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**Working Paper**

## Predicting the distribution of contest success under the illusion of proportionality

Working Paper, No. 466

**Provided in Cooperation with:**

Department of Economics, University of Zurich

*Suggested Citation:* Hefti, Andreas; Shen, Peiyao (2025) : Predicting the distribution of contest success under the illusion of proportionality, Working Paper, No. 466, University of Zurich, Department of Economics, Zurich, <https://doi.org/10.5167/uzh-276543>

This Version is available at:

<https://hdl.handle.net/10419/316345>

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University of Zurich  
Department of Economics

Working Paper Series  
ISSN 1664-7041 (print)  
ISSN 1664-705X (online)

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# **Predicting the Distribution of Contest Success under the Illusion of Proportionality**

Andreas Hefti and Peiyao Shen

March 2025

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# Predicting the Distribution of Contest Success under the Illusion of Proportionality

Andreas Hefti\* and Peiyao Shen

## Abstract

Understanding disparities in contest success is central to explaining how competition shapes the distribution of rewards, influence, or market shares. We introduce the Proportional Play Equilibrium (PPE), a boundedly rational alternative to Nash Equilibrium (NE) grounded in the Illusion of Proportionality, and show that it results in more unequal outcomes by exaggerating the success chances of stronger contestants. Laboratory evidence strongly supports PPE's predictions for success dispersion while rejecting those of NE. Our results highlight how equilibrium analysis under full rationality may mischaracterize the inequality-generating effects of competition, with further implications for understanding inequality in markets or political contests.

**Keywords:** Illusion of Proportionality; Bounded Rationality; Contest Success; Market Share and Inequality; Behavioral Contest Theory

**JEL Classification:** D01, D91, D72, C72

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# 1 Introduction

Many economic and social interactions take the form of contests in which the involved parties compete to secure scarce resources. Examples include firms vying for monopoly franchises, political candidates seeking electoral victory, employees competing for promotions, researchers pursuing funding, athletes striving for championships, and firms battling for market share. Central to these competitions is the principle that contestants' actions shape their likelihood of success, contingent on the investments they make.

While contestants choose effort, it is the resulting success probabilities that determine how rewards, influence, or opportunities are allocated. Contest theory has traditionally emphasized effort levels and rent dissipation as key outcome metrics. Yet in many real-world settings, contestants are primarily concerned with their chances of success – how their actions translate into the probability of achieving a desired outcome. In business, companies prioritize market share as a key performance metric, shaping competitive strategies and investment decisions to secure long-term positional advantages (Schmalensee, 1982; Lieberman and Montgomery, 1988). In political elections, candidates adjust their communication strategies in response to their perceived chances of success, narrowing their outreach to specific constituencies as their probability of winning increases (Murayama et al., 2023). In sports, athletes and teams often strategize based on their perceived chances of victory rather than solely on effort (Palacios-Huerta, 2014).

Beyond contestants' incentives, understanding what drives the distribution of success probabilities is crucial in settings where balance, system legitimacy, or mar-

ket concentration are central concerns. In political elections, skewed success probabilities may erode public trust in the democratic process (Grant et al., 2021). In promotion or compensation decisions, a highly unequal distribution can foster dissatisfaction and weaken organizational cohesion (Bloom, 1999; Hofmann, 2023). In innovation contests, skewed success distributions – where large firms persistently manage to secure patents – can discourage smaller firms from innovating, thus stifling competition and progress (Galasso and Schankerman, 2018). In sports, tournament designers aim to maintain a competitive balance to sustain unpredictability and public engagement (Csató, 2021).

Because success probabilities play a central role in competitive environments, it is essential to have a framework that accurately rationalizes their distribution. Nash Equilibrium (NE) – the canonical solution concept in contest theory – provides the rational benchmark for predicting strategic behavior under full information and optimal reasoning. Yet NE does not always align with observed behavior. A well-documented limitation is its systematic tendency to underpredict average effort levels in imperfectly discriminatory contests. This raises the possibility that NE may likewise offer an incomplete account of the resulting success probabilities, potentially mischaracterizing how competition shapes disparities in outcomes.

Any contest model must capture how players perceive the mapping from their effort to the probability of success. This cognitive task involves probabilistic reasoning, an area in which human judgment is known to exhibit systematic distortions. In this paper, we propose an alternative equilibrium concept – Proportional Play Equilibrium (PPE) – that builds on psychological research into the Illusion of Proportionality (IoP), a cognitive bias whereby individuals misconceive complex

relationships as proportional. In the context of contests, this implies that players base their strategies on a presumed proportional relationship between their efforts and success probabilities, neglecting the true, nonlinear nature of this mapping. When embedded in strategic reasoning, this bias alters players' effort choices and, consequently, the resulting distribution of success probabilities.

Our contribution is threefold.

First, we develop PPE as a boundedly rational equilibrium framework that incorporates the IoP as the sole deviation from NE in an otherwise standard “Tullock” Contest Success Function. Under the IoP, players neglect the feedback effect that their own effort has on their success probability, leading them to overestimate their impact on success and to exert higher effort than predicted by NE.

Second, we show that PPE predicts a more unequal dispersion of success probabilities compared to NE, disproportionately favoring players capable of achieving higher success probabilities. This pattern reflects the core bias induced through the IoP: by neglecting the feedback effect and its diminishing returns to efforts, stronger players are more inclined to target high success probabilities, thereby amplifying success inequality. To establish this result, and to derive further comparative-static implications, we recast the contest as a Market Share Game (MSG) – an equivalent representation in which players compete directly in their aspired success probabilities. This reformulation enables the direct application of optimality conditions to characterize comparative-static properties of the success distribution. Specifically, this reveals that the success distributions under PPE and NE, while substantially different, are homotopic, connected through a single behavioral parameter that governs the extent to which players internalize the feedback effect of their own effort.

Moreover, the MSG framework allows us to relate equilibrium outcomes to standard inequality metrics, such as the Gini index. As the MSG representation is formally equivalent to one in which agents compete over deterministic market shares, our theoretical results extend directly to such environments, highlighting a broader applicability of our inequality analysis beyond purely probabilistic contests.

Third, we provide an empirical assessment of PPE’s differential predictions in a laboratory setting. Our results offer strong support for PPE: while NE significantly misrepresents the distribution of success probabilities, PPE’s predictions exhibit near-perfect alignment with observed data. This finding underscores that relying on NE underestimates how competition amplifies disparities among contestants, while also overestimating contest efficiency by underpredicting average effort and rent dissipation. While PPE corrects this by matching the observed distribution of success, it tends to overpredict aggregate effort. This discrepancy reveals an intriguing tension: although PPE accurately captures success probabilities, or relative efforts, it overstates absolute efforts.

To address this deviation, we consider a behavioral refinement based on the idea that individuals may evaluate deterministic and variable components of payoffs differently. Specifically, we incorporate a form of risk aversion – variability-based risk aversion – where players apply diminishing marginal utility only to the uncertain reward, while treating the deterministic cost of effort at face value. This asymmetric evaluation, motivated by the theory of mental accounting, yields a modified version of PPE that preserves its key prediction about the success distribution while aligning average effort with observed behavior.

In sum, PPE provides a tractable, boundedly rational alternative to NE that bet-

ter captures the distributional outcomes observed in our experimental data. The IoP reflects a pervasive difficulty humans face in interpreting how their actions translate into chances or shares of success in competitive settings. Strategic environments often feature such mappings – how campaign effort translates into voter support, how price discounts affect market share, or how application efforts convert into hiring probability. In such cases, the IoP predicts that individuals systematically simplify the underlying relationship in a proportionalistic manner. By embedding this bias into strategic reasoning, PPE offers an equilibrium framework for analyzing how proportional misperception shapes inequality, efficiency, and strategic misallocation across a wide range of competitive settings.

The article is structured as follows. Section 2 reviews the relevant literature. Section 3 introduces the formal model, defines the Proportional Play Equilibrium, and presents the equilibrium analysis. Section 4 outlines the experimental design, and Section 5 details the empirical analysis. Finally, Section 6 concludes the study.

## 2 Related Literature

Theoretical contest models typically presume fully rational players who maximize expected payoffs under a given Contest Success Function. While the existing contest literature, with Nash Equilibrium (NE) as the canonical benchmark prediction, provides rich insights into rent-seeking behavior and aggregate behavior, it largely abstracts from how players cognitively process the relationship between effort and success. Moreover, both the traditional and the behavioral contest literature focus on aggregate effort and rent dissipation rather than on the distribution of success



probabilities. This paper complements the existing literature by studying a novel boundedly rational equilibrium concept – Proportional Play Equilibrium (PPE) – which formalizes how proportional reasoning, a well-documented psychological bias rooted in the Illusion of Proportionality (IoP), distorts the perceived effort-success relationship. To our knowledge, this is the first paper that examines the implications of the IoP for the distribution of success probabilities and average efforts in asymmetric contests, both theoretically and empirically.

Our equilibrium analysis shifts attention to the distribution of success probabilities because this is the key outcome metric that directly determines how rewards, influence, or market shares are allocated through competition. We thereby depart from much of the theoretical contest literature, which has been developed primarily in symmetric settings, where non-uniform success dispersions typically cannot arise.<sup>1</sup> While symmetry facilitates tractability, it abstracts from the heterogeneity that characterizes many real-world contests – in economics, politics, and sports – where contestants differ in abilities, costs or incentives. We therefore study a setting with heterogeneous players, enabling a richer account of how the IoP interacts with structural asymmetries to shape contest outcome and inequality.

In case of Tullock contests, the most extensively studied type of contest (Konrad, 2009), the empirical literature provides consistent evidence that NE systematically underpredicts average effort and expenditure levels in contests, as compactly reviewed by Dechenaux et al. (2015). Behavioral contest theory has proposed various explanations for overbidding, several of which introduce behavioral elements into contest theory by modifying agents’ preferences or objective functions. These in-

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<sup>1</sup>Symmetric contests fall into a category of games that generally lack asymmetric equilibria (Hefti, 2017).

clude joy of winning, risk aversion, loss or inequality aversion and relative payoff maximization; Bruner et al. (2022) contains a comprehensive overview. In contrast, our baseline approach maintains standard preferences but assumes a distorted perception between efforts and probabilities.

By formalizing this probabilistic distortion within an equilibrium framework, our paper aligns with the growing literature on boundedly rational equilibrium models, in which agents depart from full rationality in psychologically grounded ways. Examples include asymmetric probability weighting (Parco et al., 2005; Baharad and Nitzan, 2008) or unsystematic evaluation errors modeled through Quantal Response Equilibrium (QRE) (Sheremeta, 2011), possibly combined with level-k reasoning (Lim et al., 2014). These models preserve the canonical structure of the game and introduce bounded rationality through noise, iterative belief hierarchies, or subjectively weighted probabilities. PPE follows this tradition but departs from these approaches in a critical way: it embeds bounded rationality directly in the perception of the strategic environment by distorting the mapping from effort to success probabilities. Unlike QRE or probability weighting, which introduce stochastic noise or distort how probabilities enter expected payoffs, PPE models a misconception in the underlying relationship between agents' actions and success. This causes a distortion that directly alters strategic incentives, leading to distinct behavioral implications: PPE consistently predicts aggregate overbidding, while probability weighting can result in either over- or underbidding depending on the weighting function (Baharad and Nitzan, 2008).

Analyzing equilibrium success dispersion is challenging, as it requires comparing entire distributions rather than single point predictions such as average effort. As a

result, only a few studies, notably March and Sahm (2018), have explicitly examined the dispersion of success probabilities, focusing on how risk attitudes and effort costs shape outcomes in two-player settings. Moreover, the limited set of studies with heterogeneous agents, surveyed by March and Sahm (2017), primarily address the discouragement effect – how asymmetries reduce individual and aggregate effort – rather than the distribution of success.

We address the analytical challenges of comparing success distributions across equilibrium concepts by representing the contest as a Market Share Game (MSG; Hefti and Teichgräber, 2022) – a framework originally developed to study inequality in aggregative, non-strategic environments. This representation highlights that proportional reasoning corresponds to a failure to internalize the feedback effect of one’s own efforts on contest success – a behavior traditionally associated with large games and zero-mass agents (Corchon, 2021). The MSG approach uncovers a tight connection between the success distributions under PPE and NE, embedding both within a unified comparative-static structure, governed by a single behavioral parameter that captures the degree of feedback internalization. Moreover, it allows us to relate these distributions directly to Lorenz-consistent inequality measures, like the Gini index. These insights are particularly relevant given that the MSG we study is formally equivalent to a setting in which agents compete over deterministic market shares. By integrating the IoP into this framework, our paper offers a structured and generalizable method for analyzing how bounded rationality reshapes the equilibrium distribution of success and inequality in strategic environments.

### 3 Proportional Play and Contest Success

Contest theory provides a framework for analyzing the interactions among contestants competing for some objective, which each of  $n \in \mathbb{N}$  contestants can achieve with a certain probability of success. In its simplest form, this objective entails obtaining a fixed reward, the “prize”  $V > 0$ . The expected net payoff of contestant  $j \in \mathcal{J}_n \equiv \{1, \dots, n\}$  is  $\Pi_j = p_j V - C_j$ , where  $p_j$  is the probability of success, and  $C_j$  represents the costs incurred by  $j$  in pursuit of this probability. Contestants influence  $p_j$  by exerting effort  $e_j \geq 0$  according to the “Tullock” *Contest Success Function* (CSF)  $p_j = e_j / \Sigma$ , where  $\Sigma = \sum_i e_i$  is total effort. The simplicity of this CSF has made it a cornerstone in contest theory (Konrad, 2009), and its theoretical foundations have been extensively explored (e.g., Jia et al., 2013). With contestant-specific effort costs  $c_j C(e_j)$ ,  $j$ ’s payoff takes the form

$$\Pi_j = \frac{e_j}{\sum_i e_i} V - c_j C(e_j). \quad (1)$$

The function  $C(\cdot)$  is twice continuously differentiable and satisfies:<sup>2</sup> 1) inactivity entails zero costs,  $C(0) = C'(0) = 0$ , and 2) marginal costs are positive and increasing,  $C'(e), C''(e) > 0$  if  $e > 0$ . A key example are homogeneous effort costs, represented by  $C(e) = e^\eta$ ,  $\eta > 1$ . Homogeneous costs imply that proportional changes in effort result in proportionally consistent changes in marginal costs at any positive effort level. This commonality in responsiveness irrespective of contestant involvement makes power function costs a central benchmark.

A key property in (1) is the principle of replicability, whereby contestants can achieve equal success probabilities. The simple, symmetric form of the CSF is not

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<sup>2</sup>These standard requirements ensure existence of a unique equilibrium.

restrictive in this regard, as heterogeneous costs effectively capture differences in contestants’ abilities to influence success probabilities. As elaborated in Appendix A.2, this framework accommodates asymmetries in efficiency or biases favoring certain contestants.

### 3.1 Equilibrium under Proportional Reasoning

The CSF in (1) captures a complex, nonlinear relationship between success chances and own effort. We now analyze the equilibrium implications when contestants misperceive this relationship as proportional, consistent with the *Illusion of Proportionality* (IoP).

#### 3.1.1 Illusion of Proportionality (IoP)

In contests, players choose their actions to influence their probability of success, making how individuals process probabilistic information a central concern. A growing body of research shows that individuals frequently misjudge probabilities due to cognitive biases, with the IoP being a persistent and well-documented distortion in probabilistic reasoning (Van Dooren et al., 2003). The IoP, a term introduced by Freudenthal (1983), refers to a common misconception where individuals impose proportionality on inherently non-linear relationships. This bias has been widely documented in various tasks. For example, individuals often incorrectly generalize changes in linear dimensions to area and volume (De Bock et al., 1998; Esteley et al., 2010), and this misconception persists in routine-based tasks (Duma, 2021; Christodoulou, 2022). Importantly, Van Dooren et al. (2003) demonstrate that proportional reasoning is the concept that best explains common misconceptions in

probability, where this bias remains robust even after formal training in probability theory. This resilience suggests IoP is a firmly ingrained heuristic, rather than a simple error attributable to lack of knowledge or experience.

### 3.1.2 Proportional Play Equilibrium (PPE)

Under proportional reasoning, a contestant perceives her success probability as directly scaling with her effort, such that  $\tilde{p}_j(e_j) = e_j k_j$ , where  $k_j > 0$  is a constant. Accordingly, proportional reasoning introduces a single degree of freedom in the evaluation – the marginal sensitivity to efforts,  $k_j$ .

We now introduce our notion of a *Proportional Play Equilibrium* (PPE) by outlining how players make and validate their strategic decisions under proportional reasoning. First, players maximize their payoffs based on their perceived success probabilities,  $\tilde{p}_j(e_j) = e_j k_j$ .<sup>3</sup> Second, we impose the consistency condition that perceived success probabilities match true equilibrium success probabilities:  $\tilde{p}_j = p_j = P(e_j, \Sigma)$ . A key implication is that players hold correct beliefs about the average equilibrium effort,  $\bar{e} = \Sigma^*/n$ . This follows because the marginal sensitivity to efforts must equal the reciprocal of total effort,  $k_j = 1/(n\bar{e})$ ; see Appendix A.3.1. Moreover, this consistently ensures that players' conceptions could not be systematically invalidated by repeated observations of the game. The above two conditions define a PPE:

*A PPE is an effort profile  $\mathbf{e}^* = (e_1^*, \dots, e_n^*)$  with total effort  $\Sigma^* = \sum_i e_i^*$  such that each  $e_j^*$  maximizes  $\Pi_j = \tilde{p}_j(e_j)V - c_j C(e_j)$ , and the perceived success probability is*

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<sup>3</sup>Proportional thinking implies that the perceived success probability  $\tilde{p}_j(e_j)$  can, in principle, extrapolate beyond 1, aligning with evidence from Support Theory (Rottenstreich and Tversky, 1997), where human subjects sometimes assign probabilities exceeding one in certain contexts. Nevertheless, this remains purely extrapolative in our setting, as equilibrium success probabilities will invariably fall within  $[0, 1]$ .

consistent with actual contest success:  $\tilde{p}_j(e_j^*) = P(e_j^*, \Sigma^*)$  for each  $j = 1, \dots, n$ .

### 3.2 Equilibrium Analysis

To analyze the implications of proportional play on equilibrium success probabilities and contrast these with Nash Equilibrium (NE), we study a version of (1) that integrates both equilibrium notions. Consider the CSF  $p_j = \frac{e_j}{\Sigma_j + \alpha e_j}$ , where  $\Sigma_j > 0$  is the total efforts of all contestants except  $j$ . The payoff function is

$$\Pi_j(e_j; \Sigma_j) = \frac{e_j}{\Sigma_j + \alpha e_j} V - c_j C(e_j). \quad (2)$$

Parameter  $\alpha \in [0, 1]$  quantifies the extent to which contestants account for the feedback effect of their own effort relative to the total efforts of their rivals,  $\Sigma_j$ . Contestants fully respect this feedback effect when  $\alpha = 1$ , as in NE, and disregard it when  $\alpha = 0$ , as in PPE. For given  $\alpha$ , we define an equilibrium as follows:

**Definition 1** *An equilibrium is an effort profile  $\mathbf{e}^* = (e_1^*, \dots, e_n^*)$  such that each  $e_j^*$  maximizes (2) for given  $\Sigma_j^*$ , where  $\Sigma_j^* = n\bar{e}^* - \alpha e_j^*$  and  $\bar{e}^* = \sum_i e_i^*/n$ .*

In a conventional NE, each  $j$  maximizes (1), taking her rivals' total effort  $\Sigma_j$  as given. Recalling that  $k_j = 1/(n\bar{e})$  in a PPE, Definition 1 encapsulates PPE and conventional NE for  $\alpha = 0$  and  $\alpha = 1$ , respectively. Besides offering a unified equilibrium framework, this underscores that the key bias introduced by PPE, relative to NE, arises solely from proportional reasoning. Notably, in both cases, contestants consistently anticipate the average equilibrium effort  $\bar{e}^*$ .

#### 3.2.1 Market Share Game

A distribution of success probabilities is represented as a lottery  $\mathbf{p} = (p_1, \dots, p_n)$ , or equivalently as a density  $p : \mathcal{J}_n \rightarrow [0, 1]$ , where  $p(j) = p_j$ . While the effort-based

formulation in (2) accommodates PPE and NE, it is highly impractical for analyzing the equilibrium distribution of success probabilities. Inferring these probabilities indirectly from equilibrium efforts is difficult, as closed-form solutions do not exist without imposing specific numerical assumptions about players' costs. Given our focus on how success distributions may vary with the equilibrium concept, this calls for a more direct approach.

To address this challenge we reformulate the contest model within the broader framework of a *Market Share Game* (MSG; Hefti and Teichgräber, 2022). Noting that the CSF  $\frac{e_j}{\Sigma_j + \alpha e_j}$  is bijective in  $e_j$ , where  $e_j = \frac{p_j}{1 - \alpha p_j} \Sigma_j$ , we can restate (2) as a function of  $p_j$ :

$$\Pi_j(p_j; \Sigma_j) = p_j V - c_j C \left( \frac{p_j}{1 - \alpha p_j} \Sigma_j \right). \quad (3)$$

The main idea of an equilibrium in an MSG is that each  $j$  directly chooses an aspired success probability (or market share)  $p_j$  for given  $\Sigma_j$  to maximize (3) under the equilibrium requirement that these probabilities integrate to one:

**Definition 2** *An equilibrium in the MSG with parameter  $\alpha \in [0, 1]$  is a density  $\hat{p}_\alpha : \mathcal{J}_n \rightarrow [0, 1]$  and an average effort  $\bar{e}_\alpha$  such that each  $p_j \equiv \hat{p}_\alpha(j)$  maximizes (3) for given  $\Sigma_j \in \mathbb{R}_+$ , where  $\Sigma_j$  is determined as  $\Sigma_j = (1 - \alpha p_j) n \bar{e}_\alpha$ .*

The last condition in Definition 2 is the market share analogue of  $\Sigma_j = n\bar{e} - \alpha e_j$ . In Appendix A.3, we prove that the equilibrium set of the MSG is equivalent to that of the original effort-based model in Definition 1. This equivalence allows us to rely on the MSG equilibrium to compare success distributions under PPE and NE. The key advantage of the MSG representation is that it provides a tractable framework for analyzing the comparative statics of success probability distributions, avoiding the need for cumbersome examination of the full system of equations via Implicit



Function Theorem or similar approaches.<sup>4</sup> Moreover, as discussed in Appendix A.2, the comparative statics for success probabilities extend directly to strategic environments in which players compete for deterministic shares of a divisible prize – such as product market share or shares of seats in a legislative body.

### 3.2.2 PPE and NE: Comparative Analysis

The key equilibrium outcomes are the success density  $\hat{p}_\alpha$  and average equilibrium effort  $\bar{e}_\alpha = \Sigma(\alpha)/n$ . The boundary cases of  $\hat{p}_\alpha$  correspond to the equilibrium densities of PPE ( $\hat{p}_0$ ) and NE ( $\hat{p}_1$ ), respectively. Understanding how the function  $\hat{p}_\alpha$  evolves with  $\alpha \in [0, 1]$  not only clarifies how PPE and NE differ but also reveals how these differences unfold as contestants shift from fully rational ( $\alpha = 1$ ) toward proportional reasoning ( $\alpha = 0$ ). Under our assumptions on effort costs, payoff (3) is strictly concave in  $p_j \in [0, \frac{1}{\alpha})$  for any given  $\Sigma_j > 0$ . Incorporating the equilibrium condition  $\Sigma_j = (1 - \alpha p_j)\Sigma$ , the first-order optimality condition becomes

$$V(1 - \alpha p_j) = c_j C'(p_j \Sigma) \Sigma, \quad \Sigma \equiv n \bar{e}. \quad (4)$$

The formal challenge lies in the implicit nature of  $\Sigma$  and  $p_j$  within (4), which provides only local information about  $p_j$  rather than about the entire probability distribution. However, as equation (4) is uncoupled in contestants' success probabilities, it meets the requirements in Hefti and Teichgräber (2022), allowing the inequality effects caused by a parameter like  $\alpha$  to be extracted from a representative equation like (4). As shown in Appendix A.3, the MSG admits a unique equilibrium  $(\hat{p}_\alpha, \bar{e}_\alpha)$ , with  $\bar{e}_\alpha > 0$ ,  $\hat{p}_\alpha(j) > 0$  for all  $\alpha$ . The equilibrium effects of  $d\alpha$  implied by

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<sup>4</sup>Traditional methods from Monotone Comparative Statics are inapplicable, as best replies in the studied contest model are highly non-monotonic.

(4) can then be decomposed in a direct-aggregative and an indirect effect, where the former is informative about if and how the equilibrium dispersion  $\hat{p}_\alpha$  changes. Notably, the success distributions of NE and PPE can be continuously deformed into one another via  $\alpha$ ; they are homotopic as shown in Appendix A.3.2. We next emphasize the main insights and intuition of this comparative analysis.

Suppose, without loss of generality, that contestants are ordered according to  $c_1 \leq c_2 \leq \dots \leq c_n$ . Our first result establishes an intuitive yet important commonality between PPE and NE: heterogeneous costs are both necessary and sufficient for non-uniform equilibrium success distributions, and all equilibria satisfy a “No-Leap-Frogging” property.

**Proposition 1** *For any  $\alpha \in [0, 1]$ ,  $\hat{p}_\alpha(i) = \hat{p}_\alpha(j)$  iff  $c_i = c_j$ . Thus  $p^{PPE} = p^{NE}$  iff all contestants have identical cost functions. Conversely,  $p^{PPE}(j)$  and  $p^{NE}(j)$  are non-constant, decreasing densities iff at least two contestants have different cost functions.*

The “No-Leap-Frogging” property, stating that contestants with higher  $c_j$  achieve lower success probabilities, arises from individual rationality, independent from any bias introduced by proportional thinking: a low-cost contestant can always replicate the success probability of a higher-cost rival at a lower expense, but optimally seeks a higher success chance due to marginal reasoning.

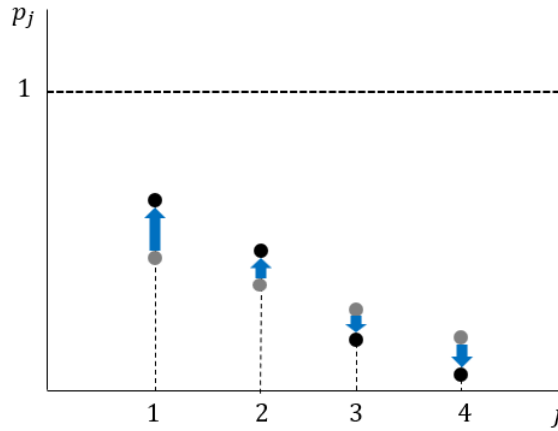
To gauge the impact of proportional reasoning on average effort, (3) shows that a lower  $\alpha$  reduces the perceived costs of achieving a given success probability  $p_j$ . This reflects that contestants increasingly neglect the diminishing sensitivity of the CSF to own efforts due to the feedback effect. Consequently, all contestants aim to secure higher success probabilities, leading to an aggregate increase in effort levels, and hence more socially inefficient rent dissipation, relative to NE. The following

result confirms this effect as an equilibrium outcome.

**Proposition 2** *Average effort  $\bar{e}_\alpha$  is strictly decreasing in  $\alpha$ . Thus  $\bar{e}^{PPE} > \bar{e}^{NE}$ .*

The fact that contestants collectively aim for higher success probabilities if they overlook the feedback effect suggests an outward shift of  $\hat{p}_\alpha$  as  $\alpha$  decreases. However, as  $\hat{p}_\alpha$  is a density, a mere outward shift is impossible. Instead, with heterogeneous contestants, this outward pressure resolves through a redistribution of success probabilities where some contestants gain success probability at the expenses of others. Our main result establishes that this transformation takes the form of a *monotonic clockwise rotation* as we move from NE to PPE, under the benchmark case of homogeneous effort costs  $c_j e^\eta$  with at least two cost types ( $c_1 < c_n$ ). This

Figure 1: Monotonic Rotations



*Notes.* The black distribution is a clockwise rotation of the gray distribution. The rotation is monotonic as indicated by the blue arrows, which increase in length towards the extremes.

means that  $p^{PPE}(i)/p^{PPE}(j) > p^{NE}(i)/p^{NE}(j)$  for any two  $i, j$  with  $c_i < c_j$ .<sup>5</sup> As illustrated in Figure 1, contestants with lower  $c_j$  achieve disproportionately higher success probabilities under PPE compared to NE. Furthermore, contestants at the

<sup>5</sup>If  $c_i = c_j$  then  $\hat{p}_\alpha(i) = \hat{p}_\alpha(j)$  for all  $\alpha \in [0, 1]$  by Proposition 1.

extremes of the advantage spectrum display the most pronounced change in success probabilities.

**Theorem 1**  *$p^{PPE}$  is a monotonic clockwise rotation of  $p^{NE}$ , entailing greater disparity than  $p^{NE}$  according to any Lorenz-consistent inequality measure.*

The key mechanism behind this result is that Nash players fully internalize the diminishing marginal return of effort on contest success due to the feedback effect, whereas proportional thinkers do not. As a consequence, Nash players who could afford higher success probabilities – owing to their favorable costs – face stronger disincentives to exert additional effort, mirroring the way how risk aversion reduces the marginal utility of wealth as wealth increases. We show in Appendix A.3, Corollary 3, that inequality in the success distribution  $\hat{p}_\alpha$  increases smoothly as  $\alpha$  decreases from 1 to 0 by any Lorenz-consistent inequality measure. More generally, this is assured if  $C(\cdot)$  exhibits a non-increasing elasticity of marginal costs, ensuring that the increasing sensitivity to effort, driven by a stronger recognition of the feedback effect (lower  $\alpha$ ), is not outweighed by increasingly sensitive marginal costs.

### 3.2.3 Contest Reward and the Dispersion of Success

Understanding how changes in key parameters shape strategic behavior and outcomes is a cornerstone of contest theory. Traditionally, the literature has focused on aggregate efforts and rent dissipation as key metrics (Konrad, 2009). We complement this perspective by examining how variations in the stakes of competition,  $V$ , influence the dispersion of success probabilities – a relevant concern in many real-world scenarios. For example, in labor markets, bonuses tied to effort-based criteria create contest-like settings. If higher rewards increase the skewness of success,

this can heighten perceptions of inequality and exacerbate social tensions among employees. Similarly, in advertising contests, where firms compete for consumer attention and  $p_j$  represents market shares, higher returns to attention – such as platforms offering access to lucrative markets – may disproportionately benefit the most advantaged firms, thereby amplifying market concentration. For the leading case of power function costs, we establish that the equilibrium success distribution is invariant to changes in contest rewards, irrespective of whether PPE or NE applies.

**Proposition 3**  *$\hat{p}_\alpha$  is invariant and  $\bar{e}_\alpha$  strictly increasing in  $V > 0$  for every  $\alpha \in [0, 1]$ .*

An increase in  $V$  sets an incentive for all contestants to aim for a higher success chance, increasing their respective efforts. The concomittant increase in total effort  $\Sigma$ , however, reduces the marginal chance of success for each individual contestant. This makes the reability of marginal effort costs  $C'(\cdot)$  the decisive factor for how increasing contest reward ultimately shapes market inequality. If marginal costs are equally sensitive to effort changes, as is the case under power function costs, the effects level off, leaving  $\hat{p}_\alpha$  unchanged. In Appendix A.3, Corollary 4, we show that if  $C'(\cdot)$  feature a decreasing (increasing) effort elasticity, then an increase in  $V$  increases (decreases) the inequality in the success distribution  $\hat{p}_\alpha$  for all  $\alpha \in [0, 1]$  and any Lorenz-consistent inequality measure. These findings highlight the importance of carefully considering the cost structure when using contest theory to evaluate how changes in rewards impact the dispersion of success. Beyond its direct implications for inequality, Proposition 3 plays a key role in our later discussion of effort levels in Section 5.2.

## 4 Experimental Design and Procedures

We test the comparative predictions of PPE and NE through an oTree-based experimental contest (Chen et al., 2016), implementing payoff function (1). Each subject chooses an effort, an integer  $e \in [0, 100]$ , to compete for a prize of  $V = 30,000$  points with a quadratic effort cost function  $c_j e^2$ . We implement asymmetric contests with different parameter constellations, using group sizes  $N = 2$  and  $N = 3$ , and assigning different cost coefficients  $c_j$  to subjects. For  $N = 2$ , cost coefficients were (1, 3), while for  $N = 3$ , two sets were used: (1, 3, 6) and (1, 2, 3). These conditions were implemented in three treatments, summarized in the upper part of Table 1.

Table 1: Contest conditions and equilibrium predictions

	Group size (Treatment)		
	N=2	N=3	N=3
Cost coefficient	(1, 3)	(1, 3, 6)	(1, 2, 3)
No. of groups /round	15	10	5
No. of round	20	20	18
	Equilibrium predictions		
<i>Winning probability</i>			
NE	(0.63, 0.37)	(0.55, 0.29, 0.17)	(0.47, 0.31, 0.23)
PPE	(0.75, 0.25)	(0.67, 0.22, 0.11)	(0.55, 0.27, 0.18)
<i>Average Effort</i>			
NE	93.1	111.6	130.6
PPE	141	150	165.8
<i>Individual effort</i>			
NE	(59, 34.06)	(60.97, 31.97, 18.66)	(61.11, 39.89, 29.61)
PPE	(106.06, 35.56)	(100, 33.3, 16.67)	(90.45, 45.23, 30.15)

*Notes.* Parameters of cost coefficient, equilibrium predictions of winning probability, and individual effort in a group are shown in tuples in parentheses.

Each session consisted of about 20 rounds, with subjects randomly re-matched into new groups each round.<sup>6</sup> Subjects only participated in one treatment. Subjects

<sup>6</sup>Our design thus yields between 90 and 300 independent group-level observations, depending on treatment. Since the goal is not to make population-level inferences but to test whether observed success distributions align with the theoretical benchmarks implied by NE and PPE, this sample size provides sufficient statistical precision to detect meaningful deviations from these

were informed about their group size, the own cost coefficient and the distribution of coefficients within their group before choosing their effort levels. Effort choices determined success probabilities through the CSF in (1). After each round, participants received feedback on their group’s total effort, their success probability, the round outcome, expenditures, and their payoff. All subjects were endowed with 60,000 experimental points at the beginning of each round to prevent losses.

At the beginning of each session, the instructions were distributed to subjects and read aloud by an experimenter. Before the actual experiment started, subjects had to successfully complete a set of online comprehension questions. The experiment was conducted at the Center for Behavior and Economic Research in Wuhan University. Final earnings consisted of a randomly selected round-payoff, converted to Chinese Yuan at a rate of 1,800 points per CNY, plus a 15 CNY show-up fee. A total of 75 subjects participated across three sessions, with an average earning of 65 CNY. Each session lasted approximately one hour.

## 5 Empirical Analysis of Success and Effort

This section evaluates the extent to which the experimental outcomes align with the theoretical predictions of PPE and NE. Specifically, we examine the distribution of success probabilities and average effort levels, focusing on the ability of each equilibrium concept to capture the key features of the data.

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predictions (see Casella and Berger, 2002, Ch. 9, for tests of distributional alignment).

## 5.1 Empirical Success Probabilities and Theoretical Fit

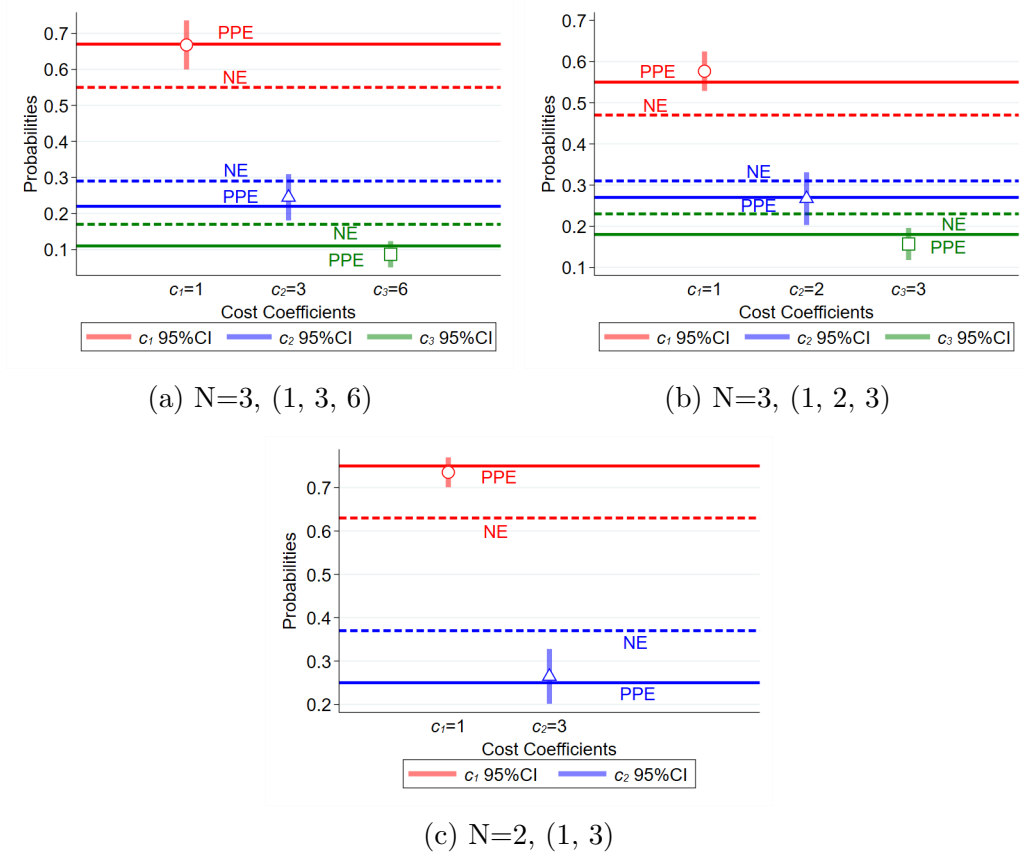
This section investigates which equilibrium notion – PPE or NE – better describes the observed distribution of success probabilities in each experimental treatment. Table 1 operationalizes the key prediction of Theorem 1 – that PPE yields a more extreme dispersion of success probabilities than NE – by deriving the precise equilibrium benchmarks for the specific parameter constellation of our experiment.

Figure 2 displays the average success probabilities for each cost type across all three treatments, pooled over rounds. Vertical bars indicate 95% confidence intervals, with standard errors clustered at the individual level to account for within-subject correlation across repeated rounds. The predictions from NE (dashed lines) and PPE (solid lines) from Table 1 are included for comparison. Success probabilities are inversely ordered to cost coefficients, validating both models’ rank-order predictions, as entailed in Proposition 1. However, the striking result lies in PPE’s exceptional fit with the data. Across all treatments, empirical success probabilities align almost perfectly with PPE predictions, which becomes particularly evident for subjects with the lowest and highest costs. PPE predictions consistently fall within the 95% confidence intervals, while NE underestimates success for the lowest-cost subjects and overestimates it for the highest-cost subjects. Figure 3 further underscores the robustness of PPE’s predictive accuracy, as average success probabilities for low- and high-cost subjects consistently match PPE predictions closely over all rounds. This resonates with prior research on the IoP, identifying proportional reasoning as a *persistent* bias that endures despite feedback and repeated experience.

To statistically evaluate the predictive accuracy of PPE and NE, we regress in-



Figure 2: Average Success Probabilities, by Contest Treatment



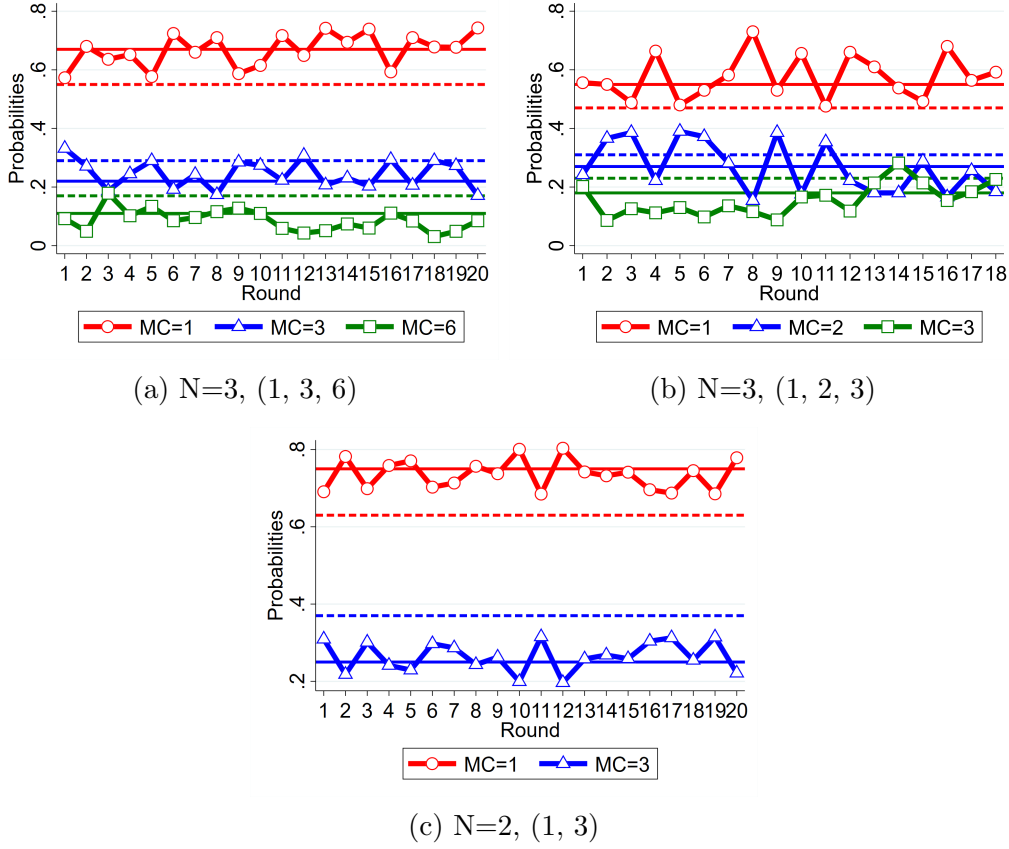
*Note.* Each figure shows mean winning probabilities by cost type, with 95% CI's. Solid horizontal lines represent PPE predictions, and dashed lines represent NE predictions.

dividual success probabilities on cost dummies, clustering standard errors at the individual level. Table 2 summarizes the results. The upper panel confirms that the estimated success probabilities follow the predicted rank-order. The lower panel reports  $F$ -tests for the null hypothesis that estimated and predicted success probabilities are equal. The results fully support PPE, with no significant differences between the empirical probabilities and its predictions, while the null is decisively rejected for NE predictions for both the lowest- and highest-cost types.<sup>7</sup>

**Result 1** *PPE provides an exceptionally precise fit to the observed success probabilities across all treatments. In contrast, NE systematically underestimates the success of*

<sup>7</sup>These results are robust to including a round variable or round fixed effects.

Figure 3: Average Success Probabilities Over Rounds



*Notes.* Each figure shows the average winning probabilities per round by cost type, with solid horizontal lines for PPE predictions and dashed lines for NE predictions.

*low-cost contestants and overestimates the success of high-cost contestants, leading to significant mispredictions.*

## 5.2 Average Efforts and Variability-Based Risk Aversion

This section examines the relationship between observed average effort levels and theoretical equilibrium predictions. Figure 4 shows the average effort per round for the three treatment conditions. Efforts frequently exceed NE predictions, consistent with prior research, but also fall short of PPE predictions, as PPE overestimates average effort. Thus, neither equilibrium notion seems to capture average effort levels accurately: corresponding  $F$ -tests reject equality between the observed efforts

Table 2: Success Probability Estimations

	N=3, (1, 3, 6)	N=3, (1, 2, 3)	N=2, (1, 3)
Cost coeff=1	0.67 (0.033)	0.58 (0.024)	0.74 (0.017)
Cost coeff=2		0.27 (0.031)	
Cost coeff=3	0.24 (0.031)	0.157 (0.019)	0.26 (0.031)
Cost coeff=6	0.09 (0.018)		
R <sup>2</sup>	0.798	0.866	0.826
Observ.	600	270	600
<i>Winning probability = NE?</i>			
Cost coeff=1	$F=12.46^{***}$	$F=20.52^{***}$	$F=39.05^{***}$
Cost coeff=2		$F=1.89$	
Cost coeff=3	$F=2.05$	$F=14.67^{***}$	$F=11.58^{***}$
Cost coeff=6	$F=21.63^{***}$		
<i>Winning probability = PPE?</i>			
Cost coeff=1	$F=0.00$	$F=1.27$	$F=0.75$
Cost coeff=2		$F=0.01$	
Cost coeff=3	$F=0.63$	$F=1.47$	$F=0.22$
Cost coeff=6	$F=1.67$		

*Notes.* OLS regressions of individual success probabilities on cost coefficients for the three contest treatments. Each cell reports the mean success probability, with standard errors clustered at the individual level. The lower panel reports  $F$ -tests comparing the estimated coefficients to NE and PPE predictions.

with NE and PPE predictions in all but one case, where NE cannot be rejected for N=3, (1, 3, 6).

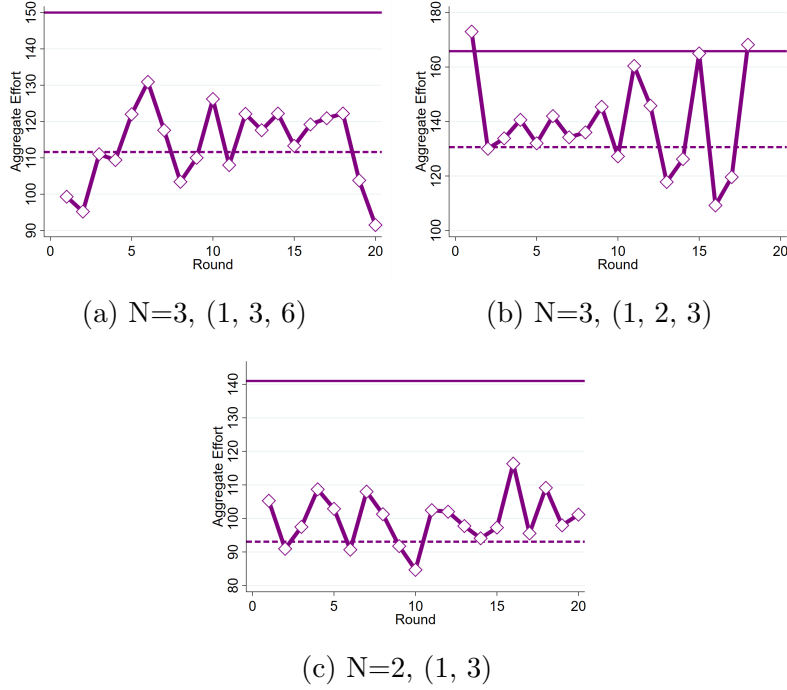
**Result 2** *PPE overpredicts average effort, while NE mostly underpredicts it. Observed effort levels lie between these two benchmarks.*

This result raises a key question: how can PPE provide an exceptional fit for success probabilities while simultaneously overpredicting effort? This suggests that another behavioral factor may influence the absolute level of efforts.

A natural candidate to account for is risk aversion, as standard models of risk aversion typically predict lower average efforts (Cornes and Hartley, 2009). The challenge, however, is that conventionally incorporating risk aversion into payoff function (2) would distort the predicted success probabilities, undermining PPE's central empirical strength in capturing contest success.<sup>8</sup> One way to address this

<sup>8</sup>Under standard risk aversion, payoffs are evaluated with diminishing marginal utility over stochastic income, depending jointly on the prize and effort expenditures. This transforms the structure of the equilibrium condition and distorts the predicted success probabilities.

Figure 4: Average Efforts



*Notes.* Average effort per round and treatment. Horizontal solid (dashed) lines indicate PPE (NE) predictions.

tension is to consider a more selective form of risk sensitivity, in which players evaluate stochastic and deterministic components of the payoff differently. Drawing on the concept of mental accounting (Thaler, 1985), we propose that agents compartmentalize risky and non-risky prospects, applying diminishing marginal utility only to the uncertain reward – the prize  $V$  – while evaluating the deterministic cost of effort at face value. We refer to this behavioral pattern as *variability-based risk aversion*, highlighting that risk sensitivity applies solely to the variable (i.e., probabilistic) component of the payoff.

Specifically, let contestants evaluate the possible revenues –  $V$  or 0 – with CRRA utility  $u(x) = x^\gamma$ , where  $\gamma \in (0, 1]$  quantifies the degree of risk aversion. The MSG version of this payoff function is

$$\Pi_j(p_j; \Sigma_j) = p_j V^\gamma - c_j C \left( \frac{p_j}{1 - \alpha p_j} \Sigma_j \right). \quad (5)$$

Variability-based risk aversion lowers the perceived value of winning the prize relative to the deterministic investment made.<sup>9</sup> But this leads to a comparative-static prediction as captured by Proposition 3:

**Corollary 1** *For every  $\alpha \in [0, 1]$ , the equilibrium success distribution associated with (5) is invariant to  $\gamma \in (0, 1]$ , while average effort increases strictly in  $\gamma$ .*

Corollary 1 shows that variability-based risk aversion preserves the success probability predictions of any equilibrium with parameter  $\alpha \in [0, 1]$  and adjusts only the average effort level. This property allows us to reconcile PPE’s strong predictive accuracy for success probabilities with its systematic overprediction of absolute effort levels. Although Corollary 1 also applies to NE, it offers limited value in that context: NE already underpredicts average effort and provides a poor fit for the observed distribution of success probabilities.

Table 3: Calibrated Risk Aversion

Cost coefficients	Observed average efforts	Risk Aversion ( $1 - \gamma$ )
(1,3)	99.76	0.07
(1,3,6)	113.29	0.05
(1,2,3)	139.24	0.03

*Notes.* The table presents the degrees of risk aversion required to match average efforts in each treatment.

The CRRA formulation in (5) offers a parsimonious representation of risk attitudes that we can use to estimate the coefficient of relative risk aversion,  $1 - \gamma$  required to align our PPE predictions with the actually observed average effort in the three treatments. This back-of-the-envelope calculation yields consistently low degrees of risk aversion across all three contest conditions, as summarized in Table 3. These values align with findings from the broader literature on risk attitudes in low-stake choices, which consistently document low levels of risk aversion and

<sup>9</sup>This holds for any strictly concave utility with  $u(0) = 0$  and  $u(x) < x$  for some  $x < V$ .

near risk-neutral behavior in such settings (Fehr-Duda et al., 2010; Bombardini and Trebbi, 2012; Rabin, 2013).

**Result 3** *Variability-based risk aversion provides an explanation for the observed effort-success pattern. An adjusted version of PPE incorporating this refinement reconciles its overprediction of effort while preserving its accurate fit for success probabilities. Calibrated estimates indicate a low but nonzero degree of risk aversion, consistent with prior findings in low-stakes decision-making.*

## 6 Conclusion

This paper develops and tests Proportional Play Equilibrium (PPE), a boundedly rational solution concept grounded in the Illusion of Proportionality (IoP), in an otherwise standard contest framework. According to this bias, individuals misconceive the non-linear relationship between effort and success as proportional, distorting strategic behavior in a systematic way.

Our paper links the distortions caused by the IoP to the broader question of how strategic choices mediate or exacerbate pre-existing asymmetries among contestants. Using the representation as a Market Share Game, we establish that the IoP not only elevates average efforts but also causes an inherently more unequal dispersion of success probabilities than predicted by the fully rational benchmark of Nash Equilibrium (NE). This disparity originates from the extrapolative tendencies captured by the IoP, which are disproportionately prevalent for players capable of targeting high success chances.

Our results underscore that relying exclusively on NE may underestimate the extent to which competition exacerbates disparities, leading to overly optimistic

assessments about the balance and fairness of competitive outcomes. This misestimation is problematic in domains such as promotion contests, political campaigns, or advertising markets, where outcome equality and concentration are key indicators of market performance. In such settings, a highly unequal success distribution can have downstream effects on market structure, access to future opportunities and perceived overall system legitimacy.

Our experimental data strongly supports PPE’s predictions on success dispersion, rejecting those of NE. While PPE tends to over-predict average effort, integrating the idea that individuals account for risk only in payments they perceive as uncertain – variability-based risk aversion – can reconcile predicted efforts with observed behavior, while preserving PPE’s accuracy in determining success probabilities.

The IoP is not confined to the specific contest analyzed in this paper, but reflects a broader failure of agents to internalize non-linearities in how their actions translate into chances or shares of success. This cognitive distortion may affect behavior across a wide range of environments where rewards are distributional and strategically mediated, including procurement, lobbying, customer acquisition, and performance-based incentive schemes. As illustration, consider digital marketplaces, where firms invest in visibility – through advertising, tagging, timing, or content production – with the goal of increasing engagement and market share. Algorithmic constraints, attention scarcity, and competitive congestion imply diminishing returns to own visibility investments, which agents subject to the IoP tend to ignore. This can lead to systematic overinvestment and highly skewed success distributions, resulting in greater concentration of attention and market power among a few players than would arise under the rational benchmark. Even in well-studied

environments such as first-price auctions, the IoP may offer new insights about bidding behavior and auction outcome. Under the IoP, participants would mistakenly believe that doubling their bid doubles their chance of winning. This extrapolative tendency may provide a psychologically grounded explanation for systematic overbidding relative to Nash predictions – a robust empirical regularity in auction settings (Kagel and Levin, 2016). If such misperceptions persist over time, repeated auctions may yield increasingly skewed success distributions, entrenching disparities in allocation and undermining competitive balance.

Bringing these insights together, PPE offers a tractable framework for analyzing how boundedly rational agents generate and reinforce disparities in success, with potential implications for understanding inequality and efficiency in competitive environments. This behavioral bias suggests that equilibrium predictions under full rationality may understate the inequality generated by competition – not because of institutional failures, but because of how agents systematically misperceive strategic environments. PPE may thus serve as a stepping-stone toward reevaluating the normative properties of competitive settings, including the extent to which observed disparities reflect merit, structural asymmetries, or bounded rationality.



# A Appendix

Appendix A.1 provides the formal proofs for the main theoretical results. Appendix A.2 discusses how the model accommodates heterogeneity and alternative interpretations of the Contest Success Function. Appendix A.3 provides formal equilibrium properties and technical results not included in the main text for brevity.

## A.1 Proofs for Main Results

**Proof Proposition 1** The claim is a direct consequence of the broader analysis provided in Hefti and Teichgräber (2022), because the equilibrium equation (4) is consistent with the general equilibrium equation of their Proposition A.1, assuring that the equilibrium sorting of success chances (or market shares) must align with the ex-ante sorting of the agents. That result directly implies that  $\hat{p}_\alpha(i) = \hat{p}_\alpha(j)$  iff  $c_i = c_j$ , which, by extension, implies that  $\hat{p}_\alpha(i) = 1/n \ \forall i \in \mathcal{J}_n$  iff  $c_1 = \dots = c_n$ . Conversely, if  $c_i < c_j$  for two agents  $i < j$ , then Proposition A.1 in Hefti and Teichgräber (2022) further assures that  $\hat{p}_\alpha(i) > \hat{p}_\alpha(j)$  across the entire range  $\alpha \in [0, 1]$ . As contestants are ex-ante sorted by increasing costs, the previous result necessitates that the equilibrium density  $\hat{p}_\alpha(j)$  is decreasing in  $j$ , and non-constant whenever  $c_1 < c_n$ . ■

**Proof Proposition 2** The proof draws from the following Lemma:

**Lemma 1** *The equilibrium aggregate  $\Sigma(\alpha)$  is a  $C^1$ -function of  $\alpha \in [0, 1]$  and  $\Sigma'(\alpha) < 0$ .*

Proof: We follow the general procedure outlined in Hefti and Teichgräber (2022) for how to derive the comparative-statics of the aggregate quantity in a Market Share Game. For any  $j \in \mathcal{J}$  the optimality condition is (4). Treating  $\Sigma > 0$  and  $\alpha \in [0, 1]$  as parameters, this equation has a unique solution  $p_j(\Sigma, \alpha) > 0$ , and the Implicit Function Theorem assures that this solution is continuously differentiable in  $\alpha$  and  $\Sigma$ , where  $\partial_\Sigma p_j(\Sigma, \alpha) < 0$  and  $\partial_\alpha p_j(\Sigma, \alpha) < 0$  again by the Implicit Function Theorem. Define  $G(\Sigma, \alpha) = \sum_i p_i(\Sigma, \alpha)$ , and note that the equilibrium  $\Sigma(\alpha)$  is determined by  $G(\Sigma, \alpha) = 1$  for any given  $\alpha \in [0, 1]$ . Applying the Implicit Function Theorem to this equation yields  $\Sigma'(\alpha) < 0$ . □

Now, Proposition 2 follows from Lemma 1 as  $\bar{e}(\alpha) = \Sigma(\alpha)/n$  and  $\Sigma(0) = \Sigma^{PPE}$  and  $\Sigma(1) = \Sigma^{NE}$ . ■

**Proof Theorem 1** Let  $i, j$  be such that  $c_i < c_j$ . Then, by Proposition 1,  $\hat{p}_\alpha(i) > \hat{p}_\alpha(j)$  for any  $\alpha \in [0, 1]$ . Note that to establish that  $p^{PPE}$  is a monotonic clockwise rotation of  $p^{NE}$ , it suffices to show that  $\frac{\hat{p}_0(i)}{\hat{p}_0(j)} > \frac{\hat{p}_1(i)}{\hat{p}_1(j)}$ .<sup>10</sup> Now, for  $C(e) = e^\eta$ , (4) implies that

$$\frac{\hat{p}_0(i)}{\hat{p}_1(i)} = \left( \frac{\Sigma(1)}{\Sigma(0)} \right)^{\frac{\eta}{\eta-1}} \left( \frac{1}{1 - \hat{p}_1(i)} \right)^{\frac{1}{\eta-1}}. \quad (6)$$

Because  $\hat{p}_1(i) > \hat{p}_1(j)$ , (6) implies that  $\frac{\hat{p}_0(i)}{\hat{p}_1(i)} > \frac{\hat{p}_0(j)}{\hat{p}_1(j)}$ , which yields the requested condition. For the remaining assertion it is straightforward to verify that the Lorenz curve associated with  $p^{NE}$  must Lorenz-dominate the one implied by  $p^{PPE}$ , which completes the proof. ■

**Proof of Proposition 3** Proposition 5 in Hefti and Teichgräber (2022) establishes that a change in a level variable, such as  $V$ , that affects marginal equilibrium revenues associated with (3) in a proportionally identical manner for all agents has no impact on the equilibrium distribution if marginal costs follow a power function. As these conditions are fulfilled in the current setting,  $\hat{p}_\alpha$  must remain unchanged. ■

## A.2 Replicability, Heterogeneity and Market Shares

This section discusses how the CSF in (1) extends to settings with heterogeneity and varying degrees of replicability. While the main analysis assumes a common functional form, many real-world contests involve asymmetric influences on success probabilities.

The principle of replicability in the CSF of (1) ensures equal success probabilities under equal efforts. However, real-world contests rarely exhibit such a strong symmetry. E.g., in political competition, voters may favor one candidate over another, leading to unbalanced success probabilities even with equal campaign efforts. Nevertheless, replicability remains a key benchmark, requiring that balanced success

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<sup>10</sup>See Definition 4 of Hefti and Teichgräber (2022).

probabilities could arise if contestants manage to replicate each other's overall influences. By allowing for heterogeneous costs, our model accommodates situations with asymmetric influences on success probabilities. To make this evident, consider the distinction between investment expenditures  $f_j$  and impact  $e_j$ , where impact represents the influence of investments on success probabilities, as determined by the Tullock CSF  $e_j / \sum_i e_i$ . Impacts are related to investments according to an increasing function  $e_j = h_j(f_j)$ , where the functions  $h_i$  encapsulate contestants' heterogeneity in converting investments into success.<sup>11</sup> Substituting the investment  $C_j(e_j) \equiv h_j^{-1}(e_j)$  of achieving impact  $e_j$  into the CSF recovers (1) under the functional form  $h_j(f) = h(f/c_j)$ .<sup>12</sup>

This framework's flexibility allows to capture a wide range of competitive scenarios. The traditional interpretation of (1) involves  $n$  risk-neutral contestants in a winner-takes-all competition. E.g.,  $V$  represents the value of winning office in political campaigns, or the market advantage of securing a patent in innovation races. The model also applies to deterministic revenue-sharing settings, where  $p_j$  represents a *market share* rather than probabilistic success.<sup>13</sup> Examples are digital advertising for attention (Hefti and Lareida, 2022) or electoral contests, where  $V$  could represent total legislative seats, with  $p_j$  as the share won by party  $j$  and  $C_j(e_j)$  reflecting campaign costs.

### A.3 Equilibrium Properties and Comparative Statics

In this section, we provide formal results that complement the intuitive discussions in the main text. Their proofs are collected in Appendix A.4.

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<sup>11</sup>Whether replication is among the *feasible* options depends on the  $h_j$ -function. For instance, one could assume that  $h_j(f_j)$  is bounded from above, such that a certain impact is not achievable, even at infinite investment. Similarly, one could include a budget constraint  $f_j \in [0, \bar{f}_j]$ .

<sup>12</sup>Power function costs arise for  $h_j(f) = \theta_j f^\mu$ , first introduced by Tullock et al. (1980) in a completely symmetric setting. It can be shown that the ratio  $h_j(f_j) / \sum_i h_i(f_i)$  is zero homogeneous in investments  $(f_1, \dots, f_n)$  iff  $h_j$  has the power function property. Thus, the power function form of  $h_j$  implies that the CSF is zero homogeneous also in investments, not only in impacts.

<sup>13</sup>Surprisingly many competitive models, including Walrasian or monopolistic competition, can be subsumed within this generalized contest framework (Hefti and Teichgräber, 2022).

### A.3.1 Consistency Condition and Effort Sensitivity

The following result formalizes the relationship between the consistency condition  $\tilde{p}_j = p_j$  and the marginal sensitivity to own efforts as discussed in Section 3.1.

**Lemma 2** *Let  $\mathbf{e} > 0$  and  $\Sigma = \sum e_i$ . Then  $\tilde{p}_j(e_j) = P(e_j, \Sigma)$  iff  $k_j = 1/\Sigma$  for  $j = 1, \dots, n$ .*

### A.3.2 Equilibrium Structure: Existence, Uniqueness, Representation

We now establish the formal equilibrium properties of the game, including existence, uniqueness, and equivalence between the Original Game and the Market Share Game (MSG). The first result establishes that, for any given  $\alpha \in [0, 1]$ , the equilibrium sets of the Original Game and the MSG coincide.

**Theorem 2** *If  $(\mathbf{e}, \bar{e})$  is an equilibrium in the Original Game, then the success distribution  $\hat{p}$  determined by  $p_j = \frac{e_j}{\Sigma_j + \alpha e_j}$  for each  $j \in \mathcal{J}_n$ , along with  $\bar{e}$ , constitutes an equilibrium in the MSG. Conversely, if  $(\hat{p}, \bar{e})$  is an equilibrium in the MSG, then the effort profile  $\mathbf{e}$  derived from  $e_j = \frac{p_j}{1 - \alpha p_j} \Sigma_j$ ,  $p_j = \hat{p}(j)$ , for each  $j \in \mathcal{J}_n$ , alongside  $\bar{e}$ , is an equilibrium in the Original Game.*

We next show that for every given  $\alpha \in [0, 1]$  the MSG with payoff function (3) has a unique and strictly positive equilibrium  $(\hat{p}_\alpha, \bar{e}_\alpha)$  under the assumptions imposed on the cost function  $C(\cdot)$ . By Theorem 2, the success distribution  $\hat{p}_\alpha$  must then correspond to the one implied by the unique equilibrium of the Original Game.

**Theorem 3** *For any  $\alpha \in [0, 1]$ , the MSG has a unique equilibrium  $(\hat{p}_\alpha, \bar{e}_\alpha)$ , where  $\hat{p}_\alpha(j), \bar{e}_\alpha > 0$  for each  $j \in \mathcal{J}_n$ .*

By Theorem 3, the MSG with payoff (3) has a unique equilibrium  $(\hat{p}_\alpha, \bar{e}_\alpha)$  for every  $\alpha \in [0, 1]$ . Define the functions  $\mathbf{p} : [0, 1] \rightarrow \Delta^{n-1}$ ,  $\pi_j(\mathbf{p}(\alpha)) \equiv \hat{p}_\alpha(j)$ ,  $\bar{e} : [0, 1] \rightarrow \mathbb{R}_+$ ,  $\bar{e}(\alpha) \equiv \bar{e}_\alpha$ , where  $\Delta^{n-1}$  is the  $(n - 1)$ -dimensional simplex. These mappings are well-defined by Theorem 3, and the following result shows their smooth dependence on  $\alpha$ .

**Corollary 2**  *$\mathbf{p}(\alpha)$  and  $\bar{e}(\alpha)$  are  $C^1$ -functions of  $\alpha$ .*

The equilibrium density for each  $\alpha$  is represented by the bivariate function  $p : \mathcal{J}_n \times [0, 1] \rightarrow [0, 1]$ , defined as  $p(j, \alpha) \equiv \pi_j(\mathbf{p}(\alpha))$ . Because  $\mathbf{p}(\alpha)$  is continuous in  $\alpha$ ,  $p(j, \alpha)$

is continuous.<sup>14</sup> As the endpoints satisfy  $p(j, 0) = p^{PPE}(j)$  and  $p(j, 1) = p^{NE}(j)$ , the functions  $p^{PPE}(j)$  and  $p^{NE}(j)$  are homotopic.

### A.3.3 Comparative Statics: How $\alpha$ and $V$ Affect Success Inequality

Our final two results establish the comparative statics of inequality in the success distribution, as discussed in Sections 3.2.2 and 3.2.3. In both cases, the elasticity of marginal costs with respect to effort plays a crucial role, as it determines how effectively contestants with larger or smaller market shares can adjust their equilibrium behavior in response to exogenous changes.

**Corollary 3** *If  $C(e)$  has a non-increasing elasticity of marginal costs, then inequality in  $\hat{p}_\alpha$  increases as  $\alpha$  decreases from 1 to 0 by any Lorenz-consistent inequality measure.*

**Corollary 4** *If the cost function  $C(e)$  has decreasing (increasing) elasticity of marginal costs, an increase in  $V$  increases (decreases) inequality in  $\hat{p}_\alpha$  according to any Lorenz-consistent inequality measure  $\forall \alpha \in [0, 1]$ .*

## A.4 Proofs for Section A.3

**Proof Lemma 2** If  $k_j = 1/\Sigma$ , then  $\tilde{p}_j(e_j) = e_j/\Sigma = P(e_j, \Sigma)$ . Conversely, if  $\tilde{p}_j(e_j) = P(e_j, \Sigma)$  then  $e_j k_j = e_j/\Sigma^*$  and hence  $k_j = 1/\Sigma$  for each  $j$ . ■

**Proof Theorem 2** Fix  $\alpha \in [0, 1]$ . For given  $\Sigma_j > 0$ , define the function  $h(e_j; \Sigma_j) = \frac{e_j}{\Sigma_j + \alpha e_j}$ . Note that  $h(\cdot; \Sigma_j) : \mathbb{R}_+ \rightarrow [0, 1/\alpha)$ ,  $\frac{1}{\alpha} \geq 1$  is bijective and  $p_j = h(e_j; \Sigma_j)$ , with inverse  $h^{-1}(p_j; \Sigma_j) = \frac{p_j}{1 - \alpha p_j} \Sigma_j = e_j$ . That is, to every given  $p_j \in [0, \frac{1}{\alpha})$  we can assign a unique  $e_j \in [0, \infty)$  and vice-versa. Define the function  $\hat{\Pi}(p_j; \Sigma_j) \equiv \Pi(h^{-1}(p_j; \Sigma_j); \Sigma_j) = \Pi(e_j(p_j); \Sigma_j)$ . Note that  $\hat{\Pi}(p_j; \Sigma_j)$  amounts to the payoff function (3) in the Market Share Game.

Let  $(\mathbf{e}^*, \bar{e}^*)$  be equilibrium in the Original Game. Let  $\Sigma^* = n\bar{e}^*$  and note that  $\Sigma_j^* = \Sigma^* - \alpha e_j^*$ . The implied success probabilities are  $p_j^* = \frac{e_j^*}{\Sigma_j^* + \alpha e_j^*}$ . By contradiction, suppose that  $(p_1^*, \dots, p_n^*)$  formed in this way and  $\bar{e}^*$  do not form an equilibrium of

<sup>14</sup>Continuity of  $p(j, \alpha)$  follows from continuity of  $p(j, \cdot)$  because  $\mathcal{J}_n$  is a discrete set. Here,  $\mathcal{J}_n$  is a discrete metric space,  $[0, 1]$  is equipped with the natural metric and  $X \times [0, 1]$  is equipped with the metric defined by the sum of the two metrics.

the Market Share Game. Then  $\hat{\Pi}_j(p_j; \Sigma_j^*) > \Pi_j(p_j^*; \Sigma_j^*)$  for some  $p_j \in [0, \frac{1}{\alpha})$ . But as  $e_j = h^{-1}(p_j; \Sigma_j^*)$  we have

$$\hat{\Pi}_j(p_j; \Sigma_j^*) = \Pi_j(e_j(p_j); \Sigma_j^*) > \hat{\Pi}(p_j^*; \Sigma_j^*) = \Pi_j(e_j^*; \Sigma_j^*)$$

as  $e_j^* = e_j(p_j^*)$ . That is, there is  $e_j$  such that  $\Pi_j(e_j; \Sigma_j^*) > \Pi_j(e_j^*; \Sigma_j^*)$  which contradicts that  $(\mathbf{e}^*, \bar{e}^*)$  is an equilibrium in the Original Game.

For the converse, let  $(\hat{p}, \bar{e})$  be equilibrium in the Market Share Game, with  $p_j^* = \hat{p}(j)$ ,  $\Sigma^* = n\bar{e}^*$  and  $\Sigma_j^* = (1 - \alpha p_j)\Sigma^*$ . For every  $p_j^*$  there is a unique impact  $e_j^* = e_j(p_j^*)$ . Suppose that the corresponding impact profile  $(e_1^*, \dots, e_n^*)$  together with average effort  $\bar{e}$  and  $\Sigma_j^*$  as determined above do not constitute an equilibrium in the Original Game. Then  $\Pi(e_j; \Sigma_j^*) > \Pi(e_j^*; \Sigma_j^*)$  for at least one contestant  $j$  and some effort  $e_j \neq e_j^*$ . Hence also

$$\Pi(e_j; \Sigma_j^*) = \hat{\Pi}(p_j(e_j); \Sigma_j^*) > \Pi(e_j^*; \Sigma_j^*) = \hat{\Pi}(p_j^*; \Sigma_j^*)$$

as  $p_j^* = e_j(p_j^*)$ . Thus there is  $p_j \neq p_j^*$  such that  $\hat{\Pi}(p_j; \Sigma_j^*) > \hat{\Pi}(p_j^*; \Sigma_j^*)$ , contradicting that  $(\hat{p}, \bar{e})$  is an equilibrium in the Market Share Game.  $\blacksquare$

**Proof Theorem 3** Fix an arbitrary  $\alpha \in [0, 1]$ . Note that for any given  $\Sigma_j \geq 0$ ,  $p_j = 0$  cannot maximize (3) because  $V > 0$  and  $C'(\cdot)$  is continuous with  $C'(0) = 0$ . Next, note that  $\Pi(p_j; \Sigma_j)$  is strictly quasi-concave in  $p_j \in (0, \frac{1}{\alpha})$  for any given  $\Sigma_j \geq 0$ . Thus, the FOC pertaining to maximizing (3) are sufficient, and given by

$$V = c_j C' \left( \frac{p_j}{1 - \alpha p_j} \Sigma_j \right) \frac{\Sigma_j}{(1 - \alpha p_j)^2}. \quad (7)$$

Thus, if  $p_j^*$  solves (7), then  $p_j^*$  is the global maximizer of  $\Pi_j(p_j; \Sigma_j)$  on  $[0, \frac{1}{\alpha})$ . In equilibrium, the quantity  $\Sigma_j$  by definition is determined by  $\Sigma_j = (1 - \alpha p_j)\Sigma$ , where  $\Sigma \equiv n\bar{e}$  is defined as the aggregate impact. Plugging this value in (7) yields the equilibrium condition (4). The remainder of the proof is organized in the following three steps. We first establish that this equation has a unique solution  $p_j(\Sigma) > 0$  for each  $j \in \mathcal{J}$  and any given  $\Sigma \geq 0$ . This solution is a  $C^1$ -function that verifies  $p_j'(\Sigma) < 0$ . Next, we will show that there is a unique value  $\Sigma^* > 0$  such that  $\sum_i p_i(\Sigma^*) = 1$ . Finally, we will establish that  $p_j(\Sigma^*) = p_j^*$ , i.e.,  $p_j(\Sigma^*)$  indeed maximizes  $\Pi(p_j; \Sigma_j)$  at  $\Sigma_j = (1 - \alpha p_j^*)\Sigma^*$ .

Let  $\Sigma \geq 0$ , and note that  $p_j = 0$  cannot solve (4). Hence, any solution to (4) must verify  $p_j > 0$ . Next, note that, for any given  $p_j > 0$ , the RHS of (4) is continuous in  $\Sigma$ , equal to zero for  $\Sigma = 0$ , strictly increasing in  $\Sigma$  and grows arbitrarily large as  $\Sigma \rightarrow \infty$ . The LHS of (4) is continuous and non-increasing in  $p_j$ . If  $\Sigma = 0$  there either is no solution of (4) (if  $\alpha = 0$ ) or the solution is  $p_j(0) = \frac{1}{\alpha} \geq 1$  (if  $0 < \alpha \leq 1$ ). For any  $\Sigma > 0$ , the above arguments assure the existence of a unique solution  $p_j(\Sigma) > 0$ . Moreover, the Implicit Function Theorem assures that this solution is a  $C^1$ -function of  $\Sigma$  and satisfies  $p_j'(\Sigma) < 0$ . Finally, we have that  $\lim_{\Sigma \rightarrow 0} p_j(\Sigma) \geq 1$  and  $\lim_{\Sigma \rightarrow \infty} p_j(\Sigma) = 0$ . For the solutions  $p_1(\Sigma), \dots, p_n(\Sigma)$  to be an equilibrium, they must integrate to one. Define  $G(\Sigma) = \sum_{i=1}^n p_i(\Sigma)$  and note that  $G'(\Sigma) < 0$  for any  $\Sigma > 0$ . The previous arguments about  $p_j(\Sigma)$  imply that  $\lim_{\Sigma \rightarrow 0} G(\Sigma) > 1$  and  $\lim_{\Sigma \rightarrow \infty} G(\Sigma) = 0$ . The existence of a unique  $\Sigma^* > 0$  that solves  $G(\Sigma) = 1$  follows from these facts. By construction,  $p_1(\Sigma^*), \dots, p_n(\Sigma^*)$  must maximize each  $\Pi_j(p_j; \Sigma_j^*)$  if the value of  $\Sigma_j^*$  is given by  $\Sigma_j^* = (1 - \alpha p_j(\Sigma^*))$ . This concludes the proof.  $\blacksquare$

**Proof Corollary 2** For any  $\alpha \in [0, 1]$  define the aggregate impact by  $\Sigma(\alpha) \equiv n\bar{e}(\alpha)$ . Thus  $\bar{e}(\alpha)$  is  $C^1$  in  $\alpha$  iff  $\Sigma(\alpha)$  is  $C^1$  in  $\alpha$ . Lemma 1 in Appendix A.1 establishes that the solution function  $\Sigma(\alpha)$  must be  $C^1$  with  $\Sigma'(\alpha) < 0$ . For each  $j \in \mathcal{J}_n$ , define the function

$$F_j(p_j; \alpha) \equiv V(1 - \alpha p_j) - c_j C'(p_j \Sigma(\alpha)) \Sigma(\alpha).$$

Thus,  $F_j(p_j \alpha) = 0$  amounts to equilibrium equation (4), and it is easy to check that  $F_j(p_j; \alpha)$  is  $C^1$  on  $\mathbb{R}_{++} \times [0, 1]$  with

$$\partial p_j F_j(p_j(\alpha); \alpha) = -\alpha V - c_j C'''(p_j(\alpha) \Sigma(\alpha)) \Sigma(\alpha)^2 < 0. \quad (8)$$

The solution  $\mathbf{p}(\alpha) = (p_1(\alpha), \dots, p_n(\alpha))$  solves the system of equations

$$F(\mathbf{p}; \alpha) \equiv \begin{pmatrix} F_1(p_1; \alpha) \\ \vdots \\ F_n(p_n; \alpha) \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Because

$$D_p F(\mathbf{p}(\alpha); \alpha) = \begin{pmatrix} \partial p_1 F_1(p_1(\alpha); \alpha) & 0 & 0 & 0 \\ 0 & \partial p_2 F_2(p_2(\alpha); \alpha) & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \partial p_n F_n(p_n(\alpha); \alpha) \end{pmatrix},$$

condition (8) implies  $|D_p F(\mathbf{p}(\alpha); \alpha)| \neq 0$  for every  $\alpha \in [0, 1]$ , and the Implicit Function Theorem therefore assures that the solution  $\mathbf{p}(\alpha)$  must be continuously differentiable at each given  $\alpha \in [0, 1]$ .  $\blacksquare$

**Proof Corollary 3** We show that the direct-aggregative effect  $R_{ij}$  implied by (4) is negative for any two distinct players  $i, j$  with  $c_i < c_j$  and any  $\alpha \in [0, 1]$ . In Hefti and Teichgräber (2022), it is established that this condition ensures that equilibrium inequality, as measured by any Lorenz-consistent inequality index, strictly decreases in  $\alpha$ . Therefore, inequality in  $\hat{p}_\alpha$  must increase as  $\alpha$  decreases from 1 to 0. Let  $G(i) \equiv \frac{V(1-\alpha p_j)}{c_j C'(p_j \Sigma) \Sigma}$  denote the ratio of marginal benefits to marginal costs as given by (4). The direct-aggregative effect due to a marginal change in  $\alpha$  then is defined by

$$R_{ij}(\alpha) = \frac{1}{\gamma_i} \left( \frac{G_\alpha(i)}{G(i)} - \frac{G_\alpha(j)}{G(j)} \right) + \frac{1}{\gamma_i} \left( \frac{G_\Sigma(i)}{G(i)} - \frac{G_\Sigma(j)}{G(j)} \right) \Sigma'(\alpha), \quad (9)$$

where  $\gamma_i > 0$  is the elasticity of  $G(i)$  with respect to  $p_i$ , and all expressions are evaluated at (4). Evaluating the partial derivatives in (9) at (4), we directly find that, for any  $\alpha \in [0, 1]$ , the first bracket in (9) is negative, while the second bracket is non-negative if  $C(e)$  has a non-increasing elasticity of marginal costs. As  $\Sigma'(\alpha) < 0$  by Lemma 1, it follows that  $R_{ij}(\alpha) < 0$  for all  $\alpha \in [0, 1]$ , completing the proof.  $\blacksquare$

**Proof Corollary 4** We proceed as in the proof of Corollary 3, and show that the conditions of Corollary 4 ensure that the direct-aggregative effect  $R_{ij}(V)$  implied by (4) has the same sign for any two distinct players  $i, j$  with  $c_i < c_j$  and any  $\alpha \in [0, 1]$ . Corollary 4 then follows directly from Hefti and Teichgräber (2022), who show that equilibrium inequality – measured by any Lorenz-consistent inequality index – strictly increases (decreases) in  $V$  if  $R_{ij}(V) > (<) 0$  consistently. The direct-aggregative effect  $R_{ij}(V)$  again is given by (9), with  $G_\alpha(\cdot)$  and  $\Sigma'(\alpha)$  replaced by  $G_V(\cdot)$  and  $\Sigma'(V)$ , respectively. Following the argument in Lemma 1, replacing  $\alpha$



with  $V$  directly establishes that  $\Sigma'(V) > 0$  whenever  $C''(e) > 0$ . Evaluating the partial derivatives in (9) at (4) shows that the first bracket in (9) is zero, while the second bracket is positive (negative) if  $C(e)$  has a decreasing (increasing) elasticity of marginal costs. As  $\Sigma'(V) > 0$ , it follows that  $R_{ij}(V) > (<)0$  under the respective cost elasticity conditions, completing the proof. ■

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