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Credit Markets, Intermediate Production and the Business Cycle

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Credit Markets, Intermediate Production and the Business Cycle

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Abstract

This paper builds an RBC model with an endogenous mechanism for firm defaults and credit spreads. The model assumes a productive sector made of a class of intermediate producers and a class of final producers. The intermediate producers borrow to fund their operations and can default when large enough negative shocks affect their revenues. The intermediate/final production structure implies that during periods of low economic activity, the demand for the intermediate good is lower. This depresses the price of the intermediate good and in turn depresses the revenues of the borrowing firms. Hence, higher default rates during the lows of the business cycle. Inversely, default rates are lower when the economy is improving: default rates are countercyclical. Intermediate producers are financed by banks that take future defaults into account when setting lending rates. This guarantees that credit spreads are countercyclical too.

JEL classifications: E32

Keywords: RBC; Credit; Credit Spreads; Financial Frictions

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1 introduction

I present a dynamic stochastic general equilibrium model with a sector of firms that borrow to finance the production of a single intermediate good, banks providing the required financing and an endogenous mechanism for the borrowing firms to bankrupt on their debt obligations. The intermediate good is used as an input by a representative firm that hires labour and rents capital to produce a final good that can be consumed by households. Within the studied framework, intermediate production firms raise financing using the information available to them at the financing stage, including their assessment of their own production efficiency and expected sale prices for the produced intermediate good. When a borrowing firm realises its actual production efficiency and the prevailing sale prices, its assessment of its revenue might drop to levels below what was initially expected. If this drop is large enough, the firm's revenues may not be enough to repay the debt obligations the firm committed to at the financing stage, the firm would then fail to honour its financial obligations. As a result, the lending bank takes over the firm and a fraction of production is lost to reflect the costs of the bankruptcy workouts and the fact that banks are less knowledgeable about the production process.

Intermediate production firms sell the good they produce to a set of final good producing firms using a Cobb-Douglas type technology with labour, capital and the intermediate good as inputs. The productivity of the intermediate production firms is decomposed into an idiosyncratic part that is specific to the firm and a common part that is shared with the productivity of the final good producing firms. This guarantees two desirable outcomes of the model. First, bankruptcy is limited to a subset of the intermediate production firms, namely the subset with low enough idiosyncratic productivity. In addition, the importance of the systemic part of the intermediate producers' efficiencies helps control the contribution of the fluctuation of intermediate production to total the variance of other aggregate variables.

I also assume that intermediate production firms are subject to a quadratic cost of changing the production level. This limits the changes in loans' demand by the intermediate production firms following fluctuations in total factor productivity. The inability of the borrowing firm to change its production levels without incurring a cost maintains borrowing at relatively high levels during economic slumps thus pushing default rates higher in periods of recession. This causes wider credit spreads during periods of recession as banks need to hike loan rates further to reflect higher

probabilities of default because firms borrow more than what they would have done if changing their production level was frictionless. Similarly, this friction causes lower default rates and tighter spreads when the economy is performing well. The costly change of the borrowing firm production level helps reproduce quantitatively realistic dynamics of default rates and credit spreads.

Banks are assumed to be competitive and face no entry costs. This implies that, in expectation, banks make no profit and no loss from extending loans. To compensate for the future default of a fraction of the intermediate production firms, banks charge a credit spread on the top of the interest rate they expect to pay depositors. When macroeconomic conditions worsen, both the sale price of the intermediate good and the efficiency of the intermediate production are lower. This depresses the revenues of the borrowing firms and, as a result, causes the proportion of firms expected to default to increase. Banks take this into account and raise the interest rates of loans. This in turn depresses the cycle further in an effect reminiscent of the financial accelerator effect studied by Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1998) and other authors. The difference between the two mechanisms being that within the framework studied in this paper, banks readjust the interest rate they charge firms to reflect the relationship between business cycles and bankruptcy rates as while typical financial accelerator models assume rationing of credit following a worsening in the market value of assets used as collateral by borrowers.

The division of the productive sector into one group of firms that borrows to produce an intermediate good and a final representative firm that uses this good to produce a final consumption good is crucial to the functioning of the model's main mechanism. Because the output of the intermediate producers is an input in the final production process, the price of the intermediate good displays a procyclical behaviour. When aggregate productivity, defined as the productivity of the final good producer, is high, demand for the intermediate good increases and so does its price. This improves the revenues of the borrowing firms and, in turn, decreases default rates. Similarly, revenues are depressed when aggregate productivity is deteriorating so that default rates are higher and credit spreads are wider. Countercyclical default rates and credit spreads are thus generated even when one assumes no correlation between intermediate and aggregate productivities.

There is a rich literature on general equilibrium models with endogenous bankruptcies. Carlstrom and Fuerst (1997) build on the Bernanke and Gertler (1989) model whereby lending agency costs arise endogenously and introduce a class of capital

transforming entrepreneurs that rely on their net-worth as collateral to raise external debt financing. These entrepreneurs can default on their debt obligations after a large enough negative shock affects their ability to transform capital. The possibility of defaults interacts with the dynamics of the entrepreneurs' net-worth over the business cycle to replicate the empirically observed positive autocorrelation of output at short horizons. In this paper, I ignore the net-worth effects on the ability of borrowers to secure external financing. Instead, I link default probabilities to the aggregate TFP driving the business cycle—Mostly through the variation of the intermediate good price. This generates countercyclical costs of financing that increase output when aggregate TFP is improving and worsen it when TFP is lower. As noted in Gomes, Yaron, and Zhang (2003) the costly financing general equilibrium model in Carlstrom and Fuerst (1997) fails to reproduce the empirically observed countercyclical default premiums.

The bankruptcy mechanism used in this paper is similar to the one in Pesaran and Xu (2016). A major difference resides in the way the two papers deal with employment. While Pesaran and Xu (2016) choose a specific consumer utility function to disentangle the problem of default from that of labour, I choose to dissociate defaults from labour demand by assuming that some firms borrow to produce an intermediate good (intermediate production firms) and others hire labour, rent capital and use the intermediate good to produce the final good (final good production firms). Another major difference lies in the modelling of the banking sector, a mere multiplier in Pesaran and Xu (2016), the model presented here assumes that banks set interest rates through balance sheet optimization. Banks provide the intermediate good producing firms with loans that carry an interest rate that reflect the probability of defaults expected by the bank at the time of the loan issuance. Finally, the productivity of intermediate producers is assumed to be correlated with an aggregate total productivity factor driving the productivity of the firm producing the final good. Unlike in Pesaran and Xu (2016) where default probabilities do not fluctuate with the cycle, the combination of the way banks "price" the credit risk of loans and the fact that the revenues of the intermediate good producing firms fluctuate with the cycle imply that default rates and credit spreads fluctuate with the cycle in a countercyclical fashion, thus accelerating growth during highs of the business cycle as well as through recession periods.

Christiano, Motto, and Rostagno (2010) develop a New Keynesian DSGE model with a mechanism for endogenous defaults. This is achieved through the assumption

that a class of entrepreneurs transforms capital by investing their net worth as a source of self-financing and securing the rest of the required financing through bank loans. Similarly, to what I assume in this paper, the authors assume that the efficiency of the entrepreneur's ability to transform capital is subject to idiosyncratic shocks. A sufficiently unfavourable shock can lead to the borrower's bankruptcy. In addition, the authors assume that the variance of the idiosyncratic shocks that hit the entrepreneur's return is the realisation of a time-varying process. As this variance changes through time, the cross-sectional distribution of returns also changes, thus producing time variation in credit risk and thereby time-varying credit spreads. In the model I consider in this paper, the variations in default rates and credit spreads are generated by procyclical behaviour of the borrowing firms' revenues. This guarantees a countercyclical behaviour of bankruptcy rates and credit spreads without having to assume the existence of an additional exogenous process driving credit risk.

Falato and Xiao (2020) argue that learning from noisy information is an important propagation mechanism for understanding credit and business cycles. They build a general equilibrium model with information asymmetries between lenders and borrowers whereby lenders interpret deteriorations in short-term profit outlook as bad news about firms' default risks. In turn, firms perceive debt as underpriced and cut investments. The authors develop a model of credit-market investors' learning that generates countercyclical default rates and credit spreads and can quantitatively account for the long-lasting widening in spreads and contraction in aggregate investment during the 2007-09 financial crisis. The model I develop in this paper requires no asymmetries in the information available to the lender and borrower at the time the loan is contracted. Countercyclical default rates and credit spreads are generated by the combination of cyclical sale prices of the intermediate good and the fact that intermediate productivity is positively correlated with aggregate TFP. This guarantees that revenues are cyclical which is sufficient to make default rates move inversely with the cycle. Because credit spreads reflect the cost due to future defaults, they too display a countercyclical behaviour.

In the general equilibrium model presented in section 2, I also assume that intermediate producers face an adjustment cost when varying their production levels. This and the correlation of intermediate production productivity with aggregate total factor productivity (TFP) help generate realistic dynamics of defaults and credit spreads while keeping the variance of output in line with the historical experience of the U.S. economy. The steady state, calibration method and dynamic findings are

presented in section 3.

2 General Equilibrium

In this section, I study a general equilibrium model with an intermediate good producing sector that provides a final representative firm with input. Intermediate production is financed through debt financing provided by a competitive banking sector with no entry costs that uses households' deposit to fund its lending operations. Some of the borrowing firms can default on their debt obligations when their revenues are too low to cover debt payments. In order to achieve no profit and no loss in expectation, the competitive banks reflect the default losses they expect in the interest rates they charge the borrowing firms. Final production firms use capital and labour to produce a final good that is consumed by households, transformed into new capital or used as an input to the intermediate production process. Households set their consumption, labour supply, capital investments and deposits to maximise expected utility, discounted over their lifetime.

2.1 Intermediate good producing firms

I assume the existence of a sector with firms that transform output before its use as an intermediate good by final production firms. The intermediate production process is fully financed by banks through loan issuance. It takes an intermediate good producing firm a single time period to produce a quantity y_{t+1}^M , to do so it issues a single time period maturity loan to the banks of principal X_t and uses the production function below to transform the invested quantity of final good X_t into intermediate good y_{t+1}^M

$$y_{i,t+1}^M = Z_{i,t+1} f(X_{i,t}) \left(1 - \lambda \left(\frac{X_{i,t}}{X_{i,t-1}} - 1 \right)^2 \right), \quad (2.1)$$

where the index i denotes the firm, $Z_{i,t+1}$ is a heterogeneous and stochastic efficiency factor, λ is a parameter reflecting the cost of changing the level of production and $f(\cdot)$ represents the deterministic component of the intermediate production technology before incurring adjustment costs. Furthermore, I assume that intermediate

production efficiencies follow a log-normal processes

$$z_{i,t} := \ln Z_{i,t} = \sigma_M [\rho_M z_t^a + \sqrt{1 - (\rho_M)^2} \epsilon_i]. \quad (2.2)$$

z_t^a is a common efficiency factor, representing aggregate productivity, that is time-dependent and that follows an AR(1) process: $z_t^a = \rho_a z_{t-1}^a + e_t$ where e_t are normal and *i.i.d.* shocks. The terms ϵ_i reflect the idiosyncratic firm efficiency and are normally distributed, independent across firms and independent from the common efficiency factor z_t . The parameter σ_M is a volatility parameter representing the riskiness of the intermediate production projects and ρ_M reflects the interdependence of intermediate production across different firms. The type of the firm ϵ_i is not known before the loan maturity so that all intermediate production firms are a priori identical and they all face the same cost of financing R_t and raise the same loan principal X_t . They also produce the same intermediate good, sold at the unique price Q_t .

In the case where the intermediate production firms do not default, their profit is

$$\pi_{i,t+1}^M = Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) - R_t X_t,$$

where by definition $g(X_t, X_{t-1}) := f(X_t) \left(1 - \lambda \left(\frac{X_t}{X_{t-1}} - 1\right)^2\right)$. The intermediate good producing firms walk away on their debt if their non-default profit is negative. In the case where a firm defaults, the lending bank takes over intermediate good production and loses a fraction θ of the produced intermediate goods in the process, reflecting a cost for the bank to go through bankruptcy workouts and the fact that the firms' managers possess more knowledge about the production process than banks. In the spirit of Carlstrom and Fuerst (1997), an intermediate production firm chooses to default when its profit becomes negative, i.e. when $Z_{i,t+1} < \frac{R_t X_t}{Q_{t+1} g(X_t, X_{t-1})}$. In other words, defaults happen when standardised version of the Gaussian variable $z_{i,t+1}$ is below a certain value, denoted $-\xi_{t+1}$.¹ The default probability of the intermediate production firms is given by

$$DP_{t+1} = \Phi(-\xi_{t+1}), \quad (2.3)$$

where Φ denotes the normal cumulative distribution function. Following the nomen-

¹The standardised version of Gaussian variable $X \sim N(\mu, \sigma^2)$ is $\frac{X-\mu}{\sigma}$.

clature inspired by Merton (1974), ξ_{t+1} is called the "distance to default". In the current set-up, the distance to default depends on the common factor z_{t+1}^a , the current loan and previous loan sizes (X_{t-1}, X_t) , the gross rate of interest R_t and the sale price Q_{t+1}

$$\xi_{t+1} = \frac{1}{\sqrt{1 - \rho_M^2}} \left\{ \rho_M z_{t+1}^a + \frac{1}{\sigma_M} \ln \left(\frac{Q_{t+1} g(X_t, X_{t-1})}{R_t X_t} \right) \right\}. \quad (2.4)$$

Note here that DP_{t+1} is meant to be the default probability at the maturity of the loan but before the firm realizes its own type ϵ_i . Once the type ϵ_i is known to the borrowing firm, default or survival are immediately determined and are not random any more. The used correlation structure and the assumption that $0 < \rho_M < 1$ guarantee a positive correlation of efficiencies across firms. During recessions, the common factor z_{t+1}^a is low and all firms have lower efficiencies, while in intermediate producers' efficiency is high when z_{t+1}^a is high. This in turn means that when the aggregate efficiency factor is low, default rates are higher in the economy. Inversely, default rates are low when aggregate productivity is high. In addition to the direct effect of aggregate efficiency, there are indirect effects that operate through the other variables affecting the distance to default. These variables are the price of the intermediate good Q_{t+1} , the size of the loan X_t and the charged interest rate R_t . While it is clear that, everything else being equal, default rates increase with higher loan levels X_t and higher interest rates R_t and decrease with higher sale prices Q_{t+1} , the net combined effect of these variables will be assessed through simulation in section 3. Section 3 will also show the effect of the cost of adjusting investment as reflected by the parameter λ . The fact that changing the level of intermediate production is costly has two important implications. First, it dampens the changes in the levels of intermediate good productions. Second, the costly adjustment in intermediate production levels provides a motivation for intermediate producers to maintain relatively high levels of loan demand when productivity is low. This, in turn, means that the loan market clears at a higher interest rate R_t , thus implying a higher credit spreads during recession periods. This and the fact that the presence of investment adjustment costs does not impact the steady state of the model implies that the parameter λ is key to generating realistic credit spreads dynamics.

The intermediate goods' producing firms maximise profit, taking defaults into

account, to set their demand of loans X_t

$$\max_{X_t} \mathbb{E}_t \left[\pi_{i,t+1}^{M,d} \right], \quad (2.5)$$

where \mathbb{E}_t denotes the expectation operator conditional on the information set available at time t and $\pi_{i,t+1}^{M,d}$ is the profit taking into account the possibility of future default

$$\pi_{i,t+1}^{M,d} = Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) \mathbf{1}_{\epsilon_i > -\xi_{t+1}} - R_t X_t \mathbf{1}_{\epsilon_i > -\xi_{t+1}}. \quad (2.6)$$

Given the independence between the idiosyncratic shocks ϵ_i and the aggregate productivity common factor z_{t+1}^a , one can rewrite expected profits as follows

$$\mathbb{E}_t \pi_{i,t+1}^{M,d} = g(X_t, X_{t-1}) e^{\Sigma_M^2/2} \mathbb{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \Phi(\xi_{t+1} + \Sigma_M) \right] - R_t X_t \mathbb{E}_t \Phi(\xi_{t+1}), \quad (2.7)$$

where $\Sigma_M := \sigma_M \sqrt{1 - \rho_M^2}$ is the volatility of the idiosyncratic part of the intermediate production efficiency. One can therefore write the loan demand equation in a form that is common to all firms and where the idiosyncratic shock ϵ_i plays no role

$$\begin{aligned} R_t \mathbb{E}_t \left[\Phi(\xi_{t+1}) + X_t \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1}) \right] &= \frac{\partial g(X_t, X_{t-1})}{\partial X_t} e^{\Sigma_M^2/2} \mathbb{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \Phi(\xi_{t+1} + \Sigma_M) \right] \\ &+ g(X_t, X_{t-1}) e^{\Sigma_M^2/2} \mathbb{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1} + \Sigma_M) \right]. \end{aligned} \quad (2.8)$$

The latter formulation of the loan demand equation confirms the fact that all firms face the same gross interest rate R_t and raise the same level of loan X_t . Writing $\frac{\partial g(X_t, X_{t-1})}{\partial X_t}$ in the form below clarifies the role of the parameter λ in moderating the fluctuations of the loan demand

$$\frac{\partial g(X_t, X_{t-1})}{\partial X_t} = f'(X_t) \frac{g(X_t, X_{t-1})}{f(X_t)} - 2 \frac{\lambda}{X_{t-1}^2} (X_t - X_{t-1}) f(X_t). \quad (2.9)$$

The parameter λ shifts loan demand lower when it is increasing in comparison to the previous period loan principal ($X_t > X_{t-1}$) and shifts it higher when it is decreasing ($X_t < X_{t-1}$).

2.2 Production Firms

Production of the final good is performed by a representative firm that is constrained by a Cobb-Douglass technology that uses labour (H_t), capital (K_t) and the intermediate good (M_t) produced by the intermediate producers, studied in the previous subsection, as inputs

$$Y_t = (Z_t^a H_t^{1-\alpha} K_t^\alpha)^{1-\zeta} M_t^\zeta, \quad (2.10)$$

where ζ is the share of the intermediate good and $(1 - \zeta)\alpha$ is the share of capital. The production sector productivity Z_t^a is completely driven by the systemic factor defined in the previous section $\ln(Z_t^a) = \sigma_a z_t^a$. The demand for capital, labour and the intermediate input are set to maximize the final producer's profit

$$\max_{H_t, K_t, M_t} Y_t - r_t^K K_t - w_t H_t - Q_t M_t, \quad (2.11)$$

where w_t is the wage and r_t^K is the rental cost of labour. The first order condition for investments in capital, labour and the intermediate good are

$$r_t^K = \alpha(1 - \zeta) \frac{Y_t}{K_t}, \quad (2.12)$$

$$w_t = (1 - \alpha)(1 - \zeta) \frac{Y_t}{H_t}, \quad (2.13)$$

and

$$Q_t = \zeta \frac{Y_t}{M_t}. \quad (2.14)$$

2.3 Households

Households like to consume and dislike work as per the utility function

$$U(C_t, L_t) = \frac{C_t^{1-\sigma_H}}{1-\sigma_H} - \chi \frac{L_t^{1+\eta}}{1+\eta}. \quad (2.15)$$

In addition, households are assumed to accumulate capital K_t and invest in deposits D_t . They decide consumption C_t , labour supply L_t , deposits D_t and new capital K_t by maximising their expected discounted lifetime utility

$$\max_{C_u, L_u, D_u, K_u} \mathbb{E}_t \sum_{u=0}^{\infty} \beta^u U(C_{t+u}, L_{t+u}) \quad (2.16)$$

where $0 < \beta < 1$ denotes the preferences discount factor. The household optimisation is subject to the budget constraint

$$C_t + D_t + K_t = w_t L_t + R_{t-1}^D D_{t-1} + (1 - \delta + r_t^K) K_{t-1} + \Pi_t \quad (2.17)$$

where δ is the depreciation rate of capital, r_t^K is the rental rate of capital, R_t^D is the gross deposit rate and Π_t is the profit distributed to households. Π_t is the combination of the profit distributed by the banks Π_t^B and the profit distributed by the intermediate good producing firms Π_t^M (the final good producing firms makes no profit and no loss)

$$\Pi_t = \Pi_t^B + \Pi_t^M. \quad (2.18)$$

Finally, one can derive the Euler equations for deposits and capital and the labour supply condition as follows

$$C_t^{-\sigma_H} = \beta R_t^D \mathbb{E}_t C_{t+1}^{-\sigma_H}, \quad (2.19)$$

$$C_t^{-\sigma_H} = \beta \mathbb{E}_t C_{t+1}^{-\sigma_H} (1 - \delta + r_{t+1}^K), \quad (2.20)$$

$$\chi L_t^\eta = w_t C_t^{-\sigma_H}. \quad (2.21)$$

2.4 Banks

Banks hold a balance sheet composed of loans issued to finance the operations of the intermediate production firms and finance these loans using households' deposits. The representative bank invests in a large enough portfolio of loans such as the final fraction defaulting is $\Phi(-\xi_{t+1})$, where $t+1$ is the loans' maturity and ξ_{t+1} the distance to default.² The bank recovers a fraction $1 - \theta$ of the production proceeds after the borrower's default. In the case of the model I study, one can calculate the final recovery and its expectations as follows

$$Rec_t := (1 - \theta) Q_t g(X_{t-1}, X_{t-2}) \int_{-\infty}^{\infty} Z_{i,t} \mathbf{1}_{\pi_{i,t}^M < 0} d\epsilon_i. \quad (2.22)$$

Given the defaulting fraction of loans $\Phi(-\xi_{t+1})$, the bank's expected profit from loans operations is

$$\pi_t^B = \mathbb{E}_t (\Phi(\xi_{t+1}) R_t X_t + Rec_{t+1} - R_t^D X_t). \quad (2.23)$$

²This is a direct consequence of the law of large numbers.

The profit function includes the return from the non-defaulting loans $\Phi(\xi_{t+1})R_tX_t$, the recovery from the defaulting loans Rec_{t+1} and the cost of borrowing from households $R_t^D X_t$. Banks are assumed to be competitive and face no entry cost so that the representative bank runs no profit and no loss in expectation. The zero expected profit condition yields a crucial link between the rates charged by banks and the expected default rates

$$R_t \mathbb{E}_t[\Phi(\xi_{t+1})] = \mathbb{E}_t \left[R_t^D - \frac{Rec_{t+1}}{X_t} \right]. \quad (2.24)$$

Clearly, the charged interest rate R_t increases with the default probability $\Phi(-\xi_{t+1})$, with the fractions of production lost conditional on default θ and with the deposit rates R_t^D . It is useful to note that in the absence of defaults the bank would simply charge the deposit rate when issuing loans to the intermediate production firms ($R_t = R_t^D$). The difference between the gross loan rate R_t and the deposit rate R_t^D reflects the extra spread banks charge the borrowing firms in order to compensate for losses due to future defaults. I therefore define credit spreads as an annualised measure of the gap between default-risky and default-free gross rates

$$CS_t = \left(\frac{R_t}{R_t^D} \right)^{1/dt} - 1, \quad (2.25)$$

where dt is the time length separating two consecutive time periods ($dt = 0.25$ in the case of a quarterly frequency). The idea that banks charge the borrowing firms the expected cost of future defaults is made clearer by replacing for the loan rate in the expression of intermediate production firms' profit 2.6 using condition 2.24 to derive the expression below the expected profits of the borrowing firm

$$\mathbb{E}_t \pi_{i,t+1}^{M,d} = \mathbb{E}_t Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) - R_t^D X_t - \theta \mathbb{E}_t Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) \mathbf{1}_{\epsilon_i < \xi_{t+1}}. \quad (2.26)$$

This shows that when there is no loss of intermediate production because of defaults $\theta = 0$, the expected profits of the intermediate producers are not influenced by credit spreads or the probabilities of future defaults. The loan demand function remains impacted by credit spreads as the borrowing firms are price takers and the banks' pricing equation 2.24 is external to their profit maximisation problem. However, we will show in section 3 that, for the calibrated model parameters, credit spreads have little impact on credit markets when $\theta = 0$.

2.5 Aggregation and Market Clearing

In this subsection, I clarify the market clearing conditions. These are

- i The clearing of the final goods market.

$$Y_t = C_t + X_t + K_t - (1 - \delta)K_{t-1}. \quad (2.27)$$

- ii The clearing of the labour market.

$$H_t = L_t. \quad (2.28)$$

- iii The clearing of the bank loan market where supply of loans by banks meets the demand of the intermediate production firms.
- iv The clearing of the intermediate good market, taking into account the effect of bankruptcies in reducing intermediate production

$$M_t = \mathbf{E}_{\epsilon_i}[y_{i,t}^M] - \theta \mathbf{E}_{\epsilon_i}[y_{i,t}^M \mathbf{1}_{\epsilon_i < -\xi_t}]. \quad (2.29)$$

- v The clearing of the deposits market

$$D_t = X_t. \quad (2.30)$$

The remaining of this paper is dedicated to the calibration and simulation of the model presented in this section. The calibration and simulation procedures are presented in section 3. This section also presents and comments on the dynamic effects of each of the model's main assumptions.

3 Findings

3.1 Steady State equilibrium and calibration

The steady state of the model exists and is unique for the set of model parameters chosen in our calibration³. The steady state default rate is $\Phi(-\bar{\xi})$ with a steady state

³See appendix B for more on the steady state determination.

distance to default given by

$$\bar{\xi} = \frac{1}{\Sigma_M} \ln \left(\bar{Q} \frac{f(\bar{X})}{\bar{R}\bar{X}} \right). \quad (3.1)$$

The quantity $\Sigma_M := \sigma_M \sqrt{1 - \rho_M^2}$ is crucial to default rates in the steady state. Given the remaining parameters of the model, the volatility Σ_M is calibrated to match the historical U.S. corporate default rate at 3.44%. The dislike for work parameter χ is chosen to match a steady-state labour at $\bar{L} = 0.3$. The loss in production following default θ is chosen to match the historical U.S. recovery rates at 40%. The preferences discounting parameter β is chosen so that the model's steady-state deposit rate matches the average historical deposit rates in the U.S. following Christiano, Motto, and Rostagno (2010). The parameter describing the share of intermediate goods in the production function is set to $\zeta = 0.5$ following Basu (1995) and Jones (2011).

The cost of readjusting intermediate production λ and the correlation parameter ρ_M are key to the reaction of output, default rates and credit spreads following shocks. These two parameters are jointly calibrated to match the volatility of output and the volatility of credit spreads in U.S. data. The remaining model parameters are standard. They are either chosen to match U.S. data or borrowed from the literature. Table 1 provides a summary of the model parameters and how they are chosen and table 2 provides the values of key variables of the model in the steady state.

3.2 Dynamic effects

In this subsection, I show the dynamic effects of the main features of the model through the study of the impulse response functions following negative shocks to the aggregate total factor productivity Z_t^a . I start by showing the effect of defaults in accelerating the business cycle before studying the effects of costly adjustment of intermediate production and the impact of the correlation between the efficiency of intermediate production and aggregate TFP. The displayed simulations are realised, for quarterly time periods, in Dynare, using third-order approximations.

3.2.1 The effect of default rates and credit spreads

Figure 1 shows the impulse response function (IRF) of the main variables of the model after one standard deviation unpredicted negative shock to logarithmic TFP ($\ln Z^a$).

	Value	Source
Households preferences		
σ_H risk aversion	1	Christiano, Motto, and Rostagno (2010)
η curvature on labour	1	Christiano, Motto, and Rostagno (2010)
β discount factor	0.996	Christiano, Motto, and Rostagno (2010)
χ disutility of labour	9.55	steady state labour at 0.3
Technology		
α	0.36	Christiano, Motto, and Rostagno (2010)
ζ	0.5	Basu (1995) and Jones (2011)
ρ_a	0.79	U.S. data
σ_a	1.1%	U.S. data
δ depreciation rate of capital	2.5%	Christiano, Motto, and Rostagno (2010)
Intermediate production		
Σ_M idiosyncratic volatility	2.18%	Steady state annual default rate 3.44%
θ loss of production upon default	60.1%	Steady state loss upon default 60%
ρ_M correlation with systemic factor	7.4%	ρ_M and λ are set to match volatility of log spreads
λ cost of varying investments	10.1%	at 30% and the volatility of log output at 1.5%

Table 1: Assumed and calibrated model parameters.

Variable	Steady state value
Debt level \bar{X}	0.112
Output \bar{Y}	0.237
Consumption \bar{C}	0.088
Capital \bar{K}	1.47
Labour \bar{L}	0.3
Intermediary good \bar{M}	0.106
Deposit rate $\bar{R}^D - 1$	0.40%
Default Rate $\bar{D}P$	0.87%
Distance to default $\bar{\xi}$	2.38
Credit Spread $\bar{C}S$	2.13%
Price of intermediate good \bar{Q}	1.12

Table 2: Steady state variables.

The IRFs are shown for: (i) the main model as calibrated in section 3.1 (ii) a version of the model assuming no loss in intermediate production upon default ($\theta = 0$, dashed lines) and (iii) a simple RBC model with no intermediate good production ($\zeta = 0$, dotted line). It is important to note that the model with no production losses due to defaults assumes the same cost of adjusting intermediate production λ and the same correlation among intermediate producers ρ_M as the main model while the remaining parameters are recalibrated as in section 3.1 except for the parameter θ that is set to zero. This guarantees that all the model parameters, with the exception of θ , are the same as in the main model. The assumption $\theta = 0$ implies no immediate loss in intermediate production because of defaults. In addition, figure 1 shows that despite an increase in default probabilities following a negative TFP shock this implies little fluctuation in credit spreads. In fact, steady-state results show that when $\theta = 0$, the steady-state credit spread remains positive but is very close to zero.⁴ There is therefore little impact on credit markets and future intermediate production because of credit spreads when $\theta = 0$. The differences between the reaction of the model variables under the assumptions of the main calibration and when θ is set to zero can therefore be associated to the effects of defaults and credit spreads. On the other hand, the results of the RBC model represent the behaviour of the main model in absence of intermediate production and credit markets.

The presence of defaults related production losses implies a larger drop in loan levels, investments, capital, labour and output. When defaults impact credit markets, default probabilities increase following a negative productivity shock. This is a consequence of the combined effect of lower efficiency of intermediate production ($\rho_M > 0$) and lower prices of intermediate goods. Both these effects depress the revenues of intermediate good producers causing higher default rates. This impact on default probabilities has two main consequences. First, higher default rates increase the proportion of intermediate production lost because of the bankruptcy workouts. This causes a larger immediate drop in intermediate good production and, as a result, depresses output, labour, capital investments and consumption further relative to the case where $\theta = 0$. Second, credit spreads increase as banks adjust the interest rates they charge intermediate producers to compensate for future defaults and loan demand is lower as the borrowing firms take into account the higher likelihood of defaults on their future profits. This depresses the size of loans issued further and in turn, worsens the impact of lower TFP on future output relative to the model where

⁴When $\theta = 0$, the recovery is very close to the face value of the loan but remain less than the face value.

defaults do not impact credit markets.

The RBC model produces the familiar dynamics with an immediate drop in output, investment and hours following a surprise negative shock to productivity. The models that assume the existence of an intermediate production sector relying on loan financing display reverse hump-shaped a reaction of intermediate production and output. This is due to the drop in the size of the loans issued, which lowers future intermediate production and negatively impact the final production in the second time period after the shock. The reverse hump-shape reaction of intermediate production to negative shocks implies a similarly shaped reaction of hours worked as labour is less productive when there are fewer intermediate goods available in the economy. In the current set-up, I assume a single-period maturity of the loans. A version of the current model with longer loan maturities would propagate the effect of changes in TFP fluctuations for multiple time periods after the shock. Longer maturity debt instruments would therefore cause more persistence in the response of output and investments.

Similarly to Carlstrom and Fuerst (1997), positive autocorrelation of outputs is generated by assuming the possibility of bankruptcy in parts of the economy's productive sector. However, there is a major difference between the mechanisms of both models. In Carlstrom and Fuerst (1997) investments cuts are delayed following negative shocks as it takes time for the shock to affect the net-worth of borrowers thereby affecting their ability to raise external financing. In the model I study, a systemic increase in default probabilities is triggered by the worsening of aggregate productivity. Higher future default probabilities are reflected by banks in the credit spreads. This discourages intermediate production firms from borrowing and reduces the level of loans in the economy. Which, in turn, reduces future intermediate production and future output. The model presented in this paper produces countercyclical credit spreads and default rates in a way that delays part of the reaction of intermediate production and output. The countercyclical behaviour of credit spreads is a documented feature of business cycles that is not generated by models of the type in Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1998). As explained in Gomes, Yaron, and Zhang (2003), in these agency cost models, the net-worth of the borrowers takes some time to deteriorate after a negative shock. This dampens the reaction of internal funds and implies a lower reliance on external funds as the financing needs drop faster than internal funds. Monitoring costs decrease as a result, thus reducing default premiums after negative shocks. On the

other hand, the slow reduction in net-worth also pushes the borrower to delay some of the investment cuts, generating a reverse hump-shaped reaction of investments and output. The ability of popular agency cost models to generate a persistent autocorrelation of output and investment is linked to the fact that they display cyclical default premiums.

3.2.2 Effect of investment adjustment costs

Figure 2 shows the dynamic effect of costly adjustment of intermediate production. The figure shows the impulse response functions of the model assuming that no costs are incurred by intermediate producers when changing the level of production ($\lambda = 0$). This "no-adjustment cost" model assumes otherwise the same correlation parameter ρ_M as the main model while the parameters χ , Σ_M and θ are recalibrated as in subsection 3.1.

The presence of investment adjustment costs especially affects the reaction of default rates, credit spreads and the size of the loan to a deterioration in the aggregate productivity, with a minor impact on the remaining model variables. When it is costly for intermediate producers to amend their loan demand, loan principals react less to aggregate TFP shocks. Following an unpredicted negative shock to TFP, the size of the loan issued decrease but remain at relatively higher levels when compared to the response of the model where it is costless to adjust intermediate production. This relatively high level of debt implies a higher likelihood of defaults in the next time period. Higher expectations of defaults imply higher credit spreads.

The way the parameter λ impacts credit spreads more than consumption, real investments and output is the reason it is used to help the model match the historical volatility of credit spreads as per the calibration process described in section 3.1.

Effect of the covariance of intermediate productivity TFP

Figure 3 shows the impact of the correlation between the productivity of the intermediate producers and aggregate productivity (ρ_M) on the response of the main variables of the model to a negative aggregate productivity shock. The "no correlation" model assumes that the correlation parameter is set to zero ($\rho_M = 0\%$) and the same intermediate production adjustment cost parameter as for the main model. The remaining parameters are recalibrated as in section 3.1.

A positive intermediate efficiency correlation parameter $\rho_M > 0$ means that the

efficiency of intermediate production deteriorates following negative shocks to TFP and implies that fewer intermediate goods are produced. This, in turn, implies a larger drop of output causing lower investments, consumption and hours. In addition, a positive correlation parameter $\rho_M > 0$ worsens the deterioration of intermediate producers' revenues following negative shocks and causes higher default rates.⁵ This, in turn, causes higher credit spreads following negative shocks.

The difference in the reaction of credit spreads between the model with $\rho_M > 0$ and the model with no correlation between intermediate production efficiency and TFP reflect the difference in future expectations of default rates. This difference is less important than the difference in default rates immediately following the shock to productivity leading to a muted effect of the correlation parameter on credit spreads. The correlation parameter ρ_M has less effect on credit spreads than on output, while the cost of adjusting intermediate production λ affects credit spreads more than output. This justifies the calibration choice made in subsection 3.1 where the pair (λ, ρ_M) is calibrated for the model to simultaneously match the historical volatilities of output and credit spreads.

The positive correlation assumption ($\rho_M > 0$) is useful to replicate historical values of the second moments of some of the aggregate variables but is not crucial to the functioning of the model's main mechanism.⁶ Even when the efficiency of intermediate production is independent of aggregate TFP ($\rho_M = 0$), default rates and credit spreads remain countercyclical and accelerate the business cycle by affecting loan issuance and intermediate production thus causing output, consumption and hours to drop further. The countercyclical behaviour of default rates (and by extension that of credit spreads) is chiefly a consequence of the procyclical behaviour of the price of the intermediate good. This procyclical behaviour is rooted in the fact that the good produced by the borrowing firms is an input in the final production process and implies a cyclical behaviour of the revenues of intermediate producers, which generates countercyclical default rates and credit spreads.

⁵As predicted by the distance to default formula 2.21.

⁶Alternatively, one can calibrate for the value of ρ_M to match the historical volatility of investments or hours.

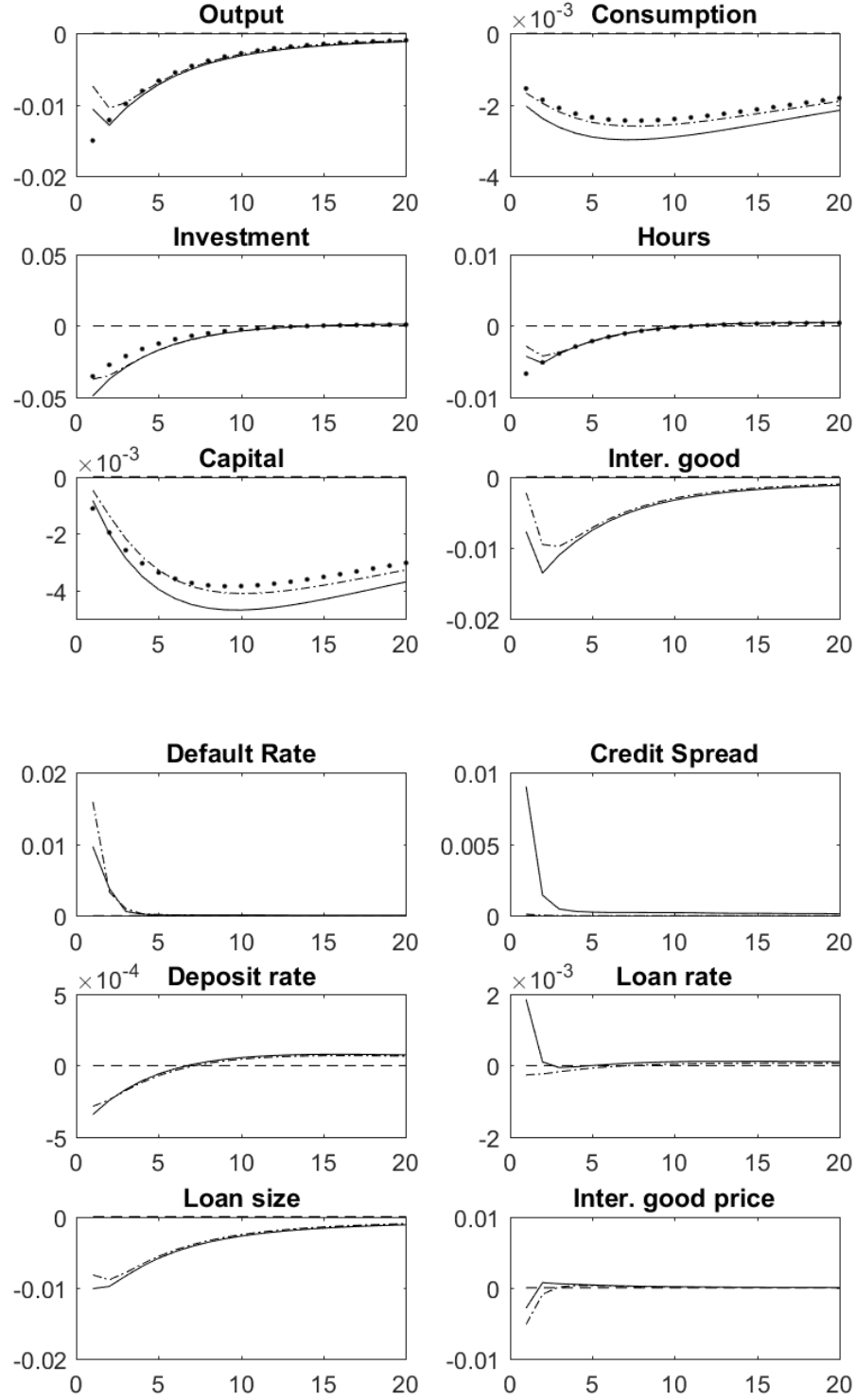


Figure 1: Impulse response functions following a negative shock to TFP ($-1 \times$ standard deviation) of: (i) the main model, (ii) a version of the model assuming no loss in intermediate production upon default ($\theta = 0$, dashed lines) and (iii) a simple RBC model with no intermediate good production ($\zeta = 0$, dotted line). All variables but deposit rates, credit spreads and default probability are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

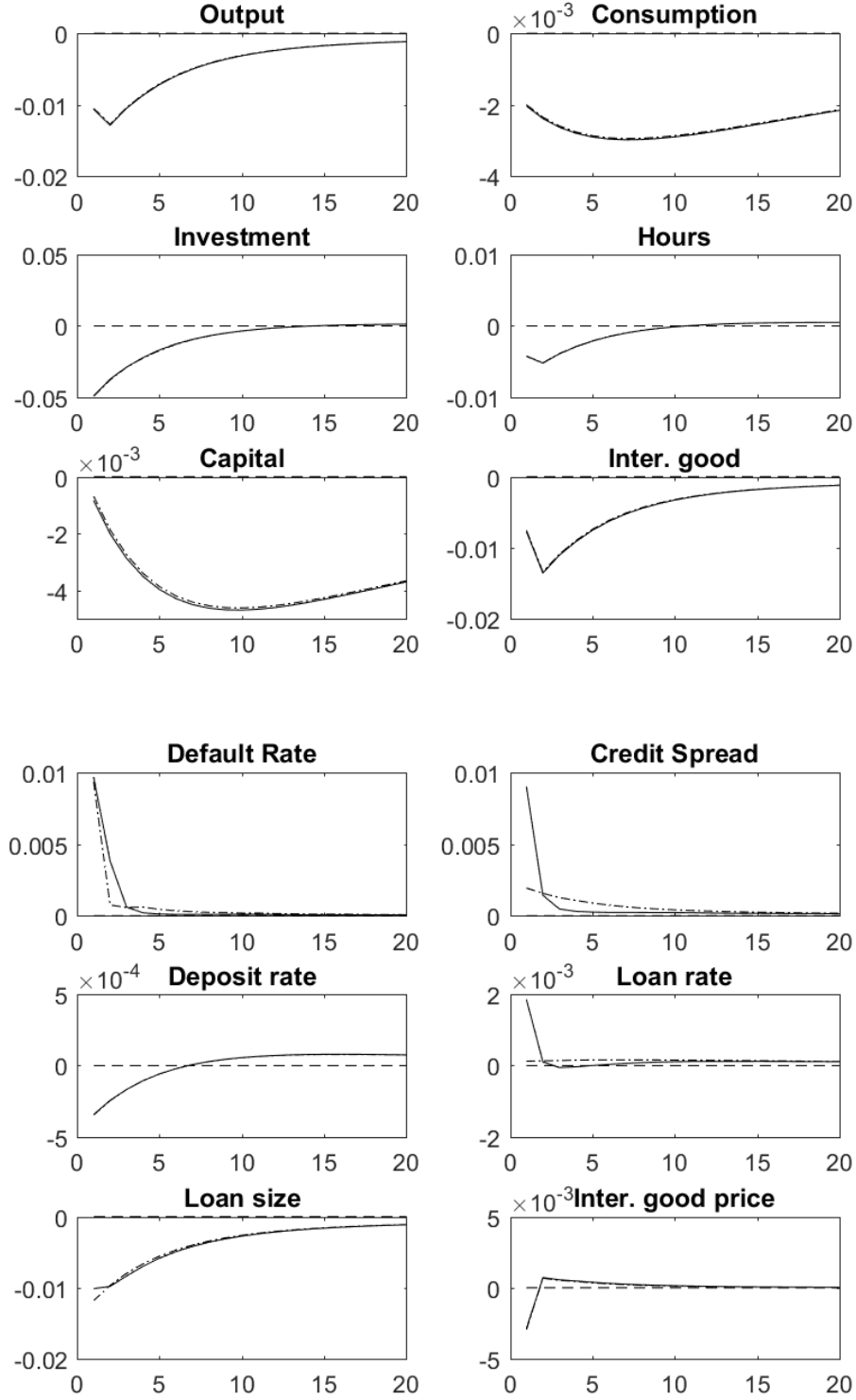


Figure 2: Impulse response functions following a negative shock to TFP ($-1 \times$ standard deviation) of: (i) the main model and (ii) a version of the model assuming no cost of changing intermediate production ($\lambda = 0$, dashed lines). All variables but deposit rates, credit spreads and default probability are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

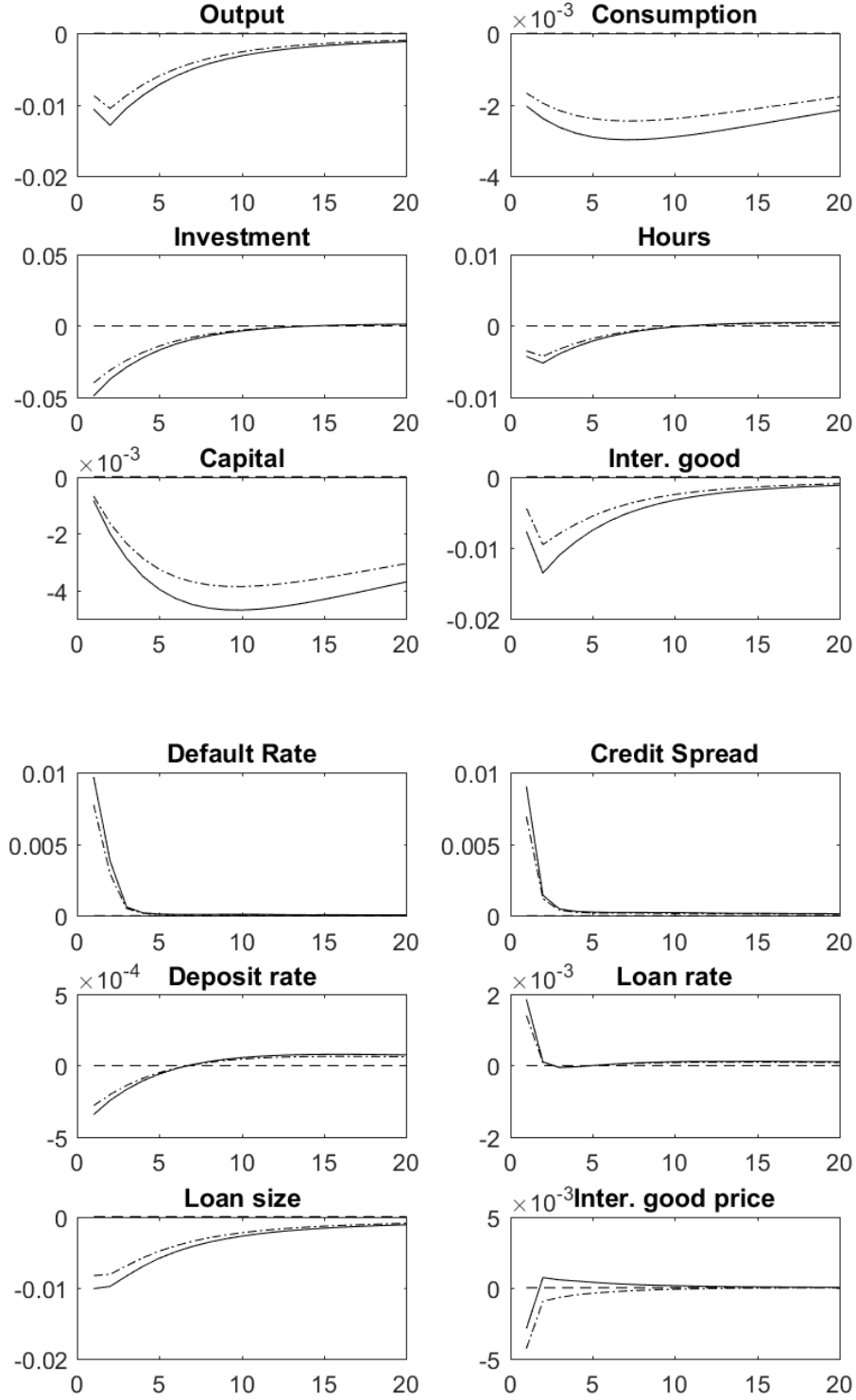


Figure 3: Impulse response functions following a negative shock to TFP ($-1 \times$ standard deviation) of: (i) the main model and (ii) a version of the model assuming that the efficiency of intermediate production is independent of aggregate TFP ($\rho_M = 0$, dashed lines). All variables but deposit rates, credit spreads and default probability are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

4 Concluding remarks

I present a general equilibrium model with endogenous defaults that reproduces the counter-cyclical fluctuations of default rates and credit spreads. This is achieved by assuming the existence of a sector of firms that borrow from banks to produce an intermediate good used by a representative final production firm. The model displays procyclical behaviour of the intermediate good prices. This depresses the revenues of intermediate production firms during slumps and increases these revenues when the economy is performing well, thus generating countercyclical default rates and, in turn, countercyclical credit spreads. This mechanism is made more potent by assuming a positive correlation between the efficiency of the intermediate producers and aggregate TFP. In addition, I assume that the borrowing firms face a quadratic adjustment cost when adapting their level of production to take productivity shocks into account. I show that these adjustment costs are key to generating quantitatively realistic dynamics of credit spreads while the correlation between the productivity of the borrowing firms and the aggregate productivity is important to the fluctuation of aggregate output, investment and consumption. These features inform the calibration process of the model so that the model is able to generate reasonable dynamics for the aggregate quantities that are well captured by the usual real business cycle models while generating realistic dynamics of default rates and credit spreads.

The model considered in this paper provides a simple framework for the modelling of endogenous bankruptcies in a dynamic stochastic general equilibrium framework. The model can be extended to capture the behaviour of simple economies with long term debt contracts as well as economies with multiple debt maturities. Such modelling effort would help analyse the effect of the use of long-term debt contract on the dynamic stability of the general equilibrium describing the economy and in the modelling of various other term structure effects. This will be the subject of future research efforts.

Because of the relative simplicity of the used bankruptcy mechanism, it can be used in the context of larger general equilibrium models. For instance, monetary DSGE models can be augmented to reproduce countercyclical default rates and credit spreads and their effects on other aggregates through the introduction of an intermediate good production sector relying on debt financing as it is described in this paper.

A Model equations

The model equations describing the general equilibrium are presented in this appendix.

$$z_t^a = \rho_a z_{t-1}^a + e_t, \quad (\text{A.1})$$

$$Z_t^a = e^{z_t^a}, \quad (\text{A.2})$$

$$\xi_{t+1} = \frac{1}{\sqrt{1 - \rho_M^2}} \left\{ \rho_M z_{t+1}^a + \frac{1}{\sigma_M} \ln \left(\frac{Q_{t+1} g(X_t, X_{t-1})}{R_t X_t} \right) \right\}, \quad (\text{A.3})$$

$$DP_{t+1} = \Phi(-\xi_{t+1}), \quad (\text{A.4})$$

$$\begin{aligned} R_t \mathbb{E}_t \left[\Phi(\xi_{t+1}) + X_t \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1}) \right] &= \frac{\partial g(X_t, X_{t-1})}{\partial X_t} e^{\Sigma_M^2/2} \mathbb{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \Phi(\xi_{t+1} + \Sigma_M) \right] \\ &+ g(X_t, X_{t-1}) e^{\Sigma_M^2/2} \mathbb{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1} + \Sigma_M) \right]. \end{aligned} \quad (\text{A.5})$$

$$Y_t = (Z_t^a H_t^{1-\alpha} K_t^\alpha)^{1-\zeta} M_t^\zeta, \quad (\text{A.6})$$

$$r_t^K = (1 - \zeta) \alpha \frac{Y_t}{K_t}, \quad (\text{A.7})$$

$$w_t = (1 - \zeta)(1 - \alpha) \frac{Y_t}{H_t}, \quad (\text{A.8})$$

$$Q_t = \zeta \frac{Y_t}{M_t}, \quad (\text{A.9})$$

$$C_t^{-\sigma_H} = \beta R_t^D \mathbb{E}_t C_{t+1}^{-\sigma_H}, \quad (\text{A.10})$$

$$\chi L_t^\eta = w_t C_t^{-\sigma_H}, \quad (\text{A.11})$$

$$C_t^{-\sigma_H} = \beta \mathbb{E}_t C_{t+1}^{-\sigma_H} (1 - \delta + r_{t+1}^K), \quad (\text{A.12})$$

$$Rec_t = (1 - \theta) Q_t g(X_{t-1}, X_{t-2}) \int_{-\infty}^{\infty} Z_{i,t} \mathbf{1}_{\pi_{i,t}^M < 0} d\epsilon_i = (1 - \theta) Q_t g(X_{t-1}, X_{t-2}) \Phi(-\xi_t - \Sigma_M) e^{\rho_M \sigma z_t^a + \Sigma_M^2/2}, \quad (\text{A.13})$$

$$R_t \mathbb{E}_t [\Phi(\xi_{t+1})] = \mathbb{E}_t \left[R_t^D - \frac{Rec_{t+1}}{X_t} \right], \quad (\text{A.14})$$

$$CS_t = \left(\frac{R_t}{R_t^D} \right)^{1/dt} - 1, \quad (\text{A.15})$$

$$M_t = \mathbf{E}_{\epsilon_i}[y_{i,t}^M] - \theta \mathbf{E}_{\epsilon_i}[y_{i,t}^M \mathbf{1}_{\epsilon_i < -\xi_t}] = g(X_{t-1}, X_{t-2}) e^{\rho^M \sigma z_t^a + \Sigma_M^2/2} (1 - \theta \Phi(-\xi_t - \Sigma_M)), \quad (\text{A.16})$$

$$D_t = X_t, \quad (\text{A.17})$$

$$L_t = H_t, \quad (\text{A.18})$$

$$Y_t = C_t + X_t + K_t - (1 - \delta)K_{t-1}. \quad (\text{A.19})$$

B Steady state

In this appendix, I provide the equations that uniquely determine the steady state of the model. First, the SS deposit and capital rental rates follow directly from the Euler equations

$$\bar{R}^D = 1/\beta, \quad (\text{B.1})$$

$$\bar{r}^K = 1/\beta - 1 + \delta. \quad (\text{B.2})$$

I will express the remaining SS variables as a direct or indirect function of the SS loan level \bar{X} , the SS distance to default $\bar{\xi}$ and the SS price of the intermediate good \bar{Q} . First the SS intermediate good quantity is derived from the clearing condition 2.29

$$\bar{M} = f(\bar{X}) e^{\Sigma_M^2/2} (1 - \theta \Phi(-\bar{\xi} - \Sigma_M)), \quad (\text{B.3})$$

where $\Sigma_M = \sigma_M \sqrt{1 - \rho_M^2}$. The intermediate good first order condition yields the SS output

$$\bar{Y} = \frac{1}{\zeta} \bar{M} \bar{Q}. \quad (\text{B.4})$$

This and the first order condition for capital provides an expression for SS capital

$$\bar{K} = \frac{(1 - \zeta)\alpha}{\bar{r}^K} \bar{Y}. \quad (\text{B.5})$$

The final good clearing condition yields SS consumption

$$\bar{C} = \bar{Y} - \bar{M} - \delta \bar{K}. \quad (\text{B.6})$$

Combining the first conditions for labour provision 2.21 and demand 2.13 yields SS labour

$$\bar{L}^{1+\eta} = \frac{(1 - \zeta)(1 - \alpha)}{\chi} \bar{Y} \bar{C}. \quad (\text{B.7})$$

One can then deduce the SS wages from the labour demand first order condition

$$\bar{w} = (1 - \zeta)(1 - \alpha) \frac{\bar{Y}}{\bar{L}}. \quad (\text{B.8})$$

The recovery in the SS is

$$\bar{Rec} = (1 - \theta) \bar{Q} \Phi(-\bar{\xi} - \Sigma_M) e^{\Sigma_M^2/2} f(\bar{X}). \quad (\text{B.9})$$

The expression of the recovery combined with the bank loan pricing condition leads to an expression for the SS loan rate

$$\bar{R} = \frac{1}{\Phi(\bar{\xi})} \left(\frac{1}{\beta} + \bar{Rec} \right). \quad (\text{B.10})$$

The SS credit spread are by definition

$$\bar{CS} = (\beta \bar{R})^{1/dt} - 1. \quad (\text{B.11})$$

The SS distance to default expression 2.4, demand for loans condition 2.8 and technology constraint 2.10 provide three equations to solve for \bar{X} , \bar{Q} and $\bar{\xi}$ using the above to express other variables as a function of \bar{X} , \bar{Q} and $\bar{\xi}$

$$\bar{\xi} = \frac{1}{\Sigma_M} \ln \left(\frac{\bar{Q} f(\bar{X})}{\bar{R} \bar{X}} \right), \quad (\text{B.12})$$

$$\begin{aligned} \bar{R} \left[\Phi(\bar{\xi}) + \bar{X} \frac{\partial \bar{\xi}}{\partial \bar{X}} \phi(\bar{\xi}) \right] &= f'(\bar{X}) e^{\Sigma_M^2/2} \bar{Q} \Phi(\bar{\xi} + \Sigma_M) \\ &+ f(\bar{X}) e^{\Sigma_M^2/2} \bar{Q} \frac{\partial \bar{\xi}}{\partial \bar{X}} \phi(\bar{\xi} + \Sigma_M), \end{aligned} \quad (\text{B.13})$$

$$\bar{Y} = (\bar{L}^{1-\alpha} \bar{K}^\alpha)^{1-\zeta} \bar{M}^\zeta. \quad (\text{B.14})$$

References

- BASU, S. (1995): “Intermediate Goods and Business Cycles: Implications for Productivity and Welfare,” *American Economic Review*, 85(3), 512–31.
- BERNANKE, B., AND M. GERTLER (1989): “Agency Costs, Net Worth, and Business Fluctuations,” *The American Economic Review*, 79(1), 14–31.
- BERNANKE, B., M. GERTLER, AND S. GILCHRIST (1998): “The Financial Accelerator in a Quantitative Business Cycle Framework,” Working Paper 6455, National Bureau of Economic Research.
- CARLSTROM, C. T., AND T. S. FUERST (1997): “Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis,” *The American Economic Review*, 87(5), 893–910.
- CHRISTIANO, L., R. MOTTO, AND M. ROSTAGNO (2010): “Financial factors in economic fluctuations,” *Working Papers Series, European Central Bank*, 47(1).
- FALATO, A., AND J. XIAO (2020): “Credit Markets, Learning, and the Business Cycle,” Discussion paper.
- GOMES, J. F., A. YARON, AND L. ZHANG (2003): “Asset prices and business cycles with costly external finance,” *Review of Economic Dynamics*, 6(4), 767 – 788, Finance and the Macroeconomy.
- JONES, C. I. (2011): “Intermediate Goods and Weak Links in the Theory of Economic Development,” *American Economic Journal: Macroeconomics*, 3(2), 1–28.
- KIYOTAKI, N., AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105(2), 211–248.
- MERTON, R. C. (1974): “ON THE PRICING OF CORPORATE DEBT: THE RISK STRUCTURE OF INTEREST RATES*,” *The Journal of Finance*, 29(2), 449–470.
- PESARAN, M. H., AND T. XU (2016): “Business Cycle Effects of Credit Shocks in a DSGE Model with Firm Defaults,” *Working Paper*.