Heyen, Daniel; Tavoni, Alessandro

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Public GoBs

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Daniel Heyen, Alessandro Tavoni

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Abstract

GoBs are goods for which agents have non-monotonic preferences: more is beneficial only up to an ideal level, beyond which additional quantities become undesirable. We analyze public GoBs (non-excludable and non-rival) through a theoretical framework applicable to diverse contexts such as solar geoengineering, wildlife management, and defense spending of European countries. The private provision of public GoBs proves inefficient due to both free-rider and free-driver externalities. Contribution costs and heterogeneity in ideal levels determine equilibrium outcomes. Surprisingly, reducing contribution costs can decrease welfare when agents' preferences diverge significantly.

JEL-Codes: D010, D620, H230, H410, Q540, Q590.

Keywords: private provision of public goods, GoBs, heterogeneity, externalities, free-riding, free-driving.

*Daniel Heyen**
School of Business and Economics
University of Kaiserslautern-Landau
Kaiserslautern / Germany
daniel.heyen@wiwi.rptu.de

Alessandro Tavoni
Department of Economics
University of Bologna / Italy
alessandro.tavoni@unibo.it

*corresponding author

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1 Introduction

GoBs ("good-or-bad") are goods for which agents have non-monotonic preferences: more of the good is welcome only up to some point, after which additional quantities become undesirable, even at zero cost. We study public GoBs – those that are non-excludable and non-rival in consumption. Weitzman (2015), who coined the term "GoB", introduced the concept in the context of solar geoengineering, i.e. rapid global cooling by technological means, such as the injection of sulfur particles into the upper atmosphere (National Academies of Sciences 2021). Solar geoengineering is non-excludable and non-rival: the changes to the climate cannot be confined to one country or region; rather, they would be felt by everyone. Solar geoengineering is a GoB because, in a world of significant climate change, virtually every country would benefit from some level of global cooling; but every country has *non-monotonic* preferences in the level of global cooling: even the hottest and driest country would eventually find it too cool. In other words, there is an ideal level of global cooling and that ideal GoB level may well be different for different countries (hot countries presumably would have a higher ideal solar geoengineering level). In the context of solar geoengineering, Weitzman (2015) brought up the concern of "free-driving". In contrast to the usual problem of public under-provision due to free-riding, the low deployment costs of solar geoengineering may well lead to over-provision of the public GoB because the agent with the strongest preferences for cooling the globe might just go ahead, thus imposing an externality – in the form of too cold temperatures – on the rest of the world.

While solar geoengineering represents a global-scale example of a public GoB, this framework applies to many other contexts at different scales. Consider, for instance, wildlife management in agricultural communities. Think of farmers residing around a forest populated by wolves who, from time to time, kill some sheep of the farmers. At the same time, wolves keep the ecosystem intact and might also provide some direct economic value in terms of tourism. This demonstrates that 'wolf management' – the deliberate culling of wolves – is a public GoB: every farmer benefits from some reduction in wolf population, but excessive culling would eliminate ecological benefits and tourism revenue. Different farmers are differently affected by the wolf population (e.g., because some of them live closer to the forest and are therefore more prone to wolf attacks on their sheep) and hence have different ideal GoB levels.

For another instructive example, consider European countries investing resources in joint defense against Russia. Defense spending by individual nations accumulates to an aggregate level that matters collectively, as these countries pledge mutual support through alliances. This defense capability is both non-excludable and non-rival – all European nations benefit from the collective security, with Baltic states typically deriving greater benefits due to their geographic proximity. However, European defense spending exemplifies a public GoB because there exists a point beyond which additional expenditure becomes undesirable, even at zero cost. This occurs because excessive aggregate defense spending carries the perceived risk of provoking Russia, potentially escalating tensions rather than enhancing security. Whether this concern is empirically justified matters less than its perception among policymakers. This negative component creates non-monotonic utility functions, with different European nations having different ideal levels: Baltic states, feeling more directly threatened, likely prefer higher levels of military readiness, while countries farther from Russia's borders might prioritize avoiding escalation, preferring more modest aggregate expenditures.

In summary, public GoBs represent a widespread yet understudied topic. While economic theory has extensively examined the provision of traditional public goods, the decentralized provision of public GoBs – with their distinctive non-monotonic utility structures – raises fundamentally different questions about strategic behavior and efficiency. This paper addresses this gap by developing a theoretical framework for analyzing how self-interested agents navigate the complex externalities created when providing goods that can be both beneficial and harmful.

We make three contributions. First, we develop a general framework for public GoBs that extends existing models of private public good provision. Within this framework, we identify two distinct types of externalities: The *free-rider externality*, where self-interested agents fail to account for the positive effects their additional provision has on others, leading to the classic underprovision of public goods; and the *free-driver externality*, where agents disregard the *negative* effects their provision imposes on others. This second externality, emphasized by Weitzman (2015) in the context of solar geoengineering, can result in overprovision of the public GoB.

Our second contribution is to characterize both efficient and non-cooperative provision of public GoBs within a tractable version of our general framework, featuring quadratic benefit and cost functions. We demonstrate that the unique Nash equilibrium can take different forms: of the N total agents, m agents contribute while the remaining $N-m$ agents abstain because they perceive the public GoB as already overprovided from their perspective. We show that contribution costs and heterogeneity in ideal GoB levels critically determine the equilibrium structure. Specifically, lower contribution costs and greater heterogeneity result in fewer contributing agents.

Our third contribution is to analyze the welfare implications of non-cooperative public GoB provision. A key finding reveals that reducing contribution costs is universally beneficial when the public GoB is provided efficiently. This welfare improvement also holds under private non-cooperative provision when agents share the same ideal GoB level. However, when agents have divergent preferences regarding the ideal GoB level, reducing contribution costs can actually decrease overall welfare.

Related literature. Our paper connects to several research areas. The concept of public GoBs has been most extensively studied in the context of solar geoengineering.

Following the seminal work of Weitzman (2015), there has been a significant increase in research. Recent comprehensive surveys are Heyen and Tavoni (2024) and Moreno-Cruz et al. (2025). Our contribution extends beyond solar geoengineering by developing a general theoretical framework for analyzing abstract public GoB settings.

A few comparable papers have examined public goods with both beneficial and detrimental characteristics. Buchholz et al. (2018) and Giraudet and Guivarch (2018) study such goods, though in their frameworks whether a good is beneficial or detrimental is fixed for each individual rather than dependent on the provision level. Similar to our setting, Ansink and Weikard (2025) study public GoBs but focused on cooperation incentives; their model allows for negative contributions, which prevents the emergence of our different equilibrium types.

Our work also connects to fundamental social choice theory, where single-peaked preferences (with different ideal levels) are a standard concept (Black 1948; Inada 1969). However, while social choice literature typically focuses on centralized provision and collective decision-making mechanisms, our approach examines private provision without social choice procedures.

Finally, our paper contributes to the established literature on the private provision of public goods, see Bergstrom et al. (1986). We extend this tradition by incorporating the "GoB" aspect, i.e. that agents' utility functions are non-monotonic in the level of the public good.

The paper proceeds as follows. Section 2 presents the general framework and defines free-rider and free-driver externalities. Section 3 introduces quadratic costs and benefits and puts the focus on two important parameters, contribution costs and the heterogeneity in ideal GoB levels. Section 4 determines the best-response functions and contrasts selfish to altruistic best-response functions. Section 5 determines the benchmark of efficient public GoB provision, whereas Section 6 uses the best-response functions to determine the Nash equilibrium of public GoB provision. Section 7 studies comparative statics. Section 8 studies the welfare implications of non-cooperative GoB provision. Section 9 discusses the robustness of our findings and concludes.

2 A framework of public GoB provision

2.1 Public GoBs

The basic setting is in line with the literature on the private provision of public goods: There are N agents, each contributing a non-negative amount $q_n \geq 0$ to a public good. We denote the vector of contributions by $\boldsymbol{q} = (q_n)_{n=1,\dots,N}$ and the total level of the public good by $Q = \sum_n q_n$.[1] We assume that agents have quasi-linear utility over money

[1] We leave the analysis of other aggregator rules for future research. See Cornes and Sandler (1996) and Barrett (2007)) for related work.

and the public good. Therefore, wealth effects can be ignored, and agents maximize $u_n(\boldsymbol{q}) = B_n(Q) - C_n(q_n)$. Here, $C_n(q_n)$ denotes the cost that agent n incurs from their contribution q_n. Cost function C_n is differentiable and strictly increasing, $C_n' > 0$, for all agents. As usual, B_n captures how agent n is affected by the total public good level Q.

The novel aspect is that the public good we study is a "GoB", i.e. it has characteristics of a good and a bad in the following sense: agent n has the ideal GoB level $0 < \alpha_n < \infty$. Benefit function B_n is increasing in Q for $Q < \alpha_n$, decreasing for $Q > \alpha_n$, and α_n is the unique maximizer of B_n. This includes the setting of Weitzman (2015) as a special case. The standard literature on public goods assumes increasing benefit functions, corresponding to the corner case $\alpha_n = \infty$ for all n.[2] We denote the vector of ideal GoB levels by $\boldsymbol{\alpha}$ and label agents from highest to lowest ideal level, $\alpha_1 \geq \ldots \geq \alpha_N$. It is important to stress again that agent n's ideal interior level α_n does not stem from a trade-off between contribution costs and benefits from a public good. Even if the agent could have more of the public good for free, at level $Q = \alpha_n$ she would prefer not to have more of it.[3]

2.2 Free-rider and free-driver externality

Consider total welfare $W(\boldsymbol{q}, \boldsymbol{\alpha}) = \sum_{n=1}^{N} u_n(\boldsymbol{q})$ and an arbitrary contribution scheme $\boldsymbol{q}$ with total public GoB level $Q = \sum q_n$. We are interested in the welfare effect from a small additional contribution Δq by one of the agents, say agent n. Agent n incurs additional cost of $\Delta C_n = C_n(q_n + \Delta q) - C_n(q_n)$. The effect on the benefit functions is more intricate. The additional contribution by agent n increases the total GoB level to $Q + \Delta q$ and thus not only affects their own benefit, $\Delta B_n(Q) = B_n(Q + \Delta q) - B_n(Q)$, it affects all other agents as well. The set of other agents $\mathcal{N}_{-n} = \{1, \ldots, N\} - \{n\}$ consists of two sets of agents: first, those for which the initial public GoB level Q was below their ideal level, $\mathcal{N}_{-n}^{<}(Q) = \{\nu \in \mathcal{N}_{-n} \mid Q < \alpha_\nu\}$, and second, those agents for which the public GoB already met or exceeded their ideal level, $\mathcal{N}_{-n}^{\geq}(Q) = \{\nu \in \mathcal{N}_{-n} \mid Q \geq \alpha_\nu\}$.

[2] While not essential for our analysis, one possible reason for the GoB structure is that the public good Q is essentially the superposition of a good and a bad, both of which are non-excludable and non-rival. Take the European defense example: the higher the level of defense spending QQ Q, the greater the collective security against potential aggression, a clear benefit for all countries in the alliance. And because one country's defense contribution benefits all other European countries, it constitutes a standard monotone public good. At the same time, defense spending also has a public bad characteristic. The higher the aggregate military expenditure, the larger the (perceived) risk of provoking Russia and escalating tensions. This is a monotone public bad. Now, if for instance the 'good' characteristic grows linearly in Q and the 'bad' characteristic grows quadratically in Q, then the superposition of good and bad, reflected in the benefit function $B_n(Q)$, is that of a GoB with an ideal GoB level of $0 < \alpha < \infty$.

[3] We emphasize that public GoBs are not entirely new objects, they are special cases of the public good literature (Samuelson 1954; Bergstrom et al. 1986). Usually however the classical contributions to the literature at some point assume monotonicity, i.e. that agents prefer to have more of the good: Bergstrom et al. (1986) does so when assuming that the demand is strictly greater than zero. Samuelson (1954) assumes that the partial derivative of utility with respect to every argument is positive.

The total effect on the benefit functions is

$$\Delta B_n(Q) + \sum_{\nu \in \mathcal{N}^{\leq}_{-n}(Q)} \Delta B_\nu(Q) + \sum_{\nu \in \mathcal{N}^{\geq}_{-n}(Q)} \Delta B_\nu(Q) \tag{1}$$

By construction of the two sets, $\Delta B_\nu(Q)$ is positive for $\nu \in \mathcal{N}^{\leq}_{-n}(Q)$ (provided Δq is sufficiently small) and negative for $\nu \in \mathcal{N}^{\geq}_{-n}(Q)$. From the social standpoint, the additional GoB contribution by agent n is a welfare improvement if and only if the additional costs ΔC_n are smaller than the total additional benefits in (1).

At the heart of the social dilemma of the provision of public goods is that individuals do not take into account the effects on others. Here, a self-interested agent n who only looks at their own utility function would take into account the first term in (1) but ignore the last two terms. The first of those, $\sum_{\nu \in \mathcal{N}^{\leq}_{-n}(Q)} \Delta B_\nu(Q)$, is positive and we call it the **free-rider externality**. Agent n does not take into account the positive benefit of marginally increasing the public GoB level on those that still benefit from more of it. This is the usual externality in public good provision that results in free-riding and underprovision. The other term, $\sum_{\nu \in \mathcal{N}^{\geq}_{-n}(Q)} \Delta B_\nu(Q)$, is negative and we call it the **free-driver externality**.[4] Here, agent n does not take into account the negative impact of increasing the public GoB level on those that already consider the GoB overprovided. The usual literature on the private provision of public goods has $\alpha_n = \infty$ and therefore only studies free-rider externalities.

3 Tractable functional forms

In this section we present a tractable version of the general framework presented in the previous section. We discuss generality in section 9. Contribution costs are quadratic,

$$C_n(q_n) = \frac{c}{2} q_n^2, \tag{2}$$

where $c > 0$ is the cost parameter. Benefits are quadratic in the public GoB level Q,

$$B_n(Q) = -\frac{b}{2}(\alpha_n - Q)^2, \tag{3}$$

where $b > 0$ is the benefit parameter. As introduced above, $\alpha_n > 0$ is agent n's ideal GoB level. That agents can have different ideal levels is a key element of our framework. Clearly the assumptions from above are met for the concrete functional form of B_n: The benefit function B_n is increasing in Q for $Q < \alpha_n$, decreasing in Q for $Q > \alpha_n$, and the ideal level α_n is the unique maximizer of B_n.

With the quadratic specification it is easy to see that we can write total welfare for

[4] In memoriam of Weitzman who introduced the term "free-driver".

an arbitrary contribution profile $\boldsymbol{q}$ with total GoB level $Q = \sum_n q_n$ as

$$W(\boldsymbol{q}, \boldsymbol{\alpha}) \;=\; -\frac{\mathrm{b}_{\mathrm{Soc}}}{2}\left[(\bar{\alpha} - Q)^2 \,+\, \sigma^2\right] \;-\; \frac{c}{2}\sum_{n=1}^{N} q_n^2, \tag{4}$$

where $\mathrm{b}_{\mathrm{Soc}} := bN$ is the slope of the marginal social benefit curve, $\bar{\alpha} = (\sum_n \alpha_n)/N$ is the average ideal level, and $\sigma^2 = var(\alpha) = (\sum_n (\alpha_n - \bar{\alpha})^2)/N$ is the variance of the ideal levels. In particular, welfare depends on the vector of ideal GoB levels $\boldsymbol{\alpha}$ only in terms of average ideal GoB level $\bar{\alpha}$ and variance σ^2. Expression (4) is intuitive: society's benefit is determined by two components: the deviation of the total GoB level from the average ideal level, $\bar{\alpha} - Q$, and heterogeneity in ideal levels, σ^2.

Our analysis of public GoBs in this paper will focus mostly on two key parameters. The first is cost parameter c. It is well known that contribution costs shape both the efficient as well as non-cooperative public good provision. Our results below confirm that this is also true for public GoBs, and in some sense even more pronounced so. The second key parameter is the heterogeneity of ideal levels σ. Heterogeneity in ideal levels, $\sigma > 0$, is the interesting element in public GoBs and it is insightful to compare outcomes to the traditional public good setting in which agents agree on the ideal level, $\sigma = 0$.

The model introduced in this section puts the focus on heterogeneity in the ideal GoB levels and thus restricts to symmetric cost and benefit parameters c and b. The appendix covers the general case with agent-specific b_n and c_n.

4 Best-response functions

In this section we study best-response functions within the tractable framework developed in the previous section. This will help us better understand public GoB provision incentives and will be the basis for determining the Nash equilibrium.

4.1 Selfish best-response function

We first study standard best-response functions, i.e. when agents only care about their own utility. The net benefit of agent n is

$$\begin{aligned} u_n(Q, q_n) &= B_n(Q) - C_n(q_n) \\ &= -\frac{b}{2}(\alpha_n - Q)^2 \;-\; \frac{c}{2}q_n^2 \\ &= -\frac{b}{2}(\alpha_n - q_n - Q_{-n})^2 \;-\; \frac{c}{2}q_n^2, \end{aligned}$$

where $Q_{-n} = \sum_{\nu \neq n} q_\nu$ is the public GoB level provided by agents other than n. The marginal net benefit for agent n is $du_n/dq_n = b(\alpha_n - q_n - Q_{-n}) - cq_n$. This implies for the best response function

$$q_n(Q_{-n}) = \begin{cases} \frac{b}{b+c}(\alpha_n - Q_{-n}) & Q_{-n} < \alpha_n \\ 0 & Q_{-n} \geq \alpha_n. \end{cases} \tag{5}$$

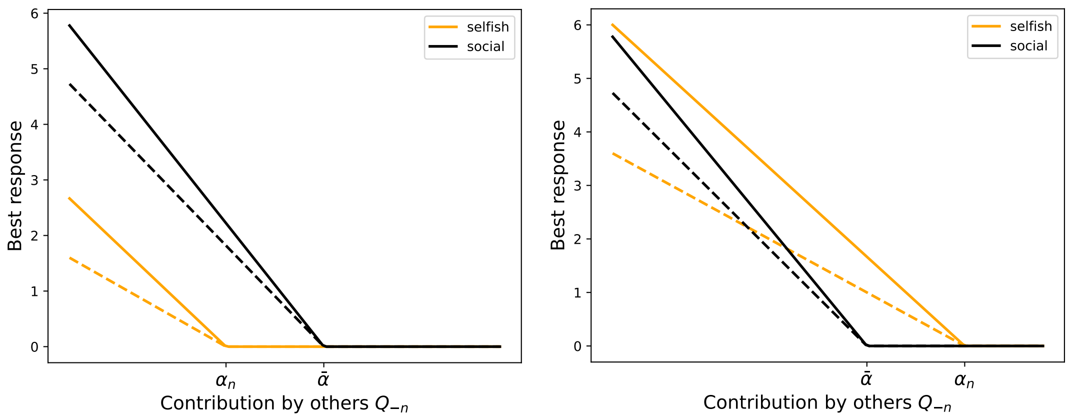
The interpretation is straightforward. If the contribution by others is lower than her ideal level α_n, agent n provides fraction $b/(b+c)$ of the remaining gap $\alpha_n - Q_{-n}$; that fraction is increasing in benefit b and decreasing in cost c. If the contribution by others already exceeds her ideal GoB level α_n, any further GoB level increase would not only be costly but but even harmful, so the agent contributes zero.

4.2 Social best-response function

In order to illustrate the difference between private and social optimum, we study "social best-response functions". That is, what is the best response to GoB provision Q_{-n} of an agent interested in social welfare? Taking the derivative of total welfare (4) with respect to q_n, we get $dW/dq_n = \mathrm{b}_{\mathrm{Soc}}(\bar{\alpha} - q_n - Q_{-n}) - cq_n$. This directly leads to the social best-response function

$$q_n^{\mathrm{eff}}(Q_{-n}) = \begin{cases} \frac{\mathrm{b}_{\mathrm{Soc}}}{\mathrm{b}_{\mathrm{Soc}}+c}(\bar{\alpha} - Q_{-n}) & Q_{-n} < \bar{\alpha} \\ 0 & Q_{-n} \geq \bar{\alpha}. \end{cases} \tag{6}$$

Instead of the personal ideal GoB level α_n, cf. expression (5), agent n interested in maximizing social welfare would focus on the average ideal level $\bar{\alpha}$. Furthermore, the agent compares marginal contribution costs c not to individual marginal benefits b, but rather social marginal benefits $\mathrm{b}_{\mathrm{Soc}} = Nb$.



(a) $\alpha_n < \bar{\alpha}$

(b) $\alpha_n > \bar{\alpha}$

Figure 1: Best-response functions $q_n(Q_{-n})$ (in orange, cf. expression (5)) in comparison to welfare-maximizing best-response functions $q_n^{\mathrm{eff}}(Q_{-n})$ (in black, cf. expression (6)). The solid (resp. dashed) line corresponds to low costs $c = 0.5$ (resp. high costs $c = 1.5$). The other parameter settings are $b = 1$, $\bar{\alpha} = 6.5$, and $\alpha_n = 4$ (resp. $\alpha_n = 9$) in Panel (a) (resp. Panel (b)).

Figure 1 shows best response function and social best response function and essentially illustrates the interplay of free-rider and free-driver externalities defined in section 2.2. Black lines represent the social best-response functions that take the external effects into account, whereas orange lines represent the (selfish) best-response functions of an agent that ignores the external effects.

An agent with an ideal level lower than the average ideal level, $\alpha_n < \bar{\alpha}$ (left panel), always under-provides the public GoB, the selfish best-response function $q_n(Q_{-n})$ always lies below the welfare-maximizing best-response function $q_n^{**}(Q_{-n})$. Such an agent, therefore, always contributes less than what would be best from a societal point of view. In contrast, for an agent with an ideal level higher than the average ideal level, $\alpha_n > \bar{\alpha}$ (right panel), the picture is more subtle. Due to the free-driver externality, over-provision of the public GoB is possible; this is the case when the selfish best-response function lies above the social best-response function. But because the selfish best-response function is flatter than the social best-response function, under-provision is also possible. It is intuitive that high contribution costs (dashed lines) tend to result in under-provision, whereas low contribution costs (solid lines) favor over-provision.

5 Efficient public GoB provision

An important baseline is the efficient provision of a public GoB, i.e. the contribution schedule $\boldsymbol{q}^{\text{eff}}$ that maximizes expression (4). A necessary condition for a welfare maximum is that agents provide the GoB in a cost-efficient way. Due to symmetric cost functions (the general case is analysed in the Appendix), cost efficiency requires $q_n = Q/N$ for all n. We can therefore write (4) as

$$W(\boldsymbol{q}, \boldsymbol{\alpha}) \;=\; -\frac{\mathrm{b_{Soc}}}{2}\Big[(\bar{\alpha} - Q)^2 \,+\, \sigma^2\Big] \;-\; \frac{\mathrm{c_{Soc}}}{2} Q^2, \tag{7}$$

where $\mathrm{c_{Soc}} = c/N$ is the slope of society's marginal cost curve due to cost-sharing. This expression only depends on the total GoB level Q and we directly get the welfare maximum

$$Q^{\text{eff}} = \frac{\mathrm{b_{Soc}}}{\mathrm{b_{Soc}} + \mathrm{c_{Soc}}}\bar{\alpha}, \tag{8}$$

and accordingly $q_n^{\text{eff}} = Q^{\text{eff}}/N$ for all $n = 1, \ldots, N$. GoB level Q^{eff} is the quantity a decision-maker with benefit parameter $\mathrm{b_{Soc}}$, cost parameter $\mathrm{c_{Soc}}$ and ideal level $\bar{\alpha}$ would choose. The negative welfare impact of heterogeneity in ideal levels enters welfare in expression (7) additively and is thus not decision-relevant.

Figure 2 illustrates efficient public GoB provision for two agents, $N = 2$, with total GoB level Q on the horizontal axis. The blue line shows the societal marginal cost curve $\mathrm{MC_{Soc}}$, which has a slope of MC_n/N as all agents jointly provide the GoB in a cost-efficient way. The green and red line shows the individual marginal benefit functions

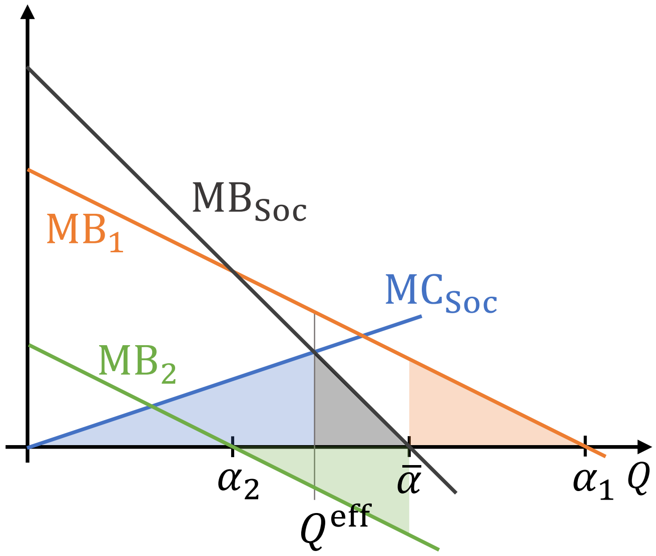


Figure 2: Efficient public GoB provision. All shaded areas correspond to negative welfare components as described in (7). Note that the red and green area together correspond to N times the variance with slope of b, which is the same as the variance with slope of $\mathrm{b}_{\mathrm{Soc}}$.

MB_1 and MB_2, respectively. They equal zero at the respective ideal level and are negative for $Q > \alpha_n$. The black line is the societal marginal benefit function $\mathrm{MB}_{\mathrm{Soc}}$ that is zero at the average ideal level $\bar{\alpha}$ and has slope $\mathrm{b}_{\mathrm{Soc}} = bN$. Starting from $Q = 0$, increasing the public GoB level has low marginal societal cost and high marginal societal benefits. But marginal societal benefits fall steeply, $\mathrm{b}_{\mathrm{Soc}} = bN$.

The efficient GoB level Q^{eff} is the level at which marginal cost and benefits intersect and strikes a balance between the agents. Agent 1 would have liked a higher GoB level, whereas agent 2 considers the GoB over-provided, $Q^{\mathrm{eff}} > \alpha_2$. The shaded areas correspond to the negative welfare components in expression (7). It shows that the negative benefit components can be written as negative benefits from the deviation between public GoB level Q and ideal level $\bar{\alpha}$ (black area) plus the heterogeneity term $-\mathrm{b}_{\mathrm{Soc}}\sigma^2/2$, the sum of the green and the red shaded area.

6 Non-cooperative public GoB provision

The (selfish) best-response functions (5) give rise to a unique Nash equilibrium. In this section we are going to determine this unique equilibrium. First note that, due to vanishing marginal costs at the point of non-contribution, $MC_n(0) = 0$ for all n, an agent contributes a strictly positive amount if and only if the aggregative contribution by others is below the agent's ideal GoB level α_n. This also implies that if agent n contributes a strictly positive amount, then so will all agents $1 \leq \nu \leq n$. In other words, a Nash equilibrium must be of the form where the first m agents contribute a strictly positive amount and the remaining $N - m$ agents contribute zero. We will see that all equilibrium types $m \in \{1, \dots, N\}$ are possible.

Crucial for determining the equilibrium type are the cost boundaries

$$c^{(m)} := mb\frac{\bar{\alpha}^{(m)} - \alpha_m}{\alpha_m} \quad , \; 1 \le m \le N, \tag{9}$$

where $\bar{\alpha}^{(m)} = (\sum_{\nu=1}^{m} \alpha_\nu)/m$ is the average ideal GoB level among the m agents with the highest GoB levels. It is obviously $c^{(1)} = 0$ and $c^{(m)} \ge 0$ for all m. We additionally define $c^{(N+1)} = \infty$.

Proposition 1. *Consider the non-cooperative public GoB game of agents characterized by their ideal GoB levels $\alpha_1 \ge \dots \ge \alpha_N$. Let $m \in \{1, \dots, N\}$ be the unique number with $c^{(m)} < c \le c^{(m+1)}$. Then in the unique Nash equilibrium, the first m agents, $1 \le n \le m$ contribute*

$$q_n^{*(m)} = \frac{b}{c}\left(\alpha_n - \frac{mb}{mb+c}\bar{\alpha}^{(m)}\right) > 0 \tag{10}$$

with aggregate contribution $Q^{*(m)} = \frac{mb}{mb+c}\bar{\alpha}^{(m)}$. For the remaining $N-m$ agents, $m+1 \le n \le N$, the GoB is (weakly) overprovided, $Q^{*(m)} \ge \alpha_n$, and hence they do not contribute, $q_n^{*(m)} = 0$. For all n, $q_n^{*(m)}$ is continuous in c and differentiable outside of the cost boundaries.

Proof. The proof works in two steps. The first step is to demonstrate that (10) is the Nash equilibrium quantities if exactly the first m agents contribute strictly positive quantities. The second step is to demonstrate that if $c^{(m)} < c \le c^{(m+1)}$, then indeed the first m agents – and only those – contribute strictly positive amounts. For details, see Appendix C. □

Figure 3 illustrates how the equilibrium type m depends on our key parameters, cost parameter c and heterogeneity parameter σ. We write the cost boundaries $c^{(m)}$ as a function of heterogeneity parameter σ. With $\boldsymbol{\alpha} = \bar{\alpha} + \sigma\boldsymbol{\delta}$, see Appendix A.2, we get

$$c^{(m)} = mb\sigma\frac{\bar{\delta}^{(m)} - \delta_m}{\bar{\alpha} + \sigma\delta_m}. \tag{11}$$

For symmetric ideal levels, $\sigma = 0$, it is $c^{(m)} = 0$ for all m, which implies that in symmetric settings, all agents contribute, irrespective of cost c. One can read Figure 3 in two ways. There are two possibilities for the "free-driver" problem of few contributing agents to emerge. First, holding fixed the level of heterogeneity σ, a *decrease* in contribution cost c (weakly) decreases the number of contributing agents. Second, holding fixed the contribution cost c, an *increase* in heterogeneity σ (weakly) decreases the number of contributing agents.

In the Nash equilibrium described by Proposition 1 there is over- and underprovision at the same time. The m contributors do not take into account the negative effect

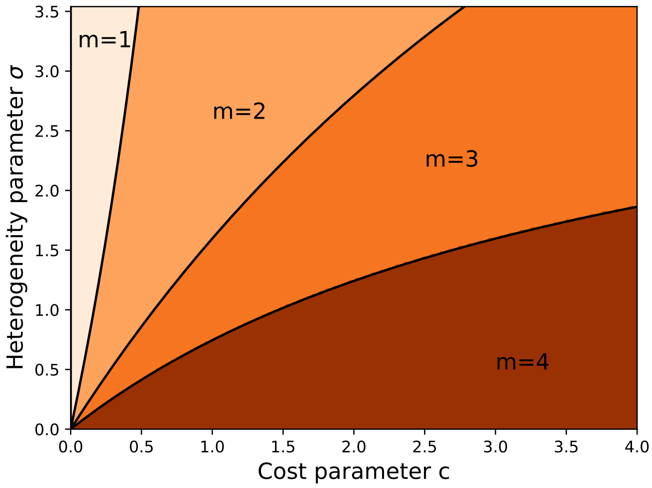


Figure 3: Equilibrium type m as function of cost parameter c and heterogeneity parameter σ for $N = 4$ agents. The black lines represent the cost boundaries $c^{(m)}$. Parameter settings are $b = 1$, $\bar{\alpha} = 5$. The heterogeneity in ideal levels is captured by $\delta = \frac{1}{\sqrt{20}} \cdot (-3, -1, 1, 3)$.

of overprovision on the $N - m$ agents with the lowest ideal GoB levels (free-driver externality). Among themselves, however, the usual free-riding incentives apply as the following remark clarifies.

Remark 1. *For a non-cooperative equilibrium of type m, the aggregate non-cooperative public GoB level falls short of the level that would be efficient for that group of m agents*

$$Q^{*(m)} = \frac{mb}{mb + c}\bar{\alpha}^{(m)} < \frac{mb}{mb + c/m}\bar{\alpha}^{(m)}, \tag{12}$$

cf. expression (8).

7 Comparative statics

In this section we analyze how public GoB provision levels depend on cost parameter c and heterogeneity parameter σ. We begin with efficient public GoB provision.

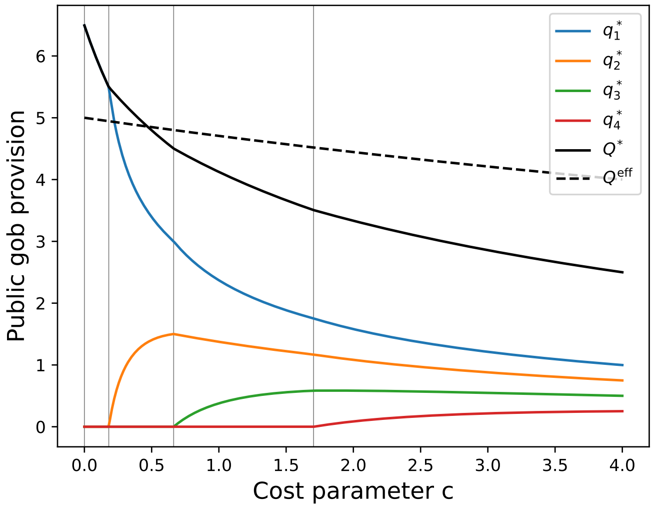
Proposition 2. *Efficient GoB provision depends on parameters as follows:*

(i) An increase in cost parameter c strictly decreases the efficient GoB level Q^{eff} and accordingly all individual provision levels $\boldsymbol{q}^{\text{eff}} = Q^{\text{eff}}/N$.

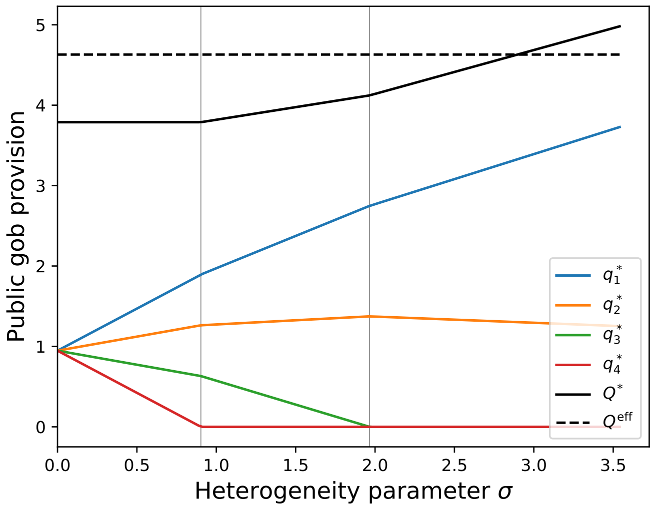
(ii) An increase in heterogeneity parameter σ does not affect the efficient GoB level Q^{eff}.

Proof. See Appendix D.1. □

The dashed black lines in both panels of Figure 4 illustrate Proposition 2. The intuition behind the result is easy to grasp. Higher marginal provision costs imply that



(a) Provision levels as a function of cost parameter c for constant heterogeneity parameter $\sigma = 1.113$.



(b) Provision levels as a function of heterogeneity parameter σ for constant cost parameter $c = 1.279$.

Figure 4: Public GoB provision levels. Individual Nash equilibrium levels (colored solid lines), aggregate Nash equilibrium (solid black line), and efficient aggregate GoB level (dashed black line). Parameter settings are $b = 1$, $\bar{\alpha} = 5$. The heterogeneity in ideal levels is captured by $\delta = \frac{1}{\sqrt{20}} \cdot (-3, -1, 1, 3)$.

a lower public GoB level is socially optimal. A mean-preserving spread in ideal GoB levels negatively affects welfare, but does not affect the efficient gob level.

Proposition 3. *Non-cooperative public GoB provision depends on parameters as follows:*

(i) An increase in cost parameter c unambiguously reduces aggregate GoB level $Q^{*(m)}$. Individual provision levels $q_n^{*(m)}$ are typically not monotone in cost parameter c.

(ii) An increase in heterogeneity parameter σ strictly increases aggregate public GoB

level $Q^{*(m)}$ when $1 \leq m < N$ and has no effect when $m = N$. Individual provision levels are in general not monotone in heterogeneity parameter σ. Among the m contributors, agent 1's level increases in σ whereas agent m's level decreases in σ.

Proof. See Appendix D.2. □

The solid black lines in both panels of Figure 4 show the effect on aggregate GoB level $Q^{*(m)}$, the solid colored lines show individual provision levels. The aggregate public GoB level in the Nash equilibrium (solid black line) shows a monotone decrease in cost parameter c and (weak) monotone increase in heterogeneity parameter σ, whereas individual provision levels show a non-monotone behavior. Note that the individual provision of agent m is zero at cost $c = c^{(m)}$, where the other agents just contribute agent m's ideal GoB level α_m; increasing cost parameter c implies that other agents provide less so that agent m decides to chip in, too. Further cost parameter increases however eventually cause individual contribution to decrease.

Finally we provide a first answer on whether non-cooperative public GoB provision results in under- or overprovision, measured against the baseline of the efficient GoB level. In Figure 4 we already see regions of over-provision (solid black line above the dashed black line) and under-provision (solid black line below the dashed black line). Figure 5 studies the ratio Q^*/Q^{eff} more systematically. Underprovision (ratio below one) is shown in blue, overprovision (ratio above one) in red. We are able to show the following result.

Proposition 4. *The ratio of aggregate non-cooperative over aggregate efficient public GoB provision strictly decreases in cost parameter c and weakly increases in heterogeneity parameter σ.*

Proof. See Appendix D.3. □

The interesting observation is that, generally speaking, the "traditional" public good case of high contribution costs and low ideal level heterogeneity is characterized by underprovision, represented by blue areas. The interpretation is that free-riding externalities dominate. In contrast, overprovision occurs for low contribution costs and high ideal level heterogeneity, represented by red areas. Here, free-driver externalities dominate.

Such analysis of whether the total GoB level exceeds or falls short of the efficient total GoB level is informative, but only tells a limited part of the story. The white area in Figure 5, where the equilibrium GoB level coincides with the total GoB level under efficient provision, hardly means that GoB provision is efficient. In the concrete example the white area mostly lies within the $m = 2$ equilibrium region. That means only the two agents with the strongest preference for the GoB contribute in the non-cooperative equilibrium. Efficient provision, in contrast, would require all agents to contribute. A full assessment of the private vs. efficient provision of public GoBs requires a welfare analysis. This is what we are going to do in the next section.

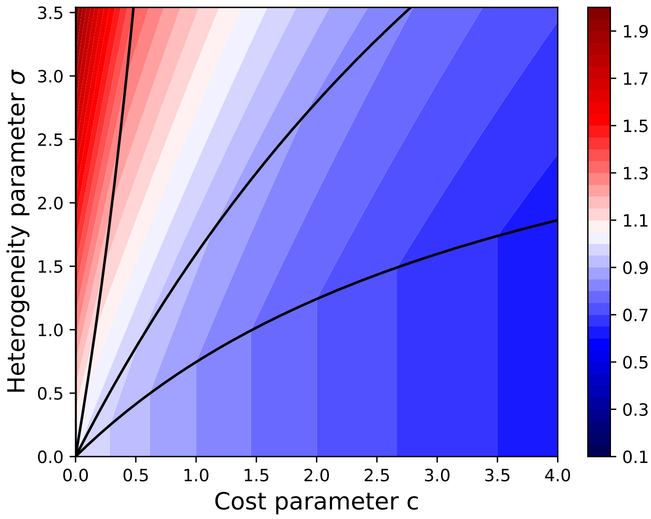


Figure 5: Ratio Q^*/Q^{eff} of the aggregate non-cooperative over efficient aggregate GoB level. Underprovision relative to efficient baseline is indicated in blue, overprovision in red. The black lines depict the equilibrium type boundaries, see Figure 3. Parameter settings are $b = 1$, $\bar{\alpha} = 5$. The heterogeneity in ideal levels is captured by $\delta = \frac{1}{\sqrt{20}} \cdot (-3, -1, 1, 3)$.

8 Welfare analysis

In this section we study several welfare-related topics of public GoB provision. As before, our focus is on the the effect of cost parameter c and heterogeneity parameter σ. We begin with efficient GoB provision.

Proposition 5. *The welfare under efficient GoB provision depends on parameters as follows:*

(i) An increase in cost parameter c strictly decreases welfare, although individual agents can be better off.

(ii) An increase in heterogeneity parameter σ strictly decreases welfare.

Proof. See Appendix E.1. □

The following Proposition shows the stark differences when public GoB provision is non-cooperative.

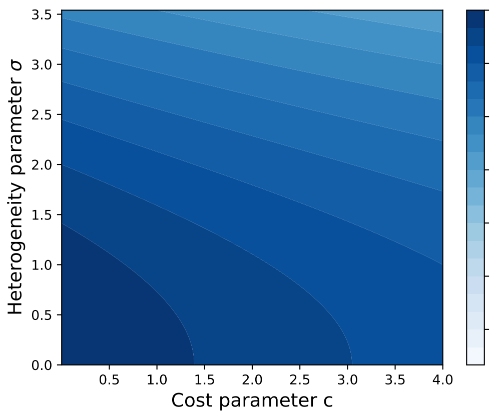
Proposition 6. *The welfare under non-cooperative GoB provision depends on parameters as follows:*

(i) An increase in cost parameter c has ambiguous welfare implications. If all agents have the same ideal GoB level, $\sigma = 0$, welfare strictly decreases. When $\alpha_1 > \alpha_2$, in a neighborhood of $c = 0$ an increase in cost parameter c increases total welfare. In terms of individual effects, an increase in c is always beneficial for agents whose ideal GoB level is already exceeded, $n = m + 1, \ldots, N$. Among the others, an

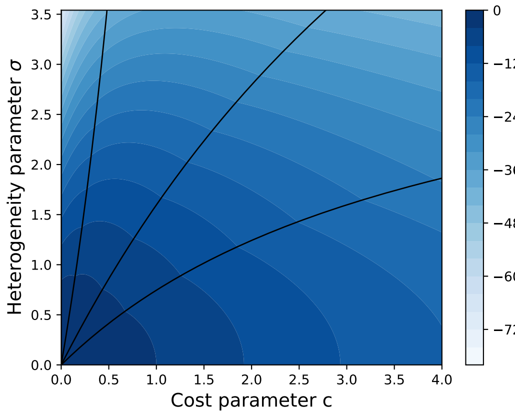
increase in c can benefit some and even can increase aggregate welfare among the m contributors.

(ii) Non-contributors suffer from an increase in ideal GoB level heterogeneity, while some individual contributors may benefit. The aggregate welfare of all contributors unambiguously decreases and therefore also total welfare.

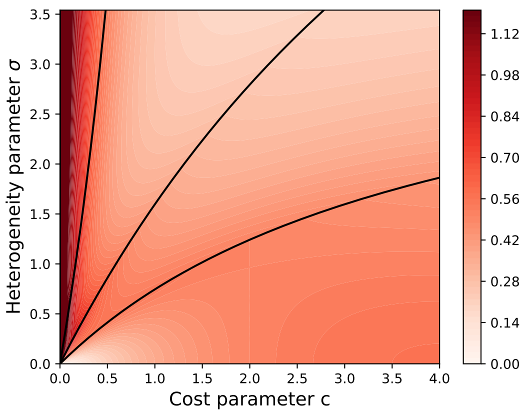
Proof. See Appendix E.2. □



(a) Efficient GoB provision.



(b) Non-cooperative GoB provision.



(c) Relative welfare loss

Figure 6: Welfare as a function of cost parameter c and heterogeneity parameter σ. We show aggregate welfare under efficient GoB provision (panel a), under non-cooperative GoB provision (panel b), and the relative welfare loss $(W(\boldsymbol{q}^{\text{eff}}) - W(\boldsymbol{q}^*))/|W(\boldsymbol{q}^{\text{eff}})|$ (panel c). The black lines in panel b and c show the boundaries of different equilibrium types as in Figure 3. Parameter settings are $b = 1$, $\bar{\alpha} = 5$. The heterogeneity in ideal levels is captured by $\delta = \frac{1}{\sqrt{20}} \cdot (-3, -1, 1, 3)$.

Figure 6 illustrates the main findings of Proposition 5 and Proposition 6. Instead of an increase in c, we interpret the findings for cost-reducing innovation that moves us from right to left. This movement is always beneficial under efficient GoB provision (first panel) but can be detrimental under non-cooperative GoB provision (second panel). For a given cost parameter c, a setting characterized by higher heterogeneity in ideal GoB levels is worse – for the efficient as well as the non-cooperative provision.

Panel c shows the relative welfare loss from non-cooperative instead of efficient provision, $(W(\boldsymbol{q}^{\text{eff}}) - W(\boldsymbol{q}^*))/|W(\boldsymbol{q}^{\text{eff}})|$. This relative welfare loss is a measure of how problematic the strategic incentives of self-interested agents are. We see roughly two areas of elevated relative welfare losses. The first is area is that of high cost parameter c and low or intermediate levels of heterogeneity σ. This area is the one typically studied in the public good literature where free-riding and hence underprovision of the public good dominates, cf. Figure 5. The second area of elevated relative welfare loss is for low cost parameter c and (sufficiently) high heterogeneity σ. This corresponds to the idea of overprovision of the public GoB due to free-driving, cf. Figure 5. In the other areas, the negative effects from free-riding and free-driving at least partially offset. Finally, we see that non-cooperative GoB provision achieves the efficient provision only in the very specific case of no contribution costs and complete absence of preference heterogeneity, $c = \sigma = 0$.

Finally, we briefly comment on winners and losers from non-cooperative public GoB provision. Figure 7 shows individual utility levels $u_n(\boldsymbol{q}^*)$ under the parameter settings of Figure 4a. It confirms that agent 1 is in the best position when $c = 0$: agent 1 can implement their ideal GoB level and thus over-provides the public GoB for everyone else. This reflects the "free-driver" concern raised in the context of geoengineering. The main insight from Figure 7, however, is that this is not the typical outcome when $c > 0$. In fact, a high ideal GoB level also means a high gap between actual and ideal GoB level, which implies high damages. This is why agent 1 fares worst for large ranges of cost parameter c.

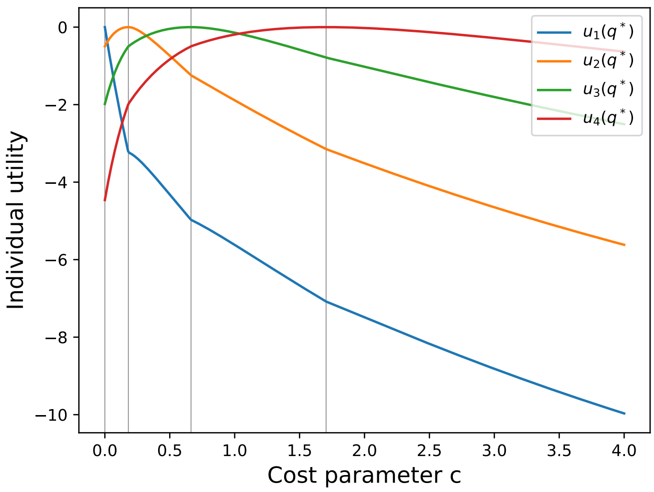


Figure 7: Individual utility under non-cooperative public GoB provision. Heterogeneity parameter $\sigma = 1.113$ as in Figure 4a. Other parameter settings are $b = 1$, $\bar{\alpha} = 5$. The heterogeneity in ideal levels is captured by $\delta = \frac{1}{\sqrt{20}} \cdot (-3, -1, 1, 3)$.

9 Concluding discussion

We have studied public GoBs, i.e. public goods over which agents have non-monotonic preferences. We have demonstrated that the private provision of public GoBs is prone to several strategic problems. In addition to free-riding and under-provision, public GoBs may suffer from free-driving and over-provision.

We comment on the **robustness** of our results. First, within the setting of quadratic costs and benefits but agent-specific cost and benefit parameters, the Appendix demonstrates that it is possible to determine the efficient provision scheme as well as the Nash equilibrium and that the findings presented above continue to hold.

Second, one might ask how our results change when we depart from quadratic functions and consider other benefit and cost structures. As long as marginal costs at non-contribution are zero, we expect most of our findings to go through. In particular, we still expect the non-cooperative equilibrium to feature non-contributors for which the GoB is already over-provided as well as contributors that provide strictly positive contributions. The reason is that, due to zero marginal contribution cost at the point of non-contribution, an agent inevitably contributes a little as long as the contribution by others is strictly less than that agent's ideal GoB level. If marginal contribution costs are not zero at the point of non-contribution, then non-contribution can either be the case because of over-contribution by others or because marginal costs are higher than marginal benefits and we expect more variation in possible equilibrium types.

Third, an important assumption in our framework is that contributions to the public GoB are non-negative, i.e. undoing the GoB provision by others is not possible. Existing studies (Heyen et al. 2019; Ansink and Weikard 2025) suggest that the possibility of negative contributions has large implications for the outcome. We have deliberately focused in this paper on the case of non-negative contributions. Whether that case is realistic or not depends on the specific application. In any case, it is an important benchmark to study.

Finally, when we leave the framework of quasi-linear preferences we expect wealth effects, as in Bergstrom et al. (1986), to play a role. While these wealth effects are certainly interesting, we leave them for future research as we consider them of secondary importance to heterogeneity in ideal GoB levels. It is the heterogeneity in ideal GoB levels that we wanted to focus on in our parsimonious model structure.

A concrete **implication** of our work is to rethink the role of cost-reducing innovation in public GoB provision. Making it cheaper to contribute to a public GoB is always beneficial if the GoB is provided in a centralized, efficient way that can balance diverse preferences. Similarly, when agents agree on the ideal gob level, cost reductions remain welfare-enhancing even under decentralized private provision. In contrast, for private provision scenarios where agents significantly disagree about the ideal gob level, cost-reducing innovation can be detrimental. This occurs because lower costs enable agents with higher ideal levels to increase provision beyond what is optimal for others. For contested technologies such as solar geoengineering, where centralized efficient provision is unlikely in the foreseeable future and ideal levels vary dramatically across countries, there might be a reasoned argument to approach research into cost reductions cautiously.

Our analysis also reveals important nuances in the welfare implications of non-cooperative public gob provision. While Weitzman (2015) highlights the possibility that agents with the highest preference for the GoB implement their ideal level at others' expense, the distribution of winners and losers is, in general, more complex. When no public GoB is provided, those with the largest ideal gob levels suffer the greatest loss, being furthest from their preferred provision point. With positive contribution costs, the "free-driver" behavior is constrained, leading to a situation where the agent with the highest ideal level bears disproportionate costs while others free-ride on their contributions. This dynamic has direct relevance to contemporary issues like European defense against Russia, where Baltic countries and Poland – with the most at stake – shoulder greater burdens relative to more distant nations. Similarly, in climate intervention scenarios, countries in the Global South already experiencing severe climate impacts might have stronger incentives to pursue solar geoengineering. Some might call this reckless "free-driving", but one should acknowledge the significant damages that result from being far from the ideal GoB level.

Future research can build on our framework to examine cooperation incentives in the provision of public GoBs. The non-monotone preferences and resulting dual externality problem we identify create unique challenges and prospects for coalition formation and stability that extend beyond traditional public goods theory. Existing contributions in this direction (e.g., Heyen et al. 2019; Heyen and Lehtomaa 2021; McEvoy et al. 2024; Ansink and Weikard 2025) suggest promising approaches for understanding how heterogeneous preferences affect cooperation possibilities and what institutional mechanisms might effectively address both free-rider and free-driver problems simultaneously.

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A General framework

A.1 Setting

As in the main body, agent n's utility function is $u_n(\boldsymbol{q}) = B_n(Q) - C_n(q_n)$. Different from the main text, we here consider general benefit and cost parameters, i.e. $B_n(Q) = -\frac{b_n}{2}(\alpha_n - Q)^2$ with $b_n > 0$ and $C_n(q_n) = \frac{c_n}{2}q_n^2$ with $c_n > 0$. We use the notation $\theta_n = b_n/c_n$ for benefit-cost ratio of agent n and denote $\Theta_m = \sum_{\nu=1}^{m} \theta_\nu$. As in the main body, the most important model element is the vector of ideal GoB levels $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)$, where agents are labeled such that $\alpha_1 \geq \ldots \geq \alpha_N$.

A.2 Weighted mean and heterogeneity of ideal GoB levels

We now define a generalized weighted mean of ideal GoB levels. Let $\boldsymbol{\lambda} = (\lambda_n)_{n=1,\ldots,N}$ be any vector with $\lambda_n > 0$ for all n. We write

$$\bar{\alpha}_{\boldsymbol{\lambda}}^{(m)} = \frac{\sum_{\nu=1}^{m} \lambda_\nu \alpha_\nu}{\sum_{\nu=1}^{m} \lambda_\nu} \tag{13}$$

for the weighted mean of the first m ideal levels with weighting vector $\boldsymbol{\lambda}$. We use $\bar{\alpha}_{\boldsymbol{\lambda}} = \bar{\alpha}_{\boldsymbol{\lambda}}^{(N)}$ for the weighted mean of all ideal levels. Definition (13) generalizes the definition in the main text, $\bar{\alpha}^{(m)} = \bar{\alpha}_{\boldsymbol{\lambda}}^{(m)}$ for any constant vector $\boldsymbol{\lambda}$.

We now clarify mean-preserving spreads of ideal GoB levels and a corresponding measure of heterogeneity. Starting from any vector of ideal levels $\boldsymbol{\alpha}$, we can write

$$\alpha_n = \bar{\alpha}_{\boldsymbol{\lambda}} + \sigma_{\boldsymbol{\lambda}} \delta_n, \tag{14}$$

where $\sigma_{\boldsymbol{\lambda}} > 0$. Vector $\boldsymbol{\delta}$ inherits the ordered structure, $\delta_1 \geq \ldots \geq \delta_N$, has $\boldsymbol{\lambda}$-weighted mean zero, $\bar{\delta}_{\boldsymbol{\lambda}} = (\sum_n \lambda_n)^{-1} \sum_n \lambda_n \delta_n = 0$, and $\boldsymbol{\lambda}$-weighted unity variance, $(\sum_n \lambda_n)^{-1} \sum_n \lambda_n \delta_n^2 = 1$. We also write $\bar{\delta}_{\boldsymbol{\lambda}}^{(m)}$ for the $\boldsymbol{\lambda}$-weighted mean of the first m components of δ, and it is a simple proof to show that $\bar{\delta}_{\boldsymbol{\lambda}}^{(m)}$ decreases in m with $\bar{\delta}_{\boldsymbol{\lambda}}^{(N)} = 0$.

With these definitions, $\sigma_{\boldsymbol{\lambda}}^2 > 0$ is the $\boldsymbol{\lambda}$-weighted variance of $\boldsymbol{\alpha}$ and a suited measure of heterogeneity of ideal GoB levels. This fits with the main body: there, benefit and cost parameters are equal across agents and hence the relevant weighting vectors (either $\boldsymbol{b}$ or $\boldsymbol{\theta}$) constant. For a constant vector $\boldsymbol{\lambda}$, heterogeneity measure $\sigma_{\boldsymbol{\lambda}}^2$ simplifies to the unweighted variance $\sigma^2 > 0$.

While cost parameter c can take any strictly positive value, heterogeneity parameter $\sigma_{\boldsymbol{\lambda}}$ is bounded. We see that $0 \leq \sigma_{\boldsymbol{\lambda}} < \sigma_{max}$, where $\sigma_{max} = -\bar{\alpha}_{\boldsymbol{\lambda}}/\delta_N > 0$ stems from the assumption that all ideal GoB levels are positive. In terms of notation, note that we also

A.3 General expression for welfare

With the general quadratic specification it is easy to see that we can write total welfare $W = \sum_n u_n(\boldsymbol{q})$ for an arbitrary contribution profile $\boldsymbol{q}$ with total GoB level $Q = \sum_n q_n$ as

$$W(\boldsymbol{q}, \boldsymbol{\alpha}) \;=\; -\frac{\mathrm{b_{Soc}}}{2}\Big[(\bar{\alpha}_{\boldsymbol{b}} - Q)^2 \,+\, \sigma_{\boldsymbol{b}}^2\Big] \;-\; \frac{1}{2}\sum_{n=1}^{N} c_n q_n^2, \tag{15}$$

where $\mathrm{b}_{\mathrm{Soc}} = \sum_\nu b_\nu$ is the slope of the marginal social benefit curve, $\bar{\alpha}_{\boldsymbol{b}}$ is the weighted mean and $\sigma_{\boldsymbol{b}}^2 = (\sum_\nu b_\nu)^{-1} \sum_\nu b_\nu(\alpha_\nu - \bar{\alpha}_{\boldsymbol{b}})^2$ the weighted variance of the ideal levels with weight vector $\boldsymbol{b} = (b_1, \ldots, b_N)$, see above. Note that this expression simplifies to (4) when $b_n = b$ for all n.

A.4 Best response functions

Selfish best-response function The net benefit of agent n reads

$$
\begin{aligned}
u_n(Q, q_n) &= B_n(Q) - C_n(q_n) \\
&= -\frac{b_n}{2}(\alpha_n - Q)^2 \; - \; \frac{c_n}{2} q_n^2 \\
&= -\frac{b_n}{2}(\alpha_n - q_n - Q_{-n})^2 \; - \; \frac{c_n}{2} q_n^2
\end{aligned}
$$

where $Q_{-n} = \sum_{\nu \neq n} q_\nu$ is the public GoB level provided by agents other than n. The marginal net benefit for agent n is $du_n/dq_n = b_n(\alpha_n - q_n - Q_{-n}) - c_n q_n$. This implies for the best response function

$$
q_n(Q_{-n}) = \begin{cases} \frac{b_n}{b_n + c_n}(\alpha_n - Q_{-n}) & Q_{-n} < \alpha_n \\ 0 & Q_{-n} \geq \alpha_n. \end{cases} \tag{16}
$$

The similarity with the best-response function (5) in the main text is clear. The individual agent considers their own benefit parameter b_n and cost parameter c_n in choosing their best response to contributions Q_{-n} by others.

Social best-response function We can write total welfare as

$$
\begin{aligned}
W(\boldsymbol{q}) &= \sum_{\nu=1}^{N} (B_\nu(Q) - C_\nu(q_\nu)) \\
&= -\sum_{\nu=1}^{N} \frac{b_\nu}{2}(\alpha_\nu - Q_{-n} - q_n)^2 \; - \; \sum_{\nu=1}^{N} \frac{c_\nu}{2} q_\nu^2,
\end{aligned}
$$

where we have written $Q = Q_{-n} + q_n$ for any agent n. We get

$$
\frac{dW}{dq_n} = \sum_{\nu=1}^{N} b_\nu(\alpha_\nu - Q_{-n} - q_n) \; - \; c_n q_n, \tag{17}
$$

which leads to the social best-response function

$$
q_n^{\mathrm{eff}}(Q_{-n}) = \begin{cases} \frac{\mathrm{b}_{\mathrm{Soc}}}{\mathrm{b}_{\mathrm{Soc}} + c_n}(\bar{\alpha}_{\boldsymbol{b}} - Q_{-n}) & Q_{-n} < \bar{\alpha}_{\boldsymbol{b}} \\ 0 & Q_{-n} \geq \bar{\alpha}_{\boldsymbol{b}}, \end{cases} \tag{18}
$$

where $\mathrm{b}_{\mathrm{Soc}} = \sum_{\nu=1}^{N} b_\nu$ is the slope of the societal marginal benefit curve. Note that (18) simplifies to (6) in the main body when $b_n = b$ and $c_n = c$ for all n. To interpret (18), the benefit-cost comparison is between individual marginal cost c_n and *social* marginal benefit $\mathrm{b}_{\mathrm{Soc}}$ of one more unit of public good. The cutoff-value for whether more GoB should be provided or not is $\bar{\alpha}_{\boldsymbol{b}}$, which simplifies to $\bar{\alpha}$ in the main text.

The difference between (16) and (18) can be understood as follows. A selfish individual looks at differences of total GoB level Q from their individual ideal GoB level α_n, whereas from a social point of view the weighted mean GoB level $\bar{\alpha}_{\boldsymbol{b}}$ is the relevant yardstick. Both selfish and social best response function have c_n as the marginal cost of agent n for the provision of the public GoB. The crucial difference is that the selfish individual weighs deviations from the ideal GoB level with their individual benefit parameter b_n, whereas from a social point of view a deviation affects all, $\mathrm{b}_{\mathrm{Soc}} = \sum_\nu b_\nu$.

B Efficient public GoB provision

We determine the welfare-maximizing GoB provision contribution $\boldsymbol{q}^{\mathrm{eff}}$. The social welfare function W is strictly concave in contributions $\mathbf{q}$. It is

$$\frac{\partial W}{\partial q_n}(\mathbf{q}) = \sum_{\nu=1}^{N} b(\alpha_\nu - Q) \; - \; c_n q_n. \tag{19}$$

The condition for the unique maximum is $\partial_n W(\boldsymbol{q}^{\mathrm{eff}}) = 0$ for all n. As the first term on the RHS of (19) is independent of n, this implies $c_n q_n^{\mathrm{eff}} = c_1 q_1^{\mathrm{eff}}$ for all n. With $Q^{\mathrm{eff}} = \sum_\nu q_\nu^{\mathrm{eff}}$ it is straightforward to calculate

$$q_n^{\mathrm{eff}} = \gamma_n Q^{\mathrm{eff}}, \quad \gamma_n = \frac{H(\mathbf{c})}{c_n} \frac{1}{N}, \tag{20}$$

where $H(\mathbf{c}) = N \left(\sum_\nu c_\nu^{-1}\right)^{-1}$ is the harmonic mean of $c_1, \ldots, c_N$, see Heyen and McGinty (2025). It is $\sum_\nu \gamma_\nu = 1$. Those agents with a cost parameter smaller than the harmonic mean contribute more than the equal share Q^{eff}/N, and vice versa. Condition (20) states that for efficiency, agents need to contribute in proportion to how their cost parameter compares to the harmonic mean.

We can now determine the efficient level Q^{eff}. First, we determine the marginal cost to society of providing one more unit of the public GoB Q. The function we are interested in is $Q \mapsto \mathrm{C}_{\mathrm{Soc}}(Q) = \sum_\nu C_\nu(\gamma_\nu Q)$ with derivative $\mathrm{MC}_{\mathrm{Soc}}(Q) = \sum_\nu \gamma_\nu \cdot MC_\nu(\gamma_\nu Q)$. With $MC_n(\gamma_n Q) = c_n \gamma_n Q$, we get the marginal cost of (equimarginal) public GoB provision

$$\mathrm{MC}_{\mathrm{Soc}}(Q) = \mathrm{c}_{\mathrm{Soc}} Q, \tag{21}$$

where the societal cost parameter is $\mathrm{c}_{\mathrm{Soc}} = H(c)/N$. The left part in Figure 8 shows the horizontal aggregation of marginal cost curves under the assumption of the equimarginal principle. If all agents have the same cost parameter c, as in the main text, then $H(c) = c$ and every agent contributes Q/N. Similarly, we consider the societal benefit function $Q \mapsto \mathrm{MC}_{\mathrm{Soc}}(Q) = \sum_\nu B_\nu(Q)$. The marginal benefits are straightforward to calculate. The marginal impact of increasing the public GoB on the benefit of agent n is $MB_n(Q) = b_n(\alpha_n - Q)$, and therefore the total marginal benefit $\mathrm{MB}_{\mathrm{Soc}} = \sum_\nu MB_\nu$ reads

$$\mathrm{MB}_{\mathrm{Soc}}(Q) = \mathrm{b}_{\mathrm{Soc}}(\bar{\alpha}_{\boldsymbol{b}} - Q), \tag{22}$$

where $\bar{\alpha}_{\boldsymbol{b}}$ is the average ideal level, weighted with the vector $\boldsymbol{b}$ of benefit parameters, and $\mathrm{b}_{\mathrm{Soc}} = \sum_\nu b_\nu$.

From the condition $\mathrm{MB}_{\mathrm{Soc}}(Q^{\mathrm{eff}}) = \mathrm{MC}_{\mathrm{Soc}}(Q^{\mathrm{eff}})$ we calculate the efficient public GoB level as

$$Q^{\text{eff}} = \frac{\text{b}_{\text{Soc}}}{\text{b}_{\text{Soc}} + \text{c}_{\text{Soc}}}\bar{\alpha}_{\boldsymbol{b}}. \tag{23}$$

This is intuitive: It corresponds to the choice an individual with cost parameter c_{Soc}, benefit parameter b_{Soc} and ideal GoB level $\bar{\alpha}_{\boldsymbol{b}}$ would take. If providing the GoB was costless, $\text{c}_{\text{Soc}} = 0$, the efficient GoB level would be the weighted mean ideal level, $Q^{\text{eff}} = \bar{\alpha}_{\boldsymbol{b}}$. The higher the provision costs, the lower the efficient GoB level.

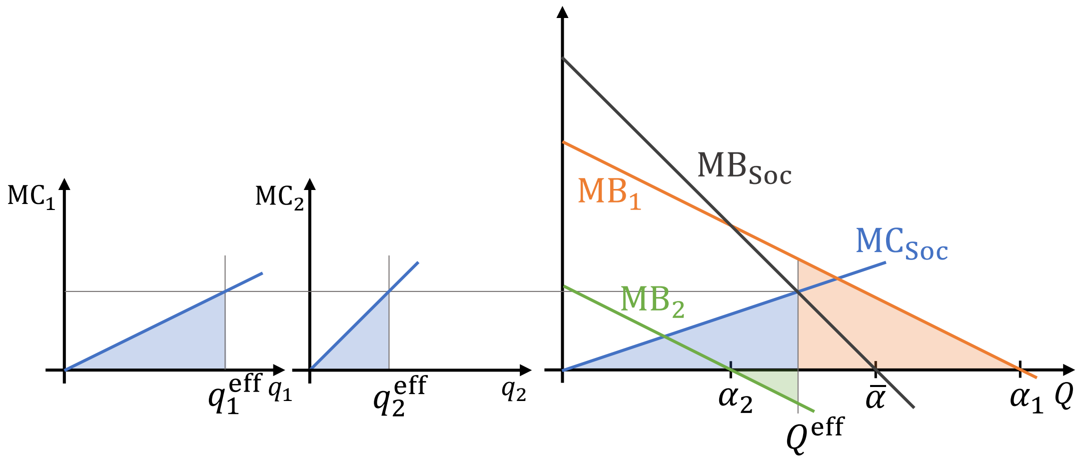

Figure 8: Efficient public GoB provision. Individual marginal cost curves are aggregated to the societal marginal cost curve MC. The marginal benefit curve has slope $\text{b}_{\text{Soc}} = \sum_\nu b_\nu$ and is zero at $Q = \bar{\alpha}_{\boldsymbol{b}}$. The efficient GoB level Q^{eff} is where marginal societal benefits equal marginal societal costs.

Figure 8 shows the efficient public GoB provision, illustrated for two agents, $N = 2$. It shows the societal marginal cost curve MC_{Soc}, which is the horizontal aggregation from the individual marginal cost curves MC_n (i.e. assuming equimarginal contribution costs across agents, a necessary condition for efficiency). It also shows individual benefit functions MB_n and the societal benefit function MB_{Soc}. Starting from $Q = 0$, increasing the public GoB level has low marginal societal cost and high marginal societal benefits. But marginal societal benefits fall steeply, $\text{b}_{\text{Soc}} = \sum_\nu b_\nu$. The efficient GoB level is where marginal cost and benefits intersect, $\text{MC}_{\text{Soc}}(Q^{\text{eff}}) = \text{MB}_{\text{Soc}}(Q^{\text{eff}})$.

C Non-cooperative public GoB provision

We determine the Nash equilibrium in the case of general cost c_n and benefit parameters b_n. The argumentation from the main body still applies: if agent n contributes a strictly positive amount in equilibrium, so do all agents $1 \leq \nu \leq n$. Therefore we can focus on the non-cooperative public GoB game among the first m agents and later determine under what conditions this assumption actually holds.

Recall that we write the benefit-cost ratio of agent n as $\theta_n = b_n/c_n$ and the vector of benefit-cost ratios as $\boldsymbol{\theta}$. That the first m agents provide strictly positive quantities implies that their best responses are described by $Q^{(m)} - \alpha_n + \theta_n^{-1} q_n = 0$, $n = 1, \ldots, m$, see (16). In matrix notation,

$$\boldsymbol{M}_{\boldsymbol{\theta}}^{(m)} \cdot \boldsymbol{q}^{*(m)} = \boldsymbol{\alpha}^{(m)}, \tag{24}$$

where the $m \times m$ matrix $\boldsymbol{M}_{\boldsymbol{\theta}}^{(m)}$ has $1 + \theta_n^{-1}$ as the n-the diagonal element and 1 everywhere outside the diagonal. Matrix $\boldsymbol{M}_{\boldsymbol{\theta}}^{(m)}$ has determinant

$$\det \boldsymbol{M}_{\boldsymbol{\theta}}^{(m)} = \left(\prod_{\nu=1}^{m} \theta_\nu \right)^{-1} \cdot (1 + \Theta_m), \quad \text{where } \Theta_m := \sum_{\nu=1}^{m} \theta_m. \tag{25}$$

Clearly, $\det \boldsymbol{M}_{\boldsymbol{\theta}}^{(m)} > 0$ for all m because $\theta_\nu > 0$ for all ν. The inverse matrix is given by

$$\boldsymbol{M}_{\boldsymbol{\theta}}^{(m)^{-1}} = (1 + \Theta_m)^{-1} \cdot \boldsymbol{A}, \tag{26}$$

where the $m \times m$ matrix $\boldsymbol{A}$ has the elements

$$a_{ij} = \begin{cases} \theta_i \cdot (1 + \Theta_m - \theta_i) & i = j \\ -\theta_i \theta_j & i \neq j. \end{cases} \tag{27}$$

We get $\boldsymbol{q}^{*(m)} = \boldsymbol{M}_{\boldsymbol{\theta}}^{(m)^{-1}} \cdot \boldsymbol{\alpha}^{(m)}$ and we easily calculate

$$q_n^{*(m)} = \theta_n \alpha_n - \theta_n \frac{\Theta_m}{1 + \Theta_m} \bar{\alpha}_{\boldsymbol{\theta}}^{(m)}, \; n = 1, \ldots, m. \tag{28}$$

It is easy to see that (28) simplifies to (10) when $c_n = c$ and $b_n = b$ for all n.

To complete the Nash equilibrium analysis, we need to understand under what conditions it is in the interest of the first m agents to contribute strictly positive amounts and in the interest of the remaining $N - m$ agents not to contribute. Agent $m + 1$ considers the public good over-provided if the total contribution by the first m agents already (weakly) exceeds this agent's optimal level, $Q^{*(m)} \geq \alpha_{m+1}$. To rewrite this condition, we make use of the following definition

$$\psi_m := \sum_{\nu=1}^{m-1} \theta_\nu \frac{\alpha_\nu - \alpha_m}{\alpha_m} \quad m = 2, \ldots, n, \tag{29}$$

where we additionally define $\psi_1 = 0$ and $\psi_{N+1} = \infty$. It is easy to show that the ψ_m are increasing in m. The condition for over-provision from agent $m+1$, $Q^{*(m)} \geq \alpha_{m+1}$, is equivalent with $\psi_{m+1} \geq 1$. Similarly, we can derive the condition under which agent m contributes a strictly positive amount; this is the case if and only if the total contribution by the first $m - 1$ agents is strictly smaller than agent m's ideal level, $Q^{*(m-1)} < \alpha_m$, or, equivalently, $\psi_m < 1$. The following proposition summarizes these findings.

Taken together, let m be the unique number for which $\psi_m < 1 \leq \psi_{m+1}$. Then, the unique Nash equilibrium is of type m, i.e. the first m agents make the strictly positive contribution given by (28) with aggregate GoB level of

$$Q^{*(m)} = \frac{\Theta_m}{1 + \Theta_m} \bar{\alpha}_{\boldsymbol{\theta}}^{(m)}, \tag{30}$$

and all remaining agents contribute zero in equilibrium. Finally, note that condition $\psi_m < 1 \leq \psi_{m+1}$ is equivalent to the cost boundary condition in the main body, $c^{(m)} < c \leq c^{(m+1)}$, when

$\theta_\nu = b/c$ for all ν.

D Comparative statics of quantities

We are going to analyze how quantities depend on the cost parameter c and heterogeneity parameter σ. The quantities we analyze are efficient GoB provision, non-cooperative GoB provision, and the ratio of aggregate Nash provision over aggregate efficient provision, where the latter is a measure of over- or underprovision.

D.1 Comparative statics of efficient GoB provision

Increasing any cost parameter c_n increases $\mathrm{c}_{\mathrm{Soc}} = H(\boldsymbol{c})/N$ and thus decreases the efficient aggregate GoB level Q^{eff}, see expression (23). In terms of the contribution shares, it is straightforward to calculate that $\partial_{c_n}\gamma_n < 0$ and $\partial_{c_n}\gamma_k > 0$ for any $k \neq n$, i.e. the agent with the cost increase contributes relatively less and everybody else contributes relatively more. For the analysis in the main body, where $c_n = c$ for all n, it is clear that the efficient aggregate GoB level decreases in c and all agents keep contributing the same share of $1/N$.

In terms of heterogeneity, a spread the preserves the $\boldsymbol{b}$-weighted mean of ideal GoB levels keeps the efficient GoB level constant. When $b_n = b$ for all n, then a (standard) mean-preserving spread preserves the $\boldsymbol{b}$-weighted mean and thus the efficient GoB level remains constant.

D.2 Comparative statics of non-cooperative GoB provision

We use the Implicit Function theorem to derive insights about comparative statics. Again, we focus on the game among the first $m \leq N$ agents. The parameters are $\boldsymbol{\eta}^{(m)} = (\boldsymbol{\alpha}^{(m)}, \boldsymbol{\theta}^{(m)}) \in \mathbb{R}^{2m}$. We define the function $f : \mathbb{R}^{2m} \times \mathbb{R}^m \to \mathbb{R}^m$ by

$$f_n(\boldsymbol{\eta}^{(m)}, \boldsymbol{q}^{(m)}) = Q - \alpha_n + \theta_n^{-1} q_n, \ n = 1, \ldots, m, \tag{31}$$

where as above $Q^{(m)} = \sum_{\nu=1}^m q_\nu$ is the total GoB level. The m conditions $f(\boldsymbol{\eta}^{(m)}, \boldsymbol{q}^{(m)}) = \mathbf{0}$ are equivalent to the best-response functions and thus jointly determine the Nash equilibrium $\boldsymbol{q}^{(m)}$. From the Implicit Function theorem we know that the matrix of derivatives of f with respect to parameters η is

$$\partial_{\boldsymbol{\eta}} \boldsymbol{q}^{*(m)} = -{\boldsymbol{J}_{\boldsymbol{f},\boldsymbol{q}}}^{-1} \cdot \boldsymbol{J}_{\boldsymbol{f},\boldsymbol{\eta}}, \tag{32}$$

where the ij element of $\boldsymbol{J}_{\boldsymbol{f},\boldsymbol{q}}$ is $\partial_{q_j} f_i$ and similar for $\boldsymbol{J}_{\boldsymbol{f},\boldsymbol{\eta}}$. It turns out that $\boldsymbol{J}_{\boldsymbol{f},\boldsymbol{q}} = \boldsymbol{M}_{\boldsymbol{\theta}}^{(m)}$, see above. It is

$$\boldsymbol{J}_{\boldsymbol{f},\boldsymbol{\eta}} = \begin{pmatrix} -1 & 0 & 0 & \cdots & 0 & -\theta_1^{-2} q_1^* & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 & 0 & -\theta_2^{-2} q_2^* & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots & 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & -1 & 0 & \vdots & \vdots & \ddots & -\theta_{m-1}^{-2} q_{m-1}^* & 0 \\ 0 & 0 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 & -\theta_m^{-2} q_m^* \end{pmatrix} \tag{33}$$

Derivatives with respect to cost parameters. We calculate

$$\partial_{c_k} q_n^{*(m)} = \begin{cases} -c_n^{-1}\left(1 - \frac{\theta_n}{1+\Theta_m}\right) q_n^{*(m)} < 0 & k = n \\ c_k^{-1} \frac{\theta_n}{1+\Theta_m} q_k^{*(m)} > 0 & k \neq n, \end{cases} \tag{34}$$

where we have used $\partial_{c_k} = -c_k^{-1}\theta_k \partial_{\theta_k}$. From there we immediately calculate

$$\partial_{c_k} Q^{*(m)} = \sum_{\nu=1}^{m} \partial_{c_k} q_\nu^{*(m)} = -c_k^{-1} q_k^{*(m)} \frac{1}{1+\Theta_m} < 0. \tag{35}$$

The implications for the analysis in the main body, when $b_n = b$ and $c_n = c$ for all n, is as follows. We get

$$\partial_c q_n^{*(m)} = \sum_{k=1}^{m} \partial_{c_k} q_n^{*(m)} = -\frac{b}{c^2}\alpha_n + \left(\frac{b}{c(mb+c)} + \frac{b}{c^2}\right) Q^{*(m)} \tag{36}$$

with unclear sign. For c converging to c^m from above, $Q^{*(m)}$ converges to α_n and the derivative $\partial_c q_n^{*(m)}$ turns positive; for very large c, in contrast, the derivative is negative. Unambigous, however, is that the aggregate public GoB level decreases when cost parameter c goes up,

$$\partial_c Q^{*(m)} = \sum_{k=1}^{m} \partial_{c_k} Q^{*(m)} = -\frac{1}{1+\Theta_m} \sum_{k=1}^{m} c_k^{-1} q_k^{*(m)} < 0. \tag{37}$$

Derivatives with respect to ideal level heterogeneity. We write $\alpha_n = \bar{\alpha}_{\boldsymbol{\theta}} + \sigma_{\boldsymbol{\theta}}\delta_n$ and $\bar{\alpha}_{\boldsymbol{\theta}}^{(m)} = \bar{\alpha}_{\boldsymbol{\theta}} + \sigma_{\boldsymbol{\theta}}\bar{\delta}_{\boldsymbol{\theta}}^{(m)}$, see section A.2. This together with expression (28) directly yields

$$\partial_{\sigma_{\boldsymbol{\theta}}} q_n^{*(m)} = \theta_n\delta_n - \theta_n \frac{\Theta_m}{1+\Theta_m} \bar{\delta}_{\boldsymbol{\theta}}^{(m)} \tag{38}$$

with unclear sign. Obviously, the derivative is highest and positive for agent $n = 1$ and lowest and negative for agent $n = m$. The aggregate GoB level has derivative

$$\partial_{\sigma_{\boldsymbol{\theta}}} Q^{*(m)} = \frac{\Theta_m}{1+\Theta_m} \bar{\delta}_{\boldsymbol{\theta}}^{(m)} \geq 0, \tag{39}$$

strictly positive for $1 \leq m < N$ and equal zero for $m = N$. In terms of the analysis in the main paper, all these statements in particular apply to standard mean-preserving spreads with heterogeneity parameter σ when $\theta_n = \theta$ for all n.

D.3 Comparative statics of provision ratio

We analyze the ratio R only under the conditions of the main body, i.e. $b_n = b$ and $c_n = c$ for all n. Then we see that

$$R = \frac{Q^{*(m)}}{Q^{\text{eff}}} = \frac{m(N^2 b + c)}{N^2(mb+c)} \cdot \frac{\bar{\alpha} + \sigma\bar{\delta}^{(m)}}{\bar{\alpha}} \tag{40}$$

and it is straightforward to verify that $\partial_c R < 0$ and $\partial_\sigma R \geq 0$. It is $\partial_\sigma R > 0$ when the equilibrium is of type $m < N$ and $\partial_\sigma R = 0$ when the equilibrium is of type $m = N$.

E Welfare analysis

In this section we analyze the welfare effect of changes in cost parameter c and heterogeneity parameter σ on welfare. The relevant expression for welfare is (15).

E.1 Efficient public GoB provision

We ask how a change in any cost parameter c_n impacts welfare. Due to the Envelope Theorem we only need to check the direct effect and immediately see that

$$\frac{dW(\boldsymbol{q}^{\text{eff}})}{dc_n} = -c_n q_n^{\text{eff}} < 0. \tag{41}$$

It is easy to see that individual agents are better of from such an increase in c_n. Consider an agent $k \neq n$ with $\alpha_n < Q^{\text{eff}}$. Then an increase in c_n brings the aggregate GoB level closer to that agent's ideal GoB level, see Appendix D.1, and reduces the cost that agent k incurs. Together, the effect on agent k is unambiguously positive.

When we look at the derivative of welfare under efficient contribution with respect to the (joint) cost parameter c as in the main body, then (41) directly shows $dW(\boldsymbol{q}^{\text{eff}})/dc = -cQ^{\text{eff}} < 0$. That individual agents can be better off, despite the increase in cost parameter c, can be seen as follows. We calculate

$$\frac{du_n(\boldsymbol{q}^{\text{eff}})}{dc} = \frac{\mathrm{b}_{\text{Soc}}^2 \bar{\alpha}(\bar{\alpha} - 2\alpha_n)}{2N^2(\mathrm{b}_{\text{Soc}} + \mathrm{c}_{\text{Soc}})^2}, \tag{42}$$

which is positive if α_n is sufficiently small.

In terms of increasing heterogeneity in ideal levels, we directly see that a $\boldsymbol{b}$-weighted mean preserving spread of ideal GoB levels, i.e. in increase in $\sigma_{\boldsymbol{b}}$, strictly decreases welfare (15). A standard mean-preserving spread, i.e. an increase in σ as in the main body, preserves the $\boldsymbol{b}$-weighted mean when $b_n = b$ for all n.

E.2 Non-cooperative public GoB provision

We go separately through increases in cost parameter c and heterogeneity parameter σ.

Increase in cost parameter c. We show the three different statements of the first part in Proposition 6. First, if all agents have the same ideal GoB level, $\alpha_n = \bar{\alpha}$ for all n, all contribute in equilibrium, $m = N$, and we get $u_n = -\frac{bc(b+c)}{2(Nb+c)^2}\bar{\alpha}^2$ and therefore $\frac{du_n}{dc} = -\frac{b^2((b+2c)N-c)}{2(Nb+c)^3}\bar{\alpha}^2 < 0$. Every agent benefits from cost-reducing innovation and thus also the entire group.

Second, we study the case of very low cost parameter c and $\alpha_1 > \alpha_2$. Even though formally not part of our framework, we can consider the case $c = 0$ (which is well-defined because $\alpha_1 > \alpha_2$). Due to $\alpha_1 > \alpha_2$, we have $c^{(2)} > 0$ and only agent 1 contributes in equilibrium, $m = 1$. Then, for general $c \geq 0$, $Q^* = q_1^* = b\alpha_1/(b+c)$ and $q_n^* = 0$ for $n = 2, \ldots, N$. An increase in c decreases the GoB, $\frac{dQ^*}{dc} = -b\alpha_1/(b+c)^2 < 0$. This improves utility for all non-contributors, their marginal effect is $\frac{du_n}{dc} = b(\alpha_n - Q^*)\frac{dQ^*}{dc} > 0$ because $Q^* > \alpha_n$. Combining the effects on all agents, we get at $c = 0$

$$\frac{dW}{dc}(0) = -bN(\alpha_1 - \bar{\alpha}) \cdot (-\frac{\alpha_1}{b}) - \frac{1}{2}\alpha_1^2 \tag{43}$$

This is positive if and only if $\alpha_1 > N\bar{\alpha}/(2N-1)$ as stated in the proposition. This is fulfilled if ideal level heterogeneity is sufficiently large.

Finally, we construct one setting where all but the first agent have the same ideal GoB level, $\alpha_2 = \dots \alpha_N = \bar{\alpha} - \frac{\alpha_1 - \bar{\alpha}}{N-1}$. We focus on the case that all agents contribute, $m = N$, which is true if $c > c^{(N)}$. At $c = c^{(N)}$, agents $n = 2, \dots, N$ have their bliss point: No contribution costs while the total GoB level exactly matches their ideal level, $Q^* = \alpha_n$. Therefore, welfare goes through a maximum at that point with $\frac{du_n}{dc}(c = c^{(N)}) = 0$. On the other hand, we calculate for agent 1 that

$$\frac{du_1}{dc}(c^{(N)}) = \frac{(N\bar{\alpha} - 2\alpha_1)(N\bar{\alpha} - \alpha_1)^2}{2(N-1)^2 N\bar{\alpha}} \tag{44}$$

which is positive for $N > 2$ and α_1 sufficiently close to $\bar{\alpha}$.

Increase in heterogeneity parameter σ. We now show part (ii). We first show that total costs among contributors increase when σ increases. The change in costs among contributors is

$$\frac{d}{d\sigma} \sum_{n=1}^{m} C_n(q_n^*) = c \sum_{n=1}^{m} q_n^* \frac{dq_n^*}{d\sigma}. \tag{45}$$

We know that the derivative of the equilibrium quantities with respect to σ is ordered, $\frac{dq_n^*}{d\sigma} \geq \frac{dq_{\tilde{n}}^*}{d\sigma}$ when $n < \tilde{n}$. Because the derivative is definitely positive for agent 1, there is a highest index $\nu \in \{1, \dots, N\}$ for which the derivative is positive. We then have

$$\begin{aligned}
\frac{d}{d\sigma} \sum_{n=1}^{m} C_n(q_n^*) &\geq c \sum_{n=1}^{m} q_\nu^* \frac{dq_n^*}{d\sigma} \\
&= c q_\nu^* \sum_{n=1}^{m} \frac{dq_n^*}{d\sigma} \\
&= c q_\nu^* \frac{mb}{mb + c} \bar{\delta}^{(m)} \geq 0,
\end{aligned}$$

and this is what we wanted to show.

Similarly, we will now prove that the total benefit among contributors decreases when σ increases. The change in benefits is

$$\frac{d}{d\sigma} \sum_{n=1}^{m} B_n(\boldsymbol{q}^*) = -b \sum_{n=1}^{m} (\alpha_n - Q^*) \frac{d}{d\sigma} \left[\alpha_n - Q^*\right]. \tag{46}$$

Similar to the argument above, let ν be the highest index for which $\frac{d}{d\sigma}\left[\alpha_n - Q^*\right] > 0$. We then have

$$\begin{aligned}
\frac{d}{d\sigma} \sum_{n=1}^{m} B_n(\boldsymbol{q}^*) &\leq -b \sum_{n=1}^{m} (\alpha_n - Q^*) \frac{d}{d\sigma} \left[\alpha_\nu - Q^*\right] \\
&= -b \frac{d}{d\sigma} \left[\alpha_\nu - Q^*\right] \sum_{n=1}^{m} (\alpha_n - Q^*) \\
&= -b \frac{d}{d\sigma} \left[\alpha_\nu - Q^*\right] m(\bar{\alpha}^{(m)} - Q^*) \leq 0,
\end{aligned}$$

and this is what we wanted to show. Together, welfare among contributors, $\sum_{n=1}^{m}(B_n(\boldsymbol{q}^*) - C_n(q_n^*))$ decreases in σ. Obviously, all non-contributors are worse off under an increase in σ as the total GoB level Q^* further moves away from their ideal level. This then also implies that total welfare among all agents must decrease in σ. That individual agents can be better off when

σ increases is obvious when we consider agents around their ideal GoB levels: an increase in σ causes them to contribute less in equilibrium but total GoB levels to move closer to their ideal level.