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**Working Paper**

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Working Paper, No. 2025-01

**Provided in Cooperation with:**

Department of Economics, The University of Utah, Salt Lake City

*Suggested Citation:* Fu, Bowen; Mendieta-Muñoz, Ivan (2025) : Trend inflation and structural shocks, Working Paper, No. 2025-01, The University of Utah, Department of Economics, Salt Lake City, UT

This Version is available at:

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DEPARTMENT OF ECONOMICS WORKING PAPER SERIES

**Trend inflation and structural shocks**

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Working Paper No: 2025-01

January 2025

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# Trend inflation and structural shocks\*

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January 17, 2025

This paper studies the effects of key underlying macroeconomic variables on the trend inflation rate in the USA. To do so, we consider eight structural shocks that incorporate a broad set of information for the US economy and that can be regarded as the main structural determinants of the latter. Using a Bayesian estimation procedure, we estimate the effects of these structural shocks on the trend inflation rate via an unobserved components model with stochastic volatility and structural shocks. We document the following results. First, four structural shocks have significant and quantitatively important effects on the trend inflation rate. Price mark-up and government policy shocks increase trend inflation, which suggests that these shocks tend to have long-run inflationary effects. Finance and productivity shocks decrease trend inflation, thus suggesting that these shocks tend to have long-run deflationary effects. Second, during the Global Financial Crisis of 2007-9, the trend in inflation became more volatile because of the combined effects derived from these four structural shocks.

**Keywords:** trend inflation, structural shocks, state space models, unobserved components.

**JEL Classification:** C11, C32, E30, E31.

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\*We have benefited from comments by Sacha Gelfer, Chris Gibbs, and seminar participants at conferences in Calgary, Milan, Palma, Paris, and Toledo.

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# 1. Introduction

The trend inflation rate is a latent variable that captures the underlying, persistent long-run path of inflation dynamics. As such, it represents a key objective of monetary policy since central banks aim to ensure price stability over the long-run (see, *e.g.*, [Bernanke, 2007](#), [Mishkin, 2007](#) and [Draghi, 2015](#)). However, almost all research in this field has focused on developing alternative econometric models and methods aimed at improving the measurement of trend inflation; while the underlying determinants of long-run inflation have remained largely unexplored in the extant literature.

This paper aims to fill this gap by studying the effects of key underlying macroeconomic variables on the trend inflation rate in the USA. In order to do so, we consider eight structural shocks that can be regarded as the main structural determinants of the US economy. These structural shocks are derived from the dynamic stochastic general equilibrium-dynamic factor model (DSGE-DFM) of [Gelfer \(2019\)](#), which incorporates a broad set of information for the USA. Using a Bayesian estimation procedure, we estimate the effects of these eight structural shocks on the trend inflation rate via an unobserved components model with stochastic volatility and structural shocks (UCSV-X model).

Our main findings can be summarized as follows. First, we document that only four structural shocks have significant and quantitatively important effects on trend inflation. In order of magnitude, price mark-up and government policy shocks tend to increase trend inflation; while finance and productivity shocks tend to reduce trend inflation. Therefore, price mark-up and government policy shocks tend to have long-run inflationary effects; while finance and productivity shocks tend to have long-run deflationary effects.

Second, using a Bayesian model comparison exercise, we find that there is strong evidence in favor of the UCSV-X model that includes the four structural shocks over a standard unobserved components model with stochastic volatility (UCSV model). This indicates that the inclusion of the structural shocks provides relevant additional information for understanding the behavior of trend inflation.

Third, although the estimated trend inflation obtained from the UCSV-X model tends to be statistically similar to the one obtained from the UCSV model, the trend inflation estimates obtained from the UCSV-X model became more volatile during some quarters that correspond to the Global Financial Crisis (GFC) of 2007-9. This suggests that the combined effects of the

four structural shocks increased the volatility of the trend inflation rate during this recession.

Our article is mainly related to two strands of literature. Firstly, the seminal work of [Stock and Watson \(2007\)](#) introduced the UCSV model, which has become one of the most prominent tools for estimating trend inflation. Ever since then, various modeling approaches aimed at improving the trend inflation estimates derived from the original univariate UCSV model have been developed. [Chan et al. \(2016\)](#) and [Chan et al. \(2018\)](#) considered bivariate UCSV models for inflation and unemployment and for inflation and a survey-based long-run forecast of inflation, respectively. [Mertens \(2016\)](#) estimated a multivariate generalization of the UCSV model by considering the information contained in monthly data on realized inflation, survey expectations, and the term structure of interest rates. [Stock and Watson \(2016\)](#) proposed a dynamic factor model estimated using disaggregated US data on sectoral inflation. [Hwu and Kim \(2019\)](#) considered a univariate unobserved components model with Markov-switching volatility that allows for nonzero correlation between innovations to trend inflation and the inflation gap. [Mertens and Nason \(2020\)](#) enriched the UCSV model by allowing for time-variation in the inflation gap persistence and in the frequency of forecast updating, which incorporates a sticky-information forecast mechanism. [Nason and Smith \(2021\)](#) extended the UCSV model by combining inflation predictions from survey-based forecasts of inflation—which can be treated either as rational expectations or updating according to a sticky inflation law of motion—with realized inflation.

Overall, the statistical procedures implemented by this first body of literature have focused on exploring alternative models and methods in which the measurement of trend inflation can be improved. These methodologies typically assume that the trend in inflation follows a random walk, which implies that the underlying dynamics of long-run inflation expectations are treated as a black box.

Secondly, other recent papers have explicitly tried to investigate the relevant factors that drive the trend in inflation. The estimation results presented by [Cecchetti et al. \(2017\)](#) show that labor market slack has a statistically significant—although quantitatively small—effect on trend inflation in the USA; while inflation expectations have no effect at all. [Correa-López et al. \(2019\)](#) found a significant but quantitatively small role of short-term inflation expectations, economic slack, and openness variables in twelve Euro Area countries. [Kamber and Wong \(2020\)](#) focused on how global factors (namely, foreign shocks) affect both trend inflation and the inflation gap, finding that these have only a marginal role in driving the former but an important influence

on the latter.

In summary, the results found by this second body of literature show that, although some factors have been found to effect trend inflation, there is still considerable uncertainty about the relevant variables that can influence its path.

Our article contributes to these two strands of literature by considering an UCSV model that incorporates a set of structural shocks that can be regarded as the main independent structural determinants of the US economy in order to understand the dynamics of trend inflation. In this sense, our empirical approach follows some of the suggestions of [Barnichon and Mesters \(2020\)](#), who have proposed utilizing well-chosen independently identified shocks derived from structural models to mitigate the endogeneity issues in macroeconomic equations. Importantly, the eight structural shocks identified by the DSGE-DFM of [Gelfer \(2019\)](#) have been used to address similar potential endogeneity problems in other empirical contexts, such as in the estimation of Okun’s law ([Gelfer, 2020](#)) and the uncovered interest parity condition ([Fu et al., 2025](#)).<sup>1</sup>

The rest of the paper is organized as follows. Section 2 presents a summary of the UCSV-X model and describes the relevant aspects of the Bayesian estimation approach, as well as the Bayesian model comparison strategy. Section 3 summarizes and discusses the main empirical results. Finally, section 4 concludes the article.

## 2. Model and estimation

This section is composed of three parts. First, section 2.1 introduces the UCSV-X model, which we use to study the effects of the DSGE-DFM structural shocks on the trend component of inflation. Second, we summarize the implementation of the Bayesian estimation procedure in section 2.2. Third, we describe the Bayesian model comparison strategy for comparing the UCSV-X and UCSV models in section 2.3.

### 2.1. An unobserved components model with stochastic volatility and structural shocks

The proposed UCSV-X model aims at studying the effects of a set of relevant DSGE-DFM structural shocks for trend inflation. Assuming that  $\pi_t$  denotes the inflation rate, we construct

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<sup>1</sup>Section 2.1 discusses further the relevance of the DSGE-DFM of [Gelfer \(2019\)](#) as well as the structural shocks derived from the latter.

the following model:

$$\pi_t = \tau_t + \epsilon_t^\pi, \quad \epsilon_t^\pi \sim \mathcal{N}(0, e^{h_t}), \quad (1)$$

$$\Delta\tau_t = \beta_{0,\tau} + \Delta x_t' \beta_{1,\tau} + \Delta x_{t-1}' \beta_{2,\tau} + \epsilon_t^\tau, \quad \epsilon_t^\tau \sim \mathcal{N}(0, e^{g_t}), \quad (2)$$

$$h_t = h_{t-1} + \epsilon_t^h, \quad \epsilon_t^h \sim \mathcal{N}(0, \omega_h^2), \quad (3)$$

$$g_t = g_{t-1} + \epsilon_t^g, \quad \epsilon_t^g \sim \mathcal{N}(0, \omega_g^2), \quad (4)$$

where  $\tau_t$  is the trend inflation rate or the permanent component of inflation;  $\epsilon_t^\pi$  is the inflation gap or the transitory component of inflation;  $x_t$  is a vector that contains the eight structural shocks identified by the DSGE-DFM of [Gelfer \(2019\)](#);  $h_t$  and  $g_t$  are the stochastic volatilities associated with the inflation gap and trend inflation, respectively; and  $\epsilon_t^\tau$ ,  $\epsilon_t^h$ , and  $\epsilon_t^g$  are mutually and serially uncorrelated error terms.

We clarify the following characteristics of the proposed UCSV-X model depicted by equations (1) through (4). Firstly, following the UCSV model of inflation developed by [Stock and Watson \(2007\)](#), our model decomposes the inflation rate  $\pi_t$  into  $\tau_t$  and  $\epsilon_t^\pi$ , where each unobserved component exhibits time-varying volatility evolving according to random walk processes—as shown by equations (3) and (4).

Secondly, as shown in equation (2), our model quantifies the response of the changes in trend inflation rate,  $\Delta\tau_t = \tau_t - \tau_{t-1}$ , to the contemporaneous changes in each of the eight structural shocks,  $\Delta x_t = x_t - x_{t-1}$ , via the estimated  $n \times 1$  vector of parameters  $\beta_{1,\tau}$ . Similarly, the  $n \times 1$  estimated vector of parameters  $\beta_{2,\tau}$  measures the response of  $\Delta\tau_t$  to the lagged changes in each of the eight structural shocks,  $\Delta x_{t-1} = x_{t-1} - x_{t-2}$ . By incorporating the effects of both  $\Delta x_t$  and  $\Delta x_{t-1}$ , we consider the possibility of time delays in the effects of structural shocks on trend inflation, or, alternatively, that the response of trend inflation to the structural shocks can exhibit different dynamics.<sup>2</sup>

Our specification, which is written in terms of the first difference of  $\tau_t$ , is consistent with the assumption that  $\tau_t$  is a random walk with drift. This is also consistent with the regression equations used by [Cecchetti et al. \(2017\)](#) and [Correa-López et al. \(2019\)](#). In their research, they first estimated  $\tau_t$  via different UCSV models. Then, in a second step, they estimated how  $\Delta\tau_t$  is affected by: (i) the first differences of various measures of inflation expectations; (ii)

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<sup>2</sup>Importantly, we also estimated an UCSV-X model that incorporated  $\Delta x_t$ ,  $\Delta x_{t-1}$  and  $\Delta x_{t-2}$ . However, the estimated 68% credible intervals for all the posterior means of the parameters associated with  $\Delta x_{t-2}$  enclosed zero, so there is no strong evidence suggesting that the effects of the shocks in  $\Delta x_{t-2}$  are different from zero. The results obtained from this model are available on request.

measures of economic activity capturing the extent of slack in the economy—namely, the unemployment gap and the output gap; and (iii) the first differences of other explanatory variables—such as changes in financial conditions (the Chicago Fed’s National Financial Conditions Index, monetary aggregates M2 and M3, private non-financial debt, yields on 10-year government bonds, and stock indexes), changes in labor costs (hourly earnings and unit labor costs), and changes in trade and openness indicators (import prices, world export prices, real effective exchange rates, Brent oil price, and commodity price indexes).

In contrast to these contributions, our UCSV-X model: (i) considers that the changes in the key macroeconomic variables that can affect the trend inflation rate  $\tau_t$  are the different dynamics of the DSGE-DFM structural shocks captured by  $\Delta x_t$  and  $\Delta x_{t-1}$ ; and (ii) comprises a single-step estimation procedure, which, overall, tends to be a more efficient modeling approach relative to a two-step methodology.

Thirdly, the DSGE structural shocks in  $x_t$  correspond to the ones identified by [Gelfer \(2019\)](#), who extended the Federal Reserve Bank of New York (FRBNY) dynamic stochastic general equilibrium (DSGE) model with financial frictions of [Del Negro and Schorfheide \(2013\)](#) by considering a data-rich environment.<sup>3</sup> In brief, he constructed a DSGE-DFM to identify the main shocks that drive the structural determinants of the US economy. The DSGE-DFM has two main advantages. First, the series that are not directly incorporated inside the DSGE model are allowed to load on economic variables and structural processes that are inside the DSGE model. Second, it produces better forecasts of variables that are directly modeled inside the DSGE model (including GDP, consumption, investment growth, inflation, and interest rates). Therefore, [Gelfer \(2019\)](#)’s model provides more robust identified structural shocks than the original FRBNY DSGE model.

The eight structural shocks derived from [Gelfer \(2019\)](#)’s DSGE-DFM are the following. First, a productivity shock, which corresponds to a total factor productivity shock faced by intermediate firms that affect firms’ production. Second, an investment shock—that is, a shock that affects the marginal efficiency of investment of capital producers. Third, a preference shock—a shock to the discount rate that alters households’ consumer and savings decisions.

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<sup>3</sup>The FRBNY DSGE model by [Del Negro and Schorfheide \(2013\)](#) represents an extension of the [Smets and Wouters \(2003, 2007\)](#) New Keynesian DSGE model that incorporates credit market frictions, which follow the financial accelerator model developed by [Bernanke et al. \(1999\)](#). Following also the work of [Boivin and Giannoni \(2006\)](#) and [Kryshko \(2011\)](#), [Gelfer \(2019\)](#) considered the model of [Del Negro and Schorfheide \(2013\)](#) and proceeded to incorporate a large set of economic and financial series (ninety-seven quarterly data series) in the estimation of the state parameters and the structural DSGE parameters using an adaptive Metropolis-within-Gibbs sampling algorithm.



Fourth, a government policy shock, which corresponds to a spending shock to the government portion of GDP driven by the fiscal authority. Fifth, a finance shock driven by entrepreneurs and banks, which corresponds to a risk shock that affects the spread between the bank deposit rate and the bank lending rate. Sixth, a monetary policy shock—which corresponds to an unexpected shock to the risk-free interest rate driven by the monetary authority. Seventh, a price mark-up shock—a shock to the mark-up above marginal costs that monopolistically competitive intermediate firms charge final good producing firms. Eighth, a wage mark-up shock—that is, a shock to the monopolistic power households have over their specialized labor.

However, it is important to point out that, as discussed by [Kilian and Lütkepohl \(2017\)](#), in DSGE models the variables often labeled as “shocks” (such as the ones considered in our paper) are, in fact, exogenous state variables driven by an underlying source of randomness. By contrast, from an econometric perspective the structural shocks are the white-noise disturbances that feed into these exogenous variables. Nevertheless, following the standard convention in the DSGE literature, we simply refer to the entire exogenous processes used in this paper as structural shocks.

Fourthly, the proposed UCSV-X model considers that, consistent with the Beveridge-Nelson decomposition, the trend in inflation,  $\tau_t$ , corresponds to the infinite-horizon forecast of the inflation rate,  $\pi_t$ , conditional on an information set available in period  $t$ ,  $\Omega_t$ :

$$\tau_t = \lim_{j \rightarrow \infty} E[\pi_{t+j} | \Omega_t]. \quad (5)$$

As discussed by [Mertens \(2016\)](#), defining  $\tau_t$  as an expectation has important consequences for the dynamics of  $\pi_t$  since differencing equation (5) yields a unit root process for  $\tau_t$ :

$$\tau_t = \tau_{t-1} + \lim_{j \rightarrow \infty} E[\pi_{t+j} | \Omega_t] - \lim_{j \rightarrow \infty} E[\pi_{t+j} | \Omega_{t-1}] = \tau_{t-1} + \bar{e}_t, \quad (6)$$

where  $\bar{e}_t$  denotes the trend shocks, which form a martingale-difference sequence. Therefore, from equations (2) and (6) we observe that:

$$\Delta\tau_t = \tau_t - \tau_{t-1} = \bar{e}_t = \beta_{0,\tau} + \Delta x_t' \beta_{1,\tau} + \Delta x_{t-1}' \beta_{2,\tau} + \epsilon_t^\tau, \quad (7)$$

so that  $\tau_t$  follows a random walk given the trend shocks  $\bar{e}_t$  that can also influence its trajectory.

Finally, we point out that our empirical approach also shares some elements related to the

literature on proxy structural vector autoregressions and the use of external instruments for the identification of shocks, which has grown to become influential in empirical macroeconomics. The applications of this strand of literature have mainly discussed the identification of monetary shocks using proxies constructed from high-frequency financial data (Caldara and Herbst, 2019), the dynamic effects of consumption and investment total factor productivity shocks as well as the effects of personal income tax shocks (Arias et al., 2021), and the effects of oil-supply shocks (Montiel-Olea et al., 2021).

Compared to these methodological and empirical contributions, our approach simply uses a set of external shocks as the relevant variables for the estimation of the structural drivers of trend inflation, without explicitly discussing whether these external shocks are correlated with possible target shocks and uncorrelated with other shocks in the model. Nonetheless, by considering a broad set of information summarized by the eight DSGE-DFM structural shocks, the proposed UCSV-X model aims at providing informative estimates regarding the main structural drivers of trend inflation. To the best of our knowledge, our contribution is the first study that has carried out this type of analysis in the context of UCSV models.

## 2.2. Bayesian estimation

We assume the following prior distributions:  $\tau_0 \sim \mathcal{N}(\mu_{\tau_0}, V_{\tau_0})$ ,  $h_0 \sim \mathcal{N}(\mu_{h_0}, V_{h_0})$ , and  $g_0 \sim \mathcal{N}(\mu_{g_0}, V_{g_0})$ , where  $\mu_{\tau_0} = \mu_{h_0} = \mu_{g_0} = 0$  and  $V_{\tau_0} = V_{h_0} = V_{g_0} = 10$ . Let  $\beta_\tau = (\beta_{0,\tau}, \beta_{1,\tau}, \beta_{2,\tau})'$ . We choose relatively non-informative priors for  $\beta_\tau$ , thus assuming that  $\beta_\tau \sim \mathcal{N}(\beta_0^\tau, V_{\beta_\tau})$ , where  $\beta_0^\tau = 0_{k \times 1}$ ,  $V_{\beta_\tau} = 0.1I_k$ , and  $k = 2n + 1$ . We also assume that  $\omega_h^2$  and  $\omega_g^2$  follow inverse gamma distributions:  $\omega_h^2 \sim \mathcal{IG}(\nu_h, S_h)$  and  $\omega_g^2 \sim \mathcal{IG}(\nu_g, S_g)$ , such that  $\nu_h = 3$ ,  $S_h = 1 * (\nu_h - 1)$ ,  $\nu_g = 3$ , and  $S_g = 1 * (\nu_g - 1)$ .

We estimate the UCSV-X model depicted in equations (1) through (4) using Markov chain Monte Carlo (MCMC) methods.<sup>4</sup> To sum up, we use the precision sampler developed by Chan and Jeliazkov (2009) to sample  $\tau$ . Therefore, we first stack equation (1) over  $t$  to obtain  $\pi = \tau + z$ .

Next, let  $\Delta \mathbf{x}_t = (1, \Delta x'_t, \Delta x'_{t-1})$ , then we write equation (2) as  $\Delta \tau_t = \Delta \mathbf{x}_t \beta_\tau + \epsilon_t^\tau$ . We stack this equation over  $t$  and obtain  $H\tau = \alpha_\tau + \Delta X \beta_\tau + \epsilon^\tau$ . Left-multiplying the latter by  $H$  we have

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<sup>4</sup>Further technical details regarding the implementation of the sampling algorithm are presented in appendix A.

that  $\tau = H^{-1}\alpha_\tau + H^{-1}\Delta X\beta_\tau + H^{-1}\epsilon^\tau$ , where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad \text{and} \quad \alpha_\tau = \begin{pmatrix} \tau_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Let us denote  $\Sigma_h = \text{diag}(e^{h_1}, e^{h_2}, \dots, e^{h_T})$  and  $\Sigma_g = \text{diag}(e^{g_1}, e^{g_2}, \dots, e^{g_T})$ . The conditional likelihood given the states  $\tau$  corresponds to:

$$(2\pi)^{-\frac{T}{2}} |\Sigma_h|^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\tau)' \Sigma_h^{-1} (y-\tau)}. \quad (8)$$

The prior density of  $\tau$  implied by equation (2) is given by:

$$(2\pi)^{-\frac{T}{2}} |(H' \Sigma_g^{-1} H)^{-1}|^{-\frac{1}{2}} e^{-\frac{1}{2}(\tau - H^{-1}\alpha_\tau - H^{-1}\Delta X\beta_\tau)' H' \Sigma_g^{-1} H (\tau - H^{-1}\alpha_\tau - H^{-1}\Delta X\beta_\tau)}. \quad (9)$$

Therefore, combining equations (8) and (9) we have that:

$$(\tau|y, \Delta X, h, g, \tau_0, \beta_\tau) \sim \mathcal{N}(\hat{\tau}, K_\tau^{-1}),$$

where  $K_\tau = \Sigma_h^{-1} + H' \Sigma_g^{-1} H$  and  $\hat{\tau} = K_\tau^{-1}(\Sigma_h^{-1}y + H' \Sigma_g^{-1}(\alpha_\tau + \Delta X\beta_\tau))$ .

Finally, the prior density of  $\beta_\tau$  is given by:

$$(2\pi)^{-\frac{n}{2}} |V_{\beta_\tau}|^{-\frac{1}{2}} e^{-\frac{1}{2}(\beta_\tau - \beta_0^\tau)' V_{\beta_\tau}^{-1} (\beta_\tau - \beta_0^\tau)}, \quad (10)$$

so combining equations (8) and (10) we have:

$$(\beta_\tau|\tau, \Delta X, g, \beta_0^\tau, V_{\beta_\tau}) \sim \mathcal{N}(\hat{\beta}_\tau, K_{\beta_\tau}^{-1}),$$

such that  $K_{\beta_\tau} = V_{\beta_\tau}^{-1} + \Delta X' \Sigma_g^{-1} \Delta X$  and  $\hat{\beta}_\tau = K_{\beta_\tau}^{-1}(V_{\beta_\tau}^{-1}\beta_0^\tau + \Delta X' \Sigma_g^{-1}(\tau - H^{-1}\alpha_\tau))$ .

### 2.3. Model comparison

We follow a Bayesian strategy to carry out the model comparison between the proposed UCSV-X model and a standard UCSV model for the US inflation rate. We believe that this comparison

is important to study whether the UCSV-X provides useful information for understanding the determinants of trend inflation in the USA.

Assuming that  $y$  is the actual observed data, the posterior odds ratio, *i.e.*, the ratio of the two posterior model probabilities, can be specified as:

$$\frac{p(\text{UCSV-X}|y)}{p(\text{UCSV}|y)} = \frac{p(y|\text{UCSV-X})}{p(y|\text{UCSV})} \times \frac{p(\text{UCSV-X})}{p(\text{UCSV})},$$

where  $p(\text{UCSV-X}|y)$  and  $p(\text{UCSV}|y)$  are the posterior probabilities for the UCSV-X and UCSV models, respectively;  $p(y|\text{UCSV-X})$  and  $p(y|\text{UCSV})$  are the marginal likelihoods under the UCSV-X and UCSV models, respectively; and  $p(\text{UCSV-X})$  and  $p(\text{UCSV})$  are the prior probabilities for the UCSV-X and UCSV models, respectively.

In other words, the ratio  $p(\text{UCSV-X}|y)/p(\text{UCSV}|y)$  corresponds to the posterior odds ratio;  $p(y|\text{UCSV-X})/p(y|\text{UCSV})$  is the Bayes factor (BF); and  $p(\text{UCSV-X})/p(\text{UCSV})$  is the prior odds ratio.

Assuming that the UCSV-X and UCSV models are equally probable *a priori*, the prior odds ratio is equal to one, so the posterior odds ratio is equal to the BF. Since the UCSV model is nested in the UCSV-X model, we use the Savage-Dickey density ratio to compute the BF (Verdinelli and Wasserman, 1995):

$$\text{BF} = \frac{p(\beta_\tau = 0)}{p(\beta_\tau = 0|y)}. \quad (11)$$

Hence, the BF in favor of the unrestricted UCSV-X model shown in equation (11) is the density ratio  $p(\beta_\tau = 0)/p(\beta_\tau = 0|y)$ . If  $\beta_\tau = 0$  is more likely under the prior relative to the posterior—that is, the numerator is larger than the denominator in equation (11), then there is evidence in favor of the unrestricted UCSV-X model. Specifically, we compute  $2\ln(\text{BF})$  and compare the result with the scale reported in Kass and Raftery (1995).

### 3. Results and discussion

We computed the inflation rate  $\pi_t$  as the annualized log percentage change of the core personal consumption expenditures (PCE) index, *i.e.*,  $\pi_t = 400 \ln(P_t/P_{t-1})$ , where  $P_t$  is the quarterly core PCE index extracted from the Federal Reserve Bank of St. Louis Economic Database (FRED). We considered the PCE inflation rate because, as mentioned by Chan et al. (2018),

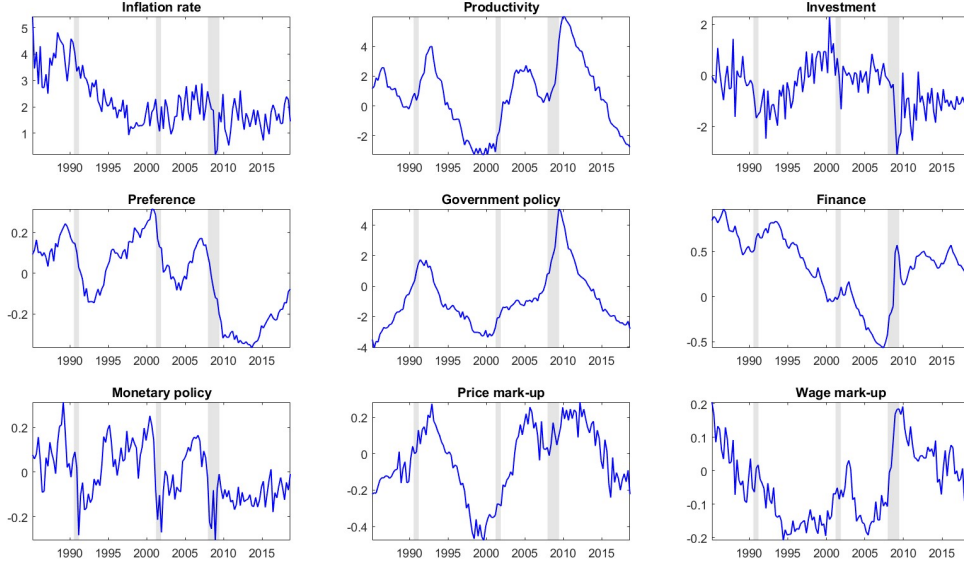


Figure 1: USA, 1985:Q1-2018:Q3. PCE inflation rate and posterior medians of the eight structural shocks obtained from the DSGE-DFM of [Gelfer \(2019\)](#). Shaded gray areas indicate NBER recessions dates.

its historical data has been revised to reflect methodology changes and the Federal Reserve’s long-run inflation objective is stated in terms of PCE inflation. The estimation period was 1985:Q1-2018:Q3 since the structural shocks identified by the DSGE-DFM of [Gelfer \(2019\)](#) are only available for this period. Figure 1 plots  $\pi_t$ , together with the posterior medians of the eight structural shocks used to estimate the UCSV-X model.

Table 1 summarizes the results obtained from the UCSV-X model by showing the effects of a one standard deviation change of all the structural shocks contained in  $\Delta x_t$  and  $\Delta x_{t-1}$  on the change in trend inflation  $\Delta \tau_t$ , measured by the parameters  $\beta_{1,\tau}$  and  $\beta_{2,\tau}$ , respectively. The estimation results show important effects of almost all shocks on trend inflation, the only exceptions being the monetary policy and wage mark-up shocks. However, the Bayesian model comparison exercise between the UCSV-X and UCSV models described in section 2.3 indicated that  $2\ln(\text{BF})$  is -0.618. Following [Kass and Raftery \(1995\)](#), we interpret this result as “negative” evidence in favor of the UCSV-X model or, alternatively, that the UCSV model is preferred over the UCSV-X model estimated in table 1.

In order to improve this result, we estimated a reduced UCSV-X model. Specifically, we considered a simplified model from the original UCSV-X model in table 1 that only incorporated the shocks in  $\Delta x_t$  and  $\Delta x_{t-1}$  that showed strong evidence suggesting that their effects on  $\Delta \tau_t$  were different from zero according to their respective credible intervals for the posterior means

Table 1: EFFECTS OF CHANGES IN STRUCTURAL SHOCKS ON CHANGES  
IN TREND INFLATION OBTAINED FROM THE UCSV-X MODEL

Changes in structural shocks	$\beta_{1,\tau}$	$\beta_{2,\tau}$
Productivity	<b>-0.169</b> (0.064) [-0.231, -0.105]	<b>0.152</b> (0.070) [0.082, 0.220]
Investment	<b>0.113</b> (0.095) [0.015, 0.215]	-0.051 (0.069) [-0.116, 0.016]
Preference	<b>0.124</b> (0.091) [0.031, 0.216]	-0.087 (0.089) [-0.177, 0.002]
Government policy	<b>0.180</b> (0.071) [0.109, 0.250]	<b>-0.151</b> (0.072) [-0.221, -0.079]
Finance	<b>-0.204</b> (0.060) [-0.261, -0.144]	<b>0.114</b> (0.068) [0.046, 0.183]
Monetary policy	-0.043 (0.066) [-0.104, 0.024]	-0.015 (0.057) [-0.074, 0.043]
Price mark-up	<b>0.224</b> (0.054) [0.170, 0.277]	<b>-0.079</b> (0.057) [-0.133, -0.025]
Wage mark-up	0.011 (0.057) [-0.044, 0.065]	0.015 (0.056) [-0.043, 0.072]

*Notes:* As shown in equation (2), the parameters  $\beta_{1,\tau}$  and  $\beta_{2,\tau}$  measure the response of  $\Delta\tau_t$  to each of the shocks in  $\Delta x_t$  and  $\Delta x_{t-1}$ , respectively. We report the posterior means, the standard deviations in parentheses, and the 68% credible intervals in square brackets. Bold numbers indicate that the respective coefficient's credible interval does not include zero. For model comparison, we computed the  $\ln(\text{BF})$  using ten sampling iterations. The mean of the  $\ln(\text{BF})$  of the UCSV-X model relative to the UCSV model across these ten iterations is -0.309, with a standard deviation of 4.267.

Table 2: EFFECTS OF CHANGES IN STRUCTURAL SHOCKS ON CHANGES  
IN TREND INFLATION OBTAINED FROM THE REDUCED UCSV-X  
MODEL

Changes in structural shocks	$\beta_{1,\tau}$	$\beta_{2,\tau}$
Productivity	<b>-0.153</b> (0.055) [-0.207, -0.098]	<b>0.118</b> (0.057) [0.062, 0.173]
Government policy	<b>0.146</b> (0.054) [0.094, 0.196]	<b>-0.124</b> (0.057) [-0.176, -0.070]
Finance	<b>-0.202</b> (0.052) [-0.251, -0.153]	<b>0.101</b> (0.059) [0.043, 0.161]
Price mark-up	<b>0.228</b> (0.049) [0.179, 0.276]	<b>-0.082</b> (0.050) [-0.132, -0.034]

*Notes:* As shown in equation (2), the parameters  $\beta_{1,\tau}$  and  $\beta_{2,\tau}$  measure the response of  $\Delta\tau_t$  to each of the shocks in  $\Delta x_t$  and  $\Delta x_{t-1}$ , respectively. We report the posterior means, the standard deviations in parentheses, and the 68% credible intervals in square brackets. Bold numbers indicate that the respective coefficient's credible interval does not include zero. For model comparison, we computed the  $\ln(\text{BF})$  using ten sampling iterations. The mean of the  $\ln(\text{BF})$  of the reduced UCSV-X model relative to the UCSV model across these ten iterations is 21.799, with a standard deviation of 4.690.

of the parameters. The results obtained from this reduced UCSV-X model are summarized in table 2.

The main results can be summarized as follows. First, the Bayesian model comparison exercise between the reduced UCSV-X model and the UCSV model indicated that  $2\ln(\text{BF}) = 43.598$ . Since  $2\log(\text{BF})$  is greater than 10, following [Kass and Raftery \(1995\)](#) we interpret this result as “very strong” evidence in favor of the reduced UCSV-X model over the UCSV model. This result implies that incorporating the more restrictive set of structural shocks shown in [table 2](#) provides useful information for estimating the trend inflation rate in the USA.

Second, we found that only four structural shocks effect the trend component of the inflation rate: productivity, government policy, finance, and price mark-up. The shock with the largest cumulative effect on  $\Delta\tau_t$  is the price mark-up shock:  $0.228 - 0.082 \approx 0.15$ , followed by the finance shock ( $-0.202 + 0.101 \approx -0.10$ ), the productivity shock ( $-0.153 + 0.118 \approx -0.04$ ) and the government policy shock ( $0.146 - 0.124 \approx 0.02$ ).

The cumulative effects of the structural shocks on trend inflation are consistent with the expected effects at the theoretical level. We found that a price mark-up shock increases importantly trend inflation. In other words, a shock to the mark-up above marginal costs that intermediate firms charge final good-producing firms has significant and quantitatively important long-run inflationary effects. We believe that this result highlights the empirical relevance of the recent discussion on sellers’ inflation, which emphasizes the microeconomic origins of the price-setting behavior of firms and the ability of firms with market power to increase prices (see, *e.g.*, [Weber and Wasner, 2023](#) and [Nikiforos et al., 2024](#)).<sup>5</sup>

Trend inflation also tends to increase when the economy experiences a government policy shock, thus reflecting the long-run inflationary pressures associated with increases in fiscal policy carried out by the US Congress and the Administration. We point out, however, that the estimated long-run inflationary effects associated with government policy shocks are comparatively minor.

On the other hand, both a finance shock—that is, an increase in risk that affects the spread between the bank deposit rate and the bank lending rate—and a productivity shock—*i.e.*, a total factor productivity shock that affect firms’ production—decrease the trend inflation rate. This implies that both shocks tend to have long-run deflationary effects. Thus, our results corroborate: (i) the findings of [Gilchrist and Zakrajšek \(2012\)](#) and [Forni et al. \(2024\)](#) for the analysis of long-run inflation, who highlight that a financial shock (the excess bond premium in

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<sup>5</sup>Admittedly, the discussion on sellers’ inflation has been mainly related to the post-COVID-19 surge in inflation, which is a period that we did not consider in our estimation. Nevertheless, the fact that our empirical results show that the price mark-up shock is the one with the largest effect on trend inflation highlights the prominence of these shocks for the analysis of long-run inflationary pressures.

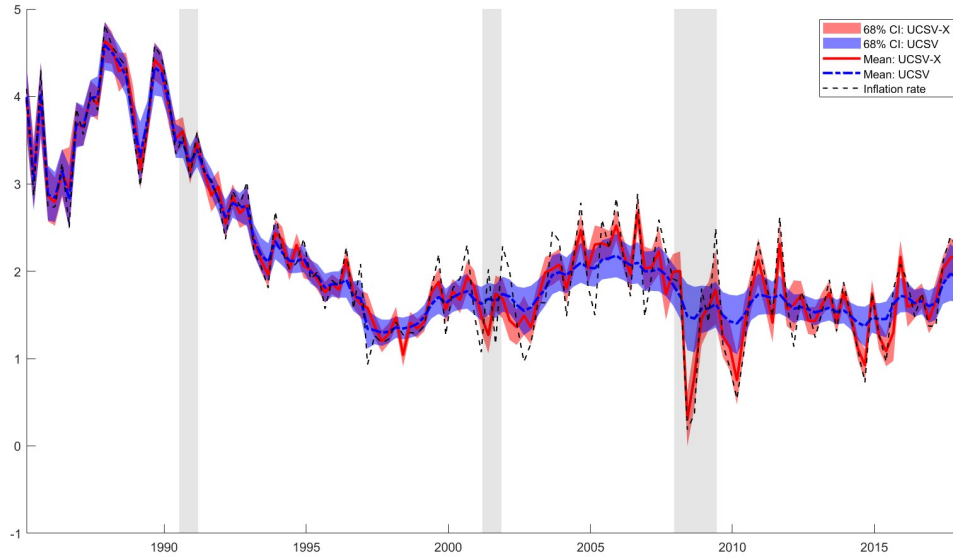


Figure 2: Posterior estimates of the trend inflation rate obtained from the UCSV-X and UCSV models. We report the posterior means obtained from each model in red and blue. Shaded red and blue areas represent the 68% credible intervals of the posterior mean estimates for each model. The black dashed line shows the PCE inflation rate. Shaded gray areas indicate NBER recession dates.

their case) behaves like a typical demand shock by reducing inflation; and (ii) that a productivity shock behaves like a typical supply shock by also reducing long-run inflationary pressures.

Third, figure 2 plots the posterior mean estimates of the trend inflation rate obtained from the reduced UCSV-X model shown in table 2 together with the ones obtained from the standard UCSV model. Our purpose is merely to show how the incorporation of the structural shocks affects the estimates of trend inflation. We observe that the two series follow closely each other and that, overall, the credible intervals associated with the posterior means tend to overlap for the great majority of the period, thus indicating that there is no strong evidence suggesting a statistical difference between the two estimates of trend inflation. However, the two estimates of trend inflation differ substantially during some quarters that correspond to the GFC of 2007-9. This result indicates that, during the GFC of 2007-9, long-run inflation tended to become more volatile because of the combined effects associated with the four relevant structural shocks identified above—namely, price mark-up, finance, productivity and government policy.

Finally, in figures 3 and 4 we compare the stochastic volatility components of the trend inflation rate and the inflation gap, respectively, obtained from both the reduced UCSV-X and UCSV models. We find that the time-varying volatilities obtained from both models are virtually identical for the whole period, including during the GFC of 2007-9. This suggests that the more volatile trend inflation obtained from the reduced UCSV-X model is more likely associated with



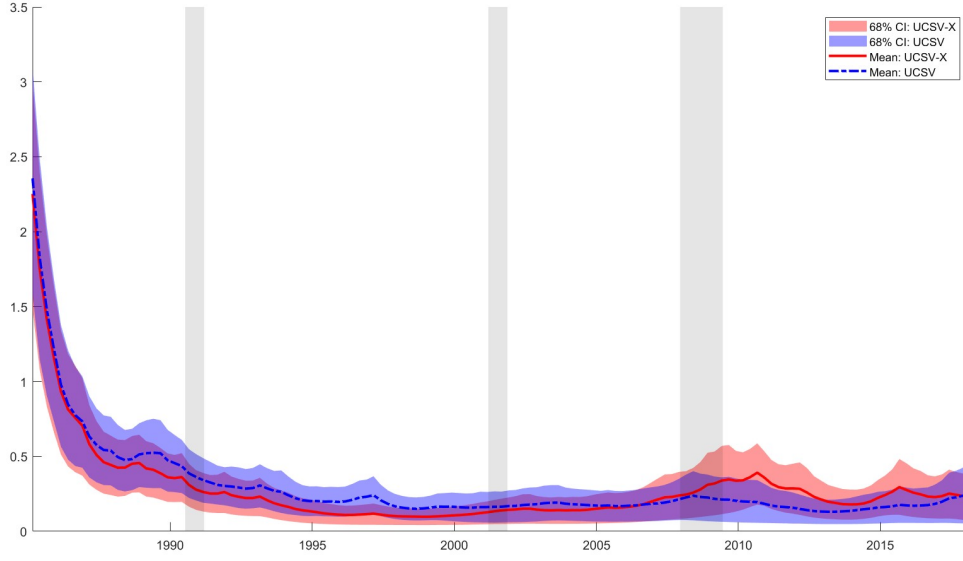


Figure 3: Posterior estimates of the time-varying standard deviation of the trend inflation rate,  $e^{gt}/2$ , obtained from the UCSV-X and UCSV models. We report the posterior means obtained from each model in red and blue. Shaded red and blue areas represent the 68% credible intervals of the posterior mean estimates for each model. Shaded gray areas indicate NBER recession dates.

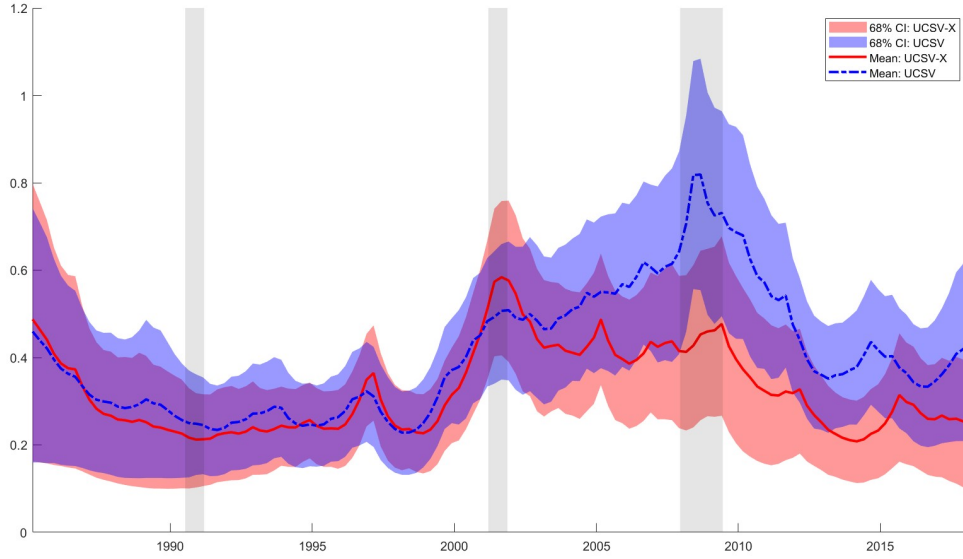


Figure 4: Posterior estimates of the time-varying standard deviation of the inflation gap,  $e^{ht}/2$ , obtained from the UCSV-X and UCSV models. We report the posterior means obtained from each model in red and blue. Shaded red and blue areas represent the 68% credible intervals of the posterior mean estimates for each model. Shaded gray areas indicate NBER recession dates.

the relevant structural shocks rather than due to higher stochastic volatility estimates derived from the proposed model specification.

## 4. Concluding remarks

This paper aims at identifying the relevant structural shocks that can influence the trend in inflation in the USA. In order to do so, we construct an UCSV-X model. The latter consists in estimating a variant of the UCSV model that incorporates eight structural shocks that can be regarded as the main determinants of the US economy into the estimation of trend inflation. Using a Bayesian sampling algorithm, we estimate the proposed UCSV-X model for the period 1985:Q1-2018:Q3.

The main results can be summarized as follows. First, we identify that the trend component of the inflation rate responds mainly to four structural shocks. In order of magnitude, price mark-up shocks, finance shocks, productivity shocks, and government policy shocks effect the trend inflation rate. Second, a Bayesian model comparison exercise shows evidence in favor of the UCSV-X model that includes the four structural shocks over a standard UCSV model. This implies that the inclusion of the structural shocks provides relevant additional information for understanding the dynamics of trend inflation. Third, during the GFC of 2007-9, the trend inflation estimates obtained from the UCSV-X model became more volatile than the ones obtained from a standard UCSV model. This indicates that the combined effects associated with these four structural shocks increased the volatility of the trend inflation rate during this period.

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## Appendix A Further details on the Bayesian sampling estimation procedure

This appendix provides further details regarding the Bayesian sampling estimation procedure described in section 2.2. Our MCMC sampling scheme comprises the following steps:

1. Draw  $\tau$  from  $p(\tau|y, \Delta X, h, g, \tau_0, \beta_\tau)$ .
2. Draw  $\beta_\tau$  from  $p(\beta_\tau|\tau, \Delta X, g, \beta_0^\tau, V_{\beta_\tau})$ .
3. Draw  $h$  from  $p(h|y, \tau, h_0, \omega_h^2)$ .
4. Draw  $g$  from  $p(g|\tau, \Delta X, \tau_0, \beta_\tau, \omega_g^2)$ .
5. Draw  $\tau_0$  from  $p(\tau_0|\tau, \Delta X, g, \beta_\tau, \mu_{\tau_0}, V_{\tau_0})$ .
6. Draw  $g_0$  from  $p(g_0|g, \omega_g^2, \mu_{g_0}, V_{g_0})$ .
7. Draw  $h_0$  from  $p(h_0|h, \omega_h^2, \mu_{h_0}, V_{h_0})$ .
8. Draw  $\omega_h^2$  from  $p(\omega_h^2|h, h_0, \nu_{\omega_h^2}, S_{\omega_h^2})$ .
9. Draw  $\omega_g^2$  from  $p(\omega_g^2|g, g_0, \nu_{\omega_g^2}, S_{\omega_g^2})$ .

The main purpose of our paper consists in exploring whether the structural shocks in  $\Delta x_t$  and  $\Delta x_{t-1}$  affect the trend inflation rate  $\tau_t$ . Hence, the technical details summarized in section 2.2 describe only steps 1 and 2 of the sampling algorithm outlined above. This is so because the implementation of steps 3 through 9 is standard. We summarize the latter below.

First, in steps 3 and 4, we draw  $h$  and  $g$  from  $p(h|y, \tau, h_0, \omega_h^2, \beta_\tau, \rho)$  and  $p(g|\tau, \Delta X, \tau_0, \beta_\tau, \omega_g^2)$ , respectively, following the auxiliary mixture sampler of Kim et al. (1998).

Second, with respect to steps 5 through 7, we have that:

$$\begin{aligned} (\tau_0|\tau, \Delta X, g, \beta, \mu_{\tau_0}, V_{\tau_0}) &\sim \mathcal{N}(\hat{\tau}_0, K_{\tau_0}^{-1}), \\ (g_0|g, \omega_g^2, \mu_{g_0}, V_{g_0}) &\sim \mathcal{N}(\hat{g}_0, K_{g_0}^{-1}), \\ (h_0|h, g, \mu_{h_0}, V_{h_0}) &\sim \mathcal{N}(\hat{h}_0, K_{h_0}^{-1}), \end{aligned}$$

where  $K_{\tau_0} = \frac{1}{V_{\tau_0}} + \frac{1}{e^{g_1}}$  and  $\hat{\tau}_0 = K_{\tau_0}^{-1}(\frac{\mu_{\tau_0}}{V_{\tau_0}} + \frac{\tau_1 - \Delta x_1 \beta_\tau}{e^{g_1}})$ ;  $K_{g_0} = \frac{1}{V_{g_0}} + \frac{1}{\omega_g^2}$  and  $\hat{g}_0 = K_{g_0}^{-1}(\frac{\mu_{g_0}}{V_{g_0}} + \frac{g_1}{\omega_g^2})$ ;  $K_{h_0} = \frac{1}{V_{h_0}} + \frac{1}{\omega_h^2}$  and  $\hat{h}_0 = K_{h_0}^{-1}(\frac{\mu_{h_0}}{V_{h_0}} + \frac{h_1}{\omega_h^2})$ .

Finally, to implement steps 8 and 9, we point out that the conditional densities of  $\omega_h^2$  and  $\omega_g^2$  are:

$$\begin{aligned}(\omega_h^2|h, h_0) &\sim \mathcal{IG}(\nu_{\omega_h^2} + \frac{T}{2}, (S_h + (h - h_0)'H'H(h - h_0))/2), \\(\omega_g^2|g, g_0) &\sim \mathcal{IG}(\nu_{\omega_g^2} + \frac{T}{2}, (S_g + (g - g_0)'H'H(g - g_0))/2).\end{aligned}$$