

Althanns, Markus; van Buggenum, Hugo

**Working Paper**

## Strategic debt in a monetary economy

KOF Working Papers, No. 524

**Provided in Cooperation with:**

KOF Swiss Economic Institute, ETH Zurich

*Suggested Citation:* Althanns, Markus; van Buggenum, Hugo (2025) : Strategic debt in a monetary economy, KOF Working Papers, No. 524, ETH Zurich, KOF Swiss Economic Institute, Zurich, <https://doi.org/10.3929/ethz-b-000737259>

This Version is available at:

<https://hdl.handle.net/10419/318545>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Strategic Debt in a Monetary Economy

**Working Paper****Author(s):**

Althanns, Markus  van Buggenum, Hugo

**Publication date:**

2025-05

**Permanent link:**

<https://doi.org/10.3929/ethz-b-000737259>

**Rights / license:**

In Copyright - Non-Commercial Use Permitted

**Originally published in:**

KOF Working Papers 524

# KOF Working Papers

## Strategic Debt in a Monetary Economy

Markus Althanns and Hugo van Buggenum

No. 524, 05 / 2025

# Strategic Debt in a Monetary Economy<sup>\*</sup>

Markus Althanns<sup>†</sup>

Hugo van Buggenum<sup>‡</sup>

KOF Swiss Economic Institute at ETH Zurich,  
Leonhardstrasse 21, 8092 Zürich, Switzerland.

May 2025

## Abstract

Producers can leverage their bargaining power vis-à-vis consumers by entering bargaining with debt. We discover novel general-equilibrium effects of such strategic debt by developing a money-search framework featuring heterogeneous consumers. Debt distorts trade along two margins: it destroys matches with low-preference consumers and it tightens liquidity constraints within matches. While the fiscal authority can fully eliminate strategic debt through taxation, in its absence, monetary policy can partially curb it by deviating from the Friedman rule—raising nominal rates up until 0.51%. Finally, we show that producers can leverage their bargaining power even more effectively with contracts different from debt.

**Keywords:** bargaining, money search, strategic debt.

**JEL Classification:** C78, D86, E52, G32.

**Declaration of Interests:** None.

---

<sup>\*</sup>We thank Lukas Altermatt, Hans Gersbach, Cyril Monnet, Dirk Niepelt, Kieran Walsh, Eric Young, Randall Wright as well as the participants at the 2024 Summer Workshop on Money, Banking, Payments, and Finance at the Federal Reserve Board of Governors; the SSES Annual Congress 2024 at the University of Lucerne; the 75th European Meeting of the Econometric Society at the University of Rotterdam; the 2024 Annual Conference of the Austrian Economic Association at BOKU University, Vienna; the 2024 EL Summer School at Bocconi University, Milan; the seminar series at the University of Berne; the University of Navarra, and the Budapest University of Technology and Economics; and the Astute Modelling Seminar at ETH Zurich for valuable comments and suggestions.

<sup>†</sup>[althanns@kof.ethz.ch](mailto:althanns@kof.ethz.ch)

<sup>‡</sup>[vanbuggenum@kof.ethz.ch](mailto:vanbuggenum@kof.ethz.ch)

# 1 Introduction

Why do firms issue debt? Traditional answers are that firms do so to finance their operations, to minimize agency costs, or to shield their cash flows from taxation. Empirical evidence suggests that firms also use debt to strengthen their bargaining position vis-à-vis their customers. For instance, U.S. hospitals with higher debt-to-equity ratios receive higher reimbursement rates for medical procedures when bargaining with health insurers (Towner, 2020). Also, firms are particularly likely to issue this *strategic* debt if the market power of their suppliers and customers is high, pointing towards firms compensating a lack of bargaining power with debt (Kale and Shahrur, 2007).

The distributive effect of strategic debt in bargaining is effectively illustrated through a simple example in which two agents, A and B, bargain over eight pieces of cake (see Hennessy and Livdan, 2009, for a similar example). Both agents exert equal bargaining power and receive nothing if they fail to reach an agreement. In the absence of debt, the surplus is split evenly, with each agent receiving four pieces. Now suppose that, prior to bargaining, A enters into a limited-liability debt contract with a third party, agent C. In this contract, A promises to transfer the first two pieces of cake from tomorrow's bargaining to C, in exchange for receiving two pieces from C today. Because A is only obligated to repay if she secures a positive number of pieces in bargaining, the debt effectively introduces a fixed cost to reaching an agreement with B. Since A must be compensated for this fixed cost, the bargaining outcome shifts: A receives five pieces (of which two are transferred to C), while B receives three. Ex post, both A and B consume three pieces, reflecting their equal bargaining power. However, debt dynamically enhances A's bargaining position, as the ex-post repayment of her ex-ante transfer is partially borne by B. Figure 1 illustrates the distribution of cake with and without debt.

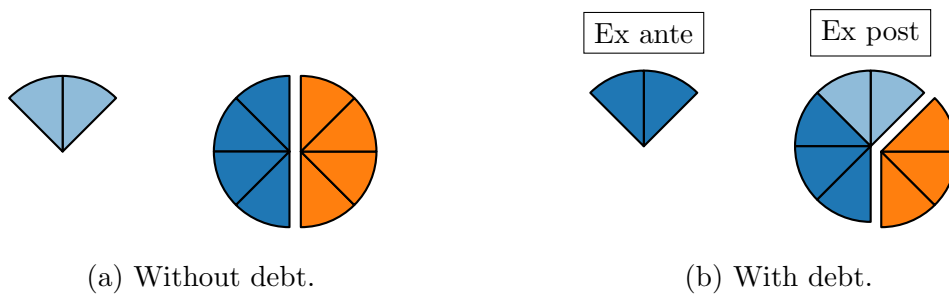


Figure 1: Cake consumption of agents A(●), B(●), and C(●).

In this paper, we ask whether strategic debt merely redistributes surplus, or whether it is also distortionary through its general-equilibrium effects. Framed in our cake example, does strategic debt merely alter how the eight pieces of cake are distributed, or does it also affect the *intensive margin* of trade—by influencing the number of pieces produced—and the *extensive margin*—by affecting how frequently agents trade? Moreover, if debt is indeed distortionary, how should fiscal and monetary policy respond? To

address these questions, we incorporate strategic debt into a money-search framework with heterogeneous customers and identify a novel externality: when firms issue debt, they make it more difficult for customers to find trading partners they are willing to compensate for debt repayment. As a result, some bargaining matches fail to result in trade. This externality can be eliminated through a Pigouvian tax on debt. In the absence of such a direct tax, we show that a deviation from the Friedman rule can partially mitigate the externality. Specifically, a modest inflation tax makes customers reduce their money holdings, which—through general-equilibrium effects—discourages firms from issuing distortionary strategic debt. Calibrating the model to U.S. data, we find that the optimal deviation from the Friedman rule corresponds to policy interest rates of up to 0.51%.

While the empirical literature stresses that debt strengthens a firm’s bargaining position, we conclude by asking whether alternative contracts might achieve this objective more effectively. We show that a firm can deviate from any debt contract to a superior contract that increases the firm’s probability of success in bargaining, enhancing extensive-margin efficiency. Similarly, for any equity contract, a firm can deviate to a superior contract that increases the bargained quantities and payments, improving intensive-margin efficiency. Consequently, firms balance the extensive-margin costs of debt against the intensive-margin costs of equity when choosing the financial contract that optimally leverages their bargaining power.

**Model and results.** Our model is based on the New-Monetarist framework with alternating decentralized and centralized trade and search frictions, developed by [Lagos and Wright \(2005\)](#) and [Rocheteau and Wright \(2005\)](#). Producers and consumers meet bilaterally in a decentralized market, in which consumers need a means of payment to purchase goods from producers. The government meets the consumers’ liquidity demand by issuing fiat money in a centralized market. Consumers are ex-ante identical when they choose their money holdings, but incur idiosyncratic preference shocks once they have entered the decentralized market. In bilateral matches, agents bargain over the terms of trade à la [Kalai \(1977\)](#). Our model is innovative in that it endows producers with the ability to leverage their bargaining power through commitment in financial contracts; producers can borrow from financiers in the current centralized market and they repay in the subsequent centralized market. [Figure 2](#) illustrates the sequence of events.

A financial contract specifies an ex-ante transfer from a financier to a borrower and an ex-post transfer to the financier contingent on the borrower’s revenues from decentralized trade. Financial contracts are backed by the borrowers’ revenues from the decentralized market, so that only producers can write financial contracts with the financiers; consumers cannot. A financial contract is written subject to: (i) limited liability of the producer; (ii) the expected return requirements of the financiers who are risk-neutral and perfectly competitive; and (iii) the anticipated ex-post bargaining solutions. We assume that

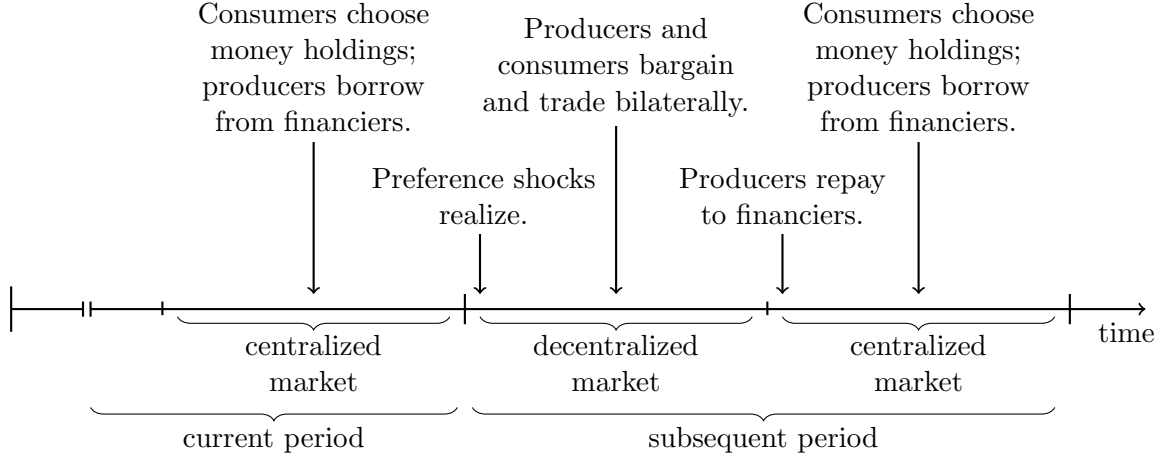


Figure 2: Timeline.

*Note:* The figure shows an extract from the timeline.

financial contracts are illiquid in that financiers cannot intermediate them as money.

Given the empirical evidence for the use of debt in bargaining, we first restrict the space of financial contracts to debt contracts. Debt affects a producer through one beneficial and two detrimental channels. The beneficial distributive channel is analogous to the cake example in Figure 1: the producer borrows in advance against her expected bargaining income but ultimately bears only part of the debt burden, shifting the remainder onto her bargaining partner by demanding more favorable terms of trade. On the detrimental side, debt tightens the liquidity constraint in every successful match. Since a certain share of the consumer's money holdings is used to compensate the producer for the fixed cost of debt repayment, there is less money left to pay the producer for actual production, so that the traded quantity decreases—an intensive-margin effect. Moreover, the producer loses successful matches with consumers who incur low preference shocks since these consumers value the producer's goods so little that the match surplus net of debt repayment would be negative for all production levels. The producer thus refrains from producing altogether—an extensive-margin effect—and defaults on her debt.

Consistent with empirical evidence, our model predicts that producers tend to issue more debt when they have less bargaining power (Dasgupta and Nanda, 1993; Hennessy and Livdan, 2009; Kale and Shahrur, 2007). In the extreme case where consumers make take-it-or-leave-it offers to producers, producers issue much debt since debt is their only means of appropriating match surplus. Conversely, if producers make take-it-or-leave-it offers, they issue no debt at all. In this case, debt offers no strategic advantage, only potential downsides, since producers command of maximum bargaining power already.

The distortionary effects of debt on both the intensive and extensive margins of decentralized trade stem from a pecuniary externality: financiers absorb the producers' debt at interest rates that fail to reflect the negative effect of debt on consumers. The

fiscal authority can perfectly resolve this externality with a Pigouvian debt tax. In the absence of such taxation, we adopt the Friedman rule as our benchmark monetary policy. Through slight deflation, the Friedman rule ensures that money earns a real return equal to the rate of time preference. Hence, consumers hold money balances that render the liquidity constraints slack in all successful matches. A deviation from the Friedman rule reduces the real return on money, prompting consumers to hold less of it. Consequently, liquidity constraints in matches with high preference shocks tighten, leading producers to reduce their debt issuance to relax these constraints. Crucially, the net surplus in the match that was marginally successful under the Friedman rule is now positive, so that more matches succeed—a positive extensive-margin effect of first-order importance for welfare. Of course, tighter liquidity constraints also have a negative intensive-margin effect. However, this effect is only of second-order importance for welfare since trade is intensively efficient under the Friedman rule if bargaining succeeds—a standard result of [Kalai \(1977\)](#) bargaining. Overall, welfare thus increases. While consumers bear the burden of the externality caused by debt issuance, they also directly bear the costs of the corrective monetary policy, as inflation implicitly taxes their money holdings. Nonetheless, they ultimately benefit from the policy’s general-equilibrium effects, reaping its full welfare gains.

The positive extensive-margin effect of a deviation from the Friedman rule is similar in outcome to the *hot-potato effect* of inflation: “When [...] depreciation [of money] is anticipated, there is a tendency among owners of money to spend it speedily” ([Fisher, 1920](#), p. 146). However, the underlying mechanism in our model differs. We account for the fact that producers also anticipate inflation, as consumers want to get rid of money faster in response to inflation. As a result, producers become less willing to accept money in trade, neutralizing Fisher’s reasoning. Instead, the driving force behind the positive extensive-margin effect in our model is that inflation discourages debt issuance through general-equilibrium effects.

To quantify optimal monetary policy, we calibrate our model to U.S. data. We use the method of simulated moments to let our model match empirical moments of money demand and firm-level markups from 1980 to 2015. We find optimal nominal interest rates ranging from 0.03% to 0.51%, depending on the heterogeneity among consumers.

The final part of our analysis asks whether debt is the optimal contract design when commitment in financial contracts is used to leverage bargaining power? We find that it is not. Specifically, we demonstrate that for any debt contract, there is a superior contract that yields a higher payoff for the producer by allowing for more successful bargaining agreements with low-preference consumers. We also examine outside equity and show that it, too, is suboptimal. In equity contracts, producers’ repayment obligations are proportional to their bargaining revenues, which results in extensive-margin efficiency by ensuring that repayment is never prohibitively high. However, equity involves intensive-



margin costs. When a producer bound by an equity contract bargains with a consumer, both parties internalize that a portion of the marginal payment goes to the financier rather than the producer. This discourages them from exchanging large payments and quantities. While we characterize the qualitative features of the contract firms optimally choose, determining whether this contract admits a closed-form solution remains an open question and lies beyond the scope of this paper.

**Outline.** The remainder of this paper is organized as follows. We embed our contributions in the literature in Section 2 and develop the model in Section 3. In Section 4, we describe agents’ optimal choices, define our equilibrium concept, and introduce welfare. In Section 5, we discuss the long-run transmission of monetary policy to debt issuance, and we calibrate the model in Section 6. We discuss optimal contract design in Section 7 and conclude in Section 8. Proofs are in Appendix A.

## 2 Literature

**Corporate finance.** We contribute to a growing literature on the impact of debt on firms’ relationships with *non-financial* stakeholders (Titman, 1984) by disentangling its effects on both the likelihood of success in bargaining and the size of bargaining surpluses in dynamic general equilibrium. The role of debt in firm-worker relationships has caught particular attention (Dasgupta and Sengupta, 1993; Matsa, 2010; Perotti and Spier, 1993). Bronars and Deere (1991) for instance show how debt protects firms’ surplus from extraction by workers’ unions: by issuing debt, a firm diverts future cash flows into current cash flows, reducing the future surplus that a union can extract without driving the firm into bankruptcy.

Dasgupta and Nanda (1993), Hennessy and Livdan (2009), and Kale and Shahrur (2007) examine supplier-customer relationships and find a main feature of our model: firms with less bargaining power tend to issue more debt. Dasgupta and Nanda (1993) develop a model in which producers, by issuing debt, skim surplus from consumers in a regulated product market. The authors portray the regulator as a bargaining protocol: the regulator distributes surplus between producers and consumers through price-setting. They find that firms issue more debt when regulated by commissions that favor consumers, and they confirm their theoretical results with empirical evidence in U.S. electricity markets where public utility commissions set prices. Hennessy and Livdan (2009) analyze strategic debt issuance in bargaining between upstream and downstream firms. The optimal level of debt balances efficiency losses—arising from distorted ex-post incentives—against distributive gains, which are particularly large for firms with little bargaining power. Kale and Shahrur (2007) investigate bargaining along the supply chain across industries. They find a positive relationship between firms’ debt levels and

the degree of concentration in the industries in which their suppliers and customers operate. Interpreting industry concentration as a proxy for bargaining power, their findings fully align with the predictions of our model.

**New Monetarism.** We uncover a new reason for the sub-optimality of the Friedman rule in a New-Monetarist framework based on the fact that producers issue excessive debt if consumers carry abundant money holdings. The New-Monetarist framework of [Lagos and Wright \(2005\)](#) and [Lagos and Rocheteau \(2005\)](#) is a natural choice for our study since it embeds bilateral trade with bargaining in a general-equilibrium framework. Our finding that the Friedman rule is not optimal particularly relates to the New-Monetarist literature on the hot-potato effect. [Lagos and Rocheteau \(2005\)](#) find that the effect of inflation on matching frequencies hinges on the pricing protocol. With bargaining, inflation acts demonetizing, agents search less, and monetary exchange decelerates. With directed and competitive search, inflation however accelerates matching.<sup>1</sup> [Liu, Wang and Wright \(2011\)](#) generate the hot-potato effect through the extensive margin rather than the intensive margin. In response to inflation, some consumers leave the search market, so that the market tightness changes in favor of the remaining consumers whose matching probabilities consequently increase. While a deviation from the Friedman rule distorts welfare in the model of [Lagos and Rocheteau \(2005\)](#), it improves welfare in [Liu et al. \(2011\)](#)—at least when quantities exchanged in bilateral trade are close to efficient—through a first-order extensive-margin effect, akin to the effect in our model.

[Dong and Jiang \(2014\)](#) study a model with price posting and undirected search in which consumers incur idiosyncratic preference shocks as in our model.<sup>2</sup> However, these shocks are private information, so that producers post incentive-compatible price-quantity schedules that favor high-preference consumers and render trade with low-preference consumers impossible. Inflation makes consumers reduce their money holdings, leading producers to attenuate their price-discrimination scheme. In spirit very much related to our model, the effective decrease in producers’ market power in response to inflation increases the mass of matches in which trade occurs. The welfare effect yet remains ambiguous.

**Contract theory.** The non-optimality of debt and equity for firms when leveraging their bargaining power contributes to the literature on firms’ ownership structure. [Jensen](#)

<sup>1</sup>[Althanns, van Buggenum and Gersbach \(2024\)](#) generalize the competitive-search model of [Lagos and Rocheteau \(2005\)](#) by endogenizing not only consumers’ search but also producers’ search. They find that the effects of inflation on consumers’ and producers’ search cancel out if consumers’ and producers’ search elasticities are symmetric. [Ennis \(2009\)](#) depicts the hot-potato effect in [Lagos and Rocheteau \(2005\)](#) as an artefact that cannot capture [Fisher’s \(1920\)](#) rationale. Instead, he adopts a bargaining model where producers are better than consumers in rebalancing their money holdings, so that producers are also better able to protect themselves against inflation. When inflation increases, consumers frontload consumption and search more intensely to shift the inflation-tax burden to producers.

<sup>2</sup>[Ennis \(2008\)](#), [Nosal \(2011\)](#), and [Peterson and Shi \(2004\)](#) also study the hot-potato effect in search models with match-specific preference shocks.

---

and Meckling (1976) are among the first to relate the decomposition of firms' outside finance to agency costs. Specifically, they develop two principal-agent models in which entrepreneurs rely on equity and debt, respectively. Equity involves moral hazard regarding entrepreneurs' effort choices, while debt is affected by moral hazard concerning their risk choices. Innes (1990) further develops the idea that entrepreneurs' project outcomes depend on their unobservable effort, and he shows that debt is optimal among all standard contracts.<sup>3</sup> Since debt involves low obligations in high-revenue states, it incentivizes entrepreneurs to exert high effort ex post, thereby reaping large surpluses.

Hellwig (2009) integrates the two dimensions of moral hazard in Jensen and Meckling (1976) into a single model and shows that a piecewise linear mix of debt and equity is the optimal ownership structure under certain regularity assumptions on outcome distributions. Our model features these two dimensions of moral hazard as well: equity is affected by moral hazard regarding the bargained quantity, akin to entrepreneurs' effort choice; and debt is affected by moral hazard concerning the probability of success in bargaining, akin to the entrepreneurs' risk choice. The most important difference between those models following Jensen and Meckling (1976) and ours is that in those models the acquisition of outside finance is necessary for entrepreneurs to engage in profitable investment projects, whereas our producers have deep pockets and write financial contracts for the sole purpose of extracting bargaining surplus.<sup>4</sup>

### 3 Model

Time  $t \in \{0, 1, \dots\}$  is discrete and goes on forever. Each period is divided into two consecutive subperiods: a decentralized market  $DM_t$  and a centralized market  $CM_t$  (see Figure 2). The economy starts in  $CM_0$ . There are two types of perfectly divisible and non-storable goods: *DM goods* and *CM goods* (treated as the numéraire). The economy is populated by a unit mass of infinitely-lived agents called *consumers*. In each  $CM_t$ , a unit mass of agents called *producers* is born, and these producers die at the end of  $CM_{t+1}$ .<sup>5</sup> Moreover, there is a unit mass of infinitely-lived *financiers* born in  $CM_0$ . All agents have the same time-discount factor  $\beta \in (0, 1)$ .

In  $DM_t$ , producers can produce DM goods but cannot consume them, whereas con-

---

<sup>3</sup>Strictly speaking, this result only holds when considering the class of contracts for which the financier's payoff function is constrained to be non-decreasing in firm revenues.

<sup>4</sup>Our model also contrasts with the costly-state-verification literature initiated by Townsend (1979). Gale and Hellwig (1985) and Williamson (1987) find in models with asymmetric information that debt is optimal. Similarly, Diamond (1984) and Wang and Williamson (1998) prove the optimality of debt in models with ex-post monitoring decisions and adverse selection, respectively. The conceptually most important difference between those models and ours is that in those models the borrowers' capital structure itself does not impact their revenue processes.

<sup>5</sup>In general equilibrium, producers are borrowers. Their finite lifespan allows us to abstract from reputation effects and debt-overhang problems.

sumers wish to consume DM goods but cannot produce them. Producers and consumers trade DM goods in a decentralized market, in which a unit mass of bilateral matches between consumers and producers is randomly arranged. In a match, a consumer and a producer determine the terms of trade through proportional [Kalai \(1977\)](#) bargaining. In  $\text{CM}_t$ , all agents can produce and consume CM goods, and they trade them in a centralized Walrasian market.

**Preferences.** The periodic utility of a consumer is

$$U_t^c = \epsilon_t u(q_t) + x_t, \quad (1)$$

where  $q_t \geq 0$  is DM-goods consumption and  $x_t \in \mathbb{R}$  is CM-goods net consumption. If  $x_t < 0$ , the consumer produces CM-goods. The function  $u$  is twice continuously differentiable on  $(0, \infty)$  and satisfies  $u' > 0$ ,  $u'' < 0$ ,  $u(0) = 0$ ,  $\lim_{q \rightarrow 0} u'(q) = \infty$ , and  $\lim_{q \rightarrow \infty} u'(q) = 0$ . The consumer incurs preference shock  $\epsilon_t \stackrel{\text{i.i.d.}}{\sim} G$  at the beginning of  $\text{DM}_t$ .  $G$  is the cumulative distribution function of a continuous probability law with compact support  $[0, \bar{\epsilon}] \subset [0, \infty)$ , and we write  $g = G'$  for the corresponding probability density function. A consumer's lifetime utility is  $\sum_{t=0}^{\infty} \beta^t U_t^c$ .

The lifetime utility of a producer born in  $\text{CM}_t$  is

$$U_t^p = x_t + \beta[-c(q_{t+1}) + x_{t+1}], \quad (2)$$

where  $q_{t+1} \geq 0$  is DM-goods production and  $x_t, x_{t+1} \in \mathbb{R}$  is CM-goods net consumption. The function  $c$  is twice continuously differentiable on  $[0, \infty)$  and satisfies  $c'(q) > 0$  for  $q > 0$ ,  $c'' \geq 0$ , and  $c(0) = 0$ .

The periodic utility of a financier is  $U_t^{fi} = x_t$ , where  $x_t \in \mathbb{R}$  is CM-goods net consumption. Its lifetime utility is  $\sum_{t=0}^{\infty} \beta^t U_t^{fi}$ .

**Money and financial contracts.** The consumers' anonymity in the DM necessitates a payment instrument. To this end, the government issues *fiat money*: a perfectly divisible, intrinsically worthless, and storable asset. Money supply at the beginning of period  $t$  is  $M_t$ , and  $M_t$  grows at gross rate  $\gamma$ :  $M_{t+1} = \gamma M_t$ . New money is injected (or withdrawn if  $\gamma < 1$ ) through lump-sum transfers (resp. taxes)  $\tau_t = \phi_t(M_{t+1} - M_t)$  to consumers in  $\text{CM}_t$ , where  $\phi_t$  is the  $\text{CM}_t$ -price of money.<sup>6</sup>

Financiers are perfectly competitive, so we consider a representative financier. We define a one-period *financial contract* between the financier and a borrower as a repayment schedule  $r_{t+1} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  that specifies a transfer  $r_{t+1}(p_{t+1})$  in  $\text{CM}_{t+1}$  from the borrower to the financier, contingent on the borrower's real balances  $p_{t+1}$  at the beginning of  $\text{CM}_{t+1}$ .

---

<sup>6</sup>One unit of money thus costs  $\phi_t$  units of CM goods. Hence,  $1/\phi_t$  is the nominal price of CM goods.

We argue when turning to the equilibrium that the producers borrow from the financier to improve their bargaining position, so that  $p_{t+1}$  is their bargaining revenues. The financier can perfectly observe  $p_{t+1}$  and enforce repayment  $r_{t+1}(p_{t+1}) \leq p_{t+1}$ , where the inequality entails limited liability. We define the set of limited-liability financial contracts as<sup>7</sup>

$$F \equiv \left\{ r \in \mathbb{R}_+^{\mathbb{R}_+} : r(p) \leq p \text{ for all } p \in \mathbb{R}_+ \right\}. \quad (3)$$

A competitive pricing kernel  $T_t : F \rightarrow \mathbb{R}_+$  determines the transfer  $T_t(r_{t+1})$  in  $\text{CM}_t$  that the borrower obtains from the financier in exchange for committing to  $r_{t+1}$ . The pricing kernel  $T_t$  is an equilibrium object that internalizes the financier's return requirements and the borrower's anticipated bargaining revenues in  $\text{DM}_{t+1}$ , which the borrower carries into  $\text{CM}_{t+1}$ . The financier does not intermediate its claims arising from financial contracts.

## 4 Equilibrium and Welfare

For convenience, we suppress the time subscript  $t$  and shorten the subscript  $t + 1$  to  $+1$  where no confusion arises.

**Value functions.** Let  $V^c(m'|\epsilon)$  be a consumer's value of entering the DM with real balances  $m'$  and incurring preference shock  $\epsilon$ . Consumers do not have any future income they could borrow against and thus cannot commit to financial contracts. Let  $W^c(\hat{m})$  denote the consumer's value of entering the CM with real balances  $\hat{m}$ , so that

$$\begin{aligned} W^c(\hat{m}) = \max_{m'_{+1} \geq 0, x \in \mathbb{R}} & \left\{ x + \beta \int_0^{\bar{\epsilon}} V_{+1}^c(m'_{+1}|\epsilon_{+1}) G(d\epsilon_{+1}) \right\}, \\ \text{s.t. } & x = \hat{m} - \frac{\phi m'_{+1}}{\phi_{+1}} + \tau. \end{aligned} \quad (4)$$

The consumer chooses next-period real balances  $m'_{+1}$  and current CM-goods consumption  $x$  to maximize the sum of  $x$  and the time-discounted expected value from entering  $\text{DM}_{+1}$ . The constraint in problem (4) captures that  $x$  equals the gains from adjusting money holdings, plus the government transfer  $\tau$ . Notably,  $W^c$  is affine in  $\hat{m}$ , so that all consumers choose the same money holdings  $m'_{+1} = m_{+1}$ . We specify  $V^c$  further below.

Let  $V^p(r'|\epsilon)$  be a producer's value of entering the DM with financial contract  $r'$  and being matched with a consumer incurring preference shock  $\epsilon$ <sup>8</sup>. Producers do not carry money across periods, as doing so would weaken their bargaining position in decentralized

<sup>7</sup>We write  $\mathbb{R}_+^{\mathbb{R}_+}$  for the space of functions mapping from  $\mathbb{R}_+$  into  $\mathbb{R}_+$ .

<sup>8</sup>Since all consumers enter the DM with the same money holdings  $m$ , consumers are only heterogeneous with respect to their preference shocks on the equilibrium path.

trade. The value of a producer being born in the CM is

$$W^{p,0} = \max_{r'_{+1} \in F} \left\{ T(r'_{+1}) + \beta \int_0^{\bar{\epsilon}} V_{+1}^p(r'_{+1}|\epsilon_{+1}) G(d\epsilon_{+1}) \right\}. \quad (5)$$

The producer chooses financial contract  $r'_{+1}$  to maximize the sum of the transfer received from the financier and the time-discounted expected value from entering DM<sub>+1</sub>. Since all newborn producers face the same optimization problem and since we focus on symmetric equilibria, all producers choose the same financial contract  $r'_{+1} = r_{+1}$  on the equilibrium path. We specify  $V^p$  further below.

The value of entering CM<sub>+1</sub> with revenues  $p_{+1}$  from DM<sub>+1</sub> and financial contract  $r_{+1} \in F$  is

$$W_{+1}^{p,1}(p_{+1}, r_{+1}) = p_{+1} - r_{+1}(p_{+1}), \quad (6)$$

where limited liability ensures  $W_{+1}^{p,1}(p_{+1}, r_{+1}) \geq 0$ .

We assume, without loss of generality, that the representative financier fully diversifies its portfolio of financial contracts. Consequently, the financier's value of entering the CM with a portfolio that has expected payoff  $a$  is given by

$$W^{fi}(a) = a + \max_{a'_{+1} \geq 0} \left\{ -\frac{a'_{+1}}{R^f} + \beta W_{+1}^{fi}(a'_{+1}) \right\}. \quad (7)$$

The financier writes new financial contracts with an expected payoff  $a'_{+1}$  to maximize the difference between the time-discounted expected value from entering CM<sub>+1</sub> net of the cost of writing new financial contracts. Notably, financiers remain passive in the DM.

**Bargaining.** Once a consumer with real balances  $m$  and preference shock  $\epsilon$  meets a producer with financial contract  $r$  in the DM, they negotiate the terms of trade  $(q, p) \in \mathbb{R}_+^2$ . In this trade, the producer transfers quantity  $q$  of DM-goods to the consumer and receives payment  $p$  in return. The terms of trade  $(q, p)$  are determined through proportional bargaining à la [Kalai \(1977\)](#), so that

$$(q, p) = \arg \max_{q, p \geq 0} \{ \epsilon u(q) + W^c(m - p) - W^c(m) \}, \quad (8)$$

$$\text{s.t. } p \leq m \quad \text{and}$$

$$\theta [\epsilon u(q) + W^c(m - p) - W^c(m)] = (1 - \theta) [-c(q) + W^{p,1}(p, r) - W^{p,1}(0, r)],$$

where  $\theta \in [0, 1]$  denotes the producer's bargaining power, and where  $p \leq m$  is the liquidity constraint. Substituting  $W^c$  and  $W^{p,1}$ , the bargaining problem simplifies as

$$(q, p) = \arg \max_{q, p \geq 0} \{ \epsilon u(q) - p \}, \quad (9)$$

$$\text{s.t. } p \leq m \quad \text{and} \quad \theta [\epsilon u(q) - p] = (1 - \theta) [-c(q) + p - r(p)].$$

Since  $(\epsilon, m, r)$  fully characterizes  $(q, p)$ , we identify a match with  $(\epsilon, m, r)$ . We denote the negotiated terms of trade as  $q(\epsilon, m, r)$  and  $p(\epsilon, m, r)$ . A match  $(\epsilon, m, r)$  is considered *successful* if  $q(\epsilon, m, r) > 0$ , meaning that a bargaining agreement has been reached.

We define the *net surplus* of a match as  $\epsilon u(q) - c(q) - r(p)$ , and we define the consumer's and the producer's net surpluses,  $v^c(\epsilon, m, r)$  and  $v^p(\epsilon, m, r)$ , as their respective portions of the net surplus. We obtain

$$v^c(\epsilon, m, r) = (1 - \theta)[\epsilon u(q) - c(q) - r(p)] \quad \text{and} \quad v^p(\epsilon, m, r) = \theta[\epsilon u(q) - c(q) - r(p)], \quad (10)$$

where  $q = q(\epsilon, m, r)$  and  $p = p(\epsilon, m, r)$ . The producer's commitment to  $r$  translates into an additional cost for the producer to engage in trade, which reduces the net surplus.

We define the *gross surplus* of a match as  $\epsilon u(q) - c(q)$ . The consumer's and the producer's gross surpluses are defined as  $\epsilon u(q) - p$  and  $-c(q) + p$ . While the consumer's gross surplus coincides with her net surplus, the producer's gross surplus is given by

$$-c(q) + p = \theta[\epsilon u(q) - c(q)] + (1 - \theta)r(p). \quad (11)$$

We will demonstrate further below that the agents' gross surpluses in all successful matches determine their lifetime utilities.

Given the bargaining solutions, the DM value functions for the consumers and producers read as  $V^c(m'|\epsilon) = v^c(\epsilon, m', r) + W^c(m')$  and  $V^p(r'|\epsilon) = v^p(\epsilon, m, r')$ .

**Pricing kernel.** Since financiers are risk neutral, they must earn the risk-free rate  $R^f$  in expectation with any financial contract between CM and CM<sub>+1</sub>. The pricing kernel  $T$  thus satisfies

$$T(r'_{+1}) = \frac{1}{R^f} \int_0^{\bar{\epsilon}} r'_{+1}(p(\epsilon_{+1}, m_{+1}, r'_{+1})) G(d\epsilon_{+1}), \quad \forall r'_{+1} \in F, \quad (12)$$

where  $m_{+1}$  denotes consumers' equilibrium money holdings. Equation (12) captures a no-arbitrage condition: the transfer  $T(r'_{+1})$  a borrower receives from the financier for committing to  $r'_{+1} \in F$  is equal to the expected repayment, discounted by  $R^f$ .

**Equilibrium.** We define our equilibrium concept in

**Definition 1.** *Given the process of money supply  $\{M_t\}_{t=0}^\infty$ , a symmetric monetary equilibrium is a process of real money holdings  $\{m_t\}_{t=1}^\infty$ , financial contracts  $\{r_t\}_{t=1}^\infty$ , the financiers' expected payoffs  $\{a_t\}_{t=1}^\infty$ , money prices  $\{\phi_t\}_{t=0}^\infty$ , pricing kernels  $\{T_t\}_{t=0}^\infty$ , risk-free rates  $\{R_t^f\}_{t=0}^\infty$ , and government transfers  $\{\tau_t\}_{t=0}^\infty$ , so that:*

- (i)  $m_{t+1}$  solves the consumers' optimization problem (4);



- (ii)  $r_{t+1}$  solves the producers' optimization problem (5);
- (iii)  $a_{t+1}$  solves the financiers' optimization problem (7);
- (iv)  $T_t$  solves Equation (12) for  $m_{t+1}$ ;
- (v) the financial market clears:  $T_t(r_{t+1}) = a_{t+1}/R_t^f$ ;
- (vi) the money market clears:  $\phi_t M_t = m_t$ ;
- (vii) the government's budget is balanced:  $\tau_t = \phi_t(M_{t+1} - M_t)$ ;
- (viii) the equilibrium is monetary:  $m_t > 0$ .

The linearity of the financiers' optimization problem (7) when choosing their next-period portfolio of financial contracts with expected payoff  $a_{+1}$  implies that

$$R^f \leq 1/\beta \quad \text{with "=" if } a_{+1} > 0. \quad (13)$$

If  $R^f > 1/\beta$ , financiers would write contracts with  $a_{+1} = \infty$ ; if  $R^f < 1/\beta$ , they would not write any contracts at all,  $a_{+1} = 0$ . Since financiers and producers have the same time-discount factor  $\beta$ , the expression for pricing kernel  $T$  in Equation (12) implies that the process of writing a financial contract  $r'_{+1} \in F$  *per se* is utility neutral: a producer's net utility of obtaining transfer  $T(r'_{+1})$  in the CM net of the expected repayment associated with  $r'_{+1}$  in  $\text{CM}_{+1}$  is zero. However,  $r'_{+1}$  affects the producer's utility rather indirectly through bargaining outcomes in  $\text{DM}_{+1}$ .

**Welfare.** Utilitarian welfare  $\mathcal{W} \equiv W_0^{fi} + W_0^c + \sum_{t=0}^{\infty} \beta^t W_t^{p,0}$  is the aggregate utility of financiers, consumers, and producers.

**Lemma 1.** *Welfare reads as*

$$\mathcal{W} = \sum_{t=1}^{\infty} \beta^t \int_0^{\bar{\epsilon}} [\epsilon_t u(q) - c(q)]_{q=q(\epsilon_t, m_t, r_t)} G(d\epsilon_t), \quad (14)$$

given equilibrium real balances  $\{m_t\}_{t=1}^{\infty}$  and financial contracts  $\{r_t\}_{t=1}^{\infty}$ .

Lemma 1 demonstrates that overall welfare is determined by the aggregate of gross surpluses in each period. Due to the quasi-linearity of preferences and uniform discounting, consumption and trade in the CM do not affect welfare. Welfare is maximized if two conditions hold: (i) every match is successful, ensuring *extensive-margin* efficiency; and (ii) in each match with a preference shock  $\epsilon$ , agents trade  $q_\epsilon^*$ , where  $\epsilon u'(q_\epsilon^*) = c'(q_\epsilon^*)$ , ensuring *intensive-margin* efficiency.



## 5 Debt

The producers' problem (5) is an optimization problem over the space of financial contracts  $F$ . Since the empirical literature documents the strategic use of *limited-liability debt* as a specific contract design, we restrict producers to debt contracts

$$D \equiv \left\{ r \in \mathbb{R}_+^{\mathbb{R}_+} : \exists b \geq 0, \text{ so that } r(p) = \min\{p, b\} \text{ for all } p \in \mathbb{R}_+ \right\} \subset F \quad (15)$$

instead of  $F$ . In Section 7, we relax this assumption again.

We identify each  $r \in D$  with the corresponding face value of debt  $b \geq 0$ . In a match  $(\epsilon, m, b)$ , the terms of trade solve

$$\begin{aligned} (q, p) &= \arg \max_{q, p \geq 0} \{ \epsilon u(q) - p \}, \\ \text{s.t.} \quad p &\leq m \quad \text{and} \quad \theta[\epsilon u(q) - p] = (1 - \theta)[-c(q) + \max\{p - b, 0\}]. \end{aligned} \quad (16)$$

It is crucial that debt is subject to limited rather than full liability for it to influence bargaining. Under full liability, where the producer's repayment would not depend on the bargaining outcome, the producer's net surplus would be  $-c(q) + p$ , instead of  $-c(q) + \max\{p - b, 0\}$ , since her threat point in bargaining would be  $-b$  rather than zero. Hence, for debt to affect bargaining, some level of commitment must be available, but excessive commitment is counterproductive.

### 5.1 Extensive margin

A successful match requires

$$p - c(q) - b \geq 0 \quad \text{and} \quad p = \theta \epsilon u(q) + (1 - \theta)[c(q) + b]. \quad (17)$$

In particular, either the match is successful, with the producer fully repaying her debt in the subsequent CM, or the match is unsuccessful, leading to the producer fully defaulting on her debt. Partial default does not occur, as it would result in a producer payoff of  $-c(q)$ , which is worse than the producer's threat point of simply refraining from any agreement. Thus, full repayment or complete default are the only possible outcomes.

Debt makes it more difficult for a producer to obtain a successful match in the DM. This is because the consumer she is matched with may not be willing to compensate her for both the cost of producing DM goods  $c(q)$  and the face value of debt  $b$ . We define

$$\tilde{q}(\epsilon, b) \equiv \inf\{q \in [0, \infty) : \epsilon u(q) - c(q) - b \geq 0\} \quad (18)$$

as the smallest DM-goods quantity that allows for a non-negative net surplus in a match

with preference shock  $\epsilon$  and the face value of debt  $b$ .<sup>9</sup>

**Lemma 2.** *A match  $(\epsilon, m, b)$  with  $m > 0$  is successful if and only if the following conditions are satisfied:*

(i) *Net-surplus condition:*  $\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - b \geq 0$ .

(ii) *Liquidity condition:*  $m \geq c(\tilde{q}(\epsilon, b)) + b$ .

Condition (i) requires that the net surplus at the intensively-efficient level of production is non-negative, imposing an upper bound on  $b$ . Condition (ii) requires that the consumer's real balances  $m$  cover the payment associated with trading  $\tilde{q}(\epsilon, b)$ , imposing a lower bound on  $m$ . If either condition is violated, the producer refrains from producing altogether and defaults on her debt. Both constraints relax as the consumer's preference shock  $\epsilon$  increases, leading to

**Corollary 1.** *A match  $(\epsilon, m, b)$  is successful if*<sup>10</sup>

$$\epsilon \geq \inf\{\epsilon \in [0, \bar{\epsilon}] : q(\epsilon, m, b) > 0\} \equiv \hat{\epsilon}(m, b). \quad (19)$$

The continuity of the bargaining problem ensures that  $\hat{\epsilon}(m, b)$  is the smallest preference shock for which a match is successful, given  $m$  and  $b$ . To streamline notation, we write  $\hat{\epsilon} = \hat{\epsilon}(m, b)$  and  $\hat{q} = q(\hat{\epsilon}(m, b), m, b)$  when no confusion arises. We refer to  $(\hat{\epsilon}, m, b)$  as the *marginally-successful* match.

**Lemma 3.** *It holds that  $\hat{q} = \tilde{q}(\hat{\epsilon}, b)$ .*

Lemma 3 states that the net surplus of the marginally-successful match  $(\hat{\epsilon}, m, b)$  is zero. Next, Lemma 4 characterizes the extensive-margin effect of debt issuance.

**Lemma 4.** *It holds that*

$$\frac{\partial \hat{\epsilon}}{\partial b} = \frac{1}{u(\hat{q})} \left[ 1 + \frac{\hat{\epsilon} u'(\hat{q}) - c'(\hat{q})}{c'(\hat{q})} \right] > 0. \quad (20)$$

When  $b$  increases, the conditions for a match to be successful, as outlined in Lemma 2, become harder to meet, so that the preference-shock threshold  $\hat{\epsilon}$  increases.

<sup>9</sup>We write  $\tilde{q}(\epsilon, b) = \infty$  if  $\{q \in [0, \infty) : \epsilon u(q) - c(q) - b \geq 0\} = \emptyset$ .

<sup>10</sup>Following the convention, we write  $\hat{\epsilon}(m, b) = \infty$  if  $\{\epsilon \in [0, \bar{\epsilon}] : q(\epsilon, m, b) > 0\} = \emptyset$ .

## 5.2 Consumers' money holdings

Recall that the equilibrium distributions of money holdings  $m_{+1}$  and debt  $b_{+1}$  at the beginning of DM<sub>+1</sub> are degenerate. A consumer's DM value function is given by<sup>11</sup>

$$V^c(m|\epsilon) = m + v^c(\epsilon, m, b) + \tau + \max_{m'_{+1} \geq 0} \left\{ -\frac{\phi m'_{+1}}{\phi_{+1}} + \beta \int_0^{\bar{\epsilon}} V_{+1}^c(m'_{+1}|\epsilon_{+1}) G(d\epsilon_{+1}) \right\}. \quad (21)$$

The necessary first-order condition for the consumer's money demand  $m'_{+1}$  is

$$0 \geq -\iota_{+1} + \int_{\hat{\epsilon}(m'_{+1}, b_{+1})}^{\bar{\epsilon}} \mathcal{L}(\epsilon_{+1}, m'_{+1}, b_{+1}) G(d\epsilon_{+1}), \quad \text{with “=” if } m'_{+1} > 0, \quad (22)$$

where we define the *liquidity premium* in a match  $(\epsilon, m, b)$  as

$$\mathcal{L}(\epsilon, m, b) \equiv \left[ \frac{(1 - \theta)[\epsilon u'(q) - c'(q)]}{\theta \epsilon u'(q) + (1 - \theta)c'(q)} \right]_{q=q(\epsilon, m, b)} \quad (23)$$

and the *Fisher rate* as  $\iota_{+1} \equiv \phi/\beta\phi_{+1} - 1$ . The liquidity premium  $\mathcal{L}(\epsilon, m, b)$  is the Lagrange multiplier of the liquidity constraint in match  $(\epsilon, m, b)$ . The Fisher rate  $\iota_{+1}$  is the hypothetical nominal interest rate that compensates for inflation and time-discounting, capturing the opportunity cost of holding money.

## 5.3 Producers' debt issuance

A producer repays debt  $b'_{+1}$  if and only if her match  $(\epsilon_{+1}, m_{+1}, b'_{+1})$  is successful, given the equilibrium real balances  $m_{+1}$ . The equilibrium pricing kernel  $T$ , as characterized in Equation (12), thus satisfies

$$T(b'_{+1}) = \beta[1 - G(\hat{\epsilon}(m_{+1}, b'_{+1}))]b'_{+1}, \quad \forall b'_{+1} \geq 0. \quad (24)$$

For any  $b'_{+1} \geq 0$ ,  $T(b'_{+1})$  is equal to the discounted expected value of debt repayment. A producer's value of being born in the current CM, as expressed in problem (5), simplifies as

$$W^{p,0} = \max_{b'_{+1} \geq 0} \left\{ \beta \int_{\hat{\epsilon}(m_{+1}, b'_{+1})}^{\bar{\epsilon}} \left[ \theta[\epsilon_{+1} u(q) - c(q)]_{q=q(\epsilon_{+1}, m_{+1}, b'_{+1})} + (1 - \theta)b'_{+1} \right] G(d\epsilon_{+1}) \right\}. \quad (25)$$

Hence, the producer issues debt to maximize her expected gross surplus as defined in Equation (11). The tradeoff the producer thereby faces becomes clear in

---

<sup>11</sup>Recall that we denote a consumer's individual choice of real balances as  $m'_{+1}$  (as in problem (4)) and the equilibrium level as  $m_{+1}$ . Similarly, we use  $b'_{+1}$  and  $b_{+1}$  (or  $r'_{+1}$  and  $r_{+1}$ ).

**Lemma 5.** *The necessary first-order condition for the producer's optimal face value of debt  $b'_{+1} \geq 0$  reads as*

$$0 \geq (1 - \theta)[1 - G(\hat{\epsilon}(m_{+1}, b'_{+1}))] - \theta \int_{\hat{\epsilon}(m_{+1}, b'_{+1})}^{\bar{\epsilon}} \mathcal{L}(\epsilon_{+1}, m_{+1}, b'_{+1}) G(d\epsilon_{+1}) - b'_{+1} g(\hat{\epsilon}(m_{+1}, b'_{+1})) \frac{\partial \hat{\epsilon}(m_{+1}, b'_{+1})}{\partial b}, \quad (26)$$

with “=” if  $b'_{+1} > 0$ .

We disentangle three channels through which an increase in the face value of debt  $b'_{+1}$  affects the producer's lifetime utility. The first term on the right-hand side of Inequality (26) captures the distributive effect of debt. For each additional unit of debt issued, the producer is compensated in every successful match through an increase in the exchanged payment by  $1 - \theta$ , as can be seen in Equation (17). The producer thus dynamically appropriates match surplus since the transfer she obtains ex ante is backed by the full debt repayment, while she only partially bears the debt burden ex post. The larger the probability  $1 - G(\hat{\epsilon}(m_{+1}, b'_{+1}))$  of a successful match, and the greater the compensation  $1 - \theta$  the producer receives for the debt repayment, the stronger is this distributive effect.

The second term on the right-hand side of Inequality (26) captures the negative intensive-margin effect of debt. Since in every successful match, the consumer uses a share of her money balances to compensate the producer for debt repayment, there is less money left to cover the actual production costs. As a result, the liquidity constraint tightens. In that sense, debt issuance has a similar effect on the bargaining set as a reduction of real balances, which is why the expected liquidity premium matters.

The third term captures the negative extensive-margin effect of debt. As  $b'_{+1}$  increases, the preference-shock threshold  $\hat{\epsilon}(m_{+1}, b'_{+1})$  increases as well, as discussed in Lemma 4, so that the producer loses matches with consumers who incur low preference shocks. Inequality (26) thus highlights the tradeoff the producer faces: while issuing more debt leverages her bargaining power through a positive distributive effect, it also exacerbates the negative intensive- and extensive-margin effects. The examples below illustrate the importance of bargaining power  $\theta$  for this tradeoff.

**Example 1.** When consumers make take-it-or-leave-it offers ( $\theta = 0$ ), producers obtain zero net surplus in every successful match and thus do not account for how debt affects liquidity constraints. By issuing debt, producers however appropriate gross surplus ex ante, forcing the consumers to bear the entire debt burden ex post. Hence, the positive distributive effect is particularly pronounced, resulting in a strong incentive to issue debt.

**Example 2.** When producers make take-it-or-leave-it offers ( $\theta = 1$ ), debt does not directly affect the negotiated payment  $p$  since  $p = \epsilon u(q)$ .<sup>12</sup> Consequently, debt does not have any positive distributive effect but makes producers only worse off through its negative intensive- and extensive-margin effects. Therefore, producers issue zero debt, and the economy's outcome equals that of a standard money-search model where debt is absent and producers have full bargaining power.

The examples above show that producers issue more debt when consumers have much bargaining power. Commitment thus leverages the producers' inherent bargaining power  $\theta$ . Debt and bargaining power thereby act as substitutes: if producers have substantial bargaining power already, the additional benefit from commitment is less pronounced than when their bargaining power is low. The substitutability of debt and bargaining power aligns with the positive relation between firms' debt levels and the degree of concentration in their supplier and customer industries, as empirically documented by [Kale and Shahrur \(2007\)](#). It is also in line with the work by [Bronars and Deere \(1991\)](#) on management-labor union bargaining and by [Hennessy and Livdan \(2009\)](#) on contract efficiency in bargaining. We next explore the policy implications of this phenomenon.

## 5.4 Welfare and policy

Debt distorts welfare at the intensive margin of decentralized trade by tightening liquidity constraints, and at the extensive margin by reducing the mass of successful matches. The economics underlying these inefficiencies involve a pecuniary externality: financiers absorb the producers' debt at interest rates that do not capture the negative effect of debt on consumers. Hence, the provision of debt lowers the borrowing rate below the societal cost of debt, entailing a role for policy intervention. In our policy analysis, we focus on monetary and fiscal policy in the steady state. We view monetary policy as the Fisher rate  $\iota = \gamma/\beta - 1$  since controlling the Fisher rate is equivalent to steering inflation  $\phi/\phi_{+1} = \gamma$  through the money-growth rate  $\gamma$ . As fiscal policy, we consider a proportional tax  $\kappa \geq 0$  on the face value of producers' debt.<sup>13</sup>

**Fiscal policy.** The optimal fiscal debt tax is effectively a Pigouvian tax that addresses the externality involved in debt issuance. It fully eliminates debt issuance since we identified debt to be distortionary.

**Lemma 6.** *Given monetary policy  $\iota$ , the smallest Pigouvian debt tax eliminating debt issuance is  $\kappa = \beta[1 - \theta(1 + \iota)]$  if  $\lim_{q \rightarrow 0} u(q)u''(q)/u'(q)^2 < \infty$ .*<sup>14</sup>

<sup>12</sup>We note that if  $\theta = 1$ , a monetary equilibrium exists only if  $\iota_{+1} = 0$ .

<sup>13</sup>A producer committing to debt repayment  $b' > 0$  in  $\text{CM}_{+1}$  must pay a tax  $\kappa b'$  to the government in the CM.

<sup>14</sup>This regularity assumption on  $u$  is weak. For instance, a CRRA utility function  $u(q) = q^{1-\sigma}/(1-\sigma)$ ,

The smallest Pigouvian debt tax  $\kappa$  that eliminates debt issuance decreases in the producers' bargaining power; the weaker the producers' incentive to issue debt, the less fiercely debt need to be taxed. Moreover,  $\kappa$  is decreasing in  $\iota$ , suggesting that monetary policy can substitute the debt tax in curbing debt issuance, as we discuss next.

**Monetary policy.** We consider the *Friedman rule* (FR), defined as  $\iota = 0$ , as our benchmark monetary policy since it is optimal in plain vanilla money-search models. Subsequently, we assume the absence of debt taxation ( $\kappa = 0$ ) to isolate the effect of monetary policy on equilibrium variables. The equilibrium at the FR and the effects of a deviation from the FR are characterized in

**Proposition 1.** *Let  $\theta < 1$  and let the FR apply.*<sup>15</sup>

1. *Debt and money: Producers issue positive debt  $b > 0$  and consumers hold abundant real balances  $m \geq \theta \bar{e} u(q_\epsilon^*) + (1 - \theta)[c(q_\epsilon^*) + b]$ , where*

$$\left. \frac{db}{d\iota} \right|_{\iota=0} < 0 \quad \text{and} \quad \left. \frac{dm}{d\iota} \right|_{\iota=0} = -\infty. \quad (27)$$

2. *Extensive margin: Trade is inefficient at the extensive margin, meaning  $\hat{\epsilon} > 0$ , where*

$$\left. \frac{d\hat{\epsilon}}{d\iota} \right|_{\iota=0} < 0. \quad (28)$$

3. *Intensive margin: Trade is efficient at the intensive margin, meaning  $q(\epsilon, m, b) = q_\epsilon^*$  for all  $\epsilon \in [\hat{\epsilon}, \bar{\epsilon}]$ , where*

$$\left. \frac{dq(\epsilon, m, b)}{d\iota} \right|_{\iota=0} = 0, \quad \forall \epsilon \in [\hat{\epsilon}, \bar{\epsilon}]. \quad (29)$$

Since the FR removes the opportunity cost of holding money, consumers render the liquidity constraints slack in all successful matches by carrying abundant real balances into the DM. Hence, trade is intensively efficient in those matches—a standard feature of monetary models with proportional [Kalai \(1977\)](#) bargaining.<sup>16</sup> If debt were ruled out by Pigouvian taxation, the FR would thus induce the efficient outcome. In the absence of Pigouvian taxation, however, producers issue positive debt at the FR, rendering trade inefficient at the extensive margin.

---

with  $\sigma \in (0, 1)$ , satisfies it.

<sup>15</sup>Producers having full bargaining power ( $\theta = 1$ ) is not compatible with a deviation from the FR since any positive Fisher rate  $\iota > 0$  would render a monetary equilibrium infeasible.

<sup>16</sup>This result does not generalize when considering [Nash \(1950\)](#) instead. With [Nash \(1950\)](#) bargaining, the consumers' match surplus is non-monotonic in their real balances, so that even if holding money is not costly, they choose real balances that render the liquidity constraints tight ([Aruoba, Rocheteau and Waller \(2007\)](#)).

A deviation from the FR imposes a cost on holding money, prompting consumers to reduce their money holdings. This renders the liquidity constraints tight in matches where the preference shock  $\epsilon$  is close to  $\bar{\epsilon}$ , thereby reducing the traded DM-goods quantities—a negative intensive-margin effect. Producers respond by reducing their debt to relax these liquidity constraints. Importantly, this debt reduction also renders the net surplus in the match that was marginally successful under the FR positive. Hence,  $\hat{\epsilon}$  decreases and more matches become successful—a positive extensive-margin effect.

The latter effect is similar in outcome to the *hot-potato effect* of inflation, which captures the intuition that inflation makes consumers spend money faster. Yet, this intuition does not apply in our model since both producers and consumers anticipate inflation, so that consumers cannot simply pass the inflation tax on to producers by spending money faster. Instead, inflation makes producers issue less debt, which crowds in matches with low-preference consumers, thereby accelerating trade.

The welfare implications of a deviation from the FR are characterized in

**Proposition 2.** *Let  $\theta < 1$  and let the FR apply. It holds that*

$$\left. \frac{dW}{d\iota} \right|_{\iota=0} = - \left( \frac{\beta}{1-\beta} \right) bg(\hat{\epsilon}) \left. \frac{d\hat{\epsilon}}{d\iota} \right|_{\iota=0} > 0. \quad (30)$$

The positive extensive-margin effect of a deviation from the FR on decentralized trade is of first-order importance for welfare because it adds matches with positive gross surplus. On the other hand, the negative intensive-margin effect of a deviation from the FR is only of second-order importance for welfare because trade is intensively efficient under the FR. The overall welfare effect is thus positive. This result is a manifestation of the *theory of the second best*, which states that if one out of multiple efficiency conditions cannot be fulfilled, a second-best optimum is achieved only by departing from all other efficiency conditions (Lipsey and Lancaster, 1956). A Pigouvian debt tax would achieve efficiency at the extensive margin but not at the intensive margin (unless at the FR), while vice versa, the FR achieves efficiency at the intensive margin but not at the extensive margin (unless the optimal Pigouvian debt tax is levied).

**Corollary 2.** *Let  $\theta < 1$  and let the FR apply. It holds that*

$$\left. \frac{dW^{p,0}}{d\iota} \right|_{\iota=0} = 0, \quad \left. \frac{dW^{fi}}{d\iota} \right|_{\iota=0} = 0, \quad \text{and} \quad \left. \frac{dW^c}{d\iota} \right|_{\iota=0} = \left. \frac{dW}{d\iota} \right|_{\iota=0} > 0. \quad (31)$$

Although it is the consumers whose money holdings are taxed by inflation, it is also the consumers who benefit from this tax due to the general-equilibrium effects that curb producers' debt issuance. Monetary policy essentially compensates for a lack of coordination among consumers. If consumers could coordinate, they would collectively maintain a moderate level of money holdings, even under the FR, to keep liquidity constraints tight,

which would discourage producers from issuing debt. However, an individual consumer would deviate from such coordination by holding more money to increase trade in her own matches. A deviation from the FR mitigates this coordination problem through an opportunity cost that incentivizes consumers to individually reduce their money holdings.

Unsurprisingly, producers are not positively affected by a deviation from the FR, as there is less surplus to skim through debt issuance when consumers hold less money. Less obvious, they are not negatively affected with first-order importance. This stems from the fact that a slight deviation from the FR induces a tightening of the liquidity constraint only in the match with  $\epsilon = \bar{\epsilon}$ , which occurs with zero probability. Moreover, the reduction of  $b$ , although affecting all successful matches, is only of second-order importance for producers' utility due to the envelope theorem— $b$  is their optimal choice. Corollary 2 thus clarifies that a deviation from the FR induces an economy-wide Pareto improvement.

## 6 Calibration

We calibrate our model with debt to U.S. data to quantify the effects of long-run inflation on equilibrium outcomes, and to determine the welfare-maximizing fiscal and monetary policies. Specifically, we conduct three separate calibrations, each using a different preference-shock distribution  $G$ , to assess the robustness of our results. Apart from that, the procedure is identical across all three calibrations. The unit of time is one year, and we use quarterly data from January 1980 to December 2015. We infer some primitives directly from the data and impose functional forms, while we calibrate the remaining parameters using the method of simulated moments.

**External calibration.** We let the yearly time-discount factor be  $\beta = 1/(1 + \rho)$  with  $\rho = 0.02$  and impose the functional forms  $c(q) = q$  and  $u(q) = q^{1-\sigma}/(1 - \sigma)$ , with  $\sigma \in (0, 1)$ . We calibrate  $\sigma$  internally and let the preference shocks be beta-distributed on the unit interval for shape parameters  $x, y > 0$ , i.e.,  $G = B(x, y)$ . The smaller  $x$  ( $y$ ), the fatter is the left (resp. right) tail of  $G$ . By conducting the subsequent internal calibration for different values of  $(x, y)$ , we check for the robustness of the calibrated parameters to functional-form assumptions on  $G$ .

**Internal calibration.** We calibrate the remaining parameters  $\theta$  and  $\sigma$  by using firm-level markups and the empirically-observed money demand, which we understand as the relationship between the Fisher rate and money holdings scaled by output. We measure money by M1 since checks and demand deposits have similar liquidity properties as currency. Following Lucas and Nicolini (2015), we further augment M1 by money-market



deposit accounts, obtaining the data series NewM1 as the relevant statistic.<sup>17</sup>

For the Fisher rate, we use the 3 Month Treasury Bill Rate (T-bill rate).<sup>18</sup> We extract the trend components of the series of T-bill rates and logarithmic money demand by applying the HP filter with smoothing parameter  $\lambda = 1600$  (Hodrick and Prescott, 1997). We do so for two reasons. First, we use steady-state relationships from our model as theoretical counterparts of the empirical data. This approach makes us rely on empirical long-run relationships, so that we need to filter out cyclical components from our data series. Second, the T-bill rate might behave uncorrelated to the Fisher rate in the short run since it prices a comparatively liquid asset. Therefore, the T-bill rate is only a suitable measure for the Fisher rate when its long-run behavior is extracted.

The empirical money demand reads as  $MD_t^{em} \equiv \text{NewM1}_t / \text{GDP}_t$ . As theoretical counterpart, we use

$$MD_t^{th} \equiv \frac{m}{\int_{\bar{\epsilon}}^{\bar{\epsilon}} p(\epsilon, m, b) G(d\epsilon) + F} \Big|_{\iota=\iota_t}, \quad (32)$$

where the subscript  $t$  refers to the steady-state equilibrium that corresponds to the Fisher rate  $\iota_t$  observed at time  $t$ , assuming the debt tax to be zero. Empirical GDP comprises the aggregate payment and the value  $F$  of goods and services in GDP that our model does not account for.  $F$  is internally calibrated along with  $\sigma$  and  $\theta$ .

We also match the model with firm-level markups to determine the producers' bargaining position, captured both by their bargaining power  $\theta$  and by their debt issuance. De Loecker, Eeckhout and Unger (2020) estimate an average markup  $\bar{\mu}^{em} = 36\%$  for U.S. firms across all sectors in the period from 1955 to 2016. The theoretical firm-level markup that our model suggests is

$$\bar{\mu}_t^{th} \equiv \frac{1}{1 - G(\bar{\epsilon})} \int_{\bar{\epsilon}}^{\bar{\epsilon}} \left[ \frac{p - b - c(q)}{c'(q)q} \right]_{(q,p)=(q,p)(\epsilon,m,b)} G(d\epsilon) \Big|_{\iota=\iota_t}, \quad (33)$$

which is the average relation of the per-unit price of DM goods, net of debt repayment, to the marginal cost of production in successful matches. We define  $\bar{\mu}^{th} \equiv \sum_{t \leq T} \bar{\mu}_t^{th} / T$  as the theoretical counterpart of  $\bar{\mu}^{em}$ , where  $T$  denotes the number of observations.

We calibrate  $(\theta, \sigma, F)$  to three targets: the estimated parameters  $(a_0^{em}, a_1^{em})$  of the

<sup>17</sup>Money-market accounts were designed to circumvent the interest-rate cap of Regulation Q when competition for deposits significantly increased during the Great Inflation in the 1970s. Money-market accounts are however economically equivalent to deposits and can help explain an inverse relationship of money demand to the T-bill rate (Gorton and Zhang, 2023).

<sup>18</sup>It is common to identify the Fisher rate, i.e., the opportunity cost of holding money, with the difference between the T-bill rate and the interest rate on demand deposits. However, until the Dodd-Frank Wall Street Reform and Consumer Protection Act was effective in 2011, Regulation Q was in place, prohibiting the payment of interest on demand deposits. Moreover, from 2011 to 2019, deposit rates were negligibly low, so that we can identify the opportunity cost with the T-bill rate from 1980 to 2015 without loss of generality.

Table 1: Calibration results.

Parameter	Description	Value		
		$B(0.5, 0.5)$	$B(1, 1)$	$B(2, 2)$
$\theta$	producers' bargaining power	0.4595	0.4054	0.3840
$\sigma$	consumers' relative risk aversion	0.5456	0.5794	0.6108
$F$	production not accounted for by the model	4.5606	4.2059	3.8425

(a) Parameters.

Target	Description	Data	Model		
			$B(0.5, 0.5)$	$B(1, 1)$	$B(2, 2)$
$\bar{\mu}$	average price markup	0.3600	0.3600	0.3600	0.3600
$a_0$	level of money demand	-1.1267	-1.1267	-1.1267	-1.1267
$a_1$	interest-rate semi-elasticity	-5.6491	-5.6491	-5.6491	-5.6491

(b) Targets.

empirical regression

$$\log \text{MD}_t^{\text{em}} = a_0^{\text{em}} + a_1^{\text{em}} \iota_t + v_t \quad (34)$$

as well as  $\bar{\mu}^{\text{em}}$ .

**Calibration results.** Table 1 gives the calibration results for the preference-shock distribution  $G$  being equal to  $B(0.5, 0.5)$ ,  $B(1, 1)$ , and  $B(2, 2)$ . It shows that all targets are hit in each calibration. In particular, we match the interest-rate semi-elasticity of money demand at  $a_1^{\text{em}} = -5.65\%$ . Figure 3 shows empirical money demand  $\text{MD}_t^{\text{em}}$  in levels, before and after HP-filtering, as well as theoretical money demand  $\text{MD}_t^{\text{th}}$  obtained from our calibration with the uniform distribution  $G = B(1, 1)$ ; the theoretical money-demand curves corresponding to  $G = B(0.5, 0.5)$  and  $G = B(2, 2)$  are almost indistinguishable from the money-demand curve for  $G = B(1, 1)$ .

**Quantitative discussion.** Figure 4 shows how the equilibrium variables respond to a deviation from the FR in the three calibrations, assuming the debt tax to be zero. All three calibrations exhibit similar patterns and extend the local analytical results in Proposition 1 for all Fisher rates. Figures 4a and 4b show that real balances  $m$  and the face value of debt  $b$  decrease, as the Fisher rate increases. Consumers reduce their real balances when the opportunity cost of holding money increases, so that liquidity constraints tighten, which prompts producers to counteract this tightening by reducing their debt issuance. Figure 4c captures the resulting positive extensive-margin effect. Figures 4d and 4e show that aggregate DM-goods production  $\int_{\epsilon}^{\bar{\epsilon}} q(\epsilon, m, b)G(d\epsilon)$  and GDP  $\int_{\epsilon}^{\bar{\epsilon}} p(\epsilon, m, b)G(d\epsilon) + F$  fall in the Fisher rate, meaning that the decline in output at the intensive margin dominates the increase in output at the extensive margin. Welfare

exhibits a different pattern. The aggregate gross surplus

$$\int_{\hat{\epsilon}}^{\bar{\epsilon}} [\epsilon u(q) - c(q)]_{q=q(\epsilon, m, b)} G(d\epsilon) = \left( \frac{1 - \beta}{\beta} \right) \mathcal{W}, \quad (35)$$

as shown in Figure 4f, is hump-shaped in the Fisher rate, so that the welfare-maximizing Fisher rate  $\iota^*$  is away from the FR, as implied by Proposition 2. Table 2 shows  $\iota^*$  for each specification of  $G$ . The reason that  $\iota^*$  increases with the fatness of  $G$ 's tails is that the extensive-margin cost of debt is comparatively small if the probability mass assigned to intermediate preference shocks is also small. Then, producers are more inclined to issue debt, which requires a higher inflation tax to curb debt issuance.

Our findings parallel the results of Lagos and Wright (2005), who calibrate a standard money-search model to empirical money demand under different levels of bargaining power. Although they can match money demand equally in either calibration, they find very different values for the welfare cost of inflation. We likewise find that models with different  $G$  can fit money demand equally well and have similar calibrated parameters (see Table 1a). Yet, they induce quite different welfare-maximizing levels for the Fisher rate. This reinforces the argument of Lagos and Wright (2005) that “[k]nowing the empirical ‘money demand’ curve is not enough: one really needs to understand the micro foundations [...] in order to correctly estimate the welfare cost of inflation” (p. 480).

Table 2 also shows the smallest Pigouvian debt taxes  $\kappa$  that eliminate debt issuance in the three calibrations, respectively. As indicated by Lemma 6,  $\kappa$  decreases in  $\iota$  since fiscal and monetary policy are substitutes in curbing debt issuance.

**Extensive and intensive margin effects.** Figure 5 illustrates how monetary policy affects the intensive and extensive margins of DM trade in the calibration with  $G = B(1, 1)$ . Figure 5a shows the bargained quantities and the preference-shock threshold  $\hat{\epsilon}$  for different Fisher rates. Under the FR, agents trade intensively-efficient quantities in all successful matches. However, when  $\iota = 10\%$ , the liquidity constraint binds for large  $\epsilon$ , creating a wedge between the bargained quantity and the efficient quantity.<sup>19</sup> For  $\iota = 50\%$ , consumers hold such low real balances that the liquidity constraint binds in every successful match.

Figure 5 also demonstrates that the preference-shock threshold  $\hat{\epsilon}$  decreases in the Fisher rate. Figure 5b moreover provides insights into what causes matches to be unsuccessful for  $\epsilon < \hat{\epsilon}$ . For  $\iota = 0\%$  and  $\iota = 10\%$ , agents trade  $\hat{q} = q_{\hat{\epsilon}}^*$  in the marginally-successful match, so that  $\hat{\epsilon}u(q_{\hat{\epsilon}}^*) - c(q_{\hat{\epsilon}}^*) - b = 0$ , and the liquidity constraint is slack. Cast in the context of Lemma 2, Condition (i) binds and Condition (ii) is slack. For  $\iota = 50\%$ , agents

<sup>19</sup>Note that the bargained quantities decline in  $\epsilon$  once the liquidity constraint binds. This is because of the bargaining protocol. Since a consumer with a high  $\epsilon$  commands of the same money holdings as all other consumers, the matched producer appropriates the utility gains of this consumer by producing less and thus facing lower production costs.

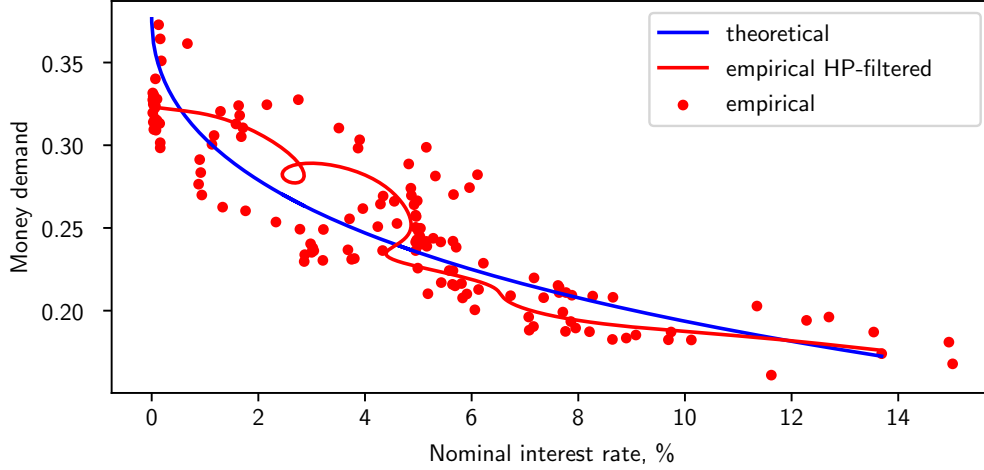


Figure 3: Money demand.

*Note:* This figure shows the empirical money demand and the theoretical money demand in levels in the calibrated model with  $G = B(1, 1)$ .

Table 2: Optimal Fisher rates and debt taxes.

Preference-shock distribution $G$	$B(0.5, 0.5)$	$B(1, 1)$	$B(2, 2)$
Optimal Fisher rate $\iota^*$ without debt tax	0.51%	0.25%	0.03%
Optimal debt tax $\kappa$ at $\iota = 0\%$	53.00%	58.29%	60.39%
Optimal debt tax $\kappa$ at $\iota = 10\%$	48.49%	54.32%	56.63%

*Note:* This table shows the welfare-maximizing Fisher rates in the absence of a debt tax and the optimal debt taxes for  $\iota = 0\%, 10\%$  in the calibrated models for different specifications of preference-shock distribution  $G$ .

trade  $\hat{q} < q_\epsilon^*$  in the marginally-successful match, so that  $\hat{e}u(q_\epsilon^*) - c(q_\epsilon^*) - b > 0$ , and the liquidity constraint binds. In this case, Condition (i) is slack and Condition (ii) binds.

## 7 Non-Optimality of Debt and Equity

We have focused so far on limited-liability debt due to its documented empirical relevance in bargaining. However, it is not obvious that debt is the optimal contract design when leveraging bargaining power. In fact, we show in this section how a producer can profitably deviate from both debt and equity to superior contracts, respectively, that strictly increase her lifetime utility. We focus on the FR since it renders the liquidity constraints in all bargaining problems slack and thus simplifies producers' contract choices.

**Proposition 3.** *Under the FR, a producer can profitably deviate from any debt contract  $r^b \in D$  to a superior contract  $\tilde{r}^b \in F$ .*

Debt distorts a producer's ex-post incentives to produce when being matched with a consumer with a small preference shock  $\epsilon < \hat{\epsilon}_b$ . Even though the consumer's real balances

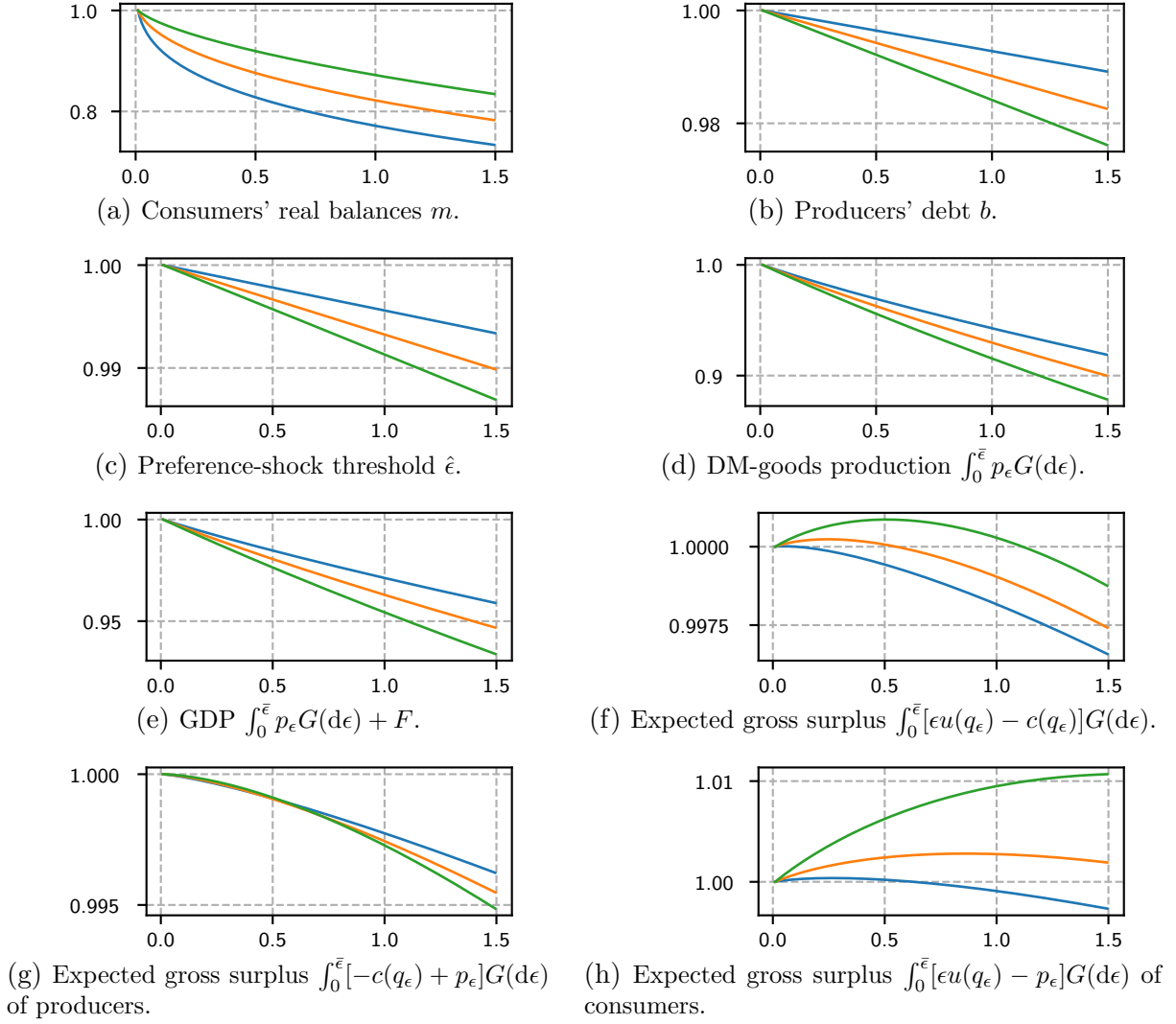


Figure 4: Responses of equilibrium variables to a deviation from the Friedman rule.

*Note:* The horizontal axes show the Fisher rate in percent. The vertical axes show the equilibrium variables in the calibrated economies with  $G = B(0.5, 0.5)$  (●),  $G = B(1, 1)$  (●), and  $G = B(2, 2)$  (●), normalized at the Friedman rule. We write  $q_{\epsilon} = q(\epsilon, m, b)$  and  $p_{\epsilon} = p(\epsilon, m, b)$ .

would allow for trade, enabling the producer to partially repay her debt, she does not produce at all and fully defaults. Hence, the producer's and the financier's interests do not align ex post, resulting in a moral-hazard problem that limits the producer's ability to leverage her bargaining power.<sup>20</sup>

For any debt contract  $r^b$  with face value  $b > 0$ , we can construct an alternative contract  $\tilde{r}^b \in F$  to which a producer can profitably deviate. We choose  $a \in (0, b)$  close

<sup>20</sup>Jensen and Meckling (1976) find that debt involves agency costs due to entrepreneurs' tendency to undertake excessively risky projects. In our framework, the non-optimality of debt is somewhat similar: limited liability causes producers to shirk from production in meetings with low-preference consumers, leading to high default probabilities and consequently high borrowing costs.

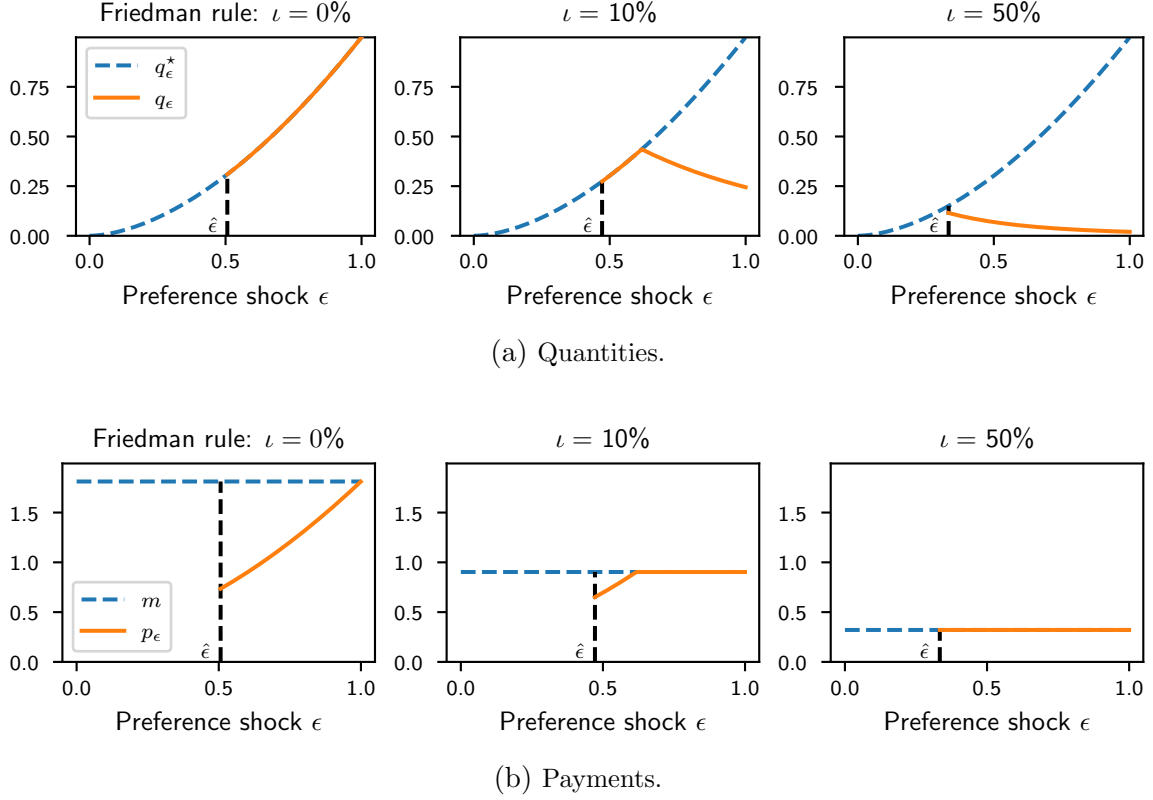


Figure 5: Negotiated terms of trade in the calibrated economy with  $G = B(1, 1)$ .

*Note:* We write  $q_\epsilon = q(\epsilon, m, b)$  and  $p_\epsilon = p(\epsilon, m, b)$ .

to  $b$  and define

$$\tilde{r}^b(p) = \begin{cases} \min\{p, a\}, & \text{if } p \leq \hat{p}, \\ b, & \text{otherwise,} \end{cases} \quad (36)$$

where  $\hat{p}$  is the payment exchanged if the producer is committed to  $r^b$  and the matched consumer incurs preference shock  $\hat{\epsilon}_b$ . Contract  $\tilde{r}^b$ , as illustrated in Figure 6, is almost identical to  $r^b$ , but requires only repayment  $a < b$  if  $p$  is smaller but close to  $\hat{p}$ . A deviation from  $r^b$  to  $\tilde{r}^b$  affects the producer's expected gross surplus in two ways, as illustrated in Figure 7. There is an  $\epsilon_a^+ \in (\hat{\epsilon}_b, \bar{\epsilon})$ , so that agents exchange only payments  $p \leq \hat{p}$  when solving bargaining problem (9) for  $\epsilon \in [\hat{\epsilon}_b, \epsilon_a^+)$ , to keep the repayment to the financier at  $a$  rather than  $b$ . The producer consequently realizes less bargaining revenues if  $\epsilon \in [\hat{\epsilon}_b, \epsilon_a^+)$  than when committed to  $r^b$ . However, trade becomes successful for  $\epsilon \in [\hat{\epsilon}_a, \hat{\epsilon}_b]$  due to the producer's smaller repayment obligations when realizing small bargaining revenues  $p \leq \hat{p}$ . The producer's gains are of higher order importance compared to her losses since her losses shrink in two dimensions for  $a$  converging to  $b$ , as also  $\epsilon_a^+$  converges to  $\hat{\epsilon}_b$ , while her gains shrink only in one dimension.

Equity contracts typically mitigate moral-hazard problems as occurring with debt by

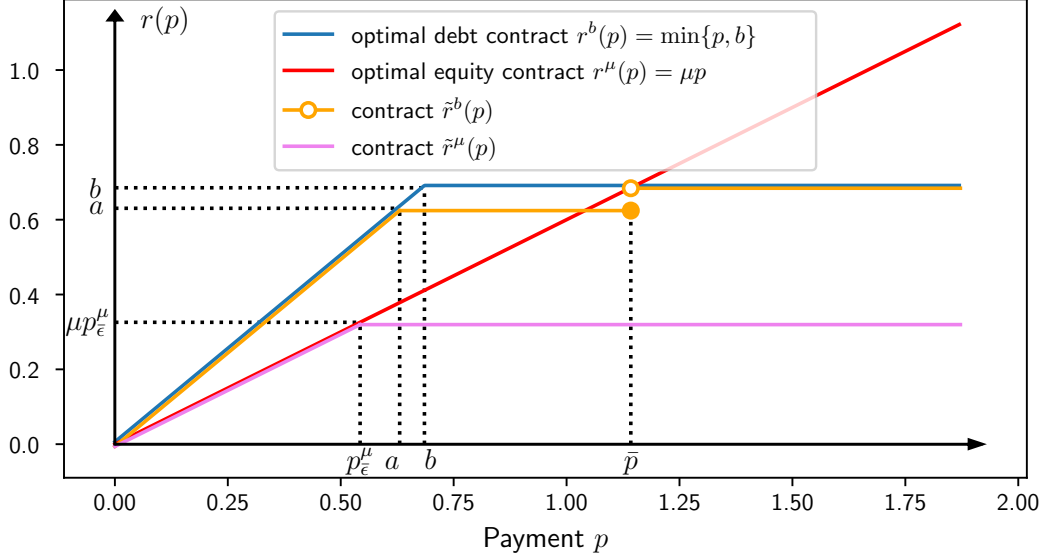


Figure 6: Contracts.

*Note:* This figure shows the optimal debt contract  $r^b$ , the optimal equity contract  $r^\mu$ , and contracts  $\tilde{r}^b$  and  $\tilde{r}^\mu$ . The underlying parameters are  $u(q) = q^{0.4}/0.4$ ,  $c(q) = q$ ,  $\theta = 0$ ,  $\beta = 0.98$ ,  $[0, \bar{\epsilon}] = [0, 1]$ , and uniform distribution  $G = U(0, 1)$ .

ensuring that borrowers have “skin in the game”, so that they do not shirk. We define

$$E \equiv \left\{ r \in \mathbb{R}_+^{\mathbb{R}_+} : \exists \mu \in (0, 1), \text{ so that } r(p) = \mu p \text{ for all } p \in \mathbb{R}_+ \right\} \subset F \quad (37)$$

as the set of equity contracts. When writing equity contract  $r^\mu$ , a producer sells off a portion  $\mu \in (0, 1)$  of the total of claims on her bargaining revenues to the financier. This equity is outside equity in that the financier does not acquire any voting rights in the producer’s bargaining behavior. We obtain

**Proposition 4.** *Under the FR, a producer can profitably deviate from any equity contract  $r^\mu \in E$  to a superior contract  $\tilde{r}^\mu \in F$ .*

A consumer and a producer committed to an equity contract internalize in bargaining that any additional unit of exchanged payment is partially used to compensate the financier. This consideration distorts trade at the intensive margin, so that the producer’s bargaining revenues against which she can borrow are comparatively small.

A producer can profitably deviate from any equity contract  $r^\mu$  to contract  $\tilde{r}^\mu(p) \equiv \mu \max\{p, p_\epsilon^\mu\}$ , where  $p_\epsilon^\mu$  is the payment exchanged in bargaining under equity contract  $r^\mu$  if the matched consumer incurs preference shock  $\bar{\epsilon}$ . Contract  $\tilde{r}^\mu$ , as illustrated in Figure 6, is a hybrid of equity and debt:  $\tilde{r}^\mu$  is identical to  $r^\mu$  if revenues  $p$  are small, but  $\tilde{r}^\mu$  takes the form of debt with repayment  $\mu p_\epsilon^\mu$  if revenues  $p$  are large. Since  $\tilde{r}^\mu$  implies small repayments  $\tilde{r}^\mu(p) < r^\mu(p)$  for  $p > p_\epsilon^\mu$ , a deviation from  $r^\mu$  to  $\tilde{r}^\mu$  incentivizes the producer to exchange payment  $p_\epsilon^\mu > p_\epsilon^\mu$  if being matched with consumers with high preference

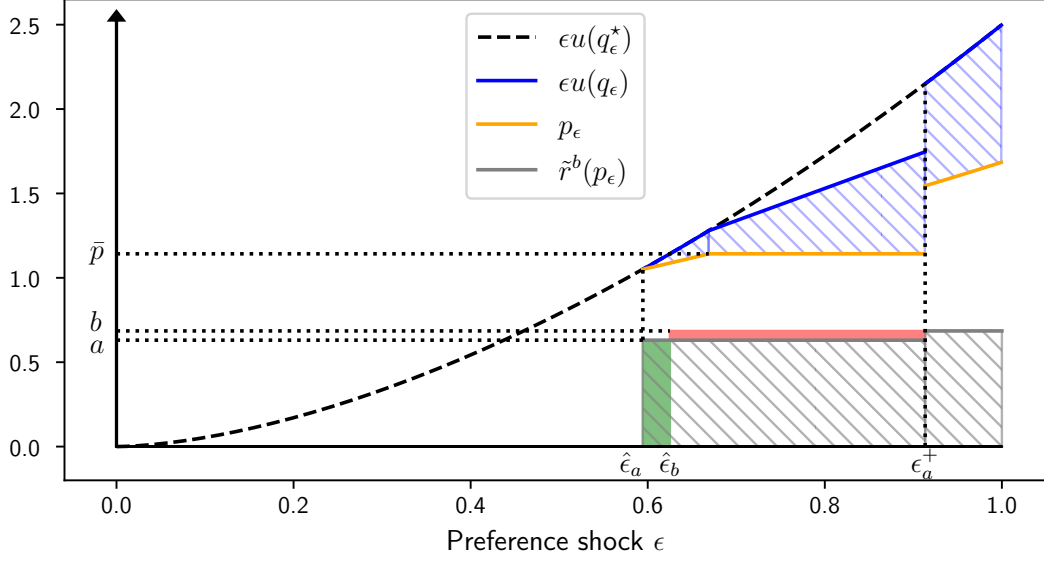


Figure 7: Deviation from debt.

*Note:* This figure shows a producer's gains (green) and losses (red) in her expected gross surplus when deviating from the optimal debt contract  $r^b$  to contract  $\tilde{r}^b$ . The gray-shaded area captures the expected gross surplus of the producer when committed to  $\tilde{r}^b$ . The blue-shaded area captures the expected gross surplus of the consumer matched with the deviating producer. The underlying parameters are  $u(q) = q^{0.4}/0.4$ ,  $c(q) = q$ ,  $\theta = 0$ ,  $\beta = 0.98$ ,  $[0, \bar{\epsilon}] = [0, 1]$ , and uniform distribution  $G = U(0, 1)$ . We write  $q_\epsilon = q(\epsilon, m, b)$  and  $p_\epsilon = p(\epsilon, m, b)$ .

shocks  $\epsilon > \tilde{\epsilon}^\mu$  for  $\tilde{\epsilon}^\mu \in (0, \bar{\epsilon})$ . Loosely speaking, this deviation shifts probability weights from small payments to large payments. Figure 8 illustrates the producer's resulting utility gains.<sup>21</sup>

Propositions 3 and 4 together suggest that the optimal financial contract is neither debt nor equity. While the benefits of equity are realized at the extensive margin for small values of  $\epsilon \in [0, \bar{\epsilon}]$ , debt facilitates efficient trade at the intensive margin when  $\epsilon$  is sufficiently large. Therefore, the optimal financial contract balances these benefits at the extensive and intensive margins. Whether this contract admits a parametric closed-form solution remains an open question and falls beyond the scope of this paper.

## 8 Conclusion

This paper uncovers novel general-equilibrium effects of limited-liability debt issued by producers to leverage their bargaining power vis-à-vis heterogeneous consumers. Producers borrow against their expected bargaining revenues and pass the resulting fixed cost of

<sup>21</sup>The intuition for why  $\tilde{r}^\mu$  dominates  $r^\mu$  is akin to the reason for which debt is superior to equity in the principal-agent model of Innes (1990). In that model, entrepreneurs borrow to engage in a stochastic investment project that relies on their unobservable effort. Since debt involves small repayment obligations when the entrepreneur realizes high revenues, it incentivizes entrepreneurs to devote high effort, resulting in large surpluses.



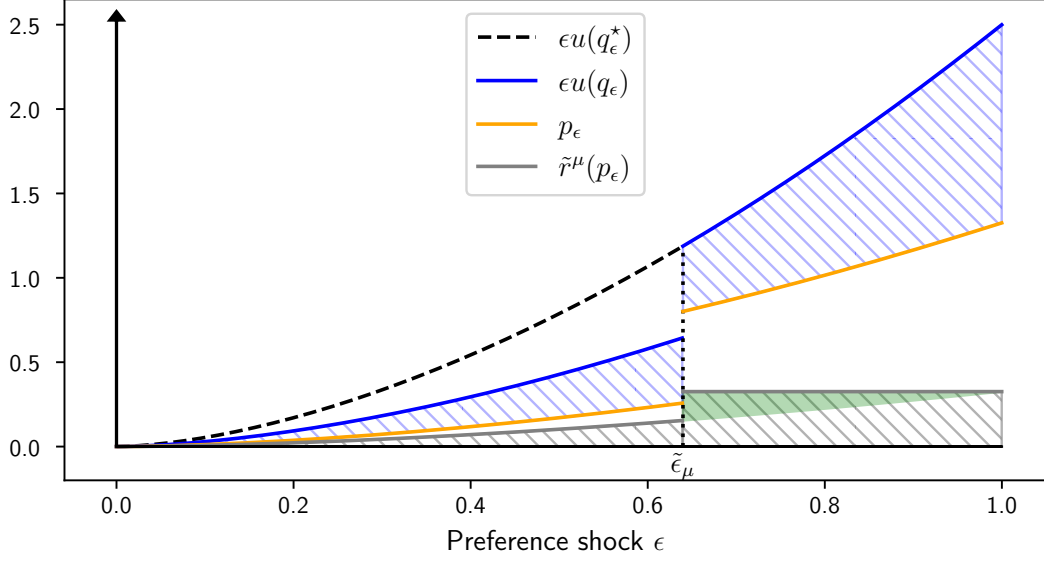


Figure 8: Deviation from equity.

*Note:* This figure shows a producer's gains (green) when deviating from the optimal equity contract  $r^\mu$  to contract  $\tilde{r}^\mu$ . The gray-shaded area captures the expected gross surplus of the producer when committed to  $\tilde{r}^\mu$ . The blue-shaded area captures the expected gross surplus of the consumer matched with the deviating producer. The underlying parameters are  $u(q) = q^{0.4}/0.4$ ,  $c(q) = q$ ,  $\theta = 0$ ,  $\beta = 0.98$ ,  $[0, \bar{\epsilon}] = [0, 1]$ , and uniform distribution  $G = U(0, 1)$ . We write  $q_\epsilon = q(\epsilon, m, b)$  and  $p_\epsilon = p(\epsilon, m, b)$ .

debt repayment onto consumers in the bargaining process. Such debt renders bargaining agreements infeasible in matches where consumers derive low utility from the producers' goods, as these consumers are unwilling to bear the producers' debt burden. Moreover, debt issuance tightens liquidity constraints: consumers must allocate part of their money holdings to debt repayment, leaving less liquidity available to compensate producers for actual production. We identify a pecuniary externality at the core of these distortions: borrowing rates fail to reflect the costs producers impose on consumers by issuing debt. To address this, optimal fiscal policy includes a Pigouvian tax on debt. In the absence of such a tax, we find that deviating from the Friedman rule to nominal interest rates of up to 0.51% partially mitigates the externality, though it does not fully eliminate it. Importantly, debt is not only suboptimal from a policy standpoint but also from the perspective of individual producers. Contrary to established results in contract theory, we show that producers can achieve better outcomes by committing to contracts beyond pure debt or equity, thereby more effectively leveraging their bargaining power.

## References

ALTHANNS, M., VAN BUGGENUM, H. and GERSBACH, H. (2024). Credit enforcement and monetary-policy transmission. *CEPR Discussion Paper*, **DP19607**. [7](#)

- ARUOBA, S. B., ROCHETEAU, G. and WALLER, C. (2007). Bargaining and the value of money. *Journal of Monetary Economics*, **54** (8), 2636–2655. [19](#)
- BRONARS, S. G. and DEERE, D. R. (1991). The threat of unionization, the use of debt, and the preservation of shareholder wealth. *Quarterly Journal of Economics*, **106** (1), 231–254. [6](#), [18](#)
- DASGUPTA, S. and NANDA, V. (1993). Bargaining and brinkmanship: Capital structure choice by regulated firms. *International Journal of Industrial Organization*, **11** (4), 475–497. [4](#), [6](#)
- and SENGUPTA, K. (1993). Sunk investment, bargaining and choice of capital structure. *International Economic Review*, **34** (1), 203–220. [6](#)
- DE LOECKER, J., EECKHOUT, J. and UNGER, G. (2020). The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics*, **135** (2), 561–644. [22](#)
- DIAMOND, D. W. (1984). Financial Intermediation and Delegated Monitoring. *Review of Economic Studies*, **51** (3), 393–414. [8](#)
- DONG, M. and JIANG, J. H. (2014). Money and price posting under private information. *Journal of Economic Theory*, **150** (C), 740–777. [7](#)
- ENNIS, H. M. (2008). Search, money, and inflation under private information. *Journal of Economic Theory*, **138** (1), 101–131. [7](#)
- (2009). Avoiding the inflation tax. *International Economic Review*, **50** (2), 607–625. [7](#)
- FISHER, I. (1920). *The Purchasing Power of Money: Its Determination and Relation to Credit, Interest and Crises*. Macmillan Company, New York City, NY, available at <https://fraser.stlouisfed.org/title/3610>, retrieved on September 11, 2024. [5](#), [7](#)
- GALE, D. and HELLWIG, M. (1985). Incentive-compatible debt contracts: The one-period problem. *Review of Economic Studies*, **52** (4), 647–663. [8](#)
- GORTON, G. B. and ZHANG, J. (2023). Taming wildcat stablecoins. *University of Chicago Law Review*, **90** (3), 909–971. [22](#)
- HELLWIG, M. F. (2009). A reconsideration of the Jensen-Meckling model of outside finance. *Journal of Financial Intermediation*, **18** (4), 495–525. [8](#)
- HENNESSY, C. A. and LIVDAN, D. (2009). Debt, bargaining, and credibility in firm-supplier relationships. *Journal of Financial Economics*, **93** (3), 382–399. [2](#), [4](#), [6](#), [18](#)
- HODRICK, R. J. and PRESCOTT, E. C. (1997). Postwar U.S. business cycles: An empirical investigation. *Journal of Money, Credit and Banking*, **29** (1), 1–16. [22](#)
- INNES, R. D. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, **52** (1), 45–67. [8](#), [29](#)
- JENSEN, M. C. and MECKLING, W. H. (1976). Theory of the firm: Managerial behavior,

- agency costs and ownership structure. *Journal of Financial Economics*, **3** (4), 305–360. [7](#), [8](#), [26](#)
- KALAI, E. (1977). Proportional solutions to bargaining situations: Interpersonal utility comparisons. *Econometrica*, **45** (7), 1623–1630. [3](#), [5](#), [9](#), [11](#), [19](#)
- KALE, J. R. and SHAHRUR, H. (2007). Corporate capital structure and the characteristics of suppliers and customers. *Journal of Financial Economics*, **83** (2), 321–365. [2](#), [4](#), [6](#), [18](#)
- LAGOS, R. and ROCHETEAU, G. (2005). Inflation, output, and welfare. *International Economic Review*, **46** (2), 495–522. [7](#)
- and WRIGHT, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, **113** (3), 463–484. [3](#), [7](#), [24](#)
- LIPSEY, R. G. and LANCASTER, K. (1956). The general theory of second best. *Review of Economic Studies*, **24** (1), 11–32. [20](#)
- LIU, L. Q., WANG, L. and WRIGHT, R. (2011). On the “hot potato” effect of inflation: Intensive versus extensive margins. *Macroeconomic Dynamics*, **15** (S2), 191–216. [7](#)
- LUCAS, R. E. and NICOLINI, J. P. (2015). On the stability of money demand. *Journal of Monetary Economics*, **73**, 48–65. [21](#)
- MATSA, D. A. (2010). Capital structure as a strategic variable: Evidence from collective bargaining. *Journal of Finance*, **65** (3), 1197–1232. [6](#)
- NASH, J. F. (1950). The bargaining problem. *Econometrica*, **18** (2), 155–162. [19](#)
- NOSAL, E. (2011). Search, welfare, and the “hot potato” effect of inflation. *Macroeconomic Dynamics*, **15** (S2), 313–326. [7](#)
- PEROTTI, E. C. and SPIER, K. E. (1993). Capital structure as a bargaining tool: The role of leverage in contract renegotiation. *American Economic Review*, **83** (5), 1131–1141. [6](#)
- PETERSON, B. and SHI, S. (2004). Money, price dispersion and welfare. *Economic Theory*, **24**, 907–932. [7](#)
- ROCHETEAU, G. and WRIGHT, R. (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, **73** (1), 175–202. [3](#)
- TITMAN, S. (1984). The effect of capital structure on a firm’s liquidation decision. *Journal of Financial Economics*, **13** (1), 137–151. [6](#)
- TOWNER, M. (2020). Debt and bargaining outcomes: Evidence from U.S. hospitals. *Management Science*, **66** (5), 2083–2098. [2](#)
- TOWNSEND, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, **21** (2), 265–293. [8](#)
- WANG, C. and WILLIAMSON, S. D. (1998). Debt contracts and financial intermediation with costly screening. *Canadian Journal of Economics*, **31** (3), 573–595. [8](#)
- WILLIAMSON, S. D. (1987). Costly monitoring, loan contracts, and equilibrium credit

## A Proofs

**PROOF OF LEMMA [1](#).** The lemma is a consequence of market clearing. ■

**PROOF OF LEMMA [2](#).** If a match is successful, there must exist a  $q > 0$ , so that  $\epsilon u(q) - c(q) - b \geq 0$ . This definitely holds for  $q = q_\epsilon^*$ , implying Condition [\(i\)](#). Moreover, the consumer's real balances  $m$  must weakly exceed the payment corresponding with the smallest  $q$  guaranteeing a non-negative surplus,  $\tilde{q}(\epsilon, b)$ , implying Condition [\(ii\)](#).

Conversely, Condition [\(i\)](#) implies that  $\tilde{q}(\epsilon, b) \in (0, q_\epsilon^*]$ . Condition [\(ii\)](#) then yields that  $q(\epsilon, m, b) \geq \tilde{q}(\epsilon, b) > 0$ . ■

**PROOF OF COROLLARY [1](#).** The proof immediately follows from Lemma [2](#). ■

**PROOF OF LEMMA [3](#).** It is clear that  $\hat{q} \geq \tilde{q}(\hat{\epsilon}, b)$ , since the net surplus would be negative otherwise. Suppose that  $\hat{q} > \tilde{q}(\hat{\epsilon}, b)$ . Since  $\partial \tilde{q}(\epsilon, b)/\partial \epsilon < 0$ , there is an  $\epsilon < \hat{\epsilon}$  close to  $\hat{\epsilon}$ , so that  $\hat{q} > \tilde{q}(\epsilon, b)$ . It thus holds that

$$\begin{aligned} \theta \epsilon u(\tilde{q}(\epsilon, b)) + (1 - \theta)(c(\tilde{q}(\epsilon, b)) + b) &< \theta \epsilon u(\hat{q}) + (1 - \theta)(c(\hat{q}) + b) \\ &< \theta \hat{\epsilon} u(\hat{q}) + (1 - \theta)(c(\hat{q}) + b) \leq m, \end{aligned} \quad (38)$$

so that a match for  $\epsilon$  would be successful. This contradicts the definition of  $\hat{\epsilon}$ . ■

**PROOF OF LEMMA [4](#).** We distinguish two cases:  $\hat{q} < q_\epsilon^*$  and  $\hat{q} = q_\epsilon^*$ .

If  $\hat{q} < q_\epsilon^*$ , the liquidity constraint binds in the match  $(\hat{\epsilon}(m, b'), m, b')$  for all  $b'$  in a (small) neighborhood of  $b$ . We infer that  $\hat{\epsilon}$  and  $\hat{q}$  are differentiable at  $b$ , so that

$$m = \theta \hat{\epsilon} u(\hat{q}) + (1 - \theta)[c(\hat{q}) + b] \quad \Rightarrow \quad 0 = \theta u(\hat{q}) \frac{\partial \hat{\epsilon}}{\partial b} + [\theta \hat{\epsilon} u'(\hat{q}) + (1 - \theta)c'(\hat{q})] \frac{\partial \hat{q}}{\partial b} + 1 - \theta. \quad (39)$$

Since  $\hat{q} = \tilde{q}(\hat{\epsilon}, b)$  according to Lemma [3](#), it moreover holds that

$$\hat{\epsilon} u(\hat{q}) - c(\hat{q}) - b = 0 \quad \Rightarrow \quad u(\hat{q}) \frac{\partial \hat{\epsilon}}{\partial b} + [\hat{\epsilon} u'(\hat{q}) - c'(\hat{q})] \frac{\partial \hat{q}}{\partial b} - 1 = 0. \quad (40)$$

Combining the equations above, we obtain  $\partial \hat{q}/\partial b = -1/c'(\hat{q})$  and Equation [\(20\)](#) follows.

If  $\hat{q} = q_\epsilon^*$ , we immediately infer from  $\hat{e}u(\hat{q}) - c(\hat{q}) - b = 0$  and  $\hat{e}u'(\hat{q}) - c'(\hat{q}) = 0$  that  $\hat{e}$  is differentiable at  $b$ —but  $\hat{q}$  is not necessarily—with  $\partial\hat{e}/\partial b = 1/u(\hat{q})$ . ■

**PROOF OF LEMMA 5.** The first-order condition (FOC) w.r.t. the face value of debt  $b'_{+1}$ , corresponding to the producer's problem (25), reads as

$$0 \geq \int_{\hat{e}(m_{+1}, b'_{+1})}^{\bar{\epsilon}} \left[ \theta \frac{\partial}{\partial b} [\epsilon_{+1}u(q) - c(q)]_{q=q(\epsilon_{+1}, m_{+1}, b'_{+1})} + 1 - \theta \right] G(d\epsilon_{+1}) - b'_{+1}g(\hat{e}(m_{+1}, b'_{+1})) \frac{\partial\hat{e}(m_{+1}, b'_{+1})}{\partial b}, \quad (41)$$

with “=” if  $b'_{+1} > 0$ . We used that the gross surplus of the marginally-successful match is zero (see Lemma 3). Using the liquidity constraint in bargaining problem (9) to infer  $\partial q(\epsilon, m, b)/\partial b$ , we obtain

$$\frac{\partial}{\partial b} [\epsilon u(q) - c(q)]_{q=q(\epsilon, m, b)} = - \left[ \frac{(1 - \theta)[\epsilon u'(q) - c'(q)]}{\theta \epsilon u'(q) + (1 - \theta)c'(q)} \right]_{q=q(\epsilon, m, b)} = -\mathcal{L}(\epsilon, m, b), \quad (42)$$

so that Lemma 5 follows immediately. ■

**PROOF OF LEMMA 6.** If the fiscal authority charges a debt tax  $\kappa$ , a producer's FOC w.r.t. the face value of debt  $b$  in Lemma 5 reads in equilibrium as

$$0 \geq (1 - \theta)[1 - G(\hat{e})] - \theta\iota - bg(\hat{e}) \frac{\partial\hat{e}}{\partial b} - \frac{\kappa}{\beta}, \quad (43)$$

with “=” if  $b > 0$ , where we used the consumers' FOC w.r.t. their money holdings. We search for the smallest  $\kappa$ , so that Inequality (43), evaluated at  $b = 0$ , holds. We note

$$\left[ b \frac{\partial\hat{e}}{\partial b} \right]_{b=0} = \left[ \frac{b}{u(\hat{q})} \right]_{b=0} = \lim_{\tilde{b} \rightarrow 0} \left[ u'(\hat{q}) \frac{\partial\hat{q}}{\partial b} \right]_{b=\tilde{b}}^{-1} = - \lim_{\tilde{b} \rightarrow 0} \left[ \frac{u(\hat{q})[\hat{e}u''(\hat{q}) - c''(\hat{q})]}{u'(\hat{q})^2} \right]_{b=\tilde{b}} = 0. \quad (44)$$

We inferred from  $\hat{e}u'(\hat{q}) = c'(\hat{q})$  in the marginally-successful match for  $\tilde{b}$  close to zero that  $\partial\hat{e}/\partial b = 1/u(\hat{q})$  and  $\partial\hat{q}/\partial b = -u'(\hat{q})/[\hat{e}u''(\hat{q}) - c''(\hat{q})]u(\hat{q})$ , and used that  $\lim_{\tilde{b} \rightarrow 0} \hat{q}|_{b=\tilde{b}} = 0$ . Hence, Inequality (43), evaluated at  $b = 0$ , reads as

$$0 \geq 1 - \theta(1 + \iota) - \frac{\kappa}{\beta} \quad \Leftrightarrow \quad \kappa \geq \beta[1 - \theta(1 + \iota)]. \quad (45)$$

If  $\kappa$  satisfies this inequality,  $b = 0$  is featured in equilibrium. ■

**PROOF OF PROPOSITION 1.** We first determine the economy's outcome at the FR and then investigate how it responds to a deviation from the FR.

**Economy at the FR.** Since the opportunity cost of holding money is zero at the FR, consumers choose real balances  $m \geq \theta \bar{e} u(q_\epsilon^*) + (1 - \theta)[c(q_\epsilon^*) + b]$ , facilitating the exchange of  $q(\epsilon, m, b) = q_\epsilon^*$  for all  $\epsilon \in [\hat{\epsilon}, \bar{\epsilon}]$ . Substituting the consumers' FOC for money in Inequality (22) into the producers' FOC for debt in Inequality (26) thus yields

$$0 \geq (1 - \theta)[1 - G(\hat{\epsilon})] - bg(\hat{\epsilon}) \frac{\partial \hat{\epsilon}}{\partial b}, \quad (46)$$

with “=” if  $b > 0$ . This implies  $b > 0$  and  $\hat{\epsilon} \in (0, \bar{\epsilon})$ .

**Deviation from the FR.** Since the equilibrium objects  $m$  and  $b$  are continuous in  $\iota$ , the liquidity constraint in the marginally-successful match is slack not only at the FR but also in its neighborhood. It thus holds that  $d[\hat{\epsilon}u'(\hat{q}) - c'(\hat{q})] = 0$  around the FR, from which we infer

$$d\hat{q} = - \left[ \frac{u'(\hat{q})}{\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})} \right] d\hat{\epsilon}. \quad (47)$$

We infer from Lemma 2 that  $d[\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b] = 0$ , which implies  $db = u(\hat{q}) d\hat{\epsilon}$ .

To sign  $d\hat{\epsilon}/d\iota$ , we consider the producers' FOC for debt in Equation (26), reading as

$$0 = (1 - \theta)[1 - G(\hat{\epsilon})] - \theta \iota - \frac{g(\hat{\epsilon})b}{u(\hat{q})}, \quad (48)$$

where we used  $\partial \hat{\epsilon}/\partial b = 1/u(\hat{q})$  from Lemma 4. The differential of this FOC reads as

$$\begin{aligned} 0 &= -(1 - \theta)g(\hat{\epsilon}) d\hat{\epsilon} - \theta d\iota - \frac{g(\hat{\epsilon})db}{u(\hat{q})} + \frac{g(\hat{\epsilon})bu'(\hat{q})d\hat{q}}{u(\hat{q})^2} - \frac{g'(\hat{\epsilon})b d\hat{\epsilon}}{u(\hat{q})} \\ \Leftrightarrow \quad \theta d\iota &= - \left[ (1 + (1 - \theta))g(\hat{\epsilon}) + \frac{u'(\hat{q})^2 g(\hat{\epsilon})b}{u(\hat{q})^2 [\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]} + \frac{g'(\hat{\epsilon})b}{u(\hat{q})} \right] d\hat{\epsilon}, \end{aligned} \quad (49)$$

where we used the expressions for  $d\hat{q}$  and  $db$  from above. To sign the term in the square brackets in Equation (49), we determine the producers' second-order condition (SOC) for debt, corresponding to their problem (25), at the FR. This SOC reads as

$$\begin{aligned} 0 &> - \left[ (1 - \theta)g(\hat{\epsilon}) + \frac{g'(\hat{\epsilon})b}{u(\hat{q})} \right] \frac{\partial \hat{\epsilon}}{\partial b} - \frac{g(\hat{\epsilon})}{u(\hat{q})} + \frac{g(\hat{\epsilon})bu'(\hat{q})}{u(\hat{q})^2} \frac{\partial \hat{q}}{\partial b} \\ \Leftrightarrow \quad 0 &> - \left[ (1 + (1 - \theta))g(\hat{\epsilon}) + \frac{u'(\hat{q})^2 g(\hat{\epsilon})b}{u(\hat{q})^2 [\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]} + \frac{g'(\hat{\epsilon})b}{u(\hat{q})} \right], \end{aligned} \quad (50)$$

where we used that  $\hat{\epsilon}u'(\hat{q}) - c'(\hat{q}) = 0$  to obtain  $\partial \hat{q}/\partial b = -u'(\hat{q})/u(\hat{q})[\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]$ , and that  $\partial \hat{\epsilon}/\partial b = 1/u(\hat{q})$ . We obtain  $d\hat{\epsilon}/d\iota < 0$ .

We immediately infer  $db/d\iota < 0$  from  $db = u(\hat{q}) d\hat{\epsilon}$ . Money holdings  $m$  also clearly

decrease in  $\iota$ . Suppose that  $dm/d\iota \in (-\infty, 0)$ . Since  $\mathcal{L}$  is continuous in  $m$  and  $b$ —but not necessarily differentiable—it follows that

$$\frac{d}{d\iota} \left[ \int_{\bar{\epsilon}}^{\bar{\epsilon}} \mathcal{L}(\epsilon, m, b) G(d\epsilon) \right] = 0, \quad (51)$$

contradicting Equation (22). We infer  $dm/d\iota = -\infty$ . ■

**PROOF OF PROPOSITION 2.** We consider the economy at the FR. Lemma 1 implies that

$$\mathcal{W} = \frac{\beta}{1-\beta} \int_{\bar{\epsilon}}^{\bar{\epsilon}} [\epsilon u(q) - c(q)]_{q=q(\epsilon, m, b)} G(d\epsilon). \quad (52)$$

We cannot apply *Leibniz's rule* to obtain  $d\mathcal{W}/d\iota$  directly since the differentiability of the integrand is not guaranteed for  $\epsilon = \bar{\epsilon}$  due to  $dm/d\iota = -\infty$  (Proposition 1). We thus pin down  $d\mathcal{W}/d\iota$  by determining the limit of the difference quotient  $[\mathcal{W}|_{\zeta} - \mathcal{W}|_0]/\zeta$ , where  $\mathcal{W}|_{\zeta}$  denotes the equilibrium welfare level for Fisher rate  $\zeta > 0$  close to the FR.

For notational convenience, we define  $q_{\epsilon}(\zeta) \equiv q(\epsilon, m|_{\zeta}, b|_{\zeta})$  and  $\mathcal{L}_{\epsilon}(\zeta) \equiv \mathcal{L}(\epsilon, m|_{\zeta}, b|_{\zeta})$  as the exchanged quantity and the liquidity premium in a match with preference shock  $\epsilon$  if the Fisher rate is equal to  $\zeta$ . We then let

$$\tilde{\epsilon}|_{\zeta} \equiv \max\{\epsilon \in [\bar{\epsilon}|_{\zeta}, \bar{\epsilon}] : q_{\epsilon}(\zeta) = q_{\epsilon}^{\star}\} \quad (53)$$

be the largest preference shock for which the liquidity constraint is slack. We write

$$\begin{aligned} \left( \frac{1-\beta}{\beta} \right) \frac{\mathcal{W}|_{\zeta} - \mathcal{W}|_0}{\zeta} &= \frac{1}{\zeta} \int_{\tilde{\epsilon}|_{\zeta}}^{\bar{\epsilon}} ([\epsilon u(q) - c(q)]_{q=q_{\epsilon}(\zeta)} - [\epsilon u(q) - c(q)]_{q=q_{\epsilon}^{\star}}) G(d\epsilon) \\ &\quad + \frac{1}{\zeta} \int_{\tilde{\epsilon}|_{\zeta}}^{\bar{\epsilon}|_0} [\epsilon u(q) - c(q)]_{q=q_{\epsilon}^{\star}} G(d\epsilon). \end{aligned} \quad (54)$$

**Intensive margin.** The first term on the right-hand side of Equation (54) captures an intensive-margin effect. For all  $\epsilon \in (\tilde{\epsilon}|_{\zeta}, \bar{\epsilon}]$ , the mean-value theorem applied to  $\epsilon u(q) - c(q)$  as a function in  $q$  guarantees the existence of a  $\bar{q}_{\epsilon}(\zeta) \in (q_{\epsilon}(\zeta), q_{\epsilon}^{\star})$ , so that

$$\begin{aligned} [\epsilon u(q) - c(q)]_{q=q_{\epsilon}(\zeta)} - [\epsilon u(q) - c(q)]_{q=q_{\epsilon}^{\star}} &= [\epsilon u'(q) - c'(q)]_{q=\bar{q}_{\epsilon}(\zeta)} [q_{\epsilon}(\zeta) - q_{\epsilon}^{\star}] \\ &= [\epsilon u'(q) - c'(q)]_{q=\bar{q}_{\epsilon}(\zeta)} \left[ \frac{q_{\epsilon}(\zeta) - q_{\epsilon}^{\star}}{[\epsilon u'(q) - c'(q)]_{q=q_{\epsilon}(\zeta)}} \right] \\ &\quad \times \left( \frac{1}{1-\theta} \right) \left[ \frac{(1-\theta)[\epsilon u'(q) - c'(q)]}{\theta \epsilon u'(q) + (1-\theta)c'(q)} \right]_{q=q_{\epsilon}(\zeta)} [\theta \epsilon u'(q) + (1-\theta)c'(q)]_{q=q_{\epsilon}(\zeta)}. \end{aligned} \quad (55)$$

We obtain the estimate

$$\begin{aligned}
& \left| \frac{1}{\zeta} \int_{\bar{\epsilon}|\zeta}^{\bar{\epsilon}} ([\epsilon u(q) - c(q)]_{q=q_\epsilon(\zeta)} - [\epsilon u(q) - c(q)]_{q=q_\epsilon^*}) G(d\epsilon) \right| \\
& \leq \sup_{\epsilon \in [\bar{\epsilon}|\zeta, \bar{\epsilon}]} \{ [\epsilon u'(q) - c'(q)]_{q=q_\epsilon(\zeta)} \} \sup_{\epsilon \in [\bar{\epsilon}|\zeta, \bar{\epsilon}]} \left\{ \frac{|q_\epsilon(\zeta) - q_\epsilon^*|}{[\epsilon u'(q) - c'(q)]_{q=q_\epsilon(\zeta)}} \right\} \\
& \quad \times \left( \frac{1}{1-\theta} \right) \left( \frac{1}{\zeta} \int_{\bar{\epsilon}|\zeta}^{\bar{\epsilon}} \mathcal{L}_\epsilon(\zeta) G(d\epsilon) \right) \sup_{\epsilon \in [\bar{\epsilon}|\zeta, \bar{\epsilon}]} \{ [\theta \epsilon u'(q) + (1-\theta)c'(q)]_{q=q_\epsilon(\zeta)} \}.
\end{aligned} \tag{56}$$

Note that

$$\lim_{\zeta \rightarrow 0} \sup_{\epsilon \in [\bar{\epsilon}|\zeta, \bar{\epsilon}]} \left\{ \frac{|q_\epsilon(\zeta) - q_\epsilon^*|}{[\epsilon u'(q) - c'(q)]_{q=q_\epsilon(\zeta)}} \right\} \leq -\frac{1}{[\bar{\epsilon} u''(q) - c''(q)]_{q=q_\epsilon^*}}, \tag{57}$$

and that the consumers' FOC in Equation (22) for money holdings implies that

$$\frac{1}{\zeta} \int_{\bar{\epsilon}|\zeta}^{\bar{\epsilon}} \mathcal{L}_\epsilon(\zeta) G(d\epsilon) = \frac{1}{\zeta} \int_{\bar{\epsilon}|\zeta}^{\bar{\epsilon}} \mathcal{L}_\epsilon(\zeta) G(d\epsilon) = 1. \tag{58}$$

It clearly follows that

$$\lim_{\zeta \rightarrow 0} \left| \frac{1}{\zeta} \int_{\bar{\epsilon}|\zeta}^{\bar{\epsilon}} ([\epsilon u(q) - c(q)]_{q=q_\epsilon(\zeta)} - [\epsilon u(q) - c(q)]_{q=q_\epsilon^*}) G(d\epsilon) \right| = 0. \tag{59}$$

**Extensive margin.** We can apply *Leibniz's rule* to derive the limit of the second term on the right-hand side of Equation (54) since the integrand is differentiable. We obtain

$$\begin{aligned}
\lim_{\zeta \rightarrow 0} \left( \frac{1}{\zeta} \int_{\bar{\epsilon}|\zeta}^{\bar{\epsilon}|\zeta_0} [\epsilon u(q) - c(q)]_{q=q_\epsilon^*} G(d\epsilon) \right) &= \frac{d}{d\hat{\epsilon}} \left[ \int_{\hat{\epsilon}}^{\bar{\epsilon}} [\epsilon u(q) - c(q)]_{q=q_\epsilon^*} G(d\epsilon) \right] \\
&= -[\hat{\epsilon} u(\hat{q}) - c(\hat{q})] g(\hat{\epsilon}) \frac{d\hat{\epsilon}}{d\zeta} > 0.
\end{aligned} \tag{60}$$

Proposition 2 follows since  $\hat{\epsilon} u(\hat{q}) - c(\hat{q}) = b$  at the FR. ■

**PROOF OF COROLLARY 2.** With Lemma 1, we can express the producers' value of being born in Equation (25) as

$$W^{p,0} = \theta(1-\beta)\mathcal{W} + \beta(1-\theta)(1-G(\hat{\epsilon}))b. \tag{61}$$

Using  $db = u(\hat{q}) d\hat{\epsilon}$  and the producers' FOC in Lemma 5, evaluated at the FR, yields

$$d[(1-G(\hat{\epsilon}))b] = (1-G(\hat{\epsilon})) db - g(\hat{\epsilon})b d\hat{\epsilon} = [(1-G(\hat{\epsilon}))u(\hat{q}) - g(\hat{\epsilon})b] d\hat{\epsilon} = \left( \frac{\theta g(\hat{\epsilon})b}{1-\theta} \right) d\hat{\epsilon}. \tag{62}$$



Proposition 2 with Equation (61) then yields  $dW^{p,0}/d\iota = 0$ .

The consumers' CM value function reads as

$$W^c = \frac{\beta}{1-\beta} \int_{\bar{\epsilon}}^{\bar{\epsilon}} (1-\theta)[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)} G(d\epsilon) = \mathcal{W} - \frac{W^{p,0}}{1-\beta}. \quad (63)$$

Hence,  $dW^c/d\iota = d\mathcal{W}/d\iota$ . It trivially holds that  $dW^{fi}/d\iota = 0$ . ■

**PROOF OF PROPOSITION 3.** Consider a debt contract  $r^b(p) = \min\{p, b\}$  with  $b > 0$ . We define  $\tilde{r}^b$  as in Equation (36) and prove that an individual producer appropriates additional expected gross surplus when deviating from  $r^b$  to  $\tilde{r}^b$  if  $a$  is sufficiently close to  $b$ . Figure 7 illustrates our proof strategy.

**Preference thresholds.** We characterize some useful preference-shock thresholds to determine the outcome of bargaining problem (9), given  $\tilde{r}^b$ . We define

$$\epsilon_a^- \equiv \max\{\epsilon \in [0, \bar{\epsilon}] : \theta \epsilon u(q_\epsilon^*) + (1-\theta)[c(q_\epsilon^*) + a] \leq \hat{p}\} \quad (64)$$

as the largest preference shock  $\epsilon$  for which the agents can trade  $q_\epsilon^*$ , while exchanging a payment weakly smaller than  $\hat{p} \equiv \theta \hat{\epsilon}_b u(q_{\hat{\epsilon}_b}^*) + (1-\theta)[c(q_{\hat{\epsilon}_b}^*) + b]$ .

We define  $q_\epsilon^a$  through  $\theta \epsilon u(q_\epsilon^a) + (1-\theta)[c(q_\epsilon^a) + a] = \hat{p}$  for any  $\epsilon \in [\epsilon_a^-, \bar{\epsilon}]$  and let

$$\epsilon_a^+ \equiv \max\{\epsilon \in [0, \bar{\epsilon}] : \epsilon u(q_\epsilon^a) - \hat{p} \geq (1-\theta)[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - b]\} \quad (65)$$

be the largest preference shock  $\epsilon$  for which the agents can attain a weakly larger net surplus by trading  $q_\epsilon^a$  and transferring  $a$  to the financier than by trading  $q_\epsilon^* > q_\epsilon^a$  and transferring  $b > a$ . Note that  $\epsilon_a^+ > \epsilon_a^- > \hat{\epsilon}_b$  and  $\lim_{a \rightarrow b} \epsilon_a^+ = \hat{\epsilon}_b$ .

**Solution to the bargaining problem.** We derive the terms of trade  $(q_\epsilon, p_\epsilon)$  that solve bargaining problem (9), given  $\tilde{r}^b$ , by distinguishing four possible ranges of  $\epsilon$ . We neglect the liquidity constraint since the FR is in place.

1. Let  $\epsilon \in [0, \hat{\epsilon}_a)$ . No trade occurs, meaning  $(q_\epsilon, p_\epsilon) = (0, 0)$ .
2. Let  $\epsilon \in [\hat{\epsilon}_a, \hat{\epsilon}_b]$ . By the definition of  $\hat{\epsilon}_b$ , a payment  $p > \hat{p}$  would not facilitate a non-negative match surplus. Hence, in every successful match, agents exchange  $p \leq \hat{p}$  and the producer repays  $a$  to the financier. The optimal terms of trade read as

$$(q_\epsilon, p_\epsilon) = (q_\epsilon^*, \theta \epsilon u(q_\epsilon^*) + (1-\theta)[c(q_\epsilon^*) + a]). \quad (66)$$

3. Let  $\epsilon \in (\hat{\epsilon}_b, \epsilon_a^-]$ . By the definition of  $\epsilon_a^-$ , agents can trade  $q_\epsilon^*$  by exchanging a

payment  $p_\epsilon$  that triggers repayment  $\tilde{r}^b(p_\epsilon) = a$ . Agents thus agree on

$$(q_\epsilon, p_\epsilon) = (q_\epsilon^*, \theta \epsilon u(q_\epsilon^*) + (1 - \theta)[c(q_\epsilon^*) + a]). \quad (67)$$

4. Let  $\epsilon \in (\epsilon_a^-, \bar{\epsilon}]$ . Conditional on exchanging  $p_\epsilon > \hat{p}$ , the consumer's optimal surplus is  $(1 - \theta)(\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - b)$ . Conditional on exchanging  $p \leq \hat{p}$ , the consumer's optimal surplus is  $\epsilon u(q_\epsilon^a) - \hat{p}$ . Note that

$$\frac{\partial[(1 - \theta)(\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - b)]}{\partial \epsilon} = (1 - \theta)u(q_\epsilon^*), \quad (68)$$

and that

$$\begin{aligned} \frac{\partial[\epsilon u(q_\epsilon^a) - \hat{p}]}{\partial \epsilon} &= u(q_\epsilon^a) + \epsilon u'(q_\epsilon^a) \frac{\partial q_\epsilon^a}{\partial \epsilon} = u(q_\epsilon^a) \left[ 1 - \frac{\theta \epsilon u'(q_\epsilon^a)}{\theta \epsilon u'(q_\epsilon^a) + (1 - \theta)c'(q_\epsilon^a)} \right] \\ &= u(q_\epsilon^a) \left[ \frac{(1 - \theta)c'(q_\epsilon^a)}{\theta \epsilon u'(q_\epsilon^a) + (1 - \theta)c'(q_\epsilon^a)} \right] < (1 - \theta)u(q_\epsilon^a). \end{aligned} \quad (69)$$

Since moreover

$$\epsilon u(q_\epsilon^a) - \hat{p} < (1 - \theta)[\bar{\epsilon} u(q_\epsilon^*) - c(q_\epsilon^*) - b], \quad (70)$$

the construction of  $\epsilon_a^+$  implies that agents trade  $(q_\epsilon, p_\epsilon) = (q_\epsilon^a, \hat{p})$  if  $\epsilon \in (\epsilon_a^-, \epsilon_a^+]$ , and  $(q_\epsilon, p_\epsilon) = (q_\epsilon^*, \theta \epsilon u(q_\epsilon^*) + (1 - \theta)[c(q_\epsilon^*) + b])$  if  $\epsilon \in (\epsilon_a^+, \bar{\epsilon}]$ .

The terms of trade solving bargaining problem [\(9\)](#) can be expressed as

$$(q_\epsilon, p_\epsilon) = \begin{cases} (0, 0), & \text{if } \epsilon \in [0, \hat{\epsilon}_a), \\ (q_\epsilon^*, \theta \epsilon u(q_\epsilon^*) + (1 - \theta)[c(q_\epsilon^*) + a]), & \text{if } \epsilon \in [\hat{\epsilon}_a, \epsilon_a^-], \\ (q_\epsilon^a, \hat{p}), & \text{if } \epsilon \in (\epsilon_a^-, \epsilon_a^+], \\ (q_\epsilon^*, \theta \epsilon u(q_\epsilon^*) + (1 - \theta)[c(q_\epsilon^*) + b]), & \text{if } \epsilon \in (\epsilon_a^+, \bar{\epsilon}]. \end{cases} \quad (71)$$

The producer's repayment is consequently given by

$$\tilde{r}^b(p_\epsilon) = \begin{cases} 0, & \text{if } \epsilon \in [0, \hat{\epsilon}_a), \\ a, & \text{if } \epsilon \in [\hat{\epsilon}_a, \epsilon_a^+], \\ b, & \text{if } \epsilon \in (\epsilon_a^+, \bar{\epsilon}]. \end{cases} \quad (72)$$

**Producer's lifetime utility.** Given the repayment schedule in Equation (72), the producer's lifetime utility  $\tilde{W}^{p,0}(\tilde{r}^b)$  when writing alternative contract  $\tilde{r}^b$  satisfies

$$\begin{aligned} \frac{1}{\beta} \tilde{W}^{p,0}(\tilde{r}^b) &\equiv \int_{\hat{\epsilon}_a}^{\epsilon_a^-} [\theta[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - a] + a] G(d\epsilon) \\ &+ \int_{\epsilon_a^-}^{\epsilon_a^+} [\theta[\epsilon u(q_\epsilon^a) - c(q_\epsilon^a) - a] + a] G(d\epsilon) + \int_{\epsilon_a^+}^{\bar{\epsilon}} [\theta[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - b] + b] G(d\epsilon). \end{aligned} \quad (73)$$

The producer's lifetime utility  $\tilde{W}^{p,0}(r^b)$  of writing debt contract  $r^b$  satisfies

$$\frac{1}{\beta} \tilde{W}^{p,0}(r^b) \equiv \int_{\hat{\epsilon}_b}^{\bar{\epsilon}} [\theta[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - b] + b] G(d\epsilon). \quad (74)$$

We obtain that

$$\begin{aligned} \frac{1}{\beta} [\tilde{W}^{p,0}(\tilde{r}^b) - \tilde{W}^{p,0}(r^b)] &= \int_{\hat{\epsilon}_a}^{\hat{\epsilon}_b} [\theta[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - a] + a] G(d\epsilon) \\ &- (1 - \theta)[b - a][G(\epsilon_a^+) - G(\hat{\epsilon}_b)] - \theta \int_{\epsilon_a^-}^{\epsilon_a^+} [\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - (\epsilon u(q_\epsilon^a) - c(q_\epsilon^a))] G(d\epsilon). \end{aligned} \quad (75)$$

Lemma 4 yields

$$\lim_{a \rightarrow b} \left( \frac{1}{b - a} \int_{\hat{\epsilon}_a}^{\hat{\epsilon}_b} [\theta[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - a] + a] G(d\epsilon) \right) = bg(\hat{\epsilon}_b) \frac{\partial \hat{\epsilon}}{\partial b} > 0, \quad (76)$$

and it holds that

$$\lim_{a \rightarrow b} \left( (1 - \theta)[G(\epsilon_a^+) - G(\hat{\epsilon}_b)] + \frac{\theta}{b - a} \int_{\epsilon_a^-}^{\epsilon_a^+} [\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - (\epsilon u(q_\epsilon^a) - c(q_\epsilon^a))] G(d\epsilon) \right) = 0. \quad (77)$$

Hence,  $\tilde{W}^{p,0}(\tilde{r}^b) - \tilde{W}^{p,0}(r^b) > 0$  for  $a$  sufficiently close to  $b$ .  $\blacksquare$

**PROOF OF PROPOSITION 4.** We consider an arbitrary equity contract  $r^\mu$ . The terms of trade  $(q_\epsilon^\mu, p_\epsilon^\mu)$ , solving bargaining problem (9), given  $r^\mu$ , are

$$(q_\epsilon^\mu, p_\epsilon^\mu) = \left( q_{(1-\mu)\epsilon}^*, \frac{\theta \epsilon u(q_{(1-\mu)\epsilon}^*) + (1 - \theta)c(q_{(1-\mu)\epsilon}^*)}{\theta + (1 - \theta)(1 - \mu)} \right) \quad (78)$$

for all  $\epsilon \in [0, \bar{\epsilon}]$ . We define contract  $\tilde{r}^\mu(p) = \mu \min\{p, p_\epsilon^\mu\}$  as a hybrid of equity and debt; contract  $\tilde{r}^\mu$  takes the form of equity for  $p < p_\epsilon^\mu$  and the form of debt for  $p \geq p_\epsilon^\mu$ . We show that a producer increases her expected gross surplus when deviating from  $r^\mu$  to  $\tilde{r}^\mu$ .

If the terms of trade  $(q_\epsilon^h, p_\epsilon^h)$  that solve bargaining problem (9), given  $\tilde{r}^\mu$ , feature

$p_\epsilon^h < p_\epsilon^\mu$ , they are  $(q_\epsilon^h, p_\epsilon^h) = (q_\epsilon^\mu, p_\epsilon^\mu)$ ; if  $p_\epsilon^h \geq p_\epsilon^\mu$ , they are  $(q_\epsilon^h, p_\epsilon^h) = (q_\epsilon^*, \theta \epsilon u(q_\epsilon^*) + (1 - \theta)(c(q_\epsilon^*) + \mu p_\epsilon^\mu))$ . The terms of trade thus feature  $p_\epsilon^h \geq p_\epsilon^\mu$  if and only if

$$\begin{aligned} \epsilon u(q_\epsilon^*) - [\theta \epsilon u(q_\epsilon^*) + (1 - \theta)(c(q_\epsilon^*) + \mu p_\epsilon^\mu)] \\ \geq \epsilon u(q_{(1-\mu)\epsilon}^*) - \frac{\theta \epsilon u(q_{(1-\mu)\epsilon}^*) + (1 - \theta)c(q_{(1-\mu)\epsilon}^*)}{\theta + (1 - \theta)(1 - \mu)}. \end{aligned} \quad (79)$$

Inequality (79) holds when being evaluated at  $\epsilon = \bar{\epsilon}$  since

$$\begin{aligned} \bar{\epsilon} u(q_{\bar{\epsilon}}^*) - [\theta \bar{\epsilon} u(q_{\bar{\epsilon}}^*) + (1 - \theta)(c(q_{\bar{\epsilon}}^*) + \mu p_{\bar{\epsilon}}^\mu)] \\ = \bar{\epsilon} u(q_{(1-\mu)\bar{\epsilon}}^*) - p_{\bar{\epsilon}}^\mu + (1 - \theta)[\bar{\epsilon} u(q_{\bar{\epsilon}}^*) - c(q_{\bar{\epsilon}}^*) - (\bar{\epsilon} u(q_{(1-\mu)\bar{\epsilon}}^*) - c(q_{(1-\mu)\bar{\epsilon}}^*))] \\ > \bar{\epsilon} u(q_{(1-\mu)\bar{\epsilon}}^*) - p_{\bar{\epsilon}}^\mu, \end{aligned} \quad (80)$$

and it does not hold when being evaluated at  $\epsilon = 0$ . By the mean-value theorem, there is an  $\tilde{\epsilon}_\mu \in [0, \bar{\epsilon}]$ , so that Inequality (79) holds with equality. More specifically, Inequality (79) holds if and only if  $\epsilon \in [\tilde{\epsilon}_\mu, \bar{\epsilon}]$  since the first-order derivative of the left-hand side of Inequality (79) w.r.t.  $\epsilon$  is  $(1 - \theta)u(q_\epsilon^*)$  and the derivative of the right-hand side is

$$\frac{(1 - \theta)(1 - \mu)u(q_{(1-\mu)\epsilon}^*)}{\theta + (1 - \theta)(1 - \mu)} < (1 - \theta)u(q_\epsilon^*). \quad (81)$$

When deviating from  $r^\mu$  to  $\tilde{r}^\mu$ , a producer thus realizes lifetime-utility gains satisfying

$$\begin{aligned} \frac{1}{\beta} [\tilde{W}^{p,0}(\tilde{r}^\mu) - \tilde{W}^{p,0}(r^\mu)] &= \int_{\tilde{\epsilon}_\mu}^{\bar{\epsilon}} [-c(q_\epsilon^*) + p_\epsilon^h - (-c(q_{(1-\mu)\epsilon}^*) + p_\epsilon^\mu)] G(d\epsilon) \\ &= \int_{\tilde{\epsilon}_\mu}^{\bar{\epsilon}} [\theta[\epsilon u(q_\epsilon^*) - c(q_\epsilon^*) - (\epsilon u(q_{(1-\mu)\epsilon}^*) - c(q_{(1-\mu)\epsilon}^*))] + (1 - \theta)\mu(p_\epsilon^\mu - p_\epsilon^\mu)] G(d\epsilon) > 0. \end{aligned} \quad (82)$$

■

# KOF

ETH Zurich  
KOF Swiss Economic Institute  
LEE G 116  
Leonhardstrasse 21  
8092 Zurich, Switzerland

Phone +41 44 632 42 39  
[kof@kof.ethz.ch](mailto:kof@kof.ethz.ch)  
[www.kof.ch](http://www.kof.ch)

© KOF Swiss Economic Institute

