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Müller, Lars; Karos, Dominik

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Lars Müller and Dominik Karos



Center for Mathematical Economics (IMW) Bielefeld University Universitätsstraße 25 D-33615 Bielefeld · Germany

e-mail: imw@uni-bielefeld.de uni-bielefeld.de/zwe/imw/research/working-papers ISSN: 0931-6558



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Welfare Effects of a Concealed Information Exchange^{*}

Lars Müller[†] Don

Dominik Karos[‡]

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Abstract

This paper analyzes the welfare effects of private and unilateral disclosure of sensitive information in a sequential bargaining context. We consider a model where two sellers each propose a take-it-or-leave-it price for a homogeneous good to a single buyer. The buyer accepts or rejects the first seller's offer before the second seller proposes her price. Crucially, the second seller might learn the first seller's price and whether it was accepted, allowing her to update her belief about the buyer's willingness to pay and optimize her pricing strategy. The welfare effects caused by this information exchange are evaluated under general conditions. We show that it benefits the buyer if a rejection is revealed but might harm him if an acceptance is revealed. Additionally, the information exchange improves the societal welfare by reducing inefficiencies and promoting additional trade. This paper strengthens the theoretical framework for assessing the welfare effects of information exchanges by offering new insights and providing tools to assess causality for alleged damages.

Keywords: Information Exchange; Collusion; Unawareness **JEL Classification:** D82; D83; L41

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[†]Center for Mathematical Economics, Bielefeld University, Postfach 100131, 33501 Bielefeld, Germany. Imueller33@uni-bielefeld.de, corresponding author. Lars Müller gratefully acknowledges financial support by the German Research Foundation (DFG) [RTG 2865/1 – 492988838].

[‡]Center for Mathematical Economics, Bielefeld University, Postfach 100131, 33501 Bielefeld, Germany. dominik.karos@uni-bielefeld.de

1 Introduction and Motivation

The exchange of information among competing firms plays a crucial role in shaping market outcomes. Additional information can contribute to market efficiencies, but can also influence pricing behavior to the consumers' detriment. Because of the latter, antitrust law is quite restrictive with respect to information exchange, and the last few years have seen several cases where firms have faced substantial fines for exchanging sensitive information. For instance, the European Commission imposed fines of about 32 million euros on two metal packaging companies (European Commission, 2022), and the German Federal Cartel Office fined steel forging companies approximately 35 million euros for exchanging information in violation of antitrust law (German Federal Cartel Office, 2021).¹

It seems to be an established principle to assume that private announcements among competing firms can only be motivated by assisting price-collusion and do not enhance efficiency, as argued by the OECD (2012). This paper demonstrates that such a principle must be questioned. We show that even private and unilateral disclosure of information can enhance efficiency and enable welfare-improving trade, which might even benefit third party consumers. Such an exchange among competing firms is nothing extraordinary: for instance, trade associations are platforms where information is easily and often shared.² With their sheer ubiquity of almost 60,000 in the U.S. alone,³ the importance of correctly assessing the positive and negative effects of information exchange becomes obvious.

This paper analyzes these effects in a sequential trading game and assesses the potential harm for a consumer unaware of any information exchange. This provides a natural worst-case analysis from their perspective. In this model one buyer sequentially bargains with two different sellers who offer a homogeneous good. Both sellers know the distribution underlying the buyer's willingness to pay but not its realization when offering a take-it-or-leave-it price for their good. The buyer accepts or rejects the first offer before the second seller proposes her price. There are no satiation effects and the buyer might buy zero, one, or two goods. The important feature of the model is that the subsequent seller might additionally receive information about the course of the preceding negotia-

¹In numerous other cases, severe fines were imposed on firms accused of having exchanged commercially sensitive information. See European Commission (2020), where ethylene purchasers were fined 260 million euros, and European Commission (2021), where investment banks were fined 371 million euros.

²See, for example, fines imposed on the Association of the German Confectionery Industry for encouraging an anti-competitive exchange of information (German Federal Cartel Office, 2019).

³See IRS (2024), the IRS refers to professional and trade associations as business leagues.

tion. Two scenarios are compared: a benchmark scenario in which the second seller does not receive any additional information and an information exchange scenario in which the first seller reveals to the second seller both her offered price and whether or not it was accepted. In both scenarios, the buyer is unaware of the possibility of an information exchange between the two sellers.

The welfare effects caused by this information exchange are evaluated under general conditions. Most notably, it is shown that the buyer benefits if a rejection is revealed but might be harmed if an acceptance is revealed. From an ex-ante point of view the information exchange can be positive or negative for the consumer. Societal welfare is always increased by the information exchange as it reduces inefficiencies and promotes additional trade.

The remainder of this paper is structured as follows. In Section 2 some related literature is presented. Section 3 introduces the bilateral trading stage game before Section 4 sets up the sequential trading model and analyses the equilibria following an acceptance or rejection. In Section 5 the welfare effects caused by the information exchange are analysed from different timing perspectives. Section 6 concludes. All proofs are relegated to the appendix.

2 Related Literature

While competition law in the United States and the European Union clearly prohibits firms from colluding on prices, there is no unified position on the sole exchange of prices among firms. Harrington (2022) criticizes this lack of a common treatment and attributes it, at least in part, to the "absence of a well-established theory of harm". He establishes a theoretical foundation that shows how a private exchange of prices between competitors results in higher consumer prices in a standard duopoly setting. Our paper differs as it assesses the economic effects of information exchanges between firms, who act as information providers and receivers.⁴ Albeit from a different perspective, our paper hence contributes to the establishment of a solid theory of harm of information exchanges by distinguishing general conditions under which third parties are harmed by or benefit from a private exchange of sensitive information.

 $^{^{4}}$ See Kirby (1988) for a discussion of the incentives of oligopolistic firms to use trade associations as an information exchange mechanism.

Information exchanges and their impact on competition have been thoroughly examined within oligopolistic models in which firms ex-ante voluntarily disclose private information. Kühn and Vives (1995) survey this strand of literature. Our paper differs from this perspective as the analyzed information exchange can not alter the welfare of the information provider. This also distinguishes the current paper from the information design literature as surveyed by Bergemann and Morris (2019).

The bilateral trading stage game in this paper resembles that of Myerson and Satterthwaite (1983) with independently distributed valuations and costs and asymmetric information. They show that no incentive-compatible and individually rational trading mechanism can be ex-post efficient. The minimal efficiency loss in this setting when jointly using a mechanism and the optimal information structure is quantified by Schottmüller (2023). In our paper, however, the trading mechanism is fixed, and we compare the welfare effects that result from the information exchange.

Bergemann, Brooks, and Morris (2015) evaluate the welfare consequences of additional information that is provided to a monopolist. The monopolist uses this information about the valuation of consumers and engages in third degree price discrimination. They characterize the possible consumer and producer surplus that can be implemented in this setting. The limits of these characterizations are also the limits for the welfare of the buyer and the second seller in this model. Ali, Kleiner, and K. Zhang (2024) provide an equivalence result between the achievable payoff profiles given by these limits and the payoffs supported by an equilibrium in a game of voluntary disclosure.

Glode and Opp (2016) is the closest paper in terms of the model setting. They also analyze sequential bilateral trading games with sellers making take-it-or-leave-it offers but focus on the effect of intermediation chains between a seller and a buyer. They show that the addition of moderately informed intermediaries improves trade efficiency by reducing the incentive for the seller to inefficiently screen the buyer for his valuation. This counterincentive to screen for the buyer's valuation is also at the core of the welfare-enhancing effects of the information exchange in our paper. However, their analysis focuses on implementing first-best trade and assumes that trade is always mutually beneficial. In a very similar setting, Glode, Opp, and X. Zhang (2018) analyze the incentives for a buyer to voluntarily disclose private information both in an ex-ante and in an interim stage of the game. Yet again, their focus is on the implementation of first-best trade. Our paper hence differs by scrutinizing offers that might be rejected with positive probability. Within this paper, players consider information about their negotiations by default as private to avoid distorting the analysis by effects that may arise from the anticipation of disclosed information. On the one hand side, this allows us to achieve an unbiased assessment of the buyer's worst-case welfare effects. On the other hand side, the buyer's unawareness ensures that we do not need to be concerned about reputation building as in Milgrom and Roberts (1982) or Kreps and Wilson (1982).

3 The Bilateral Trading Game

To analyze the welfare effects of an information exchange in a sequential bargaining context, it is necessary to first consider the individual stage game, which is played repeatedly. This section defines this stage game and analyses its equilibrium strategies and payoffs.

3.1 Notation and Preliminaries

In a bilateral trading game with incomplete information, a risk-neutral seller (she) offers a single indivisible good at a price $p \ge 0$ to a risk-neutral buyer (he) who either accepts or rejects that offer. The buyer's private valuation for the good is denoted by $v \ge 0$, and the seller's (opportunity) cost for producing it is given by $c \ge 0$. If the buyer accepts, the good is exchanged, and payoffs are v - p for the buyer and p - c for the seller. In case of a rejection, players obtain zero payoffs. Both v and c are independently distributed according to commonly known probability distributions F and G with densities f and g. Thus, any bilateral trading game is uniquely determined by the pair (f, g), and, with slight abuse of notation, we identify the game by this pair.

We denote the (set-theoretic) supports of f and g by

$$\mathcal{V} := \{ v \in \mathbb{R}_{\geq 0} : f(v) > 0 \} \text{ and } \mathcal{C} := \{ c \in \mathbb{R}_{\geq 0} : g(c) > 0 \},\$$

respectively. To ease notation, let $\underline{v} := \inf(\mathcal{V}), \ \overline{v} := \sup(\mathcal{V}), \ \underline{c} := \inf(\mathcal{C}), \ \text{and} \ \overline{c} := \sup(\mathcal{C}).$ Throughout this paper, all densities are assumed to be continuous on their supports.⁵

A pure strategy for the seller is a map $\sigma : \mathcal{C} \to \mathbb{R}$ that maps any realized cost c to a take-it-or-leave-it price p. A pure strategy for the buyer assigns to any pair (v, p) of a valuation and an offered price a rejection or acceptance.

⁵For technical reasons, f is w.l.o.g. assumed to be right-continuous at \underline{v} , i.e., $f(\underline{v}) = \lim_{v \searrow \underline{v}} f(v)$.

The following standard assumption is imposed on the buyer's distribution.⁶

Assumption 1. The hazard rate $h : \mathcal{V} \setminus \{\overline{v}\} \to \mathbb{R}_{\geq 0}$, defined by $h(v) := \frac{f(v)}{1 - F(v)}$, is non-decreasing for all $v \in \mathcal{V} \setminus \{\overline{v}\}$.

Note that Assumption 1 ensures the convexity of \mathcal{V} and, therefore, guarantees that F(v) is strictly increasing for all $v \in [\underline{v}, \overline{v})$.

3.2 Equilibrium Strategies

In a perfect Bayesian equilibrium (henceforth, just equilibrium), the buyer will accept any offer p < v and reject any offer p > v.⁷ Given this strategy, the seller's expected payoff from offering p, given her own cost c, is

$$\Pi(p;c) = (1 - F(p)) (p - c).$$
(1)

Hence, if $c > \overline{v}$, there is no scope for trade in equilibrium; and if c is sufficiently low such that it is not worthwhile for the seller to risk a rejection, she will simply offer \underline{v} and make a profit of $\underline{v} - c$ with probability 1.

In the middle ground, the seller faces the tradeoff that higher prices are rejected with higher probability. One can easily check that an interior point p that maximizes (1) must satisfy the first order condition

$$h(p)(p-c) = 1.$$
 (2)

We denote the solution of (2) by $\hat{p}(c)$. Then, there exists a unique c with $p = \hat{p}(c)$ for all $p \in (\underline{v}, \overline{v})$ by Assumption 1. Indeed, solving (2) for c quickly gives $\hat{p}^{-1}(v) = v - \frac{1}{h(v)}$.

⁷In order to avoid uninteresting case distinctions, we assume that for p = v the offer will be accepted. ⁸It might be worth mentioning here that the inverse \hat{p}^{-1} takes the form of a virtual valuation in

⁶The assumption is, for instance, satisfied by a log-concave density (Bagnoli and Bergstrom, 2005, Corollary 2) and therefore by a wide range of probability distributions, e.g., the uniform distribution on any convex set, the exponential distribution, and the gamma distribution for shape parameters greater or equal to one. It is also satisfied by the Kumaraswamy distribution $\mathcal{K}(a, b)$ with parameters $a \geq 1$ and b > 0 that allows for a wide range of densities and, in contrast to the related Beta-distribution, can be expressed in closed form (Kumaraswamy, 1980). Assumption 1 is also tightly connected to the omnipresent regularity assumption in auction theory, which ensures strictly increasing virtual valuations (Myerson, 1981) as well as to Assumption 1 in Glode and Opp (2016) and equivalent to the second hazard rate condition in McAfee (2002).

Thus, define

$$\tau = \tau(f) \equiv \lim_{v \searrow \underline{v}} \hat{p}^{-1}(v) = \begin{cases} \underline{v} - \frac{1}{h(\underline{v})}, & \text{if } f(\underline{v}) > 0\\ -\infty, & \text{if } f(\underline{v}) = 0. \end{cases}$$
(3)

This provides us with a lower bound of the domain of \hat{p} . The following Lemma summarizes all properties of \hat{p} and is used for later reference.

Lemma 3.1. The map $\hat{p} : (\tau(f), \overline{v}) \to (\underline{v}, \overline{v})$ is a continuous and strictly increasing bijection with inverse $\hat{p}^{-1}(v) = v - \frac{1}{h(v)}$ for all $v \in (\underline{v}, \overline{v})$. In particular, $\hat{p}(c) > c$ for all $c \in (\tau(f), \overline{v})$.

With slight abuse of notation, we shall write $\hat{p}^{-1}(\underline{v}) = \tau(f)$ and $\hat{p}^{-1}(\overline{v}) = \overline{v}$, keeping in mind that $\tau(f)$ and \overline{v} might be $-\infty$ and $+\infty$, respectively. After having defined \hat{p} , we can now describe the seller's equilibrium strategies in the area where they are unique.

Lemma 3.2. Every bilateral trading game (f, g) has an equilibrium. In any equilibrium, the seller's strategy $\hat{\sigma}$ satisfies for all $c \in C$

$$\hat{\sigma}(c) = \hat{\sigma}(c; f, g) = \begin{cases} \underline{v}, & \text{if } c \le \tau(f) \\ \hat{p}(c), & \text{if } \tau(f) < c < \overline{v}. \end{cases}$$

Moreover, $\hat{\sigma}$ is continuous in c for all $c \in \mathcal{C}$ with $c < \overline{v}$.

The monotonicity of \hat{p} implies, unsurprisingly, that higher cost lead to higher proposed prices. So far, no assumptions about the relation of f and g were made. The following remark captures the trivial cases.

Remark 3.3. In a bilateral trading game (f, g) the following hold:

- i) If $\overline{c} < \tau(f)$, then $\hat{\sigma} \equiv \underline{v}$ and trade happens with certainty at price \underline{v} .
- ii) If $\underline{c} > \overline{v}$, then mutually beneficial trade is ex-ante impossible. In equilibrium, any price $p \ge c$ can be chosen, and any offer will be rejected.

classical auction theory (see Myerson, 1981). As in this strand of literature, the strict monotonicity of \hat{p} and \hat{p}^{-1} is ensured by Assumption 1.

3.3 Equilibrium Payoffs

Consider the bilateral trade game (f, g). The ex-post equilibrium payoffs are

$$\hat{u}(v,c) = \hat{u}(v,c;f,g) \equiv \begin{cases} v - \hat{\sigma}(c), & \text{if } v \ge \hat{\sigma}(c) \\ 0, & \text{otherwise} \end{cases}$$
(4)

for the buyer, and the seller obtains

$$\hat{\pi}(v,c) = \hat{\pi}(v,c;f,g) \equiv \begin{cases} \hat{\sigma}(c) - c, & \text{if } v \ge \hat{\sigma}(c) \\ 0, & \text{otherwise.} \end{cases}$$
(5)

Together with the description of the seller's equilibrium strategies in Lemma 3.2, we can derive the buyer's interim payoffs for all $v \in \mathcal{V}$, namely

$$\hat{U}(v) = \hat{U}(v; f, g) \equiv \mathbb{E}_{G} \left[\hat{u} \left(v, c; f, g \right) \right]
= \int_{-\infty}^{\tau(f)} g(c) \left(v - \underline{v} \right) dc + \int_{\tau(f)}^{\hat{p}^{-1}(v)} g(c) \left(v - \hat{p}(c) \right) dc.$$
(6)

In congruence with Remark 3.3, if either $\overline{c} < \tau(f)$ or $\underline{c} > \overline{v}$, then $\hat{U}(v) \equiv 0$ for all $v \in \mathcal{V}$. The following Lemma establishes some properties of \hat{U} for the non-trivial case.

Lemma 3.4. Let (f,g) be a bilateral trading game. Then \hat{U} is non-negative, continuously differentiable, non-decreasing, and convex. In the non-trivial case with $\overline{c} \ge \tau(f)$ and $\underline{c} \le \overline{v}$ the following hold:

- 1. If $\underline{c} < \tau(f)$, then \hat{U} is strictly increasing.
- 2. If $\underline{c} \ge \tau(f)$, then \hat{U} is strictly increasing for $v > \hat{p}(\underline{c})$.

We conclude this section with the following standard example of Myerson and Satterthwaite (1983). It provides simple illustrations for the ex-post occurrence of trade and equilibrium utility of the buyer in the bilateral trading game.

Example 3.5. Let v and c be uniformly distributed on [0, 1]. Then, trade happens if and only if $v \ge \frac{1+c}{2}$, which means that not all beneficial trades occur as depicted in Figure 1a. The buyer's interim payoff, depicted in Figure 1b, is $\hat{U}(v) = \frac{(2v-1)^2}{4}$ for $v \ge \frac{1}{2}$ and $\hat{U}(v) = 0$ for $v \le \frac{1}{2}$.



black: no beneficial trade possible; grey: beneficial trade possible but not occurring.

(b) The buyer's interim payoff function.

Figure 1: Equilibrium outcomes and payoffs in a simple bilateral trade game.

Sequential Bilateral Trading 4

This section provides a model to analyze the welfare effects caused by the unilateral and private disclosure of sensitive information in a sequential bargaining context. The setting is chosen to appropriately model a stylized situation in which several manufacturers in a trademark association negotiate one after the other with one particular retailer, matching the context of the information exchanges described in this paper's introduction.

Consider two sequentially played bilateral trading games with one fixed buyer with valuation v and two different sellers with independently distributed costs c_1 and c_2 , respectively.⁹ The buyer is indifferent between the objects of both sellers and not prone to satiation effects. Hence, he may buy zero, one, or two goods depending on his valuation being higher than the respective offers. We denote such a sequential trading game by $(f, g_1, g_2).$

Suppose the second seller observes the outcome of the first game, that is, she observes the offered price p_1 and whether it was accepted or rejected. This might reveal additional information about the buyer's valuation v, and hence affect the offered price. The purpose

⁹Note that the two costs need not be identically distributed. Therefore, we denote by G_i the distribution according to which c_i is drawn and by g_i its density with corresponding support C_i .

of this analysis is to describe and measure the welfare effects caused by such a unilateral information disclosure that remains invisible to the buyer.

There are trivial cases where no effects occur: if $c_1 > \overline{v}$, some arbitrary offer above \overline{v} will be made and rejected, the second seller learns nothing about f. If $c_1 \leq \tau(f)$, then the offer \underline{v} will be made and accepted with certainty, but again the second seller learns nothing. In order to avoid these trivial cases, which provide nothing but heavy notation, we make the following assumption.

Assumption 2. It holds that $\tau(f) < c_1 < \overline{v}$ for all $c_1 \in \mathcal{C}_1$.

Under Assumption 2, $\hat{\sigma}(c_1) = \hat{p}(c_1)$ for all $c_1 \in C_1$. Moreover, any action profile (p, s) with $p \in \hat{p}(C_1)$ and $s \in \{A(ccept), R(eject)\}$ can occur in equilibrium. Thus, any action profile reveals some information to the second seller about the buyer's valuation.

Remark 4.1. We make two important premises here. First, the buyer does not consider the possibility that any information concerning his first game becomes available to the second seller. In this sense, his strategy is misspecified. Yet, this unawareness is necessary to assess the welfare effects of a secret information disclosure: If the buyer were aware of it, he could strategically reject offers in the first game, in order to feign a low valuation. This would skew the assessment as pricing effects that are caused by the information disclosure would interact with those due to strategic rejections. Our approach also provides a natural worst-case scenario from the buyer's perspective. We will see later that despite his unawareness, there are many parameter constellations where, at the ex-post, interim, or ex-ante level, the buyer is not harmed at all by the information disclosure.

Our second premise is more standard. Namely, that the second seller is a perfect Bayesian player who interprets every additional information correctly. \Box

Remark 4.2. It is natural to question the incentives for the first player to share information with the second seller. To rationalize such an information exchange, consider two sellers who know that they will bargain with the same buyer but do not know with whom the buyer will negotiate first. By committing to disclose the information about their respective outcome if they are the first seller, they improve their expected profit: The information provider's welfare is not affected by the information exchange, but the information receiver can only profit by exploiting her additional information.¹⁰ Even without

¹⁰This follows from Blackwell's informativeness theorem (Blackwell, 1953; de Oliveira, 2018).

commitment such behavior is possible if both sellers interact repeatedly with the buyer but change their roles from time to time, and the draws of variables across rounds are independent. In this case there is no reason not to share information, yet at the same time, the information disclosure generates a Pareto improvement across sellers. \Box

We make the standard assumption that a buyer with valuation v accepts the first offer if $\hat{p}(c_1) = v$.¹¹ Recall that \hat{p} is a bijection. Thus, as the second seller knows that the first player is playing her equilibrium strategy, observing p_1 allows her to recover $c_1 = \hat{p}^{-1}(p_1)$. To ease notation, the conditional probability that the buyer's valuation is at most v, given that the offer $\hat{p}(c_1; f, g_1)$ was accepted (A) or rejected (R) is respectively denoted by

$$F_{c_1}^A(v) = F(v|V \ge \hat{p}(c_1; f, g_1)),$$

$$F_{c_1}^R(v) = F(v|V < \hat{p}(c_1; f, g_1)),$$

where V is the random variable governing the buyer's valuation. Analogously, we denote the corresponding densities, supports, and hazard rates by $f_{c_1}^A$, $f_{c_1}^R$, $\mathcal{V}_{c_1}^A$, $\mathcal{V}_{c_1}^R$, $h_{c_1}^A$, and $h_{c_1}^R$.

The truncation of F has no effect on the monotonicity of the hazard rates. As this result is standard, we do not provide a proof for our next Lemma but refer to Lemma 1 in Glode and Opp (2016).

Lemma 4.3. Both $h_{c_1}^A$ and $h_{c_1}^R$ are non-decreasing on their respective support.

Lemma 4.3 ensures that all our results from Section 3 apply to the trading game between the second seller and the buyer as well.¹² In particular, we can define maps

$$\hat{p}_{c_{1}}^{A}:\left(\tau\left(f_{c_{1}}^{A}\right),\overline{v}\right)\to\left(\hat{p}\left(c_{1};f,g_{1}\right),\overline{v}\right),\quad\hat{p}_{c_{1}}^{A}\left(c_{2}\right)=\hat{p}\left(c_{2};f_{c_{1}}^{A},g_{2}\right)\\\hat{p}_{c_{1}}^{R}:\left(\tau\left(f_{c_{1}}^{R}\right),\hat{p}\left(c_{1};f,g_{1}\right)\right)\to\left(\underline{v},\hat{p}\left(c_{1};f,g_{1}\right)\right),\quad\hat{p}_{c_{1}}^{R}\left(c_{2}\right)=\hat{p}\left(c_{2};f_{c_{1}}^{R},g_{2}\right)$$

that describe the second seller's equilibrium behavior as in Lemma 3.2 following an acceptance or rejection, respectively. To ease notation, for i = 1, 2, let $\hat{p}(c_i) = \hat{p}(c_i; f, g_i)$

¹¹One might argue that in case of indifference, he should reject the offer in order to get a better offer in the second round. However, note first that he does not know about the information disclosure, so this argument is invalid. Second, it does not make significant differences in the analysis: at the ex-post and interim stage, weak inequalities and strict inequalities need to be exchanged; at the ex-ante stage there is no difference at all as $v = \hat{p}(c_1)$ is a null event.

¹²In fact, repeated application of Lemma 4.3 proves that even several consecutive bargaining games, in which sellers observe the previous outcomes, retain this structure.

for all $c_i \in [\tau(f), \overline{v}]$, and let $\hat{\sigma}(c_i) = \hat{\sigma}(c_i; f, g_i)$ for all $c_i \in C_i$. These maps describe the equilibrium behavior in case that no information has been disclosed. The next corollary is now clear and stated without proof.

Corollary 4.4. Let (f, g_1, g_2) be a sequential trading game. For every $c_1 \in C_1$ the bilateral trading games $(f_{c_1}^A, g_2)$ and $(f_{c_1}^R, g_2)$ have an equilibrium. The second seller's equilibrium strategies, denoted by $\hat{\sigma}_{c_1}^A$ and $\hat{\sigma}_{c_1}^R$, satisfy for all $c_2 \in C_2$

$$\hat{\sigma}_{c_{1}}^{A}(c_{2}) = \hat{\sigma}\left(c_{2}; f_{c_{1}}^{A}, g_{2}\right) = \begin{cases} \hat{p}\left(c_{1}\right), & \text{if } c_{2} \leq \tau\left(f_{c_{1}}^{A}\right) \\ \hat{p}_{c_{1}}^{A}\left(c_{2}\right), & \text{if } \tau\left(f_{c_{1}}^{A}\right) < c_{2} < \overline{v}, \end{cases}$$
$$\hat{\sigma}_{c_{1}}^{R}\left(c_{2}\right) = \hat{\sigma}\left(c_{2}; f_{c_{1}}^{R}, g_{2}\right) = \begin{cases} \underline{v}, & \text{if } c_{2} \leq \tau\left(f_{c_{1}}^{R}\right) \\ \hat{p}_{c_{1}}^{R}\left(c_{2}\right), & \text{if } \tau\left(f_{c_{1}}^{R}\right) < c_{2} < \hat{p}\left(c_{1}\right) \end{cases}$$

Moreover, $\hat{\sigma}_{c_1}^A$ is continuous for all $c_2 \in \mathcal{C}_2 \cap (-\infty, \overline{v})$, and $\hat{\sigma}_{c_1}^R$ is continuous for all $\mathcal{C}_2 \cap (-\infty, \hat{p}(c_1))$.

4.1 The Equilibrium After an Acceptance

Intuitively, we would not expect the buyer to gain from the information disclosure if he has accepted the first offer: such information would make the second seller only more optimistic about the price she can charge. Our first proposition gives a crisp answer to when there is an effect at all.

Proposition 4.5. Let (f, g_1, g_2) be a sequential trading game, and let $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$, and $c_2 \in \mathcal{C}_2$ with $v \ge \hat{p}(c_1)$, then $\hat{\sigma}(c_2; f_{c_1}^A, g_2) \ge \hat{\sigma}(c_2; f, g_2)$. More precisely,

$$\hat{\sigma}(c_2; f_{c_1}^A, g_2) = \begin{cases} \hat{p}(c_1), & \text{if } c_2 \le c_1 \\ \hat{p}(c_2), & \text{if } c_1 < c_2 < \overline{v}. \end{cases}$$

After observing that some offered price p_1 was accepted in the previous game, the second seller will either offer exactly the observed price $\hat{p}(c_1)$ (namely, if $c_2 \leq c_1$), or she will not change her offer at all (if $c_2 > c_1$). Hence, as expected, observing an acceptance weakly increases the price that the second seller will offer. This follows from the structure of the equilibrium pricing strategy and the fact that any equilibrium offer that is accepted with probability strictly between 0 and 1 is determined by the hazard rate of the buyer's distribution. Since the seller observes an acceptance, she truncates the buyer's distribution from below at $\hat{p}(c_1)$. However, this leaves its hazard rate unchanged. Thus, $h(v) = h_{c_1}^R(v)$ for all $v \ge \hat{p}(c_1)$ and the equilibrium price coincides with the benchmark.

4.2 The Equilibrium After a Rejection

Suppose that the second seller observes that some offer $\hat{p}(c_1) < \bar{v}$ has been rejected. If $c_2 \ge \hat{p}(c_1)$, beneficial trade will be impossible in the second period, and would have been in the benchmark as well. In the following we consider the case that $c_2 < \hat{p}(c_1)$. Recall first that for the posterior distribution $F_{c_1}^R$, we have $\underline{v}_{c_1}^R = \underline{v}$, and $\overline{v}_{c_1}^R = \hat{p}(c_1)$. A quick computation also delivers

$$\tau\left(f_{c_1}^R\right) = \left(\hat{p}_{c_1}^R\right)^{-1}(\underline{v}) = \begin{cases} \tau(f) + \frac{1 - F(\hat{p}(c_1))}{f(\underline{v})} & \text{if } f(\underline{v}) > 0\\ -\infty & \text{if } f(\underline{v}) = 0. \end{cases}$$

The next proposition shows that the second seller's offer can never be larger than in the benchmark. In fact, for any cost c_2 that would have caused a rejection in the benchmark equilibrium with positive probability, the second seller will now make a lower offer.

Proposition 4.6. Let (f, g_1, g_2) be a sequential trading game and let $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$, and $c_2 \in \mathcal{C}_2$, such that $v, c_2 < \hat{p}(c_1)$. Then $\hat{\sigma}_{c_1}^R(c_2) \leq \hat{\sigma}(c_2)$. If, additionally, $c_2 > \tau(f)$, then the inequality is strict.

Recall that Lemma 4.3 allows us to consider more than two consecutive games. Suppose now a buyer rejects an offer $\hat{\sigma}(c_2)$ in a first game and $\hat{\sigma}_{c_1}^R(c_2)$ in the consecutive game. A potential third seller who holds the information that $\hat{\sigma}_{c_1}^R(c_2)$ was rejected does not receive additional information from the fact that $\hat{\sigma}(c_2)$ has been rejected in the first place. This follows from the fact that among multiple rejected offers only the lowest one truncates her belief about the buyer's willingness to pay. However, by Proposition 4.6 her offer has to be lower than the lowest observed rejected offer. This reasoning delivers the following corollary, whose proof is omitted.

Corollary 4.7. Let (f, g_1, g_2) be a sequential trading game and let $v \in \mathcal{V}$ and $c_1, c'_1 \in \mathcal{C}_1$ be such that $c_1 < c'_1$ and $v < \hat{p}(c_1)$. Then $\hat{\sigma}^R_{c_1}(c_2) < \hat{\sigma}^R_{c'_1}(c_2)$ for all c_2 with $\tau(f^R_{c_1}) < c_2 < \hat{p}(c_1)$. The results of Proposition 4.6 and Corollary 4.7 are not that surprising: if the second seller has observed a rejection, she will offer a lower price; and the lower the rejected price she observed, the lower her price will be. This has two effects: First, there is a higher chance that her offer will be accepted. Second, in case her offer is accepted, the buyer will pay a lower price than in the benchmark. As the first effect only appears for buyer valuations that are sufficiently small, the buyer's gain depends on v.

5 Welfare Effects

We shall now investigate the gains and losses of the buyer and the second seller from the unilateral information disclosure by the first seller. We consider four stages: ex post, i.e., after all parameters have been revealed; interim, i.e., after all players know their own parameters, both before and after the information of the first seller has been disclosed; and ex ante, i.e., before players even know their own parameter. As there is no effect on the first seller, we only investigate the effects on the buyer's and the second seller's payoff.

5.1 Ex-post Welfare Effects

In this subsection, we investigate welfare effects after the buyer's valuation and the sellers' costs have realized. Proposition 4.5 and Proposition 4.6 show that the buyer can be harmed by the information disclosure only in case of an acceptance, and that he can profit from the information disclosure only in case of a rejection in the first game. The following proposition formalizes these insights.

Proposition 5.1. Let (f, g_1, g_2) be a sequential trading game, and let $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$, and $c_2 \in \mathcal{C}_2$.

- 1. If $v \ge \hat{p}(c_1)$, then $\hat{u}(v, c_2; f_{c_1}^A, g_2) \le \hat{u}(v, c_2; f, g_2)$. The inequality is strict if and only if $c_2 < c_1$. Moreover, $\hat{u}(v, c_2; f_{c_1}^A, g_2)$ is decreasing in c_1 and strictly decreasing in c_1 if $c_2 < c_1$.
- 2. If $v < \hat{p}(c_1)$, then $\hat{u}(v, c_2; f_{c_1}^R, g_2) \ge \hat{u}(v, c_2; f, g_2)$. The inequality is strict if and only if $\tau(f) < c_2 < (\hat{p}_{c_1}^R)^{-1}(v)$. Moreover, $\hat{u}(v, c_2; f_{c_1}^R, g_2)$ is decreasing in c_1 and strictly decreasing in c_1 if $\tau(f_{c_1}^R) < c_2 < \hat{p}(c_1)$.

The overall welfare that is generated by trade is $v - c_2$ and independent of the information disclosure. Thus, the information exchange only affects overall welfare if it changes whether or not a trade occurs. By Proposition 4.5, an accepted offer in the first round does not influence whether or not the buyer and the seller in the second round trade. By Proposition 4.6, a rejected offer in the first round either has no effect or increases the amount of trade. Thus, we obtain the following results and omit the proofs.

Theorem 5.2. Let (f, g_1, g_2) be a sequential trading game, and let $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$, and $c_2 \in \mathcal{C}_2$.

1. If
$$v \ge \hat{p}(c_1)$$
, then $\hat{u}(v, c_2; f_{c_1}^A, g_2) + \hat{\pi}(v, c_2; f_{c_1}^A, g_2) = \hat{u}(v, c_2; f, g_2) + \hat{\pi}(v, c_2; f, g_2)$.

2. If $v < \hat{p}(c_1)$, then $\hat{u}(v, c_2; f_{c_1}^R, g_2) + \hat{\pi}(v, c_2; f_{c_1}^R, g_2) \ge \hat{u}(v, c_2; f, g_2) + \hat{\pi}(v, c_2; f, g_2)$. This inequality is strict if and only if $\tau(f) < c_2 < (\hat{p}_{c_1}^R)^{-1}(v)$.

Interestingly, the information disclosure by the first seller might also harm the second seller from an ex-post perspective. This happens if she adapts and lowers her price after observing a rejection, although her benchmark offer would have been accepted.

Corollary 5.3. Let (f, g_1, g_2) be a sequential trading game, and let $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$, and $c_2 \in \mathcal{C}_2$. If $\tau(f) < c_2 < \hat{p}^{-1}(v) < c_1$, then $\hat{\pi}(v, c_2; f_{c_1}^R, g_2) < \hat{\pi}(v, c_2; f, g_2)$.

5.2 Interim Welfare Effects After Information Disclosure

We next consider interim effects after the information disclosure, i.e., effects after both the buyer and the second seller know their own type and the seller has updated her belief about the buyer's type. As the outcome of the first game is not affected by the disclosure of information, the effect on the buyer's payoff after an accepted or rejected offer are

$$\begin{split} \Delta \hat{U}^{A}(v;c_{1}) &\equiv \begin{cases} \hat{U}\left(v;f_{c_{1}}^{A},g_{2}\right) - \hat{U}(v;f,g_{2}), & \text{if } v \geq \hat{p}\left(c_{1}\right) \\ 0, & \text{if } v < \hat{p}\left(c_{1}\right), \end{cases} \\ \Delta \hat{U}^{R}(v;c_{1}) &\equiv \begin{cases} 0, & \text{if } v \geq \hat{p}\left(c_{1}\right) \\ \hat{U}\left(v;f_{c_{1}}^{R},g_{2}\right) - \hat{U}(v;f,g_{2}), & \text{if } v < \hat{p}\left(c_{1}\right), \end{cases} \end{split}$$

respectively. (Recall that $\hat{U}(v; f, g)$ denotes the buyer's interim expected payoff in the bilateral trading game (f, g), defined in Equation (6).)

Proposition 5.4. Let (f, g_1, g_2) be a sequential trading game, let $v \in \mathcal{V}$ and $c_1 \in \mathcal{C}_1$.

- 1. Let $v \ge \hat{p}(c_1)$. Then $\Delta \hat{U}^A(v; c_1) \le 0$. The inequality is strict if and only if $c_1 > \underline{c}_2$. Moreover, $\Delta \hat{U}^A(v; c_1)$ does not depend on v.
- 2. Let $v < \hat{p}(c_1)$. Then $\Delta \hat{U}^R(v; c_1) \ge 0$. The inequality is strict if and only if $\overline{c}_2 > \tau(f)$ and $G_2\left(\left(\hat{p}_{c_1}^R\right)^{-1}(v)\right) > G_2(\tau(f))$. Moreover, $\Delta \hat{U}^R(v; c_1)$ is weakly increasing in v.

At the interim stage after the information disclosure revealed an acceptance in the first game, the first part of Proposition 5.4 shows that the buyer can only suffer from the information disclosure. However, maybe surprisingly, the size of the damage is independent of his own valuation v. The reasoning is that an acceptance of the first offer has no effect on the likelihood that the second offer will be accepted as well. Indeed:

- (i) If $c_2 \leq \tau(f)$, then in the benchmark the seller would have offered \underline{v} and this would have been accepted with certainty. With the additional information she offers $\hat{p}(c_1)$, which is accepted with certainty as well;
- (ii) If $\tau(f) < c_2 < c_1$, then in the benchmark she would have offered $\hat{p}(c_2) < \hat{p}(c_1)$ and the offer would have been accepted. After obtaining the additional information she offers $\hat{p}(c_1)$, which is still accepted with certainty.
- (iii) If $c_2 \ge c_1$, the behavior of the second seller is not affected at all.

In contrast, the realized cost of the first seller c_1 has, conditional on an acceptance in the first round, two effects: First, if c_1 increases, the probability that c_1 exceeds c_2 increases as well, which is exactly the probability that the second seller increases her price due to the obtained information. Second, it increases the (conditional) expected difference $\hat{p}(c_1) - \hat{p}(c_2)$, which, if the second offer is accepted as well, increases the buyer's loss due to the information disclosure.

At the interim stage after the information disclosure revealed a rejection, the second part of Proposition 5.4 shows that the buyer can only profit:

- (i) If $c_2 \leq \tau(f)$, the second seller will offer \overline{v} , either way, so the information disclosure makes no difference.
- (ii) If $\tau(f) < c_2 \leq \tau(f_{c_1}^R)$, the seller would have offered $\hat{p}(c_2)$ in the benchmark compared to the lower and certainly accepted \underline{v} after observing a rejection. Since $\hat{p}(c_2)$ is

rejected with strictly positive probability, the buyer profits in two ways: a higher probability of trade and lower prices given trade.

- (iii) If $\tau(f_{c_1}^R) < c_2 \leq \hat{p}^{-1}(v)$, the seller offers $\hat{p}_{c_1}^R(c_2) < \hat{p}(c_2)$ after observing a rejection. As $v \geq \hat{p}(c_2)$, no additional trade (compared to the benchmark) emerges, nevertheless the buyer profits from the lower prices.
- (iv) If $\hat{p}^{-1}(v) < c_2 \leq (\hat{p}_{c_1}^R)^{-1}(v)$ the seller again offers $\hat{p}_{c_1}^R(c_2) < \hat{p}(c_2)$, but now inducing additional trade, as the benchmark offer would have been rejected while the actual offer is not. This additionally generated trade is the reason that the interim expected profit $\Delta \hat{U}^R(v; c_1)$ after observing a rejection, is increasing in v.
- (v) If $c_2 > (\hat{p}_{c_1}^R)^{-1}(v)$, then the second seller's offer will be rejected with and without information disclosure.

In Proposition 5.4 no assumptions on the second seller's distribution G_2 were made. If G_2 is strictly increasing, i.e., if the support of g_2 is convex, then the condition in part 2 simplifies significantly.

Corollary 5.5. Let (f, g_1, g_2) be a sequential trading game, let $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$ with $\hat{p}(c_1) > v$, and let \mathcal{C}_2 be convex. Then $\Delta \hat{U}^R(v; c_1) > 0$ if and only if $\bar{c}_2 > \tau(f)$ and $\underline{c}_2 < (\hat{p}_{c_1}^R)^{-1}(v)$.

We close this subsection with the following example, which illustrates the findings in Proposition 5.4.

Example 5.6. Consider the sequential trading game (f, g_1^1, g_2) , where $f = g_1^1 = g_2$ are the uniform distribution on [0, 1]. In this case we have $\hat{p}(c_1) = \frac{1+c_1}{2}$ and

$$\Delta \hat{U}^{A}(v;c_{1}) = \begin{cases} -\frac{c_{1}^{2}}{4} & \text{if } v \geq \frac{1+c_{1}}{2} \\ 0 & \text{if } v < \frac{1+c_{1}}{2}, \end{cases}$$
$$\Delta \hat{U}^{R}(v;c_{1}) = \begin{cases} 0 & \text{if } v \geq \frac{1+c_{1}}{2} \\ \frac{(4v-c_{1}-1)^{2}}{16} & \text{if } \frac{1+c_{1}}{4} < v < \frac{1}{2} \\ \frac{(1-c_{1})(8v-c_{1}-3)}{16}, & \text{if } \frac{1}{2} \leq v < \frac{1+c_{1}}{2}. \end{cases}$$



Figure 2: Interim effect for the buyer after result of first trading game has been disclosed.

The overall effect of the information disclosure on the buyer's welfare is given by $\Delta \hat{U}^A + \Delta \hat{U}^R$ and depicted in Figure 2 for several values of c_1 . As shown in Proposition 5.4, the effect is positive for $v < \hat{p}(c_1)$, where a discontinuity occurs. For $v \ge \hat{p}(c_1)$, the effect is negative (strictly if $c_1 > 0$) and independent of v. Moreover, a buyer with fixed valuation v who is harmed by the information exchange will be harmed more severely for larger c_1 . Similarly, a buyer (with fixed v) who benefits from the information exchange will profit less for larger c_1 .

5.3 Interim Welfare Effects Before Information Disclosure

We next consider the expected welfare effect on the buyer before the cost of the first seller has been revealed, which is given by

$$\Delta \hat{U}(v) \equiv \mathbb{E}_{G_1} \left[\Delta \hat{U}^A(v; \cdot) + \Delta \hat{U}^R(v; \cdot) \right] = \int_{-\infty}^{\infty} g_1(c_1) \left(\Delta \hat{U}^A(v; c_1) + \Delta \hat{U}^R(v; c_1) \right) dc_1.$$

In terms of Figure 2, this function is a weighted average of the functions depicted there. The following example shows this function for different distributions G_1 . **Example 5.7.** Recall the sequential trading game (f, g_1^1, g_2) from Example 5.6, where all distributions are uniform on [0, 1]. Then,

$$\Delta \hat{U}^{1}(v) \equiv \mathbb{E}_{G_{1}^{1}} \left[\Delta \hat{U}^{A}\left(v; \cdot\right) + \Delta \hat{U}^{R}\left(v; \cdot\right) \right] = \begin{cases} 0 & \text{if } v < \frac{1}{4} \\ \frac{(4v-1)^{3}}{48} & \text{if } v < \frac{1}{4} \le v < \frac{1}{2} \\ \frac{v^{3}}{6} - v^{2} + v - \frac{1}{4} & \text{if } v \ge \frac{1}{2} \end{cases}$$

which is depicted in Figure 3b. Figure 3a depicts the corresponding density g_1^1 (= $g_2 = f$).

Observe that $\Delta \hat{U}^1(v) \equiv 0$ for sufficiently small v. More specifically, if $v < \hat{p}_{\underline{c_1}}^R(\underline{c_2}) = \frac{1}{4}$, there is no trade at all and the interim welfare in the benchmark and the information exchange scenario are both zero. On the other hand, $\Delta \hat{U}^1(v) > 0$ for intermediary values of v and $\Delta \hat{U}^1(v) < 0$ for sufficiently high valuations. This follows as for all v such that $\hat{p}_{\underline{c_1}}^R(\underline{c_2}) < v < \hat{p}(\underline{c_1}) = \frac{1}{2}$, the benchmark offer will be rejected with certainty and the buyer expects a strictly positive profit. For v slightly above $\hat{p}(\underline{c_1})$, the expected gain following a rejection still outweighs the expected loss following an acceptance, in particular as the probability of an acceptance is low. However, even though, by Proposition 5.4, the expected gain after a rejection increases in v, for large v this effect is outweighed by the increasing probability that the first offer is accepted, which would result in a loss.

Figure 3c depicts the density function g_1^2 with $g_1^2(c_1) = 4 - 8c_1$ for all $c_1 \in [0, \frac{1}{2}]$ and 0 otherwise. The corresponding function $\Delta \hat{U}^2$ is shown in Figure 3d. For $v < \hat{p}(\underline{c_1}) = \frac{1}{2}$, the interim expected gains are ordered: smaller c_1 , which are more likely given g_1^2 than g_1^1 , imply lower prices by the second seller after a rejection (Corollary 4.7) and, hence, higher gains (Figure 2). Thus, $\Delta \hat{U}^2(v)$ increases faster than $\Delta \hat{U}^1(v)$ for $v < \frac{1}{2}$.

A third, maybe a bit pathological, density is given by $g_1^3(c_1) = 1 + \sin\left(6\pi c_1 + \frac{\pi}{12}\right)$ for all $c_1 \in \left[0, \frac{1}{2}\right]$ and 0 otherwise, which is depicted in Figure 3e. The corresponding function $\Delta \hat{U}^3$ in Figure 3f illustrates two points: First, given $\Delta \hat{U}^3 < 0$, it is not monotonic. That is, unlike Figures 3b and 3d suggest, buyers with larger v do not automatically suffer more. Furthermore, $\Delta \hat{U}^3$ has several roots. Hence, even if some buyer with valuation vsuffers at the interim level, there might be another buyer with a larger valuation v' who gains from the information disclosure.

All functions $\Delta \hat{U}^{j}(v)$ for $j \in \{1, 2, 3\}$ are not differentiable for $v = \hat{p}(\underline{c_1})$, indeed, differentiability at that point would require that $g_1(\underline{c_1}) = 0$. Both $\Delta \hat{U}^2$ and $\Delta \hat{U}^3$ can be derived analytically. The respective expressions can be found in Appendix B.



Figure 3: Interim buyer welfare effects before information disclosure for different distributions.

As the previous example shows, there is little that can be said about $\Delta \hat{U}$ in terms of comparative statics. There are certain areas though, where crisp results can be obtained.

Proposition 5.8. Let (f, g_1^1, g_2) and (f, g_1^2, g_2) be two sequential trading games such that $C_1^1 = C_1^2$ and g_1^1 first-order stochastically dominates g_1^2 .

- 1. For all $v < \hat{p}(\underline{c}_1^1)$ it holds that $0 \le \Delta \hat{U}^1(v) \le \Delta \hat{U}^2(v)$.
- 2. For all $v > \hat{p}(\overline{c}_1^1)$ it holds that $\Delta \hat{U}^1(v) \leq \Delta \hat{U}^2(v) \leq 0$.

5.4 Ex-Ante Welfare Effects

From an overall welfare perspective, we have seen in Theorem 5.2 that the disclosure of information by the first seller is beneficial, even at the ex-post level. From the buyer's perspective this is not the case: ex post, the effect can be both positive or negative, and the same is true at the interim stage. The buyer's ex-ante effect is given by

$$\mathbb{E}_F\left[\Delta \hat{U}(\cdot)\right] = \int_{-\infty}^{\infty} f(v)\Delta \hat{U}(v)dv,$$

which again can be positive or negative as the following example illustrates.

Example 5.9. Recall the sequential trading games (f, g_1^1, g_2) and (f, g_1^2, g_2) from Example 5.7. We find $\mathbb{E}_F\left[\Delta \hat{U}^1(\cdot)\right] = -\frac{1}{768} < 0$ and $\mathbb{E}_F\left[\Delta \hat{U}^2(\cdot)\right] = \frac{67}{15360} > 0$

From an ex-ante perspective, there is also not much that can be said with respect to changes in the distributions. Figure 2 from Example 5.6 illustrates that $\Delta \hat{U}^A + \Delta \hat{U}^R$ only depends on the values for c_1 but not their likelihood. It can also be seen that if g'_1 first order stochastically dominates g_1 the effect on $\Delta \hat{U}(v)$ can be positive or negative depending on v. Hence, we can construct probability distributions F such that a first order stochastic shift from g_1 to g'_1 can have a positive or a negative effect on $\mathbb{E}_F \left[\Delta \hat{U}(v) \right]$.

5.5 When Costs are not Independent

Throughout this paper, we have assumed that the costs of the two sellers are independent. As they produce a homogeneous good, this might not be the case. Let $\tilde{g}(c_1, c_2)$ be a joint probability density function with marginal densities $\tilde{g}_1(c_1)$ and $\tilde{g}_2(c_2)$, and allow for the possibility that $\tilde{g}(c_1, c_2) \neq \tilde{g}_1(c_1)\tilde{g}_2(c_2)$. Consider now the sequential trading game $(f, \tilde{g}_1, \tilde{g}_2)$ as before. Clearly, the equilibrium strategies do not change compared to the previous analysis, as they do not depend on any dependency structure between the two costs. The following corollary follows directly from Proposition 5.1 and Theorem 5.2.

Corollary 5.10. Let $(f, \tilde{g}_1, \tilde{g}_2)$ be a sequential trading game with $\tilde{g}(c_1, c_2) = 0$ for all $c_2 < c_1$. Then for all $v \in \mathcal{V}$, $c_1 \in \mathcal{C}_1$, and $c_2 \in \mathcal{C}_2$ it holds that

- 1. $\hat{u}(v, c_2; f_{c_1}^A, \tilde{g}_2) = \hat{u}(v, c_2; f, \tilde{g}_2) \text{ and } \hat{\pi}(v, c_2; f_{c_1}^A, \tilde{g}_2) = \hat{\pi}(v, c_2; f, \tilde{g}_2).$
- 2. $\hat{u}(v, c_2; f_{c_1}^R, \tilde{g}_2) \geq \hat{u}(v, c_2; f, \tilde{g}_2)$ with strict inequality if and only if both $v < \hat{p}(c_1)$ and $\tau(f) < c_2 < (\hat{p}_{c_1}^R)^{-1}(v)$ hold.

Corollary 5.10 shows that it is not possible for the buyer to get harmed by the information exchange, if the cost of the second seller is (almost) always higher than the cost of the first seller. Since, it is guaranteed that the buyer can not be harmed by the information exchange from this ex-post perspective, there is no possibility of damage for the buyer from all other timing perspectives as well.

Note that, analogous to Corollary 5.3, it is still possible that the second seller is harmed by the information exchange from an ex-post perspective.

6 Discussion and Conclusion

The objective of this paper was to analyze the welfare effects of private and unilateral disclosure of sensitive information in a sequential bargaining context.

We develop a sequential trading model in which one buyer bargains with two different sellers. Our results show that the buyer profits from the private and unilateral disclosure among the sellers if it reveals a rejection but might be harmed by a revealed acceptance. It is shown that the buyer cannot be harmed but might profit from the information exchange, if the cost of the seller that receives the message is higher than the cost of the information providing seller. This result holds from an ex-post, interim, and ex-ante perspective. It is also shown that the exchange of information improves the societal welfare by reducing inefficiencies and potentially generating additional trade.

Additionally, the buyer can only be harmed by the information exchange if the seller in the second game holds the entire bargaining power as assumed in the paper. To see this, consider a reversion of the bargaining power such that the buyer proposes a take-itor-leave-it price in the second game. The second seller accepts every cost-covering offer irrespective of any information exchange between the two sellers. Thus, the unilateral disclosure of information has no effect on any players' welfare.

Overall, this paper strengthens the theoretical framework for assessing the welfare effects of information exchanges by offering new insights and providing tools to assess causality for alleged damages. This does not only question existing economic principles as described above, but also contributes to questions that are at the core of follow-on damages claims, in which consumers must demonstrate a causal link between alleged anticompetitive practices and their claimed damages.

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A Proofs

Proof of Lemma 3.1. The strict monotonicity of \hat{p} follows immediately from Assumption 1. It has already been shown that the inverse exists and has the desired form. Since $\lim_{v \searrow v} \hat{p}^{-1}(v) = \tau(f)$ and $\lim_{v \nearrow v} \hat{p}^{-1}(v) = \overline{v}$, the map $\hat{p} : (\tau(f), \overline{v}) \to (\underline{v}, \overline{v})$ is a bijection. Since h(p) > 0 for all $p \in (\underline{v}, \overline{v})$, it follows from (2) that $\hat{p}(c) > c$ for all $c \in (\tau(f), \overline{v})$.

Proof of Lemma 3.2. The first order derivative of the seller's objective function is

$$\frac{\partial \Pi}{\partial p}(p,c) = \begin{cases} 1, & \text{if } p < \underline{v} \\ 1 - F(p) - f(p)(p-c), & \text{if } \underline{v} < p < \overline{v} \\ 0, & \text{if } p > \overline{v}. \end{cases}$$
(7)

Note that if there is a point $p^* \in (\underline{v}, \overline{v})$ such that $\frac{\partial \Pi}{\partial p}(p^*, c) = 0$, then by Assumption 1 it must hold that $\frac{\partial \Pi}{\partial p}(\underline{p}, c) > 0$ for all $\underline{p} \in (\underline{v}, p^*)$.

Suppose first that $c \leq \tau(f)$. Then, since in this case $f(\underline{v}) > 0$,

$$\lim_{p \searrow \underline{v}} \frac{\partial \Pi}{\partial p}(p;c) = 1 - f(\underline{v}) (\underline{v} - c) \le 1 - f(\underline{v}) \left(\underline{v} - \left(\underline{v} - \frac{1}{f(\underline{v})} \right) \right) = 0.$$

Thus, by the previous argument there does not exist a $p^* \in (\underline{v}, \overline{v})$ such that $\frac{\partial \Pi}{\partial p}(p^*, c) = 0$ and $\frac{\partial \Pi}{\partial p}(p, c) < 0$ for all $p \in (\underline{v}, \overline{v})$, which means that Π is strictly decreasing on this interval. Thus, choosing $\hat{\sigma}(c) = \underline{v}$ is the unique optimal choice.

Let now $\tau(f) < c < \overline{v}$. Then, $\lim_{p \searrow v} \frac{\partial \Pi}{\partial p}(p;c) > 0$ and the optimal price must satisfy $v . In this case the first order condition derived from (7) is equivalent to (2), so that the unique optimal choice is given by <math>\hat{\sigma}(c) = \hat{p}(c)$.

As Π is continuous in p and c, Berge's maximum theorem implies upper hemicontinuity of the optimizing function $\hat{\sigma}$. As the optimizer is unique for all $c < \overline{v}$, the map $\hat{\sigma}$ is continuous in such c.

Proof of Lemma 3.4. If either $\overline{c} < \tau(f)$ or $\underline{c} > \overline{v}$, $\Delta U(v) \equiv 0$ and the proof is trivial as seen in Remark 3.3. So, let $\overline{c} \ge \tau(f)$ and $\underline{c} \le \overline{v}$. Non-negativity of $\hat{U}(v)$ is clear. By Lemma 3.1 and 3.2

$$\hat{U}(v) = \begin{cases} G(\tau(f)) \left(v - \underline{v}\right) + \int_{\tau(f)}^{\hat{p}^{-1}(v)} g(c) \left(v - \hat{p}(c)\right) dc, & \text{if } \hat{p}^{-1}(v) < \overline{c} \\ G(\tau(f)) \left(v - \underline{v}\right) + \int_{\tau(f)}^{\overline{v}} g(c) \left(v - \hat{p}(c)\right) dc, & \text{if } \hat{p}^{-1}(v) \ge \overline{c} \end{cases}$$

For the partial derivative we find

$$\begin{aligned} \frac{\partial}{\partial v} \hat{U}(v) &= \begin{cases} G(\tau(f)) + \int_{\tau(f)}^{\hat{p}^{-1}(v)} g(c) dc, & \text{if } \hat{p}^{-1}(v) < \bar{c} \\ G(\tau(f)) + 1 - G(\tau(f)), & \text{if } \hat{p}^{-1}(v) > \bar{c} \end{cases} \\ &= \begin{cases} G\left(\hat{p}^{-1}\left(v\right)\right), & \text{if } \hat{p}^{-1}(v) < \bar{c} \\ 1, & \text{if } \hat{p}^{-1}(v) > \bar{c} \end{cases}. \end{aligned}$$

Suppose first that $\bar{c} > \bar{v}$. Then $\hat{p}^{-1}(v) \ge \bar{c}$ is impossible, and \hat{U} is continuously differentiable. Suppose next that $\bar{c} \le \bar{v}$. Then $\hat{p}(\bar{c})$ is well defined. As $\lim_{v \nearrow \hat{p}(\bar{c})} G(\hat{p}^{-1}(v)) = G(\bar{c}) = 1$, \hat{U} is continuously differentiable. Since $\frac{\partial}{\partial v}\hat{U}(v) \ge 0$ is weakly increasing for all $v \in \mathcal{V}$, \hat{U} is non-decreasing and convex. Moreover, \hat{U} is strictly increasing in v if $G(\hat{p}^{-1}(v)) > 0$. Now note for the final part of the lemma that

- 1. If $\underline{c} < \tau(f)$, then $G(\hat{p}^{-1}(v)) > 0$ as $\hat{p}^{-1}(v) > \tau(f)$.
- 2. If $\underline{c} \ge \tau(f)$, then $G\left(\hat{p}^{-1}\left(v\right)\right) > 0$ for all $v > \hat{p}\left(\underline{c}\right)$.

Proof of Proposition 4.5. By Corollary 4.4 it is sufficient to show two things, namely that $\tau\left(f_{c_1}^A\right) = c_1$, and that $\hat{p}_{c_1}^A(c_2) = \hat{p}(c_2)$ for all $c_2 \in (c_1, \overline{v})$. For the first claim note that indeed

$$\tau\left(f_{c_{1}}^{A}\right) = \hat{p}^{-1}\left(\hat{p}\left(c_{1}\right)\right) = c_{1}.$$
(8)

In order to prove the second claim, note that for all $c_2 \in (c_1, \overline{v})$ it holds that

$$h_{c_1}^A(c_2) = \frac{f_{c_1}^A(c_2)}{1 - F_{c_1}^A(c_2)} = \frac{\frac{1}{1 - F(\hat{p}(c_1))}f(c_2)}{\frac{1}{1 - F(\hat{p}(c_1))}(1 - F(c_2))} = h(c_2).$$

Thus, by the definition of \hat{p} and $\hat{p}_{c_1}^A$ it holds that $\hat{p}_{c_1}^A(c_2) = \hat{p}(c_2)$ for all $c_2 \in (c_1, \overline{v})$. *Proof of Proposition 4.6.* We consider three cases.

1. If $c_2 \leq \tau(f)$, then $\hat{\sigma}_{c_1}^R(c_2) = \underline{v} = \hat{\sigma}(c_2)$.

2. If
$$\tau(f) < c_2 \le \tau(f_{c_1}^R)$$
, then $\hat{\sigma}(c_2) = \hat{p}(c_2) > \underline{v} = \hat{\sigma}_{c_1}^R(c_2)$.

3. Let finally $c_2 > \tau \left(f_{c_1}^R\right)$. Then, $\hat{\sigma}_{c_1}^R(c_2) = \hat{p}_{c_1}^R(c_2)$ (recall that $c_2 < \hat{p}(c_1)$) and $\hat{\sigma}(c_2) = \hat{p}(c_2)$. It is therefore sufficient to show that $\hat{p}(c_2) > \hat{p}_{c_1}^R(c_2)$. To this end observe that

$$h\left(\hat{p}\left(c_{2}\right)\right)\left(\hat{p}\left(c_{2}\right)-c_{2}\right) = 1 = h_{c_{1}}^{R}\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)\right)\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)-c_{2}\right)$$

$$= \frac{\frac{1}{F\left(\hat{p}\left(c_{1}\right)\right)}f\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)\right)}{1-\frac{F\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)\right)}{F\left(\hat{p}\left(c_{1}\right)\right)}}\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)-c_{2}\right)$$

$$= \frac{f\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)\right)}{F\left(\hat{p}\left(c_{1}\right)\right)-F\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)\right)}\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)-c_{2}\right)$$

$$> h\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)\right)\left(\hat{p}_{c_{1}}^{R}\left(c_{2}\right)-c_{2}\right)$$

since $\hat{p}(c_1) < \overline{v}$ by Assumption 2 and Lemma 3.1. As the map $h(p)(p-c_2)$ is increasing in p by Assumption 1, we have $\hat{p}(c_2) > \hat{p}_{c_1}^R(c_2)$ as required.

Proof of Proposition 5.1. 1. Let $v \ge \hat{p}(c_1)$. The inequalities follow immediately from Proposition 4.5 and the definition of \hat{u} . Recall that

$$\hat{u}\left(v,c_2;f_{c_1}^A,g\right) = \begin{cases} v - \hat{\sigma}_{c_1}^A(c_2), & \text{if } v \ge \hat{\sigma}_{c_1}^A(c_2) \\ 0, & \text{otherwise} \end{cases}$$

By Proposition 4.5 and Lemma 3.1, the map $\hat{\sigma}_{c_1}^A(c_2)$ is continuous in c_1 , strictly increasing in c_1 , whenever $c_2 < c_1$, and constant in c_1 , whenever $c_2 > c_1$.

2. Let $v < \hat{p}(c_1)$. The first part of the claim follows immediately from Proposition 4.6. The monotonicity of the payoff function follows from Corollary 4.7.

Proof of Proposition 5.4. 1. Let $v \ge \hat{p}(c_1)$. By Proposition 4.5 and Lemma 3.2,

$$\Delta \hat{U}^{A}(v;c_{1}) = \int_{-\infty}^{c_{1}} g_{2}(c_{2}) \left(v - \hat{p}(c_{1})\right) dc_{2} + \int_{c_{1}}^{\hat{p}^{-1}(v)} g_{2}(c_{2}) \left(v - \hat{p}(c_{2})\right) dc_{2} - \int_{-\infty}^{\tau(f)} g_{2}(c_{2}) \left(v - \underline{v}\right) dc_{2} - \int_{\tau(f)}^{\hat{p}^{-1}(v)} g_{2}(c_{2}) \left(v - \hat{p}(c_{2})\right) dc_{2} = \int_{-\infty}^{\tau(f)} g_{2}(c_{2}) \left(v - \hat{p}(c_{1})\right) dc_{2} + \int_{\tau(f)}^{c_{1}} g_{2}(c_{2}) \left(v - \hat{p}(c_{1})\right) dc_{2} - \int_{-\infty}^{\tau(f)} g_{2}(c_{2}) \left(v - \underline{v}\right) dc_{2} - \int_{\tau(f)}^{c_{1}} g_{2}(c_{2}) \left(v - \hat{p}(c_{2})\right) dc_{2} = \int_{-\infty}^{\tau(f)} g_{2}(c_{2}) \left(\underline{v} - \hat{p}(c_{1})\right) dc_{2} + \int_{\tau(f)}^{c_{1}} g_{2}(c_{2}) \left(\hat{p}(c_{2}) - \hat{p}(c_{1})\right) dc_{2}, \quad (9)$$

where the middle equality uses that $c_1 > \tau(f)$ by Assumption 2. By the definition and the monotonicity of \hat{p} , both integrals on the right-hand side of (9) have strictly negative integrands. Thus, the sum on the right hand-side is strictly negative if and only if G_2 has positive mass on $(-\infty, c_1)$. This is the case if and only if $c_1 > \underline{c_2}$. Finally, (9) is independent of v.

2. Let $v < \hat{p}(c_1)$. If $\bar{c}_2 \le \tau(f)$, then the second seller will offer \underline{v} in either case, so that $\Delta \hat{U}^R(v, c_2) = 0$. So, let $\bar{c}_2 > \tau(f)$. If $v < \hat{p}_{c_1}^R(c_2)$, then $v < \hat{p}(c_2)$ by Proposition

4.6, so that there is no trade after the information disclosure. Hence,

$$\Delta \hat{U}^{R}(v;c_{1}) = \int_{-\infty}^{\tau(f_{c_{1}}^{R})} g_{2}(c_{2})(v-\underline{v}) dc_{2} + \int_{\tau(f_{c_{1}}^{R})}^{(\hat{p}_{c_{1}}^{R})^{-1}(v)} g_{2}(c_{2})(v-\hat{p}_{c_{1}}^{R}(c_{2})) dc_{2} - \int_{-\infty}^{\tau(f)} g_{2}(c_{2})(v-\underline{v}) dc_{2} - \int_{\tau(f)}^{\hat{p}^{-1}(v)} g_{2}(c_{2})(v-\hat{p}(c_{2})) dc_{2} = \int_{\tau(f)}^{\tau(f_{c_{1}}^{R})} g_{2}(c_{2})(v-\underline{v}) dc_{2} + \int_{\tau(f_{c_{1}}^{R})}^{(\hat{p}_{c_{1}}^{R})^{-1}(v)} g_{2}(c_{2})(v-\hat{p}_{c_{1}}^{R}(c_{2})) dc_{2} - \int_{\tau(f)}^{\hat{p}^{-1}(v)} g_{2}(c_{2})(v-\hat{p}(c_{2})) dc_{2}.$$
(10)

We consider two cases.

(a) Suppose that $\hat{p}^{-1}(v) \leq \tau \left(f_{c_1}^R\right)$. Then, by (10),

$$\Delta \hat{U}^{R}(v;c_{1}) = \int_{\tau(f)}^{\hat{p}^{-1}(v)} g_{2}(c_{2}) \left(\hat{p}(c_{2}) - \underline{v}\right) dc_{2} + \int_{\hat{p}^{-1}(v)}^{\tau\left(f_{c_{1}}^{R}\right)} g_{2}(c_{2}) \left(v - \underline{v}\right) dc_{2} + \int_{\tau\left(f_{c_{1}}^{R}\right)}^{\left(\hat{p}_{c_{1}}^{R}\right)^{-1}(v)} g_{2}(c_{2}) \left(v - \hat{p}_{c_{1}}^{R}(c_{2})\right) dc_{2} \\ \ge 0.$$

If $v = \underline{v}$, then $\tau(f) = \hat{p}^{-1}(v)$ and $\tau(f_{c_1}^R) = (\hat{p}_{c_1}^R)^{-1}(v)$. Thus, the respective upper and lower integral bounds coincide for the first and last integral, and the integrand for the second integral is 0. Thus, in this case the inequality is satisfied with equality.

For $v > \underline{v}$, all integrands are strictly positive for $c_2 > \tau(f)$, so that the sum is positive if and only if G_2 has positive mass on $\left(\tau(f), \left(\hat{p}_{c_1}^R\right)^{-1}(v)\right)$. The last integral is weakly increasing in v. Since $\hat{p}(\hat{p}^{-1}(v)) = v$ and $\hat{p}(v) > v$, the sum of the first two integrals is weakly increasing in v as well.

(b) Suppose next that $\hat{p}^{-1}(v) > \tau \left(f_{c_1}^R\right)$. Then, by (10),

$$\Delta \hat{U}^{R}(v;c_{1}) = \int_{\tau(f)}^{\tau(f_{c_{1}}^{R})} g_{2}\left(c_{2}\right)\left(\hat{p}\left(c_{2}\right) - \underline{v}\right) dc_{2}$$

$$+ \int_{\tau(f_{c_1}^R)}^{\hat{p}^{-1}(v)} g_2(c_2) \left(\hat{p}(c_2) - \hat{p}_{c_1}^R(c_2) \right) dc_2 + \int_{\hat{p}^{-1}(v)}^{\left(\hat{p}_{c_1}^R \right)^{-1}(v)} g_2(c_2) \left(v - \hat{p}_{c_1}^R(c_2) \right) dc_2 \geq 0,$$

where the non-negativity of the three integrands follows from $\hat{p}(c_2) \geq \underline{v}$ for all $c_2 \geq \tau(f)$, Proposition 4.6, and $\hat{p}_{c_1}^R(c_2) \leq v$ for all $c_2 \leq (\hat{p}_{c_1}^R)^{-1}(v)$, respectively. Since all integrands are strictly positive in the interior of the respective intervals, $\Delta \hat{U}^R(v; c_1) > 0$ if and only if $G_2((\hat{p}_{c_1}^R)^{-1}(v)) > G_2(\tau(f))$. Finally, the first integral does not depend on v, and the sum of the last two integrals is weakly increasing by the same reasoning as before.

Proof of Proposition 5.8. 1. Let $v < \hat{p}(\underline{c}_1^1) = \hat{p}(\underline{c}_1^2)$. Then $\Delta \hat{U}^A(v; c_1) = 0$ for all $c_1 \in \mathcal{C}_1^1 = \mathcal{C}_1^2$. Therefore,

$$\Delta \hat{U}^{j}(v) = \int_{-\infty}^{\infty} g_{1}^{j}(c_{1}) \Delta \hat{U}^{R}(v;c_{1}) dc_{1} \ge 0 \text{ for } j \in \{1,2\}.$$

Consider now $c_1, c'_1 \in \mathcal{C}^1_1 = \mathcal{C}^2_1$ with $c_1 < c'_1$ and note that by Proposition 5.1 $\hat{u}(v, c_2; f^R_{c_1}, g_2) \geq \hat{u}(v, c_2; f^R_{c'_1}, g_2)$ for all $c_2 \in \mathcal{C}_2$ and, hence, $\Delta \hat{U}^R(v; c_1) \geq \Delta \hat{U}^R(v; c'_1)$. Since g^1_1 first-order stochastically dominates g^2_1 , we thus have

$$\Delta \hat{U}^{1}(v) = \int_{-\infty}^{\infty} g_{1}^{1}(c_{1}) \Delta \hat{U}^{R}(v;c_{1}) dc_{1} \leq \int_{-\infty}^{\infty} g_{1}^{2}(c_{1}) \Delta \hat{U}^{R}(v;c_{1}) dc_{1} = \Delta \hat{U}^{2}(v).$$

2. Let $v > \hat{p}(\bar{c}_1^1)$. Then $\Delta \hat{U}^R(v; c_1) = 0$ for all $c_1 \in \mathcal{C}_1^1$ and, thus,

$$\Delta \hat{U}^{j}(v) = \int_{-\infty}^{\infty} g_{1}^{j}(c_{1}) \Delta \hat{U}^{A}(v;c_{1}) dc_{1} \leq 0 \text{ for } j \in \{1,2\}.$$

Again, Proposition 5.1 implies that $\Delta \hat{U}^A(v; c_1) \geq \Delta \hat{U}^A(v; c'_1)$ for all $c_1, c'_1 \in \mathcal{C}^1_1$ with $c_1 < c'_1$ and, hence, $\Delta \hat{U}^1(v) \leq \Delta \hat{U}^2(v)$ by first-order stochastic dominance.

B Complementary Functions Example 5.7

$$\Delta \hat{U}^{2}(v; f, g_{1}^{2}) = \begin{cases} 0, & \text{if } v \in \left[0, \frac{1}{4}\right] \\ \frac{(3-4v)(4v-1)^{3}}{24}, & \text{if } v \in \left[\frac{1}{4}, \frac{3}{8}\right] \\ v^{2} - \frac{7}{12}v + \frac{11}{128}, & \text{if } v \in \left[\frac{3}{8}, \frac{1}{2}\right] \\ -\frac{2}{3}v^{4} + \frac{22}{3}v^{3} - 12v^{2} + \frac{27}{4}v - \frac{463}{384}, & \text{if } v \in \left[\frac{1}{2}, \frac{3}{4}\right] \\ -\frac{1}{96}, & \text{if } v \in \left[\frac{3}{4}, 1\right] \end{cases}$$

$$\begin{split} &\Delta \hat{U}^3(v;f,g_1^3) & \text{if } v \in \left[0,\frac{1}{4}\right] \\ &= \begin{cases} 0, & \text{if } v \in \left[0,\frac{1}{4}\right] \\ \frac{\cos\left(\frac{\pi(288v-71)}{12}\right) + 6\pi(4v-1)\left(3\pi(4v-1)\left(2\pi(4v-1) + \cos\left(\frac{\pi}{12}\right)\right) + \sin\left(\frac{\pi}{12}\right)\right) - \cos\left(\frac{\pi}{12}\right)}{1728\pi^3}, & \text{if } v \in \left[\frac{1}{4},\frac{1}{2}\right] \\ \frac{(72\pi^2v^2-5)\cos\left(\frac{\pi(144v-71)}{12}\right) + 288\pi^3v^3 - 1728\pi^3v^2 - 12\pi(3v-2)\sin\left(\frac{\pi(144v-71)}{12}\right)}{1728\pi^3} \\ &+ \frac{12\pi\sin\left(\frac{73\pi}{12}\right) - 24\pi\left(\sin\left(\frac{73\pi}{12}\right) - 72\pi^2\right)v + \cos\left(\frac{73\pi}{12}\right) + 4\cos\left(\frac{\pi}{12}\right) - 432\pi^3}{1728\pi^3}, & \text{if } v \in \left[\frac{1}{2},1\right] \end{cases} \end{split}$$