

A Service of



Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Varmaz, Armin; Fieberg, Christian; Poddig, Thorsten

Article — Published Version Portfolio optimization for sustainable investments

Annals of Operations Research

Provided in Cooperation with: Springer Nature

Suggested Citation: Varmaz, Armin; Fieberg, Christian; Poddig, Thorsten (2024) : Portfolio optimization for sustainable investments, Annals of Operations Research, ISSN 1572-9338, Springer US, New York, NY, Vol. 341, Iss. 2, pp. 1151-1176, https://doi.org/10.1007/s10479-024-06189-w

This Version is available at: https://hdl.handle.net/10419/315285

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.



http://creativecommons.org/licenses/by/4.0/

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



WWW.ECONSTOR.EU

ORIGINAL RESEARCH



Portfolio optimization for sustainable investments

Armin Varmaz¹ · Christian Fieberg¹ · Thorsten Poddig²

Received: 3 February 2023 / Accepted: 26 July 2024 / Published online: 12 August 2024 © The Author(s) 2024

Abstract

In mean-variance portfolio optimization, multi-index models often accelerate computation, reduce input requirements, facilitate understanding, and allow easy adjustment to changing conditions more effectively than full covariance matrix estimation in many situations. In this paper, we develop a multi-index model-based portfolio optimization approach that takes into account aspects of the environment, social responsibility and corporate governance (ESG). Investments in assets related to ESG have recently grown, attracting interest from both academic research and investment fund practice. Various literature strands in this area address the theoretical and empirical relation among return, risk and ESG. Our portfolio optimization approach is flexible enough to take these literature strands into account and does not require large-scale covariance matrix estimation. An extension of our approach even allows investors to empirically discriminate among the literature strands. A case study demonstrates the application of our portfolio optimization approach.

Keywords Portfolio optimization · Sustainable investment · Investor preferences

1 Introduction

Financial investment decisions that must take environmental, social, and corporate governance (ESG) issues into consideration constitute a fast-growing area in the investment and banking industry. This growth is driven by the desire of investors to assess the ESG aspects of firm conduct using nonfinancial data and institutional investors' direct engagement in ESG issues (Gillan & Starks, 2000; Grewal et al., 2016). Motivated by the increasing popularity of sustainable investments, various approaches have been proposed to incorporate ESG measures into the investment decision-making process (e.g. Bilbao-Terol et al., 2012;

 Armin Varmaz armin.varmaz@hs-bremen.de
 Christian Fieberg christian.fieberg@hs-bremen.de
 Thorsten Poddig poddig@uni-bremen.de

¹ Economics, Hochschule Bremen, Werderstr. 73, 28199 Bremen, Bremen, Germany

Christian Fieberg and Thorsten Poddig have contributed equally to this work

² Economics, Universität Bremen, Max-von-Laue-Straße 1, 28359 Bremen, Bremen, Germany

Bilbao-Terol et al., 2013; Gasser et al., 2017; Pástor et al., 2021; Steuer & Utz, 2023; Liagkouras et al., 2020; Benedetti et al., 2021). A common approach is to extend the traditional mean-variance approach (e.g., Hirschberger et al., 2013; Gasser et al., 2017; Pedersen et al., 2021). We will refer to this approach as the extended mean-variance approach.

Investors face various challenges when applying the extended mean-variance approach from the literature (e.g., Mynbayeva et al., 2022). In this paper, we will focus on three challenges. The first challenge is that the extended mean-variance approach requires investors to provide preference parameters for return, risk and ESG. However, for an empirical and practical application of the extended mean-variance approach, knowledge of investors' preferences and their parameters is key to implementing utility maximizing portfolios. From a practical perspective, the choice of preference parameters is unclear. A strand of the literature estimates the preference parameters from market data (e.g., Bollerslev et al., 2011). Nevertheless, the estimated risk aversions describe an average investor and hence cannot be easily adapted to a single investor. Another strand of the literature that we follow links the preference parameters to levels of risk and returns (e.g., Das et al., 2010; Bodnar et al., 2018a). For example, Das et al. (2010) impose a risk constraint in portfolio optimization and determine the implied risk aversion coefficient. It seems that the orientation toward risk can be more easily specified by using the level of risk than by determining the risk aversion coefficients, which are abstract measures for most investors.

The second challenge is that the extended mean-variance approach incorporates investors' attitudes toward specific firm ESG characteristics. For empirical and practical portfolio implementation, investors must assume a relation among ESG, return and risk. A first strand of the literature argues that there is a direct relation between ESG characteristics and stock returns (e.g., Friedman & Heinle, 2016; Bolton & Kacperczyk, 2020). For example, investors could favor firms with high environmental standards. A second strand of literature argues that ESG is related to a (systematic) risk factor (see Luo & Balvers, 2017; Pedersen et al., 2021; Pástor et al., 2021; Hoepner et al., 2024). For example, there could be an environmental risk factor (Bolton & Kacperczyk, 2020), and firms with high (low) exposure to such environmental risk factor will offer high (low) expected returns. The main difference between the two strands of the literature is their assumption of the relation between ESG and firm risk. The first strand of the literature assumes that the ESG characteristic (e.g., CO2 emissions) is related only to the expected return but not to the risk of firms. The second strand of the literature assumes that exposure to an ESG risk factor is related to firms' expected return and risk. If the latter literature is correct, investors will want to explicitly control this source of risk, which is not necessary if the first strand of literature is correct. The challenge for investors is how to empirically decide which literature strand is correct and whether their sustainable portfolios need to be managed by ESG characteristics or by ESG factor loadings. The literature does not provide an easy solution for how to decide whether the first or the second strand of the literature, or possibly a combination of the strands, should be followed by investors when they prefer to consider ESG aspects in their portfolio formation.

Finally, the third challenge is that due to the necessary covariance matrix estimation, the extended mean-variance approaches discussed in the literature cannot be easily adapted to unbalanced panels or an investment universe with a large number of assets because the estimated covariance matrix is unreliable (e.g., Bajeux-Besnainou et al., 2012). When the number of stocks is larger than the number of historical return observations per stock, the sample covariance matrix becomes singular (e.g., Ledoit & Wolf, 2003; Bajeux-Besnainou et al., 2012). This challenge is also faced by mean-variance investors, but extended mean-variance investors must still overcome the hurdle of estimating the covariance matrix. The literature develops a large variety of approaches for covariance regularization to improve

the quality of large-scale covariance estimation (e.g., Laloux et al., 1999; Ledoit & Wolf, 2003, 2004; Bajeux-Besnainou et al., 2012; Bodnar et al., 2018b; Plachel, 2019; Nguyen et al., 2022; Mynbayeva et al., 2022; Costola et al., 2022). However, the estimation of large-scale covariance matrices remains a common challenge for adopting traditional or extended mean-variance approaches.

In this paper, we propose a simple new portfolio optimization approach to incorporate ESG into portfolio formation addressing the three challenges discussed above. Our contribution to the literature is threefold. Our first contribution is that our approach allows investors to follow the abovementioned individual strands of literature, or a combination thereof, regarding the relation among ESG, risk and return. Our approach links return, risk and ESG preferences to the investor-desired levels of return, risk and ESG, enabling easy adoption of our portfolio optimization approach in practice. The second contribution is that after mild modifications of our portfolio optimization approach, we develop a formal test that can help investors disentangle the strands of literature. This formal test allows investors to determine whether a firm's factor loading to an ESG risk factor or ESG characteristic or both ESG factor loading and ESG characteristic explains stock return variations. It also allows investors to specify before portfolio optimization whether ESG factor loading or ESG characteristic, or both or neither, are important for the formation of their specific portfolios. Our third contribution is that we show how the challenging estimation of a very large covariance matrix can be circumvented and thus allows portfolio formation for mean-variance and extended meanvariance investors to be applied, even to unbalanced panels and in situations with larger numbers of assets than time-series observations.

Our paper develops a general methodology to form portfolios relying on the assumption of an asset return model. In our derivations and applications in the main text, we use a singleindex model (e.g., the CAPM), which gives us the advantages of concreteness and simplicity. However, the optimized portfolio approach is suitable for more general environments, as shown in the Appendix for multi-index models (e.g., APT). Additionally, for the sake of concreteness, in the main part of the paper, the optimized portfolio approach considers a single ESG firm characteristic. However, as shown in the Appendix, the optimized portfolio approach can address numerous firm characteristics. Examples of characteristics and risk factors other than ESG that are discussed in the literature include size, value and momentum (e.g., Fama & French, 1993, 2007). Even with a large number of assets, risk factors and characteristics, the optimized portfolio approach is simple to implement.

The paper proceeds as follows: Sect. 2 discusses the relation among risk, return and ESG and, based on this discussion, develops our portfolio optimization approach. Section 3 describes a formal test as a modification of the optimized portfolio approach. Section 4 demonstrates the empirical application of portfolio optimization and the test based on stocks from the S&P 500 index. The empirical section presents a case study with the aim of demonstrating the application of the portfolio approach. Section 5 summarizes and concludes.

2 Portfolio selection and asset return model

2.1 Extended mean-variance optimization program

Among others, Hirschberger et al. (2013), Utz et al. (2014), Gasser et al. (2017), Liagkouras et al. (2020), Qi et al. (2017), Qi and Steuer (2020) and Pedersen et al. (2021) proposed the objective function (1), which assumes a sustainable investor whose objective function with

regard to portfolio formation incorporates return, risk and an ESG characteristic.

$$\max_{w} \quad \alpha \mu_p - \frac{1}{2} \lambda \sigma_p^2 + \epsilon \theta_p \tag{1}$$

 μ_p is the portfolio return with $\mu_p = \boldsymbol{w}^{\mathsf{T}}\boldsymbol{\mu}$, where \boldsymbol{w} is an $N \times 1$ vector of portfolio weights of N risky assets with $\sum_i w_i = 1$; and $\boldsymbol{\mu}$ is a $N \times 1$ vector of the expected excess returns μ_i over the risk-free rate on N risky assets. σ_p^2 is the portfolio return variance with $\sigma_p^2 = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{V} \boldsymbol{w}$, where \boldsymbol{V} is an $N \times N$ positive semidefinite covariance matrix of asset returns across N firms. θ_p is the portfolio ESG rating with $\theta_p = \boldsymbol{w}^{\mathsf{T}}\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ is an $N \times 1$ vector of the ESG characteristic of each single asset. Consistent with the literature, the additivity of the ESG characteristic is assumed (e.g., Drut, 2010; Gasser et al., 2017; Pedersen et al., 2021). α , λ and ϵ (α , λ , $\epsilon \in \mathbb{R}$) are scalars that represent investor preferences for return, risk and ESG, respectively. In line with the literature, the inputs $\boldsymbol{\mu}$, \boldsymbol{V} and $\boldsymbol{\theta}$ of the optimization are assumed to be given, and the investors must specify their preferences (α , λ , ϵ). The solution of optimization (1) for optimal weights is given in Equation (2),

$$\boldsymbol{w} = \frac{\alpha}{\lambda} \boldsymbol{V}^{-1} \boldsymbol{\mu} + \frac{\epsilon}{\lambda} \boldsymbol{V}^{-1} \boldsymbol{\theta} + \frac{h}{\lambda} \boldsymbol{V}^{-1} \boldsymbol{1}$$
(2)

where h is the Lagrangian multiplier with

$$h = \lambda \left(\mathbf{1}^{\mathsf{T}} V^{-1} \mathbf{1} \right)^{-1} - \alpha \mathbf{1}^{\mathsf{T}} V^{-1} \mu \left(\mathbf{1}^{\mathsf{T}} V^{-1} \mathbf{1} \right)^{-1} - \epsilon \mathbf{1}^{\mathsf{T}} V^{-1} \theta \left(\mathbf{1}^{\mathsf{T}} V^{-1} \mathbf{1} \right)^{-1}$$

2.2 Linking preferences to levels

This section reformulates the optimization program (1). The reformulation is motivated by the challenges of specifying investor preferences for return, risk and ESG. Many investors are unable to consistently specify their preferences and have difficulty quantifying the values for α , λ , and ϵ . Without these specific quantities, the optimization program (1) will not lead to a portfolio that maximizes investors' utility. Instead of specifying preference values, investors find it easier to formulate desired portfolio properties in terms of expected return, risk and ESG. Even the determination of the preferences in experiments requires (in the first step) that the investors state their desired levels of return, risk and ESG (Harrison & Rutström, 2008).

The idea of linking the risk aversion coefficient to the risk level is not new (Das et al., 2010; Merton, 1972). The literature suggests that the attitude toward risk can be more easily specified by using the risk level than by determining an abstract risk aversion coefficient. We extend this literature by linking ESG preferences to the level of an ESG characteristic. Specifically, we consider the levels for return and ESG as constraints of the reformulated optimization. The constraints must satisfy the condition that the portfolio return and portfolio ESG reflect the respective preferences. Since portfolio return and portfolio ESG are constraints, the objective function in (3) includes only the portfolio variance:

$$\min_{\boldsymbol{w}} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{V} \boldsymbol{w} \tag{3}$$

$$\mathbf{t}. \quad \boldsymbol{w}^{\mathsf{T}} \mathbf{1} = 1 \tag{4}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\mu} = \boldsymbol{\mu}_{p}^{*} \tag{5}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\theta} = \theta_n^* \tag{6}$$

The levels of portfolio return μ_p^* and portfolio ESG rating θ_p^* are investor specific and chosen by each investor individually. Intuitively, the portfolio obtained by (3)–(6) is a minimum

s

1155

variance portfolio with a desired level of return μ_p^* and ESG characteristic θ_p^* , i.e., it is an efficient portfolio. If the levels μ_p^* and θ_p^* are consistently set to the investor's preferences α and ϵ , the optimization program (3)–(6) is equivalent to maximization program (1). The intuition behind the equivalence is that after the optimal portfolio weights from (1) are found, we can calculate the values for the return, variance and ESG of the optimal portfolio. To illustrate this point, based on the solution (2), we can write the expected excess return of an investor's portfolio μ_p^* as $\mu_p^* = \mu^{\mathsf{T}} \boldsymbol{w} = \frac{\alpha}{\lambda} \mu^{\mathsf{T}} V^{-1} \mu + \frac{\epsilon}{\lambda} \mu^{\mathsf{T}} V^{-1} \theta + \frac{h}{\lambda} \mu^{\mathsf{T}} V^{-1} \mathbf{1}$. Similarly, the portfolio ESG characteristic is $\theta_p^* = \theta^{\mathsf{T}} w = \frac{\alpha}{\lambda} \theta^{\mathsf{T}} V^{-1} \mu + \frac{\epsilon}{\lambda} \theta^{\mathsf{T}} V^{-1} \theta + \frac{h}{\lambda} \theta^{\mathsf{T}} V^{-1} \mathbf{1}$ and the portfolio variance is $\sigma_p^{2*} = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{V} \boldsymbol{w} = \frac{1}{\lambda^2} (\alpha \boldsymbol{\mu} + \epsilon \boldsymbol{\theta} + h\mathbf{1})^{\mathsf{T}} \boldsymbol{V}^{-1} (\alpha \boldsymbol{\mu} + \epsilon \boldsymbol{\theta} + h\mathbf{1})$. The key insight is that if an investor specifies the desired levels of return, risk and ESG for a portfolio, μ_n^*, θ_n^* and σ_n^{2*} , i.e., the left-hand side of the equations, and solves for the optimal weights, then the corresponding preferences α , λ and ϵ are also determined. It follows that the investor's consistent indication of both, either her preferences or her desired level for return, risk and ESG, will lead to the same portfolio. The equations for μ_p^* , θ_p^* and σ_p^{2*} can be used to solve for three unknown parameters based on the desired levels of return, risk and ESG or based on the corresponding preferences. The preferences ($\alpha, \lambda, \epsilon$) and portfolio properties $(\mu_p^*, \sigma_p^{2*}, \theta_p^*)$ must be consistent because otherwise, the portfolio would not be optimal.

By this equivalence, we refer to our reformulation of the approach in Equation (1), as shown by the optimization program in Eqs. (3)–(6). The reformulation of the portfolio optimization (1) allows for the preferences for return, risk and ESG to be replaced by the desired levels of return and ESG in the determination of a minimum variance portfolio.

2.3 Asset return models and portfolio optimization for sustainable investors

2.3.1 The relation among risk, return and ESG: a qualitative description

This section describes two strands of the literature regarding the relation among risk, return and ESG. The aim is to introduce two views on ESG from the literature and to discuss their consequences for portfolio formation. The first strand of the literature argues that there is a direct relation between ESG characteristics and stock returns (e.g., Friedman & Heinle, 2016; Li et al., 2019). For example, investors could favor firms with high environmental standards. A second strand of the literature argues that ESG is related to a (systematic) risk factor (see Luo & Balvers, 2017; Pedersen et al., 2021; Pástor et al., 2021; Hoepner et al., 2024). For example, there could be an environmental risk factor (Bolton & Kacperczyk, 2020), and firms with high (low) loadings to the environmental risk factor offer high (low) expect returns.

The key difference between these two strands of literature is easiest to explain with an example. Suppose that Firm A has a low ESG characteristic. Firm A is a supplier to a high ESG firm and consequently has cash flows that are highly correlated with the ESG risk factor. Due to the correlation, the firm has a high factor loading to the ESG risk factor with a positive risk premium. Firm B also has a high ESG characteristic and low factor loading on the ESG risk factor. Firm B engages in sustainability, but its cash flow does not depend on the ESG risk factor. Except for ESG risk factor loading values. If the first strand of the literature is correct, then Firm B will have higher returns relative to Firm A. If instead, the second strand of literature is the correct description of empirical returns, then Firm A will have higher returns relative to Firm A will contribute more to portfolio risk, and consequently, investors prefer to control for this source of risk explicitly in their portfolio optimization, which is not necessary if the first strand of the literature is correct.

Each case has a considerable influence on portfolio formation. Furthermore, each case will result in completely different portfolio allocations depending on which strand of the literature an investor follows.

The first strand of the literature assumes that there is a direct relation between ESG characteristics and asset returns (e.g., Friedman & Heinle, 2016; Li et al., 2019). According to this view, there is an ESG reward that is significantly different from zero, and there is no ESG risk premium. The covariance matrix and hence the asset risk is not related to the ESG characteristic. Consequently, a firm with a high value of ESG offers different (higher or lower) returns and the same risk relative to an otherwise identical firm with a low value of ESG. In this vein, Friede et al. (2015) conduct a meta-analysis of 60 meta-studies including more than 2,000 single studies. They document that 90% of the empirical studies found a nonnegative relation between the ESG characteristic and financial performance.¹ Based on the list of "100 Best CSR companies in the world", Li et al. (2019) construct a value-weighted portfolio that offers statistically significant annual abnormal returns of 1.81% and 1.26% by controlling for Carhart four factors and the Fama-French five-factors model, respectively. There are several approaches in the literature to explain why firms with high values of ESG characteristics earn higher returns. Bénabou and Tirole (2010) argue that the ESG activity of a firm can affect its value in three ways: (1) firms can "do well by doing good" (e.g., reducing workplace injury); (2) firms can maximize shareholder value when they exercise ESG behavior on behalf of stakeholders (e.g., customers paying more for high-priced fair-trade products); and (3) firms can destroy value when they engage in projects for the benefit of managers (e.g., managers donate to their favorite charities to benefit privately). Edmans (2011) observe that high ESG firms may enjoy satisfied employees, fewer environmental risks, good governance or loyal customers. Such firms generate higher than expected earnings due to their greater efficiency. Due to the higher earnings, these firms offer higher returns. Investors following this strand of the literature want to include assets with ESG characteristics with a positive relation to return in their portfolio. However, they do not need to control for ESG risk since the ESG characteristic is assumed to have no relation to asset comovements. In portfolio optimization, we can expect to overweight firms with high ESG characteristics because such firms contribute only to higher portfolio returns without increasing portfolio risk.

The second strand of the literature follows the idea of traditional asset pricing and assumes an ESG risk factor that governs the expected return and the covariances among assets (see Luo & Balvers, 2017; Pedersen et al., 2021; Pástor et al., 2021; Hoepner et al., 2024). Accordingly, there is an ESG risk premium, which is significantly different from zero, but no ESG characteristic reward. There are different explanations for the existence of an ESG risk factor in the literature. Bolton and Kacperczyk (2020) suggest that an ESG risk factor reflects investors' ESG preferences that affect their portfolio formation. Investor preferences for specific ESG characteristics can lead to excess demand for some stocks. Among others, Luo and Balvers (2017) and Hoepner et al. (2024) argue that a permanent shift in ESG preferences will permanently change the efficiency frontier and can lead to an ESG risk factor premium. Maiti (2021) suggests forming risk factors based on sorts on size and ESG dimensions and finds improvement in model performance for EURO Stoxx constituents when an ESG risk factor is added to Fama and French (1993) three- and four-factor models. Pástor et al. (2021) and Pedersen et al. (2021) show that a sufficiently large number of investors with preferences for ESG will lead to risk factor premiums. Thus, as long as empirical studies do not incorporate an appropriate ESG risk factor, they will observe positive abnormal returns

¹ In the subsequent text, we will assume a positive reward for the ESG characteristic. However, the ESG reward can also be negative. For example, firms with high CO2 emissions may be avoided by investors (e.g., Bolton & Kacperczyk, 2020).

for ESG stocks while in reality, these stocks earn a risk premium (Sharfman & Fernando, 2008; Albuquerque et al., 2019; Flammer, 2015). In portfolio optimization, firms with high loadings on an ESG risk factor are not necessarily overweighted because such firms increase portfolio return and risk simultaneously. Here, it depends on the return-risk trade-off of whether these firms enter the portfolio. The consequences for portfolio allocation are quite different from the first strand of the literature. In Sect. 3, we describe a formal test that helps investors empirically decide which strand of the literature they should follow in their portfolio formation because this heavily influences portfolio allocation.

2.3.2 The relation among risk, return and ESG: the formal description

This section describes our asset return model. The introduction of the asset return model helps to capture the discussion from the literature about the relation among return, risk and ESG.

Let **R** denote an $N \times F$ matrix of F observations of a system of N random variables, representing F returns on a universe of N stocks. Our assumptions follow the literature dealing with the estimation procedure for large covariance matrices in the portfolio context (e.g., Ledoit & Wolf, 2003; Bodnar et al., 2018b; Kremer et al., 2018; Plachel 2019).

Assumption 1 The number of stocks N and the number of observations F are fixed and finite.

Assumption 2 Asset returns have finite first and second moments.

Assumption 3 The return r_{it} on an asset *i* at time *t* is described by the factor model (7).

$$r_{it} = E_{t-1}(r_{it}) + \sum_{k=1}^{K-1} \beta_{ik} \tilde{f}_{kt} + e_i \tilde{f}_{et} + u_{it}$$
(7)

where β_{ik} is the loading of asset *i* on factor *k*, $\tilde{f}_{kt} \sim \mathcal{N}(0, \sigma_{\tilde{f}_k}^2)$ is the innovation of factor *k* at time *t*, e_i is the asset *i*'s loading on the ESG risk factor, and $\tilde{f}_{et} \sim \mathcal{N}(0, \sigma_{f_e}^2)$ is the factor innovation of the ESG risk factor at time t. Finally, $u_{it} \sim \mathcal{N}(0, \sigma_u^2)$ is the residual asset return. According to (7), there are *K* risk factors, where the *K*-th risk factor is the ESG risk factor. The expected return is

$$E_{t-1}(r_{it}) = r_{ft} + \sum_{k=1}^{K-1} \beta_{ik} \mu_k + e_i \mu_e + \theta_i c$$
(8)

where $\mu_k(\mu_e)$ is the premium on the risk factor k (ESG risk factor), θ_i is the ESG characteristic of the asset *i*, and $c \in \mathbb{R}$ is the ESG reward. The covariance matrix for the asset return is

$$V = BV_f B^{\mathsf{T}} + RV \tag{9}$$

where V_f is the $K \times K$ diagonal covariance matrix of orthogonal factor returns, **B** is a $N \times K$ matrix of asset factor loadings to K risk factors and **RV** is the $N \times N$ covariance matrix of residual asset returns. The residual returns are identically and independently distributed (i.i.d. residual returns) and hence $Cov(u_i, u_j) = 0$, $\forall i \neq j$ and $Cov(u_i, u_j) = \sigma_u^2$, $\forall i = j$.²

 $^{^2}$ There are some remarks on the assumption 3. For our portfolio approach, the i.i.d. residual returns are an important feature of the asset return model (7). We only need that the risk factors explain a significant part of the asset variances and covariances to approximate the i.i.d. assumption. The i.i.d. assumption is a standard assumption in the literature (e.g., Pedersen et al., 2021; Daniel et al., 2020; Ledoit & Wolf, 2003).

Assumption 4 The risk factors have positive variances, that is, $Var(f_k) > 0$, $\forall k$.

In the last subsection, we described two strands of the literature about the relation among risk, return and ESG. Applying the Eqs. (7)–(9), we can show the differences between the views in the literature in a formal way. The first strand of the literature argues that the ESG characteristic is important to investors and that there is no ESG risk factor. Consequently, we remove the ESG risk factor from the factor model (7). Then, the expected return for firms is

$$E_{t-1}(r_{it}) = r_{ft} + \sum_{k=1}^{K-1} \beta_{ik} \mu_k + \theta_i c$$

and the covariance σ_{ij} between assets *i* and *j* is

$$\sigma_{ij} = \sum_{k}^{K-1} \beta_{ik} \beta_{jk} \sigma_{r_k}^2 + \sigma_{u_{ij}}$$

where $\sigma_{r_k}^2$ describes the variance of the *k*-th risk factor return, $\sigma_{u_{ij}} = 0$, $\forall i \neq j$ and $\sigma_{u_{ij}} = \sigma_u^2$, $\forall i = j$. Thus, the ESG characteristic affects the expected return but not the risk. Note that since the ESG risk factor is not included, it affects neither the returns nor the covariances.

The second strand of the literature assumes an ESG risk factor. Accordingly, the expected return of an asset is

$$E_{t-1}(r_{it}) = r_{ft} + \sum_{k=1}^{K-1} \beta_{ik} \mu_k + e_i \mu_e$$

and the covariance σ_{ij} between assets *i* and *j* is

$$\sigma_{ij} = \sum_{k}^{K-1} \beta_{ik} \beta_{jk} \sigma_{r_k}^2 + e_i e_j \sigma_{r_e}^2 + \sigma_{u_{ij}}$$

where $\sigma_{r_e}^2$ describes the variance of the ESG risk factor return. Thus, in this literature, the asset loading to the ESG risk factor drives its expected return and risk.

2.3.3 Implications for portfolio formation

The asset return model (7)–(9) helps develop a general portfolio optimization program without the need to estimate expected returns or the covariance matrix. For the sake of simplicity and concreteness, in this subsection, we rely on a return model that is described by the market risk factor (CAPM), an ESG risk factor and an ESG characteristic reward.³ We start with the general setting where we allow for an ESG reward and an ESG risk factor premium simultaneously (which might be the case in reality), although the literature usually assumes the one or the other case. Accordingly, the expected return is

$$E_{t-1}(r_{it}) = \mu_i = r_{ft} + \beta_i \mu_{r_m} + e_i \mu_e + \theta_i c$$
(10)

and the covariance between the assets i and j is

$$\sigma_{ij} = \beta_i \beta_j \sigma_{r_m}^2 + e_i e_j \sigma_{r_e}^2 + \sigma_{u_{ij}}$$
(11)

For the sake of simplicity, it is assumed that the investors know the true β_i and e_i to avoid addressing estimation errors in the presentation of the optimized portfolio framework. The

³ The description of general cases with K risk factors and M characteristics is left to our Appendix A.

variance of investors' portfolio, which is the objective function of the optimization program (3)–(6), can be written according to the Eq. (11) as

$$\sigma_p^2 = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{V} \boldsymbol{w}$$

= $\boldsymbol{w}^{\mathsf{T}} \left(\boldsymbol{\beta} \sigma_{r_m}^2 \boldsymbol{\beta}^{\mathsf{T}} + \boldsymbol{e} \sigma_{r_e}^2 \boldsymbol{e}^{\mathsf{T}} + \boldsymbol{R} \boldsymbol{V} \right) \boldsymbol{w}$
= $\boldsymbol{w}^{\mathsf{T}} \boldsymbol{\beta} \sigma_{r_m}^2 \boldsymbol{\beta}^{\mathsf{T}} \boldsymbol{w} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{e} \sigma_{r_e}^2 \boldsymbol{e}^{\mathsf{T}} \boldsymbol{w} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{R} \boldsymbol{V} \boldsymbol{w}$
= $\beta_p \sigma_{r_m}^2 \beta_p + e_p \sigma_{r_e}^2 e_p + \sigma_u^2 \boldsymbol{w}^{\mathsf{T}} \boldsymbol{I} \boldsymbol{w}$

where $\boldsymbol{\beta}$ is the $N \times 1$ vector of asset factor loadings to the market risk factor, \boldsymbol{e} is $N \times 1$ vector of asset factor loadings to the ESG risk factor and \boldsymbol{I} is an $N \times N$ identity matrix. If the portfolio factor loadings $\beta_p = \sum_{i=1}^{N} w_i \beta_i$ and $e_p = \sum_{i=1}^{N} w_i e_i$ are constrained in the portfolio optimization to be at the investor-desired levels β_p^* and e_p^* , then the terms $\beta_p \sigma_{r_m}^2 \beta_p$ and $e_p \sigma_{r_e}^2 e_p$ from the portfolio variance calculation become constants in the optimization. Adding a constant to the objective function will not change the optimal solution for the asset weights. Consequently, we can remove the terms $\beta_p^* \sigma_{r_m}^2 \beta_p^*$ and $e_p^* \sigma_{r_e}^2 e_p^*$ from the objective function. Additionally, the scaling of the sum of squared asset weights with the constant residual variance ($\sigma_u^2 \boldsymbol{w}^{\intercal} \boldsymbol{I} \boldsymbol{w}$) does not change the optimal solution for the asset weights since σ_u^2 is a constant and identical for all assets due to the i.i.d. assumption in the asset return model. Therefore, the objective function of our optimization approach in (3)–(6) simplifies to $\boldsymbol{w}^{\intercal} \boldsymbol{w}$.

Similarly, the investor-specific levels β_p^* , e_p^* and θ_p^* determine the portfolio return since

$$\mu_p^* = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{\mu} = \boldsymbol{w}^{\mathsf{T}} \left(r_{ft} \mathbf{1} + \boldsymbol{\beta} \mu_{r_m} + \boldsymbol{e} \mu_r + \boldsymbol{\theta} c \right)$$
$$= r_{ft} \boldsymbol{w}^{\mathsf{T}} \mathbf{1} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{\beta} \mu_{r_m} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{e} \mu_{r_e} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{\theta} c$$
$$= r_{ft} + \beta_p \mu_{r_m} + e_p \mu_{r_e} + \theta_p c$$

Instead of specifying the level of expected return μ_p^* (see constraint (5)), we can also control for the portfolio factor loadings to the market risk factor (i.e., β_p^*) and to the ESG risk factor (i.e., e_p^*) as we already control for the portfolio ESG θ_p^* in the optimization program (3)–(6) (see constraint (6)).

Thus, by assuming the asset return model (7)–(9), the portfolio optimization program (3)–(6) results in

5

$$\min_{\boldsymbol{w}} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{w} \tag{12}$$

$$\mathbf{s.t.} \quad \boldsymbol{w}^{\mathsf{T}} \mathbf{1} = 1 \tag{13}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\beta} = \boldsymbol{\beta}_p^* \tag{14}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\theta} = \theta_n^* \tag{15}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{e} = \boldsymbol{e}_p^* \tag{16}$$

where the constraint (14) constrains the loading to the market risk factor β_p^* , and the constraint (16) constraints the portfolio loading to the ESG risk factor.

For the sake of clarity, we gather the variables on the left-hand side of the constraints (13)–(16) into a $N \times 4$ matrix X with

$$X = [1, \beta, \theta, e]$$

D Springer

and the variables on the right-hand side of the constraints are gathered into a 4×1 vector **b** with

$$\boldsymbol{b} = [1, \beta_p^*, \theta_p^*, e_p^*]^\mathsf{T}$$

yielding the refined optimization program in Equations (17)–(18).

$$\min_{w} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{w} \tag{17}$$

s.t.
$$X^{\mathsf{T}} \boldsymbol{w} = \boldsymbol{b}$$
 (18)

The optimal portfolio weights are obtained by minimizing the Lagrange function $L(\boldsymbol{w}, \boldsymbol{\kappa}) \equiv \frac{1}{2}\boldsymbol{w}^{\mathsf{T}}\boldsymbol{w} - \boldsymbol{\kappa}^{\mathsf{T}} (\boldsymbol{X}^{\mathsf{T}}\boldsymbol{w} - \boldsymbol{b})$, where $\boldsymbol{\kappa}^{\mathsf{T}}$ is the Lagrange multiplier. The solution for the optimal portfolio weights is given in equation (19).

$$\boldsymbol{w}^{\mathsf{T}} = \boldsymbol{b}^{\mathsf{T}} \left(\boldsymbol{X}^{\mathsf{T}} \boldsymbol{X} \right)^{-1} \boldsymbol{X}^{\mathsf{T}} \tag{19}$$

The optimal solution of Eq. (19) differs across investors because they can choose different values for the elements of b. Notably, vector b can account for any other investor-specific portfolio. Thus, if vector b is

$$\boldsymbol{b} = \begin{bmatrix} 1 \ 0.66 \ 0.5 \ 1.8 \end{bmatrix}^{\mathsf{T}}$$

the investor aims to determine a portfolio that is fully invested and exhibits a beta to the market of 0.66, an ESG factor loading of 1.8 and an ESG portfolio characteristic of 0.5. The example highlights the property of the proposed portfolio optimization program to include the combination of the strands of the discussed literature. According to our example, the assumed investor believes that both ESG characteristics and ESG factor loading are important to explain the variation of the empirical returns. The two described strands of the literature assume that either ESG characteristics or ESG risk factors are important. The portfolio formation for these investor types is described in the next subsection.

Assumptions 1–4 in Sect. 2.3.2 and the resulting asset return model (7)–(9) thus simplify the optimization problem considerably. Our assumptions follow a strand of the literature that addresses the estimation of large covariance matrices where the number of assets Nexceeds the number of observations F. In contrast to the literature on the regularization of covariance matrices, our approach does not improve the estimation of the covariance matrix, but it shows that for the purpose of portfolio formation, the inclusion of the covariance matrix in the objective function is not necessary as long as the portfolio risk is constrained by the portfolio factor loadings.

Additionally, the reformulated optimization program (12)–(16) allows easier adaptation because the reformulated optimization relies only on the desired levels of risk factor loadings and ESG characteristics of the portfolio. For investors, the specification of these quantities is often easier than the determination of the corresponding preferences. Our portfolio approach allows investors to find an optimal portfolio for unbalanced panels and a large number of assets because the estimation of the expected returns μ and the covariance matrix V is circumvented. All we need is that the factor loadings can be estimated in such a way that the residual returns are approximately i.i.d. Consequently, as long as the factor loadings can be estimated, neither a balanced data panel with the same observation number nor a larger number of observations than assets, i.e., N < F, is necessary.

2.3.4 ESG characteristic and ESG risk factor portfolios

The optimization program (12)–(16) follows from our general asset return model (7)–(9). However, the two strands of the literature assume investors that must care about either an ESG characteristic (e.g., CO2 emissions) or an ESG risk factor (e.g., carbon risk factor as proposed by Bolton and Kacperczyk (2020)). For each investor type, a slightly different optimization program is needed.

If the investors must care about an ESG characteristic but not an ESG risk factor, then they solve

$$\min_{\boldsymbol{w}} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{w} \tag{20}$$

$$.t. w^{\mathsf{T}}\mathbf{1} = 1 (21)$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\beta} = \boldsymbol{\beta}_p^* \tag{22}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\theta} = \theta_p^* \tag{23}$$

The solution for optimal weights $\boldsymbol{w}_{\theta}^{\mathsf{T}}$ from optimization program (20)–(23) is

S

$$\boldsymbol{w}_{\theta}^{\mathsf{T}} = \boldsymbol{b}_{\theta}^{\mathsf{T}} \left(\boldsymbol{X}_{\theta}^{\mathsf{T}} \boldsymbol{X}_{\theta} \right)^{-1} \boldsymbol{X}_{\theta}^{\mathsf{T}}$$
(24)

where $\boldsymbol{b}_{\theta} = [1, \beta_p^*, \theta_p^*]$ and $\boldsymbol{X}_{\theta} = [\boldsymbol{1}, \boldsymbol{\beta}, \boldsymbol{\theta}].$

If the investors must care about an ESG risk factor but not about an ESG characteristic, then they solve

$$\min_{w} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{w} \tag{25}$$

s.t.
$$\boldsymbol{w}^{\mathsf{T}} \mathbf{1} = 1$$
 (26)

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\beta} = \boldsymbol{\beta}_p^* \tag{27}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{e} = \boldsymbol{e}_p^* \tag{28}$$

The solution for optimal weights $\boldsymbol{w}_{e}^{\mathsf{T}}$ from optimization program (25)–(28) is

$$\boldsymbol{w}_{e}^{\mathsf{T}} = \boldsymbol{b}_{e}^{\mathsf{T}} \left(\boldsymbol{X}_{e}^{\mathsf{T}} \boldsymbol{X}_{e} \right)^{-1} \boldsymbol{X}_{e}^{\mathsf{T}}$$
⁽²⁹⁾

where $b_e = [1, \beta_p^*, e_p^*]$ and $X_e = [1, \beta, e]$.

3 The disentangling test between the competing ESG explanations

In the last section, the asset return model (Eqs. (7)–(9)) can incorporate views from the literature that assume that either ESG characteristics or ESG factor loading is important for the cross-section of asset returns. Our portfolio optimization program (12)–(16) even allows the investor to follow a mixed model, where ESG characteristics and ESG factor loadings are important. From an investor's perspective, it is empirically unclear what the actual relation among risk, return and ESG looks like. Making matters even worse, Berchicci and King (2020) show that the relation among return, risk and ESG is sensitive to the period, market, data provider and methods applied in the literature. Whether the first or the second strand of the literature, or a combination thereof, describes the expected returns in the cross-section of asset returns thus requires an empirical clarification. However, this distinction is crucial for the specification of the optimization problem because it determines the final portfolio

allocation. In this section, we show how our proposed portfolio optimization (12)–(16) can be applied to empirically decide whether the first (ESG characteristic) strand of the literature, the second (ESG risk factor), both strands or neither are important for asset return variation. Additionally, knowledge of the correct model can help in building more appropriate models for return and risk management for practical portfolio management and research. For example, suppose there is an ESG risk factor, but investors specify the portfolio ESG characteristic and not the portfolio factor loading to the ESG risk factor. Then, they will probably not attain the desired expected return μ_p^* because they do not consider portfolio ESG factor loading to be an important part of the expected return. The investors assume their portfolio return is $\mu_p = \beta_p \mu_{r_m} + \theta_p c$, but the actual portfolio return is $\mu_p = \beta_p \mu_{r_m} + e_p \mu_{r_{ese}}$. Consequently, the expected portfolio returns will vary depending on the portfolio ESG factor loading $(\sum w_i e_i)$. It is important to note that this fluctuation of the expected portfolio return is due to the unconsidered ESG risk factor. Additionally, the *realized* portfolio return will fluctuate around the expected value (i.e., it has a standard deviation). Similarly, investors do not control for a potential source of systematic risk because portfolio variance will depend on the variance of the ESG risk factor. Since the investors' portfolio is exposed to ESG risk, the true portfolio variance $(\sigma_p^2 = \beta_p^2 \sigma_{r_m}^2 + e_p^2 \sigma_{r_e}^2)$ will always be greater than the expected portfolio variance $(\sigma_p^2 = \beta_p^2 \sigma_{r_m}^2)$ because according to our assumption 4, the risk factor variance is always according to our assumption 4. variance is always positive.

To derive the disentangling test, for the sake of simplicity and concreteness, we rely on the optimization program (12)–(16) and maintain the assumption of a single ESG characteristic and two risk factors (market portfolio and an ESG risk factor). However, the presented disentangling test is not limited to this setup. Appendix B shows the implementation of the test for a large number of risk factors and characteristics. The solution for the optimal weights of the optimization program (12)–(16) is given in equation (19) and repeated here

$$\boldsymbol{w}^{\mathsf{T}} = \boldsymbol{b}^{\mathsf{T}} \left(\boldsymbol{X}^{\mathsf{T}} \boldsymbol{X} \right)^{-1} \boldsymbol{X}^{\mathsf{T}}$$

The solution will vary across investors depending on the choices of the elements of vector **b**. For the purpose of empirical application of the disentangling test, we must consider two additional aspects. First, we consider the time dimension of portfolio returns. Second, we form several portfolios with varying values of factor loadings to risk factors and ESG characteristics. Our optimization approach is sufficiently flexible to easily incorporate both aspects. We define W_t as an $N \times P$ matrix of the optimal weights of P portfolios at time $t, t \in \{1, 2, ..., F\}$ and r_t as the vector of realized asset returns at time t. The realized, out-of-sample returns on the portfolios at time t are

$$\boldsymbol{\lambda}_{t} = \boldsymbol{W}_{t-1}^{\mathsf{T}} \boldsymbol{r}_{t} = \boldsymbol{D} \left(\boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{X}_{t-1} \right)^{-1} \boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{r}_{t}$$
(30)

where X_t is the matrix X at time t, and λ_t is a consistent vector of the realized returns of each of the considered portfolios in t. D is a $P \times 4$ matrix, the rows of which are the right-hand sides of the optimization constraints (b^{T}). Equation (30) can thus determine the returns for P different portfolios in a single step at time t. For convenience, we define a $P \times F$ matrix Λ . Each row of Λ represents a time series of realized returns on one of P portfolios at time $t, t \in \{1, 2, ..., F\}$.

Our disentangling test builds on the idea of tracking portfolios. In our optimization program (12)–(16), we can easily form tracking portfolios by setting one value in the vector \boldsymbol{b} to one and all other values to zero. If we set the first value of \boldsymbol{b} to 1, then the tracking portfolio has zero loading to the market risk factor and to the ESG risk factor, and its ESG characteristic is also zero. Such a portfolio earns a risk-free rate at time *t* for the investors (see Eq. (8)). If we

set only the second (third, fourth) element of b to 1 and the others to zero, then the portfolio tracks the market risk premium (ESG characteristic reward, ESG risk factor premium) and is a zero-net-investment portfolio. While zero-net-investment portfolios are rare in practical portfolio management, they are useful for our disentangling test. Under the null hypothesis of a true risk factor model, the traditional approach is to test whether the regression constant from a regression of portfolio returns on risk factors is significantly different from zero (e.g., Li et al., 2019). By forming zero-net-investment tracking portfolios, we can always directly test this hypothesis because the returns on such portfolios are always without the risk-free return component of asset returns. This is also true when asset returns are not excess returns. Consequently, the realized returns on zero-net-investment tracking portfolios can be used directly in traditional approaches to test whether the regression constant is significantly different from zero.⁴ By means of zero-net-investment, we obtain tracking portfolio returns that are easily tested in the regression framework.

Consequently, the choice for the matrix **D** as a 4×4 identity matrix in our disentangling test allows the calculation of the tracking portfolio returns μ for 4 portfolios at time t,⁵ i.e.,

$$\boldsymbol{\lambda}_{t} = \left(\boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{X}_{t-1}\right)^{-1} \boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{r}_{t} \forall t \tag{31}$$

where the elements of the 4×1 vector λ_t represent the returns of tracking portfolios for the risk-free rate, the market risk factor realizations, the realization of the ESG characteristic reward and the realizations of the ESG risk factor premium, respectively.

Specifically, the literature proposes applying spanning tests (e.g., Black et al., 1972; Fama & French, 1993) to test whether (arbitrarily formed) portfolios are priced by a model. Assume that each time series of the four portfolios is regressed on the two risk factor returns r_m and r_e

$$\mathbf{\Lambda} = \begin{bmatrix} r_{p1,t} = b_{0,p1} + b_{1,p1}r_{m,t} + b_{2,p1}r_{esg,t} + \varepsilon_{p1,t}, \dots, \forall t \\ r_{p2,t} = b_{0,p2} + b_{1,p2}r_{m,t} + b_{2,p2}r_{esg,t} + \varepsilon_{p2,t}, \dots, \forall t \\ r_{p3,t} = b_{0,p3} + b_{1,p3}r_{m,t} + b_{2,p3}r_{esg,t} + \varepsilon_{p3,t}, \dots, \forall t \\ r_{p4,t} = b_{0,p4} + b_{1,p4}r_{m,t} + b_{2,p4}r_{esg,t} + \varepsilon_{p4,t}, \dots, \forall t \end{bmatrix}$$
(32)
$$= \mathbf{B}_F \mathbf{F}^{\mathsf{T}} + \mathbf{E}$$

where each row of Λ represents a time series of realized returns on one of the four portfolios at time $t, t \in \{1, 2, ..., F\}$; F is a $F \times 3$ matrix of ones (constant) and two columns of returns for the market portfolio and the ESG risk factor, respectively. B_F is a 4×3 matrix of the regression coefficients, i.e., the constant and two risk factor loadings b_1 and b_2 from the spanning test (32) for the four portfolios. Again, the four specific portfolios are considered for simplicity and concreteness. The Appendix shows a general case with many risk factors and/or characteristics. E is a $4 \times F$ matrix of the random residual returns.

The important consequence from the choice of the values in D as the identity matrix is that we know the expected values for the factor loadings from the spanning tests (32). If the two risk factor model is valid, the expected values (\mathbb{E}) of the coefficients in B_F are

$$\mathbb{E}\left(\boldsymbol{B}_{F}\right) = \begin{bmatrix} b_{0,p1} = r_{f} & 0 & 0\\ 0 & b_{1,p2} = 1 & 0\\ b_{0,p3} = 0 & 0 & 0\\ 0 & 0 & b_{2,p4} = 1 \end{bmatrix}$$
(33)

⁴ On the other hand, a full-investment tracking portfolio earns the risk-free rate and the risk premiums. Then, in the traditional approach, we must test whether the regression constant is significantly different from r_f , i.e., the test would be $b_0 - r_f = 0$, where b_0 is the regression constant.

⁵ Our Appendix **B** elaborates in more detail why the identity matrix for the purpose of the disentangling test is an appropriate choice.

The disentanglement between the two strands of the literature regarding the relation among return, risk and ESG is mainly determined by the values of b_{2,p_4} and b_{0,p_3} . Accordingly, we expect to observe no significant coefficients on the risk factor $(b_{1,p_1}, b_{2,p_1}, \text{ first row in } \boldsymbol{B}_F)$, with the first portfolio and a regression constant being close to the risk-free rate.⁶ Regarding the second portfolio, we expect a factor loading of 1 to the market portfolio returns $(b_{1,p2} = 1)$ and zero for the remaining coefficients $(b_{0,p2}, b_{2,p2})$. If the first strand of the literature (ESG characteristics) is correct, then we expect to see a significant regression constant for portfolio three $(b_{0,p3})$. Since the portfolio is formed to have no loadings on the market or ESG risk factors but a positive ESG characteristic, a significant regression coefficient likely indicates that there is a relation between return and ESG characteristic. A positive (negative) constant $b_{0,p3}$ suggests that investors are asking (bidding on) ESG stocks in excess. For the second strand of the literature to be correct, the regression constant $b_{0,p3}$ for the third portfolio must be zero. Similarly, if the second strand of the literature (ESG factor loading) is a valid description of asset return variations, we expect for the fourth portfolio a factor loading of 1 to the ESG risk factor $(b_{2,p4} = 1)$ and zero for the remaining coefficients $(b_{0,p4}, b_{1,p4})$. Consequently, the constant $(b_{0,p3})$ and the risk factor factor loadings $(b_{1,p3}, b_{2,p3})$ will be zero since the portfolio is formed to have factor loadings of zero for the ESG and market risk factors. In empirical applications, it is possible that both ESG characteristics and ESG factor loading add to the explanation of asset returns. If such a mixed model is correct, then we expect to see a significant regression constant $b_{0,p3}$ for the third portfolio and a significant regression coefficient for the ESG risk factor with a value of one $(b_{2,p4} = 1)$ for the fourth portfolio.

The use of our optimized portfolios as test assets has a key advantage compared to the common use of the Black et al. (1972) test since we can provide the expected values of the regression slope coefficients. The traditional (Black et al., 1972) test focuses only on an evaluation of the regression constant b_0 . Our approach allows us to interpret both the regression constant b_0 and the slope coefficients (b_1 and b_2) for the congruent risk factors.

4 Empirical application

4.1 Data sample

This section aims to demonstrate the empirical application of our portfolio optimization program in a case study. The case study also shows how to apply and interpret the results from our disentangling test. Importantly, the aim of the case study is to demonstrate the application and not to decide whether the cross-section of asset returns is described by an ESG characteristic or by an ESG risk factor. The case study assumes extended mean-variance investors who are seeking to maximize their utility. These investors will first apply the disentangling test to analyze whether there is a relation of asset returns to ESG characteristics and/or to ESG factor loading. After the type of relation is determined, these investors form their portfolios accordingly.

For the case studies, we rely on the database "Refinitiv ASSET4" (ASSET4) to acquire an independent external ESG measure (e.g., Gasser et al., 2017; Hoepner et al., 2024). ASSET4 offers more than 250 key indicators of environmental, social and governance performance. The key indicators from these areas are aggregated into an overall ESG measure (TESGS). We follow the literature and use the aggregated ESG measure as the ESG characteristic of

⁶ If we were to use excess returns, then the regression constant would be zero.

an asset (e.g., Gasser et al., 2017; Pedersen et al., 2021). The ESG characteristic has values ranging between 0 and 100, indicating the lowest and highest scores, respectively. Due to the demonstrative character of our case study to show the application of our portfolio optimization and the disentangling test, we assume that the ESG characteristics from ASSET4 correctly indicate the ESG level of a firm. However, Berg et al. (2022) show that the correlation between ESG characteristics from different data providers is low; hence, the results from empirical studies, including our case studies, are not comparable if they use different data providers.

We conduct our empirical study with the constituents of the S&P 500 index. The data of firms from the S&P 500 index are obtained from "Refinitiv Datastream". To obtain the ESG characteristics, we match firms from the S&P 500 index that have data in the ASSET4 database. We acquire the monthly constituent list, the monthly total return index and the aggregated ESG characteristics via the Refinitiv Datastream from December 1989 to December 2019. The monthly asset excess returns are calculated based on the total return indices, which include dividend payments and the risk-free rate. The time series length in our final data sample is limited due to the availability of ESG data. The ESG characteristics are available from January 2001 to December 2019.

The constituents of the S&P 500 index are regularly updated because the index includes shares of the 500 largest US firms by market valuation. The number of firms in the index in a given month is always 500. As the constituents of the index change over time, our data sample becomes unbalanced. However, our optimized portfolio approach does not require balanced data samples because at time t, we do not need the covariance matrix, which is otherwise estimated over the last t - H observations, whereas H typically indicates the last 5 years. It requires to include the factor loadings to risk factors and the ESG characteristics. The ESG characteristic is known at time t, and the factor loadings at time t can be estimated for each asset even if the time series of the observations do not have the same length. The solution for our portfolio optimization program requires the data on ESG characteristics and on factor loadings to be available at time t to calculate the portfolio return at time t + 1. Consequently, the number of assets in the portfolio may vary over time due to the data availability of the ESG characteristic. Our final sample consists of 686 individual assets with an ESG rating that were members of the S&P 500 index during the 2001–2019 period. The data sample includes assets without a full time series of observations. Table 1 classifies the stocks included in the data sample by ESG score levels and provides the descriptive statistics of the returns. According to Table 1, numerous assets are assigned ESG scores between 21 and 80. The return-risk reward (μ/σ) increases with the ESG characteristic because the portfolio risk decreases as the values of the ESG characteristic increase.

We assume a two-risk factor world. The risk factors are a market portfolio and an ESG risk factor. Regarding the market portfolio, we define its returns as the value-weighted returns of all assets available at the time of portfolio formation. Regarding the ESG risk factor, Maiti (2021) proposes forming a zero-net-investment portfolio in line with the factor construction of Fama and French (1993). In line with Maiti (2021), in each month, the ESG risk factor-mimicking portfolio goes long into a high-ESG portfolio and short into a low-ESG portfolio. The high-ESG portfolio consists of assets with an ESG characteristic greater than the 75th percentile in the month of portfolio formation, and its return is a simple average of the asset than the 25th percentile in the month of portfolio formation, and its monthly return is a simple average of the asset returns.

Table 2 reports the descriptive statistics of our two risk factors, ESG characteristics θ , factor loadings to the market portfolio β and ESG risk factor e. The mean μ and the standard deviation σ are time series values in % for the risk factors (r_m and r_{ESG}) and panel data values

Table 1	Descriptive statistics of
monthly	returns of assets at
different	ESG score levels in %

ESG	Median	μ	σ	P(25)	P(75)	μ/σ	N
0–20	1.53	1.14	6.69	-2.09	5.12	0.170	5
21-40	1.23	1.02	5.11	-1.51	3.55	0.199	131
41-60	1.21	0.97	4.81	-1.30	3.67	0.202	238
61-80	1.29	1.08	4.71	-1.00	3.55	0.229	258
81-100	1.36	1.01	4.19	-0.95	3.46	0.241	54

 μ is the simple average return; σ is its standard deviation; P(25) and P(75) report the 25th and 75th percentiles, respectively; and N is the time series average number of assets in each portfolio. The statistics Median, μ , σ , P(25), P(75), μ/σ and N are calculated for equally weighted portfolios, monthly sorted on ESG characteristics into portfolios with ESG ranges as indicated in the first column. We use the actual ESG values of each firm in month t. For example, a firm with an actual ESG value between 0 and 20 in month t will be assigned to the first portfolio with the ESG range 0–20. Since the firm's ESG values can change over time and become greater or less than 20, the constituents of the ESG portfolio can change over time. Then, we calculate the average return in month t for the portfolio and repeat the exercise until the last period. The procedure results in a time series of monthly portfolio returns

	r_m	rESG	θ	β	е
μ	0.74	0.15	58.94	1.08	-0.49
σ	4.06	1.72	17.75	0.51	1.30
Correlati	on of the ris	k factor time	e series		
r_m	1.00				
rESG	-0.35	1.00			
θ	β	е			

Correlation of factor loadings and ESG characteristic

θ	1.00		
β	-0.03	1.00	
е	0.16	-0.26	1.00

The return values are in %. r_m is the return on the market portfolio, r_{ESG} is the return on the ESG risk factor, θ is the firm's ESG characteristic, β is the firm's factor loading to the market return, and e is the firm's factor loading to the ESG risk factor. μ is the average, while σ is its standard deviation. The correlations are calculated for the time series (r_m, r_{ESG}) and in panel (θ, β, e) variables

for the asset- and time-specific θ , β and e. The factor loadings β and e are calculated in each month in simple time series regressions based on the previous five years of observations. The monthly average return of 0.74% and its standard deviation of 4.06% are considerably higher for the market portfolio relative to the ESG risk factor (0.15% and 1.72%, respectively). The average firm in our sample has an ESG characteristic greater than 50, a factor loading to the market portfolio of approximately 1 and a negative factor loading to the ESG risk factor of approximately -0.5. The time series correlation between the variables is calculated for the two risk factors and is found to be negative. The panel correlations are calculated for the asset- and time-specific variables θ , β and e and are found to be low, particularly the

 Table 2
 Descriptive statistics of the risk factor returns, ESG characteristics, and factor loadings to the risk factors

Table 3Results of the test of thefour portfolios based on Eq. (30)		b_0	<i>b</i> ₁	<i>b</i> ₂	<i>R</i> ²
based on empirical data	r_{p1}	0.00	-0.38	-2.26	0.10
	r_{p2}	-0.01	0.78	0.09	0.72
	r_{p3}	0.00	0.01	0.03	0.12
	r_{p4}	0.00	0.01	0.54	0.52

The table columns b_0 , b_1 , b_2 and R^2 show the estimated OLS coefficients from the regression (32), where the portfolio excess returns are regressed on the risk factors. The values of the regression coefficients marked in bold are statistically significantly different from zero at the 5% level and are calculated with HAC-robust standard errors. The standard errors are Newey-West corrected with a lag number of 4. R^2 is the adjusted R^2

correlation between the ESG characteristic and ESG factor loading. The correlation values suggest that the ESG characteristic does not simply translate into a specific value of ESG factor loading. For example, assets with high ESG values do not automatically have high ESG factor loadings.

4.2 Case study

This subsection presents the application of the disentangling test and of the portfolio optimization program. In the first step, we will test the relation of asset returns to ESG characteristics and to ESG factor loading by applying our disentangling test, which is presented in Sect. 3. In the second step, depending on the results from the disentangling test, we will form the portfolios either according to the first strand of the literature (ESG characteristic) or the second strand of the literature (ESG factor loading) or a combination of both strands of the literature or neither of them.

Table 3 reports the results from the first step, which is the disentangling test shown in Eq. (32). In each month, we determine the weights of the out-of-sample excess returns on four portfolios in the subsequent month (as specified by Eq. (30)).

Table 3 shows the coefficients from a time series regression of optimized portfolio excess returns on the two risk factors. The reported coefficients in the table are OLS coefficients, but the standard errors are corrected for heteroscedasticity and autocorrelation (HAC corrected). Due to our disentangling optimization approach, we know the expected values of the regression coefficients. If the first strand of the literature is in line with the data, then we expect to see a significant regression constant (b_0) for portfolio 3 and insignificant regression coefficients b_2 for portfolios 3 and 4. If the second strand of the literature correctly describes the asset return variation, then the risk factor model must be true, and accordingly, we expect to observe significant coefficients close to one for the market portfolio (b_1) and the ESG risk factor (b_2) in portfolios 2 and 4, respectively, and insignificant regression constants (b_0). A mix of the results would indicate that a mixed model of ESG characteristics and ESG factor loading could be true. If no regression coefficients for portfolios 3 and 4 are significant, neither strand of the literature with respect to ESG is a correct description of asset return variation.

There are three notable results of our test. First, the regression constant of portfolio 2 is significant, which suggests that the risk factor model cannot fully price portfolio returns. However, the constant is economically less relevant. Second, the returns on portfolios 1 (zero-beta portfolio) and 3 (ESG characteristic portfolio) are not significantly exposed to the

A. Investors' setting		ors' settin	g (pre-formation)	B. Port	B. Portfolio statistics (post-formation)		
Р	Budget	β	е	$\overline{\mu}$	σ	SR	
1	1	0.66	- 1.25	0.73	3.47	0.21	
2	1	0.66	-0.5	0.77	3.25	0.24	
3	1	0.66	0.25	0.82	3.08	0.27	

 Table 4
 Descriptive statistics of the monthly returns of portfolios for investors following second strand of the literature at different ESG risk factor factor loadings in %

"P" is the number of the portfolio. "Budget" is the sum of the portfolio weights, β is the factor loading to the market factor and *e* is the factor loading to the ESG risk factor. "Budget", β and *e* define the investor-specific portfolio. μ , σ and "SR" are the mean, standard deviation and Sharpe ratio of the out-of-sample portfolio returns, respectively

market risk factor. However, both portfolios are somewhat exposed to the ESG risk factor. Interestingly, the factor loading of portfolio 1 to the ESG factor is high and insignificant, while the factor loading of portfolio 3 to the ESG factor is significant but economically rather low. Third, the returns on portfolios 2 and 4 are exposed to congruent risk factors and significantly different from zero. However, our disentangling test suggests that the factor loadings are expected to be 1. Since the (untabulated) HAC standard errors are 0.08 and 0.14 for the coefficient to the market factor for portfolio $2(b_1)$ and the coefficient to the ESG risk factor for portfolio 4, respectively, only the former is statistically insignificantly different from 1. The coefficient b_2 for the ESG risk factor is significantly less than 1, which is not in line with our expectation. According to the disentangling test, the coefficient must be insignificantly different from 1. There are several explanations for the result. First, an explanation for the lower than 1 value might be that the optimization relies on the estimated factor loadings e, which are subject to the errors-in-variable problem, i.e., they are not perfectly correlated with the true factor loadings. Second, the reason for the result might be our formation of the ESG risk factor. We form the ESG risk factor in line with the literature, but it may not fully capture an ESG risk premium. Third, our model includes only two risk factors, the market and ESG risk factors. There could be additional important risk factors or characteristics that we did not include in our illustrative case study.

The results of the disentangling test suggest that the ESG characteristics do not add to the explanation of stock returns in the cross-section. However, the two risk factor models cannot fully explain the time series of the estimated risk factor premiums. Nevertheless, notably, the empirical investigation presented here is only of a demonstrative nature. In a real-world application, additional tests would be necessary to determine the additional drivers of the asset return variation. For our illustrative example, we will refrain from conducting additional tests and assume that the ESG characteristics do not add to the explanation of the return variation in our sample and that the risk factor model performs sufficiently well for our purpose. Accordingly, extended mean-variance investors will follow the second strand of the literature and consider the ESG factor loading only in the portfolio formation.

Table 4 reports descriptive statistics for three portfolio returns. The portfolios are formed according to the optimization program (25)–(28). The three portfolios are for illustrative purposes only. In portfolios one to three, the loadings to the ESG risk factor are arbitrarily set to -1.25, -0.5 and 0.25. The mean value of the ESG exposure is -0.49 for our data sample, and our choice of -0.5 roughly mimics investors with an average exposure to ESG risk. The values -1.25 and 0.25 for ESG loadings are set equidistant from the mean value. Then, in each month, we determine the weights of three portfolios according to Eq. (29)

and calculate the out-of-sample return in the next month. We calculate the weighted average return according to Eq. (30) and repeat the calculations for all months t = 1, 2, ..., F - 1. We obtain for each portfolio a time series of portfolio returns. These time series of portfolio returns are used to calculate descriptive statistics in Table 4.

Part A of Table 4 reports the setting of the portfolio optimization. Part B of Table 4 shows the out-of-sample descriptive statistics of the portfolio returns. We present the monthly average out-of-sample return (column μ), the standard deviation of the portfolio return (column σ) and the Sharpe ratio (column SR), which is calculated as μ/σ . In Table 4, all of the portfolios are set to be fully invested (column Budget) and have an arbitrarily chosen beta of 0.66 (column β). We assume risk-averse investors who are only willing to accept portfolio risks less than the market risk. The portfolios differ in how the factor loadings to the ESG risk factor (column e) are set. While these values are chosen arbitrarily, the ascending order of the factor loadings to the ESG risk factor can confirm by example the relation between the ESG factor loading and portfolio return from the disentangling test.

We observe that the portfolio return (column μ) increases with increasing values of the ESG factor loading from portfolios 1 to 3. Based on the results from the disentangling test, the increasing portfolio return is indeed not surprising. Similarly, the portfolio risk decreases from portfolios 1 to 3. Because the market factor loading is held constant and the portfolio variances increase with the squared value of the factor loading (i.e., $Var(r_p) = \beta_p^2 \sigma_{r_m}^2 + e_p^2 \sigma_{r_e}^2$), the result is plausible and is in line with the observation from the disentangling test that the returns are exposed, however not perfectly, to the ESG risk factor.

Our case study and the presented results, which are purely demonstrative, reflect the ongoing discussion in the academic literature. Our results indicate that the inclusion of ESG aspects in portfolio optimization may be beneficial for the out-of-sample return and Sharpe ratio. However, our results from the disentangling test are inconclusive. The results do not support the first strand of the literature, where ESG characteristics are important for the explanation of the asset returns. The results are only weakly in favor of the second strand of the literature, which assumes the existence of an ESG risk factor. More research than an illustrative case study is needed to determine whether the ESG characteristic, the factor loading to an ESG risk factor, a mixture of the two or neither explains the asset returns. This is all the more important as Berchicci and King (2020) show that the relation among return, risk and ESG is sensitive to the period, market, data provider and methods applied in the literature.

5 Conclusion

This paper makes several contributions to the literature. First, motivated by the increasing popularity of ESG aspects in the investment fund- and banking industries, we introduce and demonstrate an optimized portfolio approach that has technical and practical advantages over the incorporation of ESG into the traditional mean-variance approach. Additionally, based on the assumption of i.i.d. residual returns, we show how portfolio optimization is conducted without the estimation of the covariance matrix. Second, based on the proposed portfolio formation, we introduce a simple test that allows investors to distinguish between competing explanations for the relation among ESG, risk and return.

The paper uses the two-risk factor model and a single (ESG) characteristic for concreteness and simplicity. Our optimized portfolio approach is suitable for more general environments, as shown in the Appendix for arbitrage pricing theory (APT). In fact, the optimized portfolio approach can address a very large number of (arbitrarily chosen) firm characteristics. Even with a large number of assets, risk factors and characteristics, the optimized portfolio approach is simple to implement. Additionally, our approach can easily be adopted to create factor tracking portfolios (by setting the respective factor loading constraint to one), factor neutral portfolios (by setting the respective factor loading constraint to zero), style tracking portfolios (by setting the respective constraint to zero) or mixtures of these approaches. Another appealing feature of our optimization approach is that an analytical solution can be derived. For practical applications, where additional constraints and subset selection out of investment universe are important, the investors need to implement numerical portfolio optimization problems, which may be considerably more difficult so solve. Our test can also help with the more general debate (e.g. Fama & French, 1993; Daniel & Titman, 1997; Daniel et al., 2020) regarding whether the factor loadings to risk factors (such as SMB or HML) or congruent characteristics (such as firm size and the book-to-market-equity ratio) explain returns.

The models and corresponding portfolio optimizations presented in our paper consider a broad range of empirical and theoretical literature. However, there are still limitations to our approach. There is empirical evidence that tail-risk measures are related to ESG characteristics since companies with high ESG characteristics are less vulnerable to companyspecific negative events (e.g., Diemont et al., 2016). Since portfolio optimization assumes an investor who accepts variance as the appropriate risk measure, our approach cannot account for alternative risk measures (e.g., expected shortfall, value at risk).

Appendices

A Portfolio optimization with K risk factors and M characteristics

Let asset returns be generated by the asset return model under assumptions 1, 2 and 4 and the refined Assumption 3a (Eqs. (34)–(36)).

Assumption 3a The return r_{it} on asset *i* at time *t* is described by the factor model (34).

$$r_{it} = E_{t-1}(r_{it}) + \sum_{k=1}^{K} \beta_{ik} \tilde{f}_{kt} + u_{it}$$
(34)

where β_{ik} is the loading of asset *i* on factor *k* and $\tilde{f}_{kt} \sim \mathcal{N}(0, \sigma_{f_k}^2)$ is the factor's *k* innovation at time *t*. $u_{it} \sim \mathcal{N}(0, \sigma_u^2)$ is the residual asset return. The expected return is

$$E_{t-1}(r_{it}) = r_{ft} + \underbrace{\left(\sum_{k=1}^{K} \beta_{ik} \mu_k + \sum_{m=1}^{M} \theta_i c_m\right)}_{\text{excess return}}$$
(35)

where r_{ft} is the return on the risk-free asset at time t, μ_k is the premium on risk factor k, θ_i is the characteristic of asset i and $c_m \in \mathbb{R}$ is the reward for characteristic m. The covariance matrix for the asset return is

$$\boldsymbol{V} = \boldsymbol{B}\boldsymbol{V}_{f}\boldsymbol{B}^{\mathsf{T}} + \boldsymbol{R}\boldsymbol{V} \tag{36}$$

where V_f is the $K \times K$ diagonal covariance matrix of orthogonal factor returns, **B** is the $N \times K$ matrix of asset factor loadings to K risk factors, and **RV** is the $N \times N$ covariance matrix

of residual asset returns. The residual returns are identically and independently distributed (i.i.d. residual returns); hence, $Cov(u_i, u_j) = 0$, $\forall i \neq j$ and $Cov(u_i, u_j) = \sigma_u^2$, $\forall i = j$.

We again reformulate the portfolio optimization program (3)–(6). The variance of the portfolio is

$$\sigma_p^2 = \boldsymbol{w}^{\mathsf{T}} \left(\boldsymbol{B} \boldsymbol{V}_f \boldsymbol{B}^{\mathsf{T}} + \boldsymbol{R} \boldsymbol{V} \right) \boldsymbol{w}$$

= $\boldsymbol{w}^{\mathsf{T}} \boldsymbol{B} \boldsymbol{V}_f \boldsymbol{B}^{\mathsf{T}} \boldsymbol{w} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{R} \boldsymbol{V} \boldsymbol{w}$
= $\boldsymbol{\beta}^{\mathsf{T}} \boldsymbol{V}_f \boldsymbol{\beta} + \sigma_{\varepsilon} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{I} \boldsymbol{w}$

where I is the $N \times N$ identity matrix and β is a $K \times 1$ vector, whose elements represent factor loading values of the portfolio.

Similarly, the investor-specific levels b and θ determine the portfolio return since

$$\mu_p^* = \boldsymbol{w}^{\mathsf{T}}\boldsymbol{\mu} = \boldsymbol{w}^{\mathsf{T}}\left(r_f \mathbf{1} + \boldsymbol{B}\boldsymbol{\mu}_k + \boldsymbol{\Theta}\boldsymbol{c}\right) = r_f \boldsymbol{w}^{\mathsf{T}} \mathbf{1} + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{B}\boldsymbol{\mu}_k + \boldsymbol{w}^{\mathsf{T}} \boldsymbol{\Theta}\boldsymbol{c} = r_f + \boldsymbol{\beta}^{\mathsf{T}} \boldsymbol{\mu}_k + \boldsymbol{\theta}^{\mathsf{T}} \boldsymbol{c}$$

where μ_k is the $K \times 1$ vector of risk factor premiums, **B** is a $N \times K$ matrix of factor loadings, **c** is the $M \times 1$ vector of characteristic rewards, Θ is the $N \times M$ matrix of characteristics and θ is the $M \times 1$ vector, whose elements represent characteristic values of the portfolio.

By the same arguments as given in Sect. 2.2 (linking preferences to levels), the portfolio optimization program (3)–(6) results in

s.

$$\min_{\boldsymbol{w}} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{w} \tag{37}$$

$$\mathbf{t}. \quad \boldsymbol{w}^{\mathsf{T}}\mathbf{1} = 1 \tag{38}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{B} = \boldsymbol{\beta}^{\mathsf{T}} \tag{39}$$

$$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{\Theta} = \boldsymbol{\theta}^{\mathsf{T}} \tag{40}$$

where the constraint (39) constrains the loadings to risk factors and the constraint (40) constrains the portfolio characteristics.

For the sake of clarity, we gather the variables on the left-hand side of the constraints (38)–(40) into a $N \times (1 + K + M)$ matrix X with

$$X = [1, B, \Theta]$$

and the variables on the right-hand side of the constraints are gathered into a $(1 + K + M) \times 1$ vector **g** with

$$\boldsymbol{g} = [1, \boldsymbol{\beta}^{\mathsf{T}}, \boldsymbol{\theta}^{\mathsf{T}}]^{\mathsf{T}}$$

yielding the refined optimization program in Eqs. (41)–(42).

$$\min_{\boldsymbol{w}} \quad \frac{1}{2} \boldsymbol{w}^{\mathsf{T}} \boldsymbol{w} \tag{41}$$

s.t.
$$X^{\mathsf{T}} w = g$$
 (42)

The solution for the optimal portfolio weights is given in Eq. (43).

$$\boldsymbol{w}^{\mathsf{T}} = \boldsymbol{g}^{\mathsf{T}} \left(\boldsymbol{X}^{\mathsf{T}} \boldsymbol{X} \right)^{-1} \boldsymbol{X}^{\mathsf{T}}$$
(43)

The optimal solution of Eq. (43) differs across investors because they can choose different values for the elements of g.

🖄 Springer

B Disentangling test with M risk factors and M characteristics

For the purpose of the disentangling test, we must calculate the portfolio returns at time t, t = 1, 2, ..., F. Based on the solution for optimal portfolio weights in Eq. (43), the return r_{pt} on portfolio p at time t is

$$r_{pt} = \boldsymbol{w}_{t-1}^{\mathsf{T}} \boldsymbol{r}_t = \boldsymbol{g}^{\mathsf{T}} \left(\boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{X}_{t-1} \right)^{-1} \boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{r}_t$$
(44)

By the choices of the elements in vector g, the investors can specify the desired portfolio features and earn different returns.

We can calculate the returns on P different portfolios in one step when we observe that the portfolios for one investor differ only with respect to the values in g. Let $G = [g_1, g_2, ..., g_P]$ be a $(1 + K + M) \times P$ matrix of right-hand-side values of portfolio constraints. The number of portfolios P can be greater than, less than or equal to (1 + K + M). For the purpose of the disentangling test, we are interested in the special case P = 1 + K + M because we test whether the K risk factors and the M characteristics are priced while one portfolio tracks the risk-free return. In "Appendix" section A Eq. (43), we show how the optimization for a single portfolio works for the general case of K risk factors and M characteristics. We can calculate the realized return for all (1 + K + M) portfolios at time t as

$$\boldsymbol{\lambda}_{t} = \boldsymbol{W}_{t-1}^{\mathsf{T}} \boldsymbol{r}_{t} = \boldsymbol{G}^{\mathsf{T}} \left(\boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{X}_{t-1} \right)^{-1} \boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{r}_{t} \forall t$$
(45)

where λ_t is a $P = (1 + K + M) \times 1$ vector of portfolio returns at time t, and W_t is for the purpose of the disentangling test a $N \times (1 + K + M)$ matrix of asset weights. Note that the dimensions of λ_t and W_t are $P \times 1$ and $N \times P$, respectively, if we simply calculate the return on P portfolios.

To test whether the *K* risk factors and *M* characteristic portfolios are priced, among a large number of potential approaches, presumably the easiest approach is to create tracking portfolios and to test whether the created tracking portfolio loads on the corresponding risk factor. In our portfolio optimization program, the tracking portfolios are easily obtained by setting the respective element of the constraints (*g*) to the value 1 and all other elements to zero. For example, if we set the first value of *g* to 1, then the portfolio is a zero-factor-loading and zero-characteristic portfolio, which earns the risk-free rate at time *t* for the investors. If we set the second (K + 2) element of *g* to 1, then the portfolio tracks the risk premium (characteristic reward) of the first risk factor (first characteristic).

By setting the constraint for the factor loading or the characteristic to 1 in vector g, and all other elements of g being zero, the resulting tracking portfolio is a zero-net-investment. While zero-net-investment portfolios are rare in practical portfolio management, they are useful for our disentangling test. Under the null hypothesis of the true risk factor model, the traditional approach is to test whether the regression constant from a regression of portfolio returns on risk factors is significantly different from zero. As we form zero-net-investment tracking portfolios, we can always directly test this hypothesis because the returns on such portfolios are always without the risk-free return component of asset returns. To understand this point, consider a two-assets zero-investment portfolio that tracks the first risk factor. The tracking portfolio is formed to be exposed only to the first risk factor. The sum of asset weights is zero; hence, the portfolio weights are $w_1 + w_2 = 0 \Leftrightarrow w_2 = -w_1$. According to the risk factor model (34), the realized return on this portfolio is

$$\begin{aligned} r_p &= w_1 \left(r_{ft} + \beta_{11} \mu_1 + \beta_{11} \tilde{f}_{1t} \right) - w_1 \left(r_{ft} + \beta_{21} \mu_1 + \beta_{21} \tilde{f}_{1t} \right) \\ &= (w_1 - w_1) r_{ft} + w_1 \left(\beta_{11} \mu_1 + \beta_{11} \tilde{f}_{1t} - \beta_{21} \mu_1 - \beta_{21} \tilde{f}_{1t} \right) \\ &= \mu_1 \left(w_1 \beta_{11} - w_1 \beta_{21} \right) + \tilde{f}_{1t} \left(w_1 \beta_{11} - w_1 \beta_{21} \right) \\ &= \mu_1 \beta_{p1} + \tilde{f}_{1t} \beta_{p1} = \mu_1 + \tilde{f}_{1t} \end{aligned}$$

The realized returns on such a portfolio can directly be used in traditional approaches to test whether the regression constant is significantly different from zero. On the other hand, if we use the full-investment portfolio, i.e., $w_1 + w_2 = 1$, the return on such a portfolio is

$$r_{p} = w_{1} \left(r_{ft} + \beta_{11}\mu_{1} + \beta_{11}\tilde{f}_{1t} \right) + w_{2} \left(r_{ft} + \beta_{21}\mu_{1} + \beta_{21}\tilde{f}_{1t} \right)$$

$$= (w_{1} + w_{2}) r_{ft} + w_{1} \left(\beta_{11}\mu_{1} + \beta_{11}\tilde{f}_{1t} \right) + w_{2} \left(\beta_{21}\mu_{1} + \beta_{21}\tilde{f}_{1t} \right)$$

$$= r_{ft} + \mu_{1} (w_{1}\beta_{11} + w_{2}\beta_{21}) + \tilde{f}_{1t} (w_{1}\beta_{11} + w_{2}\beta_{21})$$

$$= r_{ft} + \mu_{1}\beta_{p1} + \tilde{f}_{1t}\beta_{p1} = r_{ft} + \mu_{1} + \tilde{f}_{1t}$$

In this example, the full-investment tracking portfolio earns the risk-free rate and the risk premiums. Then, in the traditional approach, we need to test whether the regression constant is significantly different from r_f , i.e., the test would be $b_0 - r_f = 0$ with b_0 being the regression constant. By means of zero-net-investment, we obtain portfolio returns that are easily tested in the regression framework.

Consequently, the choice for the matrix G as the $(1 + K + M) \times (1 + K + M)$ identity matrix in our disentangling test allows the calculation of the tracking portfolio returns μ for (1 + K + M) portfolios at time *t*, i.e.,

$$\boldsymbol{\lambda}_{t} = \left(\boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{X}_{t-1}\right)^{-1} \boldsymbol{X}_{t-1}^{\mathsf{T}} \boldsymbol{r}_{t} \forall t \tag{46}$$

where the elements of the $(1 + K + M) \times 1$ vector λ_t represent the returns of tracking portfolios for the risk-free rate, the *k*-th factor return realizations and the *m*-th characteristic reward realizations, respectively. For example, the APT factor premium μ_k is the return on a portfolio that mimics the *k*-th risk factor such that $\mu_k = \sum_{i=1}^{N} w_{ik}r_i$, where w_{ik} is the weight of asset *i* in portfolio k + 1. As the calculation of realized returns λ_t is done in each time period *t*, we can define a $(1 + K + M) \times F$ matrix $\mathbf{A} = [\lambda_1, \lambda_2, \dots, \lambda_F]$.

After forming the tracking portfolio for each time *t* according to Eq. (46), the next step in our disentangling test is the regression of the portfolio returns in Λ on the risk factor realization as in Eq. (34), i.e.,

$$\mathbf{\Lambda} = \mathbf{B}_F \mathbf{F}^\mathsf{T} + \mathbf{E} \tag{47}$$

where each row of Λ represents a time series of realized returns on one of the (1 + K + M) portfolios at time $t, t \in \{1, 2, ..., F\}$; F is a $F \times (K + 1)$ matrix of ones (constant) and K columns of returns of risk factors. B_F is a $(1 + K + M) \times (K + 1)$ matrix of regression coefficients, i.e., the constant and the risk factor factor loadings β from the spanning test (47). E is a $(1 + K + M) \times F$ matrix of the random residual returns.

The important consequence from the choice of the values in *G* as identity from Eq. (46) is that we know the expected values for the factor loadings from the spanning tests (32). If the risk factor model is valid, the expected values (\mathbb{E}) of the coefficients in B_F from the regression of portfolio returns are

$$\mathbb{E}(\boldsymbol{B}_{F}) = \begin{bmatrix} r_{f} & 0 & 0 & \dots & 0\\ 0 & \beta_{1,p2} = 1 & 0 & \dots & 0\\ 0 & 0 & \beta_{3,p3} = 1 & \dots & 0\\ 0 & 0 & 0 & \dots & \beta_{K,p(1+K+M)} \end{bmatrix}$$
(48)

or, if we use excess asset returns

$$\mathbb{E}(\boldsymbol{B}_F) = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & \beta_{1,p2} = 1 & 0 & \dots & 0 \\ 0 & 0 & \beta_{3,p3} = 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & \beta_{K,p(1+K+M)} \end{bmatrix}$$
(49)

Author Contributions Armin Varmaz: The author has no relevant financial or non-financial interests to disclose. Christian Fieberg: The author has no relevant financial or non-financial interests to disclose. Thorsten Poddig: The author has no relevant financial or non-financial interests to disclose.

Funding Open Access funding enabled and organized by Projekt DEAL.

Data Availability The data that support the findings of this study are available from Refinitiv but restrictions apply to the availability of these data, which were used under license for the current study, and so are not publicly available.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

- Albuquerque, R., Koskinen, Y., & Zhang, C. (2019). Corporate social responsibility and firm risk: Theory and empirical evidence. *Management Science*, 65(10), 4451–4469.
- Bajeux-Besnainou, I., Bandara, W., & Bura, E. (2012). A Krylov subspace approach to large portfolio optimization. *Journal of Economic Dynamics and Control*, 36(11), 1688–1699.
- Bénabou, R., & Tirole, J. (2010). Individual and corporate social responsibility. *Economica*, 77(305), 1–19.
- Benedetti, D., Biffis, E., Chatzimichalakis, F., Fedele, L. L., & Simm, I. (2021). Climate change investment risk: Optimal portfolio construction ahead of the transition to a lower-carbon economy. *Annals of Operations Research*, 299, 847–871.
- Berchicci, L., & King, A. (2020). Evidence on social and financial performance: Mapping the empirical garden of forking paths. Academy of Management Proceedings, 2020(1), 17546.
- Berg, F., Kölbel, J. F., & Rigobon, R. (2022). Aggregate confusion: The divergence of ESG rating. *Review of Finance*, 8, 1–30.
- Bilbao-Terol, A., Arenas-Parra, M., & Cañal-Fernández, V. (2012). Selection of socially responsible portfolios using goal programming and fuzzy technology. *Information Sciences*, 189, 110–125.
- Bilbao-Terol, A., Arenas-Parra, M., Cañal-Fernández, V., & Bilbao-Terol, C. (2013). Selection of socially responsible portfolios using hedonic prices. *Journal of Business Ethics*, 115(3), 515–529.
- Black, F., Jensen, M., & Scholes, M. (1972). The capital asset pricing model: Some empirical tests. In M. Jensen (Ed.), *Studies in the theory of capital markets*. Praeger Publishers Inc.
- Bodnar, T., Okhrin, Y., Vitlinskyy, V., & Zabolotskyy, T. (2018). Determination and estimation of risk aversion coefficients. *Computational Management Science*, 15(2), 297–317.
- Bodnar, T., Parolya, N., & Schmid, W. (2018). Estimation of the global minimum variance portfolio in high dimensions. *European Journal of Operational Research*, 266(1), 371–390.
- Bollerslev, T., Gibson, M., & Zhou, H. (2011). Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities. *Journal of Econometrics*, 160(1), 235–245.

- Bolton, P., & Kacperczyk, M. (2020). Do investors care about carbon risk? (Technical Report). National Bureau of Economic Research.
- Costola, M., Maillet, B., Yuan, Z., & Zhang, X. (2022). Mean-variance efficient large portfolios: A simple machine learning heuristic technique based on the two-fund separation theorem. *Annals of Operations Research*, 8, 1–23.
- Daniel, K., Mota, L., Rottke, S., & Santos, T. (2020). The cross-section of risk and returns. *The Review of Financial Studies*, 33(5), 1927–1979.
- Daniel, K., & Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns. *The Journal of Finance*, 52(1), 1–33.
- Das, S., Markowitz, H., Scheid, J., & Statman, M. (2010). Portfolio optimization with mental accounts. *Journal of Financial and Quantitative Analysis*, 45(2), 311–334.
- Diemont, D., Moore, K., & Soppe, A. (2016). The downside of being responsible: Corporate social responsibility and tail risk. *Journal of Business Ethics*, 137(2), 213–229.
- Drut, B. (2010). Sovereign bonds and socially responsible investment. Journal of Business Ethics, 92(1), 131–145.
- Edmans, A. (2011). Does the stock market fully value intangibles? Employee satisfaction and equity prices. *Journal of Financial Economics*, 101(3), 621–640.
- Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33(1), 3–56.
- Fama, E. F., & French, K. R. (2007). Disagreement, tastes, and asset prices. Journal of Financial Economics, 83(3), 667–689.
- Flammer, C. (2015). Does corporate social responsibility lead to superior financial performance? A regression discontinuity approach. *Management Science*, 61(11), 2549–2568.
- Friede, G., Busch, T., & Bassen, A. (2015). ESG and financial performance: Aggregated evidence from more than 2000 empirical studies. *Journal of Sustainable Finance & Investment*, 5(4), 210–233.
- Friedman, H. L., & Heinle, M. S. (2016). Taste, information, and asset prices: Implications for the valuation of CSR. *Review of Accounting Studies*, 21(3), 740–767.
- Gasser, S. M., Rammerstorfer, M., & Weinmayer, K. (2017). Markowitz revisited: Social portfolio engineering. European Journal of Operational Research, 258(3), 1181–1190.
- Gillan, S. L., & Starks, L. T. (2000). Corporate governance proposals and shareholder activism: The role of institutional investors. *Journal of Financial Economics*, 57(2), 275–305.
- Grewal, J., Serafeim, G., & Yoon, A. (2016). Shareholder activism on sustainability issues (Technical Report). Harvard Business School.
- Harrison, G. W., & Rutström, E. E. (2008). *Risk aversion in the laboratory. Risk aversion in experiments*. Emerald Group Publishing Limited.
- Hirschberger, M., Steuer, R. E., Utz, S., Wimmer, M., & Qi, Y. (2013). Computing the nondominated surface in tri-criterion portfolio selection. *Operations Research*, 61(1), 169–183.
- Hoepner, A. G., Oikonomou, I., Sautner, Z., Starks, L. T., & Zhou, X. Y. (2024). ESG shareholder engagement and downside risk. *Review of Finance*, 28(2), 483–510.
- Kremer, P. J., Talmaciu, A., & Paterlini, S. (2018). Risk minimization in multifactor portfolios: What is the best strategy? Annals of Operations Research, 266, 255–291.
- Laloux, L., Cizeau, P., Bouchaud, J.-P., & Potters, M. (1999). Noise dressing of financial correlation matrices. *Physical Review Letters*, 83(7), 1467.
- Ledoit, O., & Wolf, M. (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance*, 10(5), 603–621.
- Ledoit, O., & Wolf, M. (2004). A well-conditioned estimator for large dimensional covariance matrices. *Journal of Multivariate Analysis*, 88(2), 365–411.
- Li, Z., Minor, D. B., Wang, J., & Yu, C. (2019). A learning curve of the market: Chasing alpha of socially responsible firms. *Journal of Economic Dynamics and Control*, 109, 103772.
- Liagkouras, K., Metaxiotis, K., & Tsihrintzis, G. (2020). Incorporating environmental and social considerations into the portfolio optimization process. *Annals of Operations Research*, 8, 1–26.
- Luo, H. A., & Balvers, R. J. (2017). Social screens and systematic investor boycott risk. *Journal of Financial and Quantitative Analysis*, 52(1), 365–399.
- Maiti, M. (2021). Is ESG the succeeding risk factor? Journal of Sustainable Finance & Investment, 11(3), 199–213.
- Merton, R. C. (1972). An analytic derivation of the efficient portfolio frontier. Journal of Financial and Quantitative Analysis, 7(4), 1851–1872.
- Mynbayeva, E., Lamb, J. D., & Zhao, Y. (2022). Why estimation alone causes Markowitz portfolio selection to fail and what we might do about it. *European Journal of Operational Research*, 301(2), 694–707.

- Nguyen, V. A., Kuhn, D., & Peyman, M. E. (2022). Distributionally robust inverse covariance estimation: The wasserstein shrinkage estimator. *Operations Research*, 70(1), 490–515.
- Pástor, L., Stambaugh, R. F., & Taylor, L. A. (2021). Sustainable investing in equilibrium. Journal of Financial Economics, 142(2), 550–571.
- Pedersen, L. H., Fitzgibbons, S., & Pomorski, L. (2021). Responsible investing: The ESG-efficient frontier. Journal of Financial Economics, 142(2), 572–597.
- Plachel, L. (2019). A unified model for regularized and robust portfolio optimization. Journal of Economic Dynamics and Control, 109, 103779.
- Qi, Y., & Steuer, R. E. (2020). On the analytical derivation of efficient sets in quad-and-higher criterion portfolio selection. Annals of Operations Research, 293, 521–538.
- Qi, Y., Steuer, R. E., & Wimmer, M. (2017). An analytical derivation of the efficient surface in portfolio selection with three criteria. Annals of Operations Research, 251, 161–177.
- Sharfman, M. P., & Fernando, C. S. (2008). Environmental risk management and the cost of capital. Strategic Management Journal, 29(6), 569–592.
- Steuer, R. E., & Utz, S. (2023). Non-contour efficient fronts for identifying most preferred portfolios in sustainability investing. *European Journal of Operational Research*, 306(2), 742–753.
- Utz, S., Wimmer, M., Hirschberger, M., & Steuer, R. E. (2014). Tri-criterion inverse portfolio optimization with application to socially responsible mutual funds. *European Journal of Operational Research*, 234(2), 491–498.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.