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by

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Endogenous Money – On Banking Behaviour in New and Post Keynesian Models

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Abstract

In New and Post Keynesian macroeconomic models, money supply is assumed to be endogenous. The reasons for the endogeneity and the role of the financial sector in the supply process, however, are seen very differently. In this paper we explicitly derive the behaviour of the banking sector regarding the supply of loans and demand for reserves from portfolio and liquidity considerations. As a result, the money multiplier as well as the money base are endogenously determined. Although the microeconomics of bank behaviour is modelled quite simply, credit, money, and bonds demand depend on policy variables in a non-linear and non-monotonous way.

Keywords: endogenous money, loans market, bonds market, central banking

JEL Classification: E51, E44, B22

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1 Introduction

The endogeneity of money supply is a widely discussed topic, especially in New and Post Keynesian macroeconomics. It can be taken as a common conviction that individual behaviour regarding credit demand and supply as well as holding currency and deposits has an impact on the money creation process. These issues are often neglected in New Classical and Neoclassical type models. There are, however, very different approaches how endogeneity of money originates (for an extensive review see e.g. Palley (2002), see also Park (2007)). New Keynesian economics (see e.g. Mankiw and Romer (1991), or Woodford (2003) for more recent developments) is dominated by the *"New Consensus"* where the endogeneity originates only from the Taylor rule which describes the behaviour of the central bank (Taylor (1993)). We interpret this as a very weak form of endogeneity since the central bank controls the money creation process perfectly but is itself controlled by other endogenous variables like income and inflation. In Post Keynesian economics there have been developed two distinct approaches which are usually denoted as the *"accomodationist"* (or horizontalist) and the *"structuralist"* (or verticalist) approach (see e.g. Moore (1988), Pollin (1991), or Fontana (2004) for a more recent overview). Both schools have in common that the money creation process is determined by the behaviour of commercial banks and non-banks on the credit market. The accomodation approach argues that an increase in credit demand leads to a need for additional reserves. In order to ensure the liquidity of the banking sector the central bank has to respond by increasing the money base (reserves) and hence to accomodate the credit demand. In this view the microeconomic considerations of the commercial banking sector play a minor role. In contrast, the structuralist approach argues that the commercial banks respond to an increase in credit demand with structural changes of their portfolio on the asset and the liability side. This may lead to a change in the demand for reserves and hence in the interaction with the central bank. However, there is no monotone relationship between credit demand and the response of the central bank, but more complex structural effects on the interest rates and portfolio composition. While the accomodationist see the central bank's behaviour as a reflex to the non-bank public (which hence determine solely the money supply), the structuralist see a certain degree of autonomous central banking policy, hence money is endogenously generated by the interaction of the public, the central bank, and the commercial banks, where the latter play a crucial role.

We argue, that it is important to investigate these complex interactions to understand how (and if) central bank policy impulses are conducted to the real sphere, as well as to understand how the real sphere affects the money creation process. Therefore, our approach is related to the structuralist view. In a certain sense the accomodation view is similar to the New Consensus insofar as the endogeneity originates in a rule-guided behaviour of the central bank. It is always possible to employ such rules in a macroeconomic model but we argue that it is important to understand how commercial banks behave on credit, bond and money markets, and how they respond to changes on these markets as well as to changes in the central bank policy. Fixed policy rules seem not to be sufficient to understand in detail the impact of monetary policy and the interaction of real and monetary sphere. As we will see, open market operations and changes in reserve interest rates have – depending on the parametrization – complex and sometimes countervailing effects on variables like credit supply or bonds demand. Therefore, it is more reasonable to combine the building block of the financial sector, as outlined in this paper, with a complete macroeconomic model, and then – if possible – to derive a rationale for monetary policy rules.

In contrast to most Post Keynesians we develop a model of banking behaviour which is in some sense neoclassical: the (representative) bank has preferences regarding risk, return, and liquidity, and it manages its assets and liabilities via portfolio and value at risk techniques. A single commercial bank operates in competitive markets and responds to changes in market conditions as well as to changes in central bank policy. This allows for a detailed analysis of some spillover effects between credit and bonds market, the market for reserves, and the real sector (via income). Although the microeconomics of banking are portrayed in a very simplified way the results are not trivial. They confirm and prove some conclusions of the structuralist view and should be understood as a "building block" of the financial sector which can be used for macroeconomic reasoning. Since we have no real sector it is neither possible nor justifyable to classify the framework as Post or New Keynesian although it is in line with the structuralist approach.

The paper is organized as follows: Before developing our model, we briefly discuss two

sources of endogeneity by means of two approaches in the literature. In section 2 we review the model of Bernanke and Blinder (1988) who introduce the idea how portfolio considerations of the commercial bank affect the money multiplier. Section 3 discusses the less common approach of Bofinger (2001) where changes on the credit market affects the bank's demand for central bank loans. This establishes a close relation between the interest rates on the market for credits and the market for central bank money via an optimization calculus of the commercial bank. Section 4 picks up both ideas in a consistent framework and extends them with liquidity considerations. These liquidity issues are twofold: When the bank's capital is fixed the volume of risky assets (like loans) has to be restricted. On the liability side there is a risk of unperceived outflows of deposits which requires to hold a sufficient volume of liquid assets like excess reserves. All portfolio and value at risk decisions are calculated explicitly and exhibit nonlinear relationships between the central variables (e.g. loans (bonds) demand (supply)) and the interest rates. A few implications of the model are briefly discussed. Section 4.5 numerically illustrates the relationships for "reasonable" parametrizations. Section 5 concludes and poses further questions and extensions of the framework.

2 The approach by Bernanke and Blinder

In the approach by Bernanke and Blinder (1988), the commercial bank's simplified balance sheet contains reserves (R), loans (L^s), and bonds (B^b) as assets, while deposits (D) are the unique liability. There are no currencies and no central bank loans to commercial banks. The reserve requirements are rD, hence the balance sheet can be written as $E + L^s + B^b = (1-r)D$, where E are the excess reserves at the central bank with a zero interest rate. Since loans and bonds have both expected returns and a certain risk (failed credits and bonds price volatility) the commercial bank has portfolio considerations about its assets. The structure of the portfolio is given by:

$$\begin{aligned} E(i) &= \lambda_E(i)(1-r)D \qquad (1) \\ L^s(i,\rho) &= \lambda_L(i,\rho)(1-r)D \\ B^b(i,\rho) &= (1-\lambda_E(i)-\lambda_L(i,\rho))(1-r)D \end{aligned}$$

where i is the interest rate of the bonds, ane ρ is the interest rate of loans. Obviously λ_L depends positively on ρ, negatively on i, and vice versa for λ_B. For simplicity, Bernanke

and Blinder assume that variations in ρ only affect the shares of L^s and B^b in the portolio. The reserves of the commercial bank are

$$R = rD + E = rD + \lambda_E(i)(1-r)D = (r + \lambda_E(i)(1-r))D \tag{2}$$

Hence the money multiplier is $m(i) = [r + \lambda_E(i)(1-r)]^{-1}$. In contrast to the exogenous multipliers in common textbook models there is now a dependency of the multiplier on the behaviour of the commercial bank, i.e. the multiplier depends on the endogenously determined bonds interest rate i.

The equilibrium in the loans market is determined by $L^d(i, \rho, y) = L^s = \lambda_L(\rho, i)(1-r)D$. The demand for loans depends positively on i and income y, and negatively on ρ. The bonds market is not explicitely modelled in the Bernanke/Blinder approach. While the loans and the bonds market determine the money supply $D^s = m(i)R$, the money demand $D^d = D^d(i, y)$ follows the standard assumptions (positive dependency on y and negative dependency on the bonds interest rate i). Money market equilibrium is given by $D^d(i, y) = m(i)R$ which is the conventional LM curve. From these results Bernanke and Blinder construct a so-called CC curve as a substitute for the IS curve where the goods and credit markets are in equilibrium. Together with the LM curve they study the impact of monetary impulses on the real sector.

For the purpose of our paper we are not interested into the CC-LM macro model but we pick up the idea that the commercial bank's behaviour is driven by portfolio considerations, which have important implications for the loans market and the money market. The mechanistic exogenous money multiplier is replaced by an endogenous money creation process, based on the behaviour in the loans market and on portfolio considerations of the commercial bank. There are, however, some shortcomings which deserve an extension of the framework (for further critical remarks see Bajec and Graf Lambsdorff (2006)).

First, there are no central bank loans to the commercial bank, even though the interest rate policy for bank loans plays a prominent role in central banking. Changes in the central banks interest rate ρ_c for refinancing commercial banks is an important component of monetary policy. In the Bernanke/Blinder model, it is not made explicitly clear how the central bank changes the reserves R. In absence of loans for commercial banks, this must

be done by open market operations, i.e. by buying or selling bonds. If we allow central bank credits L_c with interest rate ρ_c, the commercial bank has not only to decide on the portfolio structure of a given volume $(1-r)D$, but on the amount of desired reserves R as well.

Second, the bonds market is not modelled explicitely. Bernanke and Blinder implicitly assume that the non-bank's demand for bonds is a residual from net financial wealth plus loans demand minus desired deposits (see Bajec and Graf Lambsdorff (2006), p.10). Since firms and households face budget constraints it is more reasonable to assume that they decide on the desired structure of financial assets like deposits and bonds, and then decide on the volume of the assets, financed also by loans. Thus, the loans demand L^d is not properly derived. Furthermore, if we assume that open market operations are condected by buying or selling bonds, this also affects the bonds interest rate i.

Third, the bank's portfolio considerations are reduced to risk and return decisions. However, banking management also addresses solvency and liquidity issues. These shape the loan supply, bonds demand, and the extent of refinancing the operations with central bank loans.

Fourth, there are neither domestic nor foreign currencies, and there is no market for equities and derivative financial contracts. It is clear that a model cannot include too many items without loosing the ability to derive clear analytical results. For the sake of simplicity it is admissible to neglect these things. However, the equity market and especially the demand for equities financed by loans becomes of growing importance as the recent subprime crisis indicates. Since the observed fragility of the inter-related markets challenges monetary policy, it may be worth to include them in the framework.

Fifth, all markets are assumed to be perfect. Starting from Stiglitz and Weiss (1981) there is a broad literature on credit rationing based on asymmetric information which plays a role also in equity markets (see e.g. Hellmann and Stiglitz (2000)). From Neo Keynesian theory we know that rationing changes the agent's calculus. They will adapt their plans so that e.g. rationing on the loans market probably has spillovers to other

financial markets as well as to the real sphere. As a result, the macroeconomic effective demand may depend on rationing effects.

It is not possible, of course, to address all mentioned shortcomings. This paper concentrates on the first three mentioned issues. Regarding the central bank loans for commercial banks we first summarize a model by Bofinger (2001).

3 The approach by Bofinger

In Bofinger (2001) (pp. 53) a model of the macroeconomic loans market is presented, where commercial banks are able to refinance their credit supply by central bank loans L_c, i.e. by the demand for reserves. The aim of the model is to explain the money creation process endogenously by the interaction of the market for loans and the market for reserves. There is no bonds market and no currency in this model. Hence, the simplified commercial bank's balance sheet is $R + L^s = L_c + D$. The loan supply is explicetly derived from a profit maximizing calculus:

$$\max_{L^s} \ \pi = \rho L^s - \rho_c L_c - \beta (L^s)^2 \tag{3}$$

where the term $\beta(L^s)^2$ describes the increasing risk of debt failures. This can be justified by assuming that with an expanding loan volume, the bank finances more and more risky projects, or more debitors have limited soundness. However, it is more common to assume that the debt failure probability depends on ρ rather than L. Since a central bank loan L_c extends the balance sheet of the bank and increases the reserves, the credit expansion follows the multiplier process. When $R = rD$ is subtracted from the balance sheet we have:

$$L^s = L_c + (1-r)D = L_c + (1-r)mL_c \tag{4}$$

Because for the money multiplier $m = 1/r$ holds true in absence of currency, a simple rearrangement leads to $L^s = mL_c$. Substituting $L_c = m^{-1}L^s$ into the profit function the first order condition yields the supply function:

$$L^s = L^s(\rho, \rho_c, \beta) = \frac{1}{2\beta}\left(\rho - \rho_c/m\right) \tag{5}$$

which is increasing in ρ. The demand for loans is given by $L^d(\rho, y)$. From the market equilibrium condition $L^d = L^s$ we obtain an equilibrium interest rate $\rho^*(\rho, \beta)$. Of course

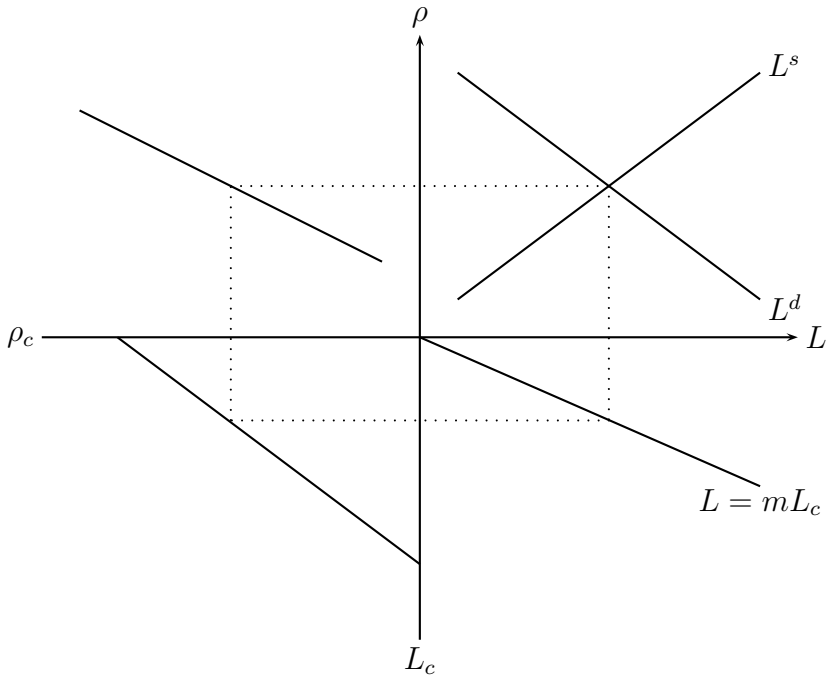

Figure 1: *The Bofinger model as described in Bofinger (2001).*

ρ^* also depends on demand parameters. The loans market equilibrium implies a profit maximizing demand for reserves, i.e. central bank loans. Substituting $L^s = mL_c$ into the profit function and maximizing with respect to L_c yields the reserve demand function:

$$L_c(\rho, \rho_c, \beta) = \frac{1}{2m\beta}(\rho - \rho_c/m) \tag{6}$$

On the market for reserves the central bank acts as a monopolist. The central bank chooses a point on the demand function L_c^d according to monetary policy goals instead of profit maximization. A change in the interest rate ρ_c for reserves and therefore a shift of the reserve demand changes the loan supply curve L^s and has therefore an impact on the loan interest rate $\rho^*(\rho_c, \beta)$.

Assuming a linear loan demand $L^d = \gamma y - \alpha\rho$ it is an easy task to derive the resulting equilibrium interest rates. Figure 1 shows the complete model for the linear case. The upper right quadrant depicts the loans market, the lower right quadrant shows the money multiplier. The loans market equilibrium hence determines the demand for reserves (lower left quadrant) via the multiplicator. The relation between the interest rates for loans and reserves ($\rho^*(\rho_c, \beta)$) is depicted in the upper left quadrant. The interest rate based transmission of monetary impulses works as usual: An increasing ρ_c shifts the loan supply curve upwards. This results in a raise of the market intrerest rate ρ (depending

on supply and demand elasticities), and a decrease of the demand for central bank money L_c. However, the money supply is no longer a policy variable, the money (credit) creation process is also determined by the loans demand.

Obviously, the Bofinger model has some shortcomings. As in the Bernanke/Blinder approach, there are no equity markets, no currencies, and no rationing effects due to market imperfections. Moreover, there is neither a bonds market nor excess reserves. So the commercial bank has no asset portfolio and therefore no portfolio considerations (which implies risk aversion while the profit function in the Bofinger model implies risk neutrality). As a consequence, the multiplier is constant. Nevertheless, the money creation process is determined by the behaviour of the commercial banks and the debitors.

Our aim is now to combine Bofinger's idea of a demand for reserves and hence an interest rate policy of the central bank with the Bernanke/Blinder approach which includes a bonds market and portfolio considerations of a (risk averse) commercial bank. Furthermore we extend the framework by liquidity and solvency considerations.

4 A model of aggregated banking behaviour

Starting from the balance sheet of the aggregated banking sector, we derive the decision of a single representative commercial bank regarding the structure and volume of its portfolio. These decisions are driven by considerations about risk, return and liquidity. We draw conclusions about how the endogenous money supply process is determined by the bank's behaviour. The results are illustrated by some numerical examples.

4.1 The balance sheet of the aggregated commercial banking sector

The balance sheet of a commercial bank contains three liabilities: deposits D, central bank loans L_c, and bank capital $\overline{BC}$, and the three assets: loans L, bonds B and excess reserves E. The required reserves rD are subtracted from both sides of the sheet. The balance sheet of the commercial bank thus reads:

$$L + B + E = (1 - r)D + L_c + \overline{BC} \tag{7}$$

where the bank capital is assumed to be fixed. Since we look at the aggregated banking sector, all inter-bank loans are subtracted from the sheet. Therefore, the market for reserves consists of the aggregated banking sector on the demand side and the central bank on the supply side. All types of reserves demanded by commercial banks which have to pay interest rates to the central bank are subsumed to L_c.

The portfolio considerations of the commercial bank are now twofold. First, the bank has to decide about the portfolio *structure*, which is determined by the shares λ_L, λ_B and λ_E which give the fractions of L, B and E in the full portfolio (with $\lambda_L + \lambda_B + \lambda_E = 1$). The second decision a commercial bank has to make is about the *volume* of the portfolio. The portfolio volume is defined by:

$$V = (1 - r)D + L_c + \overline{BC}. \tag{8}$$

Although D is determined by a multiplier process, in a competitive market a single commercial bank will take D as given. A single bank is seen as not being able to determine the outcome of the multiplier process. Therefore the volume of the portfolio is determined solely by L_c which hence is a policy variable of the bank. Thus, our analysis distnguishes the *short run perspective* where the bank responds optimally to a given D, from the *long run perspective* where D is endogenously determined by the multiplier process. Throughout the multiplier process the bank will adjust their decisions to the development of D. For a comparative static policy analysis it is appropriate to compare the long run equilibria before and after a change in policy variables. In the last section, however, we briefly address the question whether a comparative static analysis is significant when the dynamic adjustment processes are non-linear and complicated.

4.2 Management of risk and return

From the three assets L, B and E, there is one riskless asset E with expected return $\mu_E = 0$, and two risky assets L and B. For L, the expected return per unit and the variance are

$$\begin{aligned} \mu_L &= p\rho - (1 - p) \\ \sigma_L^2 &= p(\rho - \mu_L)^2 + (1 - p)(-1 - \mu_L)^2 \end{aligned} \tag{9}$$

where p is the probability for a successfully returned credit and ρ is the loans interest rate, collaterals have been neglected. As Stiglitz and Weiss (1981) argue, the probability $(1-p)$ of a credit failure may be assumed to be a positive function of ρ. In this paper, however, we take p as exogenously given. The expected return and variance for bonds is

$$\begin{aligned} \mu_B &= i \\ \sigma_B^2 &= const \end{aligned} \tag{10}$$

For the sake of simplicity we assume that covariances are not present.

The optimal portfolio structure for one riskless and two risky assets is determined in two steps (for details see Huang and Litzenberger (1988)). In the first step, the efficient portfolio frontier for a mix of the two risky assets has to be derived. The risky portfolio R is given by the shares $\tilde{\lambda}_L$ and $\tilde{\lambda}_R = (1-\tilde{\lambda}_L)$ which implies:

$$\begin{aligned} \mu_R &= \tilde{\lambda}_L\mu_L + \tilde{\lambda}_B\mu_B = \tilde{\lambda}_L(p\rho - (1-p)) + (1-\tilde{\lambda}_L)i \\ \sigma_R^2 &= \tilde{\lambda}_L^2\sigma_L^2 + \tilde{\lambda}_B^2\sigma_B^2 = \tilde{\lambda}_L^2(p^2(\rho-\mu_L)^2 + (1-p)^2(-1-\mu_L)^2) + (1-\tilde{\lambda}_L)^2\sigma_B^2 \end{aligned} \tag{11}$$

Hence, $\tilde{\lambda}_L$ determines all possible (μ_R, σ_R)-combinations, which define the portfolio frontier. In order to find the optimal risky portfolio, which is then mixed with the riskless asset E, we have to determine the tangential point of the efficient portfolio frontier with the capital allocation line (CAL) being defined as:

$$\mu_P = \mu_E + \left(\frac{\mu_p - \mu_E}{\sigma_R}\right)\sigma_P = \left(\frac{\mu_R}{\sigma_R}\right)\sigma_P \tag{12}$$

Standard portfolio methods provide the solution:

$$\tilde{\lambda}_L = \frac{\mu_L\sigma_B}{\mu_L\sigma_B + \mu_B\sigma_L} \quad \text{and} \quad \tilde{\lambda}_B = \frac{\mu_B\sigma_L}{\mu_B\sigma_L + \mu_L\sigma_B} \tag{13}$$

as there is no covariance present. In the second step, the bank decides how to mix the riskless asset E with the risky portfolio R according to its preferences. This results in a portfolio P with:

$$\begin{aligned} \mu_P &= \lambda_R\mu_R + (1-\lambda_R)\mu_E = \lambda_R\mu_R \\ \sigma_P^2 &= \lambda_R^2\sigma_R^2 + (1-\lambda_R)^2\sigma_E^2 = \lambda_R^2\sigma_R^2 \end{aligned} \tag{14}$$

To find the optimal proportion λ_R, the bank maximizes its utility function:

$$\max_{\lambda_R\in[0,1]} U(\mu_P, \sigma_P) = \mu_P - \theta\sigma_P^2, \tag{15}$$

where θ is the bank's risk-aversion parameter. It is known from portfolio theory, that the solution for λ_R is the tangential point of the bank's indifference curve with the CAL, and is in general given by:

$$\lambda_R = \arg\min \left\{ \frac{\mu_R}{2\theta\sigma_R^2}, 1 \right\} \tag{16}$$

Note, that λ_R changes as soon as additional constraints from Value at Risk are introduced. Now the bank's optimal portfolio structure is completely determined by:

$$\lambda_L = \tilde{\lambda}_L \lambda_R, \quad \lambda_B = (1 - \tilde{\lambda}_L)\lambda_R, \quad \lambda_E = 1 - \lambda_R \tag{17}$$

where the explicit form is not further revealing and thus have been omitted here. As it can be shown, the structure depends nonlinearily on the return and risk variables.

The next task consists in deriving the optimal portfolio volume. Since the (μ, σ^2)-utility function of the portfolio calculus describes the utility of one portfolio unit, we now have to consider the utility of the total profits. The portfolio volume V is determined by eq. (8). Then the profits are given by $\pi = \mu_P V$ and the variance is given by $\sigma_P^2 V^2$. The utility function is therefore:

$$\max_{L_c \geq 0} u(\pi) = \mu_P V - \theta \sigma_P^2 V^2 \tag{18}$$

The volume V can solely be changed by L_c, as explained above. The deposits are taken as an exogenous variable for a single bank. However, in the long run the deposits are endogenously determined by a multiplier process which also depends on the aggregated bank behavior.

In an optimum the bank will expand its portfolio volume until the marginal utility equals the marginal cost ρ_c, i.e. the cost of refinancing an additional marginal portfolio unit. With $\theta > 0$ the marginal utility of a portfolio is a decreasing function of the portfolio volume. Therefore, in the optimum the marginal utility must equal the marginal cost ρ_c:

$$\frac{\partial u(\pi)}{\partial L_c} = \rho_c \tag{19}$$

$$\Rightarrow \quad L_c = \arg\max \left\{ \frac{\mu_P - \rho_c}{2\theta\sigma_P^2} - ((1-r)D + \overline{BC}),\ 0 \right\} \tag{20}$$

With (14) for μ_P and σ_P^2, the demand for central bank loans L_c is given by:

$$L_c = L_c(\rho, i, \rho_c, D) \tag{21}$$

which determines the volume V. Note, that the optimal L_c is parametrized by a given D (short run perspective). As we will see later on, the deposits are determined endogenously by (27) which depends also on L_c.

With the optimal shares λ_L, λ_B and λ_E and the optimal portfolio volume, we obtain the demand for bonds B, the excess reserves E, and the supply of loans L:

$$L = \lambda_L V, \quad B = \lambda_B V, \quad E = \lambda_E V \tag{22}$$

Again, the explicit form is omitted here. The demand for B and E and the supply of L are determined by structural decisions $(\lambda_L, \lambda_B, \lambda_E)$ as well as by volume considerations (via L_c). For two reasons the dependencies on ρ and i are analytically not clear: First, since σ_L^2 depends on ρ, an increase in ρ may have ambiguous effects on λ_R, the share of the risky assets. The same effect may hold true in case of i but is omitted here by the assumption of a constant σ_B^2. Secondly, the shares λ_L, λ_B and the central bank credits L_c may have different signs in their derivatives with respect to i and ρ. The structural and the volume effects may be countervailing, thus the total effect depends on the parametrization. If, for example, i increases, then bonds become more attractive, and the portfolio is rearranged, which decreases loan supply L. On the other hand, the total portfolio becomes more attractive and will be expanded by increasing L_c and therefore L. The only clear effect is that ρ_c which has a negative impact on all assets.

4.3 Mangement of liquidity by Value at Risk (VaR)

In the last section the commercial bank's goal was to balance risk and expected returns. However, banks are also interested to keep a certain level of capital in order to stay solvent. Loans may fail and the bonds position in the portfolio is also volatile. Only the excess reserves E are risk-free. Depending on the probability distributions of μ_L, μ_B and the optimal shares λ_L, λ_B it is possible to derive a probability distribution for the losses of the portfolio. With a certain probability the losses could exceed the bank's capital $\overline{BC}$. In this case the bank would be insolvent.

We assume that the bank's management addresses this problem with the Value at Risk (VaR) approach (for details see e.g. Wahl and Broll (2003)). Let α be the probability

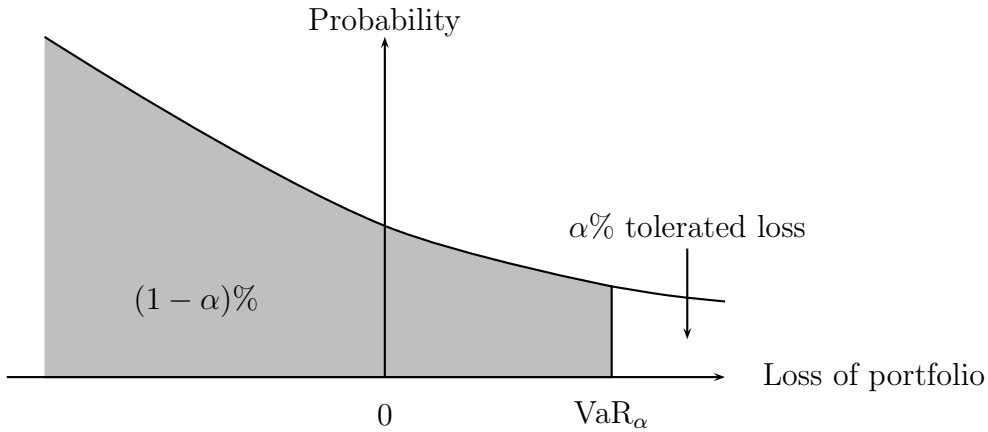

Figure 2: *Loss distribution with Value at Risk.*

that the losses exceed the bank's capital, then

$$\mathrm{VaR}_\alpha = q_\alpha V \tag{23}$$

determines the capital requirement to ensure solvency with probability $1-\alpha$ in a given period. Here V is the portfolio volume and $-q_\alpha$ is α-fractile of the probability distribution (see Figure (2)).

The VaR approach requires that the capital $\overline{BC}$ covers at least the VaR at the level α, i.e. $\overline{BC} \geq \mathrm{VaR}_\alpha = q_\alpha V$. The more risky the portfolio and the higher the desired probability $(1-\alpha)$ of staying solvent – either determined by the bank's management or by bank regulation policy – the more capital $\overline{BC}$ is required. It can be shown that, for a given VaR_α, the bank chooses an optimal structure of V and BC. In our approach, however, we take $\overline{BC}$ as a given constant. Hence, VaR_α is a constraint for the portfolio volume V, leading to:

$$\begin{aligned} \overline{BC} &\geq \mathrm{VaR}_\alpha = q_\alpha V = q_\alpha((1-r)D + L_c + \overline{BC}) \\ \Rightarrow \quad L_c &\leq \frac{1-q_\alpha}{q_\alpha}\overline{BC} - (1-r)D \end{aligned} \tag{24}$$

This is an additional restriction for determining the optimal portfolio volume via L_c as discussed in the previous section (see eq.(18)). In case that the restriction is binding, the equality sign holds true in (24) and the marginal utility of the portfolio exceeds the marginal cost ρ_c. As a consequence, the money creation process is eventually limited by the bank's solvency policy.

The liquidity management also affects the portfolio structure. If we consider the deposits D not to be a given deterministic value, but a stochastic variable with a given distribution (with D as the expected value), the bank faces the risk of deposit volatility and sudden deposit outflows (bank runs). If depositors wish to draw their deposits, the bank needs immediately liquid assets. We assume that only excess reserves E have the required liquidity (see e.g. Mishkin (2006), pp. 208). Then the VaR approach also applies to the probability to be a victim of bank runs, i.e. to become illiquid. Let β be the probability that sudden outflows of deposists exceed the excess reserves E. Then the bank avoids illiquidity with probability $1-\beta$ if for excess reserves

$$\begin{aligned} E &\geq \mathrm{VaR}_\beta = q_\beta D \\ \Rightarrow \quad \lambda_E &\geq q_\beta \frac{D}{(1-r)D + L_c + \overline{BC}} \equiv \underline{\lambda}_E \end{aligned} \tag{25}$$

holds true. Again, we have an additional constraint for the portfolio calculus as discussed in the previous section. The structure of the risky portfolio $\tilde{\lambda}_L, \tilde{\lambda}_B$ is obviously not affected by the VaR approach. But if (25) is binding then we have $\lambda_R = 1 - \underline{\lambda}_E$ which determines λ_L and λ_B.

Summing up, the VaR approach can be used on the one hand to ensure solvency by balancing V and $\overline{BC}$. This eventually has an impact on the chosen portfolio volume. On the other hand the approach is used to avoid illiquidity in case of deposit outflows by balancing E and D. This eventually has an impact on the chosen portfolio structure. In this paper we do not investigate these effects analytically. The impact of eventually binding liquidity restrictions is illustrated by the numerical examples in section 4.5.

4.4 Endogenous money supply

In a first step, we derive some money (credit) multipliers under the assumption that i and ρ are given exogenous variables. This is useful to highlight two principal ways of endogeneity in the money supply process, due to the building block "banking behaviour". Afterwards, we have to consider the other market sides on the credit, bonds and money market. This endogenizes i and ρ and makes the money creation process more complicated. This has also implications for the conduct of monetary policy.

Let $e = E/D = \lambda_E((1-r)D + L_c + \overline{BC})/D$, then the central bank's balance sheet can be expressed as

$$(MB =) \quad S + L_c = (r+e)D \tag{26}$$

with MB as the money base and S as the securities held by the central bank (e.g. bonds). For technical reasons we assume $S \geq \overline{BC}$. In contrast to L_c it is assumed that S is determined by purchases and sales of the central bank on the market for securities. Note again, that in this paper we neglect any currency. The ratio e depends on D and on endogenously determined values of λ_E and L_c. Inserting e into (26) and solving for D we obtain:

$$D = \frac{S + (1-\lambda_E)L_c - \lambda_E\overline{BC}}{r + (1-r)\lambda_E} \tag{27}$$

There are two ways to change the money base. First, the central bank could conduct open market operations on the market for securities, which leads c.p. to the multiplier:

$$\left.\frac{dD}{dS}\right|_{L_c=const} = \frac{1}{r + (1-r)\lambda_E} > 0 \tag{28}$$

where λ_E is endogenously determined but is assumed to have a given value throughout the multiplier process. This result is essentially the same as the multiplier derived by Bernanke and Blinder (1988). This is a conventional view of the multiplier process which is slightly enriched by the assumption that the fraction of excess reserves are endogenously determined by portfolio considerations. As we have seen, however, the commercial bank will also manage the portfolio volume by adjusting borrowed reserves L_c. As eq. (20) shows, a change in D leads to an adjustment of L_c which also determines the money base. The multiplier (28) is therefore valid only in case of a fixed L_c (e.g. in case of $L_c = 0$). Otherwise an expansion of the money base by $dS > 0$ induces a decrease of L_c because the bank attempts to keep its portfolio volume on an optimal level. In the long run perspective the equilibrium values of D and L_c are determined by the solution of the linear equation system (20) and (27):

$$D^{\text{long}} = \begin{cases} (S - \overline{BC}) + \frac{(1-\lambda_E)(\mu_P - \rho_c)}{2\theta\sigma_P^2} & \text{for } L_c^{\text{long}} > 0 \\ \frac{S - \lambda_E\overline{BC}}{r + (1-r)\lambda_E} & \text{for } L_c^{\text{long}} = 0 \end{cases} \tag{29}$$

where L_c^{long} is given as

$$L_c^{\text{long}} = \arg\max\left\{\frac{[(1-\lambda_E)r + \lambda_E](\mu_p - \rho_c)}{2\theta\sigma_p^2} - rBC - (1-r)S,\ 0\right\} \tag{30}$$

When the commercial bank is able to keep the portfolio volume at the chosen optimal level, the required reserve rate r does not play a role anymore. The central bank is then able to enforce an increasing money supply D but is not able to initiate a multiplier process since we have $dD^{\text{long}}/dS = 1$. The conventional multiplier (28) holds true only if the banks are not willing or able to adjust L_c. It is also possible that the bank sector has positive but very limited borrowed reserves. In this case the bank is able to damp the multiplier process to some extent until L_c becomes zero. As a result in the long run D will be between the two cases as described by (29).

The second way to change the money base, as discussed above, is a shift in the commercial bank's demand for borrowed reserves L_c: From (27) we have

$$\frac{dD}{dL_c} = \frac{1-\lambda_E}{r+(1-r)\lambda_E} > 0 \tag{31}$$

Again, λ_E is assumed to be given ceteris paribus. A change of the endogenous variable $L_c(i, \rho, \rho_c, D, \cdot)$ can only be explained by a change of one or more of the variables (e.g. D). We have discussed already the case that L_c responds to a change of D while D can be changed by S. In the long run, where D and L_c take their equilibrium values, we have

$$\frac{dD^{\text{long}}}{dS} = 1$$

according to (29), in case of a positive L_c^{long}.

The second policy variable of the central bank are the refinancing conditions ρ_c. A change in ρ_c affects the demand for L_c and therefore the money base. We do not distinguish different types of borrowed reserves like standing facilities and open market operations on the market for reserves – all reserves where commercial banks have to pay interest rates to the central bank are subsumed to central bank loans L_c. Ceteris paribus we have in the short run:

$$\frac{dD}{d\rho_c} = \frac{dD}{dL_c}\frac{dL_c}{d\rho_c} = -\frac{1}{2\theta\sigma_P^2} \cdot \frac{1-\lambda_E}{r+(1-r)\lambda_E} < 0 \tag{32}$$

The central bank's refinancing policy may fail in the case that L_c is determined by a binding VaR_α condition or a non-negativity constraint. A change of ρ_c would then have no effect on L_c and hence D. In the long run we have:

$$\frac{dD^{\text{long}}}{d\rho_c} = -\frac{1-\lambda_E}{2\theta\sigma_P^2} < 0 \tag{33}$$

in case of $L_c > 0$ in the long run and, of course, $dD^{\text{long}}/d\rho_c = 0$ otherwise. Comparing (32) and (33), it can be seen that the impact of monetary policy is lower than in the short run.

It has to be stressed that these multipliers tell only "half of the truth", since each change of S or ρ_c affects the supply and demand conditions on the loans, bonds, and money market. Therefore, we have effects on the equilibrium values of i and ρ which have an impact on λ_E and L_c. Probably some of these effects may be countervailing. In order to analyse some of these effects, we have to consider the behavior of non-banks on the loans and bonds markets. This allows for some reasonable assumptions about the endogenous determination of ρ and i, as well as assumptions about how variables like the income y determine the credit demand and hence the money supply process.

Bernanke and Blinder (1988) assume a credit demand function $L^d(\rho, i, y)$ where the partial derivative of ρ is negative, and the other two are positive. The bonds market is not explicitely modelled there. The money demand is assumed to be $D(i, y)$ with the usual signs. As Bajec and Graf Lambsdorff (2006) point out, the non-bank sector also faces a budget constraint. If money and credit demand are defined in the way of Bernanke and Blinder then the non-bank's bonds demand is a residual, which is not very convincing. Bajec and Graf Lambdsdorff argue that it is more reasonable to start by modelling the money and bonds demand and then derive the loans demand from the budget constraint. We follow this argument but with slightly different balance sheets of the sectors.

Consider that bonds represent the only asset for open market operations of the central bank, hence $S = B^{cb}$. This simplifying assumption prevents the introduction of further assets and it allows that the central bank buys or sells bonds either from/to banks, or from/to non-bank financial intermediates. Furthermore, we assume that the total amount of bonds is fixed: $\overline{B} = B^{cb} + B^b + B^p$, where B^p are the bonds held by the private (non-bank) sector and B^b are the bonds in the commercial bank's balance sheet. Adding the balance sheet restrictions of the central bank and the commercial bank we then have:

$$\overline{B} + L^s = D + B^p + \overline{BC} \tag{34}$$

For the housholds we assume that they hold the firm's capital C as well as the bank

capital $\overline{BC}$, bonds B^p, deposits D^h, and physical assets like housing H. This must be equal to the net financial wealth NFW plus bank loans L^h. For the non-bank firms we assume that they hold physical capital PC and deposits D^f, while on the liability side we have firm capital C, bonds $\overline{B}$ and credits L^f (alternatively, it is also possible to include the government's balance sheet and to interpret b as government bonds instead of firm bonds). Adding these constraints we have

$$D + B^p + PC + H + \overline{BC} = \overline{B} + L^d + NFW \tag{35}$$

with $L^d = L^h + L^f$ as the total loans demand, and $D = D^h + D^f$ as the total money demand. Note that adding (34) to (35) with $L^d = L^s$ we have the identity that net financial wealth equals the total physical capital stock of the economy $(PC + H)$, implying that in flow terms savings equal net investment. Rearranging (35), the loans demand L^d is a function of the desired D, B^p, and also of the desired physical assets PC and H.

Now we are able to apply the same logic of portfolio *structure* and portfolio *volume* as in the previous sections. However, it has to be taken into consideration that housholds and firms have different views about how to structure the asset and the liability side. Furthermore, additional goals like a solvency signalling liability structure of the firms and the transaction motive for holding money (deposits) play an important role. Once, the non-bank sector has derived the desired portfolio structure it has to decide about the portfolio volume. To finance the desired portfolio volume, non-banks demand loans L^d. Hence, also the loans interest rate ρ affect all positions on the left hand side of (35).

Without going too deep into details in this paper, we can reasonably asume that for the money, bonds, and credit market we have (see Bajec and Graf Lambsdorff (2006))

$$\begin{aligned} B^p &= B^p(\underset{-}{\rho}, \underset{+}{i}, \underset{?}{y}) \\ D &= D(\underset{-}{\rho}, \underset{-}{i}, \underset{+}{y}) \\ L^d &= L^d\underset{-}{\rho}, \underset{+}{i}, \underset{+}{y}) \end{aligned} \tag{36}$$

The bonds market equilibrium condition is $\overline{B} = B^{cb} + B^p(\rho, i, y) + B^b(\rho, i, \rho_c)$ where B^{cb} is determined by monetary policy, which is given as exogenous here, and the commercial bank's bonds demand B^b is given by (22). Obviously the equilibrium bonds rate is a

function $i^*(\rho, \rho_c, y, B^{cb})$. On the loans market we have $L(i, \rho, \rho_c) = L^d(\rho, i, y)$ where L is given by (22). Again, the equilibrium value ρ^* is a function $\rho^*(i, \rho_c, y)$.

Every change in the exogenous variables $S = B^{cb}, \rho_c$ and y affects i^* and ρ^* and therefore λ_E and L_c. Taking these market responses into consideration the multiplier effects are much more complicated than previously shown, and have to be investigated in a subsequent paper.

4.5 Numerical examples

In this section we present some numerical examples that illustrate the results given above. As it has been shown, λ_L, λ_B, λ_E and hence L, B, E as well as L_c and D are complicated functions which depend on ρ, i and ρ_c in a non-linear way. We have explicitly implemented the given constraints by truncating μ_L, λ_R and L_c. The truncation in μ_L is necessary to exclude the possibility of a negative μ_L, in which case no bank would provide loans.

The truncations of λ_R and L_c stem from liquidity considerations and non-negativity restrictions, as has been explained in section 4.3. Liquidity considerations of a commercial bank lead to two distinct effects for the portfolio decision of the bank. The first effect regards the volume of the portfolio and is expressed by the constraint $0 \leq L_c \leq \overline{L}_c = \frac{1-q_\alpha}{q_\alpha}\overline{BC} - (1-r)D$ (see eq. (24)). The second effect is related to the portfolio structure and is expressed as a truncation $\lambda_R \leq \overline{\lambda}_R = 1 - \underline{\lambda}_E$. This is is a direct consequence of the VaR constraint $\lambda_E \geq \underline{\lambda}_E$ (see eq. (25)), which expresses the bank's management's decision regarding excess reserves.

In our computations we used the following set of parameters: the probability p of a successfully returned credit is given by $p = 0.95$, the variance for bonds σ_B^2 was chosen to be $\sigma_B^2 = 0.1$, and the interest rate for central bank loans is $\rho_c = 0.02$. The required reserve rate r is set to $r = 0.1$. Commercial banks have a risk aversion parameter, which was chosen to be $\theta = 0.1$. Finally, the VaR parameter for the upper bound on λ_R was set to $\overline{\lambda}_R = 0.9$. The banking capital BC was set to $BC = 2$ and we have securities $S = 2$.

In Figure 3, we have shown the results for $\tilde{\lambda}_L$ and λ_R for the above defined set of param-

eters. One can see that $\tilde{\lambda}_L$ depends positively on ρ and negatively on i. Note that λ_R has been truncated according to equation (25). Figure 4 shows the shares λ_L, λ_B and λ_E as well as the demand for central bank loans L_c^{long}. In the long run perspective, D^{long} and L_c^{long} adjust to the equilibrium level. For small and medium ρ and i the long run demand L_c^{long} is zero. If the risky portfolio becomes more attractive (ρ and/or i increases) the short run value for L_c will increase. This induces a multiplier process which increases D. Therefore, the demand for L_c will decrease again in the long run and adjusts to the equilibrium level.

In figure 5 the long run equilibrium values for L, B and E is shown as well as the long run value for the deposits D. The dependencies of L on ρ and i are as usual, except for very small i (< 0.02) and small ρ (< 0.07) values. The risky portfolio becomes more unattractice when i decreases so that the volume effect overcompensates the structure effect, and the loans supply decreases slightly. The same effect can be seen for the bonds demand B which is increasing with a decreasing ρ, but is also decreasing when ρ falls under a certain level.

Most of the graphs exhibit some kinks due to VaR or non-negativity constraints. Figure 6 shows the parameter values where the constraints are binding. As it can be seen the kinks in the previous figures coincide with these constraints. An extensive interpretation and analysis of the policy implications will be given in a subsequent paper.

5 Discussion

In this paper we presented a simple model of the aggregated commercial banking sector. The bank is assumed to manage its assets and liabilities according to risk, return, and liquidity considerations. These have been conducted by the portfolio as well as by the Value at Risk methodology. The endogeneity of money supply is hence twofold: The *structure* of the portfolio, driven by risk and liquidity preferences, determines the money multiplier, while the acchieved portfolio *volume* determines the demand for reserves and hence the money base. Therefore, this approach is related to the structuralist approach in Post Keynesian macroeconomics. However, we see this only as one building block of

the financial sector which can be implemented also in New Keynesian (or even other) types of models. In the present model we do not consider any special type of central bank policy like the accomodation approach or the Taylor rule as in so-called "New Consensus models". Our model shows that open market policy and interest rate policy may have different impact on the volume and the structure of the bank's portfolio in the short and in the long run. Especially, the loans and bonds interest rates ρ and i depend in a different and nonlinear way on the interest rate for reserves ρ_c. These policy implications deserve further investigation.

As an extension of the framework different types of liabilities may be considered. Deposits D have been assumed to have no interest rate and a high risk of outflows and (therefore) a requirement for holding excess reserves. However, there are other types of liabilities where the bank has to pay interest rates but has no reserve requirements. This enriches the strategic possibilities to attrract deposits in order to enhance the credit volume and is hence a substitute for the central bank loans demand L_c. Since these liabilities have no reserve requirement, this may have substantial effects on the money multiplier – the abilities of the central bank to manage the expansion process are much more restricted as they are anyway.

Another important extension is to include new financial instruments like derivative contracts, inter-bank loan contracts and other instruments to obtain funds. Markets for those contracts have been grown rapidly and seem to affect the risk and liquidity position of a bank significantly. This will also have an impact on the risk of illiquidity and bankruptcy. In macroeconomic models, however, these derivative markets are rarely discussed.

We have already addressed the possibility to endogenize the credit crunch risk since the probability p may depend on the interest rate ρ. As Stiglitz and Weiss (1981) argue, this may result in credit rationing effects. Such effects are popular, but also critized (see Besanko and Thador (1987), Arnold (2005)), hence we do not include them here.

Interesting issues are the dynamical features of the process of credit and money creation. The static multiplier approach, as used in this paper, is not able to capture dynamical

effects. Consider, for example, the case that a bank decides to enhance the portfolio volume by demanding additional reserves L_c. After adapting the portfolio volume on the asset side, the multiple credit and deposit creation process starts. Additional deposits D in subsequent periods thus lead to an "overshooting" of the desired portfolio volume, so that L_c will decrease. During this process the interest rate structure changes as well which may cause real effects. Such a dynamical investigation will be addressed in a subsequent paper. Due to the nonlinearity of the central relationships between the variables, it is reasonable to expect that the money creation process will be complex.

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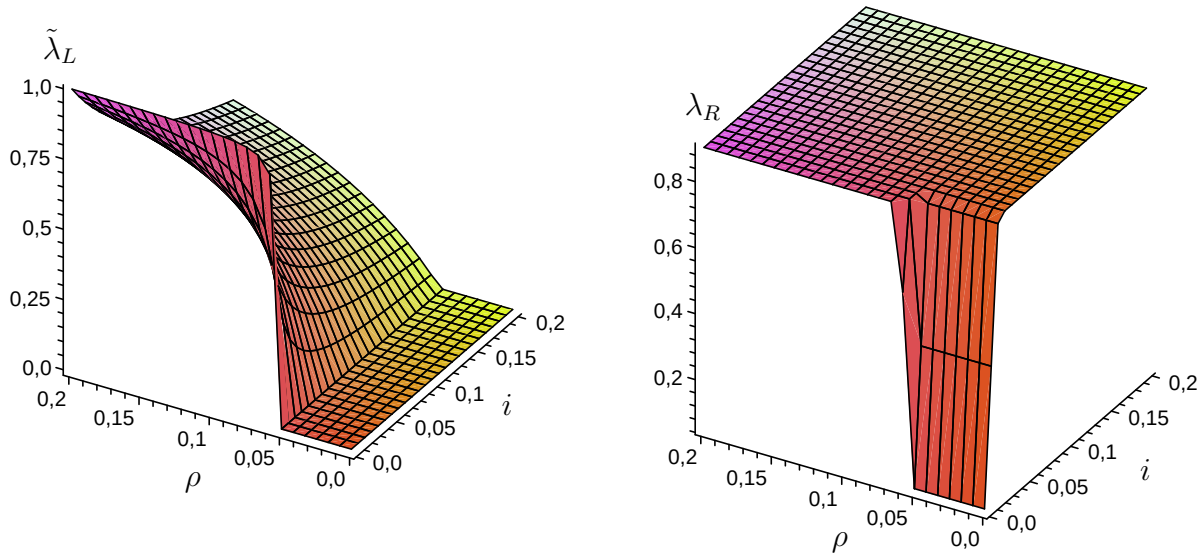

Figure 3: *The share $\tilde{\lambda}_L$ (left) of loans in the risky portfolio, as defined in equation (13). The share $\tilde{\lambda}_B$ is defined as $\tilde{\lambda}_B = 1 - \tilde{\lambda}_L$. The share λ_R (right) of the risky portfolio in the total portfolio as defined in equation (16). The results have been truncated to the region $0 \leq \tilde{\lambda}_L \leq 1$ and $0 \leq \lambda_R \leq \bar{\lambda}_R$.*

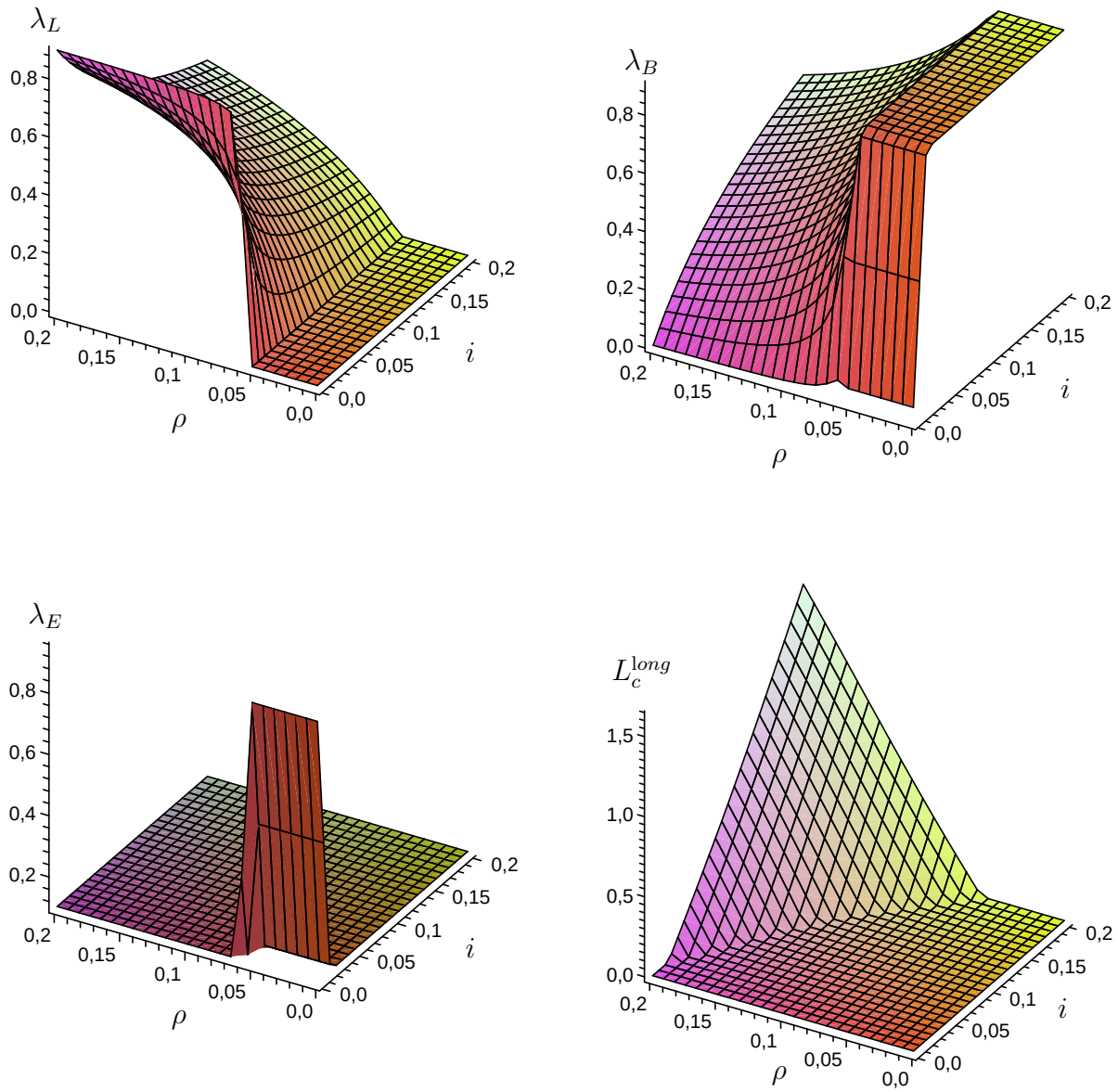

Figure 4: *The shares λ_L (top left), λ_B (top right) and $\lambda_E = (1 - \lambda_R)$ (bottom left) of the loans, bonds and excess reserves in the total portfolio, as defined in equation (17). The long run demand for central bank loans (bottom right) is defined in equation (30). The constraint defined in equation (25) has been taken into account.*

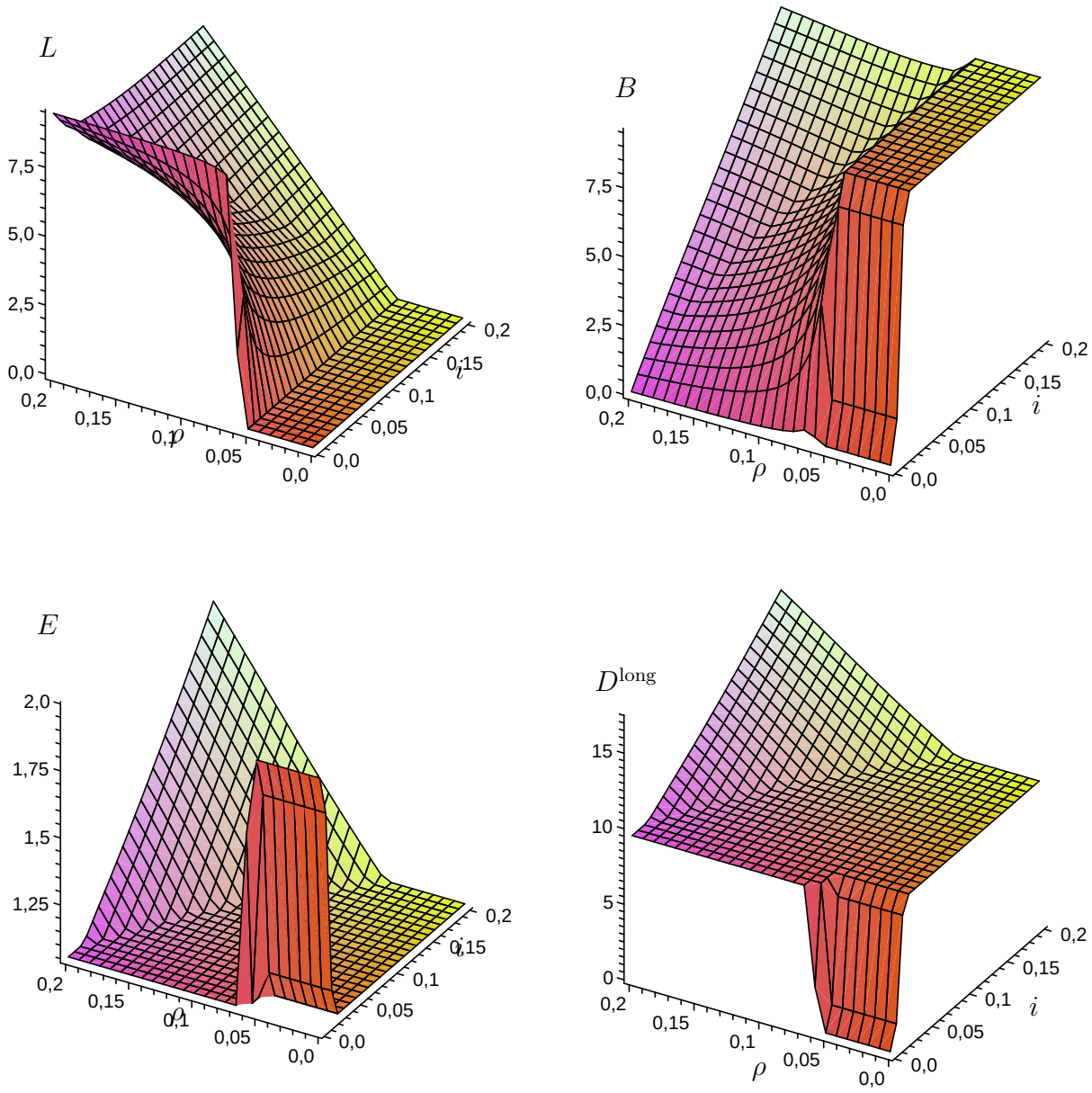

Figure 5: *Total supply of loans L (top left), demand for bonds B (top right) and excess reserves E (bottom left). These are the long run equilibrium values. The long run deposits (bottom right) are defined in equation (29).*

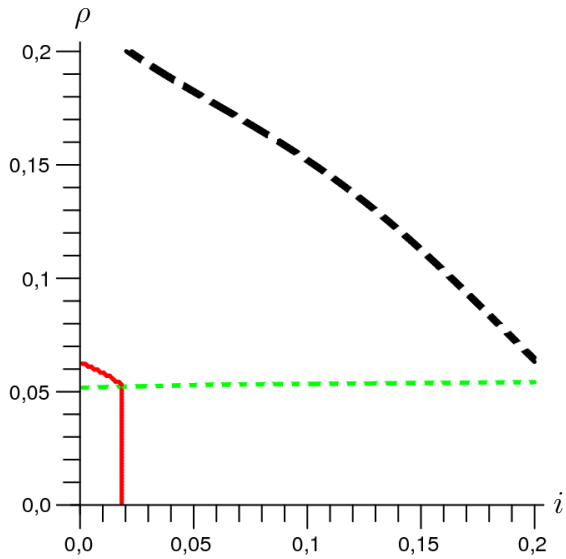

Figure 6: *The solid red line depicts the VaR constraint for λ_E, in the north-west region of the line λ_E is at its minimum. The dotted green line is the nonnegativity constraint for $\tilde{\lambda}_L$ (and therefore λ_L), for all values below this line we have $\lambda_L = 0$. In the region right of the dashed line, we have $L_c^{long} > 0$ in the long run, left of this region the non-negativity constraint applies.*