Galbraith, Craig S.

Article

Size premium in small business valuation: Analysis of closely-held firms

The Journal of Entrepreneurial Finance (JEF)

Provided in Cooperation with:
The Academy of Entrepreneurial Finance (AEF), Los Angeles, CA, USA

Suggested Citation: Galbraith, Craig S. (2025) : Size premium in small business valuation: Analysis of closely-held firms, The Journal of Entrepreneurial Finance (JEF), ISSN 2373-1761, Pepperdine University, Graziadio School of Business and Management and The Academy of Entrepreneurial Finance (AEF), Malibu, CA and Los Angeles, CA, Vol. 27, Iss. 1, pp. 31-46, https://doi.org/10.57229/2373-1761.1498

This Version is available at:
https://hdl.handle.net/10419/319786

PEPPERDINE UNIVERSITY

The Journal of Entrepreneurial Finance

Volume 27
Issue 1 *2025*

Article 2

2025

SIZE PREMIUM IN SMALL BUSINESS VALUATION: ANALYSIS OF CLOSELY-HELD FIRMS

Craig S. Galbraith
University of North Carolina - Wilmington

Size Premiums in Small Business Valuation: Analysis of Closely-Held Firms

Craig S. Galbraith
Duke Progress Entergy/Betty Cameron Distinguished Professor
University of North Carolina Wilmington, Wilmington, NC USA
galbraithc@uncw.edu

ABSTRACT

Abstract: One of the most misunderstood components of valuing a small closely-held business is how to address the impact of small size. Most closely-held enterprises are relatively small in size, with market values less than $1million. Many small mom and pop operations, or single owner-operator family businesses, often have market values even smaller. The most common method to account for size is to use the size premium reports from *Kroll* or another financial data service provider. However, these small firm premiums are determined exclusively from publicly-traded firms, where even the category of the smallest publicly-traded firms are still magnitudes larger than the typical small closely-held firm. This study examines the *Kroll* size premium data on publicly-traded firms and compares it with an analysis of size data from a proprietary database of closely-held firm transactions. We develop various models that better assess the impact of size on the cost of equity calculations for small, closely-held firms.

Key Words: Size premium; Valuation; Closely-held firms; Discount rates, Capitalization rates

1. Introduction

The valuation of small closely-held companies, most of which are single owner-operator or entrepreneurial entities, always presents a difficult estimation process. There are a variety of issues the business valuator of a small business needs to consider, such as accuracy of financial statements, adjusting for fair market expenses, and the nature of owner compensation. There are three broad categories of methods used to value small closely-held firms: asset methods, market comparable methods, and income methods. In general, the asset method calculates the minimum value of a business (approximation of net liquidation), while the comparable and income methods represent "going concern" values. The comparable methods inevitably attempt to employ transactions of similar firms and then apply various metrics such as revenue or gross profit multiples to the target firm.

Income methods, on the other hand, examine the earnings of the firm. The most common income-based valuation method used in practice is the capitalization of earnings (or free cashflows/FCFE) methods. This involves applying a capitalization rate to the adjusted earnings in order to obtain a fair market equity value of the target firm. The calculation of the appropriate capitalization rate requires an estimate of the associated discount rate, which is then adjusted for growth. In fact, the capitalization of earnings method is essentially an approximation of the present value of future earnings or cashflows under the assumption that past earnings will continue into the future at a somewhat constant growth rate.

For this reason, capitalization of earnings methods should be applied to relatively stable firms. It is generally not appropriate for early-stage technology firms, intellectual property valuations, or situations where the future financial situation might be very different from the past – in these cases a

discounted future cashflow analysis is generally used. However, in many cases of small closely-held firms, with stable past earnings, and no major expected changes in the competitive environment in the future, the capitalization of earnings, with its underlying theoretical assumption of discounted cashflows, is a reasonable and often-used method.

While income-based valuation methods have their own set of constraints, such as developing accurate earnings or cashflow forecasts for the firm, a particular requirement for calculating discounted earnings, or its associated capitalization of earnings, is the development of an appropriate discount rate, or cost of equity. Unlike the cost of debt, which can be determined from a weighted average of interest rates, the cost of equity requires significant thought and analysis. While there are many different approaches associated with this calculation, the most common way of calculating the cost of equity (r_e) for a closely-held business is the basic "build-up" method, or a similar variation (e.g, Trevino, 1997, Pinto, 2020; Damodaran, 2024b).

$$Dr_e = \text{Risk Free Rate} + \text{Equity Risk Premium} + \text{Industry Premium} + \text{Firm Specific Premium} + \text{Size Premium}$$

In the build-up approach, the risk-free rate is generally determined by the yield rate of a government backed instrument, such as T-bills or T-bonds, although there is some debate as to which is most appropriate. The calculation of the equity risk premium can be quite complicated, with different opinions regarding the appropriate methodology. Inevitably, however, the equity premium is calculated from data of publicly-traded firms. Fortunately for the business appraiser, the calculated equity risk premiums are easily obtained from published sources, such as from *Kroll* (Kroll, 2024) or by Professor Aswath Damodaran's work at New York University (Damodaran, 2024). *Kroll*, for example, currently reports a 5.0% equity premium for 2024 when developing a discount rate for U.S. based firms.

The equity premium for international markets can also vary dramatically based on the country's risk and credit profile. Damodaran (2024a), for example, reports a 2023 equity risk premium of 15.43% for Egypt, and 8.70% for Spain, while other countries such as Canada and Germany are almost identical to the U.S. Since equity premiums are almost always determined from averages obtained from the full equity markets, when developing an appropriate discount rate for a particular target firm, consideration must then be given more specific characteristics, such as industry differences and firm size adjustments. Industry premiums are also available from a variety of sources, such as Damodaran (2024a).

If the target firm is a small firm, however, the issue of an appropriate small firm premium in determining the discount rate must be given serious consideration. That is the focus of this study.

2. Commonly Used Size Premiums: Publicly-Traded Firms

The relationship between the size of the firm and the value of the firm has received significant attention. From a theoretical point of view, the argument is basically two-fold. First, larger firms are believed to have more bargaining and negotiating power in the market with their upstream suppliers and downstream customers. The issue of asymmetric bargaining power enjoyed by larger firms has been discussed for decades in the industrial organization and strategic management fields under various headings, such as "vertical quasi-integration" and "bilateral market power" (Blois, 1972; Galbraith and Stiles, 1983; Galbraith and DeNoble, 1992; Porter, 1976). Interest in the impact of asymmetric bargaining power on supply chain relationships continues to the present time, with empirical research consistently showing significant cost/price advantages for larger firms. (e.g., Picot et al, 2023; Chi and Goo, 2023; Jo, Jeong and Kim, 2023). Second, large firms are believed to have

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

scale advantages both in input components, such as hiring better workers and receiving bulk discounts, and in output components, such as size advantages in distribution and product placement. The impacts of both of these forces, market power and scale economies, should be reflected in the firm's earnings.

The impact of firm size on the cost of equity has been almost exclusively examined with publicly-traded firms, as revealed by the size premium published by various financial data providers, such as *Kroll* (2024), *Duff and Phelps* (2015-2017) and *Ibbotson* (Ibbotson and Harrington, 2020). Contrary to the findings of a disappearing size effect that some early research indicated (e.g., Foerster and Porter, 1992; Horowith, Loughran and Savin, 2000) there is substantial evidence that a small firm risk premium does exist with publicly-traded firms, in both the U.S. and international markets (e.g., Amel-Zadeh, 2011; Elyasiani, Gambarelli, and Muzzioli, 2020; Yadav, Pahi and Gangakhedkar, 2022; Kroll, 2024; Ibbotson and Harrington, 2020; Khan, Hassan and Ali, 2012). For years, *Kroll* (which merged with *Duff and Phelps* in 2018) and other financial data providers have calculated the size premium using various risk adjusted stock price data categorized by the size deciles of publicly-traded firms, with the 10th decile being the smallest 10% of the publicly-traded firms examined.

When valuing a small closely-held firm, however, there is a problem with using this "decile" premium - the average size of firms in the 10th decile of publicly-traded firm is approximately $30million in market capitalization. In comparison, the vast majority of closely-held firms have market capitalizations significantly less, particularly if the target firm is a typical owner-operated business. In fact, the median market value in the *DealStat* databases of closely-held firm sales transactions from 2020 to 2024 was approximately $400,000; with average market values ranging somewhat higher or lower depending on the industry sector. Comparing a small closely-held firm even with the 10th decile of publicly-traded firms is like comparing apples and oranges, at best.

Recognizing this*, Kroll* and others, now report using four, more detailed 10th decile categories in their analysis of U.S. publicly-traded firms - 10z, 10y, 10x, and 10w, with 10z being the smallest. According to recent *Kroll* calculations, the small firm premiums for these the different categories of publicly-traded U.S. firms are shown below for 2021 and 2022. While the small firm premium can vary from year to year (declining slightly somewhat in the last decade for all deciles) it has been relatively stable at the smallest category, 10z. For example, from 2012 to 2022 the *Kroll* 10z size premium has ranged from a high of 11.77% (2012) to 11.17% (2022). Table 1 shows the *Kroll* small firm decile calculations for 2021 and 2022.

Table 1: Size Premiums for Publicly-Traded Firms – Kroll Categories of Market Capitalization

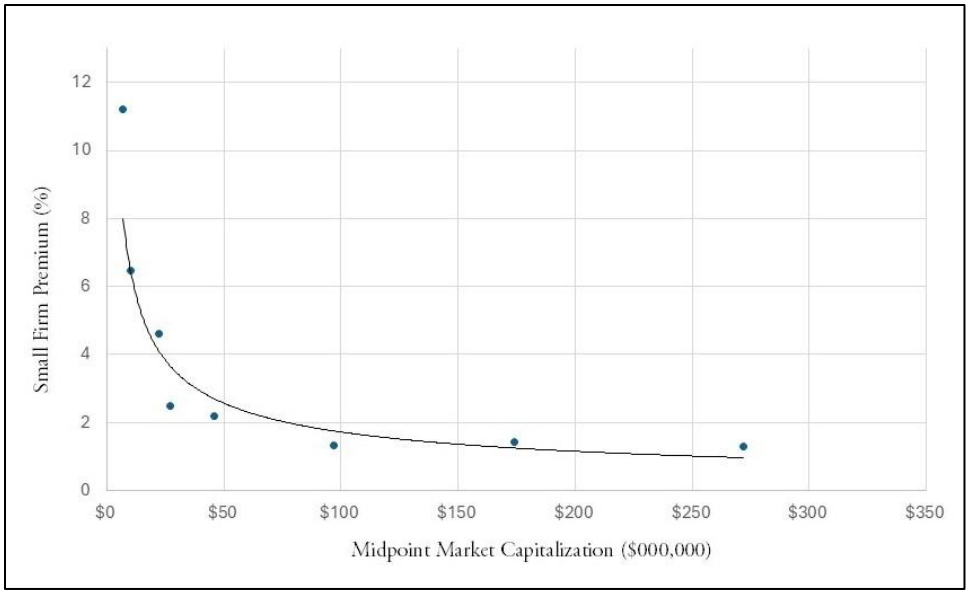
Firm Size Category	Mid-Point Capitalization	2022	2021
10z	$6,915,000	11.17%	11.29%
10y	$15,920,000	6.34%	6.60%
10x	$22,100,000	4.54%	4.65%
10w	$27,035,000	2.34%	2.60%
9	$45,890,000	2.10%	2.29%
8	$96,755,000	1.21%	1.46%
7	$173,545,000	1.34%	1.54%
6	$272,345,000	1.18%	1.37%
5	$414,235,000	0.89%	1.09%
4	$661,625,000	0.65%	0.75%
3	$1,247,740,000	0.55%	0.71%
2	$2,642,930,000	0.43%	0.49%

However, even the 10z category, which reports the smallest publicly-traded firms, has a mid-point equity capitalization near $7million, which is still many times larger than the average

capitalization reported in the databases of closely-held firm sales transactions. However, the vast majority of small business appraisers are taught in their training classes to use the 10z (some still just use the 10 category) to determine the small firm equity premium.

When valuing a typical closely-held business with a much smaller capitalization, an obvious problem is evident - one can see a clear non-linear growth in the size premium as the firm gets smaller (Table 1 and Figure 1). It is also evident from the decile calculations that the dramatic increase in size premiums appear to occur primarily in the smallest size categories. This also indicates that even the more detailed decile categorization may not capture the appropriate size premium for the smallest firms, particularly if they fall in the lower capitalization range of the 10z definition.

Figure 1: Reported Small Firm Equity Premiums: Kroll Categories 6-10z (Average 2021 & 2022)



3. Analyzing the Size Premium: Publicly-Traded Firms

We approach the problem of estimating a more accurate small firm premium for very small, closely-held firms in several ways. First, for comparison purposes we simply use a regression of the *Kroll* data. We attempted several different models (linear, logarithmic, exponential, power curve) and using different Kroll categories. Since this is an exercise in extrapolation, obtaining a model with a good fit (or R^2) is critical. After attempting several different model specifications, we found that a logarithmic regression model using the midpoint capitalization for the 7,8,9, 10w, 10x, 10y, and 10z categories provides a good fit (R^2 = 0.775). A similar model was found using the 2 to 10z categories, although the R^2 was not as high, while the 9 to 10z categories provided a slightly better fit (R^2 = 0.859), but with fewer data points. For the size premium metric for this analysis, we average the 2021 and 2022 data. It should be noted that while there are only technically seven data points in our regression, each of the *Kroll* categories is made up of hundreds of firms, for a total of approximately 2,000 publicly-traded firms for the categories 7 to 10z. The results of the logarithmic model are shown below.

Table 2: Logarithmic Model, Small Firm Premium – Kroll Categories of Market Capitalization, 7 to 10z

Variable	*B*	*T-statistic*	*Prob*
Constant	14.427	5.650	<0.002
Ln(Firm Capitalization)	-2.891	-4.148	<0.009

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

To see the difference, if we assume a firm has an approximate $300,000 equity capitalization, using our first logarithmic regression model a firm of this size would have a small firm premium of 17.91% versus 11.17% if one simply uses the 2022 10z category from *Kroll*. This significantly changes the overall discount rate calculations, which would have a major impact on the estimated market value of the target firm.

However, while the data from publicly-traded firms certainly suggest a non-linear relationship between firm size and its equity premium, there are the typical validity concerns about extrapolating a model fitted to a particular set of data to forecast something outside the range of the original data set or scope. While intuitively attractive, the risks of making predictions beyond the range of the data used to estimate the original model are well known, not only in finance but many other fields such as ecology, epidemiology and engineering. This is often called a "transportability" or generalizability validity problem where the study sample does not overlap with the target population (Lesko et al, 2020), that is, in our case, the much smaller size range of closely-held firms. In order to address this extrapolation validity issue, simulation studies are often suggested (e.g, Bartley et al, 2019). Another option is to analyze highly comparable or representative datasets that cover the data range of interest (e.g., Smith, 2019), thus bringing the treatment of external validity in line with internal validity (e.g., Lesko et al, 2020). In this paper, we are therefore interested in exploring whether the same non-linear, negative small size premium continues for smaller, closely-held firms. This is our interest for two reasons – first, to address the validity of extrapolating size premium determined from much larger publicly-traded data to much smaller, closely-held firms, and second, to develop a similar model of size premiums using actual transaction data for small closely-held firms.

4. The Impact of Size: Closely Held Firms

It is impossible to exactly replicate the analysis of publicly-traded firm size premiums with transaction data for closely-held firms for several reasons. First, unlike publicly-traded firms where daily stock prices can be examined for risk-adjusted market returns, closely-held firm transaction data is only a one-time event upon the sale of the business. Given this, our analysis cannot calculate a beta-adjusted risk associated with the small size premium. However, transaction data does represent the only fair market value determination for closely-held firms.

Second, as previously mentioned, normally transaction data for a closely-held firm is reported as a "market" value or a "transaction price". A market value or transaction price represents the total price of the deal, excluding any debt and cash (since debt and cash are rarely assumed during the sale of a closely-held firm). This is similar to a real estate sale, where a house is sold and subsequently recorded in the real estate transaction databases at a price that doesn't incorporate any outstanding mortgage against the property. On the other hand, the analysis of publicly-traded firm data, including the *Kroll* data, is based on stock price data, which is an "equity" value.

Third, in any closely-held company the owners can significantly influence reported earnings, particularly related to control over salaries paid to themselves. For example, some owners of closely held firms may pay themselves a large salary, thus reducing reported earnings, while other owners of closely held firms may not pay themselves a salary, thus resulting in higher reported earnings and taking personal income through owner "draws". When performing a business valuation using the firm's earnings, it often becomes necessary to normalize the financial statements to reflect what an arms-length investor might do if they were to invest-in, or purchase, the company.

However, we are not interested in exactly replicating the analysis from publicly-traded firms, but rather interested in two issues. Our first interest, and the focus of Analysis 1, is determining whether small, closely-held firms follow the same non-linear, logarithm pattern as seen in Table 2 using

comparable metrics. If closely-held small firms follow the same pattern as seen for publicly-traded firms, then we have addressed the major validity concerns regarding the extrapolation of the estimated logarithm regressions for the larger firm categories such as shown in the *Kroll* calculations. Second, using transaction data for closely-held firms, can we develop a model that does, in fact, reasonably estimate the size premium in determining the cost of equity discount rate for smaller firms? This is the focus of Analysis 2.

5. Data

To examine this relationship for closely-held firms, data was obtained from a proprietary database of sales transactions for closely-held firms. We use the *DealStat's* database. *DealStat* (formally *Pratt's Stats*) is the world's largest database of sales transactions for closely-held firms. Data comes from a variety of sources, including business brokers and merger and acquisitions professionals. The transaction data contains several types of information related to each transaction, including the market price, the deal structure, the NAICS codes, product description, and the number of employees. Also included are revenue, income and sometimes balance sheet items. The firms include a broad array of businesses, with the vast majority of firms under $10million in revenues.

We used sales transactions for the years 2016 to 2019, and 2022 to 2024. We exclude 2020-2021 to control for the impact of the Covid-19 years. There is ample evidence that most small businesses were dramatically impacted by the Covid-19 restrictions of 2020, while 2021 is considered a year of pent-up demand for many industry sectors (Dore and March, 2022). We examined the broad manufacturing, retail, and wholesale industry sectors (NAICS 31, 32, 33, 42, 44, 45), resulting in a total sample size of 2,092. We examined these sectors since these industries are relatively stable, they are not as strongly influenced by personal goodwill components such as the various services sectors, and they are not heavily regulated, such as utilities. While recognizing there are probably industry specific differences, since *Kroll* (and other financial service providers) pool data for their reported size premiums, and since business valuators look for a generalized small firm premium in their analysis, we also pool these sectors for the present analysis.

We perform two different analyses. The first analysis (Analysis 1) examines whether very small, closely-held firms follow the same non-linear, logarithm pattern as seen in studies of publicly-traded firms. This analysis is designed to address the validity concerns regarding the extrapolation of the estimated logarithm regressions for the larger firm categories. The second analysis (Analysis 2) develops a model that reasonably estimates the size premium in determining an equity-based discount rate for smaller firms using a subset of the transaction data.

6. Analysis 1: Small Size and Sellers Discretionary Cashflow Capitalization Rates

For our first analysis, we are initially interested in the "shape" of the relationship between firm size, as measured by market value or transaction price, and a reasonably comparable measure of value. This specifically addresses the validity issue of extrapolating small firm premium estimates from models developed from publicly-traded firm data. To control for the issue of owner's salary identified above, we use a seller's discretionary cash flow measure, or SDCF (combination of net earnings, owner's compensation, and non-cash charges, such as depreciation).

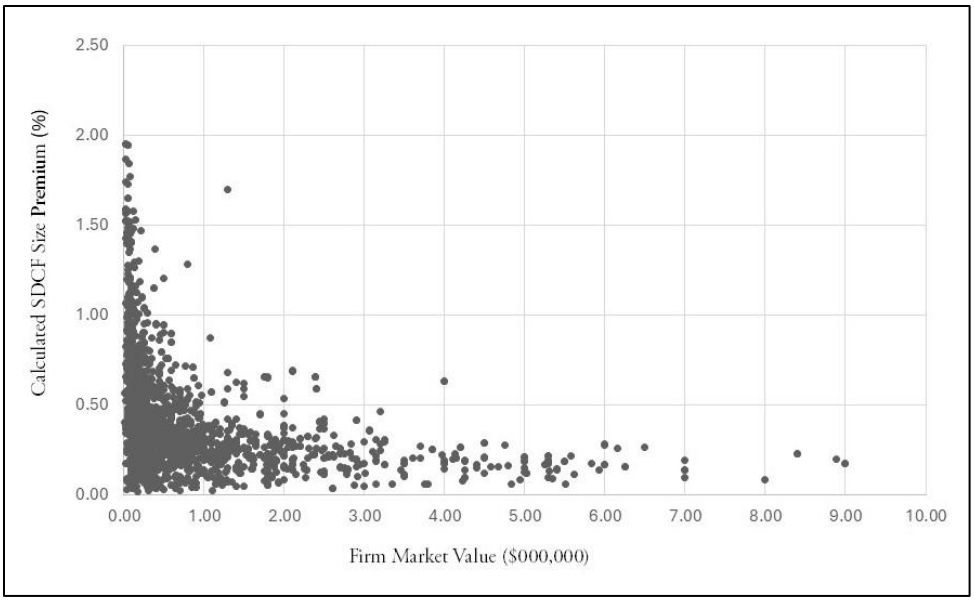
The SDCF model is attractive since it is both a commonly used metric for valuing small, closely-held firms and is regularly recorded in transaction databases of closely-held firms. We then divide SDCF by the transaction price to obtain a SDCF capitalization rate (SDCF CapRate). The SDCF CapRate is

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

therefore similar to an earnings capitalization rate, which divides a firm's adjusted earnings by the firm's equity price but rather uses a total owners cashflow measure.

Figure 2 shows the plot of our full sample, using the market value or transaction price for the size dimension and the SDCF CapRate. This graph appears to clearly indicate the same non-linear negative relationship between firm size and a capitalization rate measure as seen in the publicly-traded data examined earlier.

Figure 2: SDCF CapRates versus Firm Market Value: Small Closely-Hold Firms



We then calculated both a linear regression and a logarithm regression. For the firm size dimension, we again use the actual transaction price or market value. However, about 10% of the transactions in the database also provide balance sheet data for the firm. For these firms it is also possible to calculate an approximation of total firm equity value by simply subtracting long-term debt from the transaction price/market value. This firm total equity value is similar to the "capitalization" terminology of firm size commonly used for publicly-traded firms.

We controlled for possible outliers by limiting SDCF CapRates greater than 10% (or Transaction Price/SDE multiples<10), since 97% of the calculated SDCF multiples are less than 10). This also controls for firms reporting little or no positive cash flow, or possibly mis-entered data. We also did not include firms designated as "development stage firms", since a capitalization of earnings or cashflow approaches would generally not be appropriate for these types of firms.

Since we are interested in the size impact for small firms, we only examined U.S. firms with market values less than $10million, purposely resulting in some overlap with the 10z category of publicly-traded firms. Table 3 shows the results of our regression analysis.

Table 3: Regressions – Firm Size and SDCF Capitalization Rates

Model 1: SDCF CapRate Linear Model (Market Value)			
Variable	*B*	*T-statistic*	*Prob*
Constant	0.587	49.282	<0.001
Firm Market Value	-0.069	-10.065	<0.001
R^2=0.046; N=2092			
Model 2: SDCF CapRate Logarithmic Model (Market Value)			
Variable	*B*	*T-statistic*	*Prob*
Constant	0.366	30.990	<0.001
Ln(Firm Market Value)	-0.176	-22.819	<0.001
R^2=0.211; N=2092			
Model 3: SDCF CapRate Logarithmic Model (Equity Value)			
Variable	***B***	*T-statistic*	*Prob*
Constant	0.399	27.727	<0.001
Ln(Firm Equity Value)	-0.095	-8.474	<0.001
R^2=0.336; N=253			

In each model estimated, there is a negative relationship between the SDCF CapRate and firm size, whether firm size is measured by the full transaction price/market value or an estimate of the firm's total equity value. This indicates that the smaller the market value of the firm, the higher the SDCF CapRate – a somewhat expected result. However, another key finding from Table 3 is that the non-linear logarithmic models provide a better fit to the data than the linear model. This implies an increasingly larger size premium for smaller firms.

6.1 *Small size premium: Sellers discretionary cashflow capitalization rate*

Table 3, however, only examines the relationship between firm value and the firm's overall SDCF CapRate, and not specifically an equity size premium. In order to examine the size premium issue, we also need to adjust for the risk-free rate and an equity premium. Since our data is from multiple years, with different economic conditions, we calculate a specific SDCF CapRate size premium by subtracting the risk-free rate (we used the average 20-year t-bond yield rate for each year in the database) and the overall firm equity risk premium for each year in our sample (Damodaran, 2024a). This is similar to the build-up method of calculating the appropriate discount rate to determine equity value as discussed later in this paper. Thus, our SDCF CapRate Size Premium is calculated as follows:

SDCF CapRate Size Premium = SDCF Cap Rate – annual risk-free rate - equity premium

Our next analysis is to see if there is a relationship between firm size, as measured by market value, and the SDCF CapRate Size Premium. This is the analysis that is most analogous to the regression model of publicly-traded size deciles in Table 2 using the Kroll data.

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

We estimated a logarithm regression model, but this time with the dependent variable the specific SDCF CapRate Size Premium calculated for each transaction. Table 4 below shows the results of our analysis of the SDCF CapRate Size Premium.

Table 4: Logarithmic Model, SDCF CapRate Size Premium

Variable	*B*	*T-statistic*	*Prob*
Constant	0.283	24.009	<0.001
Ln(Firm Market value)	-0.176	-22.820	<0.001
R^2=0.211, N=2,092			

The results indicate a negative relationship between Market value and the SDCF CapRate Size Premium. It should also be noted that while the R^2 of the transaction data is lower than the models estimated from the *Kroll* data, this was expected given that the *Kroll* model estimates are based upon average values of different size categories, such as deciles, while our model estimates shown in Table 4 are based upon the data from individual firms which have much more dispersion.

As a final comparison, since we specifically selected our size data to overlap with the 10z category of publicly-traded firms we can compare similar sized firms between the 10z *Kroll* data and the larger, overlapping sized firms in our data. The midpoint of the 10z category is approximately $7million of equity capitalization. As a comparison to the 10z we therefore examine the firms that have a market value greater than $7million (which estimates equity values closer to the 10z mean). The resulting average SDCF small firm premium in our calculations for firms greater than $7million of market value is 14.21%, which compares with an 11.43% average equity small firm premium for the *Kroll* 10z category of publicly-traded firms for the same years as our data. Given the difference between our analysis using market value versus the *Kroll* model based on risk-adjusted equity values, this appears to be reasonably close, again providing additional support to our argument that the extrapolation of small-firm equity premiums from publicly-traded firms is likely valid.

6.2 Implications of Analysis 1

Since our Analysis 1 examines the relationship between market value and an SDCF CapRate Size Premium it should not be used to calculate a small firm premium for the purpose of adjusting the discount rate, which is used to determine an equity value. Rather the purpose of Analysis 1 is to address the validity issues of extrapolation by examining a similar metric using market value and a somewhat comparable small firm premium. Using a seller discretionary cashflow (SDCF) measure, the relationship between a small firm size premium and firm size appears to follow the same logarithmic pattern for small, closely held firms as seen with publicly-traded firms. Analysis 1 clearly supports the validity of using a logarithmic model, such as calculated from the *Kroll* data, and extrapolation techniques to estimate an appropriate small firm premium when calculating the equity value of small, closely-held firms.

7. Analysis 2: Equity Size Premiums for Small Closely-Held Firms

As previously mentioned, the transaction price in our database is presented as an "market value" or "transaction price" which normally assumes no debt or cash transferred. However, published size premiums are calculated from publicly-traded stock prices, which is an equity value. Thus, a comparable analysis of a small firm premium for closely-held firms needs to adjust the transaction price in our database to reflect an equity value. As previously mentioned, approximately 10% of the

transactions in our database provide a long-term debt figure for the firm. For these firms we can estimate an equity value around the date of the transaction by subtracting the long-term debt from the market value indicated by the transaction price. We recognize that there might be additional business obligations accounted for by short-term liabilities, but for our analysis we only use long-term debt to calculate an equity value.

7.1 Small size Premium: An equity analysis

We calculate an equity value by subtracting the long-term debt of the year closest to the sales transaction. In some cases, this resulted in a negative equity value. We excluded any firms that resulted in a negative equity value since this might indicate firms in distress.

To calculate a discount rate/capitalization rate related to earnings we need to determine an appropriate earnings figure for each firm. As mentioned above, in any closely held company the owners have the ability to influence reported earnings, and other elements of the financial statements by their control over salaries, salaries of family members, benefit programs, perks (such as cars and boats), investments (and corresponding depreciation charges), form of employee payment (W-2s versus 1099s), barter trades, and "sweet-heart" deals such as "non arms-length" transactions or transactions between related parties affecting rentals, leases, and sales to name a few. These strategies are often used to maximize the expense deductions while minimizing the firm's tax liability. When there is confusion about the appropriate earnings figures, many valuation experts will use an "inferred" or "normalized" earnings. Inferred or normalized earnings is a process of applying an average or expected profit level (generally obtained from industry data) to a particular business. Many buyers of small businesses often use inferred earnings to adjust a target business's financial statement when the business is reporting below average earnings since buyers recognized that the reported earnings of the target firm may not be overly accurate, or the new buyers might feel they can better manage the acquired firm to obtain at least a normal performance.

We therefore normalize each of the firm's earnings by using industry average earnings. We use "operating profit" as a percentage of revenues as reported by RMA's Annual Statement Studies. Data for the RMA Annual Statement Studies comes directly from the anonymized financial statements of small and medium-size business clients of RMA's member institutions, mostly lending institutions and credit agencies. We use data reported at the 3-digit NAICS level. Since the reported operating profit can vary somewhat from year to year in the RMA Annual Statement Studies, we average the data over the years of the study (2016-2019, 2022-2024) for the revenue ranges contained in our data. Each industry in our study reported a different average operating profit, although all the mean operating profits were less than 10% of revenues. We then apply these percentages to the reported revenues to obtain inferred earnings.

To calculate an earnings capitalization rate (EarningsCapRate) we then divide the calculated equity value by the inferred or normalized earnings. Thus EarningsCapRate = (EquityValue)/(Inferred Earnings). For this analysis we ignore tax effects and focus on operating profit, since most of these firms report their financials as pass-through entities, such as S-corporations, LLCs, partnerships or sole-proprietorships and we want to concentrate on earnings related to actual business operations.

Similar to Analysis 1, we then calculate a Small Size Premium. However, in Analysis 2 we are interested in determining a size premium related to the discount rate for an equity valuation. We therefore also include an industry growth component, since in its most basic form a capitalization rate is equal to the discount rate minus a growth rate. The growth rate was determined by the average revenue growth in the three industries (Manufacturing, Wholesale, and Retail) for the years under examination using data from *IBIS World* IBIS World, 2024. In nominal terms, this generally averaged

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

approximately 4% to 6% for the years under examination. We ran different models for different nominal growth rates. Using a standard build-up approach, the following formula is used:

DR Size Premium = Earnings Cap Rate - annual risk-free rate - equity premium + long-term growth

Duff and Phelps (2015-2017) calculated their premium data both as arithmetic means and as smoothed data. Given the nature of our data, we also use both methods. For our smoothed data, we use an exponential smoothing process (α=0.5). An α=0.5 gives somewhat equal weight to both current and past data in the smoothing process. Table 5 presents the logarithmic regression models for both the raw data and the smoothed data.

Table 5: Logarithmic Model, Equity Discount Rate Size Premium

Arithmetic Mean Data	*B*	*T-statistic*	*Prob*
Constant	0.184	14.095	<0.001
Ln(Firm Equity Value)	-0.071	-7.588	<0.001
R^2=0.314, N=205			

Smoothed Data	B	*T-statistic*	*Prob*
Constant	0.176	23.441	<0.001
Ln (Firm Equity Value)	-0.061	-11.235	<0.001
R^2=0.502, N=204			

Table 5 indicates that a negative logarithmic relationship exists between the equity discount rate size premium and the equity size of the company, with the smoothed data model presenting a better fit due to the lower variation in smoothed data. These results also support an extension of the non-linear relationship seen in the small firm premiums calculated from publicly-traded firms as discussed in Analysis 1.

For a final comparison with the size premiums often presented by publicly-trade firm deciles, we divide our data into seven equal categories of small firm (SF) size and compute the average equity-based size premium with each category. As previously mentioned, we originally estimated our equity discount rates using different nominal annual growth rates. Using a nominal average growth rate of 4%, Table 6 shows the size premiums using both the arithmetic and smoothed data means, similar to how *Duff and Phelps* calculated their tables.

Table 6: Size Premium by Category: Small Closely-Held Firms

Size Category	Lower Range	Upper Range	Median Equity Capitalization	Non-Smoothed Premium	Smoothed Premium
SF1	$3,300,000	$10,000,000	$4,420,138	7.67%	7.02%
SF2	$2,000,000	$3,300,000	$2,280,578	11.48%	12.18%
SF3	$1,100,000	$2,000,000	$1,257,867	15.34%	14.59%
SF4	$550,000	$1,000,000	$652,036	20.18%	18,44%
SF5	$330,000	$550,000	$434,856	22.46%	21.62%
SF6	$200,000	$330,000	$288,337	25,53%	27.02%
SF7	$0	$200,000	$94,732	35.21%	29.18%

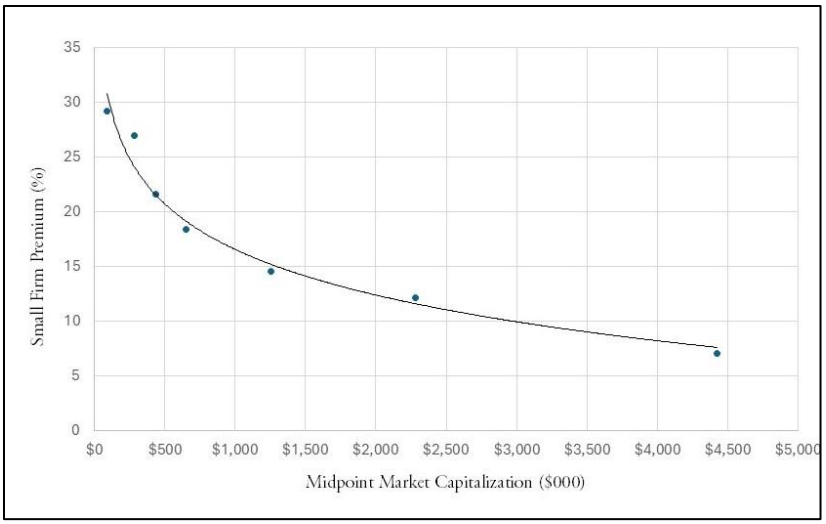
As mentioned earlier, we intentionally limited our sample to firms under $10million of market value in order to overlap with the 10z decile category for publicly-traded firms. The 10z category of publicly-traded firms would generally fall with our SF1 to SF3 ranges, although most 10z publicly-traded firms would probably fall within our SF1 category. As a comparison, if we average the small

firm smoothed premiums for the SF1 to SF3 categories in our analysis, it results in an average size premium of 11.26%. Averaging the arithmetic means results in a size premium of 11.50%. This is remarkably similar to the 11.43% average *Kroll* 10z premium for the years of our data, and the 11.23% average 10z premium for years 2021 and 2022.

As a final check, the largest firm in our SF1 category is smaller than the lower limit of the 10y decile category ($12.79million). As expected, the calculated mean premiums (7.67% and 7.02%) for our SF1 category are above the 6.47% average premium (2021 and 2022) for the *Kroll* 10y category. This provides strong support for using Table 6 to calculate the small firm premium for firms that are smaller than the 10z range. It also provides a more nuanced breakdown within the 10z category showing there are dramatic differences in the firm premium even within the 10z range.

We also plot our SF equity premium categories using smoothed data in a way similar to our previous plot of the *Kroll* categories for publicly-traded firms. It appears to be a similar relationship, but with much smaller firm capitalization values.

Figure 3: Discount Rate Size Premium versus Equity Value: Galbraith Small Firm (SF) Categories



7.2 Implications of analysis 2

Analysis 2 provides the most comparable analysis with the regularly cited decile approaches to understanding the small firm premium when calculating a discount rate. Clearly, our analysis shows that using the 10z small firm premium calculated from publicly-traded firms underestimates the size premium for smaller closely-held firms, and the differences become much more pronounced as firm size decreases. Analysis 2 also provides a more detailed analysis of the size premium for firms that overlap with the 10z category. More than anything, our Analysis 2 indicates that the firm size premiums for small closely-held firms is significantly higher than calculated from the decile categories of publicly-traded firms.

7.3 A revenue model

Since we also had revenue data for our sample, we calculated a similar model using revenues as the size variable. However, the correlation between firm revenues and firm equity values was not strong in our data, approximately 0.45. Thus, revenues and equity appear to be measuring different dimensions of size. Investigation of the data also indicated that several very low revenue firms in the

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

data had very high transaction prices, which is counter intuitive for stable firms. Although we initially eliminated "development stage firms" from our data, this very well could indicate the inclusion of some relatively early-stage post-revenue and profitable firms that have very high capitalizations. We are interested in relatively stable entities in our study. We therefore controlled these potential outliers and eliminated any firms with low revenues (<$500,000) that also had very high equity values (>$2million), that is, equity values > 4.0 times revenues. After this control, we estimated a statistically significant revenue model of y=0.2518-0.044ln(x), where y=size premium and x=revenues (in $million). This model showed a reasonable fit (R^2 =0.204), and results in estimates of the small firm premiums similar to the equity models described above. However, more analysis is needed to obtain a strong revenue model of small firm size premiums.

8. Conclusions and Implications

Most professionals who value small, closely-held firms use the basic build-up method for calculating their cost of equity and discount rates. However, when it comes to analyzing the impact of size for even the smallest of enterprises, many valuation professionals still use the 10th decile premium, or if familiar with the newer subcategories, they might use the size premium of the more refined "10z" category. As we have demonstrated, even this still significantly understates the impact of size given the average size of firms in the 10z category of publicly-traded firms far exceeds the vast majority of small closely-held firms. When valuing a small closely-held firm, simply using the 10z small firm premiums in a build-up method will certainly result in a lower than appropriate cost of equity calculation, and thus a fair market valuation estimate substantially higher than appropriate.

In our study we took two approaches using sales data from a proprietary transaction database, *DealStats*. The first approach was to address the validity of extrapolating the small firm premium calculated from publicly-traded firms. The issue of extrapolating beyond the scope of the original data always raises validity issues. Our first analysis (with over 2,000 transactions), using a seller's discretionary measure of performance and a measure of market value clearly indicates that the same logarithmic relationship between size premiums and firm capitalization continues for much smaller, closely-held firms.

Our second analysis specifically focuses on estimating an equity-based size premium for small closely-held firms. Using a methodology of calculating inferred earnings and estimating an equity value based on market value less long-term debt, our analysis resulted in a model for determining an appropriate small firm premium for much smaller firms than seen in publicly-traded markets. It is also evident that this model also fits with the extrapolated results for publicly-traded firms. This provides additional validity to using a logarithmic model in understanding the relationship between capitalization and the size premium used in determining a discount rate. Our second analysis also resulted in a categorization of size premiums that could be considered when determining an appropriate discount rate for valuing small, closely-held firms.

Finally, given the nature of the single-point transaction and financial statement data used in the present analysis, the results from our analysis of small, closely-held firms appear remarkably similar to the firm size models obtained from the theoretically sophisticated analysis of highly detailed beta-adjusted daily stock prices presented by the large financial data providers such as *Kroll, Ibbotson,* and *Duff and Phelps*. This, in itself, suggests that the issues of firm size premiums in determining the discount rates for small, closely-held firms be taken very seriously.

As a limitation, it must be noted that while our results are convincing, we only examined the broad manufacturing, wholesale, and retail markets. We did not examine certain other sectors, such as professional offices, information, agriculture, and health care. While it is very likely that these

industries also exhibit a similar relationship between size and firm premiums for determining an appropriate earnings discount rate, we did not include them in our model estimates.

https://digitalcommons.pepperdine.edu/jef/vol27/iss1/2
DOI: 10.57229/2373-1761.1498

References

Amel-Zadeh, A. (2011). The return of the size anomaly: Evidence from the German stock market. *European Financial Management*, *17(1)*, 145-182.

Bartley, M., Hanks, E., Schliep, E., Soranno, P., & Wagner, T. (2019). Identifying and characterizing extrapolation in multivariate response data. *Plus One*, 14(12). Accessed at https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6894872/

Blois, K., (1972). Vertical quasi-integration. *Journal of Industrial Economics*, July, 253-272.

Chi, P., & Goo, Y. J. (2023). The contagion and spillover effects of stock returns between quasi-vertically integrated firms in Taiwan semiconductor industry. *Journal of Business Administration, 48(4),* 67-88.

Damodaran, A. (2024a) https://pages.stern.nyu.edu/~adamodar/New_Home_Page/home.htm

Damodaran, A. (2024b). *The little Book of valuation: How to value a company, pick a stock, and profit*. NJ: John Wiley and Sons.

Dore, T., & March, T. (2022). Economic restrictions during the Covid-19 pandemic and measures of small business health, *FEDS Notes*, Board of Governors of the Federal Reserve System.

Duff & Phelps (2015-2017). *Valuation handbooks: U.S. guide to cost of capital*: Hoboken, NJ: John Wiley and Sons.

Elyasiani, E., Gambarelli, L., & Muzzioli, S. (2020). Moment risk premia and the cross-section of stock returns in the European stock market. *Journal of Banking and Finance*, *111*, 1-14.

Foerster, S., & Porter, F. (1992). The disappearing size effect: Evidence from dual class shares, *Quarterly Journal of Business and Economics, 31(4),* 80-91.

Galbraith, C., & Stiles. C. (1983). Firm profitability and relative firm power. *Strategic Management Journal*, *4*, 237-249.

Galbraith, C., & DeNoble A. (1992). Competitive strategy and flexible manufacturing: New dimensions in high-technology venture-based economic development. *Journal of Business Venturing, 7(5),* 387-404.

Horowith, J., Loughran, T., & Savin, N. (2000). The disappearing size effect. *Research in Economics,* 54, 83-100.

Ibbotson, R., & Harrington, J. (2020*). Stocks, bonds, bills and inflation (BBBI): 2020 Summary Edition*. Duff and Phelps.

IBIS World (2024). Various Industry Reports (*Wholesale trade in the U.S*. July, 2024; *Retail trade in the U.S*. July, 2024; *Manufacturing in the U.S*. October, 2023), https://my.ibisworld.com

Jo, H., Jeong, J., & Kim, C. (2023). Supplier relations: Extended quasi-vertical integration. In H. Jo, Jeong, J., & C. Kim (Eds), *Agile against lean: An inquiry into the production system of Hyundai Motor (*pp, 177-222). Singapore: Palgrave Macmillan.

Khan, F., Hassan, A, & Ali, S. (2012). Size, leverage and stocks return: Evidence from Pakistan. *International Journal of Academic Research*, *4(1),* 24-32.

Kroll (2024), *Risk premium reports*. Kroll.com/en.

Lesko, C., Ackerman, B., Webster-Clark, M., & Edwards, J. (2020). Target validity: Bringing treatment of external validity in line with internal validity. *Epidemiologic Methods*, 7, 117-124.

Picot, A., Reichwald, R., Wigand, R.., Möslein, K., Neuburger, R., & Neyer, A. (2023). Dissolution of the company: Networks. In A. Picot, Reichwald, R., Wigand, R.., Möslein, K., Neuburger, R., & A. Neyer (Eds). *The boundaryless enterprise: Information, organization and leadership* (pp. 105-117). Wiesbaden: Springer Fachmedien Wiesbaden.

Pinto, J. (2020). *Equity asset valuation*. NJ: John Wiley and Sons.

Porter, M. (1976). *Interbrand choice, strategy, and bilateral market power*. Cambridge, MA: Harvard University Press.

RMA (2924) *Annual Statement Studies*. https://www.rmahq.org/statementstudies/?gmssopc=1

Smith, T. (2019). Validation approach for statistical extrapolation. In V. Belenky, , Spyrou, K., van Walree, F., Almeida Santos Neves, M., & N. Umeda (Eds), *Contemporary ideas on ship stability: Fluid mechanics and its applications*, vol 119. Springer, Cham., 573

Trevino, G. (1997). A note on formulating and corroborating discounts rates for small firms. *Journal of Legal Economics*, 7, 45-54.

Yadav, I., Pahi, D., & Gangakhedkar, R. (2022). The nexus between firm size, growth and profitability: New panel data evidence from Asia–Pacific markets. *European Journal of Management and Business Economics*, *31*(1), 115-140.

DOI: 10.57229/2373-1761.1498