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## Nature Loss and Climate Change: The Twin-Crisis Multiplier

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# Nature Loss and Climate Change: The Twin-Crises Multiplier

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# Nature Loss and Climate Change: The Twin-Crises Multiplier

## Abstract

We study the economic effects of the interaction of nature loss and climate change in a model that incorporates important aspects of both processes. We capture the distinct ways in which they affect economic activity—with nature constituting a key factor of production and climate change destroying parts of output—but also the ways in which they interact: climate change causes nature loss, and nature provides both a carbon sink and adaptation tools to reduce climate damages. Our analysis of these feedback loops reveals a novel amplification channel—the Twin-Crises Multiplier—that systematically affects optimal climate and nature conservation policies.

Keywords: climate change, biodiversity loss.

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# Nature Loss and Climate Change: The Twin-Crises Multiplier

By STEFANO GIGLIO, THERESA KUCHLER, JOHANNES STROEBEL, OLIVIER WANG\*

Economists are increasingly interested in better understanding the many interactions between economic activity and the health of our planet. The two main areas of focus have been the economics of climate change and of nature and biodiversity loss (Nordhaus and Boyer, 2003; Dasgupta, 2021; Heal, 2000; Giglio et al., 2023, 2024). Due to the conceptually distinct economic effects of climate change and nature loss, this prior work has largely explored them independently. Yet, there are important feedback loops between climate change and nature loss, prompting policy makers to refer to them as “twin crises.” Indeed, the final text of the COP28 agreement in December 2023 highlighted the “urgent need to address, in a comprehensive and synergistic manner, the interlinked global crises of climate change and biodiversity loss.”

In this paper, we study the interaction of nature loss and climate change in a stylized model that incorporates important aspects of both processes. It captures the distinct ways in which they affect economic activity—with nature constituting a key factor of production and climate change destroying parts of output—but also the many ways in which they interact: climate change causes nature loss, and nature provides both a carbon sink and adaptation tools to reduce climate damages. Our analysis of these feedback loops reveals a novel amplification channel—the *Twin-Crises Multiplier*—that systematically affects optimal climate and nature conservation policies.

## I. Model

There are two dates,  $t = 0, 1$  and a representative agent with log utility and time

discount factor  $\beta$ . Output is produced using three factors of production. The first is physical capital,  $K_t$ , resulting from past investment. The second factor is ecosystem services provided by nature,  $E_t$ , which include productive inputs from ‘provisioning services’ (e.g., agricultural goods or timber) and ‘supporting services’ such as water and air filtration. The third factor is land, which is fixed at  $L$ , with agents choosing the share  $u$  of land to use in production. Output is given by  $F(K, uL, E)$ . The model is then described by equations (1) - (5):

$$(1) \quad W = \log(C_0) + \beta \log(C_1)$$

s.t.

$$(2) \quad C_0 = F(K_0, uL, E_0) - K_1,$$

$$(3) \quad C_1 = (1 - D(Z_1, E_1))F(K_1, \bar{u}L, E_1).$$

Equation (1) gives the agent’s utility, which depends on consumption  $C_t$  in each period. Time-0 state variables are initial capital  $K_0$  and ecosystem services,  $E_0$ . The agent has two choices at time 0: the share of land to use in production,  $u$ , and how much capital to invest for period 1,  $K_1$ .

Equation (2) shows that consumption in the first period,  $C_0$ , is the total output produced,  $F$ , minus what is invested in physical capital to be used in period 1,  $K_1$ .

Equation (3) describes consumption at time 1, when the agent produces with the pre-determined factors  $K_1$  and  $E_1$  (for simplicity, we fix land use in period 1 at  $\bar{u}$ ). However, climate change will destroy some period-1 output before it can be consumed.

The level of climate change—or, equivalently, the level of carbon emissions—in period 1 is  $Z_1$  (for simplicity, we assume there are no time-0 carbon emissions). The *damage function*  $D(Z_1, E_1)$  maps climate change to economic damages, similar to how climate change is treated in the Dynamic Integrated Climate-Economy

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(DICE) models of Nordhaus and Boyer (2003) and others. A novel aspect is that we allow the economic damages from climate change to be reduced by nature,  $E_1$ , capturing so-called ‘nature-based climate adaptation solutions,’ such as the ability of mangrove forests to reduce damages from coastal flooding (Menéndez et al., 2020), or the fact that well-preserved ecosystems prevent coastal erosion induced by climate change (Spalding et al., 2014).

Finally, we describe the determinants of the levels of climate change and ecosystem services in period 1. For analytical tractability, they are assumed to be log-linear and we describe them in logs (e.g.,  $z_1 = \log(Z_1)$ ):

$$(4) \quad z_1 = \theta^K k_1 + \theta^E e_1,$$

$$(5) \quad e_1 = e_0 - \delta u - \gamma z_1.$$

Equation (4) describes period-1 greenhouse gases,  $Z_1$ , which are emitted through production using physical capital,  $K_1$ , with the coefficient  $\theta^K > 0$  capturing the carbon intensity of capital use. The parameter  $\theta^E$  captures the net carbon emissions of nature,  $E_1$ . Based on empirical work in ecology that finds that carbon sequestration by forests, oceans, and soils makes nature a net carbon sink (Weiskopf et al., 2024), we assume  $\theta^E < 0$ . For simplicity, we set the direct emissions from land use to zero.

Equation (5) gives the evolution of ecosystem service provision. Starting from an initial level of  $E_0$ , the provision of period-1 ecosystem services will be degraded through higher period-0 land use in production that destroys nature and biodiversity ( $\delta > 0$ ), as highlighted by IPBES (2019).<sup>1</sup> Ecosystem services are also reduced by period-1 climate change ( $\gamma > 0$ ), for example because climate change drives biodiversity loss via higher and more variable temperatures (see Urban, 2024).

<sup>1</sup>Giglio et al. (2024) provide a rich model of how land use affects total ecosystem service production through species extinction, and highlight that the effects depend crucially on which species are lost and how uneven past species loss has been across different parts of the ecosystem. Here, we abstract from these dynamics to focus on the interactions between nature loss and climate change.

## II. Key Mechanisms

Our model captures several ways in which climate change and nature loss interact with each other and economic activity.

### M1: Nature as a net carbon sink.

As discussed, setting  $\theta^E < 0$  captures the fact that nature, on net, is a carbon sink.

### M2: Ecosystem services as a clean factor of production.

A key feature that emerges from jointly considering both nature loss and climate change is that ecosystem services are not just any factor of production, but a *clean* factor of production: by substituting physical capital with nature for the production of some goods, the economy can achieve the same level of output with fewer emissions. For instance, natural water filtration may achieve the same agricultural output as mechanical water filtration with lower energy use and emissions. This feature is captured in the model if the ratio of emission intensities  $\theta^E/\theta^K$  is smaller than the ratio of marginal products  $F_E/F_K$ , which is always the case if  $\theta^E < 0$  (see M1).

### M3. Ecosystem services reduce the impact of climate damages.

In addition to the so-called ‘mitigation’ effects of nature (see M1), our model also captures the ‘climate adaptation’ effects of nature, that is, nature’s ability to reduce the impact of climate damages on the economy (see above). Formally, this is incorporated by the dependence of the climate damage function  $D$  not only on emissions  $Z_1$ , but also on ecosystem services  $E_1$  as long as  $\partial D/\partial E_1 < 0$ .

### M4. Climate-Nature Feedback and the *Twin-Crises Multiplier*.

Climate change and nature loss interact to amplify the total economic damages of emissions and land-use: climate change accelerates nature loss, and nature loss means that less carbon is sequestered, which worsens climate change. This can be seen by combining equations (4) and (5) into:

$$z_1 = \frac{1}{1 + \gamma\theta^E} (\theta^K k_1 + \theta^E e_0 - \delta\theta^E u).$$

The terms inside the parenthesis capture the direct net emissions of physical capital and ecosystem services as well as the effect

on emissions of nature loss due to land use (through  $\delta$ ). The term

$$(6) \quad \Phi = \frac{1}{1 + \gamma\theta^E}$$

captures the *Twin-Crises Multiplier*, highlighting that the total effect of each factor of production on overall carbon emissions—and thus on economic damages—is larger than their direct emissions effects suggest.

To interpret  $\Phi$ , consider the empirically-relevant case  $\theta^E < 0$ . In that case, since  $\Phi > 1$ , the effective emission intensity of physical capital accumulation is higher than what is implied by the simple intensity  $\theta^K$ , since ecosystem degradation due to emissions from physical capital weakens nature’s ability to offset those very emissions.

Similarly, destroying nature through land use ( $\uparrow u$ ) has a larger effect on emissions than suggested by the initial emissions increase,  $-\Delta u \times \delta\theta^E$ . This is because those emissions accelerate climate change, which in turn causes additional destruction of nature’s ability to sequester carbon.

The *Twin-Crises Multiplier* is important because it means that climate change and ecosystem losses cannot be studied in isolation: ignoring one leads to an underestimate of the potential losses from the other. We explore this interaction through the lens of optimal policy in the next section.

### III. Social Optimum

We next study the optimal land use and physical capital choices,  $u^*$  and  $K_1^*$ , that maximize social welfare (1), and show how they reflect the feedback between climate and nature described above. For each factor of production, we denote by  $\eta_X$  the elasticity of date-1 output net of climate damages (and thus date-1 consumption) to factor  $X$ .

Agents have two ways to transfer resources between periods. The first is by reducing land-use  $u$ , which lowers output in period 0 but increases the amount of natural capital available in period 1. The second is the traditional choice of saving some of period-0 output as capital for period 1. We show in the Appendix that the ratio of optimal savings rates in nature,  $1 - u^*$ , and

the optimal savings ratio in physical capital,  $s^* = K_1^*/Y_0$ , is

$$(7) \quad \frac{1 - u^*}{s^*} = \frac{\delta\tilde{\eta}_{E,1}/\eta_{L,0}}{\tilde{\eta}_{K,1}}$$

where:

$$(8) \quad \tilde{\eta}_{E,1} = \Phi(\eta_{E,1} + \theta^E\eta_{Z,1})$$

$$(9) \quad \tilde{\eta}_{K,1} = \eta_{K,1} + \Phi\theta^K(\eta_{Z,1} - \gamma\eta_{E,1}).$$

To build intuition for this result, consider first the case without climate change, such that emissions do not affect output ( $\eta_{Z,1} = 0$ ) or ecosystem services ( $\gamma = 0$ ). In that case, the ratio of optimal savings rates is:

$$(10) \quad \frac{1 - u^*}{s^*} = \frac{\delta\eta_{E,1}/\eta_{L,0}}{\eta_{K,1}}.$$

Effectively, agents can invest in two assets to shift consumption between periods: physical capital and natural capital. They thus face a standard consumption-saving tradeoff coupled with a portfolio choice problem between the two assets. At the optimum, the marginal returns on the two forms of capital must be equalized, which, once converted to output-elasticities, translates into the optimal savings ratio (10).

Conserving nature through reducing land-use is more attractive relative to saving in physical capital when land use is more destructive to nature (higher  $\delta$ ). In addition, more productive ecosystem services (higher  $\eta_{E,1}$ ) lead to more optimal nature conservation relative to investment in physical capital, whereas higher land or physical capital productivity ( $\eta_{L,0}$  and  $\eta_{K,1}$ ) lead to less conservation.

When we consider the interactions with climate change ( $\eta_{Z,1} < 0$ ,  $\gamma > 0$ ), the optimal conservation choices also incorporate the indirect effects that conservation has on climate change. Specifically, the marginal value of conservation in equation (7) now depends on “effective” elasticities  $\tilde{\eta}$ , that incorporate the multiplier effects due to the climate-nature interactions.

Nature’s effective productivity is *higher* than without climate damages:  $\tilde{\eta}_{E,1} > \eta_{E,1}$ . Its contribution to output has three compo-

nents. The two direct effects on net output are captured by  $\eta_E \geq 0$ , which incorporates both the effects due to nature as a factor of production and the effects of nature on reducing the damages from climate change:

$$(11) \quad \eta_E = \partial \log F / \partial \log E + \partial \log(1-D) / \partial \log E.$$

There is also an indirect effect on productivity from nature's ability to absorb carbon emissions ( $\theta^E \leq 0$ ) and thus further reduce climate damages; this part of nature's marginal product grows with the strength of climate damages (i.e., with how negative  $\eta_{Z,1}$  is) and is further amplified by the feedback between climate change and nature loss via the *Twin-Crises Multiplier*,  $\Phi$ .

Similarly, the positive emission intensity of physical capital  $\theta^K \geq 0$  leads to a lower effective marginal product of capital relative to the case without climate damages:  $\tilde{\eta}_{K,1} < \eta_{K,1}$ . Specifically, physical capital becomes effectively less productive because its carbon emissions raise the amount of output destroyed by climate change. The last term in equation (9) captures that the emissions from physical capital, which has effective carbon intensity  $\Phi\theta^K$ , also destroy nature ( $\gamma > 0$ ). This hurts output in proportion to ecosystem productivity  $\eta_E$ —again capturing nature's two roles in (11), as a factor of production and a tool for adaptation to reduce climate damages—and further lowers the effective productivity of physical capital.

In total, the consideration of climate-nature feedback loops therefore raises the socially optimal level of nature conservation: the bigger the *Twin-Crises Multiplier*, the larger the optimal level of conservation.

#### IV. Optimal Carbon Tax and Nature Conservation Under Twin Crises

The planner's solution highlights two distinct but interacting effects that private agents may not internalize: the damages from climate change caused by the use of physical capital and the destruction of nature caused by the use of land. We now show how policies can achieve the social optimum in the presence of these externalities.

We implement the solution with (i) a carbon tax to internalize the climate externality, and (ii) a cap on land use (e.g., through nature protection) to address the nature loss externality from land use.<sup>2</sup>

In the presence of a carbon tax  $\tau$  on emissions  $\theta^K K_1$  and a cap on land use  $u^{\max}$ , individual agents choose physical capital  $K_1$  and land use  $u$  to maximize

$$U(F(K_0, uL, E_0) - (1 + \tau\theta^K)K_1) + \beta U(F(K_1, \bar{u}L, E_1, Z_1))$$

subject to  $u \leq u^{\max}$ . Externalities arise because individuals take  $E_1$  and  $Z_1$  as given.

We start by considering the optimal climate policy. A carbon tax  $\tau$  lowers agents' return on capital to  $F_{K,1}/(1 + \tau\theta^K)$ . Therefore, comparing with the social optimum, the optimal carbon tax  $\tau^*$  is

$$\tau^* = \frac{1}{\theta^K} \left[ \frac{\eta_{K,1}}{\tilde{\eta}_{K,1}} - 1 \right].$$

Suppose  $(-\eta_{Z,1} + \eta_{E,1}\gamma) \ll \eta_{K,1}$ .<sup>3</sup> Then the optimal carbon tax simplifies to

$$\tau^* \approx \Phi \frac{(-\eta_{Z,1} + \gamma\eta_{E,1})}{\eta_{K,1}}.$$

In a model without ecosystem services, this would further reduce to  $\tau^* = -\eta_{Z,1}/\eta_{K,1}$  with carbon taxes increasing in the strength of damages ( $-\eta_{Z,1}$ ) and decreasing in the productivity of capital ( $\eta_{K,1}$ ).

Adding nature loss introduces two forces. First, the optimal tax is amplified by the *Twin-Crises multiplier*  $\Phi$ , which reflects that the effective emission intensity of capital—taking into account feedback effects through nature loss—is  $\Phi\theta^K > \theta^K$ . Second, the term  $\gamma\eta_{E,1}$  enters as an additional source of damages to capture the destructive effects of climate change on ecosystems' productivity.

<sup>2</sup>Specifying a quantity restriction for land use is more natural in our setting as we abstract from modeling the different markets and prices for land use.

<sup>3</sup>This approximation states that damages are small relative to the capital elasticity of output, and is only made to give a simpler formula for  $\tau^*$ . The same conclusions hold without this approximation, at the cost of more complex expressions.



Turning to the optimal nature conservation policy, note that private agents ignore the cost of land use entirely and hence use land as much as possible. Implementing the social optimum thus requires setting

$$u^{\max} = u^* = 1 - \frac{\delta}{\eta_{L,0}} \frac{\beta \tilde{\eta}_{E,1}}{1 + \beta \tilde{\eta}_{K,1}}.$$

The optimal land-use cap internalizes both the direct effects of land use on production and its indirect effects through the *Twin-Crises Multiplier*. The optimal cap implies more stringent land-use restrictions as climate damages become more severe (higher  $-\eta_{Z,1}$ ) and preserving nature becomes increasingly critical due to its roles as a carbon sink, a climate adaptation tool, and a clean factor of production. Environmental policies that do not consider the feedback between climate change and nature loss may fail to account for these effects.

This optimal policy reveals a key coordination challenge. Local governments may internalize direct local benefits of production and adaptation embedded in  $\eta_{E,1}$ , while still ignoring the benefits of mitigating *global* climate change, captured by the multiplier  $\Phi$  and the term  $\theta^E \eta_{Z,1}$  in  $\tilde{\eta}_{E,1}$ . As a result, local conservation policies will tend to be too lenient, creating a need and a rationale for international coordination.

## V. Conclusion

While compact, our model can capture key higher-order effects and non-linearities not yet discussed. For instance, nature’s carbon absorption capacity may exhibit diminishing returns, with the marginal effect of nature loss on climate change growing as ecosystems become more depleted. The marginal value of nature’s adaptation services can also increase with climate change ( $\partial^2 D / \partial Z \partial E < 0$ ), as extreme weather events become more frequent. These non-linearities suggest that the *Twin-Crises Multiplier* will become more important as we move further from current conditions.

Given the feedback between the economic effects of climate change and nature loss established in this paper, we hope that future work will consider the two processes jointly.

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# Supplemental Appendix

## “Nature Loss and Climate Change: The Twin-Crises Multiplier”

Stefano Giglio, Theresa Kuchler, Johannes Stroebe, Olivier Wang\*

January 7, 2025

The planner chooses physical capital  $K_1$  and land conservation  $c = 1 - u$  to maximize

$$W = U(F(K_0, (1 - c)L, E_0) - K_1) + \beta U(Y_1(K_1, \bar{u}L, E_1, Z_1)) \quad (1)$$

where  $Y_1 = (1 - D(Z_1, E_1))F(K_1, \bar{u}L, E_1)$  is output net of climate damages, subject to the two constraints

$$\begin{aligned} E_1 &= G(c, Z_1) \\ Z_1 &= H(K_1, E_1) \end{aligned}$$

which capture how ecosystem services  $E_1$  are affected by conservation and climate change, and climate change is affected by production, respectively. Define

$$\begin{aligned} \theta^K &= \frac{\partial \log H}{\partial \log K_1} \\ \theta^E &= \frac{\partial \log H}{\partial \log E_1} \\ \delta &= \frac{\partial \log G}{\partial \log c} \\ \gamma &= -\frac{\partial \log G}{\partial \log Z_1} \end{aligned}$$

The Lagrangian is

$$W + \lambda \{G(c, Z_1) - E_1\} + \mu \{Z_1 - H(K_1, E_1)\}$$

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and the first-order conditions with respect to  $c, K_1, E_1, Z_1$  are:

$$\begin{aligned} -U'_0 F_{L,0} + \lambda G_c &= 0 \\ -U'_0 + \beta U'_1 F_{K,1} - \mu H_K &= 0 \\ \beta U'_1 F_{E,1} - \lambda - \mu H_E &= 0 \\ \beta U'_1 F_{Z,1} + \lambda G_Z + \mu &= 0 \end{aligned}$$

Without climate change, we can ignore the last equation and replace  $\lambda = \beta U'_1 F_{E,1}$  to obtain two equations

$$\begin{aligned} U'_0 &= \beta U'_1 F_{K,1} \\ U'_0 &= \beta U'_1 \frac{F_{E,1} G_c}{F_{L,0}} \end{aligned}$$

that characterize the standard optimal consumption-saving decision and equalize the marginal returns on physical capital and natural capital.

With climate change, the solution becomes

$$\begin{aligned} U'_0 &= \beta U'_1 \left\{ F_{K,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_K \right\} \\ U'_0 &= \beta U'_1 \left\{ F_{E,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_E \right\} \frac{G_c}{F_{L,0}} \end{aligned}$$

Instead of the simple marginal returns on physical capital  $F_{K,1}$  and on natural capital  $\frac{F_{E,1} G_c}{F_{L,0}}$  we have modified marginal returns taking into account the feedback loop between nature, economic activity, and climate change. These can be further simplified as

$$\begin{aligned} F_{K,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_K &= \frac{Y_1}{K_1} \tilde{\eta}_{K,1} \\ F_{E,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_E &= \frac{Y_1}{E_1} \tilde{\eta}_{E,1} \end{aligned}$$

where we define the modified capital and ecosystem elasticities as

$$\begin{aligned} \tilde{\eta}_{K,1} &= \eta_{K,1} + (\eta_{Z,1} - \eta_{E,1} \gamma) \Phi \theta^K \\ \tilde{\eta}_{E,1} &= \Phi [\eta_{E,1} + \eta_{Z,1} \theta^E] \end{aligned}$$

respectively. Therefore, in the case of log-utility  $U = \log$ , the optimality conditions become

$$\begin{aligned}\frac{s^*}{1-s^*} &= \beta \tilde{\eta}_{K,1} \\ \frac{c^*}{1-s^*} &= \beta \tilde{\eta}_{E,1} \frac{\delta}{\eta_{L,0}}\end{aligned}$$

where we denote  $s^* = K_1/Y_0$  the optimal savings rate in physical capital, which yields

$$\begin{aligned}s^* &= \frac{\beta \tilde{\eta}_{K,1}}{1 + \beta \tilde{\eta}_{K,1}} \\ c^* = 1 - u^* &= \frac{\delta}{\eta_{L,0}} \frac{\beta \tilde{\eta}_{E,1}}{1 + \beta \tilde{\eta}_{K,1}}\end{aligned}$$

Therefore the ratio of optimal savings rates is

$$\frac{1-u^*}{s^*} = \frac{\delta}{\eta_{L,0}} \frac{\tilde{\eta}_{E,1}}{\tilde{\eta}_{K,1}}.$$