterized by low and high disagreement.

As agents trade and accumulate gains or losses, the wealth distribution shifts toward those whose beliefs more closely align with realized outcomes. Asset prices are shaped by this evolving, wealth-weighted distribution of beliefs. In particular, the market price reflects a harmonic average of the terminal payoff, aggregated over all possible future news trajectories and weighted by investors' current wealth. The risk-neutral variance of market returns, in turn, captures the belief dispersion within this wealth-weighted distribution.

Frequent news arrivals affect the shape of the wealth-weighted distribution in distinct ways. When disagreement is primarily about the content of news, frequent arrivals concentrate wealth among agents with more accurate assessments, reducing implied volatility and supporting higher prices. In contrast, when disagreement centers on the frequency of news itself, frequent arrivals sustain broader dispersion in beliefs and wealth, thereby increasing implied volatility and dampening prices. Since more frequent news arrivals also translate into higher realized volatility, these two scenarios imply different volatility dynamics: mean reversion when disagreement is mainly about content, and high persistence

when it concerns arrival rates.

Exploring further, the relationship between agents' subjective return expectations and their derivative positions reveals a U-shaped pattern. Extreme optimists and pessimists hold larger derivative exposures, anticipating greater volatility, while moderate investors act as derivative sellers. This pattern underscores the speculative nature of trading driven by heterogeneous beliefs, with wealth dynamics amplifying the influence of extreme views on market outcomes.

Furthermore, we demonstrate that investors' derivative positions are closely linked to their subjective return expectations. Investors with extreme bullish or extreme bearish views on market returns tend to take large positions in the volatility derivative, reflecting strong beliefs about the frequency of news arrivals. In contrast, investors with moderate expectations act as counterparties and sell this volatility derivative. The model delivers closed-form expressions for both the wealth distribution across return beliefs and the corresponding derivative exposures, confirming this U-shaped pattern: extreme return expectations are associated with higher perceived news frequency and greater derivative investment.

This framework is particularly well-suited to environments where investors expect the data-generating process to remain stable and the nature of news to be consistent over a defined period while acknowledging the possibility of a regime shift before or after this window. The model also demonstrates flexibility by showing its equivalence to various alternative setups and interpretations. In particular, its pricing outcomes are equivalent to those of a representative agent with log utility who learns over time, starting from a prior belief distribution that mirrors the initial wealth distribution in our setup. Alternatively, the framework can be reinterpreted through the lens of momentum and contrarian trading strategies: momentum traders assign greater probability to extreme realizations, expecting trends to persist, while contrarian investors anticipate mean reversion, placing more weight on outcomes clustered around the average.

Relevant literature. Heterogeneous beliefs shape financial markets and diverse expectations drive speculative trading and volatility Harrison and Kreps (1978), Scheinkman and Xiong (2003). Our work builds on the vast literature of heterogeneous beliefs, enriching the analysis by modeling belief heterogeneity in both news content (optimistic vs. pessimistic views) and news arrival frequency—a dimension offering new insights into asset pricing, volatility, and portfolio dynamics. Another possible dimension of heterogeneity is agents' preferences, and it has been studied by Dumas (1989), Chan and Kogan (2002) although agents' belief heterogeneity is not taken into account (Shiller (1987), Ben-David, Graham and Harvey (2013)).

Earlier works in asset pricing studied the role of disagreement in market efficiency Figlewski (1978), in evolution of wealth distribution (Zapatero (1998), Jouini and Napp (2007), Bhamra and Uppal (2014)), in volatility and trading volume (Basak (2005), Banerjee and Kremer (2010), Atmaz and Basak (2018)), in the amplification of important but rare states (Kogan et al. (2006)), in the pricing of derivatives (Buraschi and Jiltsov (2006)), and in the reactions of prices to public information (Ottaviani and Sørensen (2015)). Our model extends these by incorporating news frequency and our gamma-exposed derivative isolates frequency beliefs, enhancing the understanding of derivative pricing. Cao and Ou-Yang (2008) models options trading driven by differences in signal precision. We extend that by incorporating wealth effects and analyzing how changes in wealth influence the dynamics of derivatives trading. We also show frequency disagreement spurs derivative speculation, notably among extreme optimists/pessimists.

More recently, Martin and Papadimitriou (2022) highlights how sentiment-driven trading amid belief heterogeneity amplifies both volatility and trading volume, resonating with our content dimension. Atmaz and Buffa (2023) connects volatility disagreement to variance risk premia and time varying leverage effect. By modeling news frequency with a trinomial tree and gamma derivative, we enhance those works by revealing how news frequency disagreement alongside news content disagreement drives volatility and pricing dynamics.

Most prior literature restricts to the diffusion setting. Within the papers mentioned above, one exception is Chen, Joslin and Tran (2012), where a model with heterogeneous beliefs about disaster risk is considered. The model is tractable, which allows us to study many issues analytically. The tractability partially benefits from the assumption of log-utility, which we view as a reasonable benchmark given the results of Martin (2017), Kremens and Martin (2019), Martin and Wagner (2019), Gao and Martin (2021) and Martin and Papadimitriou (2022)—representative investor's perceived risk premium is equivalent to the risk-neutral variance. It also reflects the fact that we work with a continuum of beliefs, as in Geanakoplos (2010), Atmaz and Basak (2018), Martin and Papadimitriou (2022) but unlike the two agents models of Harrison and Kreps (1978), Scheinkman and Xiong (2003), Basak (2005), Buraschi and Jiltsov (2006), Kogan et al. (2006), Dumas, Kurshev and Uppal (2009), Banerjee and Kremer (2010), Simsek (2013), Bhamra and Uppal (2014), Borovička (2020), Chabakauri and Han (2020), and Atmaz and Buffa (2023). The continuum of beliefs structure allows us to separately identify the effects of wealth-weighted disagreement and the average belief, which are non-linearly related in two-agent models.

The remainder of the paper is structured as follows. Section 2 introduces a trino-

mial tree framework to derive equilibrium asset prices and portfolio holdings. Section 3 explores how the frequency of news arrivals and belief heterogeneity affect asset prices, volatility, and cross-asset portfolio allocations through their effects on investor expectations and wealth dynamics. Section 4 demonstrates the equivalence of this framework to alternative representations and analyzes its Brownian and Poisson continuous-time limits. Section 5 provides an alternative interpretation of our model in the lens of momentum and contrarian traders. Section 6 concludes. All proofs of results are in Appendix A.

2 Model

We consider a finite-horizon discrete-time economy with dates t ranging from 0 to T. Agents could trade three non-redundant assets: 1) a risk-free asset with one-period maturity, zero net supply, and a constant gross return normalized to one; 2) two risky assets — a long-lived risky asset representing the market with one unit of supply, and a derivative asset with zero net supply. All agents have *logarithmic preferences* with respect to terminal wealth. The optimal portfolios for log utility are myopic.

The state of the world is represented by a trinomial tree, where each node branches into three potential outcomes for the subsequent period: 'high', 'middle', and 'low', corresponding to "good news", "no news", and "bad news" about the aggregate terminal payoff of risky assets. From a single investor's perspective, the probability distribution of these three outcomes remains identical and independent across all periods. The payoffs of the risky assets are structured to guarantee that the market is dynamically complete, allowing for the replication of any contingent claims.

The model's trinomial tree framework is designed to accommodate the arrival of information in a flexible manner, encompassing both anticipated and unexpected events. Notably, even scheduled announcements, such as FOMC policy statements, are categorized based on the information they convey. If a widely anticipated event introduces no meaningful deviation from previous communications, it is treated as "no news" within this structure. This distinction underscores the model's focus on informational innovation rather than the mere occurrence of events.

2.1 Agents' Beliefs

Agents hold heterogeneous and dogmatic beliefs about state evolution—they agree to disagree. Each agent's belief is represented by a two-dimensional vector $(u, v) \in (0, 1) \times (0, 1)$. Here, v denotes the probability that some news arrives (either "good" or "bad"),

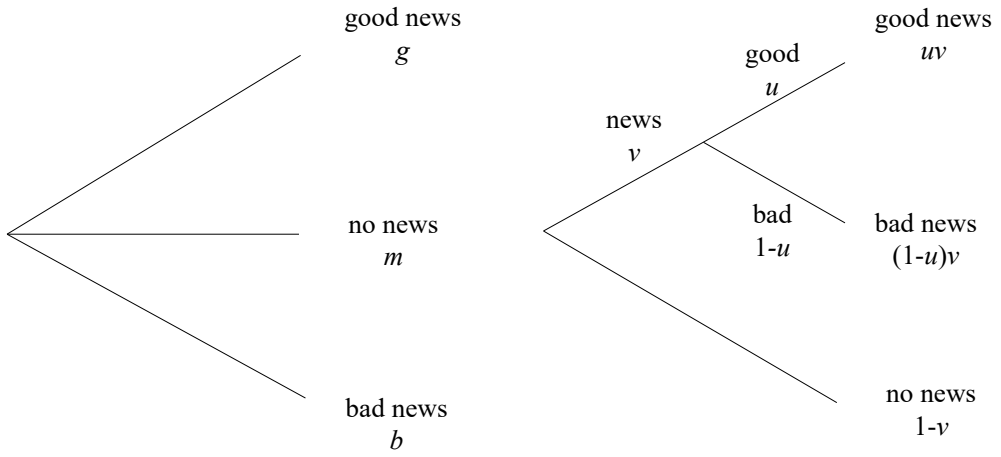


Figure 1: **Two equivalent representations of one-period trinomial tree**

and u is the probability that the news is "good," conditional on arrival. The parameter v thus reflects the agents' perceived frequency of news arrival, while u captures their relative optimism. These beliefs imply the following probabilities for "good news" (g), "no news" (m), and "bad news" (b):

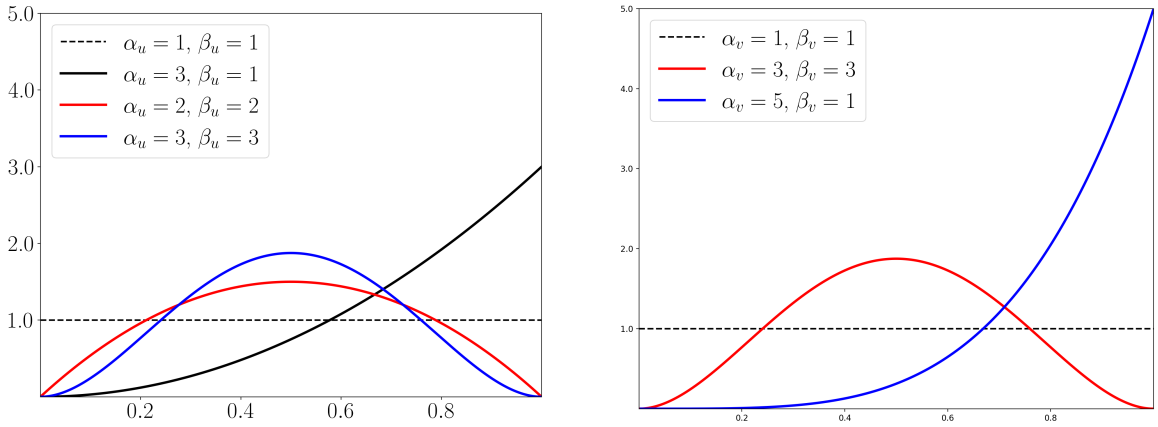
$$g = uv, \quad m = 1 - v, \quad b = (1 - u)v\,. \tag{1}$$

Naturally, the three probabilities sum up to unity: $g + m + b = 1$. Conversely, the belief parameters (u, v) can be recovered from (g, m, b) via:

$$u = \frac{g}{g + b}, \quad v = g + b\,. \tag{2}$$

Throughout the remainder of the paper, we use (u, v) to index agents according to their beliefs. Because of trading, the wealth distribution of agents across different beliefs is time varying. We use $f_t(u, v)$ to represent the wealth distribution at time t.

Our framework incorporates two distinct dimensions of belief heterogeneity. The first pertains to the perceived volatility of the fundamental, captured by differing expectations regarding the frequency of news arrival (v). Some agents anticipate a stable fundamental with "no news" dominating, whereas others expect more frequent realizations of either "good news" or "bad news." The second dimension involves disagreement on the content of fundamental news, as reflected in the relative bullishness (u), which measures the



(a) Distribution on news content u

(b) Distribution on news arrival rate v

Figure 2: **Beta distributions of beliefs in u or v**

conditional probability of "good news" relative to "bad news" when a news event occurs.

A convenient choice of the initial wealth distribution $f_0(u, v)$, which allows enough flexibility for our purposes, is a 2D-beta distribution where u and v are independent

$$f_0(u, v) = f_0(u) f_0(v) = \frac{u^{\alpha_{u0}-1}(1-u)^{\beta_{u0}-1}}{B(\alpha_{u0}, \beta_{u0})} \frac{v^{\alpha_{v0}-1}(1-v)^{\beta_{v0}-1}}{B(\alpha_{v0}, \beta_{v0})}. \tag{3}$$

The beta distribution provides a tractable formula for aggregate beliefs and allows us to characterize the equilibrium in closed form.[1]

As illustrated in Figure 2, Beta distribution provides the flexibility to host different situations of heterogeneous beliefs in news arrival rate and news contents. Two limiting cases are worth mentioning: i) When $\alpha_{v0}, \beta_{v0} \to \infty$ while α_{v0}/β_{v0} remains constant, all agents converge to a consensus on the probability of the "no news" state, given by $m_0 = \beta_{v0}/(\alpha_{v0} + \beta_{v0})$. Here, disagreement is solely focused on news content, and the dispersion in beliefs about the magnitude of the fundamental payoff diminishes as the sum $\alpha_{u0} + \beta_{u0}$ increases. ii) When only disagreement concerning news arrival rate persists and the divergence in beliefs decreases with an increase in $\alpha_{v0} + \beta_{v0}$—this happens when $\alpha_{u0}, \beta_{u0} \to \infty$ with α_{u0}/β_{u0} held constant.

[1]This 2D-beta distribution also encompasses the *Dirichlet distribution* for three states (g, m, and b) as a specific case. Notably, when the parameters of the 2D-beta distribution satisfies $\alpha_{v0} = \alpha_{u0} + \beta_{u0}$, it reduces to a Dirichlet distribution with parameters $(\alpha_{u0}, \beta_{v0}, \beta_{u0})$ governing the probabilities (g, m, b). Because Dirichlet distribution treats the three states rotationally symmetric, it associates with a rather special situation when the disagreement on news content very weakly connected with the disagreement on news arrival rate. Our framework does not necessarily adhere to this restriction.

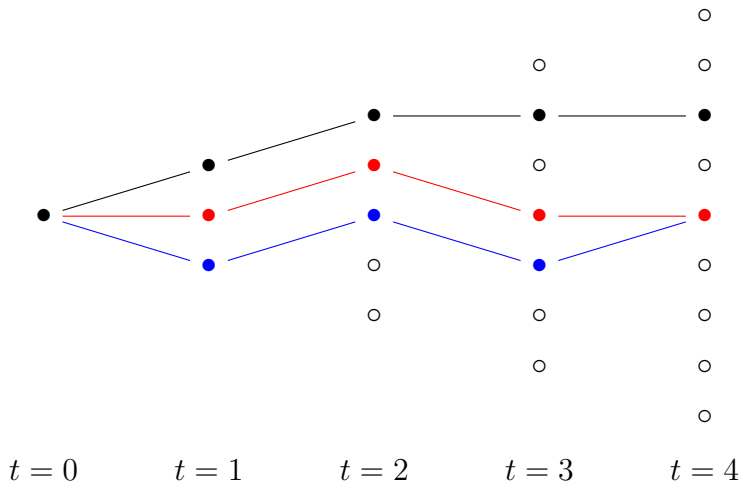


Figure 3: **Three possible sample paths at** $t = 4$

The black path is when $(n_4, \nu_4) = (2, 2)$, the red path is when $(n_4, \nu_4) = (0, 2)$, and blue path is when $(n_4, \nu_4) = (0, 4)$.

2.2 Asset Payoffs

The economy features two risky assets and a risk-free asset. The first risky asset represents *the market*, with its price at time t denoted by p_t. The second risky asset is a one-period derivative security, priced at q_t at time t, with a non-redundant payoff of x_{t+1} at time $t + 1$. The market prices and derivative payoffs at the next three nodes at time $t + 1$ are denoted by $(p_{t+1,g}, p_{t+1,m}, p_{t+1,b})$ and $(x_{t+1,g}, x_{t+1,m}, x_{t+1,b})$, respectively. Since the market is dynamically complete with three non-redundant assets, any assumptions about the derivative's payoff do not affect the agents' wealth allocation across different states. For convenience, we consider derivatives whose payoffs are uncorrelated with the market under the risk-neutral measure

$$\mathrm{Cov}_t^*[x_{t+1}, p_{t+1}] = 0\,. \tag{4}$$

This focus is without loss of generality: any derivative can be transformed into a zero-delta version by delta-hedging with the underlying asset. The hedged payoff satisfies the above condition while preserving the span of attainable payoffs. Investors hold the same quantity of the derivative before and after this transformation.

Let n_{gt}, n_{mt}, and n_{bt} represent the number of "good news," "no news," and "bad news" events, respectively, observed from time 0 to t. Because $t = n_{gt} + n_{mt} + n_{bt}$, there are only two variables needed to characterize (n_{gt}, n_{mt}, n_{bt}). Among many equivalent choices

of factors, we think the following two are intuitive: the level of fundamental news n_t and the cumulative counts of news events ν_t,

$$n_t = n_{gt} - n_{bt}\,, \qquad \nu_t = n_{gt} + n_{bt}\,. \tag{5}$$

Figure 3 illustrates the notation of (n_t, ν_t) with some sample paths. The red and black paths have the same news count but the net news on the black path is higher. The net news on the blue and black path are the same, but the news counts on blue path is higher than the black one.

2.3 Equilibrium

Since the optimal portfolios for log utility are myopic, it is sufficient to consider the one period portfolio choice problem for an agent with the belief (u, v) is to allocate her wealth in $\theta_t^{(u,v)}$ units of risky asset 1, $\phi_t^{(u,v)}$ units of risky asset 2, and the rest in the risk-free asset (long or short) to solve the following optimization problem,

$$\max_{\theta_t, \phi_t} \mathbb{E}_t^{(u,v)} \log \left[w_t^{(u,v)} + \theta_t^{(u,v)}(p_{t+1} - p_t) + \phi_t^{(u,v)}(x_{t+1} - q_t) \right]\,, \tag{6}$$

where w_t represents the time t wealth of the individual agent with beliefs (u, v).

Because the market is dynamically complete, agents' portfolios can be represented as positions in Arrow-Debreu securities. In equilibrium, agents' subjective probability weighted SDFs are equal to the Arrow-Debreu security prices, which correspond to the risk-neutral measures at each node at that time (since the gross risk-free rate is normalized to one).

$$g_t^* = g \cdot \frac{w_t^{(u,v)}}{w_{t+1,g}^{(u,v)}}\,, \quad m_t^* = m \cdot \frac{w_t^{(u,v)}}{w_{t+1,m}^{(u,v)}}\,, \quad b_t^* = b \cdot \frac{w_t^{(u,v)}}{w_{t+1,b}^{(u,v)}}\,, \tag{7}$$

where (g_t^*, m_t^*, b_t^*) are risk-neutral probabilities for the three nodes; $w_t^{(u,v)}$ denotes the wealth of the agent with beliefs (u, v) at time t; $w_{t+1,h}^{(u,v)}$, $w_{t+1,m}^{(u,v)}$, and $w_{t+1,l}^{(u,v)}$ denote this agent's wealth at the respective nodes at time $t+1$.

The optimal allocation reflects the agent's subjective belief about the future, i.e. the relative weights among the equivalent Arrow-Debreu securities match (g, m, b). The state dependent one period wealth growth of the agent (u, v) is the ratio between his own belief and the risk-neutral measure: in the good state, wealth grows by g/g_t^*; in the medium state, by m/m_t^*; and in the bad state, by b/b_t^*.

Since the aggregate wealth growth of the entire economy is the change in market portfolio, the one-period state-by-state realized return on market satisfies

$$\frac{p_{t+1,g}}{p_t} = \frac{G_t}{g_t^*} \quad \frac{p_{t+1,m}}{p_t} = \frac{M_t}{m_t^*} \quad \frac{p_{t+1,b}}{p_t} = \frac{B_t}{b_t^*}\,. \tag{8}$$

where (G_t, M_t, B_t) are average beliefs weighted by agents' time-t wealth distribution. Equation (8) also implies p_t^{-1} is a weighted harmonic mean of the prices on next three nodes

$$p_{1,t}^{-1} = G_t p_{t+1,g}^{-1} + M_t p_{t+1,m}^{-1} + B_t p_{t+1,b}^{-1}\,. \tag{9}$$

Comparing (7) and (8), we obtain a closed-form expression for the wealth distribution at time t under the 2D beta initial distribution assumption:

$$f_t(u,v) = \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut},\beta_{ut})}\,\frac{v^{\alpha_{vt}-1}(1-v)^{\beta_{vt}-1}}{B(\alpha_{vt},\beta_{vt})}\,, \tag{10}$$

where $t = n_{gt} + n_{mt} + n_{bt}$ and $(\alpha_{ut}, \beta_{ut}, \alpha_{vt}, \beta_{vt})$ evolve according to

$$\alpha_{ut} = \alpha_{u0} + \frac{n_t + \nu_t}{2}, \quad \beta_{ut} = \beta_{u0} + \frac{\nu_t - n_t}{2}, \quad \alpha_{vt} = \alpha_{v0} + \nu_t, \quad \beta_{vt} = \beta_{v0} + t - \nu_t\,. \tag{11}$$

The derivations of (10) and (11) are provided in Appendix A.

The positive and negative fundamental news respectively reallocate wealth towards more bullish agents (higher u) and more bearish agents (lower u). In contrast, the absence of news redistributes wealth in favor of agents with lower perceived likelihood of news occurring (lower v). In Figure 2, we demonstrate how the different news history would affect wealth distribution. We set the initial wealth distribution across agent (u, v) to be uniform, i.e. $\alpha_{u0} = \beta_{u0} = \alpha_{v0} = \beta_{v0} = 1$, and calculate wealth distributions at $t = 4$ for three different sample histories (see Figure 3 for visualisation). When realized news arrival rate is the same, higher net news would shift the wealth distribution on u towards more bullishness–"no news" is good news. This is reflected by comparing the red and black lines in Figure 2a. When the realized net news is the same, lower realized intensity would shift the wealth distribution towards more bullishness. This is reflected by comparing the blue and red line in Figure 2a. For the wealth distribution across v, what matters is the realized frequency of news arrival: the higher the frequency, the more wealth is shifted toward higher values of v. Notably, the red and black paths have the same distribution at $t = 4$ because they have experienced the same number of news arrivals.

We referred to those investors who allocate 100% of their wealth into the market as *Mr. Market* —his identity changes over time in our model. To compute Mr. Market's belief at time t, we simply take means of the wealth distribution in equation (10).

$$U_t(n_t, \nu_t) = \frac{\frac{1}{2}n_t + \frac{1}{2}\nu_t + \alpha_{u0}}{\nu_t + \alpha_{u0} + \beta_{u0}}, \quad V_t(n_t, \nu_t) = \frac{\nu_t + \alpha_{v0}}{t + \alpha_{v0} + \beta_{v0}}. \tag{12}$$

The first formula in (12) indicates that the 'bullishness' of Mr. Market would reflect the proportion of good news among all the news has arrived since $\frac{1}{2}n_t + \frac{1}{2}\nu_t = n_{gt}$. This also means, for a fixed time t and n_t, Mr. Market would be less bullish on sample paths with more volatile news history. The second formula in (12) implies Mr. Market's view about the news arrival rate would only be affected by the accumulative counts on the news ν_t: increased frequency of news arrival, in the sense of a higher ν_t/t, would push the wealth aggregated v up.

2.4 Asset holdings

Agents' holdings in Arrow-Debreu securities maps into positions in the market portfolio and the derivative security. For an agent with beliefs (u, v), the optimal positions are given by:[2]

$$\theta_t^{(u,v)} = w_t^{(u,v)} \frac{\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t]}{\mathrm{Var}_t^*[p_{t+1} - p_t]}, \quad \phi_t^{(u,v)} = w_t^{(u,v)} \frac{\mathbb{E}_t^{(u,v)}[x_{t+1} - q_t]}{\mathrm{Var}_t^*[x_{t+1} - q_t]}. \tag{13}$$

According to (13), agents with beliefs equal to the risk-neutral measure (u_t^*, v_t^*) would not hold any risky asset. Since the derivative is constructed to be *delta-neutral*, investors' positions in the underlying market portfolio are driven entirely by their bullishness (or lack thereof), rather than any hedging needs related to the derivative. In another word, the derivative provides only gamma exposure, introducing non-linear sensitivities in investors' wealth to market movements. The gamma exposure of agent (u, v) quantifies the curvature of wealth $w_t^{(u,v)}$ with respect to price p_t, and can be expressed as the following transformation of the wealth distribution. We provide the formula of gamma exposure for agent (u, v) in internet appendix I.A.1.

Without loss of generality, we assume the derivative is convex in the market, implying positive gamma. In a three-state setting, the derivative payoff must be either convex or concave in the market price. If the payoff is concave, we can simply redefine a short position as a long position, thereby ensuring the derivative exhibits convexity.

[2]We provide derivation in internet appendix I.A.1.

We proceed to simplify the asset holdings expressions and identify investors who holds delta-neutral or gamma-neutral portfolios.

Result 1 (Portfolio weights)**.** *The agents' portfolio weights in the underlying market and the derivative security are represented as:*

$$\frac{\theta_t^{(u,v)} p_t}{w_t^{(u,v)}} = \frac{\mathbb{E}_t^{(u,v)}[p_{t+1}/p_t - 1]}{\mathbb{E}_t^{(U_t,V_t)}[p_{t+1}/p_t - 1]}\,, \quad \frac{\phi_t^{(u,v)} q_t}{w_t^{(u,v)}} = \frac{\mathbb{E}_t^{(u,v)}[x_{t+1}/q_t - 1]}{\mathrm{Var}_t^*[x_{t+1}/q_t - 1]} \tag{14}$$

A delta-neutral position requires zero subjective expected return for the market:

$$\mathbb{E}_t^{(u,v)}[p_{t+1}/p_t - 1] = 0\,. \tag{15}$$

For a gamma-neutral position, the agent's belief must lie on the line passing through Mr. Market's belief and the risk-neutral belief:

$$(g, m, b) = \kappa(G_t, M_t, B_t) + (1-\kappa)(g_t^*, m_t^*, b_t^*)\,, \quad \textit{where } \kappa \textit{ is any scalar}\,. \tag{16}$$

By construction, Mr. Market always expect the market portfolio has a positive return. For other investors, their holdings in the market portfolio are determined by the ratio between subjective net return expectation and the market's average net return expectation. Those who expect zero market return will hold no position in the market portfolio. Investors who anticipate a positive return would hold position in the market, while those who take a short position expect a negative return. Investors who have more extreme views than Mr. Market would take leveraged position in their long or short positions —the absolute value of their portfolio weight in market will exceed 100%.

Since the derivative provides positive payoffs only in the case of "good news" or "bad news," an investor's subjective expectation of the derivative return $\mathbb{E}_t^{(u,v)}[x_{t+1}/q_t - 1]$ increases with g and b, and decreases with m. Investors who believe the fundamental is more volatile will take a positive position in the derivative, while those with opposite beliefs will take a negative position. On the diagram of (g, m, b), the boundary that separates these two groups of investors pass through both Mr. Market's belief (G_t, B_t) and the risk-neutral belief (g_t^*, b_t^*).

2.5 Asset Prices

Although aggregate beliefs are path-dependent, the pricing relationship in equation (9) enables a tractable valuation of the stock at time t. Specifically, the total value of the

market at time t can be computed using the weighted harmonic mean of its time-T value $p_T(n_T)$. The same weights also allows us to compute the risk-neutral variance of returns.

Result 2 (Market price and risk-neutral variance)**.** *Assuming the trinomial tree has T periods and the payoffs are $p_T(n_T)$ with $n_T \in [-T, T]$, the value of the market at time t is given by*

$$p_t = \left(\mathbb{E}_t \left[p_T^{-1}(n_T) \right] \right)^{-1}, \tag{17}$$

where the expectation is taken under the wealth-weighted belief distribution over terminal states n_T, given by:

$$\Pr(n_T | n_t, f_t) = \sum_{\substack{\forall n_g - n_b = n_T - n_t, \\ n_g + n_m + n_b = T - t}} \binom{T-t}{n_g, n_m, n_b} \frac{B(\alpha_{ut} + n_g, \beta_{ut} + n_b) B(\alpha_{vt} + n_g + n_b, \beta_{vt} + n_m)}{B(\alpha_{ut}, \beta_{ut}) B(\alpha_{vt}, \beta_{vt})} . \tag{18}$$

The risk-neutral variance of the return from time t to T is given by:

$$\mathrm{Var}_t^*[p_T/p_t - 1] = \mathbb{E}_t \left[p_T(n_T) \right] \mathbb{E}_t \left[p_T^{-1}(n_T) \right] - 1 . \tag{19}$$

The probability $\Pr(n_T | n_t, f_t)$ encapsulates the collective expectation—weighted by investor wealth—regarding the likelihood of reaching a terminal state with fundamental value n_T, given current fundamentals n_t and the prevailing wealth distribution f_t. It is computed by aggregating over all feasible news trajectories between t and T that lead to a cumulative net news change of $n_T - n_t$. When investor beliefs conform to a 2D-Beta distribution, this aggregation becomes analytically tractable through the Beta function, as detailed in Result 2. Ultimately, the market price and risk-neutral variance reflect this distribution, capturing the wealth-weighted consensus over all possible terminal outcomes.

On average, investors expect that the fundamental increase by $G_t - B_t$ over each period from t to T,

$$\mathbb{E}[n_T | n_t, f_t] = \sum_{n_T} n_T \Pr(n_T | n_t, f_t) = n_t + (T - t)(G_t - B_t) . \tag{20}$$

$G_t - B_t$ thus measures the wealth-weighted average expectation of fundamental growth.

Next, we examine how wealth distribution affects asset prices, holding current fundamentals n_t constant. We assume that the terminal marginal utility, $p_T^{-1}(n_T)$, is a convex function of the fundamental n_T. This condition is satisfied by a broad class of payoff

functions $p_T(n_T)$, including exponential, power or affine payoff functions with positive exponents and coefficients.

Consider two scenarios in which the average level of bullishness is identical, $U_t = U'_t$, but the degree of disagreement is greater in the first scenario, that is, $\alpha_{ut}+\beta_{ut} \leqslant \alpha'_{ut}+\beta'_{ut}$. This increased dispersion in beliefs leads to a more spread-out distribution of n_T, such that n_T is a mean-preserving spread of n'_T. As a result, the expected terminal marginal utility is higher in the first scenario:

$$\mathbb{E}[p_T^{-1}(n_T)] \geqslant \mathbb{E}[p_T^{-1}(n'_T)]\,, \tag{21}$$

which, according to equation (17), implies a lower market price. Hence, greater disagreement in news content depresses asset prices.

A higher average market expectation about news arrival rate has a similar effect. Suppose the market, on average, expects good and bad news to happen with equal probability. Consider two scenarios in which $V_t \geqslant V'_t$ while the degree of disagreement remains the same, i.e., $\alpha_{vt} + \beta_{vt} = \alpha'_{vt} + \beta'_{vt}$. The increase in average news arrival rate leads to a more dispersed distribution of n_T, making n_T a mean-preserving spread of n'_T. Consequently, the expected terminal marginal utility is higher, and the market price is lower. Thus, higher levels of news arrival rate also depress asset prices.

3 Analysis of Equilibrium

Section 3.1 analyzes how the frequency of news arrivals influences equilibrium asset prices and volatility, highlighting the contrasting effects that emerge depending on whether investors agree or disagree about news arrival rate. Section 3.2 investigates the relationship between investors' subjective return expectations and their positions in derivative markets, revealing how belief heterogeneity shapes cross-asset portfolio allocations.

3.1 How News Arrivals Shape Prices and Volatility

Asset prices are influenced by both the occurrence and absence of news events. The content of news is salient: positive news signals an improvement in fundamentals and shifts the wealth distribution in favor of optimistic investors. The frequency of news arrival also plays a crucial role. Frequent news redistributes wealth among investors with varying degrees of optimism and additionally empowers those who anticipate high news arrival rate. Consequently, the historical count of news events, ν_t, shapes market

disagreement and aggregate beliefs, and through these mechanisms, volatility expectations and asset prices.

News arrival exerts two distinct effects on the market. First, frequent news arrivals lead to a concentration of wealth among agents whose beliefs are correct ex-post, and reduce disagreement over news content. The level of "agreement" among investors, $\alpha_{ut} + \beta_{ut} = \alpha_{u0} + \beta_{u0} + \nu_t$, increases with ν_t. As demonstrated in Section 2.5, this reduction in disagreement over news content raises asset prices.

Second, a high historical count of news increases the market's average belief about future news arrival rate, V_t. This upward shift in expectations places downward pressure on asset prices. The overall impact depends on the relative strength of these two channels, which is, in turn, determined by the degree of disagreement among investors regarding news arrival rate.

When there is no disagreement over news arrival rate, the first channel dominates. We focus on a specification in which each piece of news has a multiplicative effect:

$$p_T(n_T) = e^{\sigma n_T} . \tag{22}$$

The terminal payoff takes the form of an exponential function of the fundamental. [3]

To isolate the effects of belief heterogeneity along specific dimensions, we examine a benchmark and limit scenario in which all investors share a common view about the frequency of news arrivals. In this setting, belief heterogeneity arises solely from differences in investors' assessments of the likelihood of receiving "good news" versus "bad news". The following result shows that frequent news arrivals lower expected volatility and increase asset prices.

Result 3 (News count and asset prices, no news arrival rate disagreement)**.** *Suppose all agents share a common belief about news arrival rate. Holding constant the average expectation of the fundamental growth, $G_t - B_t$, a higher historical news count ν_t reduces the risk-neutral variance* $\mathrm{Var}_t^*[p_T/p_t - 1]$ *and increases the ratio of the current asset price to the current fundamental, $p_t/e^{\sigma n_t}$.*

When investors hold similar expectations about the frequency of news arrivals, derivative trading becomes insignificant, as one key source of speculative demand – heterogeneous views on news arrival rate – is absent. In this environment, frequent news arrivals

[3] This specification lends itself naturally to a continuous-time limit, where the interval between trading opportunities shrinks to zero. Unlike an arithmetic payoff, the exponential (or geometric) form guarantees that the payoff remains strictly positive and cannot fall below zero.

spur more active trading of the underlying asset, concentrating wealth among agents whose beliefs are more closely aligned with realized fundamentals. For instance, frequent but offsetting fundamental shocks, neutral on average, tend to benefit moderate investors, while those with more extreme views are more likely to incur losses. As investor beliefs converge and expectations about the future stabilize, implied volatility declines and asset prices rise.

Notably, the lack of disagreement in news arrival rate results in mean reversion of volatility. Frequent news arrivals lead to high realized volatility in both fundamentals and asset prices, reflecting the market's continuous adjustment to new information. However, news also sustains active trading of the underlying asset, which diminishes disagreement and consequently lowers implied volatility.

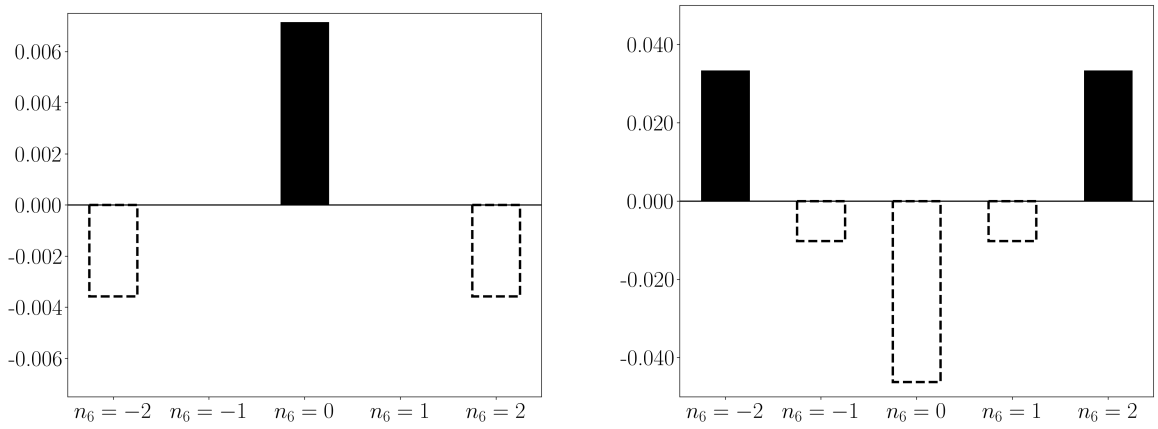
In contrast to Result 3, when disagreement about news arrival rate is strong, the second channel dominates. A comparison between disagreement in news arrival rate and disagreement in news content can be made by comparing α_v with the degree of "agreement" in news content $\alpha_u + \beta_u$. If, in the initial wealth distribution, α_{v0} is lower than $\alpha_{u0} + \beta_{u0}$, this inequality persists over time. Specifically, for any t:

$$\alpha_{vt} = \alpha_{v0} + \nu_t \leqslant \alpha_{u0} + \beta_{u0} + \nu_t = \alpha_{ut} + \beta_{ut} \tag{23}$$

Disagreement about news arrival rate remains persistently high at any point in time. The following result shows that a higher frequency of news arrival raises implied volatility and lowers asset prices.

Result 4 (News count and asset prices, prominent news arrival rate disagreement)**.** *Suppose $\alpha_{v0} \leqslant \alpha_{u0} + \beta_{u0}$ in the initial wealth distribution. Holding constant the average expectation of the fundamental growth, $G_t - B_t$, a higher historical news count ν_t increases the risk-neutral variance $\mathrm{Var}_t^*[p_T/p_t - 1]$ and lowers the ratio of the current asset price to the current fundamental, $p_t/e^{\sigma n_t}$.*

Frequent news arrivals, even when neutral in their aggregate effect, can lead to significant fluctuations in investor beliefs and substantial wealth redistribution. Derivative sellers are particularly vulnerable, as their positions are exposed to sharp market movements. These shifts often transfer wealth to investors with extreme views on market returns. This dynamic amplifies the likelihood of extreme outcomes in the wealth-weighted distribution of beliefs, thereby increasing implied volatility and depressing current asset prices. Consequently, markets tend to fare worse during periods of volatile but offsetting fundamental swings than during periods of relative informational silence or "no news."



(a) Agreement on v

(b) Disagreement on v

Figure 4: **Differences in probability across sample paths: high ν_t minus low ν_t** Each bar illustrates the difference in the probability mass assigned to state $n_{T=6}$, evaluated at time $t = 4$. Black bars above the zero indicate states where the blue path assigns higher probability than the red path. Hollow bars with dashed borders indicate states where the blue path assigns lower probability.

Result 4 highlights a critical link between volatility dynamics and the disagreement in news arrival rate. When investors disagree about the frequency of news arrivals, volatility becomes more persistent. These disagreements fuel sustained derivative trading, as opposing beliefs generate ongoing speculative demand. A history of volatile price movements signals elevated future option prices, as market participants anticipating continued turbulence gains more influence. In essence, high realized volatility today translates into high implied volatility going forward, making derivatives more expensive and reflecting the market's expectation of continued instability.

Example.—Consider a $T = 6$ period example with an initial wealth distribution in which u is uniformly distributed and all agents share the belief that $v = 1/2$. The blue path in Figure 3 features more frequent news arrivals than the red path. By time $t = 4$, the wealth-weighted distribution of the terminal payoff along the blue path becomes more concentrated relative to the red path. Figure 4a visualizes this contrast: black bars above zero indicate states where the blue path assigns higher probability than the red path, while hollow bars with dashed borders indicate states where it assigns lower probability. This reduced wealth-weighted disagreement along the blue path results in a higher price. The result is reversed under an alternative initial wealth distribution, where both u and v are uniformly distributed. At $t = 4$, the wealth-weighted distribution of the terminal

payoff under the blue path becomes more dispersed than under the red path, as shown in Figure 4b. Greater dispersion along the blue path results in a lower price.

3.2 Derivative Investments and Subjective Return Expectation

Next, we delve into the relationship between investors' position in the derivative security and their subjective return expectations for the underlying asset. As shown in (13), investors' positions in the underlying asset are linear in these subjective expectations. Consequently, this relationship also reveals how investors' holdings in the derivative and the underlying asset are connected.

We group investors according to their subjective expectations of the underlying asset's returns. Using result 1, we then derive a closed-form expression for the wealth distribution across these beliefs, denoted by $f_t(r)$.

Result 5 (Wealth share across beliefs in expected returns)**.** *Define the variable $z(r)$, k and γ,*

$$z(r) = \frac{(1+r)g_t^* m_t^* - g_t^* M_t}{G_t m_t^* - g_t^* M_t}, \quad k = \frac{G_t m_t^* - g_t^* M_t}{g_t^* M_t - \frac{B_t}{b_t^*} g_t^* m_t^*}, \quad \gamma = \frac{g_t^* m_t^*}{G_t^* m_t^* - g_t^* M_t^*}, \tag{24}$$

the wealth share of agents who believe expected return of underlying is r satisfies

$$f_t(r) = \frac{\gamma k^{\beta_{ut}} z^{\alpha_{vt}-1}(1-z)^{\beta_{ut}+\beta_{vt}-1} B(\beta_{ut},\beta_{vt})}{(k+1)^{\beta_{ut}} B(\alpha_{ut},\beta_{ut}) B(\alpha_{vt},\beta_{vt})} F_1\left(\beta_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\alpha_{ut}; \beta_{ut}+\beta_{vt}; 1-z, \frac{k(1-z)}{1+k}\right),$$

for $z > 0$, and

$$f_t(r) = \frac{\gamma k (-kz)^{\alpha_{vt}-1}(1+kz)^{\alpha_{ut}} B(\alpha_{ut},\beta_{vt})}{(1+k)^{\alpha_{ut}+\beta_{vt}-1} B(\alpha_{ut},\beta_{ut}) B(\alpha_{vt},\beta_{vt})} F_1\left(\alpha_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}; 1+kz, \frac{1+kz}{1+k}\right),$$

for $z < 0$. The function $F_1(\dots)$ represents the Appell hypergeometric function, and $(\alpha_{ut}, \beta_{ut}, \alpha_{vt}, \beta_{vt})$ follows the same definition as in (11).

The left panels of I.A.1 illustrate the path-dependent feature of the wealth distribution across beliefs of net return of market. The three colored lines correspond to the sample paths shown in Figure 3. Utilizing this closed-form solution, we examine how varying outlooks on market performance translate into their derivative positions.

Although the choice of derivative payoff structure does not impact the equilibrium allocation or wealth distribution, specifying this structure simplifies the interpretation of asset holdings presented above. A payoff structure for the derivative security uncorrelated

with the market is given by:

$$x_{t+1,g} = p_t(b_t^* - B_t)\,, \quad x_{t+1,m} = 0\,, \quad x_{t+1,b} = p_t(G_t - g_t^*)\,. \tag{25}$$

The derivative resembles a *straddle*, yielding zero payoff in the case of "no news", while providing positive payoffs in response to both "good news" and "bad news". The return of this derivative is uncorrelated with the market under the risk-neutral measure. In the context of options trading, the delta of the derivative is equal to zero. Therefore, the derivative can be interpreted as a delta-neutral combination of call and put options on the market portfolio, with a strike price at the "no news" realization $p_{t+1,m}$. This structure allows for a clear connection between the investors' view on volatility and their dollar positions in the derivatives market.

Building upon the relationship between investors' beliefs and their market positions, it follows that optimists, who anticipate positive returns on the underlying asset, will establish long positions in the market. Conversely, pessimists, expecting negative returns, will short the market. Extending this further, investors holding extreme bullish views will exhibit portfolio allocations that resemble long call options, benefiting disproportionately from significant upward price movements. Conversely, those with extreme bearish sentiments will hold portfolios akin to long put options, profiting most from substantial price declines.

More broadly, investors' market outlook correlates with their derivative positions. The subsequent result examines the average derivative position for investor groups segmented based on their subjective expectations of market returns.

Result 6 (Aggregated gamma positioning across same beliefs of return)**.** *For cohort of investors with same subjective expectation r for the market return, the wealth-weighted 'average portfolio weight' in the derivative satisfies*

$$\mathbb{E}\left[\left.\frac{\phi_t^{(u,v)} q_t}{w_t^{(u,v)}}\right| r\right] = \frac{1}{1-v_t^*}\left(\mathbb{E}[v|r] - v_t^* + \frac{(v_t^* - V_t)}{\mathbb{E}^{(U_t,V_t)}[r]}\, r\right)\,, \tag{26}$$

where $\mathbb{E}[v|r]$ denotes the wealth-weighted average news arrival rate v among investors with expectation r with the following close form expression: when $z \geqslant 0$,

$$\mathbb{E}[v|r] = z + z(1-z)\cdot\frac{\beta_{ut}}{\beta_{ut}+\beta_{vt}}\cdot\frac{F_1\left(\beta_{ut}+1;\alpha_{vt}+\beta_{vt},1-\alpha_{ut};\beta_{ut}+\beta_{vt}+1;1-z,\frac{k(1-z)}{1+k}\right)}{F_1\left(\beta_{ut};\alpha_{vt}+\beta_{vt}-1,1-\alpha_{ut};\beta_{ut}+\beta_{vt};1-z,\frac{k(1-z)}{1+k}\right)}$$

and when $z < 0$, we have

$$\mathbb{E}[v|r] = -kz + (-kz)(1+kz) \cdot \frac{\alpha_{ut}}{\alpha_{ut}+\beta_{vt}} \frac{F_1\left(\alpha_{ut}+1; \alpha_{vt}+\beta_{vt}, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}+1; 1+kz, \frac{1+kz}{1+k}\right)}{F_1\left(\alpha_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}; 1+kz, \frac{1+kz}{1+k}\right)}$$

The variable $z(r)$ and k follow the same definition in (24) and $F_1(\dots)$ represents the Appell hypergeometric function.

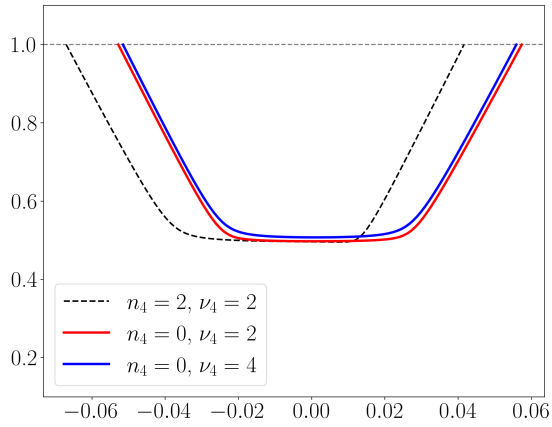
The wealth-weighted average news arrival rate, $\mathbb{E}[v|r]$, exhibits a U-shaped relationship with the subjective expected return r. Investors with highly optimistic or pessimistic expectations also tend to hold extreme views about the news arrival rate, resulting in $\mathbb{E}[v|r] = 1$ at the boundaries of possible returns, $r = G_t/g_t^* - 1$ and $r = B_t/b_t^* - 1$. In contrast, those with more moderate expectations typically assign lower average news arrival rates. Figures 5a and 5b illustrate this relationship under low and high disagreement in news arrival rates, using $(\alpha_v, \beta_v) = (100, 100)$ and $(1, 1)$, respectively. While both exhibit the U-shape, the low-disagreement case produces a noticeably flatter curve near the center, reflecting stronger consensus among investors.

Differences in news arrival rate expectations naturally lead to variation in derivative positions. Figures 5c and 5d show the derivative holdings of investor cohorts, expressed as fractions of total wealth in the economy. These holdings are obtained by multiplying the wealth distribution in (18) with the corresponding portfolio weights from (26).

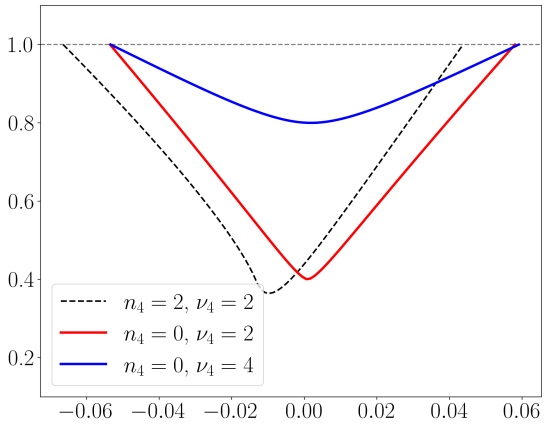
Since every derivative contract has both a buyer and a seller, aggregate net positions must sum to zero. Market clearing thus requires that the extreme positions of highly optimistic or pessimistic investors be offset by counterparties. This role is primarily filled by investors with more moderate beliefs, who act as derivative sellers. These participants, with less extreme views on market returns, provide essential liquidity and facilitate trade execution. The equilibrium in the derivatives market emerges from the interaction between investors expressing strong views on news arrival rate and those accommodating these trades through more tempered expectations. [4]

Derivative positions offer an empirically observable metric to distinguish between cases of low and high disagreement about the news arrival rate, as discussed in section 3.1. When disagreement about the news arrival rate is low, $\mathbb{E}[v|r]$ stays close to the market average V_t across most investor cohorts. Consequently, derivative portfolio weights are small, leading to limited investments in derivative markets, as illustrated in Figure 5c.

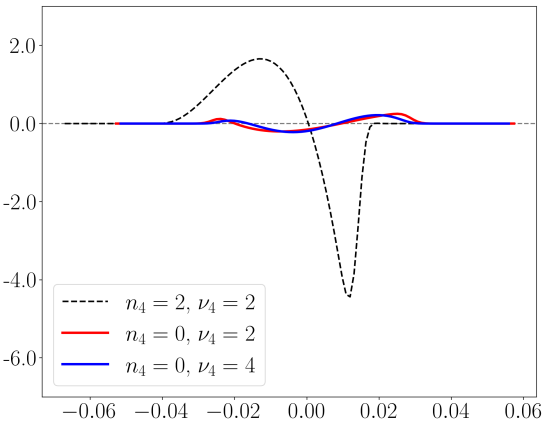
[4] In the online appendix I.A.4, we plot all the figures' time variation at time $t = 1, 2, 3, 4$ to demonstrate how different sample paths would affect the wealth distribution, gamma positioning across agents' market return expectation.



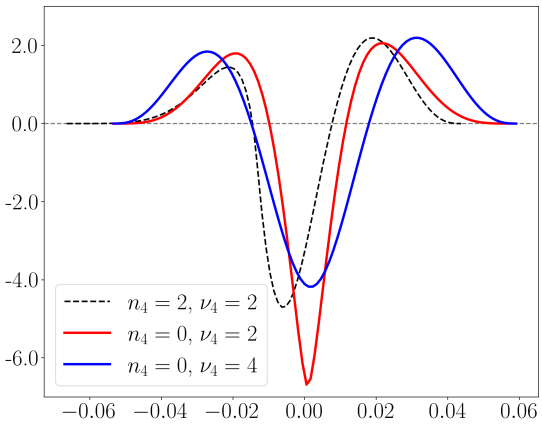
(a) Wealth weighted beliefs of v across r
Low news arrival rate disagreement



(b) Wealth weighted beliefs of v across r
High news arrival rate disagreement



(c) Gamma exposure across r
Low news arrival rate disagreement



(d) Gamma exposure across r
High news arrival rate disagreement

Figure 5: **Beliefs of market return and gamma positioning**

The figure reports wealth-weighted average news arrival rate v and Gamma exposures for the cohort of investors with expected return r. In the left panels (5a and 5c), the initial wealth distribution reflects low disagreement in the news arrival rate dimension, characterized by $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. In contrast, the right panels (5b and 5d) assume a uniform initial wealth distribution across agents, with $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$. See Figure 3 for an illustration for the possible sample paths that leads to the corresponding wealth distribution and gamma position at $t = 4$.

In contrast, high disagreement gives rise to significantly larger derivative positions, as shown in Figure 5d. These large derivative investment and the resulting shift in wealth distribution is the central force behind the volatility persistence result in 4.

4 Further discussions

This framework applies broadly but is especially relevant in scenarios where investors anticipate receiving consistent types of news over a specific period. For instance, investors expected frequent updates about the pandemic in early 2020 and about tariffs in the second quarter of 2025. The start and end of such periods map into the initial date 0 and end date T in our model.

During a finite period, investors have limited opportunities to update their beliefs based on news realizations. For simplicity, our analysis assumes that investors hold dogmatic beliefs. Nonetheless, the pricing in our model is equivalent to that of a representative agent who, starting with specific prior beliefs, gradually learns the probability governing the true fundamental news process, as demonstrated in subsequent analysis in section 4.1.

Within this period, trading generates substantial wealth redistribution while allowing the majority of investors to survive. Section 4.2 then considers the Poisson continuous-time limit, demonstrating that disagreement over the news arrival rate could persist and have a significant impact on asset prices.

4.1 The wisdom of the crowd

Pricing in the heterogeneous-agent economy is identical to pricing in an economy with a representative agent with log utility whose prior belief at time 0 about probability of future news process has the same distribution as in (3), and the agent updates his or her beliefs over time following Bayes' rule. [5]

While such a model would not be consistent with our model since heterogeneous beliefs are absent, hence no trade is happening, the comparison between those two models reveals a feature of financial markets in our model: market as a whole is "learning" about the truth over time through the redistribution of wealth — hindsight winners get wealthier and their beliefs becomes more important in pricing.

[5]The above result has been established in Martin and Papadimitriou (2022) in a binary tree setting where there is only disagreement on news content and the disagreement is modelled by 1D beta distribution. In our model, if we force all agents agree the news would always arrive with probability one, i.e. all agents agree $v = 1$, we can recover Martin and Papadimitriou (2022).

The wisdom of the crowd requires speculation for all dimensions of beliefs exist and also the presence of truth within the agent population —some agents must perceive the truth of (u, v) (or (g, m, b)). On the contrary, if agents all agree on a news arrival rate that deviates from the truth, or agree on a news content that is too bullish (or bearish), the representative agent (Mr. Market) will never learn about the truth.

Since market becomes a weighing machine in the long run—as Benjemin Graham famously said, a natural question raises when considering long term survival of agents with 'wrong beliefs'. It turns out the Kullback-Leibler divergence—defined as $KL(a, b) = a \log(a/b) + (1 - a) \log(1 - a)/(1 - b)$ for Bernoulli distributions with parameters a and b—between any agent's belief and the truth will control how fast an agent with wrong belief becomes irrelevant in the long run. This fundamental insight is a key theme in the literature on belief selection and wealth dynamics (e.g., Blume and Easley (2006)).

Result 7 (Survival)**.** *Assume there is truth (u_{true}, v_{true}) that guarantees the data generating process of fundamental news, the wealth share of agent with beliefs (u, v) will decay exponentially when $t \to \infty$ at the rate of*

$$KL(v_{true}, v) + v_{true}\, KL(u_{true}, u)\,.$$

Result 7 indicates the true news arrival rate affects survival negatively—the rarer the news is, the longer it takes for agents with wrong beliefs to be extinct. Moreover, when the true news arrival rate is low, having an accurate understanding of the flow of information is more crucial than knowing whether the news will be positive or negative. In later sections, where we discuss continuous time limits of our trinomial tree model, we would show that agents with inaccurate belief of v will survive if news of fundamental follows Poisson process while it is not the case in the Brownian limits—disagreement of v would not persist in a diffusion world.

4.2 Poisson limits

The trinomial tree model allows us to explore various realistic continuous-time limits. The information about fundamental may either comes discretely in lumps or arrives continuously in small increments. The former can be formally modeled as a Poisson process, while the latter is represented by a Brownian motion. Analysis in section I.A.2 demonstrates that in a Brownian limit, widespread disagreement about the frequency of news arrivals cannot be sustained, while minor disagreements have negligible effects on asset prices. In contrast, the Poisson limit allows disagreement over the news arrival frequency

to persist and significantly influence prices.

In this Poisson limit, we partition the interval $[0, T]$ into $N = T/\delta$ periods. Each piece of news is assumed to have a substantial impact on fundamentals, and the terminal payoff remains identical to the discrete-time case: $p_T = e^{\sigma n_T}$. With α_{u0}, β_{u0}, and α_{v0} held constant and independent of N, and β_{v0} defined as

$$\beta_{v0} = \alpha_{v0} \left(\lambda^{-1} \, \frac{N}{T} - 1 \right) ,$$

the cross-sectional averages of (u, v) are given by $\mathbb{E}\, u = \alpha_{u0}/(\alpha_{u0} + \beta_{u0})$ and $\mathbb{E}\, v = \lambda\delta$.

In the limit as $N \to \infty$ and $\delta \to 0$, the counts of "good news" and "bad news" events follow Poisson distributions. Agents with heterogeneous beliefs about the news arrival rate can survive in the market. The impact of such disagreement on expected wealth growth is captured by the Kullback–Leibler divergence:

$$KL(v_{true}, v) = v_{true} \log \left(\frac{v_{true}}{v} \right) + (1 - v_{true}) \log \left(\frac{1 - v_{true}}{1 - v} \right) . \tag{27}$$

As $N \to \infty$, both v_{true} and v approach zero. The second term, representing the expected effect during no news periods, remains bounded. Moreover, since news arrives infrequently, the cumulative impact of news events—captured by the first term, $v_{\text{true}} \log(v_{\text{true}}/v)$—is also limited. Together, these bounds ensure that agents with incorrect beliefs survive.

Taking the limit $N \to \infty$, we obtain market price and risk-neutral variance.

Result 8 (Poisson limit)**.** *In the Poisson limit, the market price at time t is given by*

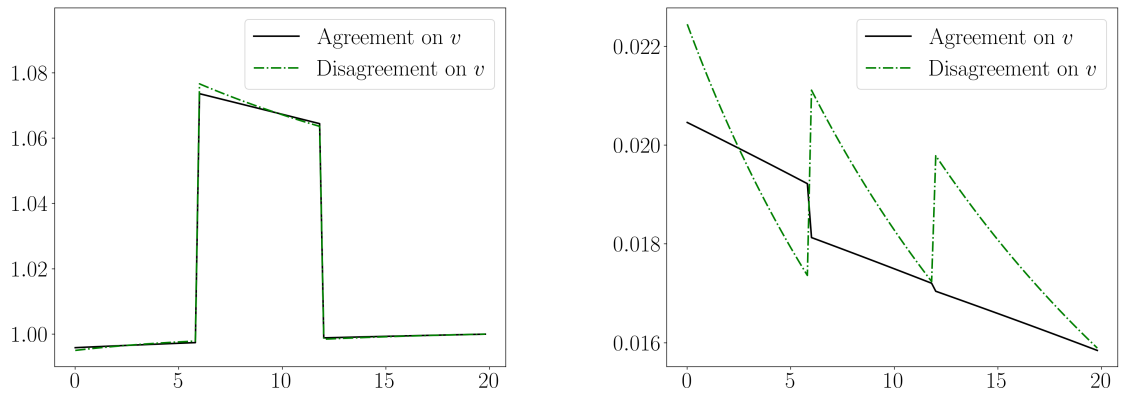
$$p_t = e^{\sigma n_t} \left[1 - \Lambda_t (e^{\sigma} - 1) \right]^{\alpha_{vt}} {}_2F_1 \left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^{\sigma} - e^{-\sigma}}{e^{\sigma} - 1 - \Lambda_t^{-1}} \right)^{-1} , \tag{28}$$

where $\Lambda_t = (T - t)/(\alpha_{v0}\lambda^{-1} + t)$, ${}_2F_1(\ldots)$ denotes the Gauss hypergeometric function, and the expressions for α_{vt}, α_{ut}, and β_{ut} are provided in (11).

The risk-neutral variance of the return from t to T is:

$$\text{Var}^*[p_T/p_t - 1] = p_t^{-1} e^{\sigma n_t} \left[1 + \Lambda_t (1 - e^{-\sigma}) \right]^{-\alpha_{vt}} {}_2F_1 \left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^{\sigma} - e^{-\sigma}}{1 + \Lambda_t^{-1} - e^{-\sigma}} \right) - 1 . \tag{29}$$

Example.— Consider $T = 20$, where we set $\alpha_{u0} = \beta_{u0} = 1$ and $\lambda = 0.1$, and the terminal payoff is specified as $e^{0.05 n_T}$. Figure 6 presents price and volatility trajectories following a "good news" event at $t = 6$ and a "bad news" event at $t = 12$, under both



(a) Market Prices

(b) Volatility Strike

Figure 6: **Price and volatility trajectories**

The figure shows price and volatility trajectories with and without disagreement about the news arrival rate v. Parameters are set as $\alpha_{u0} = \beta_{u0} = 1$ and $\lambda = 0.1$, and the terminal payoff at $T = 20$ is specified as $e^{0.05n_T}$. Solid black lines depict the case without disagreement in v, while dashed green lines represent the case with high disagreement, where $\alpha_{v0} = 2$.

no disagreement and high disagreement about the news arrival rate v. The right panel 6b shows $\sqrt{\text{Var}^*[p_T/p_t - 1]/(T-t)}$, the (risk-neutral) volatility per unit time over the remaining horizon $[t, T]$. It also represents the volatility strike of the simple variance swap, which is introduced by Martin (2011). [6] The solid black path, reflecting no disagreement about the news arrival rate, aligns with result 3, where derivative prices decline following news arrivals. In contrast, the dashed green path, representing high disagreement about the news arrival rate, reflects result 4, with derivative prices rising after news events.

[6] A well-known derivative related to the VIX index is the *variance swap*. In this contract, one party pays the other the difference between the actual variance of an asset's returns and a preset strike variance. Martin (2011) noted that the usual log-normal assumption may not hold during crises times, and introduced a jump-robust version called the *simple variance swap*. In our model, the contract payoff is

$$\left(\frac{p_{t+1} - p_t}{p_t}\right)^2 + \left(\frac{p_{t+2} - p_{t+1}}{p_t}\right)^2 + \cdots + \left(\frac{p_T - p_{T-1}}{p_t}\right)^2.$$

The time t price of this payoff is equal to the risk-neutral variance $\text{Var}^*[p_T/p_t - 1]$.

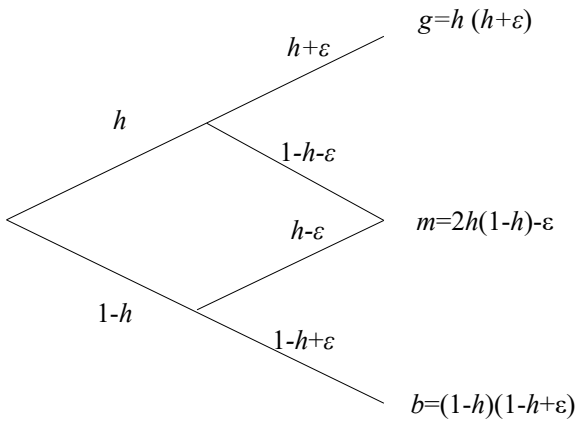


Figure 7: **Recombining binary tree with time varying probabilities**

5 Momentum and contrarian traders

Our trinomial tree model could be specialised as a recombining binary tree model, where agents beliefs about the binary distribution could switch between odd and even periods. Consider an alternative parametrisation of $(g,\, m,\, b)$:

$$g = h(h+\varepsilon), \quad m = 2h(1-h) - \varepsilon, \quad b = (1-h)(1-h+\varepsilon)\,. \tag{30}$$

where $h \in (0,1)$ and ε is small number around zero.[7] As illustrated in Figure 7, h captures agents' belief of good news and ε captures whether agent is a momentum or contrarian trader: those with positive ε are momentum traders while those with negative ε believes in reversal.

In this formulation, agents beliefs could be characterized by the pair (h, ε). A higher value of h corresponds to more bullish belief on average, while a positive value of ε indicates agent is a short term momentum trader while a negative value of ε indicates agent is a short term contrarian.

Interestingly, two agents with the same beliefs of v in the trinomial tree setting could have completely different views about (h, ε) when switching to the recombining binary

[7]The technical condition for (g, m, b) to be probability measure is when ε has to satisfy

$$\max(-h, h-1, -h^2-(1-h)^2) \le \varepsilon \le \min(h, 1-h, 2h(1-h)).$$

The more extreme h is, the narrower the range of ε is around 0.

tree setting. Using (30), we could compute

$$u = \frac{h^2 - (1-h)\varepsilon + \varepsilon}{h^2 + (1-h)^2 + \varepsilon}, \quad v = h^2 + (1-h)^2 + \varepsilon. \tag{31}$$

The more positive ε (momentum trader) is, the higher of the value of v and the same for those with very extreme views on h. Here, v but no longer measures the frequency of news arrival but continue to reflect the agent's beliefs about the likelihood of extreme outcomes. Holding h constant, a higher value of v corresponds to a belief in momentum, while a lower value reflects an expectation of reversal. Intuitively, agents who expect trends to persist assign greater probability to extreme realizations, whereas contrarian agents anticipate outcomes clustered around the mean.

If we allow agents to trade in each of those odd and even periods, the market is dynamically complete with only the underlying asset. Or, equivalently, if we allow agents to trade only at the end of those even periods, the model would be identical to the trinomial tree setting where agents would implement their views through a combination of derivatives and the underlying asset. For instance, a momentum investor may either follow a momentum-based dynamic trading strategy or take a positive gamma position using options, both of which profit from sustained directional movements. Conversely, a contrarian investor could take a negative gamma position that profits from price reversals and market stabilization.

Consistent with the analysis in Section 3, market outcomes endogenously redistribute wealth across heterogeneous investors, amplifying the influence of those whose beliefs prove ex-post correct. In environments where there is pronounced disagreement between momentum and contrarian expectations, the realized path of fundamental shocks can play a critical role in either stabilizing or destabilizing the market. Rapidly oscillating paths stabilize the market by validating reversal-oriented strategies and concentrating wealth among reversal believers. In contrast, persistent trends that revert only after a delay increase market volatility and reward momentum-oriented investors.

6 Conclusion

Our study underscores the critical role of multidimensional belief heterogeneity in shaping financial market dynamics, revealing how disagreements over news content and arrival frequency drive asset pricing, volatility, and portfolio decisions. By introducing a trinomial tree framework that incorporates a gamma-exposed derivative, our model isolates the distinct effects of those two belief dimensions, demonstrating that frequent news can both

mitigate content disagreement by redistributing wealth to accurate believers and amplify volatility when news frequency expectations diverge. The model is tractable and produces an U-shaped pattern of Gamma exposure among optimists and pessimists of market return, which highlights the speculative nature of trading driven by these heterogeneous beliefs.

These findings deepen our understanding of how complex belief structures influence behaviors of underlying and derivative markets, providing new insights on volatility persistence and asset valuation. Future research could explore empirical validations of these dynamics or extend the model to incorporate additional belief dimensions, further enriching the analysis of financial markets under heterogeneous expectations.

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A Mathematical Appendix

Proof for equations (10) and (11). Comparing the individual wealth growth and aggregate wealth growth equations in (7) and (8), we could derive the following equation for change of wealth distribution

$$\frac{f_{t+1}(g,m,b)}{f_t(g,m,b)} = \begin{cases} \frac{g}{G_t}, & \text{'good news' at } t+1 \\ \frac{m}{M_t}, & \text{'no news' at } t+1 \\ \frac{b}{B_t}, & \text{'bad news' at } t+1 \end{cases}$$

Solving this recursively back from t to 0, the wealth distribution $f_t(g,m,b)$ should satisfy

$$f_t(g,m,b) = \lambda_t \, f_0(g,m,b) \, g^{n_{gt}} m^{n_{mt}} b^{n_{bt}}$$

where the constant λ_t is path-dependent. Take integrals of both sides, we have

$$1 = \int \lambda_t \, f_0(g,m,b) \, g^{n_{gt}} m^{n_{mt}} b^{n_{bt}} \, dg \, dm \, db,$$

and it implies

$$\lambda_t = \frac{B(\alpha_{u0},\beta_{u0})B(\alpha_{v0},\beta_{v0})}{B(\alpha_{ut},\beta_{ut})B(\alpha_{vt},\beta_{vt})},$$

where $\alpha_{ut} = \alpha_{u0} + n_{gt}$, $\beta_{ut} = \beta_{u0} + n_{bt}$, $\alpha_{vt} = \alpha_{v0} + n_{gt} + n_{bt}$, and $\beta_{vt} = \beta_{v0} + n_{mt}$. Converting the notation of (n_{gt}, n_{mt}, n_{bt}) into (n_t, ν_t), we obtain result in equations (10) and (11).

□

Proof for result 1. The risk-neutral variance of $p_{t+1} - p_t$ could be expressed as:

$$\begin{aligned} \mathrm{Var}_t^*[p_{t+1} - p_t] &= g_t^* p_{t+1,g}^2 + m_t^* p_{t+1,m}^2 + B_t^* p_{t+1,b}^2 - p_t^2 \\ &= p_t\big(G_t p_{t+1,g} + M_t p_{t+1,M} + B_t p_{t+1,b} - p_t\big) = p_t \, \mathbb{E}_t^{U_t,V_t}[p_{t+1} - p_t]\,. \end{aligned} \tag{A1}$$

Multiplying the first equation in (13) by $p_t/w_t^{(u,v)}$, and the second by $q_t/w_t^{(u,v)}$, and substituting in (A1), we obtain the result in (14). A delta-neutral position entails zero investment in the market, $\theta_t^{(u,v)} = 0$, which in turn implies that the agent's subjective return for the market $\mathbb{E}_t^{(u,v)}[p_{t+1}/p_t - 1]$ is equal to zero.

Similarly, a gamma-neutral position requires zero investment in the derivative, i.e., $\phi_t^{(u,v)} = 0$, which corresponds to a zero subjective expected return on the derivative: $\mathbb{E}_t^{(u,v)}[x_{t+1}/q_t - 1] = 0$. The zero return condition, together with the probability constraint

$g + m + b = 1$, defines a line in the three-dimensional probability space. Since both Mr. Market's belief and the risk-neutral belief satisfy these conditions, the set of gamma-neutral beliefs must lie along the line connecting Mr. Market's belief to the risk-neutral belief. □

Proof for result 2. Due to (9), we could solve from T backwards to t the value $1/p_t$. It is merely the linear combination of wealth-weighted aggregate beliefs of all possible future sample paths between t and T multiplying with the one over terminal payoff $1/p_T(\mathbf{s}_t)$. To formally prove this is statement, let (a, b, c) be the number of moves that correspond to (high, middle, low) of a sample path from t to T. They must satisfy

$$a - c = n_t, \quad a + b + c = T - t\,.$$

The above could be solved in terms of b as

$$a = \frac{1}{2}(T - t + n - b), \quad c = \frac{1}{2}(T - t - n - b).$$

and (a, c) are integers only when b and $T - t - n$ are both odd or even numbers ($T - t + n$ is always the same as $T - t - n$). Since the arguments for two cases when n is positive and negative are symmetric, we focus on $n > 0$. If $T - t - n$ is odd number, b starts from t+1, otherwise it starts from t. The size of b can not exceeds $T - t - n$ for c stays non-negative. Once b is fixed, the value of a and c are chosen. There are $\frac{(T-t)!}{a!b!c!}$ possible paths that satisfies this characteristic, i.e. a times high, b times middle and c times low.

We now prove that those paths could be merged since they make equal contribution to the coefficient c_n. To show this, it suffices to show the three basic cases: 1) two trips of 'middle-low' and 'low-middle' would give the same risk-neutral probability given the same initial condition; 2) two trips of 'middle-high' and 'high-middle' are equivalent; 3) two trips of 'low-high' and 'high-low' are equivalent.

The risk-neutral variance of the return from t to T, $p_T/p_t - 1$, is given by:

$$\frac{1}{p_t^2}\,\mathbb{E}_t^*[p_T^2 - p_t^2] = \mathbb{E}_t[p_T(n_T)]/p_t - 1 \tag{A2}$$

Substituting in (17), we obtain (19).

□

Proof for result 3. The risk-neutral variance of the market return satisfies

$$\mathrm{Var}_t^*\left[p_T/p_t - 1\right] = \mathbb{E}_t^*\left[p_T^2/p_t^2\right] - 1 = \mathbb{E}_t\left[p_T|n_t, f_t\right]/p_t - 1$$

Holding news arrival rate v and the average expectation of the fundamental growth $G_t - B_t$ constant, a higher historical news count $\nu_t \geqslant \nu'_t$ reduces disagreement in bullishness u and resulting in a more concentrated distribution of fundamental growth $n_T - n_t$, such that $n'_T - n'_t$ is a mean-preserving spread of $n_T - n_t$. As a result,

$$\begin{aligned} p_t^{-1} &= \sum_{n_T - n_t} \Pr(n_T - n_t | n_t, f_t)\, e^{-\sigma(n_T - n_t)}\, e^{-\sigma n_t} \\ &\leqslant \sum_{n'_T - n'_t} \Pr(n'_T - n'_t | n'_t, f'_t)\, e^{-\sigma(n'_T - n'_t)}\, e^{-\sigma n_t} = (p'_t)^{-1}\, e^{-\sigma(n_t - n'_t)} \end{aligned} \tag{A3}$$

confirming $p_t / e^{\sigma n_t} \geqslant p'_t / e^{\sigma n'_t}$. Similarly,

$$\begin{aligned} \mathbb{E}_t\left[p_T | n_t, f_t\right] &= \sum_{n_T - n_t} \Pr(n_T - n_t | n_t, f_t)\, e^{\sigma(n_T - n_t)}\, e^{\sigma n_t} \\ &\leqslant \sum_{n'_T - n'_t} \Pr(n'_T - n'_t | n'_t, f'_t)\, e^{\sigma(n'_T - n'_t)}\, e^{\sigma n_t} = \mathbb{E}_t\left[p'_T | n'_t, f'_t\right] e^{\sigma(n_t - n'_t)} \end{aligned} \tag{A4}$$

Multiplying the above two inequalities, we obtain

$$\mathrm{Var}_t^*\left[p_T / p_t - 1\right] \leqslant \mathbb{E}_t\left[p'_T | n'_t, f'_t\right] / p'_t - 1 = \mathrm{Var}_t^*\left[p'_T / p'_t - 1\right] \tag{A5}$$

□

Proof for result 4. Suppose news arrival rate disagreement is stronger than the Dirichlet benchmark $\alpha_{v0} \leqslant \alpha_{u0} + \beta_{u0}$. Holding the average expectation of the fundamental growth $G_t - B_t$ constant, a higher historical news count $\nu_t \geqslant \nu'_t$ suggests that

1. $g + b = v$ first-order stochastically dominates $g' + b' = v'$
2. $g - b = (2u - 1)v$ second-order stochastically dominates $g' - b' = (2u' - 1)v'$

Thus, $n_T - n_t$ is a mean-preserving spread of $n'_T - n'_t$. The rest follows from the proof of Result 3. □

Proof for results 5 and 6. Consider the group of agents whose subjective expectation of the market return equals r:

$$\mathbb{E}_t^{(u,v)}[p_{t+1} / p_t - 1] = r\,. \tag{A6}$$

We can express u as a function of v and r:

$$u = \frac{p_{t+1,m} - p_{t+1,b}}{p_{t+1,g} - p_{t+1,b}} + \frac{(1+r)p_t - p_{t+1,m}}{(p_{t+1,g} - p_{t+1,b})v} \tag{A7}$$

When $r < p_{t+1,m}/p_t - 1$, u stays in $[0,1]$ if and only if $v \in [(p_{t+1,m} - (1+r)p_t)/(p_{t+1,m} - p_{t+1,b}), 1]$. When $r > p_{t+1,m}/p_t - 1$, u stays in $[0,1]$ if and only if $v \in [((1+r)p_t - p_{t+1,m})/(p_{t+1,g} - p_{t+1,m}), 1]$.

Substituting (A7) into (3), the conditional distribution of v given r satisfies

$$f_t(v|r) \propto (v + kz)^{\alpha_{ut}-1} (v - z)^{\beta_{ut}-1} v^{\alpha_{vt}-\alpha_{ut}-\beta_{ut}} (1-v)^{\beta_{vt}-1}, \tag{A8}$$

where z as a function of r represents the scaled return and k is a constant

$$z = \frac{(1+r)p_t - p_{t+1,m}}{p_{t+1,g} - p_{t+1,m}} = \frac{1 + r - M_t/m_t^*}{G_t/g_t^* - M_t/m_t^*}, \tag{A9}$$

$$k = \frac{p_{t+1,g} - p_{t+1,m}}{p_{t+1,m} - p_{t+1,b}} = \frac{G_t/g_t^* - M_t/m_t^*}{M_t/m_t^* - B_t/b_t^*}. \tag{A10}$$

Integrating the joint distribution $f_t(v,z)$ over v yields the marginal distribution of z:

$$f_t(z) = \left(\frac{k}{k+1}\right)^{\beta_{ut}} \frac{B(\beta_{ut}, \beta_{vt})}{B(\alpha_{ut}, \beta_{ut}) B(\alpha_{vt}, \beta_{vt})} z^{\alpha_{vt}-1} (1-z)^{\beta_{ut}+\beta_{vt}-1} F_1\left(\beta_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \alpha_{ut}, \beta_{ut} + \beta_{vt}; 1 - z, \frac{k(1-z)}{1+k}\right)$$

for $z > 0$ and

$$f_t(z) = \frac{k}{(k+1)^{\alpha_{ut}}} \frac{B(\alpha_{ut}, \beta_{vt})}{B(\alpha_{ut}, \beta_{ut}) B(\alpha_{vt}, \beta_{vt})} (-kz)^{\alpha_{vt}-1} (1+kz)^{\alpha_{ut}+\beta_{vt}-1} F_1\left(\alpha_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt}; 1 + kz, \frac{1+kz}{1+k}\right)$$

for $z < 0$. Substituting into $f_t(r) = p_t/(p_{t+1,g} - p_{t+1,m}) f_t(z)$, we obtain the two equations in Result 5.

From the distribution $f_t(v|r)$, the conditional expectation $\mathbb{E}[v|r]$ can be expressed in terms of ratios of Appell hypergeometric functions. This expression allows us to compute the wealth-weighted average derivative position for this group of agents. Specifically, when

$z \geqslant 0$, we obtain:

$$\mathbb{E}[v|r] = z + z(1-z) \cdot \frac{\beta_{ut}}{\alpha_{ut}+\beta_{vt}} \cdot \frac{F_1\left(\beta_{ut}+1; \alpha_{vt}+\beta_{vt}, 1-\alpha_{ut}; \beta_{ut}+\beta_{vt}+1; 1-z, \frac{k(1-z)}{1+k}\right)}{F_1\left(\beta_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\alpha_{ut}; \beta_{ut}+\beta_{vt}; 1-z, \frac{k(1-z)}{1+k}\right)}.$$

When $z < 0$, we obtain:

$$\mathbb{E}[v|r] = -kz + (-kz)(1+kz) \cdot \frac{\alpha_{ut}}{\beta_{ut}+\beta_{vt}} \frac{F_1\left(\alpha_{ut}+1; \alpha_{vt}+\beta_{vt}, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}+1; 1+kz, \frac{1+kz}{1+k}\right)}{F_1\left(\alpha_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}; 1+kz, \frac{1+kz}{1+k}\right)}.$$

For agent with belief (u, v), the expected excess payoff of the derivative is given by:

$$\begin{aligned}\mathbb{E}_t^{(u,v)}[x_{t+1} - q_t] &= p_t(b_t^* - B_t)uv + p_t(G_t - g_t^*)(1-u)v - p_t(b_t^* G_t - g_t^* B_t) \\ &= \frac{\mathrm{Var}_t^*[p_{t+1}]}{p_{t+1,g} - p_{t+1,b}}\left(v - v_t^* + \frac{(v_t^* - V_t)}{\mathbb{E}^{(U_t,V_t)}[r]}r\right) = \frac{\mathrm{Var}_t^*[x_{t+1} - q_t]}{m_t^* q_t}\left(v - v_t^* + \frac{(v_t^* - V_t)}{\mathbb{E}^{(U_t,V_t)}[r]}r\right).\end{aligned} \tag{A11}$$

Substituting this expression into (14), we obtain (26). □

Proof for result 7. For the discussion in this section, we assume there is true probability measure (u_{true}, v_{true}) that guarantees the data generating process of fundamental news. We also assume there is no learning, so agents would stick to their dogmatic beliefs.

We define the wealth share of agent (u, v) at time t with sample history (n_t, ν_t) as

$$\Omega(u, v; n_t, \nu_t) = \frac{u^{\frac{1}{2}n_t + \frac{1}{2}\nu_t}(1-u)^{\frac{1}{2}\nu_t - \frac{1}{2}n_t} v^{\nu_t}(1-v)^{t-\nu_t} f_0(u,v)}{\int u^{\frac{1}{2}n_t + \frac{1}{2}\nu_t}(1-u)^{\frac{1}{2}\nu_t - \frac{1}{2}n_t} v^{\nu_t}(1-v)^{t-\nu_t} f_0(u,v) du dv}$$

We want to compute the asymptotic decay rate (exponentially) of the above quantity, which is the following limit

$$\lim_{t\to\infty} -\frac{\log \Omega(u, v; n_t, \nu_t)}{t} = \lim_{t\to\infty} -\frac{1}{t} \log \frac{u^{\frac{1}{2}n_t + \frac{1}{2}\nu_t}(1-u)^{\frac{1}{2}\nu_t - \frac{1}{2}n_t} v^{\nu_t}(1-v)^{t-\nu_t} f_0(u,v)}{\int u^{\frac{1}{2}n_t + \frac{1}{2}\nu_t}(1-u)^{\frac{1}{2}\nu_t - \frac{1}{2}n_t} v^{\nu_t}(1-v)^{t-\nu_t} f_0(u,v) du dv}$$

Note that in large time t limit, we could write

$$\nu_t = t v_{true}, \quad n_t = n_{gt} - n_{bt} = t(u_{true} v_{true} - (1-u_{true}) v_{true}) = t(2u_{true} v_{true} - v_{true})$$

and this means we can write

$$
\begin{aligned}
&-\frac{1}{t}\log\left(u^{\frac{1}{2}n_t+\frac{1}{2}\nu_t}(1-u)^{\frac{1}{2}\nu_t-\frac{1}{2}n_t}v^{\nu_t}(1-v)^{t-\nu_t}\right)\\
&=\underbrace{-v_{true}\log v-(1-v_{true})\log(1-v)-v_{true}\left(u_{true}\log u+(1-u_{true})\log(1-u)\right)}_{-h(u_{true},v_{true};u,v)}
\end{aligned}
$$

and also compute

$$
\begin{aligned}
&\lim_{t\to\infty}\frac{1}{t}\log\int u^{\frac{1}{2}n_t+\frac{1}{2}\nu_t}(1-u)^{\frac{1}{2}\nu_t-\frac{1}{2}n_t}v^{\nu_t}(1-v)^{t-\nu_t}f_0(u,v)dudv\\
&=\lim_{t\to\infty}\frac{1}{t}\log\int\exp(t\,h(u_{true},v_{true};u,v)+\log f_0(u,v))dudv\\
&=sup_{u,v}\{h(u_{true},v_{true};u,v)\}
\end{aligned}
$$

Note that the maximum of $h(u_{true},v_{true};u,v)$ is achieved at (u_{true},v_{true}) because the following quantity is always positive when $(u,v)\neq(u_{true},v_{true})$ (this is the so called *Gibbs's inequality* in information theory, which guarantees the relative entropy is always positive between two probability measures)

$$
\begin{aligned}
&h(u_{true},v_{true};u_{true},v_{true})-h(u_{true},v_{true};u,v)\\
&=v_{true}\log\frac{v_{true}}{v}+(1-v_{true})\log\frac{1-v_{true}}{1-v}-v_{true}\left(u_{true}\log\frac{u_{true}}{u}+(1-u_{true})\log\frac{1-u_{true}}{1-u}\right)\\
&>0
\end{aligned}
$$

Thus we could conclude

$$
\begin{aligned}
&\lim_{t\to\infty}-\frac{\log\Omega(u,v;n_t,\nu_t)}{t}\\
&=v_{true}\log\frac{v_{true}}{v}+(1-v_{true})\log\frac{1-v_{true}}{1-v}+v_{true}\left(u_{true}\log\frac{u_{true}}{u}+(1-u_{true})\log\frac{1-u_{true}}{1-u}\right)\\
&=\mathrm{KL}(v_{true},v)+v_{true}\mathrm{KL}(u_{true},u)
\end{aligned}
$$

□

Proof for result 8. Let $\xi=v/\delta$ denote the continuous-time Poisson arrival rate of news. As $\delta\to 0$, the distribution of ξ at time t converges to a Gamma distribution $\xi\sim\Gamma(\alpha_{vt}\,,\alpha_{v0}\lambda^{-1}+t)$.

For an agent with belief $(u,\xi\delta)$, the arrival rate of "good news" is ξu and bad news $\xi(1-u)$. The counts of "good news" and "bad news" for the time interval $[t,T]$ respectively

follow Poisson distributions $n_{gT} - n_{gt} \sim \text{Poisson}(\,\xi u(T-t)\,)$, $n_{bT} - n_{bt} \sim \text{Poisson}(\,\xi(1-u)(T-t)\,)$.

Using the moment generating functions for $n_{gT} - n_{gt}$, $n_{bT} - n_{bt}$ and ξ, we find that

$$
\begin{aligned}
p_t^{-1} &= e^{-\sigma n_t}\,\mathbb{E}[e^{-\sigma(n_T - n_t)}|f_t] = e^{-\sigma n_t}\int e^{-\sigma(n_{gT}-n_{gt})-\sigma(n_{bT}-n_{bt})} f_t(u,\xi)d\xi du \\
&= e^{-\sigma n_t}\int \exp\left(\xi u(T-t)(e^{-\sigma}-1) + \xi(1-u)(T-t)(e^{\sigma}-1)\right) f_t(u,\xi)d\xi du \\
&= e^{-\sigma n_t}\int \left(1 - \frac{u(T-t)(e^{-\sigma}-1)+(1-u)(T-t)(e^{\sigma}-1)}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut},\beta_{ut})}du \\
&= e^{-\sigma n_t}\left(1 - \frac{(T-t)(e^{\sigma}-1)}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt},\alpha_{ut};\alpha_{ut}+\beta_{ut};-\frac{e^{\sigma}-e^{-\sigma}}{(\alpha_{v0}\lambda^{-1}+t)/(T-t)-(e^{\sigma}-1)}\right).
\end{aligned}
$$

The final step uses Euler's integral representation for the hypergeometric function ${}_2F_1$. Similarly,

$$
\begin{aligned}
\mathbb{E}[p_T] &= e^{\sigma n_t}\,\mathbb{E}[e^{\sigma(n_T - n_t)}|f_t] \\
&= e^{\sigma n_t}\int \left(1 - \frac{u(T-t)(e^{\sigma}-1)+(1-u)(T-t)(e^{-\sigma}-1)}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut},\beta_{ut})}du \\
&= e^{\sigma n_t}\left(1 + \frac{(T-t)(1-e^{-\sigma})}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt},\alpha_{ut};\alpha_{ut}+\beta_{ut};\frac{e^{\sigma}-e^{-\sigma}}{(\alpha_{v0}\lambda^{-1}+t)/(T-t)+(1-e^{-\sigma})}\right).
\end{aligned}
$$

Substituting the above expression into (19), we obtain (29). □

Internet Appendix to

When No News is Good News: Multidimensional Heterogeneous Beliefs in Financial Markets

(not for publication)

Abstract

This Internet Appendix presents supplementary material and results not included in the main body of the paper.

I.A.1 Individual's optimal portfolio choices

I.A.1.1 Agent's optimal position

Recall that we stated for an agent with beliefs (u, v), the optimal positions are given by (same as (13) in the paper)

$$\theta_t^{(u,v)} = w_t^{(u,v)} \frac{\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t]}{\mathrm{Var}_t^*[p_{t+1} - p_t]}, \quad \phi_t^{(u,v)} = w_t^{(u,v)} \frac{\mathbb{E}_t^{(u,v)}[x_{t+1} - q_t]}{\mathrm{Var}_t^*[x_{t+1} - q_t]}.$$

Derivation for Equation (13). The subjective expectation of the price change $p_{t+1} - p_t$, for an agent characterized by belief parameters (u, v) and represented by probabilities (g, m, b), is given by:

$$\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t] = g(p_{t+1,g} - p_t) + m(p_{t+1,m} - p_t) + b(p_{t+1,b} - p_t). \tag{I.A.1}$$

Using equation (7),

$$g = g_t^* \cdot \frac{w_{t+1,g}^{(u,v)}}{w_t^{(u,v)}}, \quad m = m_t^* \cdot \frac{w_{t+1,m}^{(u,v)}}{w_t^{(u,v)}}, \quad b = b_t^* \cdot \frac{w_{t+1,b}^{(u,v)}}{w_t^{(u,v)}}. \tag{I.A.2}$$

Substituting into (I.A.1) and simplifying,

$$\begin{aligned}
&\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t] \\
=&\frac{1}{w_t^{(u,v)}} \left[g_t^* w_{t+1,g}^{(u,v)} (p_{t+1,g} - p_t) + m_t^* w_{t+1,m}^{(u,v)} (p_{t+1,m} - p_t) + b_t^* w_{t+1,b}^{(u,v)} (p_{t+1,b} - p_t) \right] \\
=&\frac{1}{w_t^{(u,v)}} \left[g_t^* \theta_t^{(u,v)} (p_{t+1,g} - p_t)^2 + m_t^* \theta_t^{(u,v)} (p_{t+1,m} - p_t)^2 + b_t^* \theta_t^{(u,v)} (p_{t+1,b} - p_t)^2 \right] \\
=&\frac{1}{w_t^{(u,v)}} \theta_t^{(u,v)} \mathrm{Var}_t^*[p_{t+1} - p_t],
\end{aligned} \tag{I.A.3}$$

where the second equality follows from the fact that the risk-neutral covariance between the derivative payoff and the market price is zero.

This concludes the proof for the market demand $\theta_t^{(u,v)}$. The proof for the derivative demand $\phi_t^{(u,v)}$ follows analogously. □

I.A.1.2 Gamma exposure for individual agent

$$\Gamma^{(u,v)} = \frac{2g_t^* b_t^*}{p_t\left(G_t b_t^* - g_t^* B_t\right)}\left[\frac{g-m}{G_t - M_t} - \frac{m-b}{M_t - B_t}\right] f_t(u,v) \tag{I.A.4}$$

Recall the variables (g, m, b) in (I.A.4) can be expressed in terms of (u, v) using (1).

Derivation for (I.A.4). The expression is derived by constructing a Lagrange interpolating polynomial $w(p)$ that expresses the agent's wealth as a function of price, based on the wealth–price pairs from three scenarios: $(p_{t+1,g}, w_{t+1,g}^{(u,v)})$, $(p_{t+1,m}, w_{t+1,m}^{(u,v)})$, and $(p_{t+1,b}, w_{t+1,b}^{(u,v)})$.

$$\begin{aligned} w(p) =& w_{t+1,g}^{(u,v)} \cdot \frac{(p-p_{t+1,m})(p-p_{t+1,b})}{(p_{t+1,g}-p_{t+1,m})(p_{t+1,g}-p_{t+1,b})} + w_{t+1,m}^{(u,v)} \cdot \frac{(p-p_{t+1,g})(p-p_{t+1,b})}{(p_{t+1,m}-p_{t+1,g})(p_{t+1,m}-p_{t+1,b})} \\ &+ w_{t+1,b}^{(u,v)} \cdot \frac{(p-p_{t+1,g})(p-p_{t+1,m})}{(p_{t+1,b}-p_{t+1,g})(p_{t+1,b}-p_{t+1,m})}. \end{aligned} \tag{I.A.5}$$

The gamma exposure corresponds to the second derivative of wealth $w(p)$ with respect to the price. Since $w(p)$ is quadratic, $\Gamma^{(u,v)}$ is equal to twice the quadratic coefficient. We compute the following

$$\begin{aligned} \Gamma^{(u,v)} &= \frac{2}{p_{t+1,g}-p_{t+1,b}}\left[\frac{w_{t+1,g}^{(u,v)} - w_{t+1,m}^{(u,v)}}{p_{t+1,g}-p_{t+1,m}} - \frac{w_{t+1,m}^{(u,v)} - w_{t+1,b}^{(u,v)}}{p_{t+1,m}-p_{t+1,b}}\right] \\ &= \frac{2}{p_{t+1,g}-p_{t+1,b}}\left[\frac{\frac{w_{t+1,g}^{(u,v)}}{w_t^{(u,v)}}\frac{w_t^{(u,v)}}{p_t} - \frac{w_{t+1,m}^{(u,v)}}{w_t^{(u,v)}}\frac{w_t^{(u,v)}}{p_t}}{\frac{p_{t+1,g}}{p_t} - \frac{p_{t+1,m}}{p_t}} - \frac{\frac{w_{t+1,m}^{(u,v)}}{w_t^{(u,v)}}\frac{w_t^{(u,v)}}{p_t} - \frac{w_{t+1,b}^{(u,v)}}{w_t^{(u,v)}}\frac{w_t^{(u,v)}}{p_t}}{\frac{p_{t+1,m}}{p_t} - \frac{p_{t+1,b}}{p_t}}\right] \\ &= \frac{2f_t(u,v)}{p_{t+1,g}-p_{t+1,b}}\left[\frac{g-m}{G_t-M_t} - \frac{m-b}{M_t-B_t}\right] \end{aligned}$$

□

I.A.2 Brownian limits

To consider a Brownian limit, the interval $[0, T]$ is partitioned into $N = T/\delta$ periods, each with length δ. The terminal payoff is defined as $p_T(n_T) = e^{\sigma\sqrt{\frac{T}{N}}n_T}$, reflecting that each piece of news has a smaller effect on the fundamental, which is scaled as $\sqrt{T/N}n_T$. This is the same setting as in Cox, Ross and Rubinstein (1979). As the number of steps increases, the wealth distribution parameters $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0})$ may also be adjusted to ensure meaningful limiting behavior.

However, substantial disagreement over news arrival rate cannot persist, while minor disagreements have negligible effects on asset prices. By result (7), agents with beliefs $v \neq v_{\text{true}}$ lose wealth each period at a strictly positive rate. As the number of trading periods grows, their repeated mistakes drive their wealth share to zero for any $t > 0$.

Conversely, suppose an agent's belief is represented by a sequence $v(N) \to v_{true}$. The difference in expected fundamental between agents (u, v_{true}) and $(u, v(N))$ is given by:

$$\mathbb{E}^{u,v_{true}}\left[\sqrt{T/N}\, n_T\right] - \mathbb{E}^{u,v(N)}\left[\sqrt{T/N} n_T\right] = \sqrt{TN}(2u-1)\big[v_{true} - v(N)\big]\,. \tag{I.A.6}$$

Keeping the terminal payoff bounded requires that $\sqrt{TN}(2u-1)$ remains bounded. This ensures that the expectation gap vanishes as N grows, meaning the disagreement is too small to generate difference in perceived fundamental growth and hence no effect on prices.

The previous analysis does not rule out the possibility of preserving disagreement of news content in the Brownian limit. As an illustration, we set the parameters as following

$$\alpha_{u0} = \frac{1}{2}\lambda\theta N + \frac{1}{2}\eta\sqrt{N}\,, \quad \beta_{u0} = \frac{1}{2}\lambda\theta N - \frac{1}{2}\eta\sqrt{N}\,, \quad \alpha_{v0} = \lambda\theta N\,, \quad \beta_{v0} = \alpha_{v0}(\lambda^{-1} - 1)\,,$$

where $\alpha_{u0} + \beta_{u0} = \alpha_{v0}$ always hold and the 2D-Beta distribution in (3) would reduce to Dirichlet distribution $B(\alpha_{u0}, \beta_{u0})B(\alpha_{v0}, \beta_{v0}) = B(\alpha_{u0}, \beta_{v0}, \beta_{u0})$. The cross-sectional average of (u, v) would be

$$\mathbb{E}\, u = \frac{1}{2} + \frac{\eta\lambda}{2\theta}N^{-\frac{1}{2}}, \quad \mathbb{E}\, v = \lambda,$$

where θ captures the disagreement of news content and the model reduce to homogeneous belief case when $\theta \to \infty$. Note that the disagreement of news arrival rate does not affect $\widetilde{\mathbb{E}}v$ and those with inaccurate beliefs of v would become irrelevant in the diffusion limit—parameter θ only effectively capture the disagreement of news content.

Applying results 2 and equation (19) in the large-N limit, we obtain closed-form expressions for the prices of the market and risk-neutral variances.

Result 9 (Brownian limit)**.** *In the Brownian limit, the market price at time t is*

$$p_t = \exp\left[\frac{\eta}{\theta}\frac{1-\phi}{1+\phi}\sigma\sqrt{T} + \left(1+\frac{1}{\theta}\frac{1-\phi}{1+\phi}\right)\eta_t\sigma\sqrt{\phi T} - \frac{1+\theta^{-1}+\phi}{2}\frac{1-\phi}{1+\phi}\lambda\sigma^2 T\right]. \quad \text{(I.A.7)}$$

where $\phi = t/T$ *and* $\eta_t = n_t/\sqrt{\phi N}$*.*

The risk-neutral variance of the return from t to T is given by:

$$\mathrm{Var}^*[p_T/p_t - 1] = \exp\left((1+\theta^{-1}+\phi)\frac{1-\phi}{1+\phi}\lambda\sigma^2 T\right) - 1\,. \quad \text{(I.A.8)}$$

The above Brownian limit corresponds to that of a binary tree model with only heterogeneous beliefs about news content: everyone agrees that news always come with a fixed probability λ and everyone knows it's the truth. [8] The risk-neutral variance between t and T is deterministic and unaffected by news realizations.

Proof for result 9. By construction, from Mr. Market's point of view at time 0, the variables (n_{gT}, n_{mT}, n_{bT}) follows a Dirichlet distribution $B(\alpha,\beta,\gamma) = B(\frac{1}{2}\theta\lambda N + \frac{1}{2}\eta\sqrt{N}, \theta N(1-\lambda), \frac{1}{2}\theta\lambda N - \frac{1}{2}\eta\sqrt{N})$. It is then straight forward to compute the reciprocal of initial price using result 2. To get a close form formula, we use Paul and Plackett (1978). In the limit when $\mathbb{N}\to\infty$, the variable $n_T = n_{gT} - n_{bT}$ follows normal distribution with the mean $\mathbb{E}\, n_T = N\left(\frac{\alpha-\gamma}{\alpha+\beta+\gamma}\right) = \frac{\eta}{\theta}\sqrt{N}$ and the variance $\mathrm{var}[n_T] = N\left(\lambda - \frac{\eta^2}{\theta^2}N^{-1}\right)\frac{1+\theta}{\theta} \to \lambda\frac{1+\theta}{\theta}N$. Thus the following random variable $n_T/\sqrt{N}$ follows a normal distribution in large N limit $\frac{n_T}{\sqrt{N}} \sim \mathcal{N}\left(\frac{\eta}{\theta}, \frac{\lambda(1+\theta)}{\theta}\right)$. Consequently, the reciprocal of price is given by $p_0^{-1} = \mathbb{E}\exp\left(-\sigma\sqrt{T}\frac{n_T}{\sqrt{N}}\right)$ and it implies $p_0 = \exp\left(\frac{\eta}{\theta}\sigma\sqrt{T} - \frac{\lambda(1+\theta)}{2\theta}\sigma^2 T\right)$.

For market price at time t, we need to adjust the wealth distribution and also the number of periods. The number of periods left is $N(1-t/T) = N(1-\phi)$, where $\phi = t/T$. The random variables $(n_{gT}-n_{gt}, n_{mT}-n_{mt}, n_{bT}-n_{bt})$ follows a Dirichlet distribution at time t $B_t(\alpha',\beta',\gamma') = B_t(\frac{1}{2}\theta\lambda(1+\phi)N + \frac{1}{2}\eta\sqrt{N} + \frac{1}{2}\eta_t\sqrt{\phi N}, \theta(1+\phi)N(1-\lambda), \frac{1}{2}\theta\lambda(1+\phi)N - \frac{1}{2}\eta\sqrt{N} - \frac{1}{2}\eta_t\sqrt{\phi N})$. The parameter η_t captures the wealth transfer between 0 and t among agents because of heterogeneous beliefs about news content $\eta_t = \frac{n_t}{\sqrt{\phi N}}$. The price at time t can then be written as $p_t^{-1} = \mathbb{E}_t\, p_T^{-1} = e^{-\eta_t\sigma\sqrt{\phi T}}\,\mathbb{E}_t\left(\exp\sigma\sqrt{T/N}(n_t - n_T)\right)$.

The large N limits for mean and variance of $n_T - n_t$ (which would be Gaussian distributed), according to Paul and Plackett (1978), shall be $\mathbb{E}_t[n_T - n_t] = (1-\phi)N\frac{\alpha'-\gamma'}{\alpha'+\beta'+\gamma'} =$

[8] The case when $\lambda = 1$ has been studied extensively in Martin and Papadimitriou (2022). At time $t = 0$, equation (I.A.7) has a simple expression $p_0 = \exp\left[\frac{\eta}{\theta}\sigma\sqrt{T} - \frac{1+\theta}{2\theta}\lambda\sigma^2 T\right]$.

$\frac{\eta+\eta_t\sqrt{\phi}}{\theta}\frac{1-\phi}{1+\phi}\sqrt{N}$ and $\mathrm{var}_t[n_T - n_t] = (1-\phi)N\left(\lambda - \left(\frac{\eta+\eta_t\sqrt{\phi}}{\theta}\frac{1-\phi}{1+\phi}\right)^2 N^{-1}\right)\frac{1+\theta(1+\phi)}{\theta(1+\phi)} \to \lambda\frac{1+\theta(1+\phi)}{\theta(1+\phi)}(1-\phi)N$. We can then compute p_t as

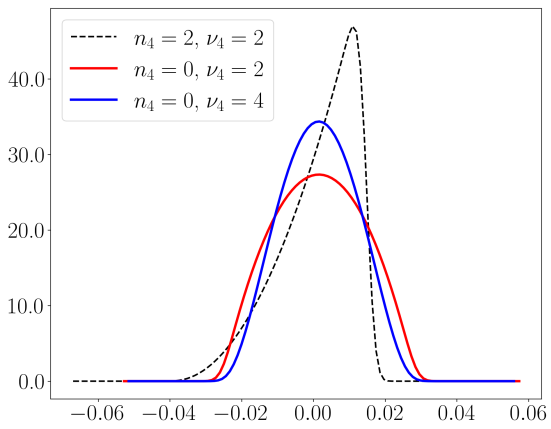
$$p_t = \exp\left(\frac{\eta}{\theta}\frac{1-\phi}{1+\phi}\sigma\sqrt{T} + \left(1+\frac{1}{\theta}\frac{1-\phi}{1+\phi}\right)\eta_t\sigma\sqrt{\phi T} - \frac{\lambda(1+\theta+\theta\phi)}{2(\theta+\theta\phi)}(1-\phi)\sigma^2 T\right)$$

The risk-neutral variance. Similar argument gives us

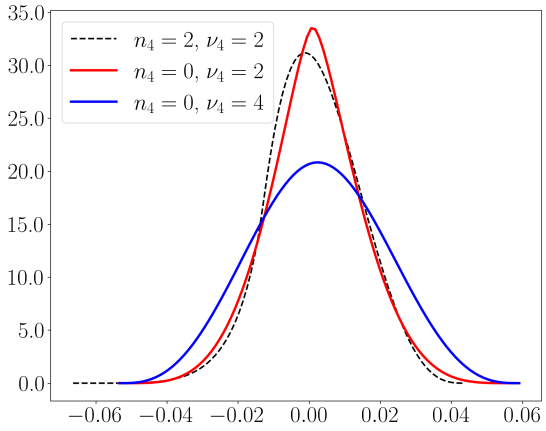
$$\mathbb{E}\, p_T(n_T) = \lim_{N\to\infty}\mathbb{E}\exp\left(\sigma\sqrt{T}\frac{n_T}{\sqrt{N}}\right) = \exp\left(\frac{\eta}{\theta}\sigma\sqrt{T} + \frac{\lambda(1+\theta)}{2\theta}\sigma^2 T\right).$$

Thus, we have $\mathcal{V}_{0\to T} = \exp\left(\frac{\lambda(1+\theta)}{\theta}\sigma^2 T\right)$. □

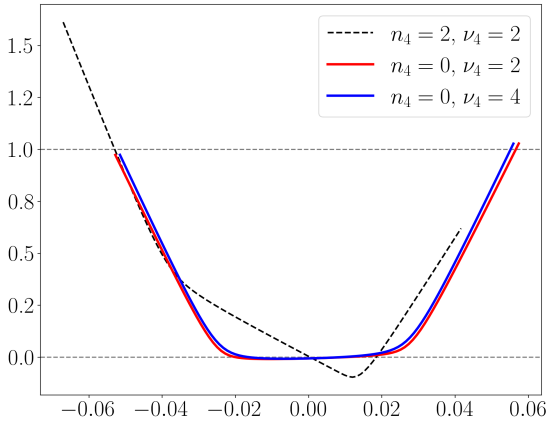
I.A.3 Wealth Distribution and Portfolio Weights



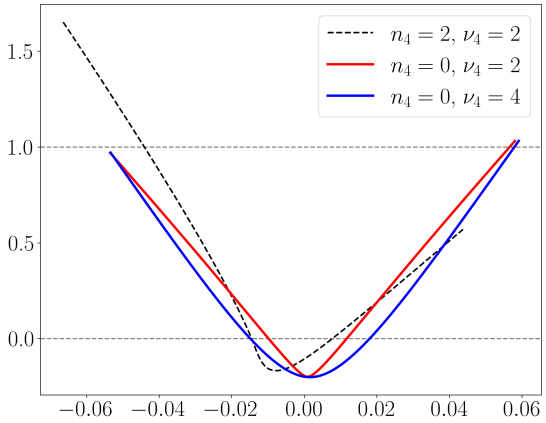
(a) Wealth distribution across r
Low news arrival rate disagreement



(b) Wealth distribution across r
High news arrival rate disagreement



(c) Portfolio weights in 'straddle' across r
Low news arrival rate disagreement

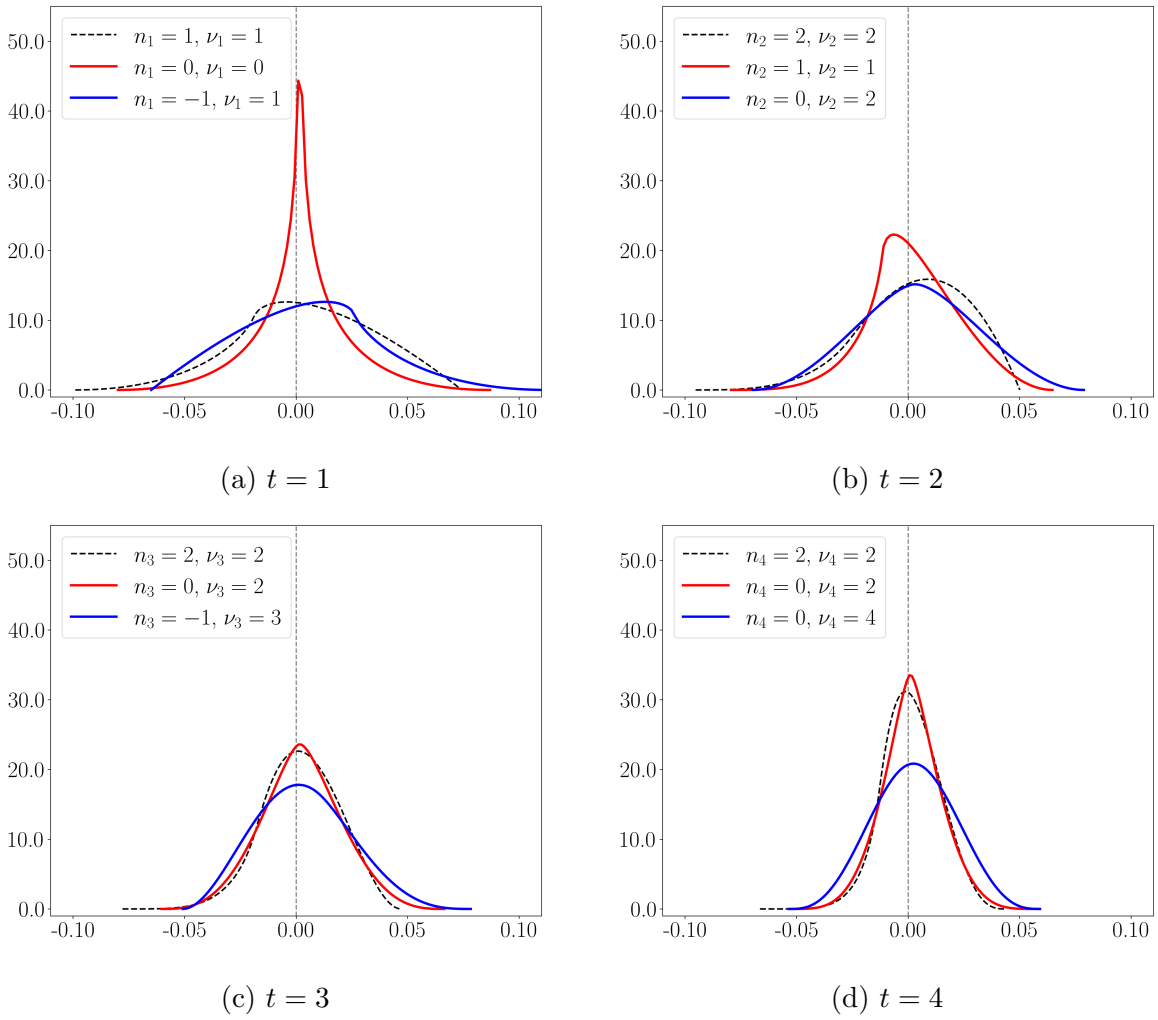


(d) Portfolio weights in 'straddle' across r
High news arrival rate disagreement

Figure I.A.1: **Low disagreement in news arrival rate**

The figure reports wealth distribution and portfolio weights for the cohort of investors with expected return r. In the left panels (I.A.1a and I.A.1c), the initial wealth distribution reflects low disagreement in the news arrival rate dimension, characterized by $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. In contrast, the right panels (I.A.1b and I.A.1d) assume a uniform initial wealth distribution across agents, with $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$. See Figure 3 for an illustration for the possible sample paths that leads to the corresponding wealth distribution and gamma position at $t = 4$.

I.A.4 Figure 5 and I.A.1 at different periods

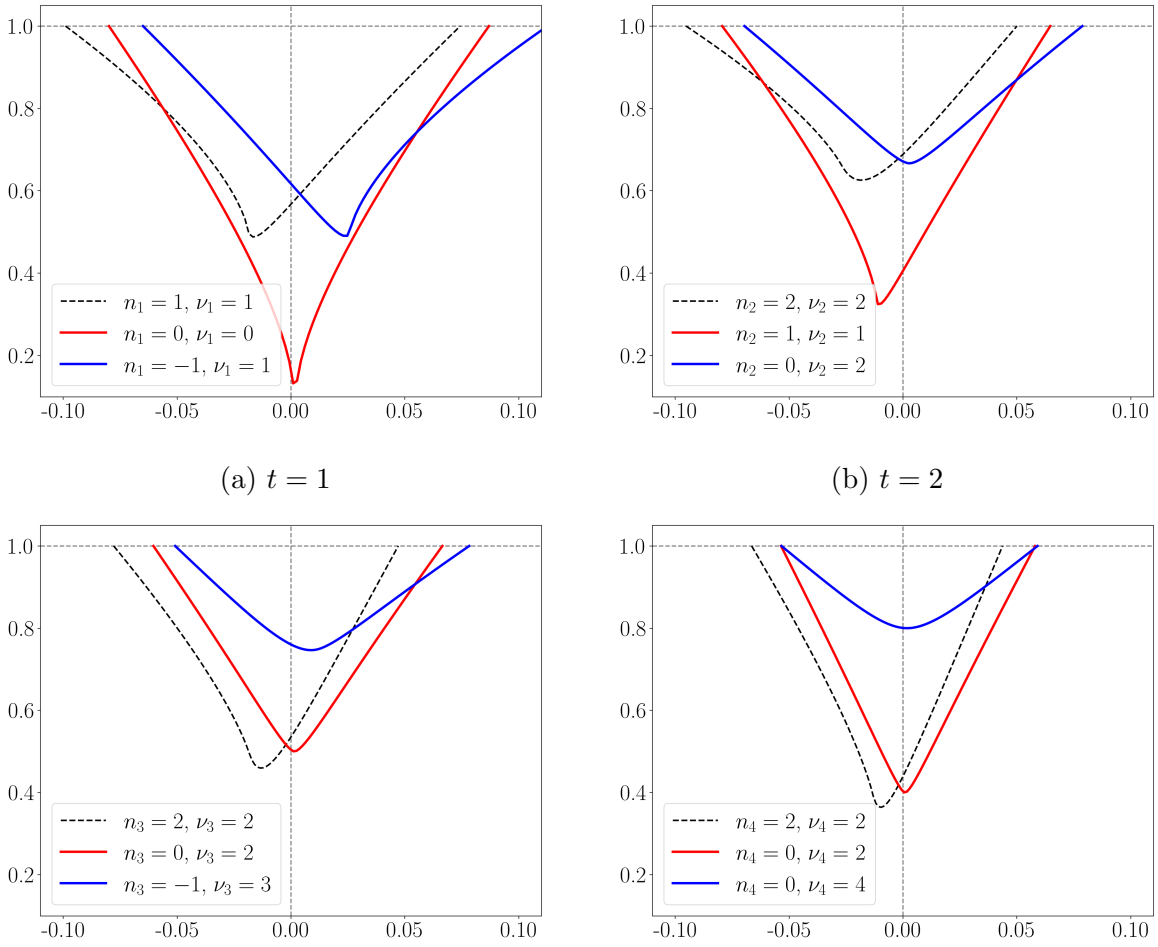


(a) $t = 1$ (b) $t = 2$

(c) $t = 3$ (d) $t = 4$

Figure I.A.2: **Wealth distribution across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 3. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.

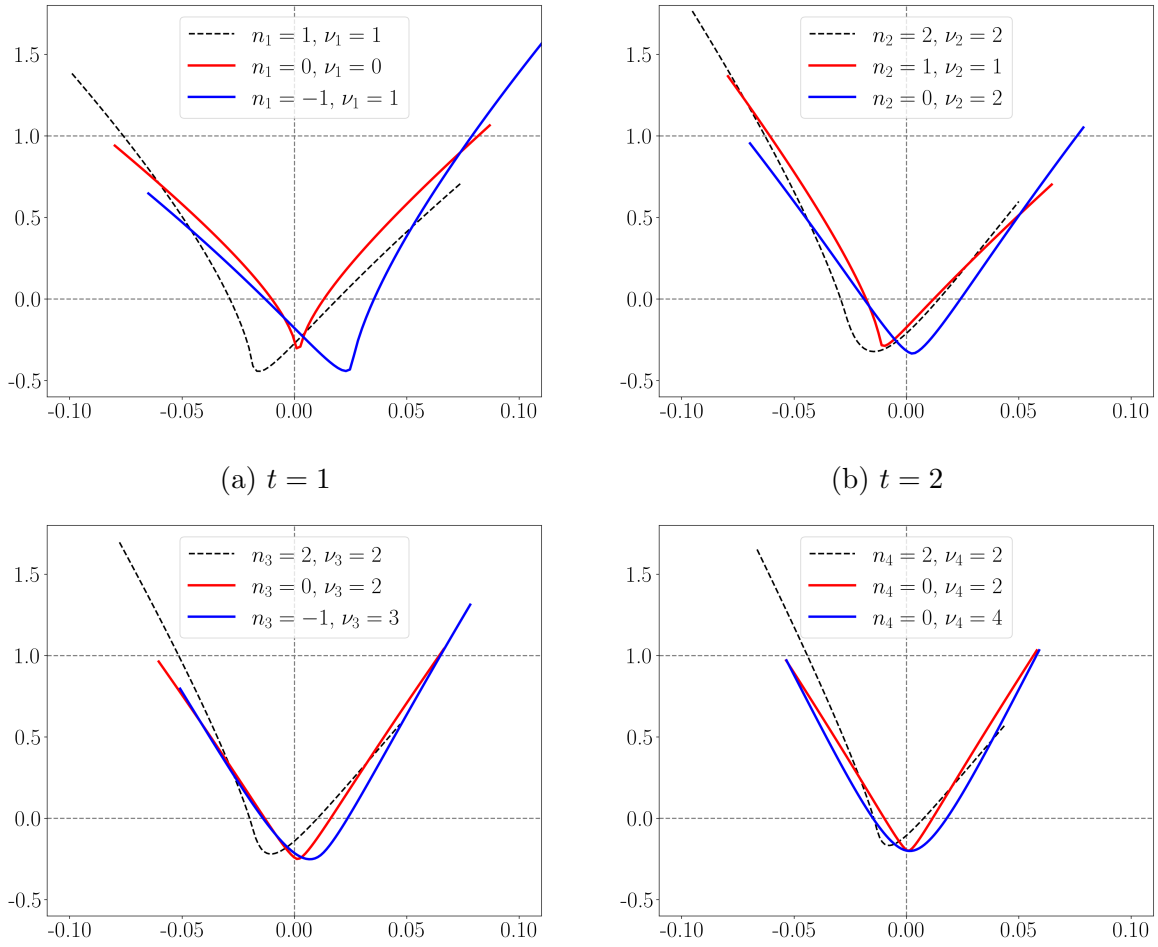


(a) $t = 1$ (b) $t = 2$

(c) $t = 3$ (d) $t = 4$

Figure I.A.3: **Wealth weighted beliefs of v across r**

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 3. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



(a) $t = 1$

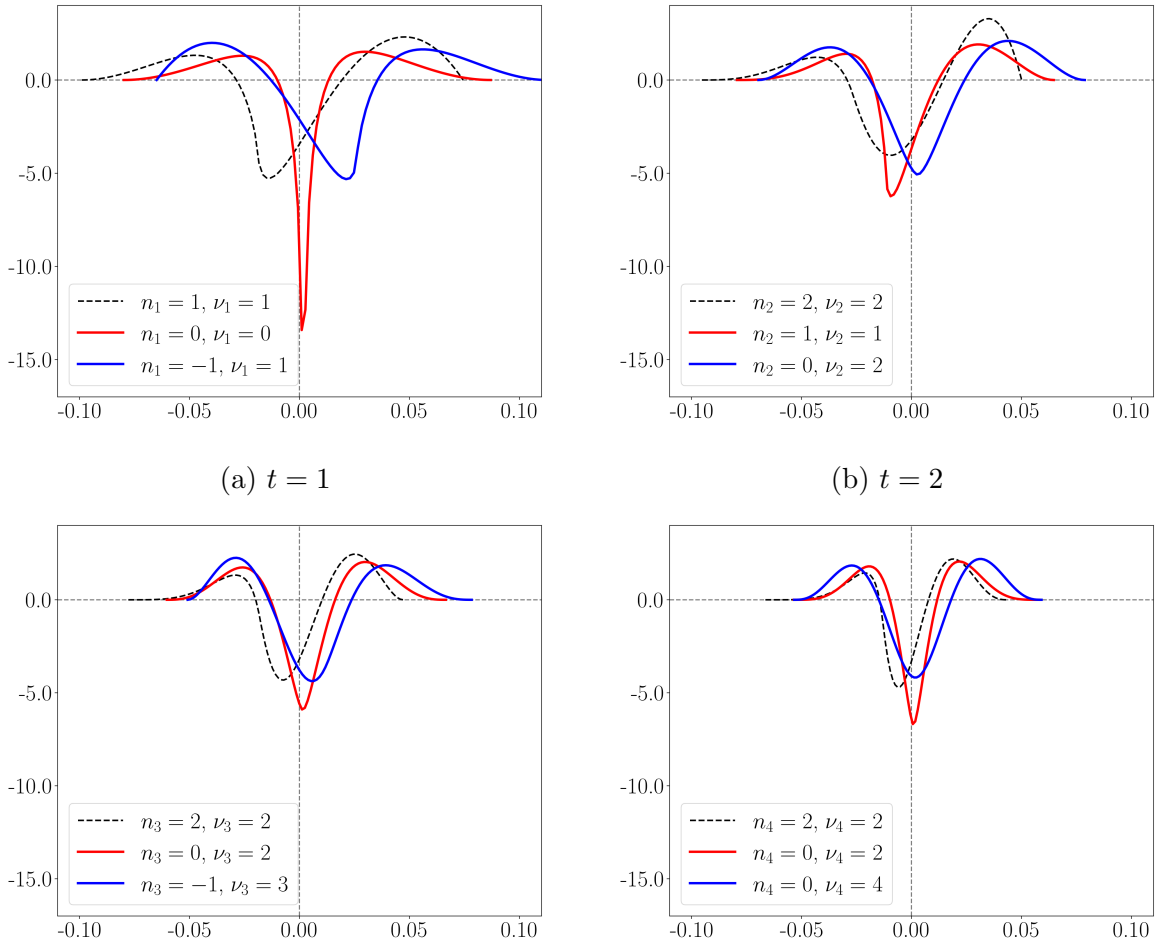
(b) $t = 2$

(c) $t = 3$

(d) $t = 4$

Figure I.A.4: **Portfolio weights in 'straddle' across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 3. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



(a) $t = 1$

(b) $t = 2$

(c) $t = 3$

(d) $t = 4$

Figure I.A.5: **Gamma exposure across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 3. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.

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