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Abstract

We consider a market in which a platform hosts third-party monopolistic complementors. Users are segmented into single-use and multi-use consumers, and services exhibit network externalities. In the agency model, the presence of multi-use consumers leads complementors to set inefficiently high prices, reducing demand and the platform's profits. Platform entry can resolve these pricing inefficiencies by targeting multi-use consumers with a bundled offering, but it then fragments the market, diminishing network benefits for consumers. We find that the platform opts to enter only when it has committed to a low commission fee, and network benefits are modest. Integration with a complementor reduces prices for consumers and enhances network benefits, thereby improving consumer welfare, but the pricing inefficiency is only partially mitigated.

JEL-Codes: L220, L860.

Keywords: platform, network externalities, platform strategy, hybrid business model.

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1 Introduction

Platforms have transformed service delivery by integrating diverse offerings within their ecosystems, providing users with a one-stop solution that offers access to various services. This agglomeration of complementary services is achieved by opening their ecosystem and hosting a wide variety of third party complementors. The success of this approach is evident by the dominance of aggregators such as *Apple's App Store* and *Google's Play Store*. A direct implication of this success is that consumers enjoy access to multiple services available on these platforms. In addition to hosting traditional services, platforms also host services that exploit network effects such as (music or video) streaming services, gaming services, social media among others. The presence of network effects in this environment implies that the users subscribing to multiple services creates network externalities across seemingly unrelated markets.¹ This results in externalities between these services which may not be fully internalized leading to inefficiencies in the market.

To manage interactions within their ecosystems, platforms employ a variety of tools. These tools can be in the form of pricing level and structure as well as decisions to enter in competition with the complementors they host. In this paper, we focus on the latter tool. While there are various rationales that provide insights into why platforms prefer a dual role in their ecosystem (e.g., value appropriation as in Gawer 2002), we identify a novel rationale for entry associated with mitigating inefficiencies arising from externalities generated by seemingly independent services.

This paper examines a platform market where users choose to avail services that exploit network externalities, for e.g., music and gaming. These services can be provided either by complementors or by the platform. Users are of different types and can be either single-use consumers, who prefer only one service, or multi-use consumers, who value both. The concurrent presence of network effects and multi-use consumers results in network externalities that affect both complementor and platform profitability. In such a setting, platform operator has to strategically decide on its organizational structure to maximize its revenue streams. Specifically, we allow the platform to choose between two business models. First, the platform can collect part of the value created by thirdparty complementors by charging them a commission fee for each transaction facilitated

¹While specific data on the overlap of consumers between services such as Spotify and Netflix subscribers is limited, both platforms have experienced significant growth in their user bases, indicating that a large number of consumers likely subscribe to both services. As of the second quarter of 2024, Spotify reported to have approximately 246 million paid subscribers, while Netflix had around 277.65 million paid subscribers. See (link)

through the platform. This traditional *agency model* allows the platform to earn a share of the revenue generated by the complementors without directly participating in the service provision and without controlling complementors' prices.² Second, the platform may decide to directly intervene in the marketplace by offering competing services to consumers. This *platform entry* intervention is not neutral since it alters the pricing strategies of the complementors.

We show that under agency, the well-known "Cournot" externality arises due to the presence of multi-use consumers of these service and network externalities exacerbate it.³ This externality results in complementors setting higher prices, which are above the monopoly level. This lowers demand and as the platform obtains revenues from transactions on its ecosystem, its profit falls for a given per-transaction fee.

To mitigate the negative effect of this externality, the platform may choose to foster competition on its ecosystem. One of the strategies available to the platform is taking a more active stance in the market and offering its own services in competition with the third-party complementors. We show that entry with a single service leads to intense price competition between the services of the platform and the complementor, and it destroys all the profits. Hence, a single-service entry is never profitable for the platform. Nevertheless, an equilibrium in which the platform enters and offers multiple services as a bundle exists.⁴ After entry, complementors focus only on the single-use demand segments and the bundled offers of the platform serves the multi-users. This breaks down complementarity across markets and, as a result, leads to lower complementor prices. Despite reducing prices and resolving the pricing inefficiency associated with the Cournot externality, there is a downside for the platform because consumers' demand is fragmented into different and smaller segments. The latter effect is detrimental to consumers as they enjoy reduced value from the network externalities. Thus, the platform's decision to enter must trade-off the resolving the inefficient pricing strategy of complementors with the reduced value generation from a fragmented network on its platform.

We find that the platform's owner prefers to enter with a bundle of services when the (ad-valorem) fee levied on complementors is small enough. In this case, the platform prefers to open a direct channel to serve users. This strategy allows the platform to

^{2}See Hagiu and Wright (2015).

 $^{^{3}}$ For more details on this refer to Tirole (1988).

⁴Such a bundling equilibrium exists when the three groups of consumers have a similar size, and the platform finds it optimal to enter with a bundle of services and cater to multi-use segment only.

mitigate the Cournot externality but becomes less appealing when network externalities get larger. The reason is that, with larger network externalities, demand fragmentation decreases the value associated with these network enabled services, and, to compensate, the platform should decrease the bundle price, i.e., the platform must be very aggressive in its pricing strategy to deter third party sellers from catering to the multi-use segment.

From consumers' perspective, there is a trade-off between the two business models. On the one hand, the agency model features the Cournot externality and thus high prices, which reduces demand and consumer surplus, but network externalities are strong. On the other hand, bundled entry by the platform intensifies competition for the multiuse segment leading to lower prices but this fragments the network benefit. We find that there exists a threshold value of the network benefit above which consumer surplus is higher with the agency model and below which it is lower if the platform enters the market. Interestingly, there is alignment between the choice of the platform and the business model that leads to a higher consumer surplus in two cases. First, when the platform's intermediation fee is high and the network benefit is high, the platform chooses the agency model and also benefits consumers as the value associated with the network size is large. Second, when the fee is small and the network benefit is also preferred by consumers. In all other cases, the decision of the platform harms consumers.

We extend our model by considering another strategy that the platform can adopt to avoid or mitigate the Cournot externality: integrate with one complementor (*integration*), e.g., via acquisitions. We find that the platform has an incentive to acquire one of the complementors if and only if the commission fee is low enough; whereas for a high fee, it can passively collect revenues without having to purchase a costly complementor. However, if an integration is profitable, the Cournot externality is only mitigated. The reason is that it can control directly the price of its own service but not the price of the rival, which increases in response. Importantly, the total price charged to multi-use consumers is now lower than in the agency model. This leads to a higher demand in this segment of the market, thereby, generating network benefits, and increasing the value also obtained by single-use consumers. Since the network benefit is boosted and the composite price is lower under integration, consumers are overall better off with integration relative to the agency model. Indeed, whenever such acquisitions occur, interests of the consumers and the platform are fully aligned.

The structure of the paper is as follows. Section 2 provides a discussion of the related literature. Section 3 presents the model setup. Section 4 presents our main analysis.

Section 5 considers the case in which the platform also enters the market in competition with the complementors and Section 6 discusses the platform's decision and its welfare effects. Section 7 extends the model to consider the case in which the platform integrates with a complementor to enter the market. Section 8 discusses the managerial and policy insights from our work and Section 9 concludes.

2 Related Literature

Our paper relates to different streams of the literature. First and foremost, it relates to the emerging literature on hybrid business model practices of digital platforms. Prominent work includes Etro (2021), Hagiu et al. (2021), Zennyo (2022), Anderson and Bedre-Defolie (2024), Madsen and Vellodi (2024) among others. In these studies, platform entry in competition with third-party complementors can have both positive or negative effects. An important and unique feature of our model is given by how consumer heterogeneity in preferences for different products or services shapes the incentives of the platform on entry choice. This aspect, which applies mostly in an app ecosystems, has not been considered so far since most of the literature has dealt with platform entry in e-commerce environments.

A recent study by Jeon and Rey (2024) highlights the role of Cournot complementarity in platform markets, particularly how the presence of economies of scale, when app developers create apps for multiple platforms, can induce platforms to inefficiently set higher commission fees. In our setting, complementarity arises within a platform ecosystem because two independent markets become linked by a group of users—multi-use users—who can potentially consume both.

Additionally, we contribute to the literature on bundling and tying strategies in multiproduct and platform markets (Prasad et al., 2010; Chao and Derdenger, 2013; Llanes et al., 2019). While these studies typically assume exogenous product complementarities, we endogenize them through consumer heterogeneity, capturing the distinct preferences of single-use and multi-use users. This distinction is particularly relevant in app platforms like the Apple Store or Google Play, where services cater to diverse consumer groups and overlap in user base drives Cournot externalities.

Recent studies have studied platform entry and its implications for market dynamics and welfare. Gawer and Henderson (2007) focuses on how platforms employ organizational

structure of their ecosystem (e.g., entry being a strategic tool) as a commitment mechanism to increase trust and efficiency among the platform participants. Sharma and Mehra (2021) examine platform entry in competition with providers of complementary access hardware, finding that such entry can simultaneously lower prices and improve quality. Carrillo and Tan (2021), on the other hand, study competing platforms and analyze the welfare effects of various strategic approaches. Unlike their focus on competition between platforms, our analysis focuses on a monopoly platform interacting with complementors in a market characterized by network externalities. We investigate how platforms can engage in bundled entry to address inefficiencies stemming from Cournot externalities caused by overlapping customer segments. Consistent with their findings, we show that platform entry can benefit both complementors and consumers by alleviating pricing inefficiencies and enhancing market outcomes.

Our paper also relates to a recent contribution by Chen and Rey (2023), who develop a theoretical framework to analyze the economic effects of conglomerate mergers, especially through the lens of demand-side "consumption synergies" created by bundling products across different markets. In their context, there are two markets and at least two firms in each market. When a conglomerate merger occurs, the merged entity can set a price for the bundle and attract consumers with evaluation of both products if synergies are sufficiently large. All other consumers can buy products from independent companies and mix-and-match products in the two markets. If the independent firms' market is competitive, then competitive pressure leads to prices at cost. However, if only one independent firm remains active in each independent market, this creates a Cournot complementarity effect and prices tend to be higher than the monopoly one. Our setting demonstrates consumption synergies in a context of traditional network externalities that emerge both among consumers within the same group and across groups if consumers in other groups use the same service. The Cournot externality is present in the case where firms are fully independent and a set of consumers mix-and-match services from the two independent firms. Platform entry into the market fragments these consumption externalities, generating a negative effect on consumers. However, entry also eliminates the Cournot externality, thus benefiting consumers.

Although the strategies that two competing firms can adopt to mitigate Cournot complementarity are well known in the literature, it is not well explored how such inefficiency can be eliminated or mitigated in a platform ecosystem, where the intermediary cares about the efficient allocation and pricing of goods. We contribute to the existing literature by examining this specific case.

3 Model Setup

Players and the environment. We consider an ecosystem operator \mathcal{P} (hereafter referred to as the platform) that mediates interactions between complementors, providing differentiated services, and users, who consume these services in exchange for a price.⁵ Complementors, indexed by 1 and 2, are active in separate markets A and B, where complementor 1 provides service A and complementor 2 provides service B to end-users. If \mathcal{P} finds it optimal to do so, it can also enter the market, offering services in competition with the complementors.

This setting captures the context of an app distribution platform (e.g., Apple Store, Google Play as well as gaming platforms) that hosts third party complementors, whose number is normalized to 2, operating in independent markets (e.g. Spotify for music, Epic Games for games) and which can also enter competition with the platform's own services.

Network Externalities Each service exhibits network externalities, as users value interactions with other users of the same service j = A, B from the same provider $i = 1, 2, \mathcal{P}$. If the same type of service (e.g., game) is offered by different parties (e.g., the platform and one of the two complementors), users subscribing to different providers cannot interact, thus missing out on the benefits associated with network externalities. We measure the strength of the network externality for each service as $\theta > 0$ and impose that $\theta \leq 0.91 \equiv \hat{\theta}$. This assumption implies that network externalities are not "too" strong and, as a result, the demands are interior in equilibrium.

Users. We consider a unit mass of consumers which are split into three groups. Consumers who prefer subscribing exclusively to one service j = A or B and multi-use users, who are interested in both services A and B.⁶ We assume that the relative mass of each single-use segment is α and mass of multi-use segment is $1 - 2\alpha$. Within each of the

⁵Note that the platform can host more than two services, but for the purposes of our analysis, we focus on services with overlapping customers.

⁶This assumption can be microfounded by considering that single-use users for service A have a strictly positive evaluation for service A and a negative evaluation for service B. Similarly, the reverse applies for single-use users for service B. This assumption captures users heterogeneity in the sense that some prefer spending their, for example, only in streaming services whereas others prefer playing video games exclusively. However, this does not imply that in equilibrium users do not subscribe to their least-preferred service, especially if it is offered as part of a bundle. Rather, they may subscribe but derive no utility from its consumption.

three demand segments, users are heterogeneous in their valuation (v) for the service they are interested in and are uniformly and independently distributed with unit support, i.e., $v \sim \mathcal{U}[0, 1]$. To focus on making our main point clear, we impose $\alpha = 1/3$ in the benchmark model.⁷

The utility of a single-use consumer of type v subscribing to a service $j = \{A, B\}$ offered by provider $i = \{1 \lor 2, \mathcal{P}\}$ is

$$u_{ji}^S(v, p_{ji}, N_{ji}^e) = v - p_{ji} + \theta N_{ji}^e,$$

where p_{ji} is the price at which the service is sold, N_{ji}^e is the expected total number of consumers using service j from provider i.⁸

The utility of a multi-use consumer subscribing to both services A and B offered respectively by provider $i = \{1, \mathcal{P}\}$ and $k = \{2, \mathcal{P}\}$ is

$$u_{ik}^{M}(v, p_{Ai}, p_{Bk}, N_{Ai}^{e}, N_{Bk}^{e}) = 2v - p_{Ai} - p_{Bk} + \theta(N_{Ai}^{e} + N_{Bk}^{e}).$$

Importantly, a multi-user consumer might find it optimal not to buy both services and only subscribe to one of them. We assume that multi-use preferences are perfectly correlated. This means that a user obtains 2v when subscribing to both services for a given price. This assumption is consistent with the one made by Chen and Rey (2023) and ensures tractability in our analysis.

Complementors Complementors pay the platform \mathcal{P} and valorem commission fee, which is denoted by $\tau \in (0, 1)$. The profit of a complementor *i* in either market structure is:

$$\pi_i = (1 - \tau) p_i D_i,$$

where D_i is the total demand for complementor *i*. As these are digital services, we normalize the marginal costs of serving users to zero.⁹

Platform Abusing notation, we use the superscript $\{NE\}$ to refer to the case in which platform \mathcal{P} is not active in the market and only collects revenues from complementors.

⁷This assumption greatly simplifies our analysis and assumes that the two independent markets have symmetric potential access to single-use users.

⁸The gross utility of a single-use consumer that values only service j from buying service -j is 0.

⁹Note that our main effect remains unchanged qualitatively in the presence of marginal costs.

In this case, the platform adopts an *agency model*. We use instead the superscript $\{E\}$ to refer to the case in which \mathcal{P} also offers services in competition with complementors, respectively. In different instances, we will abuse notation and use these superscripts for identifying equilibrium outcomes.

In the case where the platform chooses an agency model, \mathcal{P} 's profit is given by:

$$\Pi_{\mathcal{P}}^{NE} = \frac{\tau}{1-\tau} \sum_{i=1,2} \pi_i,$$

since the platform engages in revenue sharing with the complementors.

In case the platform enters the market in competition with complementors j, the platform offers either stand-alone services (in markets A and/or B) or as part of a bundle (A+B). Moreover, we assume that the services offered by the platform have the same quality as those offered by third-party complementors. Thus, \mathcal{P} 's profit is given by:

$$\Pi_{\mathcal{P}}^{E} = \sum_{j=A,B} \mathbb{1}_{j} \cdot p_{j\mathcal{P}} \cdot D_{j\mathcal{P}} + \frac{\tau}{1-\tau} \sum_{i=1,2} \pi_{i},$$

where $\mathbb{1}_j$ is an indicator function that takes a value of 1 if the platform has entered market j and sells in that market, and zero otherwise. The first term in the platform's profit function captures the revenues made from offering services directly to users, whereas the second term captures the revenues obtained by charging complementors' transactions whenever they occur.

Throughout the analysis, we assume that the platform offers its intermediation services at no marginal cost. In addition, the commission fee is fixed as platform's commission fees are set to organize the whole ecosystem and cannot be changed quickly.¹⁰ This assumption allows us to abstract from the presence of other inefficiencies, such as double marginalization, and distortions that can interplay with the Cournot complementarity externality that, as we shall see, arises in our setting. This enables us to better focus on the platform's incentives under different business configurations. Throughout the analysis, we assume τ to be exogenous and equal to the market rate.

Timing. The timing of the game is as follows.

¹⁰Note that τ does not impact complementors' prices and demands, and as such, the platform's profit is always increasing in τ . In the absence of marginal costs for serving users or complementors, the platform would always have an incentive to set the highest possible ad valorem fee, $\tau = 1$, or, more realistically, equal to any binding cap resulting from ex-ante regulation or ex-post market intervention.

- (i) The platform \mathcal{P} decides on its business model, that is whether to follow an agency model (denoted by case (NE)) or to enter directly in the market and compete with complementors (denoted by case E).
- (ii) Complementors set their prices for their services.
- (iii) Users form expectations on the value of network interactions and decide whether to buy the service and from whom.

We employ the Subgame Perfect Rational Expectations Equilibrium concept.

4 Agency model

We begin with providing an analysis of the case where the platform adopts an agency model.

Stage 3: Consumer demands. In the agency model, the two services A and B are offered exclusively by complementors 1 and 2, respectively. Since in this continuation game each complementor only provides one service, we save on notation by using the index i only.¹¹

Demand for the service offered by complementor i is the sum of both single-use and multi-use consumers.

Single-use consumer demand. In each single-use segment for service offered by complementor $i \in \{1, 2\}$, consumers are uniformly distributed according to their valuation v. A consumer of type v facing a price p_i and an expected network benefit equal to θN_i^e subscribes to the service offered by complementor i if and only if

$$u_i^S(v) > 0 \implies v > v_i^S(N_i^e, p_i) := p_i - \theta N_i^e,$$

which leads to the following demand from single-use consumers purchasing from complementor $i: 1 - v_i^S(\cdot)$.

<u>Multi-use consumer demand</u>. In this segment, a user of type-v facing prices p_1 and p_2

¹¹Note that this abuse of notation implies that i = 1 is meant for service A and i = 2 is meant for service B.

for the access to services A and B joins both services if and only if:

$$u^{M}(v) > 0 \implies v > v^{M}(p_{1}, p_{2}, N_{1}^{e}, N_{2}^{e}) := \frac{\sum_{i=1}^{2} \left[p_{i} - \theta N_{i}^{e} \right]}{2},$$

which leads to the following demand from multi-use consumers: $1 - v^M(\cdot)$.

The total demand for each service A and B is the sum of their demands in two segments, weighted by the share of users in each group. Specifically, total demand for service i is given as

$$D_i(p_i, p_{-i}, N_i^e, N_{-i}^e) = \frac{1}{3} \left(\underbrace{(1 - v_i^S(\cdot))}_{\substack{\text{single-use} \\ \text{segment}}} + \underbrace{(1 - v^M(\cdot))}_{\substack{\text{multi-use} \\ \text{segment}}} \right).$$

Note that the above total demand expression is multiplied by the relative weights of each demand segment $\alpha = 1/3$.

Imposing fulfilled expectations by setting $D_i = N_i^e$ and solving for D_i , we obtain total demand —denoted by $\hat{D}_i(\cdot)$ — as a function of prices only:

$$\hat{D}_i(p_i, p_{-i}) = \frac{18 - 9(2 - \theta)p_i - 3\theta(p_{-i} + 2)}{2\left(2\theta^2 + 9(1 - \theta)\right)}, \ i = 1, 2.$$
(1)

Note that $\frac{\partial \hat{D}_i(\cdot)}{\partial p_i} < 0$ and $\frac{\partial \hat{D}_i(\cdot)}{\partial p_{-i}} < 0$. The negative cross-derivative reflects the fact that, for users in the multi-use segments, services are *complements*. As consumers want to consume the two services, an increase in the price of one service decreases the demand for the other. This is a well-known effect called "Cournot complementarity" and we discuss it later as a source of a pricing inefficiency.

Stage 2: Service price setting. Plugging these demands into the profit expression yields profit of each complementor i as a function of prices is denoted as

$$\max_{p_i} \hat{\pi}(p_i, p_{-i}) = (1 - \tau) p_i \hat{D}_i(p_i, p_{-i}) \text{ with } i \neq -i \in \{1, 2\}.$$
(2)

Each complementor sets prices to maximize their own profits.

$$\frac{\partial \hat{\pi}_i}{\partial p_i} = \underbrace{\hat{D}_i(\cdot) + p_i \frac{\partial \hat{D}_i(\cdot)}{\partial p_i}}_{\text{(3)}} = 0.$$

Margin + Volume Effect

The optimal price for each service i is symmetric and is given by:

$$p_i^{NE} = \frac{4(3-\theta)}{21-8\theta} \ge \underbrace{\frac{1}{2}}_{\substack{\text{Joint Profit}\\\text{Maximizing price}}} (4)$$

Despite being monopolists on their respective markets, the optimal prices set by the complementors are inefficiently high from a joint profit perspective. Indeed, complementor i fails to take into account the impact of its pricing strategy on the demand for service -i. This impact is negative and stems from the presence of multi-use consumers who view the two services as complements.¹² This pricing externality is known as Cournot complementarity.

Proposition 1 (Cournot Complementarity.) Suppose the platform adopts an agency model. Then, equilibrium prices are above the joint profit maximizing level.

An implication of the above proposition is that a joint reduction in the prices of the two complementors would increase their profit, the profit of the platform and would benefit the consumers. However, the two complementors are in a prisoners' dilemma due to a lack of coordination in their pricing strategy resulting in a pricing inefficiency, which is well known as the *Cournot Complementarity*. This (pricing) inefficiency hurts the platform as it negatively affects the gross profit of each complementor, which is subject to the platform's fee.

The equilibrium profit of complementor *i* is denoted by $\pi_i^{NE} \equiv \hat{\pi}_i(p_i^{NE}, p_{-i}^{NE})$ and the platform's profit is given by $\Pi_{\mathcal{P}}^{NE} \equiv 2\tau \pi_i^{NE}/(1-\tau)$. Since the profit of the platform is increasing in the profit of complementors, an increase in θ impacts the profit of the platform and of the complementors with the same sign. The following lemma can be established after performing some simple comparative statics.

Lemma 1 (Profitability.) Suppose the platform adopts an agency model. The profit of the complementors and the platform are increasing in θ .

An increase in the network externality parameter (θ) increases the strength of the network benefits which increases their value from the services and hence also their demand.

¹²Compared to the profit maximization condition in Equation (3), joint profit (of the two complementors) maximization would include an additional (negative) term in the first-order condition: $p_{-i} \frac{\partial \hat{D}_{-i}}{\partial p_i} < 0$. The corresponding price for each service would be equal to $\frac{1}{2}$, below the prices p_i^{NE} set by each complementor when they operate independently.

This encourages services to set higher prices p_i^{NE} to extract a portion of the value created. As a consequence of this, both price and demand rise implying that the profit of complementors and the platform are increasing in θ .

5 Platform entry

Having the ability to organize its ecosystem, the platform may want to fix the pricing efficiencies, stemming from the Cournot complementarity, which also negatively hurts its profits. One strategy could be to directly intervene in the market, set pricing restraints on services. However, such an intervention may not be possible due to regulatory pressure. An alternative to pricing restraints may be fostering competition in the market. For instance, the platform may enter in the product space of complementors and offer competing services to mitigate pricing inefficiencies. However, entry of the platform in the market in which complementors operate has attracted large attention in recent years in light of the alleged anti-competitive practices that a hybrid business model might facilitate.¹³ In our work, we abstract from the presence of such conduct since we do not allow the platform set a higher fee that squeezes margins of complementors over alternatively the fact that the platform can copy third-party sellers and prioritize its own (a practice of self-preferencing). While we acknowledge that such concerns exist, in our work, we elicit the (positive) role entry plays in resolving the pricing inefficiencies discussed earlier.

In this section, we present the market outcomes when the platform has entered the market in competition with complementors. As discussed, we abstract from quality distortions and assume that the platform offers services with intrinsic value identical to the offering of complementors at the same marginal cost, which is normalized to zero. The platform can enter with service A, B or both. In the latter case, they can be sold independently or as part of a bundle. In what follows, we first rule out the case in which the platform offers services independently. Then, we describe the prices for which a bundle of A and B can be sustained at equilibrium.

 $^{^{13}}$ See e.g., Etro (2021); Zennyo (2022); Hervas-Drane and Shelegia (2022); Shopova (2023); Anderson and Bedre-Defolie (2024).

5.1 Standalone sales and mixed bundling

Suppose that the platform provides services j = A, B and sells them at a price $p_{j\mathcal{P}}$. Then, it is straightforward that the platform's version of the service is now a direct competitor of the services offered by the third-party complementors. Because users choose the cheapest service, for each service there is a Bertrand-like competition between the third party complementor and the platform, which drives down prices and, as a result, profits as well to zero. It is then immediate that it is not profitable for the platform to enter with its own version sold as standalone. Intuitively, a similar argument also holds in the case of a mixed bundle. The following lemma concludes that there cannot be an equilibrium in which the platform enters the market and provides standalone services or, by engaging in mixed bundling, provides standalone services or by engaging in mixed-bundling.

Lemma 2 (Stand-alone Sales.) For the platform, standalone sales or mixed bundling are pay-off dominated strategies.

5.2 Bundled entry

An alternative strategy for the platform to enter is to provide a bundle of different services. For example, Apple One features several services such as Apple Arcade, iCloud+, Apple Music, among others, at the individual price of \$19.95 per month. In this section, we characterize the price equilibrium when the platform enters with a bundle.

Suppose that instead of independent services, now the platform offers a bundle of services at a price $p_{\mathcal{P}}$. It is straightforward that the platform would not have incentives to attract the single-use consumer as this would entail fierce competition. Thus, the platform can choose to target the multi-use segment only, leaving the single-use segment to the thirdparty complementors on which the platform can levy a commission fee τ . Yet, this strategy is not without losses for the platform since, by separating multi-use users from the single-use ones, it reduces the benefit users obtain from cross-segment interactions (i.e., the network benefit).

In what follows, we present the bundling equilibrium and we show that, contrary to the case with single service entry, in this equilibrium, all firms realize a positive profit. Hence, bundled entry could be a substitute to the agency model. However, one must be cautious that the existence of a bundling equilibrium is not always guaranteed, especially when the multi-use segment is relatively large or relatively small. We discuss this issue in greater details in the Appendix. Here, we focus on the case where $\alpha = 1/3$ for which existence of the bundling as a strategy is not a concern.

In the bundling equilibrium, the platform sets a sufficiently low price for the bundle so that complementors prefer to focus on the demand from the single use segment. To derive the bundling equilibrium, we suppose the firms set their prices sequentially and that the platform moves first in the price game.¹⁴

Demand functions under entry. Let us first characterize the demand functions corresponding to the situation where single-use users find it optimal to subscribe to the complementors and multi-use users find it optimal to subscribe to the platform. The demand addressed to complementor i = 1, 2 by the single-user segment is defined by:

$$D_i(p_i, N_i^e) = \frac{1}{3}(1 - v_i^S(\cdot)).$$

Similarly, the demand for the bundle is defined by:

$$D_{\mathcal{P}}(p_{\mathcal{P}}, N_{\mathcal{P}}^e) = \frac{1}{3}(1 - v^M(\cdot))$$

Imposing fulfilled expectations by setting $D_i = N_i^e$ and $D_{\mathcal{P}} = N_{\mathcal{P}}^e$ and denoting these demands by \tilde{D}_i and $\tilde{D}_{\mathcal{P}}$, respectively, we have:

$$\tilde{D}_i(p_i) = \frac{1 - p_i}{3 - \theta}, \qquad \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}) = \frac{2 - p_{\mathcal{P}}}{2(3 - 2\theta)}.$$
(5)

Stage 2: Complementor price setting. In stage 2, each complementor i sets its price to maximize its profit:

$$\pi_i^E(p_i) = (1-\tau)p_i\tilde{D}_i(p_i).$$

Each complementor, in equilibrium, sets the monopoly price, which we denote as $p_i^E = \frac{1}{2}$. Thus, the corresponding demand in each single-use segment is $\tilde{D}_i^{\star} = \frac{1}{2(3-\theta)}$ and each complementor would earn $\pi_i^E := \pi_i^E(p_i^E)(1/2, 1/2) = \frac{(1-\tau)}{4(3-\theta)}$.

¹⁴For the existence of the equilibrium, the platform should be able to commit to a sufficiently low price for the multi-use segment to convince the complementors to retreat on the single-use segments. Being the first mover is a way to commit to such a price. With simultaneous pricing by firms, there is no equilibrium in pure strategies but there exists a mixed-strategy equilibrium in which all firms have a positive profit (See Gautier and Wauthy (2010) for a similar analysis).

Stage 1: Bundle pricing. At the first stage, the platform sets the bundle price $p_{\mathcal{P}}$. To attract consumers in the multi-use segment: $p_{\mathcal{P}} \leq \sum_{i=1,2} p_i^E = 1$. In addition, to be an equilibrium, $p_{\mathcal{P}}$ must be such that complementor i prefers to focus on their single-use segment and collect the profit π_i^E rather than lowering its price below 1/2 to attract the consumers in the multi-use segment. Formally, suppose that complementor j sets the equilibrium price $p_j^E = 1/2$, then complementor i has the choice between serving the single-use segment (demand equal \tilde{D}_i) at a price of 1/2 and to lower its price to \underline{p}_i in order to attract both the single-use and the multi-use consumers (demand equal \hat{D}_i). Complementor i has the same profit with these two alternatives for a price $\underline{p}_i < 1/2$ defined as:

$$\underline{p}_{i}\hat{D}_{i}(\underline{p}_{i}, 1/2) = \frac{1}{2}\tilde{D}_{i}(1/2, 1/2) = \pi_{i}^{E}.$$
(6)

Now, if the platform sets a bundle price (slightly below) $p_{\mathcal{P}}^E = 1/2 + \underline{p}_i < 1$, with \underline{p}_i being defined in Equation (6), complementor *i* prefers to serve the single-use segment only and the platform will serve the multi-use segment. We can therefore define the price for the bundle as:

$$p_{\mathcal{P}}^E = \frac{39 - 16\theta - \sqrt{225 - 96\theta}}{4(9 - 4\theta)}.$$
(7)

which decreases in θ . This is an interesting comparative static and arises from a nuanced effect of bundled prices. Specifically, an increase in the strength of the network benefit, θ , makes it more lucrative for the complementors to deviate and attract multi-use consumers to expand its network. To prevent such deviation, the platform must lower its bundle price as θ increases. Moreover, $p_{\mathcal{P}}^E \in (1/2, 1)$, meaning that multi-use consumers do not have incentives to buy services independently from those complementor and single-use consumers not to buy the bundle rather than from the complementors. Thus, having identified a bundling equilibrium, we conclude the following.

Proposition 2 (No Cournot Complementarity with Bundling.) Suppose the platform enters with the bundle. Then, complementor's equilibrium prices are at the monopoly level and the platform charges $p_{\mathcal{P}}^E \in (1/2, 1)$ for the bundle by serving multi-use consumers.

Plugging the optimal prices into the profit of complementors and the platform yields

$$\pi_i^E = (1-\tau) \frac{1}{4(3-\theta)}, \qquad \Pi_{\mathcal{P}}^E = \frac{\tau}{1-\tau} \sum_{i=1,2} \pi_i^E + p_{\mathcal{P}}^E \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}^E),$$

where the latter is formally characterized in the Appendix. Now, we are in the position to discuss how equilibrium profits are affected by changes in the network benefit θ . An increase in the network benefit θ has two opposite effects on the profit of the platform. First, an increase in the strength of the network benefit, θ , exerts a downward pressure on the bundle price, which leads to lower profits (*bundle price-reduction*). Second, for given prices in the bundling equilibrium, an increase in θ results in the expansion of the demand in each segment denoted by the *network-driven demand expansion*. This demand expansion arises for bundled offering of the platform as well as the demand for the services offered by the complementor. Formally,

$$\frac{\partial \Pi_{\mathcal{P}}^{E}}{\partial \theta} = \underbrace{\frac{\partial p_{\mathcal{P}}^{E}}{\partial \theta}}_{\substack{\text{Bundle price}\\ \text{reduction } (-)}} \underbrace{\left(\tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}^{E}) + p_{\mathcal{P}}^{E} \frac{\partial \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}^{E})}{\partial p_{\mathcal{P}}}\right)}_{(+)} + \underbrace{p_{\mathcal{P}}^{E} \frac{\partial \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}^{E})}{\partial \theta} + \frac{\tau}{1 - \tau} \left(\underbrace{\frac{\partial \pi_{i}^{E}}{\partial \theta} + \underbrace{\frac{\partial \pi_{j}^{E}}{\partial \theta}}_{(+)}}_{\substack{\text{Network-driven demand expansion } (+)}\right) > 0,$$

for $i \neq j \in 1, 2$. We find that the positive impact of platform's profit from networkdriven demand expansion effect dominates any losses from the reduction of the platform's bundle price. Thus, an increase in θ leads to a higher profit for the platform. Specifically, we conclude the following

Lemma 3 (Profitability Under Bundling.) Suppose the platform enters the market and offers a bundle. The profit of the complementors and the platform are increasing in θ .

6 Platform's Decision and Effects on Consumer Surplus.

6.1 Entry decision

We can now solve for the optimal decision in the first stage. The platform's owner can decide whether to offer a bundle or just adopt a pure agency model, as in Section 4. Entry is not only motivated by the possibility to capture downstream profits, but also by the interest of the platform in minimizing the Cournot inefficiency. In other words, entry is more than a substitute to an increase in the fee τ .

Recall that the platform earns $\Pi_{\mathcal{P}}^{E}$ if offering a bundle and $\Pi_{\mathcal{P}}^{NE}$ if operating as a mere

intermediary. From a simple comparison, we can observe that the following¹⁵

$$\Pi^{E}_{\mathcal{P}} > \Pi^{NE}_{\mathcal{P}} \iff \tau < \tilde{\tau}(\theta).$$

where

$$\tau < \tilde{\tau}(\theta) \equiv \frac{(21 - 8\theta)^2 (3 - 2\theta)(531 + 3\sqrt{225 - 96\theta} - 16\theta(33 - 8\theta))}{24(9 - 4\theta)^2 (423 - 330\theta + 64\theta^2)}$$

Note that $\tilde{\tau}'(\theta) < 0$, and the opposite arises otherwise.

Intuitively, the platform prefers to enter the market with its own service when it can appropriate only a small value of complementors' transactions. Specifically, there exists a threshold of the commission fee, denoted by $\tilde{\tau}$, such that entry is preferred to the pure agency model if $\tau \leq \tilde{\tau}(\theta)$. The following result can be established.

Proposition 3 (Bundled entry.) For the platform, entry is more profitable than no entry for $\tau \leq \tilde{\tau}(\theta)$ with $\tilde{\tau}(\theta)$ decreasing in θ . Complementors' profits are lower when the platform enters the market.

The above proposition highlights that platform entry becomes less likely as the strength of network externalities increases. This is because network externalities boost the platform's profits both when it is following an agency model and when it enters the market with the bundle, but profits increase at a faster rate when the platform does not enter. The reason is that as the strength of network benefits increases, consumers value highly interacting with a larger network. If the platform follows an agency model, demand is not fragmented and this increased consumer value allows complementors to set higher prices and extract a portion of this increased consumer value. Instead, when the platform deviating and competing in the multi-use segment. In addition, the network benefit is lower as the demand is fragmented into multiple segments. All these elements together imply that the platform's profit increases at a lower rate. Consequently, as the network benefit θ increases, the likelihood of observing platform entry decreases. The result that complementors' profit is lower after platform entry is due to increased competition and lower demand for their services.

¹⁵Note that the commission fee that does not impact prices and, therefore, the demand functions.

6.2 Consumer surplus

In this section, we study under which conditions the platform's decision is aligned with the one that maximizes consumer surplus. From the perspective of a consumer, entry entails a trade-off since it reduces all prices but networks get fragmented. When the strength of network benefits are high, consumer prefer that the platform does not enter the market with a bundle. This is the case despite prices being lower under bundled entry. The rationale for this result is as follows: when the strength of network externalities is high, consumers benefit significantly by interacting with a larger network even at the expense of higher prices. Therefore, when the platform adopts the agency business model, consumers interact with a larger network and prefer this model when the strength of network benefits is high. On the contrary, when the strength of network benefits is low, consumers prefer that the platform enters with a bundle corrects, since in such a case they gain more from the lower prices than their loss (from network fragmentation) as the network benefits are already low. In the Appendix, we formally show that there exists a critical threshold of θ above which consumer surplus increases with platform entry and decreases otherwise. Formally, we state the following proposition.

Proposition 4 (Consumer Surplus.) Consumer surplus is higher under platform entry relative to the agency model for any $\theta < \tilde{\theta} \approx 0.763$ and lower otherwise.

Figure 1 illustrates the platform's incentives to enter the market and how these align with the market configuration that maximizes consumer benefits. The x-axis represents the fee, while the y-axis captures the strength of the network benefit. The blue shaded region indicates the parameter range where the platform opts to enter the market with a bundle, whereas the orange shaded region corresponds to cases where the platform adopts an agency model. The area with vertical lines highlights instances where the platform's entry decision diverges from the market outcome that maximizes consumer surplus. In contrast, the region with diagonal lines shows alignment between the platform's entry decision and the configuration that enhances consumer surplus.

There are two regions where the platform's choice aligns with consumer choice, and two regions where the platform's choice does not align with consumer choice.

Alignment of incentives. There are two regions in the plot where the platform's choice aligns with the market structure that benefits consumers. First, when both the strength of network benefits and the fee are low, both consumers and the platform prefer bundled



Figure 1: Platform entry decision and alignment with consumer surplus.

entry. In this scenario, the platform lowers prices to address the pricing inefficiency in the agency model. This benefits consumers as the network externalities are relatively weak. Second, when the strength of network benefits is high and the fee is also high, the platform chooses not to enter. This decision benefits consumers because they derive more value from a larger network than from lower prices associated with platform entry. The platform also opts not to enter because the bundle price would need to be set too low to prevent deviation by complementors, making such a strategy unprofitable when the fee is high.

Misalignment of incentives. There are two regions in the plot where the platform's choice is not aligned with the market structure that would benefit consumers. First, when the strength of network benefits is low and the fee is high, consumers prefer the platform to enter, but the platform chooses not to enter with a bundle. In this case, the platform opts to capitalize on the high fee, even if it results in lower industry profits. Second, when the strength of network benefits is high but the fee is low, the platform enters with a bundle, but this choice harms consumers. Consumers are worse off because the network they engage with on the services is smaller. However, the platform finds entry optimal in this case, as foregoing entry would result in much lower revenues from fees.

7 Integration with a Complementor

Entry (with a bundle) is not the only strategy for the platform to intervene in its marketplace. The acquisition of a complementor is an alternative, and there are many examples of platforms acquiring firms active in their ecosystem. For example, Apple in 2018 acquired Texture (previously known as Next Issue), a digital magazine app that was later integrated into Apple News+. In this section, we focus on a scenario in which there is integration with only one complementor, which is the most realistic scenario since the acquisition of all complementors would entail a fully vertically integrated setting that would be a concern for antitrust authorities and would fall short of representing current real world practices.¹⁶ To this end, let us assume that the platform integrates with complementor 1 active in market A.

7.1 Prices and profits under integration

Integration modifies the way complementors set prices and, in turn, their profits. One advantage of acquisition over entry is that it does not fragment the demands across different sets of consumers, and, therefore, it preserves valuable network benefits. Indeed, integration can be a profitable strategy, especially if the network benefits are sufficiently strong.

The demands in stage 3 are as in Equation (1) in our baseline model. The profit of the non-integrated complementor, 2, is as in the benchmark (in Equation (2)). Under integration, profit of the merged entity is given by:

$$\hat{\Pi}_{\mathcal{P}}^{I}(p_1, p_2) = p_1 \hat{D}_1(p_1, p_2) + \frac{\tau}{1 - \tau} \hat{\pi}_2(p_2, p_1).$$
(8)

For the independent complementor, the first-order condition is unchanged compared to the agency model and given by Equation (3). Differentiating the platform's profit with respect to its own price, p_1 , yields:

$$\frac{\partial \hat{\Pi}_{\mathcal{P}}^{I}}{\partial p_{1}} = \underbrace{\hat{D}_{1}(\cdot) + p_{1}}_{\text{Margin + Volume Effect}} \underbrace{\frac{\partial \hat{D}_{1}(\cdot)}{\partial p_{1}}}_{\text{Network}} + \underbrace{\tau p_{2} \left(\frac{\partial \hat{D}_{2}(\cdot)}{\partial p_{1}}\right)}_{\text{Network}} = 0.$$
(9)

¹⁶A setting in which only the integration of one complementor can be an option for the platform can arise for example in cases in which authorities decide to block (or to sanction) cases in which there is full monopolization of the market via backward integration.

The first two terms above present the well-known *margin and volume* trade-off. The third term elicits how vertical integration changes pricing incentive of the platform which we refer to as the *network internalization effect*. Specifically, this term shows that the platform partially internalizes the impact of its price on the demand for the complementor's service and thus on its intermediation revenues. This internalization occurs because the platform accounts for how a change in its own price affects the other service's demand due to the presence of multi-use consumers, thereby influencing the intermediation revenues generated from charging a commission fee.

Following integration with one complementor, the integrated firm gains control over prices. Although the Cournot Complementarity problem persists, its magnitude is dampened. Solving the first-order conditions of the platform and complementor 2 yields the following prices.

$$p_1^I = \frac{4(3-\theta)(15-3\tau-8\theta)}{(21-8\theta)(15-8\theta)-9\tau}, \qquad p_2^I = \frac{4(3-\theta)(15-8\theta)}{(21-8\theta)(15-8\theta)-9\tau}.$$
 (10)

Comparing the prices under integration with those emerging from the equilibrium of the continuation game, wherein the platform follows an agency model. We summarize our findings in the following result.

Lemma 4 (Prices Under Integration.) An increase in τ leads to a decrease in the platform's price p_1^I and an increase in the independent complementor's price p_2^I . An increase in θ unambiguously increases p_2^I while p_1^I increases in θ if and only if $\tau < \tau^I$.

The platform internalizes the inefficiency created by Cournot complementarity and the network internalization effect shifts its best response function downwards and the magnitude of this shift depends on the value of τ . Consequently, an integrated complementor will choose a lower price for service A, and, as a response, the independent complementor 2 will choose a higher price for service B as it wants to extract a portion of the value gained by multi-use consumers and it does not internalize the pricing inefficiency.

Focusing on the comparative statics of the prices with respect to θ , we also observe that as the value of the network increases, the complementor 2 finds it profitable to increase its price to extract a portion of the value gained by consumers. Instead, we find that the comparative statics of the price of the integrated platform with respect to θ are more nuanced and depend on the value of commission fees. Specifically, as θ increases, p_1^I is rising for sufficiently low τ and falling otherwise. When the fee is low, the integrated firm cares more about the extraction of rent from consumers than enhancing the ecosystem size as θ increases. Instead, when the fee is high, the integrated firm cares about the entire ecosystem and tries to maximize the size of the network on the platform by lowering its price and reducing the pricing inefficiency arising from the Cournot complementarity effect.

Substituting these prices into the expression for profits, the profits of the platform and the complementor 2 are defined respectively as follows:

$$\Pi_{\mathcal{P}}^{I} := \hat{\Pi}_{\mathcal{P}}^{I}(p_{1}^{I}, p_{2}^{I}) \text{ and } \pi_{2}^{I} := \hat{\pi}_{2}(p_{2}^{I}, p_{1}^{I}).$$

Differentiating the above profit with respect to the strength of the network benefit θ and the commission fee τ , we obtain the following result.

Lemma 5 (Profitability Under Integration.) The profit of the integrated firm and the complementor 2 increases in the strength of the network benefit θ . As the commission fee τ increases, the profit of the platform increases, whereas the profits of complementor 2 decrease.

The first part of the lemma above shows that as the strength of network benefit increases, consumers benefit more from interacting with their network. This makes the independent complementor find it profitable to raise its price to extract a portion of this increased consumer value. As a result, its profit also increases. The profit of the integrated platform also increases with an increase in θ . This is because an increase in θ leads to an increase in the profit of its own service and the revenues from the independent complementor. The second part of the lemma discusses the effect of an increase in the commission fee: a higher τ leads higher profit for the integrated firm as more of the ecosystem rents are accruing to it while it is able to reduce any pricing inefficiencies. In contrast, the profit of the independent complementor decreases as it loses a greater portion of its (gross) profit through the commission fee even though its (gross) profit increases in θ .

7.2 Comparison: Integration vs. Agency.

In the following, we compare the agency model with the integration case. Towards this, we first compare prices, demands and profits.

Lemma 6 (Price comparison.) $p_1^I < p_i^{NE} < p_2^I$ and $p_1^I + p_2^I < \sum_{i=1,2} p_i^{NE}$.

The above result is quite intuitive and it is due to the network internalization effect of vertical integration which affects both complementors pricing strategies but in contrasting ways. Combining these two effects, we find that the total price of the two services after integration is lower and multi-use consumers benefit. This is because the direct effect of a reduction in the price on the integrated firm is larger than the increase in the price of the rival. Thus, the composite price ultimately paid by the multi-use consumers is lower in under vertical integration. Hence, integration is effective in reducing the extent of the Cournot complementarity problem. For any given quantity demanded, this effect benefits consumers.

Remark 1 (Demand Comparison.) Total demand at the two complementors is higher after integration than in the agency model.

Next, we compare consumer surplus in the agency model with the one resulting from integration of the platform with one complementor. Results are summarized in the following proposition.

Proposition 5 (Consumer Surplus Comparison.) Consumer surplus always increases with integration relative to the agency model.

From a consumer perspective, there is no trade-off, and integration always leads to a higher surplus: under agency, consumers suffer from the Cournot externality, whereas integration allows the platform to partially internalize the Cournot externality and reduce price of its own service in response. As the other complementor only reacts and does not raise prices by as much, the composite price, which is paid by the multi-use consumers, decreases (see Lemma 4). In turn, integration mitigates the Cournot externality but does not fully avoid it since the other complementors' prices increase even more above the monopoly price.

Lastly, comparing profits, integration with complementor 1 always increases the platform's profit compared to the agency model. Indeed, under integration, the platform could replicate the price p_1^{NE} under agency, leading to the same prices and profits as in the agency scenario and an extra profit due to the direct revenue from its own service. By choosing the optimal prices identified in Lemma 4, the platform's profit increases even further. By construction, integration increases the profit of the platform while also creating a positive spillover for complementor 2. Indeed, the profit of complementor 2 also increases as it serves a larger demand (Remark 1) at a higher price (Lemma 6).

Finally, note that the profit of the integrated platform is larger than the combined profit of complementor 1 and the platform in the agency model. Consequently, it is possible to find an acquisition price (T) for complementor 1 that satisfies $\Pi_{\mathcal{P}}^{I} - T \geq \Pi_{\mathcal{P}}^{NE} + \pi_{1}^{NE}$ and $T \geq \pi_{1}^{NE}$, that is a price such that both parties are better off under integration. Hence, integration benefits consumers and all firms.

8 Managerial and Policy Implications

Our work provides novel insights about platform markets with network effects that can be of interest for both managers and policymakers. We present these insights below.

Managerial Implications 1 Overlapping consumers across network-based services may lead to inefficient pricing that harms the overall ecosystem.

The above managerial implication sheds light on the existence of a pricing externality (i.e, Cournot complementarity) arising purely from the main features associated with platform markets: network effects. This is due to the fact that multi-use consumers transform independent products into related products that create externality on each other, which in turn results in prices that are even higher than the joint profit maximizing levels. This hurts total demand and may be detrimental to the well-being of a platform ecosystem. Thus, our work brings about awareness of the existence of such a pricing externality and informs managers of such platform ecosystem to devise different mechanisms to alleviate this pricing externality.

There are, of course, other ways for the platform to address the pricing externality in the agency model. One approach is to impose pricing restraints on the complementors' pricing strategies within the ecosystem (e.g., RPM clauses). However, such restraints may be viewed unfavorably by authorities, as they infringe on the pricing independence of complementors. In the following, we explore how platform owners can mitigate the effects of this externality by using non-pricing tools, specifically by entering the market in competition with their complementors.

Managerial Implications 2 Bundled entry by a platform is a profitable decision when

the strength of network benefits is low (and the commission fee is low). Else, the agency business model is optimal.

Bundling services offers a viable entry strategy for platforms to target multi-use consummers while the single-use segments is served by third party complementors. This approach reduces the pricing inefficiency as the two markets are insulated from the externality exhibited by the presence of multi-homers in their demand segment. As the platform focuses on the multi-use segment, it eliminates this externality at the expense of limiting the overall value creation through larger network at each service. Thus, the platform must trade-off solving the pricing externality with lower value creation due to demand fragmentation. Indeed, it may not be profitable for the platform managers to always resolve this pricing inefficiency through the bundled entry strategy. For example, when the value associated with network interactions is high, it is more profitable for the platforms managers to accept the pricing inefficiency in favor of benefits associated with larger network formation on the services. In contrast, when the value associated with network interactions is low, the platform should enter with a bundle as the value loss from network fragmentation are dominated by gains from resolving the pricing inefficiency. However, as shown in the Appendix, this strategy may not always be feasible to implement. For instance, when there is significant heterogeneity in the shares of singleuse and multi-use consumers. This dynamic results in fierce head-to-head competition between the platform's bundled service offering and the third parties services, eroding profits, or would not be incentive-compatible for consumers. In such circumstances, it may be useful for platform managers to consider alternative ways to resolve such externalities. In the following, we present another organization structure decision available to platforms that may help mitigate but not fully eliminate such pricing inefficiencies.

Managerial Implications 3 Integration with one complementor can help mitigate the pricing inefficiency but does not eliminate it while offering the benefit of increased network benefits.

Platforms are now under scrutiny by authorities worldwide for large-scale acquisition of services and complementors. In our work, we elicit a novel rationale for platforms acquiring a complementor. In our setting, acquisition of a complementor reduces pricing inefficiencies and thus increases welfare. Specifically, acquiring a complementor mitigates the Cournot inefficiency without fragmenting the network and benefiting from enhancing network interactions, making this strategy an attractive alternative. Yet, the pricing inefficiency is only mitigated as not all complementors can be acquired, which is expected to be the case especially when regulators are concerned about the strengthening of the platform's dominance by entry in adjacent markets (see e.g., EU DMA).

Policy Implications 1 When the value associated with network interactions is low, platform's entry with a bundle in competition with third party complementors also benefits consumers relative to the agency model. In addition, the platform also mitigates an inefficiency that (negatively) affects the total surplus in the ecosystem. Otherwise, when entry occurs it may reduce total surplus.

Platforms have been under scrutiny by competition authorities across the Atlantic and in Asia. This scrutiny helps prevent abusive conduct, given platforms' dominance as providers of digital infrastructure. Several cases have been brought by authorities regarding so-called "self-preferencing," and the EU has introduced regulations for gatekeeping platforms that prohibit favoring their own products when the platform owner competes with complementors. In this paper, we set aside concerns about such distortionary conduct and instead focus on the effects of platform entry into competition with complementors to address its underlying rationale. The motives for such entry may not stem from value appropriation but rather from mitigating pricing inefficiencies associated with the cournot complementarity effect we uncovered in this paper. When the value of network interactions is low, bundled entry by platforms should be encouraged.

Policy Implications 2 Integration of a platform with a complementors can mitigate the pricing inefficiency and increase both consumer surplus and profit of complementors in other markets.

Another policy implication of our analysis is that an alternative approach to (partially) mitigating the pricing externality is to allow the platform to integrate with one of the complementors. Compared to entry with bundling, this strategy has the advantage of preserving network benefits and reducing the price of the integrated firm's product, as the platform internalizes the pricing externality. While this creates a significant benefit for the independent complementor, which raises its prices in response, total industry profits and consumer surplus both increase. Indeed, setting aside other common distortions associated with vertical integration, such as raising rivals' costs (which we abstract from here), acquisitions in such cases could be

9 Conclusions

In this paper, we examine the incentives of a platform to enter a market in competition with the complementors it hosts. We uncover a novel rationale for the platform's entry decision, which hinges on the existence of an externality specific to markets characterized by network benefits in the presence of multi-use consumers. The presence of these network effects, combined with multi-use consumers, transforms seemingly unrelated markets into complements and introduces the well-known "Cournot Externality". This externality results in complementors setting prices inefficiently higher than the monopoly level.

In such a market structure, we discussed how platforms can strategically employ nonpricing tools to mitigate this externality. Specifically, we considered two strategies (i) bundled entry of a platform and (ii) acquisition of a complementor. Under bundled entry, the platform trades-off gains from lowered prices vis-á-vis losses from a smaller network. We argued that when the value associated with network interactions is low, platform entry decision may be aligned with the consumer surplus enhancing choice. Under acquisition of a complementor, the platform is only able to mitigate this externality and not fully eliminate it.

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Appendix

A.1 Derivation of results in Section 4

Proof of Proposition 1

As show in (4), this price is higher than 1/2, which in turn proves Proposition 1.

Proof of Lemma 1

Using the optimal price, $p_i^{NE},\, {\rm the}$ equilibrium demand at each complementor is

$$D_1^{NE} = D_2^{NE} = \frac{2(9 - 4\theta)}{16\theta^2 - 66\theta + 63}$$

The complementors' profits are

$$\pi_1^{NE} = \pi_2^{NE} = (1 - \tau) \frac{8(3 - \theta)(9 - 4\theta)}{(21 - 8\theta)^2(3 - 2\theta)}$$

which are increasing in θ .

The profit of the platform is:

$$\Pi_{\mathcal{P}}^{NE} = \frac{16(3-\theta)(9-4\theta)\tau}{(21-8\theta)^2(3-2\theta)},$$

which is increasing in θ as

$$\frac{\partial \Pi_{\mathcal{P}}^{NE}}{\partial \theta} = \frac{16(8\theta(\theta(8\theta - 63) + 162) - 1107)\tau}{(3 - 2\theta)^2(8\theta - 21)^3} > 0.$$

A.2 Derivation of results in Section 5

Proof of Lemma 2

The proof of the lemma is straightforward. Standalone sales or mixed bundling would drive profits to zero for both complementors and the platform. As such, this strategy is pay-off dominated by any other strategy that entails strictly positive profits.

Proof of Proposition 2

The proof follows from the discussion in the main text as the equilibrium bundling price

$$p_{\mathcal{P}}^E = \frac{39 - 16\theta - \sqrt{225 - 96\theta}}{4(9 - 4\theta)} < 1.$$

where 1 is the sum of the monopoly price that multi-use consumers would pay by buying services from both complementors at a monopoly price of 1/2. We show in Appendix A.5 that at these prices multi-use consumers are better off with the bundle and single use consumers are better off with the complementors' product.

Proof of Lemma 3

The equilibrium profits are as follows:

$$\pi_i^E = (1 - \tau) \frac{1}{4(3 - \theta)},$$

$$\Pi_{\mathcal{P}}^E = \frac{3(177 + \sqrt{225 - 96\theta}) + 8\tau(9 - 4\theta)^2 - 16\theta(33 - 8\theta)}{16(3 - \theta)(9 - 4\theta)^2}.$$
(A-1)

From (A-1), note that π_i^E and $\Pi_{\mathcal{P}}^E$ are both increasing in θ as

$$\frac{\partial \Pi_{\mathcal{P}}^{E}}{\partial \theta} = \frac{4\theta \left(-9 \left(\frac{135}{\sqrt{25 - \frac{32\theta}{3}}} + 216\tau + 161\right) + 16\theta \left(48 - 8\theta(\tau+1) + \frac{15}{\sqrt{25 - \frac{32\theta}{3}}} + 54\tau\right)\right)}{16(9 - 4\theta)^{3}(3 - \theta)^{2}} + \frac{\frac{6129}{\sqrt{25 - \frac{32\theta}{3}}} + 5832\tau + 3267}{16(9 - 4\theta)^{3}(3 - \theta)^{2}}$$

which is strictly positive for any $\theta > 0$ and

$$\frac{\partial \pi^E}{\partial \theta} = \frac{1-\tau}{4(3-\theta)^2} > 0$$

This proves Lemma 3.

A.3 Derivation of results in Section 6

Proof of Proposition 3

The proof is straightforward. $\tilde{\tau}'(\theta) < 0$ follow from simple comparative statics of $\tilde{\tau}$ with respect to τ .

Proof of Proposition 4

Consumer surplus under agency. In the agency model—superscript $\{NE\}$ —consumer surplus of single-use and multi-use consumers is

$$CS_i^{NE} = \frac{9(9-4\theta)^2}{2\left(16\theta^2 - 66\theta + 63\right)^2}, \qquad CS_{AB}^{NE} = \frac{1}{2}CS_i^{NE} = \frac{9(9-4\theta)^2}{\left(16\theta^2 - 66\theta + 63\right)^2}.$$

Total consumer surplus is in turn

$$CS^{NE} = \frac{1}{3}(CS_i^{NE} + CS_j^{NE}) + \frac{1}{3}CS_{AB}^{NE} = \frac{6(9-4\theta)^2}{\left(16\theta^2 - 66\theta + 63\right)^2}.$$

Consumer surplus under entry. Consumer surplus in the single-use segments is

$$CS_i^E = \frac{9}{8(3-\theta)^2},$$

whereas consumer surplus in the multi-use segment is

$$CS_{12}^E = \frac{297\Phi - 144\theta \left(5913 - 8\theta + \Phi + 36\right)}{32(9 - 4\theta)^2(3 - \theta)^2}.$$
 (A-2)

Total consumer surplus under entry is equal to

$$CS^{E} = \frac{768\theta^{2} - 48(\Phi + 72)\theta + 99\Phi + 3915}{32(9 - 4\theta)^{2}(\theta - 3)^{2}},$$

where $\Phi \triangleq \sqrt{225 - 96\theta}$.

Comparison. CS^{NE} and CS^{E} are both increasing in θ . By comparing total consumer surplus in the two cases, then $CS^{E} > CS^{NE}$ if and only if $\theta < \tilde{\theta} \approx 0.762$, else the opposite holds.

A.4 Derivation of results in Section 7

Proof of Lemma 4

The prices defined in Equation (10) decrease in τ . The price p_2^I always increases in θ . Differentiating p_2^I with respect to θ yields

$$\frac{\partial p_2^I}{\partial \theta} = \frac{12(\tau(117 - 48\theta) + (15 - 8\theta)^2)}{(315 - 9\tau - 288\theta + 64\theta^2)} > 0.$$

The above is always positive as $0 < \theta < 1$.

Similarly, differentiating p_1^I with respect to θ yields

$$\frac{\partial p_1^I}{\partial \theta} = \frac{12((15 - 8\theta)^2 - 9\tau^2 - 16\tau(3 - \theta)(9 - 4\theta))}{(315 - 9\tau - 288\theta + 64\theta^2)}$$

The sign of the above comparative static depends on the sign of the numerator. For simplicity, let us define the relevant expression in numerator as $\beta = ((15 - 8\theta)^2 - 9\tau^2 - 16\tau(3-\theta)(9-4\theta))$. Note that β is unambiguously decreasing in τ . In addition, β is positive at $\tau = 0$ — i.e., $\beta|_{\tau=0} = (15 - 8\theta)^2 > 0$. Instead, β is negative for $\beta|_{\theta=\overline{\theta}} = (15 - 8\theta)^2 + 3(\sqrt{17} - 9)(\theta - 3)(4\theta - 9) + \frac{81}{128}(9\sqrt{17} - 49) < 0$. Therefore, there exists only one threshold for τ below which p_1^I is increasing in θ and is decreasing otherwise. Solving for $\beta = 0$ with respect to τ yields the solution $\tau^I \triangleq \frac{\sqrt{48681-16\theta(4671-8\theta(333+8\theta^2-84\theta))+8\theta(21-4\theta)-216}}{9}$. As β is unambiguously decreasing in τ which implies that for all $\tau \in [0, \tau^I]$ we have $\frac{\partial p_1^I}{\partial \theta} > 0$ and negative otherwise.

Proof of Lemma 5

Under acquisition, the profits are equal to:

$$\Pi_{\mathcal{P}}^{I} = \frac{4(1+\tau)(15-8\theta)^{2}(9-4\theta)-432\tau^{2}(2-\theta)}{(315-9\tau+64\theta^{2}-288\theta)^{2}},$$

$$\pi_{2}^{I} = \frac{4(1-\tau)(15-8\theta)((9-4\theta)(15-8\theta)-9\tau)}{(315-9\tau+64\theta^{2}-288\theta)^{2}}.$$

The profits are increasing in θ . $\Pi^{I}_{\mathcal{P}}$ is increasing in τ while π^{I}_{2} is decreasing.

Proof of Lemma 6

We can write $p_1^I = p_1^{NE}\Gamma_1$, $p_2^I = p_2^{NE}\Gamma_2$, with for all $\tau > 0$, $\Gamma_1 = \frac{(15-3\tau-8\theta)(21-8\theta)}{(21-8\theta)(15-8\theta)-9\tau} < 1$ and $\Gamma_2 = \frac{(15-8\theta)(21-8\theta)}{(21-8\theta)(15-8\theta)-9\tau} > 1$. Furthermore we have $\Gamma_1 + \Gamma_2 < 1$.

Proof of Proposition 5

Lemma 6 implies that the demand for the two products increases compared to the agency case. For product 1, it is obvious as its price decreases; for product 2, it comes from the fact that the increase in price p_2^I is lower than the decrease in price p_1^I . Hence, the larger demand from the multi-use segment more than compensate the decrease in demand from the single-use segment. Hence, as we stated in Remark 1, the demand at the two complementors is higher after integration than in the agency model. Consequently, with lower total prices and a higher demand, the consumer surplus increases.

A.5 Complement to Section 5: Existence of a bundling equilibrium in the general case

In this section, we derive and discuss the existence of the bundling equilibrium for all $\alpha \in (0, 1/2)$. In this set-up, we need to restrict the admissible parameters for the network effect to and we assume that:

$$\theta \le \frac{3 - \sqrt{9 - 32\alpha(1 - \alpha)}}{8(1 - \alpha)\alpha} \equiv \hat{\theta}.$$

Demand functions First, we derive the demand functions for this general case in the entry and no entry cases:

$$\hat{D}_i(p_i, p_{-i}) = \frac{2(1 - \alpha - (1 - \alpha)\alpha\theta) - (1 - 2\alpha(1 - \alpha)\theta)p_i - (1 - 2\alpha)p_{-i}}{2((1 - \theta) + (1 - \alpha)\alpha\theta^2)}, i, -i = 1, 2.$$
$$\tilde{D}_i(p_i) = \frac{\alpha(1 - p_i)}{1 - \alpha\theta}, i = 1, 2; \qquad \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}) = \frac{(1 - 2\alpha)(2 - p_{\mathcal{P}})}{2(1 - (1 - \alpha)\theta)}.$$

The demand functions have the same properties as in the above case, and in particular $\frac{\partial \hat{D}_i}{\partial p_{-i}} < 0$ for all the relevant parameters.

Bundling equilibrium As before, we can define a candidate bundling equilibrium where the complementors serve their respective single-home segment at the monopoly price $p_i^E = \frac{1}{2}$ and the platform chooses a price such that, given p_j^E , complementor *i* prefers to serve only the single-use segment (see Equation 6). The bundle price for the general case is given by

$$p_{\mathcal{P}}^{E} = \frac{1}{2} - \frac{(1-\alpha)\alpha\theta}{1-2(1-\alpha)\alpha\theta} + \frac{(3-2\alpha) - \sqrt{(1-2\alpha)(9-16(1-\alpha)\alpha\theta - 2\alpha)}}{4-8(1-\alpha)\alpha\theta}.$$
 (A-3)

Existence We have a candidate bundling equilibrium (p_1^E, p_2^E, p_P^E) and, by construction, no firm has an incentive to deviate from these prices. They constitute the unique equilibrium if consumers do not have a *unilateral* incentive to switch from one service to another. More precisely, the following incentive-compatible constraints should be satisfied.

First, a user in the multi-use segments should prefer to buy the bundle rather than the two goods independently at each complementor. Even if this alternative would be more expensive, he might find it profitable to do so if, for example, the multi-use segment is small (α large) and the network externalities (θ) are large, implying that the network benefit of the bundle is small. Formally, the equilibrium is incentive-compatible for a

multi-use consumer if:

$$2\theta \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}^{E}) - p_{\mathcal{P}}^{E} \ge \theta \tilde{D}_{1}(p_{1}^{E}) - p_{1}^{E} + \theta \tilde{D}_{2}(p_{2}^{E}) - p_{2}^{E}.$$
 (A-4)

Second, a user in the single-use segment should prefer to buy the product from the complementor i than the bundle even if he would consume only one element of it. A single-use user may switch to the bundle when network externalities are large and the single-use segment is small. Formally, the equilibrium is incentive-compatible for a single-use consumer if:

$$\theta \tilde{D}_i(p_i^E) - p_i^E \ge \theta \tilde{D}_{\mathcal{P}}(p_{\mathcal{P}}^E) - p_{\mathcal{P}}^E.$$
(A-5)

So, the above candidate bundling price is an equilibrium if and only if it satisfies the incentive-compatibility constraints defined in Equations (A-4) and (A-5).

The condition (A-4) is satisfied if α is not too large. If the multiuse segment is small, consumers will buy from the complementors despite higher prices to interact with a larger user base. Equation (A-4) is satisfied for any θ when $\alpha \leq \overline{\alpha}$, with $\overline{\alpha} > 1/3$. For $\alpha > \overline{\alpha}$, the condition is satisfied if $\theta < \overline{\theta}(\alpha)$, with $\overline{\theta}(.)$ decreasing in α and $\overline{\theta}(\overline{\alpha}) = \hat{\theta}$ and $\overline{\theta}(1/2) = 0$.

When condition (A-4) is not satisfied, the bundle price $p_{\mathcal{P}}$ should be reduced to make the condition binding. This gives a new bundle price $p_{\mathcal{P}} = \frac{1+\theta-4\alpha\theta}{1-\alpha\theta} > 0$ that together with $p_i^E = 1/2$ constitute the unique bundling equilibrium. A lower bundle price reduces the profit of the platform while leaving unchanged those of the complementors who do not have incentives to change their price.

The condition (A-5) is satisfied if α is not too small. If the single-use segment is small, consumers will buy the bundle despite a higher price to interact with a larger user base. This condition is always satisfied for $\alpha \geq \underline{\alpha}(\theta)$ with $\underline{\alpha}(\theta) < \frac{1}{3}$. For $\alpha < \underline{\alpha}(\theta)$, the condition is satisfied if $\theta \leq \underline{\theta}(\alpha)$ with $\underline{\theta}(.)$ increasing and $\underline{\theta}(0) = 0$ and $\underline{\theta}(\underline{\alpha}) = \hat{\theta}$.

When condition (A-5) is not satisfied, complementors should lower their price below 1/2 to attract single-users. But, by doing so, they will reduce their profits below the monopoly profit level that we used in Equation (6) to determine the bundle price. In other words, if the complementor reduces its price, the bundle price should be set below $p_{\mathcal{P}}^E$. Therefore, an equilibrium should simultaneously satisfy Equations (6) and (A-5), but there is no guarantee that such an equilibrium exists and our numerical simulations indicate that there is no such equilibrium for the relevant set of parameters ($\alpha < \underline{\alpha}(\theta)$).

We summarize our findings in a lemma.

Lemma 7 (Bundling Equilibrium: Existence.) $(p_1^E, p_2^E, p_{\mathcal{P}}^E)$ is the unique equilibrium in pure strategies when

1. $\alpha \leq \underline{\alpha}$ and $\theta \leq \underline{\theta}(\alpha)$, with $\underline{\theta}(\alpha)$ increasing in α and $\underline{\theta}(0) = 0$,

- 2. $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, with $\underline{\alpha} \leq \frac{1}{3} \leq \overline{\alpha}$,
- 3. $\alpha \geq \overline{\alpha} \text{ and } \theta \leq \overline{\theta}(\alpha), \text{ with } \overline{\theta}(\alpha) \text{ decreasing in } \alpha \text{ and } \overline{\theta}(1/2) = 0.$