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RESEARCH



Computing Optimal Mitigation Plans for Force-Majeure Scenarios in Dynamic Manufacturing Chains

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Abstract

We consider force-majeure supply disruptions in a dynamic, multi-product manufacturing supply chain with time-dependent parameters. We present a linear programming model that captures a specific force-majeure scenario with respect to several objective functions that can be combined in a multi-objective framework, e.g., minimization of loss, maximization of shortage-free time, or prioritization of mitigation types. Solving this model yields an optimal mitigation plan that describes how to best (re-)allocate supply and production operations. Supported mitigation options include plant-side safety stock, supplier-side inventories, and additional production thus reflecting the manufacturing setting of a large-scale industrial player. We describe a workflow for increasing the resilience of supply chains based on risk profiles generated by our approach.

Keywords Supply chain disruptions \cdot Force-majeure scenarios \cdot Mitigation \cdot Risk analysis \cdot Business continuity planning

Mathematics Subject Classification 90B06

1 Introduction

Disrupted supply chains are an omnipresent challenge in the purchasing of raw materials and pre-products. Companies therefore proactively protect themselves against potential delivery delays and failures by diversifying their supplier base, qualifying more capacity than required (capacity-to-demand ratio C-to-D > 1), building up

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safety stocks, and maintaining mitigation plans. Nevertheless, it is difficult to prepare for major incidents. Although these certainly occur from time to time, it is generally impossible to determine the probability of occurrence of a specific event. From the recent past, examples of such events include the following:

- During the corona pandemic, there were massive production losses due to factory closures of suppliers, which resulted, for example, in a shortage of parts for German car manufacturers [1]. Also, the distribution of overseas containers was disrupted during the pandemic. Transports were delayed; on some routes, costs rose dramatically. See [2–4] for further analysis of the impact of the corona pandemic on supply chains.
- Due to the Ever Given accident in the Suez Canal in 2021, there was a considerable backlog on the canal, resulting in significantly longer shipping times [5].
- The Huthi rebels' attacks on cargo ships in the Red Sea in 2024 forced vessels to take the much longer route around the Cape of Good Hope.
- Natural disasters like earthquakes, storms, floods, fires, and explosions but also political upheavals and strikes regularly lead to considerable supply losses [6]. Climate change will further exacerbate this situation [7].

The continuous optimization of a supply chain, including the associated mitigation plans, is therefore a prerequisite for the success of manufacturing companies. The procurement of pre-products can be switched to other suppliers at short notice, and delays can be bridged with safety stocks.

In this paper, we develop a rigorous mathematical optimization model to simulate failure scenarios in a supply chain. Our work is motivated by, but not limited to, a manufacturer of fast consumer goods that produces goods at several manufacturing sites. The manufacturer requires critical raw materials for production, which it obtains from various specially selected and qualified suppliers.

The following parameters are included in our optimization model: production ramp-up times, transportation times, safety stocks, and the possibility to interchange materials. We primarily minimize the remaining shortage and secondarily maximize the time to shortage, i.e., the time it takes until a shortage arises.

By aggregating the results of many simulated failure scenarios, critical suppliers and production sites can be identified and mitigation plans can be adapted. We outline how such an analysis and associated decision-making process might look like.

With the help of our optimization model, intuitive best practices can be verified:

- 1. A broadly positioned supplier base with suppliers close to and further away from the production sites is an advantage.
- 2. Material on the transportation routes (material in transit) postpones delivery failures and enables other suppliers to ramp-up production.
- 3. A capacity-to-demand ratio significantly greater than 1 is a prerequisite to compensate for failures at all.

We deliberately neglect the costs of contingency plans in our model and focus only on their valuation in relation to supply delays and shortages. The decision of how much mitigation plans may cost and what supply risk a company should bear is the subject of management discussions beyond the model presented below.



1.1 Related Work

Mathematical optimization of supply chains considering risks is an active area of research; see the survey by Suryawanshi and Dutta [8] for an overview of different optimization approaches and the survey by Katsaliaki et al. [9] regarding the treatment of supply chain disruptions. We focus on force-majeure scenarios affecting the supply side, such as temporary supplier shutdowns, lost shipments, port strikes, and hurricanes (affecting a complete region), but also a rapid increase of demand. These events are generally of low frequency and high impact, thus occupying the upper left area in the diagram of Fig. 1.

Low-impact, low-frequency events are usually handled by a sufficient allocation of buffers and stocks and do not require a scenario-based analysis. High-frequency events allow for probabilistic models, like, e.g., Monte Carlo simulation approaches, that are able to reflect the simultaneous appearance of multiple problems [10, 11]. For optimizing the robustness of supply chains subject to these stochastic deviations, stochastic programming techniques can be used [12, 13] which require however a large running time. It should be noted that the high-impact, high-frequency area ought to be empty in any healthy supply chain.

Predicting the actual impact of some given scenarios allows to base the risk management of a supply chain on a quantitative basis, unlike many traditional approaches that take into account qualitative information from experts [14]. A scenario is characterized by two main properties: its probability of occurrence and its potential impact [15]. An estimation of probabilities for scenarios is outside the scope of this paper, but can be achieved using a systems approach [16], by simulation or with statistical models.

The assessment of risks in supply chains is often done on a qualitative basis. Lockamy [17] presents an approach for assessing disaster risks in supply chains with Bayesian models. A framework for the managerial handling of force-majeure risks is presented by Kleindorfer and Saad [18]. Craighead et al. [19] discuss the effects of various supply chain characteristics on the severity of supply disruptions and the relation to mitigation capabilities. The survey by Golan et al. [20] describes various approaches to analyze the resilience of a supply chain.

The resilience of a supply chain often needs to be optimized simultaneously with other goals. Mari et al. [21] propose an optimization model that incorporates both resilience and sustainability in the form of carbon emissions of the supply chain. The optimization model by Elluru et al. [22] combines location with routing decisions to

create disaster-resilient supply chains. Margolis et al. [23] combine the two objectives of minimizing costs and maximizing the resilience of a supply chain into a multiobjective optimization model. Chen et al. [24] integrate considerations of the product life cycle into their mixed-integer programming model for disruption recovery.

The existence of a rigorous method to compute mitigation plans is important for multiple stages of supply chain operation and planning. Obviously, in case of an actual failure, it allows to quickly generate both a preview of the impact and suggestions for most efficient counteractions. Ivanov et al. [25] give a survey on the literature of methods for planning the recovery from disruptions in a supply chain. A case study details the effects of disruption scenarios and corresponding countermeasures based on a linear programming model by Ivanov et al. [26]. The case study of Balster and Friedrich [27] discusses a model for the food supply chain and corresponding mitigation measures for several disruption scenarios.

Especially, business continuity planning at a strategic scope benefits from the ability to assess the supply chain's vulnerability to specific scenarios. Such an assessment can, on the one hand, guide successive development towards a more resilient supply chain by showing the impact of potential adjustments (such as supplier selection or the allocation of inventories) on the supply chain's robustness [28]. On the other hand, it allows to compare (and then balance) the protection level across plants, regions, or material classes, helping to reduce the cost of unnecessary robustness created by large over-capacities. An optimization approach by Parajuli et al. [29] for computing protection decisions in an attacker-defender model takes mitigation as a lower-stage problem.

For various use cases, specialized models for optimizing the resiliency of supply chains have been proposed. Ali and Nakade [30] establish a mixed-integer program (MIP) that chooses distribution center locations and order quantities so as to minimize expected (pre-mitigation) shortage. The same authors consider a single-period set-up with mitigation from a single local supplier [31]; here, a MIP minimizes the expected cost of a set of scenarios, assuming all failures can be mitigated completely. Ziegenbein and Baumgart [32] consider time-varying supply and demand, but again no mitigation. You et al. [33] assume probabilistic demand and freight rate ratio, as opposed to the scenario-based approach in our work.

Recently, the concept of resilience of a supply chain has been extended to the concept of *viability* that also takes long-term disruptions and the ecosystem around the supply chain into consideration [34].

1.2 Our Results

In this work, we present a rigorous mathematical framework for quantitative what-if analysis of force-majeure scenarios in a supply chain.

Our approach allows to compute mitigation plans that are (Pareto-) optimal for the respective scenarios, subject to several objective functions, e.g., minimizing post-mitigation losses, maximizing shortage delay, or minimizing mitigation effort. We present an extensible framework for finding the optimal mitigation plan for a scenario, centered around a time-expanded flow network that is solved with a linear program (LP).

The framework supports a supply chain with multiple materials, supplier sites, and manufacturing plants (receivers), as well as time-varying demands by these manufacturing plants and order quantities. Potential mitigation options include a short-term increase of raw-material production and the use of supplier-side inventory, both subject to various delivery constraints. Also, we properly take the mitigating effect of significant shipping times into account: the corresponding material in transit acts as an implicit inventory on road, rail, or sea that delays the impact of disruptions on the manufacturing plants. In addition, we present a detailed handling of plant-side safety stock that arises from shipments arriving a certain time before the planned consumption. These basic features can be extended by, e.g., multiple end products, material interchangeability, or a multi-tier supply chain.

We present and model multiple objective functions that quantify several goals. These are as follows:

- The *loss value*, representing the weighted sum of the shortage remaining after the mitigation. In this way, loss of profits can be modeled.
- The *shortage delay*, i.e., the time until shortage occurs. By maximizing it, the time available to restructure the supply chain can be increased.
- *Mitigation efforts and costs* that are incurred by choosing a mitigation plan. By measuring and optimizing the simplicity of the mitigation plan, organizational costs can be reduced.

Since these goals are often contradicting, a multi-objective problem ensues.

We also describe the effects of the various types of mitigation types in a case study. Our mitigation model can also be used to assess a whole set of scenarios, leading to a risk profile which highlights the vulnerable parts of the supply chain. We illustrate the use of these results in an integrated planning process to allow decision-makers to improve the robustness of their supply chains.

1.3 Failure-Aware Supply Chain Optimization

The flow chart in Fig. 2 illustrates how our proposed optimization model can be used in the overall decision-making process to create a failure-aware supply chain. In this paper, we focus on the precise modeling of the failure scenarios, the simulation of the resulting shortage, and the optimization of the corresponding mitigation measures.

- For the definition of failure scenarios, decision-makers should monitor the political, economical, societal, and natural disaster risks to ensure all currently relevant scenarios are captured.
- The simulation of shortage needs a precise model of the material dependencies and transportation times with the supply chain.
- Since the optimal choice of the mitigation measures depends on the local situation at multiple locations and is time-dependent, a time-expanded graph model is used.
- The optimized resulting shortage after considering mitigation in different failure scenarios can be used to inform needed changes on the supply chain structure.



Fig. 2 Integrated decision process using the optimization model

1.4 Organization of the Paper

The remainder of this work is organized as follows. In Section 2, we formally define what a mitigation plan is in the context of a given disruptive scenario. We then describe in Section 3 an LP based on a minimum cost flow network that can easily find an optimal ex-post strategy. To simplify notation and because every supply chain has its individual characteristics, requiring not every potential mitigation option, Sections 2 and 3 contain what we think is the core of our approach. Supplementary to that, Section 4 lists several additional features by which that core model can be augmented, if needed. Numerical results are presented in Section 5. In Section 6, we describe the use of the model in an integrated workflow to increase the resilience of supply chains based on risk profiles. Section 7 contains a conclusion and possible topics for future research.

2 Problem Description

In the following, $\mathbb{N} = \{0, 1, ...\}$ denotes the natural numbers, \mathbb{R} the real numbers, and the nonnegative real numbers are written as $\mathbb{R}^+ = \{x \in \mathbb{R} : x \ge 0\}$.

2.1 Supply Chain Model

We consider a manufacturing company that operates a set P of manufacturing plants, producing some end products not further specified here (but see Section 4.2 for how to include end products).

For doing so, the plants require a continuous supply of raw materials. We investigate the supply chain of a set of materials M. As an example, imagine the pigment supply chain of a candy-producing company, where each type of candy contains exactly one type of pigment.

Each plant has a production plan that determines the required amount of each raw material for each time step. In practice, the production plan is created from the fore-casted end product demands and depends on the specific raw-material requirements of each end product. Since our model does not include the end product layer, by the term *demand*, we always refer to the raw-material demand of some plant.

Definition 1 (Demand, Horizon). For each plant $p \in P$ and material $m \in M$, the (raw-material) *demand* of m at p at time t is denoted by $d(t, m, p) \in \mathbb{R}^+$. Here, t is assumed to be contained in a suitable *horizon* $T = \{0, ..., t_h\} \subset \mathbb{N}$ that covers the time period under investigation.

Materials are delivered by a set of supplier sites S, each of which is capable of producing some subset of M. Shipping times introduce a delay between the delivery at a supplier site and the arrival at the destination plant.

Definition 2 (Shipping Time). For a supplier $s \in S$, material $m \in M$, and plant $p \in P$, $\delta(s, m, p) \in \mathbb{N}$ is the *shipping time* (distance) required for shipping *m* from *s* to *p*.

In addition to shipping times, it is common to schedule a delay between arrival and consumption at a plant, in order to hedge against ordinary fluctuations of shipping times or minor demand fluctuations. We call this the *safety-stock delay*.

Definition 3 (Safety-Stock Delay). The scheduled *safety-stock delay* is denoted by $\sigma(s, m, p) \in \mathbb{N}$. It defines the number of time steps that a shipment of material $m \in M$ from supplier $s \in S$ to plant $p \in P$ should arrive prior to planned consumption.

Note that the safety-stock *delay* (a time span) entails the storage of a specific amount of safety-stock *volume* (an amount) at the plants. The interdependency of the "time" and "volume" notion, respectively, of safety stock is covered in Section 2.3.3. We use the time span as the leading input since it is easier to interpret for managers and because this allows a better-integrated modeling with the shipping times.

Within the time horizon T, it is assumed that supply orders have been placed to exactly cover the plants' demands.

Definition 4 (Order Quantities). For time point $t \in T$, supplier $s \in S$, material $m \in M$, and plant $p \in P$, the *order quantity* of material *m* ordered from *s* by *p* that is planned to be consumed at time *t* is denoted by $o_{\rm C}(t, s, m, p)$. The volume that is planned to *arrive* at time *t* is $o_{\rm A}(t, s, m, p)$, and finally, the volume planned to *depart* from supplier *s* at time *t* is $o_{\rm D}(t, s, m, p)$.

Note that this notation of order quantities reflects the time shift due to shipping times and safety-stock delays, yielding the relation

$$o_{\rm D}(t, s, m, p) = o_{\rm A}(t + \delta(s, m, p), s, m, p)$$

= $o_{\rm C}(t + \delta(s, m, p) + \sigma(s, m, p), s, m, p).$

We assume that the ordered quantities match the demands exactly and just-in-time (after taking safety-stock delays into account), that is,

$$\sum_{s \in S} o_{\mathcal{C}}(t, s, m, p) = d(t, m, p) \tag{1}$$

for all $t \in T$, $m \in M$, and $p \in P$. In particular, this implies that shipments arrive continuously at each time step, as opposed to being shipped in larger, less frequent bulks.

2.2 Force-Majeure Scenarios

Most examples of force-majeure scenarios given in Section 1 have in common that they lead to a certain part of planned arriving order quantities o_A failing. We model these scenarios by specifying the amount of supply material that is missing at the plant at each time point. This allows to model all types of failures, including short-term complete disruptions and long-term supply reductions. In the literature, supply chain risks are often analyzed by using such a scenario-based approach (see for example the risk model of Klibi and Martel [35]).

Definition 5 (Scenario). A (force-majeure) *scenario* is specified by a time "now" $t^* \in T$, together with failure volumes (named *pre-mitigation shortage*) at plants,

$$f(t, s, m, p)$$
 for $t \in T', s \in S, m \in M$, and $p \in P$, (2)

with $0 \le f \le o_A$. $T' = [t^*, ..., t_h] \subset T$ is the *scenario horizon*, i.e., the "future" subset of T for which decisions can yet be made.

The actual arriving volume is denoted by $\hat{o}_A = o_A - f$.

Example 6 Assume the production of supplier s^* is interrupted during the interval $T_F \subset T$, e.g., due to a fire. Then, the failure volumes are given by $f(t, s, m, p) = o_D(t - \delta(s, m, p), s, m, p)$ if $s = s^*$ and $t - \delta(s, m, p) \in T_F$ hold, and are zero otherwise. In other words, the failed arrivals at the plants are exactly the failed departures of the affected supplier, shifted by the respective shipping time. The time t^* could be set to the start of T_F , but also earlier or later, depending on when the company gets notified about the failure and can start to arrange for mitigations.

Remark 7 The framework presented in the sequel can easily be extended to also incorporate other types of scenarios, such as short-notice increase of demand (which would make the *actual demand* a part of the scenario) or delay of shipments (in which case the actually arriving volume exceeds the planned volume as soon as the shipment arrives).

For the sake of a clear presentation, however, we restrict the formal discussion on the most relevant case of failing supply.

2.3 Mitigation Options

In order to avoid throttling or even stopping of production at affected plants, several means of mitigating supply failures are available: using safety stock, shipping supplier inventory, and short-term additional production at suppliers. We model each of these mitigation options separately, additionally to the corresponding business constraints limiting their use in terms of time and volume.

It might take some time to set up a (previously inactive) supply relation in order to ship mitigation volume across it, and some relations might (temporarily) not be feasible at all for, e.g., political or regulatory reasons. The following concept of *qualification* can represent both.

Definition 8 (Qualification). For $t \in T$, $s \in S$, $m \in M$, and $p \in P$, $q(t, s, m, p) \in \{0, 1\}$ specifies whether supplier *s* is qualified to dispatch a shipment of material *m* to plant *p* at time *t*.

Example 9 Assume that five time steps of paperwork are required to establish a supply relation for material *m* from supplier *s* to plant *p*. Then, q(t, s, m, p) = 0 holds for the time points $t \in \{t^*, ..., t^* + 4\}$ whereas q(t, s, m, p) = 1 for $t \in \{t^* + 5, ...\}$.

We assume that positive planned order quantities always imply a qualified supply relation, i.e., it has to hold q(t, s, m, p) = 1 for the corresponding time point $t \in T$, supplier $s \in S$, material $m \in M$ and plant $p \in P$.

When shipping mitigation volumes, it may make sense to use alternative means of transport such as faster boats or air freight. This emergency shipping is usually more expensive but faster.

Definition 10 (Emergency Shipping Time). For $s \in S$, $m \in M$, and $p \in P$, the *emergency shipping time* for shipping material *m* from supplier *s* to plant *p* is denoted by $\delta_{\rm E}(s, m, p) \in \mathbb{N}$.

2.3.1 Additional Production

If a supplier site has some remaining capacity, it might be able to quickly produce additional material that can be shipped to affected plants in order to replace failed volumes, subject to Definitions 8 and 10. Additionally, the production itself can consume time, and the additional production is in general subject to several capacity constraints.

Definition 11 (Production Time). For supplier $s \in S$ and material $m \in M$, $\pi_T(s, m) \in \mathbb{N}$ is the number of time steps it takes after a corresponding request was made, until *s* can start delivery of additionally produced material *m*, due to production time.

Definition 12 (Capacity Constraints). A capacity constraint *C* consists of a supplier $s_C \in S$, a time window $T_C \subseteq T'$, a number of materials $M_C \subseteq M$, and a limit $u_C \in \mathbb{R}^+$, stating that the total additional production within the time window T_C of materials among M_C at supplier s_C must not exceed the amount u_C .

In the above definition, M_C might represent a set of materials for which shared equipment or raw material is used in production.

From the above, we see that plant p can receive additional production of material m from supplier s at time t using the emergency shipping time only if

$$q(t - \delta_{\mathrm{E}}(s, m, p), s, m, p) = 1$$

and $t - \delta_{\mathrm{E}}(s, m, p) \ge t^* + \pi_{\mathrm{T}}(s, m)$ (3)

are fulfilled. Additionally, all capacity constraints must be satisfied by the total additional production.

2.3.2 Inventory

The manufacturing company can contract suppliers to proactively keep inventory of material at their sites. Such supplier-side inventory differs from safety stock in two important aspects when it comes to mitigation: On the one hand, keeping the inventory at the suppliers is a form of risk pooling, because it can be shipped to a multitude of plants using already-established shipping lanes (we assume that plant-to-plant shipping of safety stock is infeasible). On the other hand, safety stock is available immediately in the event of a shortage, while inventories need to be dispatched and shipped. In general, a robust supply chain will combine both approaches, keeping just enough safety stock to guarantee a desired level of robustness while pooling as much inventory as possible at supplier sites.

Definition 13 (Inventory). In a scenario, i(s, m) denotes the amount of (supplier-side) *inventory* of material $m \in M$ at supplier $s \in S$ available at the start of the scenario horizon t^* . In addition, for each $s \in S$ and $m \in M$, there is an *inventory dispatching time* $i_T(s, m)$, denoting that the inventory can be sent no earlier than $t^* + i_T(s, m)$.

In conjunction with Definitions 8 and 10, we obtain that plant p can receive inventory of material m from supplier s at time t only if the supply relation is qualified at departure time and the needed departure is after the start of the failure, formally

$$q (t - \delta_{\mathrm{E}}(s, m, p), s, m, p) = 1$$

and $t - \delta_{\mathrm{E}}(s, m, p) \ge t^* + i_{\mathrm{T}}(s, m),$ (4)

and the total inventory of *m* shipped from *s* must not exceed the amount i(s, m).

2.3.3 Safety Stock

Due to safety-stock delay (Definition 2), a plant p generally has more material m on hand than required for immediate usage, which is an opportunity for failure mitigation. We assume the initial safety-stock level is given as an input and cannot be changed by the optimizer. This is motivated by the fact that the mitigation can only be executed when the failure already occurred. The short response time does not allow to organize a plan to adjust the general safety-stock levels, also because of limited storage space.

A direct optimization of safety stock and inventory levels to reduce shortage risks and reduce inventory costs is outside of the scope of this paper.

Because the safety-stock volume depends on the demand, which in general is fluctuating, the planned safety-stock volume fluctuates as well, i.e., there are scheduled increases and decreases (by usage). Our model, however, should let only unscheduled safety-stock usage account for mitigation volume; hence, it needs to distinguish between the two.

Let a time point $t \in T'$, a plant $p \in P$, a material $m \in M$, and a supplier $s \in S$ be given. By

$$\sigma_{\rm V}(t,s,m,p) \tag{5}$$

we denote the (planned) safety-stock volume at plant p of material m available at time t, resulting from shipments from supplier s. We assume in this notation that the safety-stock volume is measured *before* material arrives and is consumed within time period t, i.e., $\sigma_V(t, s, m, p)$ does not include $o_A(t, s, m, p)$, but the amount $\sigma_V(t, s, m, p)$ can be consumed at time t.

The planned change of safety-stock volume from t to t + 1 is the difference between arriving and consumed volume:

$$\sigma_{\Delta}(t, s, m, p) = \sigma_{V}(t+1, s, m, p) - \sigma_{V}(t, s, m, p)$$

= $o_{A}(t, s, m, p) - o_{C}(t, s, m, p).$ (6)

For fixed values of t, s, m and p, we split up σ_{Δ} into its positive and negative part,

$$\sigma_{\Delta}^{+} = \max\{\sigma_{\Delta}, 0\}$$

and $\sigma_{\Delta}^{-} = \max\{-\sigma_{\Delta}, 0\}$ (7)

such that $\sigma_{\Delta} = \sigma_{\Delta}^+ - \sigma_{\overline{\Delta}}^-$. These can be interpreted as planned safety-stock volume increase (by additional arrival) or decrease (by consuming safety stock instead of arrived volume), respectively. Note that at most, one of σ_{Δ}^+ and $\sigma_{\overline{\Delta}}^-$ is positive.

Rearranging Eqs. (6) and (7) yields

$$o_{\rm A} = \left(o_{\rm C} - \sigma_{\Delta}^{-}\right) + \sigma_{\Delta}^{+} =: o_{\rm AC} + \sigma_{\Delta}^{+}.$$
(8)

We assume here that the safety stock is a (supplier-specific) First-In First-Out buffer that receives the part σ_{Δ}^+ of incoming arrivals and emits the part σ_{Δ}^- into the plant's consumption. The remaining part o_{AC} of the arrival is immediately fed into the plant's consumption, bypassing the buffer. This model is visualized in Fig. 3.

Remark 14 We assume that, for technical reasons, a plant *p* cannot increase its safetystock volume of material *m* from supplier *s* at time *t* by more than $\sigma_{\Delta}^+(t, s, m, p)$. As a consequence, the volume o_{AC} must be consumed immediately. Furthermore, in case of failures, the safety stock can neither be replenished with materials from other suppliers nor with emergency shipping from the same supplier. Fig. 3 The interplay of arrivals, consumption, and planned safety-stock volume changes. Arrows indicate flows of material within the plan from arrival to consumption



The initial safety-stock volume is assumed to equal the planned consumption of the following $\sigma(s, m, p)$ time periods:

$$\sigma_{\rm V}(t^*, s, m, p) = \sum_{i=0}^{\sigma(s, m, p)-1} o_{\rm C}(t^* + i, s, m, p).$$
(9)

In a failure situation with $\hat{o}_A \leq o_A$, we assume that filling the safety stock takes priority over immediate consumption, because that way, the flexibility is maximized. Hence, the actually arriving part of σ_{Δ}^+ is defined as $\hat{\sigma}_{\Delta}^+ = \min\{\hat{o}_A, \sigma_{\Delta}^+\}$, and the remaining actually arriving volume for immediate usage is $\hat{o}_{AC} = \hat{o}_A - \hat{\sigma}_{\Delta}^+$, such that

$$\hat{o}_{\rm A} = \hat{o}_{\rm AC} + \hat{\sigma}_{\Delta}^+ \tag{10}$$

holds, which parallels Eq. (8).

2.3.4 Mitigation Plans

The main task of this paper is to compute a *mitigation plan* for a specific scenario, which is a detailed allocation of mitigation actions for compensating failures. We formalize this in the following definition.

Definition 15 (Mitigation Plan). Let a supply chain (as stated in Section 2.1) and a scenario (Definition 5) be given. A *mitigation plan* for the scenario is the triple $\mu = (\mu_{\pi}, \mu_i, \mu_{\sigma})$, each being a function from $T' \times S \times M \times P$ to \mathbb{R} , where

- μ_{π} is the mitigation by additional production,
- μ_i the mitigation by supplier-inventory,
- μ_{σ} is the safety-stock usage. We split μ_{σ} into the planned part $\mu_{\sigma}^{P} = \max\{\mu_{\sigma}, \sigma_{\Delta}^{-}\}$ and the unplanned part $\mu_{\sigma}^{U} = \mu_{\sigma} - \mu_{\sigma}^{P}$.

By Remark 14, \hat{o}_{AC} is always consumed immediately, such that

$$\mu_{\rm C} = \hat{o}_{\rm AC} + \mu_{\pi} + \mu_i + \mu_{\sigma} \tag{11}$$

gives the total consumed volume for a given mitigation plan; we speak of *postmitigation shortage* whenever this volume is smaller than the demand.

Definition 16 (Post-mitigation Shortage). For time $t \in T'$, material $m \in M$, and plant $p \in P$, the *post-mitigation shortage* is $f_{\mu}(t, m, p) = d(t, m, p) - \sum_{s \in S} \mu_{C}(t, s, m, p)$.

The actual safety-stock volume at time $t \ge t^*$ is

$$\hat{\sigma}_{\rm V}(t,s,m,p) = \sigma_{\rm V}(t^*,s,m,p) + \sum_{\tau=t^*}^{t-1} \left(\hat{\sigma}_{\Delta}^+(\tau,s,m,p) - \mu_{\sigma}(\tau,s,m,p) \right).$$
(12)

In our optimization problem, we impose some constraints on the mitigation to ensure it is feasible to execute in practice. These requirements are given in the following definition.

Definition 17 (Valid Mitigation Plan). A mitigation plan $\mu = (\mu_{\pi}, \mu_i, \mu_{\sigma})$ is called *valid* if it satisfies the following conditions.

- 1. The inventory arrivals μ_i are compatible with the supply relation qualification requirement for inventory Eq. (4) and do not exceed available inventory: for supplier $s \in S$ and material $m \in M$, the inequality $\sum_{t,p} \mu_i(t, s, m, p) \leq i(s, m)$ holds.
- 2. The additional production is compatible with the supply relation qualification requirement for additional production Eq. (3) and adheres to the capacity constraints: for each capacity constraint *C* in the scenario:

$$\sum_{m \in M_C} \sum_{t \in T_C} \sum_{p \in P} \left(\mu_{\pi}(t - \delta_{\mathrm{E}}(s_C, m, p), s_C, m, p) \right) \le u_C.$$
(13)

- 3. The safety-stock volume $\hat{\sigma}_{\rm V}$ as defined in Eq. (12) is (pointwise) nonnegative.
- 4. The post-mitigation shortage is nonnegative, i.e., mitigation must not overcompensate failures.

2.4 Optimization Objectives

Our goal is to compute Pareto-optimal mitigation plans, subject to a subset or a combination of the following objectives. By including multiple objective functions, our model allows a decision-maker to choose the most relevant criteria depending on the situation of the company and the characteristics of the market. The available objectives in the model are chosen to represent the different main criteria supply chain managers use to evaluate a mitigation plan.

This subsection informally discusses several relevant objective functions. A formal treatment based on the network representation of a scenario in Section 3.8 explains how these objectives can be included in the optimization model.

2.4.1 Minimize Loss Value

The main objective is to minimize the value loss caused by post-mitigation shortage. We achieve that by minimizing a weighted sum of post-mitigation shortage values. A reasonable weight could be the average profit gained by selling the end products produced from one unit of the failing material in the market targeted by the affected plant. Note that, for the above approach to be reasonable, we require that each end product has *only one* raw material $m \in M$ as an ingredient. In our example, this requirement would be violated in the case that the candy company produces a certain type of gummy bears using a color mixture of two different pigments *A* and *B*, both of the same quantity. Then, a *simultaneous* shortage of both *A* and *B* has the same impact on the plant's production as the shortage of only one of the two, but a weighted-sum objective function would count that failure twice, thus overstating the loss. In Section 4.2, we show how to properly handle multiple ingredients for end products.

2.4.2 Maximize Shortage Delay

The more time the company has to react to a force-majeure incident, the more likely it is that mitigation options beyond the scope of the model described here can be employed (shifting production plans; increasing sales prices to reduce demand...). According to this criterion, an optimal mitigation plan maximizes the time span until the first post-mitigation shortage occurs.

2.4.3 Minimize Mitigation Effort and Cost

We assume that organizing additional production is more complex and expensive than shipping inventory, which in turn comes at a higher expense than using safety stock that already is at the plant. Hence, an optimal mitigation plan should use the least complex mitigation options possible and in addition use the more expensive options as late as possible (for the same reasons as detailed in Section 2.4.2).

Moreover, a mitigation plan's complexity can be considered increasing in the number of involved suppliers. Consequently, that number should be as small as possible.

3 Computation of Optimal Mitigation Plans

In this section, we show how an optimal mitigation plan, as described in the previous section, can be computed. To that end, we describe, for a given scenario (Definition 5), a time-expanded flow network $N = (G, l, u, e^{\max}, e^{\min})$, from which we then derive in Section 3.6 a multi-objective linear program that delivers an optimal mitigation plan.

It should be noted that this "detour" is not strictly necessary: from the definition of a valid mitigation plan in Definition 17, a linear programming formulation can be derived without the intermediate network. However, the network model is much easier to create, makes the implementation less error-prone because it reduces the need of indexing, and also simplifies a consistent augmentation of the model, some examples of which are given in Section 4.

After introducing general graph and network-flow notation, the following sections describe the different types of vertices and arcs that together form the complete network. These components represent the different entities in the supply network such as suppliers, plants and shipping lanes and model their relevant properties. Unless otherwise mentioned, we assume throughout this sections that t, s, m and p denote elements from T', S, M, and P, respectively.

3.1 Graph and Network Flow Notation

We represent the flow of materials in the supply network as a flow in a mathematical graph. A flow network consists of a directed graph G = (V, A) with $A \subseteq V \times V$ together with lower and upper arc capacity bounds $l : A \to \mathbb{R}^+$ and $u : A \to \mathbb{R}^+ \cup \{\infty\}$ as well as vertex excess bounds $e^{\min}, e^{\max} : V \to \mathbb{R} \cup \{-\infty, \infty\}$. Let $\delta^+(v) = \{(s, t) \in A : s = v\}$ denote the set of outgoing arcs of a vertex $v \in V$ and $\delta^-(v) = \{(s, t) \in A : t = v\}$ the sets of arcs going into v.

Definition 18 A *feasible flow* is a map $\varphi : A \to \mathbb{R}^+$ such that

1. each vertex $v \in V$ fulfills

$$e^{\min}(v) \le e^{\varphi}(v) \le e^{\max}(v), \tag{14}$$

where $e^{\varphi}(v) = \sum_{a \in \delta^-(v)} \varphi(a) - \sum_{a \in \delta^+(v)} \varphi(a)$ is the actual excess at v; 2. for each arc $a \in A$ it holds $l(a) \le \varphi(a) \le u(a)$.

Note that, in textbooks on network flows, one usually requires $e^{\min} \equiv e^{\max} =: e$ and partitions the vertices into *sinks* (e > 0), *sources* (e < 0) and flow-conserving vertices (e = 0). While a network in the above, more general sense can be converted into a classical one by introducing auxiliary vertices and arcs, we find it more natural to think of variable excess vertices in the first place.

In the following, we assume l(a) = 0 and $u(a) = \infty$ for all $a \in A$, unless otherwise stated.

3.2 Consumption Vertices

For each time point t, material m, and plant p, the network N contains a consumption vertex $v_{t,m,p}^{C}$, with variable excess

$$e^{\min}(v_{t,m,p}^{C}) = 0 \text{ and } e^{\max}(v_{t,m,p}^{C}) = d(t,m,p).$$
 (15)

A consumption vertex is a sink that absorbs at most the nominal demand; if it receives less, there is a positive post-mitigation shortage.

3.3 Arrival and Safety-Stock Vertices

In this section, we introduce the part of the network that adds the actually arriving volumes, \hat{o}_A , which are split into the actually arriving volume for immediate usage \hat{o}_{AC} and the actually arriving volume for safety stock $\hat{\sigma}_A^+$ as in Eq. 10.

Note that the actually arriving volumes "encode" are the failure scenario (see Definition 5). Hence, this is the only part of the network that depends on the scenario.

3.3.1 Arrivals for Immediate Usage

For *t*, *s*, *m*, *p* as above, the flow network contains the arrival vertex $v_{t,s,m,p}^{AC}$ with fixed excess $e^{\min} = e^{\max} = -\hat{o}_{AC}(t, s, m, p)$ and an arc to $v_{t,m,p}^{C}$. This ensures that the part \hat{o}_{AC} of the total arriving volume is used immediately to fulfill the plant's demand, as required by Remark 14.

Note that the above forces any feasible flow to be exactly $\hat{o}_{AC}(t, s, m, p)$ on the arc $(v_{t,s,m,p}^{AC}, v_{t,m,p}^{C})$, and one could hence omit that vertex and edge instead reduce the demand of $v_{t,m,p}^{C}$ by $\hat{o}_{AC}(t, s, m, p)$. However, some of the extensions outlined in Section 4 add a degree of freedom in how the arriving volume is used at p. Therefore, we include this part of the network in our general model.

3.3.2 Safety Stock

Safety-stock vertices model the capacity for safety-stock usage. The flow on the corresponding arcs tells decision-makers how much safety stock should be used. For t, s, m, p as above, add a safety-stock vertex $v_{t,s,m,p}^{\sigma}$ with $e^{\min} = -\hat{\sigma}_{\Delta}^{+}(t, s, m, p)$ and $e^{\max} = 0$. Then, add two parallel arcs from $v_{t,s,m,p}^{\sigma}$ to $v_{t,m,p}^{C}$,

- $a_{t,s,m,p}^{\sigma-}$ with $u(a_{t,s,m,p}^{\sigma-}) = \sigma_{\Lambda}^{-}(t, s, m, p)$ for planned safety-stock usage $\mu_{\sigma}^{\rm P}$,
- $a_{t,s,m,p}^{\sigma}$ for unplanned safety-stock usage μ_{σ}^{U} .

This construction allows to distinguish between planned (flow on $a^{\sigma-}$) and unplanned (flow on a^{σ}) safety-stock consumption, respectively. In particular, it allows to impose penalty costs on the latter only; see Section 3.8.3.

Furthermore, because safety stock is allowed to be kept across periods, we add holding arcs $a_{t,s,m,p}^{\sigma-\text{hold}}$ from $v_{t,s,m,p}^{\sigma}$ to $v_{t+1,s,m,p}^{\sigma}$ if $t + 1 \in T'$. Finally, the initial safety-stock volume at t^* has to be considered. To that end, for

Finally, the initial safety-stock volume at t^* has to be considered. To that end, for each $s \in S$, $m \in M$, and $p \in P$, the network contains an initial safety-stock vertex $v_{s,m,p}^{\sigma-\text{init}}$ with variable excess $e^{\min} = -\sigma_V(t^*, s, m, p)$ and $e^{\max} = 0$ and an arc to $v_{t^*,s,m,p}^{\sigma}$.

3.4 Additional Production Vertices

The use of additional production from a supplier is represented by the flow from production vertices to the plants. For time *t*, supplier *s*, and material *m*, when $t \ge t^* + \pi_T(s, m)$, we add to *V* a *production vertex* $v_{t,s,m}^{\pi}$ with $e^{\min} = -\infty$ and $e^{\max} = 0$. For each $t \in T'$ and $p \in P$ such that q(t, s, m, p) = 1 and $t \ge t^* + \pi_T(s, m)$ hold, we add an arc $a_{t',s,m,p}^{\pi} = (v_{t,s,m}^{\pi}, v_{t',m,p}^{C})$ with $t' = t + \delta_E(s, m, p)$, that carries potential mitigation production from *s* to *p*. Note that capacity constraints are handled separately below (see Section 3.6).

3.5 Inventory Vertices

The limits to use inventory are encoded in inventory vertices and their outgoing arcs. For each supplier s and material m, there is an *inventory vertex* $v_{s,m}^i$ in V with e^{\min}



Fig. 4 Sketch of the flow network for one single plant p and material m showing the progression of time from left to right. The incoming edges (and their source vertices) are shown for a single supplier only. Supply vertices (nonpositive excess) are marked with a -, demand nodes (nonnegative excess) with a +

= -i(s, m) and $e^{\max} = 0$. For each time point $t \in T'$ and plant $p \in P$ such that $q(t - \delta_{\mathrm{E}}(s, m, p), s, m, p) = 1$ and $t - \delta_{\mathrm{E}}(s, m, p) \ge t^* + i_{\mathrm{T}}(s, m)$ are fulfilled, we include the arc $a_{t,s,m,p}^i = (v_{s,m}^i, v_{t,m,p}^{\mathrm{C}})$ that delivers potential mitigation inventory from *s* to *p* in our set of arcs *A*.

See Fig.4 for a sketch of the network's structure.

3.6 The Linear Program

From the network $N = (G = (V, A), l, u, e^{\min}, e^{\max})$ that was constructed in the previous sections, one can derive an LP by the following procedure:

- For each arc $a \in A$, introduce a variable x_a representing the flow $\varphi(a)$ with bounds $l(a) \le x_a \le u(a)$.
- For $v \in V$, add an excess variable x_v to the LP together with constraints

$$x_v = \sum_{a \in \delta^-(v)} x_a - \sum_{a \in \delta^+(v)} x_a,$$
(16a)

$$e^{\min}(v) \le x_v \le e^{\max}(v), \tag{16b}$$

which ensures that the flow-conservation condition Eq. (14) holds.

The capacity constraints from Definition 12 need to be treated separately because they are not inherently represented in the flow network. Obviously, for each such constraint,

the condition Eq. (13) translates into the inequalities

$$\sum_{m \in M_C} \sum_{t \in T_C} \sum_{a \in \delta^+ \left(v_{t,s_C,m}^{\pi} \right)} \varphi(a) \le u_C \quad \text{on the flow values, or}$$
(17a)

$$\sum_{m \in M_C} \sum_{t \in T_C} \sum_{a \in \delta^+ \left(v_{t,s_C,m}^{\pi} \right)} x_a \le u_C \quad \text{on the respective LP variables.}$$
(17b)

3.7 Obtaining the Mitigation Plan

Once have computed an optimal flow in the network, we need to create the corresponding mitigation plan that can be used to reduce shortages as much as possible. The following definitions explain this conversion.

Definition 19 Let φ be a feasible flow in *N*. Then, we define the *mitigation plan induced by* φ as follows: For *t*, *s*, *m*, *p* as above, we set

1.
$$\mu_{\pi}(t, s, m, p) = \varphi(a_{t,s,m,p}^{\pi}),$$

- 2. $\mu_i(t, s, m, p) = \varphi(a_{t,s,m,p}^i)$, and
- 3. $\mu_{\sigma}(t, s, m, p) = \mu_{\sigma}^{P}(t, s, m, p) + \mu_{\sigma}^{U}(t, s, m, p)$ with unplanned part $\mu_{\sigma}^{U}(t, s, m, p) = \varphi(a_{t,s,m,p}^{\sigma-})$ and planned part $\mu_{\sigma}^{P}(t, s, m, p) = \varphi(a_{t,s,m,p}^{\sigma-})$.

Proposition 20 If φ is a feasible flow in N that also satisfies Eq. (17a), the mitigation plan μ induced by φ is valid. Conversely, for every valid mitigation plan μ , there is a feasible flow in N inducing it that satisfies Eq. (17a).

Proof Let φ be a feasible flow in *N* that satisfies Eq. (17a) and μ the induced mitigation plan. Item 1 and the first condition of Item 2 of Definition 17 are obviously fulfilled by construction, because the graph contains exactly one dedicated edge for each entry of $\mu_{\pi}(t, s, m, p)$ and $\mu_i(t, s, m, p)$ for which Eqs. (3) and (4), respectively, are satisfied. The second condition of Item 2, namely Eq. (13), is equivalent to Eq. (17a) and fulfilled by assumption.

Also, Item 4 is satisfied by the excess bounds of the consumption vertices Eq. (15) and the fact that the part \hat{o}_{AC} of the arriving flow is necessarily consumed, as mentioned in Section 3.3.1.

As for the safety stock (Item 3), observe that the flow on a holding arc $a_{t,s,m,p}^{\sigma-\text{hold}}$ represents exactly the available safety stock at time t + 1. Therefore, the flow-conservation constraint Eq. (14) of a safety-stock vertex $v_{t,s,m,p}^{\sigma}$ reads

$$\mu_{\sigma}^{\mathrm{P}}(t,s,m,p) + \mu_{\sigma}^{\mathrm{U}}(t,s,m,p) + \hat{\sigma}_{\mathrm{V}}(t+1,s,m,p)$$
$$= \hat{\sigma}_{\mathrm{V}}(t,s,m,p) + \hat{\sigma}_{\mathrm{A}}^{+}(t,s,m,p),$$

which by recursion and the construction of $v^{\sigma-\text{init}}$ equals Eq. (12). Because $l(a_{t,s,m,p}^{\sigma-\text{hold}}) = 0$ holds, this implies a nonnegative safety-stock volume, i.e., Item 3 of Definition 17.

The converse which is omitted here for brevity follows exactly the same arguments.

3.8 Objectives

In this section, we show how to minimize the objectives from Section 2.4 with the LP described above. The choice of the suitable objective functions allows decision-makers to obtain a mitigation plan that follows their most relevant business needs. In practice, decision-makers will choose a subset of these objectives which are either ranked, leading to a lexicographic optimization problem, or left uncomparable, in which case there is a Pareto set of non-dominated solutions. For the theory of multi-objective optimization and solution techniques, we refer to Ehrgott [36]. Note that recent MIP solvers such as Gurobi [37] offer native support for weighted and lexicographic optimization with multiple objectives.

3.8.1 Minimize Lost Value

By the design of the network, a feasible flow φ fulfills the relation

$$f_{\mu}(t, m, p) = d(t, m, p) - e^{\varphi}(v_{t,m,p}^{C}),$$

i.e., the post-mitigation shortage is the gap between demand and flow into a consumption vertex. Thus, the total loss (as defined in Section 2.4.1) with weights $\lambda(m, p) \ge 0$ for the loss of material *m* at plant *p* is given by

$$\sum_{t,m,p} \left(\lambda(m,p) (d_{t,m,p} - x_{v_{t,m,p}^{\mathsf{C}}}) \right)$$
(18)

which is a suitable linear expression for minimization in the LP.

3.8.2 Maximize Shortage Delay

Direct modeling of the shortage delay (Section 2.4.2) requires the introduction of the indicator variables $x_t^f \in \{0, 1\}$ for $t \in T'$ describing whether a shortage is occurring at time *t* with the meaning $x_t^f > 0 \Leftrightarrow f_{\mu}(t, m, p) > 0$ for any *m*, *p*. To that end, add a constraint

$$\sum_{m,p} f_{\mu}(t,m,p) \le M_t x_t^f \quad \text{for all } t \in T,$$

where $M_t = \sum_{m,p} d(t, m, p)$. This constraint forces $f_{\mu}(t, m, p) = 0$ whenever $x_t^f = 0$. Further, introduce an integer variable x^{SD} giving the shortage delay. Together with the additional constraints

$$x^{\text{SD}} \le t - t^* + (1 - x_t^f) |T'| \text{ for } t \in T',$$

the objective

$$\max x^{SD} \tag{19}$$

will maximize the shortage delay.

In practice, however, one will rarely use such an objective. First, the introduction of multiple integer variables turns the problem into a mixed-integer program (MIP), which in general is computationally tractable only for small instances. Furthermore, the objective value Eq. (19) is independent of all shortages occurring after the earliest. Instead, our approach is to penalize early shortages in the course of minimizing loss value as described in Section 3.8.1. This can be accomplished by adding a penalty factor $\varepsilon(t - t^*)$ with a small constant $\varepsilon > 0$ to each term in Eq. (18).

3.8.3 Mitigation Cost

Using the flow-induced mitigation expressions from Definition 19, it is straightforward to obtain linear functionals for total (unplanned) safety-stock μ_{σ}^{U} , additional production μ_{π} , or inventory usage μ_{i} that can be minimized with any desired trade-off weights or lexicographic hierarchy.

For the case that a decision-maker is able to attach actual cost figures to each mitigation option, they can also be combined with Eq. (18) into a single cost-minimizing objective.

3.8.4 Number of Suppliers

Minimizing the number of suppliers involved in mitigation requires similar modeling as maximizing the shortage delay in Section 3.8.2: for each supplier $s \in S$, introduce a binary variable $x_s^S \in \{0, 1\}$ which is 1 if and only if *s* takes part in mitigation. Then, add a constraint

$$\sum_{t,m,p} \left(\mu_i(t,s,m,p) + \mu_\pi(t,s,m,p) \right) \le M x_s^{\mathsf{S}}$$

for each supplier, where $M = \sum_{t,s,m,p} f(t, s, m, p)$ is an upper bound for the mitigation. Now, the objective min $\sum_{s \in S} x_s^S$ will minimize the number of active suppliers.

3.8.5 Well-Shaped Mitigation Plans

Depending on the input data, the optimal solution with respect to the abovementioned objective functions might not be unique, as shown in the following example.

Example 21 Let a scenario consist of a single plant p, a single material m of which p requires 1 unit a day, and a single supplier s with shipping time $\delta(s, m, p) = \delta_{\rm E}(s, m, p) = 2$ and order quantities $o_{\rm A}(s, m, p, t) = 1$ for all time points t. The supplier has inventory i(s, m) = 3 with $i_{\rm T}(s, m) = 2$.

Assume production at *s* fails from t^* until $t^* + 9$ with a safety stock of $\sigma(s, m, p) = 4$. Then, f(t, s, m, p) = 1 for $t \in [t^*+2, t^*+11]$, and the earliest inventory arrival from *s* is $t^* + 4$. If we choose to lexicographically minimize as primary objective the

total loss (with loss factor $\lambda(m, p) = 1$), as a secondary objective the shortage delay, and with lower priority safety-stock usage and inventory usage, then all of the following mitigation plans are optimal:

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|----|----|
| s | S | i | i | i | S | S | f | f | f |
| s | s | s | S | i | i | i | f | f | f |
| s | s | i | s | i | s | i | f | f | f |

where the first line denotes the offset from t^* and each row corresponds to a mitigation plan, where the symbols *s*, *i*, and *f* stand for *safety stock*, *inventory*, and *failure*, respectively.

Multiple optimal solutions are generally undesirable because they lead to unpredictable results. Moreover, in practice, one favors mitigation plans in which the higher-prioritized mitigation options are used earlier. By default, we consider safety stock to have higher priority than inventory. In the above example, this would mean that the second plan in the table is most preferable.

In the following, we develop a single objective function that combines the prioritization of mitigation options with respect to both volume *and* time, i.e., prefers to use higher-prioritized mitigation options both *more* and *earlier* than the ones with lower priority.

It assumes that the post-mitigation shortage has already been fixed by a higherranked lexicographic objective, such that only the distribution of mitigation options is yet to be decided on.

Let $\Omega = (\omega_0, ..., \omega_{|\Omega|-1}) \in \{i, \sigma, \pi\}^{|\Omega|}$ be an ordering of (a subset of) the mitigation options, declaring their priority (where ω_0 has the highest priority). For example, if all three mitigation options are available and ranked as *safety stock*(σ), *additional production*(π), and *inventory* (*i*), then $\Omega = (\sigma, \pi, i)$ with $|\Omega| = 3$.

The mitigation-prioritizing objective is the minimization of

$$c_{\Omega}(\mu) = \sum_{i=0}^{|\Omega|-1} \sum_{t \in T'} \sum_{s \in S} \sum_{m \in M} \sum_{p \in P} \xi_{i,t} \mu_{\omega_i}(t, s, m, p)$$
(20)

where $\xi_{i,t} = i |\Omega| |T'| + (|\Omega| - i)(t - t^*)$.

While there are edge cases in which that objective does not strictly minimize the amounts of mitigation volumes in lexicographic order of priorities, we can prove the following that guarantees that there are at least no "obvious" ways to improve the optimal solution.

Lemma 22 Let $\mu = (\mu_{\pi}, \mu_i, \mu_{\sigma})$ be a mitigation plan that minimizes Eq. (20). Then:

1. It is not possible to replace a single mitigation entry with a higher-prioritized mitigation type, without making additional changes to the mitigation plan. Formally, there are no t, s, m, p, x > 0 and $\omega_i, \omega_j \in \Omega$ with i < j such that the mitigation plan $\bar{\mu}$ that equals μ , except that

$$\bar{\mu}_{\omega_i}(t, s, m, p) = \mu_{\omega_i}(t, s, m, p) + x$$

and $\bar{\mu}_{\omega_i}(t, s, m, p) = \mu_{\omega_i}(t, s, m, p) - x$

are different, is valid.

2. It is not possible to swap a single mitigation entry with an earlier one that has the same loss factor but lower priority, without making additional changes to the plan. Formally, there are no t < t', $s, s' \in S$, $m, m' \in M$, $p, p' \in P$ such that $\lambda(m, p) = \lambda(m', p'), x > 0$ and $\omega_i, \omega_j \in \Omega$ with i < j such that the mitigation plan $\overline{\mu}$ that equals μ , except that

$$\begin{split} \bar{\mu}_{\omega_i}(t, s, m, p) &= \mu_{\omega_i}(t, s, m, p) + x, \\ \bar{\mu}_{\omega_j}(t, s, m, p) &= \mu_{\omega_j}(t, s, m, p) - x, \\ \bar{\mu}_{\omega_i}(t', s', m', p') &= \mu_{\omega_i}(t', s', m', p') - x, \\ and \quad \bar{\mu}_{\omega_i}(t', s', m', p') &= \mu_{\omega_i}(t', s', m', p') + x \end{split}$$

are different, is valid.

Proof For the first item, observe that, by Eq. (20),

$$c_{\Omega}(\bar{\mu}) - c_{\Omega}(\mu) = x(\xi_{i,t} - \xi_{j,t}) = x(j-i)(t-t^* - |\Omega||T'|),$$

where we have inserted the definition of $\xi_{i,t}$. By the assumptions, x and j - i are positive, and $|\Omega| \cdot |T'| \ge |T'| > t - t^*$, such that the expression is negative. In other words, $\bar{\mu}$ would improve on the objective value of μ , which is impossible by the optimality assumption of μ .

As for the second part, we can similarly compute

$$c_{\Omega}(\bar{\mu}) - c_{\Omega}(\mu) = x(\xi_{i,t} - \xi_{j,t} + \xi_{j,t'} - \xi_{i,t'}) = x(t' - t)(i - j),$$

which is negative by the assumptions; hence, the contradiction follows like for part one. $\hfill \Box$

3.8.6 Avoiding Supplier Switches

Similarly to the problem addressed in the previous section, another unwanted effect may occur if several suppliers have the same emergency distance δ_E to a plant. In that case, there might be a large number of patterns of alternating inventory or additional supply from several suppliers, all of which are optimal with respect to the objectives discussed so far. In practice, however, unnecessary switches between suppliers are highly dissatisfactory. This issue can be solved by adding another lexicographic objective. To that end, we first compute, for each plant p and material m, a deterministic supplier ranking $r_p : S \to \mathbb{N}$ that assigns a different priority to each supplier. The ranking should reflect the "probability" that a supplier will be able to mitigate the plant; as a simple yet useful heuristic, we order the suppliers by $\delta_E(s, m, p)$ and break ties randomly. Then, the minimization of

$$\sum_{t \in T'} \sum_{s \in S} \sum_{m \in M} \sum_{p \in P} r_p(s)(\mu_{\pi}(t, s, m, p) + \mu_i(t, s, m, p))$$

removes avoidable supplier switches from the optimal solution.

4 Extensions

We list some extensions of our model in the following, that can be added on a caseby-case basis depending on the application.

4.1 Interchangeability

Assume that, for some materials $m, m' \in M$, it is possible to replace one unit of m' by $r_{m,m'}$ units of m. For example, both m and m' could be pigments of the same color but different efficiency, such that $r_{m,m'}$ units of m can color the same amount of material as 1 unit of m'. Note that interchangeability is not necessarily symmetric: it could be possible to replace a lower-quality material by its higher-quality counterpart, but not vice versa.

In the flow network, this can be accomplished by adding, for each arc $(v, v_{s,m,p}^{C})$ of the original network where the vertex v is any predecessor of $v_{s,m,p}^{C}$, an additional arc $a = (v, v_{s,m',p}^{C})$. If $r_{m,m'} \neq 1$ holds for any interchangeable pair (m, m'), then a is assigned a gain factor of $\gamma(a) = 1/r_{m,m'}$ and the problem turns into a *generalized flow problem* (see Ahuja et al. [38]). The only adjustment then required to handle gain factors can by achieved by modifying the excess constraint, i.e., replacing Eqs. (16a) and (16b) by

$$x_v = \sum_{a \in \delta^-(v)} \gamma(a) x_a - \sum_{a \in \delta^+(v)} x_a.$$

4.2 Multiple End Products and Recipes

Assume the company produces a set of end products *E*. Let $r : E \times M \to \mathbb{R}^+$ be the *recipe function* i.e., the required units of material $m \in M$ for producing one unit of the end product $e \in E$ is given by r(e, m). We now show how to include end products in the model, assuming that their production is possible only when all ingredients are at hand in the necessary quantities and that each end product has an individual loss factor when it comes to minimizing lost value.

First, we replace *M* by *E* in the definition of demand (Definition 1), i.e., d(t, e, p) gives the demand for end product $e \in E$ at plant *p* for time *t*. We also replace condition

Eq. (1) by the following:

for
$$t \in T$$
, $m \in M$, $p \in P$: $\sum_{s \in S} o_{\mathbb{C}}(t, m, p) = \sum_{e \in E} d(t, e, p) \cdot r(e, m)$.

In the network model, we introduce a new type of vertices.

Definition 23 (Multiplication Vertices). A *multiplication vertex* is a vertex v, together with a function $d_v : \delta^-(v) \to \mathbb{R}^+$.

Let

$$N = \left(G = (V \cup V^M, A), l, u, e^{\min}, e^{\max}, \{d_v\}_{v \in V^M}\right)$$

be a flow network, where V^M are the multiplication vertices. For a flow φ to be feasible, we require in addition to Definition 18 for $v \in V^M$ that

$$\sum_{a'\in\delta^+(v)}\varphi(a')=d_v(a)\cdot\varphi(a) \quad \text{for all } a\in\delta^-(v),$$

i.e., the total out-flow of v is a scalar multiple of the flow on every single in-arc of v.

Now, we add an end-product-specific layer below the consumption vertices in the network model described in Section 3. For $t \in T'$ and $p \in P$:

- For each $m \in M$, set the consumption vertices' excess to zero: $l(v_{t,m,p}^{C}) = 0$.
- For $e \in E$, add an *end product demand vertex* $v_{t,p,e}^d$ with $l(v_{t,p,e}^d) = -d(t, e, p)$ and $u(v_{t,p,e}^d) = 0$.
- For $e \in E$, add a multiplication recipe vertex $v_{t,p,e}^R \in V^M$ and arcs $a_{t,m,p,e}^R = (v_{t,m,p}^C, v_{t,p,e}^d)$ for $m \in M$, with $d_v(a_{t,m,p,e}^R) = r(e, m)$.

This part of the network is depicted in Fig. 5. It can be easily verified that a flow in the network is feasible if and only if all ingredients for the end products are available in the required amounts.



4.3 Multi-echelon Supply Chains

A multi-echelon supply chain is a supply chain in which there exists more than one level of pre-products. In such a setting, the first-tier suppliers $S_1 = S$ are manufacturers themselves, receiving raw materials from a set of second-tier suppliers S_2 . Including these in the analysis is important for two reasons: Firstly, ramp-up time and capacity restrictions of second-tier suppliers can lead to a more detailed model of when direct suppliers can deliver how much of additional production, replacing the simplified parameters in Section 2.3.1. Secondly, it is valuable to study the scenario of a failing second-tier supplier; especially if that second-tier supplier ships to multiple direct suppliers, such an analysis can reveal hidden dependencies in the supply chain.

Without going into details, it is obvious that our network-based model can be extended to a multi-echelon case by "stacking" a similar-shaped network on top of the additional production vertices. Depending on the visibility and data quality of the second level, it might however make sense to reduce the model detail, e.g., by omitting the safety-stock part in the second level.

5 Numerical Experiments

In this section, we consider some case studies to illustrate the model described in Section 3. To that end, we generate random supply chains, using the open source *scgen* supply chain generator developed by one of the authors [39].

5.1 Used Software

We implemented the optimization model and the corresponding data processing and visualization in the programming language C#, Version 9 [40]. For the corresponding user interface, the framework WinForms was used. We extended our software with the custom functionality to automatically run the experiments described below and output the results in a comma-separated-value format. The results were then visualized using TikZ/PGF within Latex.

5.2 Experiment Set-up

The parameters of random supply chain generation are carefully chosen such as to obtain non-trivial scenarios. Specifically, the goal is to have, with high probability, both positive post-mitigation shortage and positive volumes for each of the mitigation options from Section 2.3. That way, the scenarios are intuitively neither too easy nor too hard.

The supply chain generator is parameterized with the total number of sites $N = |S \cup P|$, the supplier-to-plant ratio $r_{SP} = |S|/|P|$, the average degree of plant vertices d_P and three ratio parameters *safety-stock ratio* $r_{ss} \ge 0$, *capacity ratio* r_{cap} , and *inventory ratio* r_{inv} which are explained below. For simplicity, we do not consider

any of the extensions from Section 4; in particular, there is no material interchangeability, and hence, it suffices to consider a single material, i.e., $M = \{m\}$.

For the resulting values of |S| and |P|, we generate random order quantities for a horizon $T = \{0, ..., 11\}$ of 12 months as follows. For each plant $p \in P$, a demand d(p) is sampled uniformly from [1000, 5000] and then assigned to each month: d(t, m, p) = d(p) for all $t \in T$. On average, thus, d(t, m, p) equals 3000. Next, each potential supply relation $(s, p) \in S \times P$ is set *active* with probability $d_P/|S|$, such that each plant is supplied from d_P suppliers on average. Separately for each plant p and time step t, the demand d(t, m, p) is distributed among the active suppliers according to weights $w_{s,p}$ sampled uniformly from (0, 1]:

$$o_{\mathrm{A}}(t, s, m, p) = \frac{w_{s,p}d(t, m, p)}{\sum_{(s,p) \text{ active }} w_{s,p}}$$

Shipping times are uniformly sampled between 2 and 30 days, leading to an average of 16 days. The safety-stock delay is then sampled uniformly from $\{0, ..., \lceil 32r_{ss} \rceil\}$ such that it equals r_{ss} times the shipping time on average.

Based on the random supply chain model described above, we next generate forcemajeure scenarios as follows. First, the time discretization is refined to a day-based scale (assuming each month has 30 days). We assume an arbitrary single supplier $\hat{s} \in S$ fails to ship for 30 days during the sixth month of the horizon; this avoids the influence of boundary effects. Note that the size of the relevant scenario horizon is therefore bounded by $30 + 30 \cdot (1 + r_{ss})$, i.e., the failure period plus the maximum delay due to shipping and safety stock.

Based on the above construction, the total failure volume caused by the failing supplier amounts to $F = 3000/r_{SP}$. Each non-failing supplier $s \in S \setminus \{\hat{s}\}$ is assigned inventory i(s, m) sampled uniformly from $[0, 2r_{inv}F/(|S| - 1)]$, such that the total available inventory equals $r_{inv}F$ on average. Likewise, for each non-failing supplier, we constrain the additional production on the scenario horizon to a random value taken uniformly from $[0, 2r_{cap}F/(|S| - 1)]$.

Finally, for all suppliers $s \in S$, we set the production time $\pi_T(s, m) = 10$ and inventory dispatching time $i_T(s, m) = 1$. Qualifications q(t, s, m, p) are configured in such a way that inactive relations can be used after 4 days (cf. Example 9), and the emergency shipping time is set to $\delta_E(s, m, p) = \min{\{\delta(s, m, p), 10\}}$ for all $s \in S$ and $p \in P$.

The resulting LP is optimized with respect to three objective functions, using Gurobi's built-in support for lexicographic optimization: with highest priority, the total loss is minimized, where the loss factor is $\lambda(m, p) = 1$ for all plants $p \in P$ and materials $m \in M$, and early shortages are penalized as described at the end of Section 3.8.2. Then, we prioritize mitigation types in the order of increasing complexity *safety stock, inventory*, and *additional production*, using the objective function from Section 3.8.5. Finally, supplier switches are penalized by the objective function given in Section 3.8.6.

For our experiments, the parameters are set as |N| = 90, $r_{SP} = 1/2$, $d_P = 5$, $r_{cap} = 0.3$, $r_{inv} = 0.3$, and $r_{ss} = 0.1$.



Fig. 6 Optimal mitigation plan for an example force-majeur scenario with a single supplier failing to deliver for 30 days. The plot contains the total amount of mitigation on each day aggregated over all affected plants

5.3 Discussion of Experiments

An example mitigation plan is shown in Fig. 6. The figure shows the total amount of mitigation safety stock, inventory usage, additional production, and post-mitigation shortage for each day after the first delivery failure.

Because the *x*-axis of the plot is given with respect to *arrival* times, there is no effect in the first few days, corresponding to the minimum shipping time from the failing supplier to a plant it supplies. Different shipping times between the failing supplier and the affected plants lead to a characteristic hill-like shape of the pre-mitigation shortage, which is the sum of all mitigation and post-mitigation shortages.

Furthermore, Fig. 6 shows that there are in general two reasons for post-mitigation shortage to occur: First, on days 11–14, some plants ran out of safety stock before supplier-side inventory could arrive from the closest supplier. To avoid such, one needs to increase safety-stock targets and/or decrease set-up and (emergency) shipping times for remote mitigations.

Secondly, starting from day 36, the global inventory and production capacity have been used up; this effect is independent of lead times and can be avoided only by increasing inventories and production capacities.

In order to analyze structural effects, we now consider average mitigation plans when running 100 scenarios using different random supply chains based on the above parameters. The results are shown in Fig. 7a, where the average pre-mitigation shortage has been normalized to 1 (or 0, for the time periods in which no pre-mitigation shortage occurred in any random sample).

Figure 7a exhibits a peculiar spike in safety-stock usage around day 30. This is due to the fact that order volumes (and hence also safety-stock levels) are changed on a monthly basis in our random supply chain model. Therefore, on average, half of the plants increase their safety-stock levels at the turn of the month (cf. Section 2.3.3), leading to a potential re-fill of previously empty stocks.



Fig. 7 Average mitigation plans for 100 random supply chains with different mitigation configurations, showing the relative usage of the different mitigation options inventory, safety stock, and additional production

Figures 7b to d show the effect of disabling production, inventory, and safety stock, respectively, as mitigation options. Because production has the lowest priority in our setting, removing that option does not influence the general shape, as can be seen from Fig. 7b. Without supplier-side inventories, Fig. 7c shows that additional production tends to arrive earlier; however, due to production ramp-up times, a larger early premitigation shortage cannot be avoided. Lastly, Fig. 7d shows the situation without any safety stocks. Here, we observe post-mitigation immediately with the first missing shipment.

Note the double spikes in both the inventory and production mitigation curves, which amount to the emergency shipping time plus inventory dispatching (or production ramp-up) time for mitigation, both with and without inactive link set-up time, depending on whether the respective supplier is active or not for the affected plant.



Average LP solution time

Fig. 8 Average LP solution time for a random supply chain with *N* sites in total, for three different ratios of the number of suppliers and the number of plants $|S|/|P| = \{1:1, 2:1, 1:2\}$

5.4 Performance

In this section, we give some performance results of the model described in Section 3, when solved with the commercial Gurobi LP solver (version 8.1) [37]. For this, we generate random supply chains of varying sizes in a similar way as described above.

Figure 8 shows the average LP running time depending on the size of the supply chain. The experiments were conducted on a Gurobi compute server installation that is installed on a virtual machine equipped with 7 GB RAM and a 4-core Intel Xeon CPU at 3.00 GHz. As can be seen from the figure, the algorithm performs very well, with a running time remaining in the order of 1 s even for large supply chains with 100 sites. This allows to quickly evaluate multiple scenarios, even in an interactive application. Not surprisingly, increasing the number of plants has the greatest effect on running time, because it is the plant layer of the network that contains the most vertices and arcs.

6 The Integrated Planning Workflow

Apart from the usage of a mitigation plan during an actual failure scenario or for business continuity planning, our approach can also be used for improving supply chain planning to reduce risks. The survey by Heckmann et al. [41] describes alternative approaches for measuring and optimizing risks in supply chains. Our model forms the basis of an integrated planning process that can be used to improve the resilience of supply chains. A customized software tool implementing the model supports this process with optimization and analysis capabilities. This software is used by a global producer of fast-moving consumer goods. Figure 9 illustrates the corresponding workflow.

The risk assessment is supported by tools that use our mitigation model to evaluate the severity of various scenarios. For this, a list of disruption scenarios is defined. Our model allows to quickly compute optimal mitigation plans in each of these scenarios. This enables the supply chain planner to directly distinguish between scenarios with sufficient available mitigation and un-mitigable disruptions. Structural problems in the supply chain can be identified efficiently in this way. Since the impact of a disruption can be also analyzed on a local basis, proper enhancements of the mitigation options can be found, for example, an increase of the inventories at some sites. After performing such improvements to the plan, the risks can be evaluated again to analyze the impact of these changes. This can be iterated until a suitable plan is found that satisfies the desired balance between resilience and other factors like costs or organizational simplicity. Due to the automatic risk assessment tool, the time needed for such an iteration can be significantly reduced compared to traditional methods. Thus, more iterations can be performed, leading to more resilient supply chains.

At the end of the workflow, we find a suitable supply chain plan with an acceptable risk. Other parts of the organization outside of the upstream supply chain can then be adapted accordingly, e.g., relating to insurances against risks and contracts with clients. For example, the production of plant sites that are facing a larger amount of supply risk should not be assigned to consumer markets where high reliability is required. The computed optimal mitigation plans can also be shared with operational teams for preparation and training. The availability of such plans reduces the time needed to react to a disruption which improves the execution of the mitigation and facilitates an efficient coordination.

Fig. 10 A risk profile showing the impact of disruption scenarios on various plants after taking into account the mitigation. Each *case* corresponds to one of the automatically generated disruption scenarios

6.1 Risk Profiles

To enable a quick overview on the local impact of a set of possible scenarios, we suggest to use risk profiles. A risk profile consists of the impact of a failure at each of the supply chain parts. After the user identifies an acceptable level of risk, a color-coded summary indicates whether this acceptable threshold is surpassed. The acceptable level of risk includes the maximal duration of the shortage until a proper mitigation is available. Further, the time until a first shortage occurs at a plant is taken into account. In this way, situations are identified where the shortage has an impact on the production too quickly after a disruption to react properly. An instance of a risk profile is shown in Fig. 10. Based on this risk profile, the critical disruption scenarios can be identified. Additionally, the elements of the supply chain which have a high amount of risk in many disruption scenarios are highlighted. A color-coded scheme is used depending on user-defined thresholds for the severity of disruption scenarios, to allow a quick overview. Using this information, the decision-maker can decide on the placement of additional inventories or safety stocks to increase the shortage delay to an acceptable level. This is a significant improvement over the standard analysis of disruption scenarios which only considers whether supply chain elements are affected

| Acceptable time to shortage: 6 days Toleran | | | Tolerance | e: | | Acceptable shortage dura | Acceptable shortage duration: 21 days | | |
|---|----------|----------------------|-----------|------------------|---------|--------------------------|---------------------------------------|------------------------------------|--|
| +/- | | | | 1 🚖 days | | | | +/- 4 📩 days | |
| 0 1 2 3 4 5 6 7 | 8 9 10 1 | 2 14 16 18 20 | | | | 0 2 4 6 8 10 13 | 16 19 22 25 28 31 3 | 4 37 40 | |
| Show all Mind Plants | | | | Show all Find Ma | terials | | Show all Find Supplier S | ites | |
| | Plant A | 29 cases 29 / 0 / 0 | î | | Mat. 3 | 127 cases 30 / 19 / 78 | | Supplier I 180 cases 66 / 13 / 101 | |
| | Plant G | 19 cases 17 / 2 / 0 | | | Mat. 4 | 79 cases 26 / 16 / 37 | | Supplier F 28 cases 20 / 3 / 5 | |
| 6 | Plant H | 30 cases 16 / 0 / 14 | | | Mat. 5 | 115 cases 15 / 13 / 87 | 3 | Supplier B 23 cases 8 / 8 / 7 | |
| | Plant F | 23 cases 10 / 3 / 10 | | | Mat. 6 | 36 cases 12 / 3 / 21 | 2 | Supplier K 22 cases 5 / 0 / 17 | |
| | Plant Z | 9 cases 9 / 0 / 0 | | | Mat. 2 | 69 cases 11 / 2 / 56 | | Supplier H 31 cases 3 / 3 / 25 | |
| | Plant I | 9 cases 6 / 3 / 0 | | | Mat. 1 | 9 cases 9 / 0 / 0 | | Supplier D 34 cases 1 / 12 / 21 | |
| | Plant J | 9 cases 6 / 3 / 0 | | | | | | Supplier G 30 cases 0 / 12 / 18 | |
| | Plant B | 22 cases 5 / 2 / 15 | | | | | | Supplier A 40 cases 0 / 1 / 39 | |
| | Plant N | 18 cases 3 / 0 / 15 | | | | | | Supplier C 21 cases 0 / 1 / 20 | |
| | Plant P | 14 cases 1 / 12 / 1 | | | | | | Supplier L 18 cases 0 / 0 / 18 | |

Fig. 11 A detailed risk view on the impact of disruption scenarios on the various parts of the supply chain. For each element of the supply chain, only the scenarios affecting the relevant part are considered. The color-coding distinguishes between good, acceptable, and problematic outcomes at the respective part of the supply chain after taking an optimal mitigation in the corresponding scenario

but does not take mitigation into account. The distinction allows to precisely identify the places where action needs to be taken.

To plan concrete measures to improve the resilience, a detailed view on the impact of the considered disruption scenarios on the parts of the supply chain in Fig. 11 can be used. This visualization illustrates for each of the supply chain parts how many disruption scenarios lead to a strong negative impact on the supply chain even after taking an optimal mitigation. In this way, the responsible managers can be informed on the supply chain parts for which current mitigation options are too often insufficient. The automatic analysis ensures that no important element of the supply chain is ignored in this process.

In summary, these tools enable a strategic improvement of the resilience of the supply chain configuration for a large range of potential scenarios.

7 Conclusion and Outlook

Long-term disruptions to supply chains are omnipresent challenges in the procurement of raw materials and primary products. There are many reasons for this, ranging from geopolitical tensions and social upheaval to natural disasters. The resulting delivery delays and supply shortages can cause critical effects and put affected companies in substantial difficulties. Companies therefore generally address these challenges proactively with a whole range of possible mitigation plans that they can execute if the worst comes to worst.

In this publication, we developed a rigorous mathematical optimization model for optimizing mitigation plans. On the one hand, our approach minimizes the amount of shortage and, on the other hand, maximizes the time until a shortage occurs. This allows further situation-dependent actions to be called up in an emergency. Our model describes failure scenarios as an extended flow problem in time-expanded graphs and translates these into a linear program. A numerical study proves that our approach solves realistic problem instances in an acceptable time.

With the help of the model, network planners can identify critical suppliers and production sites that would be affected by delivery failures in possible failure scenarios. We have also presented a concept for aggregating failure scenarios. Network planners can use this to optimize their supply chain structure. However, this step is not the focus of our considerations.

Decisions about supply chain structures are often based on empirical knowledge and gut feelings. Sometimes, this is due to a lack of data. In such situations, however, our quantitative approach can be used to identify blind spots and improve the quality of the data in dialog with network partners so that more informed data-based decisions can be made overall.

7.1 Future Work

A natural direction of future research is to optimize the parameters of the mitigation plans given the results of simulated failure scenarios.

This includes an optimization of inventory levels as well as a *risk-aware* purchasing, defining the planned order quantities in such a way that cost-minimization in the nominal case and loss value in the scenarios are properly balanced.

A method for optimization of the order quantities to minimize the worst-case risk in a given set of scenarios, with mitigation taken into account, is proposed by Ackermann et al. [42]. This model for the risk can be combined with the detailed optimization model for various costs by Ackermann et al. [43].

Such an approach to perform a bicriteria optimization of both costs and risks in supply chains is described by Diessel and Ackermann [44]; the underlying model however can be extended to include all mitigation options discussed above.

Additionally, in such a risk optimization model, the safety stock and inventory levels can be directly optimized to ensure that sufficient mitigation options are available in the case of a shortage.

The power of the model can be extended by allowing also random variables with a known probability distribution as input data into the model. By using techniques from stochastic optimization, this could increase the accuracy in comparison to the scenario-based approach we proposed in Section 6.1. Especially for parameters with a known variance, such as transportation time, this could be valuable.

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