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On the influence of conventional and automated market makers on market quality in cryptoeconomic systems

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Abstract

Decentralized exchanges (DEXs) have become an alternative to centralized exchanges (CEXs) for trading assets in the form of tokens in cryptoeconomic system markets. The emergence of DEXs is strongly driven by their potential to tackle challenges for market quality originating from CEXs by design, such as opaque market-making strategies and centralization of power. However, it remains unclear to what extent DEXs can enhance market quality compared to CEXs. A core reason for this is the lack of an analysis concept for investigating influences of market makers, including automated market makers (AMMs) used in DEXs and conventional market makers used in CEXs, on market quality in cryptoeconomic systems. To better understand influences of market makers on market quality in cryptoeconomic systems, we developed an analysis concept based on our formal price model grounded in established concepts of market microstructure. We demonstrate the usefulness of the analysis concept by examining conventional market makers on CEXs (i.e., Binance and Coinbase) and automated market makers (AMMs) on DEXs (i.e., Uniswap v2 and Uniswap v3). The main purpose of this work is to support the analysis of influences of different market makers on market quality in cryptoeconomic systems. This is useful to better understand how cryptoeconomic systems can ensure high market quality and safeguard market participants, when issuing tokens.

Keywords Cryptoeconomic system · Automated market maker · Centralized exchange · Decentralized exchange · Market quality

JEL Classification D40 · G10

Introduction

Cryptoeconomic systems, such as the Bitcoin and Ethereum systems, are sociotechnical systems wherein market participants (e.g., individuals, organizations, and software agents)

manage ownership of assets represented as digital tokens that are secured by cryptographic techniques and can be traded instantaneously (Sunyaev, Kannengießer, Beck, Treiblmaier, Lacity, Kranz, Luckow, 2021).

By trading tokens, markets emerge. Participants in such markets need the ability to execute trades at desirable prices and manage risks, such as quickly opening/closing large trading positions. Such needs can be fulfilled when markets exhibit high market quality. Liquidity is particularly important for market quality (Chordia, Roll, Subrahmanyam, 2008).

To reach high market quality, token issuers in cryptoeconomic systems list their tokens on centralized exchanges (CEXs) in order to achieve sufficient liquidity for token issuance and trading. CEXs often provide market making services that provide liquidity to cryptoeconomic system markets. Being able to influence the liquidity of markets, market makers of CEXs can strongly affect market quality in cryptoeconomic system markets (Barbon & Rinaldo, 2023; O'Hara, Ye, 2011).

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Driven by technological advances in distributed ledger technology (DLT) and the vision of decentralized crypto-economic systems, decentralized exchanges (DEXs) emerged (Xu et al., 2023). Instead of conventional market makers used in CEXs, DEXs typically use automated market makers (AMMs). AMMs are market makers that are implemented as software agents (based on smart contracts) that execute transparent and persistent market making strategies based on mathematical functions (Xu et al., 2023; Kirste et al., 2023). For example, AMMs like Uniswap v2 (Hayden et al., 2020) allow market participants to continuously exchange Ether (ETH) and USD Coin (USDC), while a constant-product function determines the exchange rate.

Market making strategies based on mathematical functions are transparent and assumed to be less dynamic than those of conventional market makers on CEXs (Aoyagi & Ito, 2021). AMMs seem to be useful in tackling challenges related to conventional market makers in crypto-economic system markets. Notwithstanding the assumed benefits of AMMs over conventional market makers, the extent to which AMMs influence market quality remains unclear.

AMM development is predominantly driven by practitioners focused on technological innovation. As a result, existing concepts from financial literature to analyze market quality are rarely adopted or modified to be suitable for crypto-economic system markets.

The often unclear applicability of well-known concepts prevalent in the finance literature (Amihud, 2002; Hendershott, Menkveld, 2014) make it difficult to analyze and compare the influence of conventional market makers and AMMs on market quality in crypto-economic system markets. An analysis concept is needed to better understand how AMMs can influence market quality in crypto-economic systems markets. We answer the following research questions: *What is a useful analysis concept to examine, and what are the actual influences of conventional and automated market makers on market quality in crypto-economic systems?*

Our work lays a foundation for analyzing and comparing the influence of market makers on market quality in crypto-economic systems. We have four main contributions. First, we present a formal price model based on well-known concepts in finance literature on market microstructure. The formal price model builds the foundation to understand price discovery, how the execution price of a trade is determined, and introduces concepts to analyze market quality. This is helpful in better understanding price evolution in markets. Second, we present an analysis concept that uses our formal price model to analyze the influence of market makers on market quality. The analysis concept supports analyses and comparisons of market quality and liquidity provided by market makers on CEXs and DEXs. This is useful for market partic-

ipants to assess suitable markets for token trading. Third, we analyze the influences of conventional market makers and AMMs on market quality by applying our analysis concept to historical market data during a 6-month timeframe. To be representative, the analyzed timeframe covers sideways movements of prices as moments of equilibrium and larger price downturns due to the FTX bankruptcy, as a moment of definite non-equilibrium. The analysis reveals the influence of different market makers on market quality in crypto-economic system markets and showcases the utility of our analysis concept. Fourth, by offering evidence for assumptions on market makers, we support assessing market impact, market quality, and liquidity. This is useful to optimize trade execution and reduce risks in trades.

The remainder of this work is structured into six sections. In the next section, we elucidate the foundations relevant to understanding the influence of market makers on market quality in crypto-economic system markets. In Section “[A formalized price model, data accessibility, and analysis concept](#),” we present a formalized price model and an analysis concept to measure the influences of market makers on market quality. In Section “[Methods](#),” we describe how we applied the analysis concept to analyze the influences of three different market makers types (i.e., conventional market makers on Binance and Coinbase, Uniswap v2, Uniswap v3) on market quality. We present the analysis results in Section “[Liquidity-based influences of market makers on market quality in crypto-economic systems](#).” In Section “[Discussion](#),” we discuss the principal findings from the analysis. Moreover, we explain our contributions to practice and research. Then, the limitations of the findings presented in this work are explained, and future research directions are showcased. In Section “[Conclusions](#),” we conclude this work with our personal takeaways.

Background

To better understand the interrelationships between market making strategies and *market quality* relevant to this work, we describe the foundations of *market quality*, conventional market making, and automated market making. We use ***bold-face italic characters*** for term definitions. *Italic characters* indicate the use of already defined terms elsewhere within the paper.

Market quality, market makers, and adverse selection

Price discovery and market quality

Like in conventional financial markets, traders in crypto-economic system markets aim to swiftly buy or sell assets

(e.g., shares and stocks represented as tokens) at reasonable prices and transaction costs. *Market quality* characterizes the ability to do so. We introduce the aspects that contribute to *market quality* in the following, concluding with its definition.

The *effective price* paid or received by the trader in a specific trade event is not known beforehand, but the result of a *price discovery* process. The *price discovery* process has a key role for *market quality*. It incorporates two basic price contributions to the *effective price*: first, a more or less accurate reflection of the true value of the asset, and second, the price impact of the traded volume, referred to as *market impact*. The *market impact*'s price contribution to the *effective price* depends on the *liquidity* of the market.

To better understand the interrelationships of those aspects, we give a simplistic view of the *price discovery* process before offering more details. We split the process into two hypothetical steps, according to the two basic price contributions given above. In the first step, a true value-based *reference price* is assumed to be established. For perfectly efficient markets, referred to as being *in equilibrium*, most market participants know the information about the true value of a traded good, and all available information (e.g., past prices and future expected returns) is incorporated in prices. Therefore, the (reference) price is close to the true value (Fama, 1970; Zhang, 1999).

In the second step, trade orders (i.e., orders to buy or sell a certain volume of an asset) are collected and matched. As the trade volume related to buying and selling typically does not exactly cancel at every point in time, a so-called *net order volume* remains. This *net order volume* is absorbed by *market makers* who are willing to do so but at a price premium. This price deviation typically increases for increasing *net order volume*. The *net order volume*, therefore, has a directional effect on the price. In case the *net order volume* stems from an individual trader solely, we refer to the trade as a *directional trade* (Madhavan, 2000; Farmer, 2002).

The time-resolved price impact of *net order volume* can basically be split into two components. Firstly, the overall dynamics of instantaneous price change followed by partly price recovery, referred to as *transitory price effects* (e.g., price pressure effects; Hendershott, Menkveld, 2014). Secondly, the remaining persistent price change after recovery.

A market is referred to as a *liquid market* if the price impact of (*net*) *order volume* size is small. The relationship between *net order volume* size and asset price is typically referred to as the *market impact function*. The *slope* of the *market impact function* is approximately inversely proportional to *liquidity* (Madhavan, 2000; Farmer, 2002). In other words, the more *liquid* a market, the smaller the *slope* of the *market impact function* and hence the price impact of *net order volume*. This illustrates how *liquidity* has a direct impact on the *effective price* and *market quality*.

In real-world markets, prices do not adjust to value changes instantaneously. This is due to information asymmetry, nonlinear dynamic effects, and marginal arbitrage costs (Zhang, 1999; Farmer, 2002). Continuously changing environments lead to at least some transient moments of *non-equilibrium*. Within these moments, the actual trade orders might inherently contain information about a value attribution adaption not yet publicly known, especially also not by the *market makers* (Beja, Goldman, 1980; Zhang, 1999). These information asymmetries are to be considered by the *market makers* and lead to larger price premiums and reduced *liquidity*. Therefore, the above hypothetically separated steps of *reference price* discovery and *liquidity provisioning* are, in fact, a continuous, interwoven process. Information gathering about reasonable pricing is partly trade external (we refer to this as *external price effects* in the following) and partly from trade signals.

Based on the above-defined concepts and according to O'Hara and Ye (2011), *market quality* can be defined as a market's ability to meet its dual and involved goals of efficient *price discovery* and *liquidity*.

The cost of liquidity: market makers and adverse selection risk

Market quality depends on the *liquidity* provided by economically rational market participants submitting non-matching bid/ask orders (e.g., via limit orders to the order book) at which they are willing to buy/sell assets and thereby absorb *net order volume*. These market participants are referred to as *market makers* (Madhavan, 2000). Typically, bid prices are placed below and ask prices above the estimated *reference price*, respectively, such that the market making activity under close to *equilibrium* conditions, where the price movement resembles a random walk, is in principle profitable (Kyle, 1985). The peaks in supply and demand caused by asynchronous trading activities balance out over time, and *market makers* can maintain a balanced inventory (Madhavan, 2000).

In an efficient market, *market makers* compete for the opportunity to absorb the *net order volume* and thereby extract a surplus. This competition is the driving force for a small bid/ask price gap and a small *slope* of the *market impact function*.

Real markets, however, show more or less strongly pronounced transient moments of *non-equilibrium*, where prices do not reflect all available information (Fama, 1970; Zhang, 1999). Thereby, *market makers* face an *adverse selection risk* through asymmetric information because there potentially are asymmetries between market participants in terms of information about asset valuation (Akerlof, 1978). Informed traders exploit these asymmetries by creating a surplus of supply or demand that is absorbed by the less well-informed

market makers. The price evolution under *non-equilibrium* conditions resembles a random walk with potentially large mean value drift or even jumps. Thereby, as it is often the case for cryptoeconomic system markets, the *market maker* is at risk of building up a large *inventory imbalance*, holding more of the less-worthy assets. The imbalance can only be re-balanced at a loss. These potential losses are referred to as **adverse selection cost** (Neal, Wheatley, 1998; Kyle, 1985; Akerlof, 1978).

Market makers are rational market participants depending on economic sustainability. Therefore, they need to account for the *adverse selection risk* in their typically opaque pricing strategies. This leads to larger bid/ask gaps, depending on the uncertainty in the market. In addition, *market makers* are required to adapt *liquidity* positions swiftly in dynamic market situations (leading to reduced *liquidity* associated with increased *slope* of the *market impact function*) (Bagehot, 1971; Menkveld, Wang, 2013). This adaption can even create a self-reinforcing spiral, for example, through panic selling. *Market makers* keep on removing *liquidity* when prices increasingly fluctuate. This fluctuation, however, is intensified by the continuously decreasing *liquidity*. Market participants, therefore, encounter higher transaction costs (large bid/ask spread and price impact) and significantly heightened price volatility in *non-equilibrium*. These effects severely constrain their ability to execute trades at desirable prices and manage risk by quickly opening/closing trading positions to re-balance their inventory without loss (Zhang, 2010).

To summarize, *liquidity* comes at a risk and hence at a cost, referred to as the **cost of liquidity**, the trader has to pay for. Due to the dynamics of the underlying processes, *liquidity* and *market quality* related thereto may be fragile.

Conventional and automated market making in cryptoeconomic systems

The following subsections illustrate the importance of separation of concerns between trade process operationalization (i.e., exchange operation), *initial public offering* (organized by the *underwriter*), and *market making*. Violating that separation in cryptoeconomic system markets can lead to financial losses of market participants and dramatic breakdowns of markets.

On the importance of separation of concerns: the interplay between exchanges, underwriters, and market makers

Exchanges make an important contribution to *market quality* as they provide the technical operationalization of trade processes. Exchanges bring together a reasonable amount of buyers, sellers, and *market makers*, forming the technical

manifestation of “the market.” If the technical operationalization of an exchange is provided by a single entity or a small group of entities or institutions, the exchange is referred to as **centralized exchange (CEX)** in cryptoeconomic systems.

In principle, exchange operation and market making (i.e., *liquidity provisioning*) should be separated to prevent conflict of interest. In a nutshell, the ability to analyze incoming trade orders provides an information advantage no market participant has. Intermixing market making with exchange operations may provide an unfair advantage over all other market participants and entails market manipulation risks.

In matured markets, the exchange infrastructure is fragmented into many providers competing for market participants, jointly forming a virtual overall market (O’Hara, Ye, 2011). In immature markets (e.g., early cryptoeconomic system markets), there are only a few possibilities to exchange assets, offered by a small amount of providers. This entails the risk of dominating providers exploiting their supremacy. This dominance is problematic for traders (possibly paying excess premiums) and newcomer projects because the ability to raise capital is crucially affected by the access to trading facilities, for example, being listed on an exchange and the organization of **initial public offerings (IPO)** (Madhavan, 2000). In cryptoeconomic system markets, *IPO* is also referred to as **initial coin offering (ICO)**.

IPOs are usually organized by so-called **underwriters**. The *underwriter* assumes the risk of purchasing the securities from the issuer and then selling them to the public or institutional investors. This places *underwriters*, especially in not well-developed markets, in a special position that may be exploited, as it is long known for conventional financial markets (Chen, Ritter, 2000). In addition, *underwriters* may become dominant *market makers* in the *IPO* aftermarket, giving them considerable ability to affect asset prices (Ellis, Michaely, O’Hara, 2000). This is most often the case for cryptoeconomic system markets.

To complete what could be regarded as the financial systems “hat trick” in unregulated cryptoeconomic system markets, the three roles of exchange, *underwriter*, and dominant *market maker* are typically closely entangled. This fraudulent entanglement has been shown to harm honest market participants massively. The breakdowns of *CEXs*, such as Mt. Gox in 2014 (Sidel et al., 2014; Leising, 2021), QuadrigaCX in 2019 (Deschamps, 2020; Doug, 2019; Ontario Securities Commission, 2020), and FTX in 2022 (Huang et al., 2022; Berwick et al., 2022; Scharfman, 2023), showcase the vulnerability of central parties combining *exchanges*, *underwriters*, and *market makers*. The entanglement of FTX and Alameda Research, as the main *market maker* and *underwriter* for FTX, showcases how *market makers* could fraudulently manipulate token prices (e.g., FTT, the native utility token of the FTX platform) and wrongfully use

more than half of FTX's customer funds to compensate for losses caused by risky market making (Berwick et al., 2022; Huang et al., 2022). Apparently, dependencies on fraudulently entangled intermediaries lay at odds with the core idea of cryptoeconomic systems (Nakamoto, 2008; Sunyaev, Kanengieser, Beck, Treiblmaier, Lacity, Kranz, Luckow, 2021). Decreasing the reliance of market participants on dominant parties is a key motivation for developing *automated market makers* and *decentralized exchanges*.

Automated market makers and decentralized exchanges

Automated market makers (AMMs) are *market makers* implemented as software agents that operate in DLT systems. AMMs determine asset prices in an automated and transparent manner. Market participants can trade with AMMs anytime, without requiring trust in intermediaries (Xu et al., 2023; Mohan, 2022). Depending on the specific AMM protocol, the general exchange process is mapped to the individual trader-AMM transactions (today's state of the art). Alternatively, order book functionality and batch-settlement of several trade orders against each other and the AMM in a simultaneous fashion is enabled.

Strictly speaking, AMMs combine the exchange process with market making and are therefore more generally referred to as *decentralized exchanges (DEXs)*. Making use of DLT, the shortfalls of exchange-market maker entanglement of CEXs are circumvented in a transparent and tamper-proof way by design, preventing central points of manipulation and failure.

DEXs may also provide means for holding ICOs, so-called *initial DEX offerings (IDO)*, and thereby, in addition, remove the dependency on *underwriter* intermediaries, which should resolve the possibly fraudulent "hat trick" discussed above (Zargham et al., 2020).

To cope with *adverse selection risks* (see Section 2.1.2), conventional *market makers* use opaque and highly dynamic strategies to determine prices and amounts of *liquidity* they provide to markets. In contrast to that, AMMs are fully transparent and, in many cases, mostly persistent (Kirste et al., 2023) by applying mathematically specified price functions to determine prices, typically based on their inventory and the amount of tokens that should be exchanged (Hayden et al., 2020), thereby explicitly encoding the *market impact function*.

AMM designs differ in how *liquidity* is provided and used, prices are determined, and surplus from *market making* is shared. In our previous work (Kirste et al., 2023), we present an AMM taxonomy that conceptualizes the design space of AMMs. For details, we refer the reader to our work on designs of AMM (Kirste et al., 2023) and the systematization of knowledge by Xu et al. (2023).

Contemporary AMMs commonly source required *liquidity* from deposits of *liquidity* providers. *Liquidity* providers are market participants that take the risk of *divergence loss* (also called impermanent loss) related to diverging prices and inventory of the deposited asset pairs for retrieving a proportion of the shared surplus from *market making*.

As there is no free lunch, we expect that, due to the design of most liquidity pool-based AMMs, the *cost of liquidity* might well be higher than with opaque and highly dynamic strategies.

A formalized price model, data accessibility, and analysis concept

AMMs seem to have several benefits compared to conventional *market makers*, such as transparent and persistent trading strategies. However, the extent to which AMMs can help to improve *market quality* compared to conventional *market makers* is barely understood. To better understand the influences of *market makers* on *market quality*, an analysis concept capable of analyzing *market quality* based on different data sources (e.g., trade or order book data) is needed. In the following sections, we present a concept for measuring the influence of *market makers* on *market quality*. Moreover, we explain how the analysis concept can be applied to analyze the influence of *market makers* on *market quality*.

Formalization and elucidation of the price model

Formal price model

We define the *market impact function* \mathcal{M} as depending explicitly on the *net order volume* ω (the part of the *total trade volume* V , absorbed by the *market makers*). We relate \mathcal{M} to the price P as follows:

$$P_j = P_{j-1} + \Delta P_j^{ext} + \mathcal{M}(\omega_j, j) \quad (1)$$

The price pair P_{j-1} and P_j refer to the price "before" and "after," while "before-after" has two distinct meanings in the following analysis. The first meaning is before and after event j , respectively (e.g., a trade of *net order volume* ω_j against the *market maker* or order book). The second meaning relates to a price at the beginning and end of a timeframe j (e.g., in 1-min trade data set). The effect of the value attribution adaption related *external price effects*, discussed in Section "Price discovery and market quality," is represented twofold in Eq. 1. On the one hand, the price delta ΔP_j^{ext} corresponds to a shift of the *reference price*. An example is the shift of the mid-price, the price halfway between the highest bid and lowest ask price in order book-based exchanges due to order

book position updates occurring independently from trades. On the other hand, the *market impact function* \mathcal{M} may change its shape, which is indicated by the explicit dependency on j . In the order book example, this corresponds to the change in *liquidity* distribution due to order book updates.

A usual representation of price evolution is the normalized price action referred to as **return** R :

$$R_j = \frac{P_j - P_{j-1}}{P_{j-1}} \quad (2)$$

Applying our relation (1), it follows:

$$R_j = \frac{1}{P_{j-1}} \left(\Delta P_j^{ext} + \mathcal{M}(\omega_j, j) \right) \quad (3)$$

Linearizing the *market impact function* provides the relation to the **price normalized slope** S and **liquidity** L :

$$R_j \approx \frac{\Delta P_j^{ext}}{P_{j-1}} + \omega_j \cdot \frac{1}{P_{j-1}} \cdot \frac{\partial \mathcal{M}}{\partial \omega} \Big|_{\omega=0, j} \quad (4)$$

$$\approx \frac{\Delta P_j^{ext}}{P_{j-1}} + \omega_j \cdot S_j = \frac{\Delta P_j^{ext}}{P_{j-1}} + \omega_j \cdot \frac{1}{L_j} \quad (5)$$

Note that the *slope* S and *liquidity* L absorbed the price normalization $1/P_{j-1}$, respectively.

In the following, we introduce two more concepts in the above notation. One from financial markets analysis and one from *AMM*-based *DEX* formalism. We do so to relate them to *liquidity* and discuss the similarities, differences and which parts can be extracted from the data analysis below.

In the context of trade data timeframe analysis, the concept of **illiquidity** (*ILLIQ*) (Amihud, 2002) is commonly used:

$$ILLIQ_j = \frac{|R_j|}{V_j} \quad (6)$$

The concept of *illiquidity* can also be applied as mean over a sequence of N events or timeframes:

$$ILLIQ_N = \frac{1}{N} \sum_{j=1}^N \frac{|R_j|}{V_j} \quad (7)$$

An established analysis, for example, is the yearly mean *illiquidity* from daily *returns* and volumes. *Illiquidity* is often used because it can easily be determined from price and total volume information that is widely accessible for basically any traded asset.

A standard term from the context of *AMM*-based *DEX* formalism, related to *return*, is **slippage** (*SLP*):

$$SLP_j = \frac{\bar{P}_j - P_{j-1}}{P_{j-1}} \quad (8)$$

\bar{P}_j refers to the **mean execution price** a trader trading against an *AMM* experiences. The difference between *slippage* and *return* stems from the fact that the *AMMs* typically apply a non-linear cost function prescribing a total amount of value to be paid or received for an amount of asset traded (usually referred to as **swapped**). When generalizing the cost function as a *market impact function* defined over an absolute inventory state Ω (denoted by \mathcal{M}^\dagger in the following), and assuming no parameter updates and adaptations to the *liquidity* pool occurred (indicated by the superscript *stat* in the following equation), the mean price can be given as:

$$\bar{P}_j^{stat} = \frac{1}{\omega_j} \int_{\Omega_{j-1}}^{\Omega_{j-1} + \omega_j} \mathcal{M}^\dagger(\Omega) d\Omega \quad (9)$$

Mapping parameter updates, change in *liquidity* pool volumes, and *liquidity* distribution (for *liquidity* concentrating *AMMs*) similarly to Eq. 1 provides:

$$SLP_j = \frac{\Delta P_j^{ext}}{P_{j-1}} + \frac{1}{P_{j-1} \omega_j} \int_{\Omega_{j-1}}^{\Omega_{j-1} + \omega_j} \mathcal{M}^\dagger(\Omega, j) d\Omega - 1 \quad (10)$$

Linearizing the *market impact function* as with Eq. 4,

$$SLP_j \approx \frac{\Delta P_j^{ext}}{P_{j-1}} + \omega_j \frac{S_j^\dagger}{2} = \frac{\Delta P_j^{ext}}{P_{j-1}} + \omega_j \frac{1}{2L_j^\dagger} \quad (11)$$

with

$$S_j^\dagger = \frac{1}{P_{j-1}} \cdot \frac{\partial \mathcal{M}^\dagger}{\partial \Omega} \Big|_{\Omega_{j-1}, j} \quad (12)$$

, illustrates the similarity to *return*. In the linear case, the difference lies only in a factor $1/2$.

Model-based relation between CEX and DEX

The generalized formulas allow to map different *AMM* types to standard exchanges: *DEX* parameter updates, adaptations to the *liquidity* pool volume (e.g., for function-based *liquidity*-concentrating *AMMs* like Uniswap v2) and change of *liquidity* distribution (e.g., for *liquidity* provider-based *liquidity*-concentrating *AMMs*, like Uniswap v3) are implied in the shape change of \mathcal{M} (indicated by the explicit dependence on j). This corresponds to the *liquidity* distribution change in the previous order book example.

The price adopting step of accordingly labeled *AMMs* (like Dodo) is mapped to ΔP_j^{ext} , while this term vanishes for price-discovering *AMMs* (e.g., Uniswap v2, v3).

Elucidating price model terms and accessibility from exchange data

The individual terms of the price model given in Section “Formal price model” can be elucidated based on the real-world effects and the accessibility from exchange data.

Data related to exchanges can be ordered in a sequence of accessibility. Accessibility relates to principal availability, paid access, and complexity of retrieval and processing, such as retrieving historical DEX data from blockchain archive nodes. In the following, a concise overview (also summarized in Table 1) for discussing the identifiability of the basic terms is provided. Details for the specific data used and its (pre-) processing for the analysis are given in Section “Methods.”

CEX Basic time-frame cumulated trade data typically provides price (P) and *total volume* (V) and allows to determine *return* (R , Eq. 2) and *illiquidity* ($ILLIQ$, Eqs. 6 and 7). The concept of *illiquidity* and related input does not allow to resolve for the *external price effect* (ΔP^{ext}) and *market impact function* (\mathcal{M}). In addition, *illiquidity* may diverge for small volumes and is therefore typically applied for wider time-frames and to provide an easily accessible, coarse, typically noisy, and less accurate combination of relevant effects (Amihud, 2002).

However, for trading timeframe length getting smaller and individual order volume larger, it is increasingly improbable that a matching counter-order occurs, hence the order will mainly be absorbed by the *market makers* and $V \rightarrow \omega$. Under this condition, and close to *equilibrium* ($\Delta P^{ext} \rightarrow 0$), the actual *market impact function* could, in principle, be resolved. However, it would mean that specific large trade events are required to exist and need to be isolated from the data. In addition, the approach does not allow to explicitly separate the *net order volume* dependence from the evolution over j (i.e., $\mathcal{M}(\omega_j, j)$). The reason is that in order to approximately resolve the ω dependence requires a set of N “atomic” datasets sampling different ω values, however also sampling different shapes of \mathcal{M} related to the explicit j dependence. This is indicated by *unresolved* ω_j, j in Table 1.

CEX timeframe cumulated, taker/maker volume enriched trade data adds additional information about the cumulated taker volume, which enables resolving for the *net order trade volume* (ω). Analyzing *return* vs. ω for a set of N atomic datasets allows to determine an approximate, *external price effect* noised *market impact function*, *slope* and *liquidity*, respectively, therefrom. For close to *equilibrium* conditions (i.e., $\Delta P^{ext} \rightarrow 0$), the actual *market impact function* can be resolved.

The noisiness of so determined *market impact* can indicate the lead/following character of exchanges. As discussed in Section “Price discovery and market quality,” *reference price* adaption and *liquidity* provisioning are a continuous, complex, interwoven process. Information gathering about

reasonable pricing is partly trade external (mapped by ΔP^{ext} in the formalization) and partly from trade signals (reflected in \mathcal{M} , explicit dependency on j). Extracting the *market impact function* from timeframe cumulated, taker/maker volume enriched trade data can, therefore, be expected to be noisy, with the strength of noise being related to the underlying adoption of value attribution. If the adaption is implicitly contained in trade signals, the effect is more covered by \mathcal{M} .

When comparing two exchanges of the types order book-based CEXs or price adopting AMMs, one can, therefore, expect that the exchange that has more of a lead character to have a less noisy *market impact function*, compared to the following exchange, when determined from timeframe cumulated trade data. This is because the value attribution adaption manifests implicitly in the trade data on the lead exchange, which, however, makes it explicit. For exchanges with more of a following character, the information then is explicit and hence taken into account in the pre-trade order book update or price adoption for price-adopting AMMs. Therefore, the downstream price action on these exchanges has a larger ΔP^{ext} contribution, inducing a less well mapping of the *return* as a direct function of \mathcal{M} and ω , hence a more noisy *market impact function* when determined from timeframe cumulated trade data.

CEX order book-update event-resolved data permits to access the *market impact function* as it would be experienced by a trader trading any *directional trade volume* against the *market makers*. At any given point in time, the effect of trading a volume ω_j against the order book. Hence, $\mathcal{M}(\omega_j, j)$ can be calculated from the distribution of *liquidity*.

CEX trade event data allows to directly access the *market impact function* as it was experienced by the trader trading a specific *directional trade volume*. However, it does not allow to explicitly separate the *net order volume* dependence from the evolution over j (i.e., $\mathcal{M}(\omega_j, j)$).

CEX combined order book-update and trade event data permits to differentiate the source of order book updates in trade and non-trade-related price changes. The non-trade-related change of, for example, the midprice is related to ΔP^{ext} . The order book adaption originating from trades and the trade data, respectively, provide information about the *net order volume* dependence of the *market impact function* (i.e., $\mathcal{M}(\omega_j)$). The order book evolution after trade events is related to the evolution of the *market impact function* (i.e., $\mathcal{M}(j)$). Therefore, *transitory price effects*, such as the short-term recovery of *liquidity* after trades and persistent changes, can be resolved. This allows a comprehensive analysis of micro market effects and market anomalies (Chordia, Subrahmanyam, Tong, 2014; Amihud, 2002).

For AMM-based DEXs, in principle, all data is available on a per-event basis due to the publicly distributed nature of DLT systems. Given the exact AMM design and extracting the (historic) on-chain AMM’s state allows reconstructing

Table 1 Terms based on the price model and accessibility from exchange data

Type of data	R	$ILLIQ$	SLP	ω	ΔP^{ext}	Market impact function noisy	Market impact function	Transitory price effects
CEX basic timeframe cumulated trade data	X	X	-	$(\Delta t \rightarrow 0, V_{order} \rightarrow \infty)$	-	$(V \rightarrow \omega, \Delta P^{ext} \rightarrow 0)$		-
CEX timeframe cumulated, taker/maker volume enriched trade data	X	X	-	X	-	unresolved ω_j, j	$(\Delta P^{ext} \rightarrow 0)$	-
CEX order book-update event resolved data	X	-	X	-	-	X^*	X	-
CEX trade event data	X	X	-	X	-	unresolved ω_j, j	$(\Delta P^{ext} \rightarrow 0)$	-
CEX combined order book-update and trade event data	X	X	X	X	X	X^*	X	X
DEX with price-discovering AMM trade event data	X	X	X	X	$\Delta P^{ext} \stackrel{!}{=} 0$	X	$(\Delta L \rightarrow 0)$	-
DEX with price-discovering AMM full state reconstruction	X	X	X	X	$\Delta P^{ext} \stackrel{!}{=} 0$	X^*	X	X
DEX with price-adopting AMM trade event data	X	X	X	X	-	unresolved ω_j, j	$(\Delta P^{ext} \rightarrow 0)$	-
DEX with price-adopting AMM full state reconstruction	X	X	X	X	X	X^*	X	X

- indicates that a term cannot be determined from the given data source.

X indicates that a term can be determined, while X^* hints that a more precise term is applicable.

All other texts give information about the constraints under which the term can be determined

every detail. In the following, we group sensible combinations based on the complexity of retrieval or reconstruction.

DEX with price-discovering AMM trade event data allows to determine a noisy *market impact function* without the need to know further parameters or AMM mechanisms. However, noise does not come from external price effects but from *liquidity* updates (ΔL), which cannot be extracted from trade data alone. For diminishing *liquidity* updates ($\Delta L \rightarrow 0$), the *market impact function* can be determined exactly because these types of AMMs determine prices strictly following the encoded *market impact function* (i.e., no price jumps, $\Delta P^{ext} \stackrel{!}{=} 0$). This does not mean there exists no external adaption of value attribution, but rather that such AMMs map any price adaption onto movement along the defined *market impact function*. See also the discussion of the related implications on the rational economic limit of these AMMs in Section “[Discussion](#).”

DEX with price-discovering AMM full state reconstruction corresponds to full trade and *liquidity* adoption information. For price-discovering AMMs, the *market impact function* is fully defined for every point in time (i.e., $\mathcal{M}(\omega_j, j)$), allowing to determine the ω_j dependence and the time evolution. This makes DEX with price-discovering AMM full state reconstruction data comparable to CEX combined order book-update event-resolved data.

DEX with price-adopting AMM trade event data does neither allow to determine the *external price effect* from price adoption (i.e., $\Delta P^{ext} \neq 0$), nor *liquidity* updates. The situation is comparable to CEX trade event data.

DEX with price-adopting AMM full state reconstruction allows to extract full information, including *liquidity* updates and *external price effects*, comparable to CEX combined order book-update and trade event data.

Analysis concept

The analysis concept is based on the price model given in Section “[Formal price model](#)” and measures the influence of *market makers* on *liquidity*-related aspects of *market quality*, such as the *market impact function* and especially its *slope*. The following subsections present the analysis concept for the subset of aspects relevant to this context.

Determining liquidity from timeframe cumulated taker/maker volume enriched trade data

Following the price model discussion given in Section “[Elucidating price model terms and accessibility from exchange data](#),” analyzing the influence of *net order volume* (ω) on *return* allows to determine an approximate, *external price effect* noised *market impact function* \mathcal{M} , however, without the ability to separate the *net order volume*

dependence from the *slope* evolution over j (i.e., $\mathcal{M}(\omega_j, j)$) explicitly.

In order to sample a representative range of *net order volumes*, an *overarching set* of N subsequent *atomic datasets* (i.e., timeframe cumulated and indexed by j) is used as an analysis basis. N might be windows spanning over, for example, $\Delta T = 6$ h, while j relates to $\Delta t = 1$ min cumulated datasets.

More precisely, *returns* R_j of all *atomic* timeframes j within the *overarching set* can be grouped into uniformly spaced ω -bins, based on the return’s associated *net order volume* ω_j to a collection $R_N = \{(R_0; \omega_0), (R_1; \omega_1) \dots (R_n; \omega_n)\}$.

The noisy *market impact function* \mathcal{M}_N can then be related to the distribution of the per bin determined median values $\tilde{R}_N(\omega)$, while the per bin statistics, for example, inner quantile range¹ *IQnR* indicates the per bin representativeness of such median values. The number of datasets per bin c can be used to weight individual bins, for example, $w = \sqrt{c}$ supposing normally distributed data.

Employing the linearization given in Eq. 4 allows applying a weighted straight line fit to $\tilde{R}_N(\omega)$ and extracting the *slope* \tilde{S}_N . The *weighted normalized mean squared error* (WN-MSE) can be used to evaluate the fit quality.

Determining liquidity from order book event resolved data

Given order book-update event-resolved data, the *market impact*, as it would be experienced by a trader trading any *directional trade volume* ω_j against the *market makers*, can be determined at any given point in time. Hence, $\mathcal{M}(\omega_j, j)$ can be explicitly calculated from the limit order distribution.

A sensible approach for comparing trade with order book data is to choose the same set of ω -bins for both. The resulting collection of returns with one dataset per bin can then be further processed just as the set of trade data returns discussed above. To achieve relative importance of the individual bins comparable to the trade data set, the respective weights of timeframe cumulated trade data can also be used for the order book data collection fit.

Rolling window-based time evolution

The time evolution of the (noisy²) *market impact function* and *slopes*, determined following the approaches given in the previous subsections, can be analyzed by applying a *rolling window of length* ΔT for an *analysis timeframe* ΔT , for example, spanning over several months. To indicate the vari-

¹ a given range centered around the median, for example, 5 – 95%, corresponding to the *inner quartile range* (*IQnR*) which is defined to span over 25 – 75%

² in case of timeframe cumulated trade data.

ability of the calculated *market impact function* and *slopes*, the fit quality, median, and *IQnR* values can be calculated from the time evolution.

Methods

To demonstrate the utility of the developed analysis concept, we analyzed the influence of market makers on market quality using the analysis concept and actual data. In the following, we describe how we proceeded in that analysis.

Data sources and preprocessing

To analyze *liquidity* provisioning of *market makers* in cryptoeconomic systems, we used historical trade data and order book-update event-resolved data from *CEXs* (i.e., Binance and Coinbase) and historical on-chain data from *DEXs* (i.e., Uniswap v2 and Uniswap v3). To process the data, we used standard Python libraries, such as matplotlib, numpy, pandas, and seaborn.

We analyzed representative *CEX* and *DEX* markets for Bitcoin–US dollar and Ethereum–US dollar pairs in the most liquid representation per market. We considered Wrapped Ether and Ether to be on par, as well as USDT and USDC with USD. The exchanges and pairs have been selected based on trade volume and largest value locked, where applicable (i.e., liquidity pool-based *AMMs*). Table 2 shows the analyzed trading pairs and their 24-hour volumes.

Our analysis focuses on the *liquidity*-related influence of *market makers* on *market quality*. We selected two major *CEXs*, namely, Binance and Coinbase. The historical trade event data for *CEXs* was provided by *tardis.dev*. Based on historical trade event data, we calculated timeframe cumulated open, high, close, and low prices (OHCL data). We enriched this data by volume and *net order flow* (ω_j), based on the individual trades executed on the *CEXs*. We used 1-minute timeframes to sample the trade data for the analysis.

To reconstruct the past order book for the analysis of order book-update event-based data, we used historical incremental order book data. We took snapshots of the order book every minute. Each snapshot represents the states of the order book at a specific time, including the maximum bid/ask lev-

els available at snapshot time. We calculated *returns*, *log returns*, and *illiquidity* for a range of artificial *net order volumes* executed against the order book at every snapshot to resolve $\mathcal{M}(\omega_j, j)$ and stored this data. If the *liquidity* in the order book was not sufficient to satisfy large *trade volumes*, we asserted warnings and returned NaN values.

Next, we downsampled the *returns*, *log returns*, and *illiquidity* of the 1-minute lower resolution time frames by calculating the median values over 1-hour for speeding up the subsequent data processing to conduct our analysis.

We selected Uniswap v2 and Uniswap v3 as *AMM*-based *DEXs* to be analyzed in this work. The *AMMs* used in those *DEXs* represent common implementations of two different *liquidity* provisioning approaches: function-based *liquidity* concentration (used in Uniswap v2) and liquidity provider-based *liquidity* concentration (used in Uniswap v3). Given full state reconstruction (see Section “[Elucidating price model terms and accessibility from exchange data](#)”), the difference in price determination in *AMM*-based *DEXs* (i.e., price-discovering and price-adopting) plays a subordinate role in this work because price-adoption is related to the external price effect ΔP_j^{ext} . Therefore, in our analysis, Uniswap v3, a *price-discovering AMM* with liquidity provider-based *liquidity* concentration, is representative also for *price-adopting liquidity pool-based AMMs* with automatic *liquidity* concentration, such as Dodo.

To sufficiently reconstruct *AMM* states for the timeframe in the scope of the analysis, we gathered on-chain data related to state variables of smart contracts used in Uniswap v2 and Uniswap v3. For Uniswap v2, we gathered data on the actual reserves of *token0* and *token1*. For Uniswap v3, we gathered the *slot0* data (e.g., *sqrtPriceX96* and *tick*), and *current liquidity*. Moreover, we gathered data of all neighbor ticks up to a price change of plus-minus 3% of the state’s current token price, resulting in 300 ticks with a tick spacing of 10 for the WETH/USDC pair. Because *liquidity* is less likely to fluctuate on Uniswap v2 and Uniswap v3, we reconstructed *AMM* states hourly within our analysis timeframe.

We analyzed BTC and ETH as the tokens with the largest trading volume. We selected the trading pair with the highest trading volume on the individual exchange. For example, Binance has pairs such as BTC/USDT, BTC/USDC, BTC/BUSD, and BTC/TUSD. There, we selected the pair

Table 2 Overview of the analyzed historical exchange data per trading pair

Exchange	Pair	Data type	Sampling	Average 24 h Volume
Binance	BTC/USDT	trade, order book	1 min, 1 h	5365.7M USD
Binance	ETH/USDT	trade, order book	1 min, 1 h	786.6M USD
Coinbase	BTC/USD	trade, order book	1 min, 1 h	494.0M USD
Coinbase	ETH/USD	trade, order book	1 min, 1 h	433.8M USD
Uniswap v2	WETH/USDC	full state reconstruction	1 h	3.1M USD
Uniswap v3	USDC/WETH	full state reconstruction	1 h	40.1M USD

with the highest 24-hour trading volume. For Uniswap v2 and Uniswap v3, we used pairs with Wrapped ETH because the trading volume was higher than with native ETH.

We use the *WN-MSE* to quantify the fit quality of the *market impact function* linearization to the sampled *market impact* at time j . The *WN-MSE* uses trading *net order volume* dispersion and emphasizes outlier impact, providing a comprehensive efficacy measure of the *market impact function* fit. The fit of the *market impact function* is weighted based on trading activities. Relative errors allow to compare the effectiveness of the fit of the *market impact function* between different timeframes and datasets. Additionally, taking the mean of squared errors helps to compute a singular, overall indicator of the fit quality. The mean of squared errors places greater emphasis on outliers, and negative values are eliminated. This multifaceted approach helps to compute a detailed and accurate assessment of how well our fit of the *market impact function* represents the real *market impact function*.

Timeframe selection

To demonstrate the applicability of our analysis concept and measure the influence of different *market makers* on *market quality* in cryptoeconomic systems, we selected a 6-month time frame ($\Delta T = 6M$) from 2022-09-01 to 2023-02-28 for our analysis. The timeframe includes mostly sideward movements of Bitcoin and Ether prices in 2022-09, 2022-10, 2022-12, and 2023-02, while it also covers the FTX

bankruptcy in 2022-11 as a black swan event with a massive downturn of 26% for Bitcoin and 35% for Ether within three days. Bitcoin and Ether prices fully recovered in 2023-01. The time frame of the FTX bankruptcy in 2022-11 can be regarded as a moment of *non-equilibrium* for Bitcoin, Ether, and other tokens of cryptoeconomic systems. We assume that the selected timeframe is suitable for analyzing the influence of different *market makers* on *market quality* because this timeframe covers moments of *equilibrium* and *non-equilibrium*.

Liquidity-based influences of market makers on market quality in cryptoeconomic systems

This section presents the results of the analysis concept's utility demonstration. First, we show the fit quality that is achieved by our analysis concept to validate its suitability. Second, we present the time evolution of the *market impact function's slope* based on different types of data. Finally, we compare the influence of conventional and automated market makers on *slope* evolution and, hence, *market quality*.

Validating derived market impact functions and slope metrics

Figure 1 illustrates fitted *market impact functions*, *net order volume* distributions as bar plots, and *WN-MSE* values for

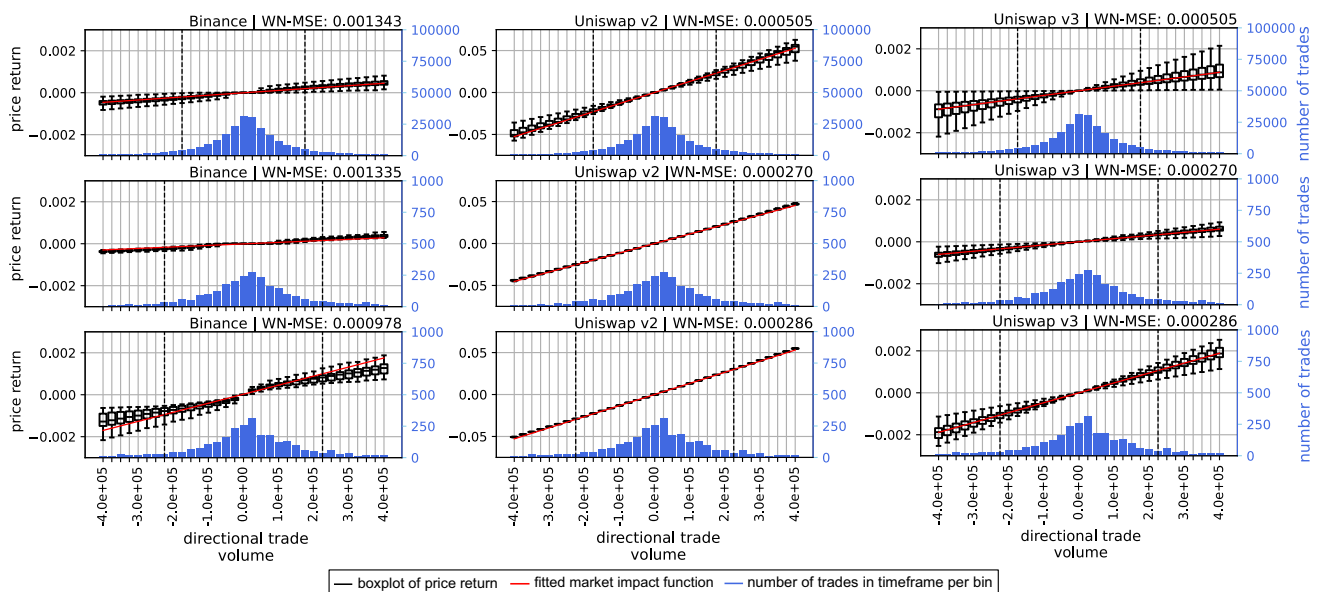


Fig. 1 Return over *net order volume* ω with fitted *market impact functions* weighted on the prevalence of ω for the ETH/USD pair. Horizontal: Binance, Uniswap v2, and Uniswap v3. Vertical: timeframes 2022-09-01 to 2023-02-28 (full timeframe), 2022-11-04 to 2022-11-05 (before FTX bankruptcy), and 2022-11-11 to 2022-11-12 (apex of the FTX bankruptcy)

Binance, Uniswap v2, and Uniswap v3, examined at three distinct timeframes: full 6 months, 1 day before, and 1 day during the FTX bankruptcy. The vertical lines illustrate the 95% $IQnR$ of the number of atomic datasets per ω -bin, indicating the region of main trading activity. The three timeframes sample overall, normal, and extreme price actions and can be regarded as representative of *equilibrium* and *non-equilibrium* conditions.

The results illustrated in Fig. 1 offer evidence for the consistent fit quality that is achieved by applying the analysis concept (see Section “[Analysis concept](#)”). This validates the chosen approach. achieved by and, hence, the validity of the chosen approach. Figure 2 shows the results of the WN - MSE for the fitted *market impact function* over time for the BTC/USD pair on Binance. The fit quality is sufficient for our analysis even during moments of strong *non-equilibrium*, as evidenced during the FTX bankruptcy (in early 2022–11). Here and for the following analyses, we indicate the fitting precision by analyzing the WN - MSE ’s median and 95% $IQnRs$. Subsequently, we report the WN - MSE median and $IQnR$ of the WN - MSE .

Results from time evolution of the market impact function’s slope metric

In the following subsections, we present the results from analyzing the time evolution of the *market impact function*’s slope metric.

Market impact function’s slope metric is in line with market microstructure theory

Figure 3 illustrates the evolution of the *market impact function*’s slope for timeframe cumulated maker/taker enriched trade data (first row), order book event-resolved data (second row), and the log price (third row) for the BTC/USD pair. The left and right columns juxtapose data from two conventional *market makers* of major *CEXs*: Binance (left) and Coinbase (right). The dashed lines indicate the respective median values over the complete analysis timeframe.

Regarding the slope metric, Figs. 1 and 3 show that the *market impacts function*’s slope is positive for all timeframes T . This empirical finding is in line with the basic hypothesis widely accepted in market microstructure economic literature: a positive *net order volume*, indicating a predominance of buy over sell orders, tends to exert upward pressure on prices and vice versa (Madhavan, 2000). This relationship underscores the interplay between market behavior and price evolution.

Median slopes from trade and order book data are comparable

The median slopes determined from timeframe cumulated maker/taker enriched trade data approximates the median slopes of order book-update event resolved data. For Binance

(top and middle in the left column of Fig. 3), the median slope values are closer ($0.56e^{-9}$ trade vs. $0.65e^{-9}$ order book data) than with Coinbase ($1.43e^{-9}$ trade vs. $1.16e^{-9}$ order book data). Overall, Binance has a 2.56 times flatter slope of the *market impact function*, indicating higher *liquidity* than Coinbase. This finding aligns with the more noisy trade data-derived slope of the *market impact function* for Coinbase. This present noise causes the large deviation of the *market impact function slopes*’ median values for Coinbase. The higher *liquidity* and the larger trade volume at Binance compared to Coinbase indicate that Binance could be regarded as a lead market. We discuss this observation in more detail in the following subsections.

Order book event-resolved data provides an accurate representation of the market impact function

For Binance, the time evolution of slopes derived from timeframe cumulated trade data and book-update event-resolved data (illustrated in Fig. 3) shows remarkable similarities. This indicates that the noisy *market impact function* approximates the real *market impact function*. Therefore, as discussed in Section “[Elucidating price model terms and accessibility from exchange data](#),” the effects from value attribution adaption leading to *external price effects* ΔP^{ext} and *market impact function* shape change can be assumed to be small in cumulated maker/taker enriched trade data on Binance. In contrast, the noisy *market impact function* on Coinbase strongly deviates from the non-noisy *market impact function* derived from order book-update event-resolved data. Therefore, the *external price effects* are larger on Coinbase.

Overall, the analyses support the price model-based argumentation given in Section “[Elucidating price model terms and accessibility from exchange data](#).” The order book event-resolved data provides a more accurate representation of the *market impact function*’s slope. Order book snapshots offer a clearer insight into the exchange-local market’s intrinsic behavior by focusing on immediate market conditions and excluding external effects.

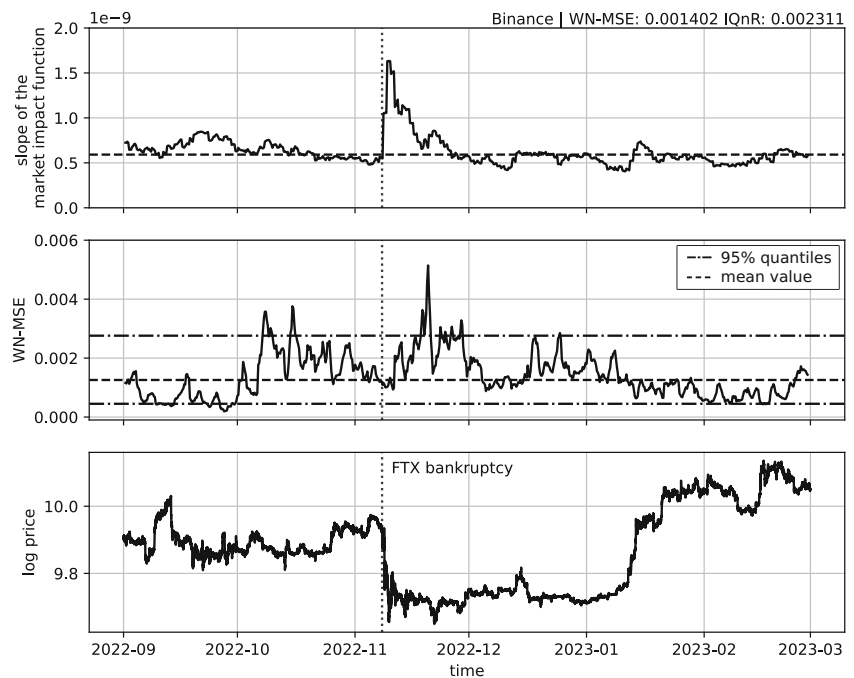
Lead markets can be identified via noisiness of the market impact function

The larger volume, higher *liquidity*, and smaller noise of the trade data derived slope for Binance, compared to Coinbase, is in line with the predictions based on the price model given in Section “[Elucidating price model terms and accessibility from exchange data](#).” Consequently, the theoretical and data-based analyses imply that Binance potentially assumes the role of a lead market for Bitcoin, while Coinbase appears to act as a following or reactive market.

Market makers avert adverse selection costs—liquidity is reduced in non-equilibrium

When examining the timeframe cumulated maker/taker enriched trade data for *CEXs* (e.g., Binance and Coinbase)

Fig. 2 Slope evolution of the market impact function over time (first row). WN-MSE with median and 95% quantiles lines (second row). Log-price evolution (third row). The BTC/USD pair on Binance with $\Delta T = 6M$ (2022-09-01 to 2023-02-28) and $\Delta T = 1d$



illustrated in Fig. 3, notable peaks in the *slope* of the market impact function can be observed in early November 2022. Those peaks correspond to an increased *slope* of the market impact function, indicating reduced market liquidity. The peak in early November is particularly noteworthy because it aligns with a significant downturn in Bitcoin's logarithmic price due to the FTX bankruptcy event. The correlation underscores the sensitivity of the market impact function's

slope to major market events, leading to liquidity shifts. This shows that market makers remove liquidity in moments of non-equilibrium to avert adverse selection costs.

Function-based liquidity-concentrating AMMs provide inferior average liquidity and market quality

Figure 4 provides a detailed visualization of the evolution of the market impact function's *slope* (first row) for order book-update event resolved data on Binance (orange lines),

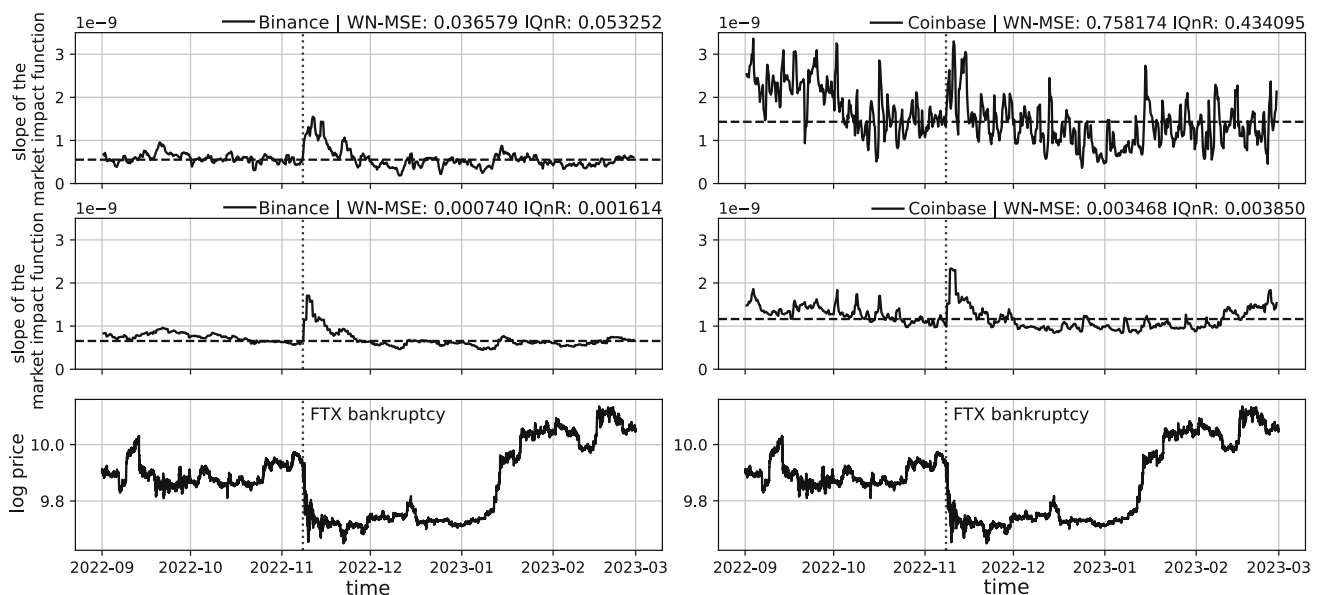


Fig. 3 Slope of market impact function for timeframe cumulated maker/taker enriched trade data (first row). The slope of market impact function for order book event resolved data (second row). Log-price evolution (third row). The BTC/USD pair on Binance (left) and Coinbase (right) with $\Delta T = 6M$ (2022-09-01 to 2023-02-28) and $\Delta T = 1d$

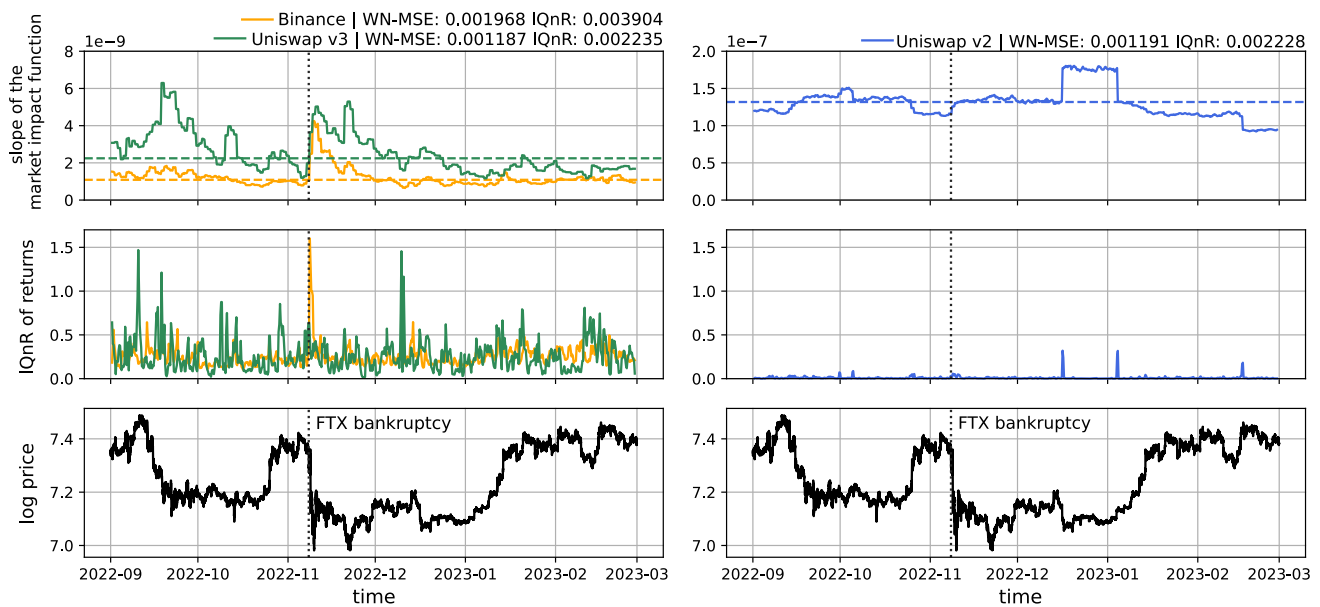


Fig. 4 Slope of *market impact function* for order book event resolved data with median lines (first row), *IQnR* of *returns* (second row) and log-price evolution (third row) for the ETH/USD pair on Binance (left, orange), Uniswap v3 (left, green) and Uniswap v2 (right, blue) for a rolling window width $\Delta T = 1d$ over the analysis period 2022-09-01 to 2023-02-28 $\Delta T = 6M$

Uniswap v2 (blue lines), and Uniswap v3 (green lines) for the ETH/USD pair. The second row illustrates the corresponding 95% *IQnRs* of the respective *slopes*. The log price of Ether is given in the third row.

Uniswap v2 is a representative of function-based liquidity-concentrating AMM implementations, which were invented way before the liquidity provider-based liquidity-concentrating AMMs. Therefore, we first compare Uniswap v2 against Binance. The median *slope* of the *market impact function* of Uniswap v2 ($1.32e^{-7}$), compared to Binance ($1.09e^{-9}$), is approx. 120 times larger. This indicates inferior average *liquidity* and *market quality* on Uniswap v2..

Function-based liquidity-concentrating AMMs show less detrimental external price effect-based liquidity dynamics

Although Uniswap v2 provides inferior average *liquidity* compared to Binance, it shows much smaller relative changes in *liquidity* over time (see the middle row in Fig. 4). These changes are uncorrelated to external events (e.g., the FTX bankruptcy). Obviously, the sharp changes in the *market impact function slope* on Uniswap v2 are caused by major *liquidity* providers temporarily removing their *liquidity*. In contrast, minor changes in the *market impact function's slope* on Uniswap v2 are correlated with external price-based trading against the AMM and follow the convex shape of the programmed *market impact function*.

One could, therefore, argue that function-based liquidity-concentrating AMMs, such as Uniswap v2, have the potential to provide more reliable *liquidity* to the market, especially in strongly *non-equilibrium* conditions, thereby ensuring certain levels of *market quality*. Nevertheless, the *liquidity* on

Uniswap v2 was way lower (approx. factor 120) than with Binance, even under strong *non-equilibrium* conditions with the FTX bankruptcy.

Liquidity provider-based liquidity-concentrating AMMs potentially provide CEX-competitive overall market quality, however may suffer stronger adverse selection costs

Function-based liquidity-concentrating AMMs (e.g., Uniswap v2) persistently distribute *liquidity* across the AMMs price range based on the implemented function. This differs from liquidity provider-based liquidity-concentrating AMMs (e.g., Uniswap v3). In these AMMs, *liquidity* providers have more individual influence on the *liquidity* distribution, for example, by specifying price ranges in which their *liquidity* will actually be distributed.

With a deviation of about a factor of 2.06, the median *slope* of the *market impact function* of Uniswap v3 (top-left subfigure, green line in Fig. 4) is about two orders of magnitude closer to Binance's *market impact function slopes* compared to Uniswap v2.

The *slope's* time evolutions of Binance and Uniswap v3 show similarities. However, the *slope* evolution for Uniswap v3 is more dynamic. This similarity indicates that *liquidity* providers of Uniswap v3 avert *adverse selection costs* by redistributing their *liquidity*, similar to CEXs. Therefore, one could argue that *liquidity* providers on liquidity provider-based liquidity-concentrating AMMs could pursue similar market making strategies as on order book-based exchanges through frequent *liquidity* reallocation. Such AMM designs inherit the disadvantages of conventional *market makers*, such as *liquidity* removal in moments of *non-equilibrium* to

avert *adverse selection costs*. The effect of *liquidity* removal in moments of *non-equilibrium* can be seen in the second row when looking at the *IQnR* of *returns*. For Binance and Uniswap v3, *returns* show a large per bin dispersion (large *IQnR* values), indicating large liquidity changes. In contrast, the *IQnR* of *returns* for Uniswap v2 show a small dispersion, indicating small liquidity changes, especially compared to Binance and Uniswap v3.

Depending on the AMM design, frequent *liquidity* reallocation can lead to even stronger overall *liquidity* fluctuation than on *CEXs*, as is the case for Uniswap v3. This worsens the *market quality* for such AMMs, especially in moments of *non-equilibrium*.

Discussion

Principal findings

Although AMM-based *DEXs* seem promising to tackle challenges of *CEXs* (e.g., separation of concerns, transparent and persistent *liquidity* provisioning), the extent to which AMMs operated in *DEXs* can enhance *market quality* compared to conventional *market makers* used in *CEXs* remains unknown. This makes the targeted use of AMMs and conventional *market makers* in cryptoeconomic systems to reach high *market quality* difficult. To analyze the influence of *market makers* on *market quality* in cryptoeconomic system markets, we present a formal price model and derive an analysis concept. We demonstrate the utility of our analysis concept by analyzing and comparing AMMs operated in *DEXs* (i.e., Uniswap v2, Uniswap v3) and conventional *market makers* on *CEXs* (i.e., Binance and Coinbase).

The analysis results show that trade data includes *external price effects* leading to noisy *market impact functions*. This is particularly observable in the comparison between *market impact functions* derived from cumulative maker/taker enriched trade data and order book-update event-based data (see Section “[Results from time evolution of the market impact function’s slope metric](#)”).

In the analysis of *external price effects* and the noisiness of the *market impact function* for Bitcoin, we observed that the *slope* of the *market impact function* on Binance is much less noisy than the *slope* of the *market impact function* on Coinbase. This indicates that for Bitcoin, *external price effects* on Binance are smaller, compared to Coinbase. Binance can be supposed to form a lead market and Coinbase a following market. This is supported by the overall smaller *slope* of the *market impact function* and larger trade volume on Binance.

For AMM-based *DEXs*, the analysis results show a substantially larger *slope* of the *market impact function* for function-based liquidity-concentrating AMMs (e.g., Uniswap v2). We argue that the *slope* and, hence, the cost of *liquidity*

of these AMMs will, in principle, always be larger compared to sufficiently adopted *CEXs* and *DEXs* with other AMM designs. The reason lies in the *price discovery* combined with the inherently opposed relationship between experienced *divergence loss* and retrieved surplus from *market making*: increasing the pool size decreases the *slope* of the *market impact function*, which is desirable in *equilibrium* conditions³. However, in *non-equilibrium* conditions, when price adaption to external value changes is necessary, a larger amount of assets is required to be traded at economically disadvantageous prices. Thereby, the absolute *divergence loss* increases with pool size. In contrast, the absolute surplus from *market making*, which depends on the trading volume transacted by the AMM, is unlikely to increase accordingly. Thus, if the *liquidity* provided increases more than the transacted volume, the *liquidity providers* receive less reward per deposited value unit, and a rational economic limit to pool size exists, which in turn hinders adoption and increase of trade volume. This chicken-egg problem prevents the *slope* of the *market impact function* from growing comparably small to *CEXs* and other AMM designs.

Contributions to research and practice

To support analyses of the influences of *market makers* on *market quality*, we applied concepts established in finance literature on market microstructure to cryptoeconomic systems. Thereby, we offer a novel theoretical lens for analyses of the performance of *market makers* in terms of their influences on *market quality*. In particular, we contribute to the better analysis and design of *market makers* for cryptoeconomic systems in four ways.

First, we present a formal price model based on well-established concepts in finance literature. Thereby, we offer a foundation to better understand price formation in markets. This supports market participants in analyzing the different components of price evolution.

Second, we present an analysis concept that uses the formal price model to investigate the influence of *market makers* on *market quality* in cryptoeconomic system markets. This is useful to assess and compare *market quality* and *liquidity* on different *CEXs* and *DEXs*. For example, market participants can assess lead markets and following markets by analyzing the external price effects and noisiness of the *market impact functions*.

Third, we describe how to use the analysis concept to analyze the influences of conventional *market makers* and AMMs on *market quality* by applying our formal price model and

³ For example, for Uniswap V2 to reach comparable market quality to Binance CEX, the pool size would have to grow by approximately factor 120, see Subsection “[Results from time evolution of the market impact function’s slope metric](#)” for details.

analysis concept on historical data. Through that analysis, we show the influences of different *market makers* on *market quality* in cryptoeconomic systems. This supports practitioners in using the analysis concept for future analysis of *market quality*.

Fourth, by offering evidence for theoretic assumptions on conventional *market makers*, we support assessing *market impact*, *market quality*, and *liquidity* in cryptoeconomic system markets. This is useful to predict *market quality* and associated risks of trade execution. Thereby, they can optimize trade execution and reduce trade risks.

Overall, this work lays a foundation for analyzing and comparing *CEXs* and *DEXs* in terms of their influences on *market quality*. This helps understand the origins of benefits and drawbacks of *AMMs* compared to conventional *market makers* and helps guide the future design of *AMMs*.

Limitations

In the scope of this work, we focused on the largest two *CEXs* (i.e., Binance and Coinbase) by 24-h trading volume because we assumed those *CEXs* to be the most representative conventional *market makers*. Conventional *market makers* used in other *CEXs* (e.g., Kraken, KuCoin, and OKX) may apply other market making strategies. Therefore, the presented findings on *CEXs* apply to Binance and Coinbase at the time of observation but cannot ultimately be generalized to any *CEXs*.

Even though we used extensive data for the *market maker* analysis, we did not differentiate between *market makers* and regular traders. We assumed that all market participants who place limit orders into the order book act as *market makers*. Thus, our results do not examine individual institutional *market makers*, but the collective behavior of *market makers* and market participants. This approach seems reasonable because, to the best of our knowledge, *liquidity* is mainly influenced by this collective behavior.

To compare conventional *market makers* with *AMMs*, we analyzed Uniswap v2 and Uniswap v3 as representatives of function-based and liquidity provider-based liquidity-concentration *AMMs*. We selected these *AMM*-based *DEXs* because they have the highest 24-hour trading volume, which is, at least, partially comparable to the 24-hour trading volume of *CEXs*.

Recent developments brought forth new *AMM* designs with unique characteristics. For example, supply-sovereign *AMMs* are promising to overcome the dependence on liquidity providers and the *liquidity* problem related to *adverse selection cost* by design. However, we do not provide any analyses for supply-sovereign *AMMs*. The reason is that to date, to the best of our knowledge, no sufficiently adopted real-world implementation exists.

Future research

Supply-sovereign *AMMs* seem promising to overcome the dependence of *AMMs* on liquidity providers. However, supply-sovereign *AMMs* are not intended to provide a means of exchange for arbitrary assets. Instead, supply-sovereign *AMMs* are envisioned to control the token supply of cryptoeconomic systems to issue and trade those tokens. By controlling the token supply, supply-sovereign *AMMs* can overcome the *adverse selection cost* of *liquidity* problem by design. This is because the *AMM* as issuer can guarantee *liquidity* without depending on external *liquidity* providers. Supply-sovereign *AMMs* form the lead market by design, with a *market maker* guaranteeing *liquidity* based on their transparent *market impact function*. In contrast, liquidity provider-based *AMMs* (e.g., Uniswap v2, Uniswap v3) must follow the lead market, typically located elsewhere due to the rational economic limit of *market impact function's slope*. Thereby, liquidity providers on liquidity provider-based *AMMs* suffer losses from buying and selling tokens at disadvantageous prices to arbitrageurs exploiting price discrepancies between the *AMM* and the lead market. With supply sovereign *AMMs*, the previous drawback is resolved in a twofold way. No liquidity providers are averting *adverse selection risk* and *divergence loss*, and there is no need to follow an external lead market. This can help to greatly enhance *market quality*, especially in *non-equilibrium* conditions.

Due to the principal differences between *supply-sovereign AMMs* and *AMMs* with other designs (e.g., Uniswap v2, Uniswap v3) and the lack of *supply-sovereign AMMs* implementations, a detailed analysis of possible advantages and drawbacks should be better understood in future analyses of influences of *supply-sovereign AMMs* on market quality. We plan to perform such analyses in subsequent work.

Conclusions

Conventional market making and exchange operations entail risks of low *market quality*. This can facilitate market manipulation by fraudulent entanglement of exchanges, *market makers*, and *underwriters*, harming honest market participants. *AMM*-based *DEXs* operating in cryptoeconomic systems seem to tackle those challenges, by employing persistent and transparent market making strategies. However, the extent to which *AMMs* can help improve *market quality* compared to conventional *market makers* is barely understood.

Drawing from finance literature on market microstructure, we developed a formal price model and an analysis concept for *market quality*. The analysis concept allows to analyze and understand the influence of *market makers* on *market quality* in cryptoeconomic systems.

We show that, depending on *AMM* designs, *AMMs* operated in *DEXs* have the potential to provide *CEX*-competitive market quality. However, the problem of low market quality due to the significant removal of liquidity in non-equilibrium conditions remains unsolved. The root cause of liquidity removal lies in adverse selection costs that strongly influence the economic sustainability of market making strategies. Considering fundamental economic principles, it seems plausible that these drawbacks will not be overcome with approaches for which adverse selection costs of liquidity are a major concern.

Supply-sovereign *AMMs* focus on the issuance and trading of own tokens of cryptoeconomic systems, while the *AMM* controls the supply. Supply-sovereign *AMMs* eliminate dependence on liquidity providers. This can help overcome the liquidity problem related to adverse selection cost by design. Thus, supply-sovereign *AMMs* are promising to become state of the art for new projects that envision keeping sovereignty over their tokens and offer markets with high liquidity and market quality. This can be regarded as well aligned with the core idea of cryptoeconomic systems to create decentralized, self-sovereign systems without the need for central authorities.

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