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Skill signaling, prospect theory, and regret theory*

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Abstract

When a risky decision involves both skill and chance, success or failure is a signal of the decision maker's skill. Adopting standard models from the career concerns literature, we show that a rational desire to avoid looking unskilled may help explain several anomalies associated with prospect theory, including probability weighting, loss aversion, and framing. Prospect theory's four-fold pattern of probability weighting predicts that decision makers favor long-shots, avoid near sure-things, buy insurance against unlikely losses, and take risky chances to win back large losses. We find that this pattern emerges because winning a gamble with a low probability of success is a strong signal of skill, while losing a gamble with a high probability of success is a strong signal of incompetence. Regarding loss aversion, a fear of looking inept provides an alternative explanation for the puzzle of why people are so risk averse for small gambles. Such behavior can arise because losing any gamble, even a "friendly bet" with little or no money at stake, reflects poorly on the decision maker's skill. Finally, we find that framing affects choices because different formulations of a question provide different information about how a decision maker's actions will be interpreted. While the theoretical predictions of skill signaling closely parallel those of prospect theory, they differ in some cases, allowing for tests between the theories. The theoretical predictions are also closely related to, but distinguishable from, those of regret theory.

JEL Classification Categories: D81; D82; C92; G11

Key Words: Prospect theory; regret theory; career concerns; probability weighting; loss aversion; dare taking; embarrassment aversion; framing

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1 Introduction

Most risky decisions involve both skill and chance. Success is therefore doubly lucky in that it brings both material gain and an enhanced reputation for skill, while failure is doubly unlucky. In many cases the gain or loss from looking skilled or unskilled can be significant. For instance, the manager of a successful project might win the confidence of her boss to oversee more projects, while the manager of a failed project might be viewed as incompetent and lose future opportunities. More generally, it is difficult to imagine a risky decision of any consequence where a person is not concerned that failure will reflect unfavorably on them.

We are interested in what anomalies are predicted if people are afraid of looking unskilled, but they are modelled as only concerned with the immediate monetary payoff. We will show that, in many cases, the result is a set of behaviors that are typically explained by the psychological theory of prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). In other cases the predicted behaviors diverge, thereby providing a basis for testing the relative explanatory power of the theories. We will also show that the predicted behaviors are closely related to, but distinguishable from, those of regret theory (Bell, 1982; Loomes and Sugden, 1982).

Economists have long recognized that actual behavior in risky environments differs substantially from standard economic models. Prospect theory's concepts of probability weighting, loss aversion, and framing provide a common structure for organizing many of these anomalies. Probability weighting argues that decision makers do not weight probabilities linearly as expected utility theory requires but instead overweight low probabilities. The theory has been used to gain insight into a range of anomalous behavior that has long puzzled economists, including the simultaneous purchase of lottery tickets and insurance (Friedman and Savage, 1948), the Allais paradox (Allais, 1953), and the preference for long shots in horse races and other gambling environments (Thaler and Ziemba, 1988). Loss aversion argues that utility depends on gains and losses relative to the status quo and that utility falls more steeply in losses than it rises in gains. It is used to explain why people appear so risk averse for small gambles when standard models predict near risk neutrality (Rabin and Thaler, 2001). Finally, framing is used to explain why seemingly irrelevant rephrasing of a situation leads to different choices (Tversky and Kahneman, 1981).

To model the idea that people do not want to look like a "loser" and to gain insight into the relationship between skill signaling² and behavioral anomalies, we borrow heavily from the principle-agent literature on career concerns starting with Holmstrom (1982/1999). This literature shows how decisions by managers affect estimates of their skill and analyzes the resulting impact on managerial behavior and contract design. We differ from the literature in that we do not explicitly model the details of the career environment, but rather derive general results for situations where individuals, for narrow career concerns or more general considerations, care

¹Prospect theory also argues that the utility function (or "value function") is concave in gains and convex in losses (Kahneman and Tversky, 1979). We follow Tversky and Kahneman (1992) and Prelec (1999) in emphasizing probability weights as the basis for observed behavior in gambles involving gains or losses.

²Our use of the term captures actions to prove one's type as in the signaling literature (Spence, 1973) and actions to hide one's type as in the signal-jamming literature (Fudenberg and Tirole, 1986).

about what other people think of their abilities.³ In particular, we derive results for the case where people are "embarrassment averse" in that they are afraid of being perceived as unskilled in the same pattern as is normally assumed for risk aversion regarding monetary outcomes.⁴

Following the career concerns literature, we investigate two types of skill. First, with "performance skill" some decision makers face better odds of success than less skilled decision makers. For instance, a project might be more likely to succeed under a skilled manager. Interest in a reputation for performance skill has been used by Holmstrom (1982/1999) to understand "rat race" career incentives, by Holmstrom and Costa (1986) to analyze excessive risk-taking, and by Zwiebel (1995) to analyze corporate conformism. Second, with "evaluation skill" some decision makers are better at identifying the exact odds of a gamble than their less skilled counterparts. For instance, a skilled manager might be better at choosing promising projects, or a skilled broker might have a talent for identifying profitable companies. Evaluation skill has been used by Holmstrom (1982/1999) to understand distorted investment decisions, by Scharfstein and Stein (1990) to understand herding behavior, by Kanodia, Bushman, and Dickhaut (1989) to analyze the sunk cost fallacy, by Prendergast and Stole (1996) to analyze both conservatism and overconfidence, and by Morris (2001) to analyze political correctness.

We find that the presence of either type of skill in a gamble can induce the three behaviors associated with prospect theory. For probability weighting, the key is that losing a low probability gamble is less embarrassing than losing a high probability gamble. When the odds of success are poor, failure is common but only slightly reduces the perceived skillfulness of the decision maker because both skilled and unskilled decision makers are expected to fail. But when success is likely, failure is rare but far more embarrassing because a person who fails is probably unskilled. Embarrassment averse decision makers are therefore more willing to take a chance on gambles that observers recognize are long shots, and reluctant to take gambles where success is expected.⁵ For loss aversion, the main result is that losing any gamble is embarrassing so losing involves a discontinuous drop in utility even if no money is at stake. Embarrassment averse people are therefore more afraid of small gambles than pure risk aversion regarding monetary payoffs would predict. For framing, in many situations there are multiple equilibria depending on how the choice to gamble or not gamble is interpreted. The framing of the question can determine what inferences a decision maker should make about observer beliefs and thereby affect which equilibrium is chosen.

These results are sensitive to the information environment, thereby providing an opportunity to distinguish

³We assume that fear of failure is entirely driven by a concern for not looking incompetent to outsiders. Neilson (2002) examines behavior when utility is affected directly by success or failure and finds similar results to ours.

⁴Of course, other cases are also of potential interest. For instance, in a career concerns environment, the gains from standing out sufficiently to earn a promotion might sometimes outweigh the risks of demotion. Embarrassment aversion does not fit such situations, just as risk aversion with respect to monetary outcomes does not fit situations where consumers are willing to take a chance to attain sufficient funds to buy an expensive, indivisible good.

⁵These results are related to the social psychology literature starting with Atkinson (1957) that examines behavior when people both hope for success and fear failure. The literature recognizes that there is little embarrassment in failing at a difficult task and finds that failure averse people tend to pick either very difficult tasks or tasks where success is nearly assured (McClelland, 1985). Our formal analysis confirms the insight regarding difficult tasks but also highlights the danger of failing at easy tasks.

them from prospect theory. For probability weighting, the results are reversed if the observer does not know the odds of the gamble. For instance, a boss might not know whether a subordinate's project is likely to succeed or not. In this case a project with a low probability of success clearly offers more opportunity for embarrassment than a project with a high probability of success. For loss aversion, the results are sensitive to the information environment because refusal to take a gamble can in some cases be perceived as an admission that one is unskilled. Rather than confirming their incompetence through refusing the gamble, people may take the gamble in the hope of getting lucky and escaping embarrassment. Hence people are not necessarily loss averse but can be "dared" into taking unprofitable gambles. For framing, in some situations there are multiple equilibria in which framing is important, while in other situations there is only one equilibrium so framing should have no effect.

To understand these results further, first consider probability weighting. In both its original form (Kahneman and Tversky, 1979) and in its later form of "cumulative prospect theory" (Tversky and Kahneman, 1992),⁶ probability weighting argues that people typically violate expected utility maximization by not weighting probabilities linearly. In particular there is a "four-fold pattern" with overweighting of small probability gains (such as taking a 10% chance of winning \$100 over \$10 for sure), underweighting of high probability gains (such as taking \$90 for sure rather than a 90% chance of winning \$100), overweighting of low probability losses (such as paying \$10 for sure rather than risking a 10% chance of losing \$100), and underweighting of high probability losses (such as taking a 90% chance of losing \$100 rather than paying \$90 for sure). This pattern corresponds to favoring long-shots, avoiding near sure-things, buying insurance against unlikely losses, and taking risky chances to win back large losses.

From the perspective of skill signaling the rather complicated four-fold pattern in gains and losses can be interpreted more simply as just overweighting of small probabilities of success and underweighting of high probabilities of success. In particular, favoring long shots and taking chances to win back large losses are both cases of overweighting small probabilities of success, and avoiding near sure things and buying insurance are both cases of underweighting of high probability of success. Therefore when skill signaling predicts that decision makers favor gambles with a low probability of success and avoid gambles with a high probability of success, the four fold pattern is predicted.

Figure 1 shows a probability weighting function that corresponds to the four-fold pattern. In the prospect theory literature there are typically two such lines of nearly identical shape and position, one for the weighting function where p is the chance of winning a positive amount and one for the weighting function where p is the chance of losing a negative amount.⁷ The close similarity of the probability weighting functions in gains and

⁶Cumulative prospect theory is an elaboration of prospect theory based on rank-dependent expected utility (Quiggin, 1982). Cumulative prospect theory can more logically incorporate gambles where there are more than two outcomes, but we restrict attention in this paper to gambles with only two outcomes so the theories give identical predictions.

⁷The original formulation of prospect theory (Kahneman and Tversky, 1979) included a discontinuity at p=1 to capture the "certainty effect" that a lottery with a near 100% chance of winning some amount is valued substantially below winning that amount for certain. Such an effect arises from simpler strategic considerations than examined here. Whereas a promise of certain gains can be pursued in court (of either the legal or public opinion kind), there is little recourse against apparent fraud if the

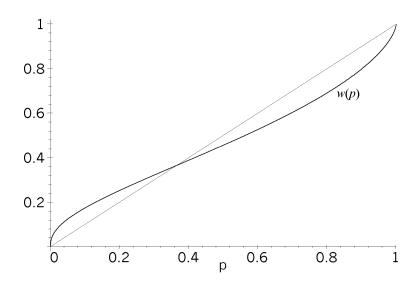


Figure 1: Prospect theory's probability weighting function

losses – the source of the four-fold pattern – is referred to as the "reflection effect". However, as discussed above, from the perspective of skill signaling the four fold-pattern only requires overweighting of gambles with a low probability of success and underweighting of gambles with a high probability of success, so in Figure 1 we interpret p as simply the probability of success.

Probability weights cannot be observed directly, but if the utility function is known to have a particular form then probability weights can be inferred from the observable risk premia that people pay to avoid gambles.⁸ The simplest approach is to assume that utility is linear in wealth since standard theories indicate that the utility function should be nearly linear for all but very large gambles (Pratt, 1964). If a person has the probability weighting function shown in Figure 1, a gamble with a p = .9 probability of winning is weighted as if the gamble really has about a p = .8 probability of winning. For instance, the person is nearly indifferent to a gamble with a 90% chance of winning \$100 and getting \$80 for sure, i.e. the gamble has a \$10 risk premium. In contrast a gamble with a p = .1 probability of winning is weighted as if the gamble really has about a p = .2 probability of winning. For instance, the person is nearly indifferent to a gamble with a 10% chance winning \$100 and winning \$20 for sure, i.e. the gamble has a negative \$10 risk premium. So the clearest testable prediction of prospect theory's probability weighting function is that risk premia are negative for gambles with low probabilities of success and positive for gambles with high probabilities of success. When

gains are advertised as probabilistic. Fear of being cheated may also be a factor in the related phenomenon of consumer aversion to "probabilistic insurance" in which compensation is not paid with certainty (Kahneman and Tversky, 1979).

⁸Even if the shape of the utility function is not known, it is still possible to test the general shape of the probability weighting function (Wu and Gonzalez, 1996).

this pattern holds we will say that the four-fold pattern is strongly satisfied. When the risk premium for gambles with a low probability of success is smaller – but not actually negative – we will say the four-fold pattern is only weakly satisfied.

First consider the connection between probability weighting and performance skill in which some decision makers are better at a given gamble than others. Because they offer less risk of embarrassment, we find that gambles with a low probability of success always have smaller risk premia than gambles with a high probability of success, so the four-fold pattern is always at least weakly satisfied. For the case where decision makers do not know their own skill, risk premia are always positive for any gamble so only weak satisfaction of the four fold pattern is possible. However, when decision makers know their own skill they may feel compelled to take gambles with negative risk premia to show that they are not trying to hide a relative lack of skill. Depending on how unfavorably they expect that refusing a gamble will be viewed, the four-fold pattern may be satisfied either weakly or strongly.

The situation is similar for evaluation skill in which some decision makers are better than others at identifying the exact odds of a gamble. Low probability gambles again have smaller risk premia than high probability gambles. Whether the four-fold pattern is satisfied weakly or strongly depends on whether the observer learns the outcome of a gamble that is not taken. If the observer does not learn the outcome then the four-fold pattern holds only weakly. But if the outcome of a gamble is observable even when the decision maker turns it down, the decision maker cannot simply refuse to take the gamble and prevent the observer from learning about her skill. If such a gamble is not taken, a good outcome is a strong indication that the decision maker failed to recognize the gamble had better than expected odds. We find that for low probability gambles this possibility is more embarrassing than taking the gamble and losing, so the risk premium is negative. The opposite situation arises with high probability gambles. If such a gamble is not taken, little about the decision maker's skill is revealed because the good outcome is likely regardless of whether the gamble's true odds were slightly better or worse than expected. The danger from taking the gamble and losing is greater, so high probability gambles have positive risk premia.

Our results on probability weighting hold for the case where the decision maker's utility function is decreasingly concave in the observer's skill estimate. This restriction has a natural interpretation based on the standard assumption of decreasing absolute risk aversion in monetary outcomes. If, for instance, future earnings are a linear function of observer skill estimates, then it immediately follows from decreasing absolute risk aversion that the utility function is decreasingly concave in the observer's skill estimate. If being perceived as incompetent is disproportionately damaging to a person's status and opportunities, financial or otherwise, then decreasing concavity is even more relevant.

⁹The standard measure of absolute risk aversion for a utility function u is -u''/u'. So decreasing absolute risk aversion occurs if $((u'')^2 - u'u''')/(u')^2 < 0$. If u' > 0 and u'' < 0, this requires u''' > 0, which is the definition of decreasing concavity. Standard utility functions used in economics, such as the exponential, logarithmic, and constant relative risk aversion utility functions, all satisfy u''' > 0. Decreasing absolute risk aversion implies that the demand for risky assets increases with wealth (Pratt, 1964) and that consumers engage in precautionary savings (Kimball, 1990).

¹⁰For instance, the structure of competition between managers may induce a strong fear of failure (Milbourn, Shockley, and

Regarding loss aversion, prospect theory argues that a loss is substantially more painful than a gain of equal size independent of the decision maker's wealth level. Whereas standard models of risk aversion assume that the first derivative of the utility function is continuous, prospect theory argues that the utility function (or "value function") has a kink at current wealth, with a steeper slope downwards than upwards. Therefore, the standard result that decision makers should be almost risk neutral for gambles with very small stakes (Pratt, 1964) does not apply. The most direct prediction of loss aversion is that gambles with an equal chance of winning or losing have a non-trivial risk premium even when the monetary stakes are very small. Rabin and Thaler (2001) show that loss aversion may explain the puzzle that common estimates of risk aversion for small gambles imply implausibly high risk premiums for large gambles (Schlaifer, 1969; Rabin, 2000).

Skill signaling implies a similar result for the basic models of performance skill and evaluation skill since decision makers have a strong dislike of being seen as a loser regardless of their wealth. Small gambles, even "friendly bets" with no money, affect estimated skill so there is a discontinuous jump in utility from winning and a discontinuous drop from losing.¹² If the decision maker is risk averse with respect to skill estimates then, consistent with loss aversion, the drop is larger.¹³

While loss aversion appears to be a common phenomenon, there are also situations where people are more risk taking than loss aversion would predict. For instance, youth are particularly prone to daring each other to risky behavior¹⁴ and entrepreneurs often show costly overconfidence.¹⁵ For performance skill, this can happen when the decision maker knows her own skill and not taking a chance is perceived as a lack of confidence in one's abilities.¹⁶ Gambles with equal probability of success or failure might then be accepted rather than rejected. For evaluation skill, loss aversion is not a factor when the outcome of the gamble is observed even if the decision maker does not take it. Because the decision maker will be judged by the outcome whether she gambles or not, she is indifferent between gambles with an equal probability of success or failure.

Finally, consider the connection between skill signaling and framing. In many situations seemingly irrel-

Thakor, 1999).

¹¹Loss aversion may also be related to the endowment effect (Kahneman, Knetsch, and Thaler, 1991). The role of signaling in the endowment effect is discussed in Posner and Fremling (1999) and the roll of bargaining with asymmetric information is analyzed in Dupont and Lee (2002). Plott and Zeiler (2003) find that the endowment effect disappears when anonymity is ensured, bargaining factors are eliminated, and likely sources of subject confusion are controlled for.

¹²This is consistent with Schlaifer's (p.161) suggestion that in some cases "nonmonetary consequences" of losing may explain high risk premia for small gambles.

¹³Just as a wealthy person is relatively unaffected by small monetary losses, it may seem that a person with a very strong reputation for skill should be relatively unaffected by losing. Accumulated savings can be used as a perfect substitute for any monetary losses, but an accumulated reputation for skill cannot fully cushion the embarrassment of losing. Skill changes with time so new information is always more important than old information, skill in a given task is unlikely to be a perfect substitute for skill in another task, and full information on a decision maker's previous successes is unlikely to be available to any given audience.

¹⁴See O'Donoghue and Rabin (2000) for a discussion of other factors in youth risk taking.

¹⁵In an experiment Camerer and Lovallo (1999) find that excess entry by subjects playing the role of entrepreneurs is larger when the subjects know that payoffs depend on skill.

¹⁶Of course, in some cases trying too hard to prove one's skill can itself signal a lack of confidence (Feltovich, Harbaugh, and To, 2002). In the simple model we consider this possibility does not arise.

evant rephrasings of a question leads to different choices. In particular, the decision to gamble or not often depends on whether the outcomes are described as gains or losses, even though the actual outcomes are the same. Consider the classic flu problem (Tversky and Kahneman, 1981). In two materially identical formulations a choice is made about the fates of 600 people. Some subjects are given the choices (A) 200 people will be saved or (B) with a $\frac{1}{3}$ probability 600 people will be saved and with a $\frac{2}{3}$ probability no people will be saved. Most of the subjects choose (A). Other subjects are given the choices (A) 400 people will die or (B) with a $\frac{1}{3}$ probability no people will die and with a $\frac{2}{3}$ probability 600 people will die. Most of the subjects choose (B).¹⁷ The decision makers know that there are 600 lives at stake so in either formulation it is clear that success at the gamble is 0 die and 600 live, failure is 600 die and 0 live, and not gambling is 400 die and 200 live. From a prospect theory perspective the answers change because people tend to behave differently in the gains and losses domains. From a skill signaling perspective, the question is what is viewed as success and what is viewed as failure. In this question do observers know what success is? Not if they just see or hear a description of the outcome without the background information. They would reasonably conclude from the newspaper headlines that "200 people saved" was a great success and that "400 people died" was at least as noteworthy a failure. Choice (A) therefore offers a sure bet at "success" in the first formulation and at "failure" in the second formulation.

Even in less extreme situations framing is still important from a skill signaling perspective. For both probability weighting and loss aversion, multiple equilibria can exist because of asymmetric information between the decision maker and the receiver. The primary case is when, with performance skill, the decision maker knows her skill and might feel pressured to signal her skill by gambling.¹⁸ Whether she will gamble depends on what the observer believes about her off-equilibrium-path behavior when she is expected to take the gamble but does not. As discussed above, these beliefs determine whether the weak or strong four-fold pattern results and whether loss aversion or dare taking results. Different beliefs are justifiable according to standard criteria such as divinity, so beliefs are very susceptible to any extra information. The observer could explicitly state that not taking a gamble will be interpreted as an admission of incompetence, or that nothing negative will be inferred from such a choice. But absent such statements, the only information to fall back on is the framing of the question.

The idea that frames often "provide information beyond their literal content" is examined empirically by McKenzie and Nelson (2002). They find that speakers consistently frame questions differently based on private information so the choice of frame provides real information to decision makers. For instance, speakers are more likely to describe a new treatment as leading to "50% of patients survive" when the standard treatment is not very successful, and to describe a new treatment as leading to "50% of patients die" when the standard treatment is very successful. The idea that framing can help choose between different equilibria has been

¹⁷Note that in this example $p = \frac{1}{3}$ so, as seen in Figure 1, the distortion from probability weighting is small. Prospect theory can explain the reversal not by probability weighting but by the additional assumption that the value (or utility) function is concave in gains and convex in losses (Kahneman and Tversky, 1979).

¹⁸As discussed in the text, multiple equilibria can arise in other cases too but they do not lead to qualitatively different predictions.

investigated empirically in coordination games by Bohnet and Cooter (2003). They show that "framing effects" can often be interpreted as equilibrium selection. Our results regarding framing have elements of both information leakage and equilibrium selection in that multiple equilibria arise because of asymmetric information and framing reveals information that leads to one of the equilibria being chosen.

Table 1 summarizes the conclusions.¹⁹ Cases where changing the information environment changes the results provide opportunities to test the comparative explanatory power of prospect theory and skill signaling. It may seem that existing tests of prospect theory do not include a skill component so any roll for skill signaling would be above and beyond the already identified effects of prospect theory. In fact, most tests starting with Kahneman and Tversky (1979) have asked hypothetical questions about whether different gambles with different probabilities of success would be accepted or rejected. As Kahneman and Tversky state, the subjects have no reason to lie and "people often know how they would behave in actual situations of choice". But it is unclear from the descriptions whether, as in almost all real world gambles, there is a skill component²⁰ or whether the gambles are meant to involve no skill at all. When probability weighting is tested using explicit randomization devices which clearly have no skill component, the four-fold pattern is substantially weakened or disappears (Harbaugh, Krause, and Vesterlund, 2002).²¹

Table 1: Summary of Conclusions on Probability Weighting, Loss Aversion, and Framing

	Performance Skill		EVALUATION SKILL	
	Decision maker	Decision maker	Observer learns	Observer learns
	does not know	knows own	outcome only if	outcome even if
	own skill level	skill level	gamble taken	gamble refused
Observer knows	Weak FFP	Weak/strong FFP	Weak FFP	Strong FFP
if gamble has high	(Prop. 1-i)	(Prop. 2-i, 2-ii)	(Prop. 3-i)	(Prop. 4-i)
or low probability	Loss averse	Dare susceptible	Loss averse	Dare neutral
of success	(Prop. 1-ii)	(Prop. 2-iii)	(Prop. 3-ii)	(Prop. 4-ii)
Observer does not know probability of success	Reverse FFP (Prop. 5)		Reverse FFP (Prop. 6)	

¹⁹Note that the case where the observer does not know the probability of success is not relevant for our simple discussion of loss aversion. Also note that we do not examine performance skill under the two different information conditions examined for evaluation skill and vice versa. Qualitatively different predictions are not generated by such permutations of the information conditions.

²⁰Perhaps the least skill dependent gambles that one can encounter in real life are lotteries and slot machines. But even those gambles offer different odds depending on how they are played, so evaluation skill is still present and winners are not entirely incorrect to brag about their (relatively) clever strategies.

²¹Eliminating performance skill in an experiment is straightforward except for the complication that decision makers (and observers) might have an "illusion of control" and still act as if skill is important (Langer, 1975). Eliminating evaluation skill is more difficult. For instance, the decision to participate in an experiment at all involves an evaluation by the subject that the likely payoff will be higher than the opportunity cost of time. Therefore even if gambles in an experiment have no explicit skill component conditional on the the subject having chosen to participate in the experiment, subjects with large earnings still have favorable information about their judgement to report to friends and family. Even conditional on participation, evaluation skill arises regarding subject estimates that the odds really are as reported and that they really will get paid if they win.

So far we have only discussed the connections with prospect theory. Regret theory (Bell, 1982; Loomes and Sugden, 1982) is similar to skill signaling in that it offers an explanation for probability weighting that does not rely on perceptual or cognitive biases. The utility function is assumed to include not just wealth, but an additively separable regret function that is increasing in the difference between the realized and unrealized outcomes. A stock might be bought because of the regret that would arise if it was not purchased and did well, and insurance might be attractive in part because of the regret that would arise if it turned out to be needed and was not taken. In particular, regret theory can explain prospect theory's weighting function if the regret function is more concave for negative outcomes (the realized outcome is the worse outcome) than for corresponding positive outcomes (the realized outcome is the better outcome).²²

Regret theory leaves open the question of why decision makers experience regret. While the internal incentive to avoid mistakes is clearly important, external incentives may be relevant also. Bell (1982) suggests that "the evaluation of others, one's bosses for example, may be an important consideration" but regret theory does not formally model how success or failure should affect such evaluations. By explicitly allowing for some decision makers to be more skilled than others we are able to model this process of how skill estimates are determined. In particular, we are able to show why losing high probability gambles is more embarrassing than losing low probability gambles under both performance skill and evaluation skill, and to analyze the different equilibria that can result. While there is a strong connection between regret and skill signaling, the models are distinct in that regret theory is driven by the relative sizes of the realized and unrealized outcomes while skill signaling is driven by how success or failure affects the observer's updating of the decision maker's skill.²³

2 Performance Skill

We first consider a model where decision makers of different skill face differing probabilities of success for the same gamble. A gamble $(p, x_w; 1 - p, x_l)$ offers x_w ("winning" or "success") with stated probability p and x_l ("losing" or "failure") with stated probability 1 - p where $x_w > x_l$ and $p \in (0, 1)$. The decision maker's type t is skilled s or unskilled u with equal probability, $t \in \{s, u\}$, $\Pr[s] = \Pr[u] = \frac{1}{2}$. If a gamble is stated to have probability p of success then the actual probability is $\Pr_p[x_w|s] = p + \varepsilon$ for the skilled decision maker and $\Pr_p[x_w|u] = p - \varepsilon$ for the unskilled decision maker for some $\varepsilon \in (0, \min\{p, 1 - p\})$. For all p, we assume ε is the same for the gamble $(p, x_w; 1 - p, x_l)$ as it is for the gamble $(1 - p, x_w; p, x_l)$. The decision maker has

²²While this case is consistent with the weighting function, Loomes and Sugden (1982) consider different cases and note "there seems to be no *a priori* reason for preferring any one of these assumptions to the others". Note that this restriction on the regret component of the utility function is equivalent to our assumption of decreasing concavity for the skill estimate component of the utility function. As argued, decreasing concavity for the skill estimate component is a natural assumption if future income depends linearly on estimated skill.

²³Although we do not model it here, in practice the size of the gamble is also likely to be relevant for skill reputation, though not in the same manner as in regret theory. If the decision maker devotes more resources to evaluating or performing well at larger gambles the observer should update the decision maker's estimated skill more strongly based on the results of such gambles. Such investments are analyzed in Dewatripont, Jewitt and Tirole (1999) and Milbourn, Shockley, and Thakor (2001).

²⁴Assuming equal proportions simplifies the analysis. The embarrassment aversion problem is less severe when most decision makers are skilled since even those who fail are still likely to be skilled. Conversely, the problem is more severe when most decision makers are unskilled.

an additively separable utility function which is linear in wealth Y and increasing in the observer's estimate of their type given all available information, 25 $U = Y + v(\Pr[s])$. The utility function is "reduced form" in that we do not explicitly model how skill signaling materially affects the decision maker. 26 We assume that a better skill estimate helps, v' > 0, that the decision maker is risk averse with respect to skill estimates, v'' < 0, and that this aversion is stronger at lower estimates, v''' > 0. As indicated, this last assumption of decreasing concavity is necessary for decreasing absolute risk aversion, a standard assumption for risk aversion with respect to monetary outcomes.

We start by assuming that the decision maker does not know her own skill so that the act of taking a gamble is not itself a signal to the observer. Since the decision maker does not know her type, the observer can infer nothing about her skill from the decision to gamble or not, so the observer believes that each type gambles with equal probability. If the gamble is taken the expected skill of a decision maker, conditioning on winning or losing, for a gamble with p average chance of winning is,

$$\Pr_{p}[s|x_{w}] = \frac{\Pr_{p}[x_{w}|s]\Pr[s]}{\Pr_{p}[x_{w}|s]\Pr[s] + \Pr_{p}[x_{w}|u]\Pr[u]} \\
= \frac{1}{2} + \frac{\varepsilon}{2p} \tag{1}$$

and

$$\Pr_{p}[s|x_{l}] = \frac{(1 - \Pr_{p}[x_{w}|s]) \Pr[s]}{(1 - \Pr_{p}[x_{w}|s]) \Pr[s] + (1 - \Pr_{p}[x_{w}|u]) \Pr[u]} \\
= \frac{1}{2} - \frac{\varepsilon}{2(1 - p)}.$$
(2)

Figure 2 shows the case where $\varepsilon = p(1-p)/2$ so that skill differences shrink as p approaches 0 or 1. For low probability gambles winning has a large impact on estimated skill as seen from the divergence of the top line $\Pr_p[s|x_w]$ from the center line representing expected skill, $p\Pr_p[s|x_w] + (1-p)\Pr_p[s|x_l]$, while losing has only a small impact as seen from the closeness of the bottom line $\Pr_p[s|x_l]$ to expected skill. Low probability gambles therefore present a chance of standing out with little downside risk. Conversely, winning at high probability gambles has only a small impact on estimated skill whereas losing has a large impact. Such gambles offer little opportunity to prove the sender's skill but carry substantial danger of embarrassment.

We will first analyze the problem from a risk premium perspective and then consider how this relates to prospect theory. Note that the risk premium is the same for both types since they do not know their own type. Since the utility function is linear in wealth, this common risk premium π is affected only by the skill

²⁵Linearity of utility in wealth allows us to isolate the effect of skill reputation. Alternative specifications, such as $U = v(Y) + v(\Pr[s])$ or $U = v(Y + \Pr[s])$, capture the same intuition but complicate the calculation of risk premia. For small gambles, wealth should be nearly linear in wealth (Pratt, 1964).

²⁶By putting estimated skill into the utility function we do not mean to imply that people receive direct utility from being perceived as skilled, any more than by putting wealth into the utility function do we mean to imply that people receive direct utility from money. A reputation for skill can help people attain the status, opportunities, and goods that they desire.

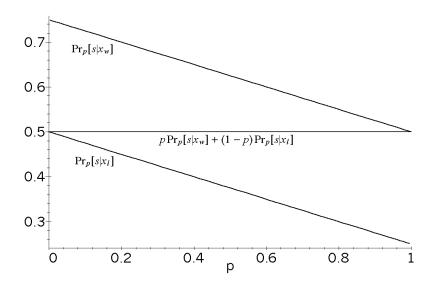


Figure 2: Performance Skill: Estimated skill from winning (top) and losing (bottom)

estimate component, so it equals

$$\pi(p) = v(\frac{1}{2}) - \left(pv(\Pr_p[s|x_w]) + (1 - p)v(\Pr_p[s|x_l]) \right).$$
 (3)

For $p < \frac{1}{2}$ the risk premium is therefore smaller for the low probability lottery $(p, x_w; 1 - p, x_l)$ than for the high probability lottery $(1 - p, x_w; p, x_l)$ if

$$pv(\Pr_{p}[s|x_{w}]) + (1-p)v(\Pr_{p}[s|x_{l}]) \ge (1-p)v(\Pr_{1-p}[s|x_{w}]) + pv(\Pr_{1-p}[s|x_{l}]).$$
(4)

Since $\Pr_p[s|x_w] > \Pr_{1-p}[s|x_w] > \Pr_p[s|x_l] > \Pr_{1-p}[s|x_l]$ for $p < \frac{1}{2}$, the choice is between a gamble with a small chance of a very good outcome and a large chance of a moderately bad outcome, and a gamble with a small chance of a very bad outcome and a large chance of a moderately good outcome. The expected values and variances are equal, but the high probability gamble has greater "downside risk" (Menezes, Geiss, and Tressler, 1980) due to the chance of a very bad outcome. Decreasing absolute risk aversion, which v''' > 0 is a necessary condition for, suggests that the decision maker is more risk averse at low values of $\Pr[s]$, so the downside risk is particularly important.

The effect of downside risk on expected utility is seen in Figure 3 for a constant relative risk aversion function $v(\Pr[s]) = \Pr[s]^{1-a}/(1-a)$ with the CRRA parameter set at a = 5.27 Skill estimates from winning and losing for a fair $p = \frac{1}{4}$ lottery (flatter line) and a fair $p = \frac{3}{4}$ lottery (steeper line) are the same as in Figure

²⁷This value is chosen for graphical convenience. Estimates of relative risk aversion for wealth, which might be confounded by embarrassment concerns and other factors, vary widely.

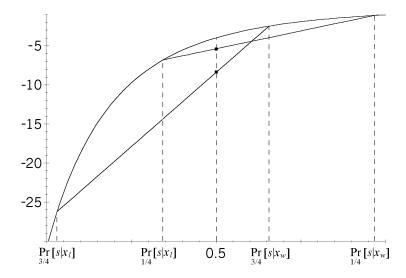


Figure 3: Performance Skill: Utility from high $(p = \frac{3}{4})$ and low $(p = \frac{1}{4})$ probability gambles

2. In each case estimated skill is $\frac{1}{2}$ on average, but the downside risk is clearly much higher for the $p = \frac{3}{4}$ gamble. Since the line for the $p = \frac{1}{4}$ gamble is higher at the average estimated skill of $\frac{1}{2}$, it offers higher utility. This would not be true if the utility function dipped less sharply for lower values of $\Pr[s]$, implying greater tolerance of risk. As the first part of the following proposition shows, our assumption of decreasing concavity of v implies the lower probability lottery is preferred. Constant relative risk aversion for a risk averse decision maker requires v''' > 0, so in this example the lower probability lottery is always preferred. The second part of the proposition shows that the risk premium is always positive for any p. This is also apparent from Figure 3.

Proposition 1 With performance skill when the decision maker is uninformed: (i) for all $p \in (0, \frac{1}{2})$, the low-probability gamble $(p, x_w; 1-p, x_l)$ has a smaller risk premium than the high probability gamble $(1-p, x_w; p, x_l)$; and (ii) for all $p \in (0, 1)$, the gamble $(p, x_w; 1-p, x_l)$ has a positive risk premium.

Proof: (i) Third-degree stochastic dominance (Whitmore, 1970) implies that, for v' > 0, v'' < 0, v''' > 0, a gamble with distribution F(z) on [0,1] is preferred to a gamble with distribution G(z) on [0,1] if

$$E_F(z) \ge E_G(z) \tag{5}$$

and

$$\int_0^x \int_0^y G(z) - F(z)dzdy \ge 0 \tag{6}$$

for all $x \in [0, 1]$. Let F(z) be the distribution of the estimated skill for the low probability lottery $(p, x_w; 1 - p, x_l)$ and G(z) be the distribution of estimated skill for the high probability lottery $(1 - p, x_w; p, x_l)$, so

$$F(z) = 0 for z < \Pr_p[s|x_l]$$

$$F(z) = 1 - p for \Pr_p[s|x_l] \le z < \Pr_p[s|x_w]$$

$$F(z) = 1 for z \ge \Pr_p[s|x_w]$$

$$(7)$$

and

$$G(z) = 0 \quad \text{for } z < \Pr_{1-p}[s|x_l]$$

$$G(z) = p \quad \text{for } \Pr_{1-p}[s|x_l] \le z < \Pr_{1-p}[s|x_w]$$

$$G(z) = 1 \quad \text{for } z \ge \Pr_{1-p}[s|x_w].$$
(8)

Therefore the distribution of skill estimates for the low probability gamble is preferred if (5) holds, or,

$$p\Pr_{p}[s|x_{w}] + (1-p)\Pr_{p}[s|x_{l}] \ge (1-p)\Pr_{1-p}[s|x_{w}] + p\Pr_{1-p}[s|x_{l}]$$
(9)

and (6) holds, or,

$$p\left(\Pr_{p}[s|x_{l}] - \Pr_{1-p}[s|x_{l}]\right) \ge 0,\tag{10}$$

$$2p\left(\Pr_{p}[s|x_{l}] - \Pr_{1-p}[s|x_{l}]\right) + p\left(\Pr_{1-p}[s|x_{w}] - \Pr_{p}[s|x_{l}]\right) \ge (1-p)\left(\Pr_{1-p}[s|x_{w}] - \Pr_{p}[s|x_{l}]\right),\tag{11}$$

and

$$3p\left(\Pr_{p}[s|x_{l}] - \Pr_{1-p}[s|x_{l}]\right) + 2p\left(\Pr_{1-p}[s|x_{w}] - \Pr_{p}[s|x_{l}]\right) + \Pr_{p}[s|x_{w}] - \Pr_{1-p}[s|x_{w}]$$

$$\geq 2(1-p)\left(\Pr_{1-p}[s|x_{w}] - \Pr_{p}[s|x_{l}]\right) + (1-p)\left(\Pr_{p}[s|x_{w}] - \Pr_{1-p}[s|x_{w}]\right). \tag{12}$$

Condition (9) holds with equality since the expected skill is $\frac{1}{2}$ in each case. Condition (10) reduces to $\frac{1}{2}\varepsilon\frac{1-2p}{1-p}\geq 0$, which holds strictly, and the last two conditions hold with equality.

(ii) Since
$$p \Pr_p[s|x_w] + (1-p) \Pr_p[s|x_l] = \frac{1}{2}$$
, the premium $\pi(p)$ in (3) is positive for $v'' < 0$.

The results on risk premia can be related to the probability weighting function by finding the different certainty equivalents c for different values of p in the gamble $(p, x_w; 1-p, x_l)$ and then inferring what probability w of winning would have induced a risk neutral decision maker unconcerned with skill signaling (or other factors) to choose that certainty equivalent.²⁸ Setting w such that $c(p) = wx_w + (1-w)x_l$, the weighting function is then

$$w(p) = \frac{c(p) - x_l}{x_w - x_l}. (13)$$

In our model with embarrassment aversion but without probability weighting (i.e. w(p) = p) the certainty equivalent is just $c(p) = px_w + (1-p)x_l - \pi(p)$. So if an experimenter estimates the probability weighting

²⁸Rather than asking for the certainty equivalent directly, the decision maker can be offered a range of different gambles and the probability weights can then be inferred from these choices. Asking for certainty equivalents forces, or attempts to force, the decision maker to reveal more information about her skill than is revealed by just accepting or rejecting a given gamble. And multiple gambles provide more information about skill than the single gamble analyzed here.

function assuming no concern for skill signaling, but the true utility function is $U = Y + v(\Pr[s])$ without probability weighting, the experimenter finds

$$w(p) = p - \frac{\pi(p)}{x_w - x_l}. (14)$$

Note that the experimenter will find overweighting if the risk premium is negative and underweighting if it is positive. Moreover, the amount of overweighting or underweighting is in direct proportion to the risk premium. The first part of Proposition 1 therefore implies that there will be disproportionate weight on low probabilities relative to high probabilities, which is consistent with prospect theory. However, from the second part of the proposition the risk premium is always positive so even low probabilities will still appear to be underweighted rather than overweighted as assumed by prospect theory. Therefore the four-fold pattern is satisfied weakly rather than strongly. The second part of the proposition also implies that for gambles with equal chances of winning or losing, $p = \frac{1}{2}$, the risk premium is positive. If an experimenter does not control for skill signaling, this behavior will appear as loss aversion.

We now consider the asymmetric information case where the decision maker knows whether she is skilled or unskilled. Choosing to gamble or not can then become a signal of the decision maker's private information about her type. The three pure strategy equilibria that can exist are a both-gamble pooling equilibrium, a neither-gamble pooling equilibrium, and a separating equilibrium where skilled decision makers gamble and unskilled decision makers do not. Note that if skill signaling is important the neither-gamble and separating equilibria are difficult to support.²⁹

Since the neither-gamble equilibrium provides no information on decision maker type, while the separating equilibrium provides complete information, the both-gamble equilibrium is most important for understanding the relationship between skill signaling and probability weighting. In the both-gamble equilibrium the observer believes that both types gamble and, given these beliefs, both types choose to gamble. Therefore if the gamble is taken the expected skill of a decision maker conditioning on winning or losing for a gamble with p average chance of winning is still $\Pr_p[s|x_w]$ and $\Pr_p[s|x_l]$ as already defined. If, however, a decision maker deviates from the equilibrium path and refuses to gamble there is no clear implication for what exactly the observer should believe. Let μ_r represent the observer's belief regarding the probability that a person who refuses a gamble is skilled. The first part of the following proposition shows that for a given μ_r , the average risk premium for skilled and unskilled decision makers is smaller for low probability gambles, implying weak satisfaction of the four-fold property.^{30,31} The second part of the proposition addresses when the four-fold property might

²⁹In the neither-gamble equilibrium the skilled type always has stronger incentives to deviate than the unskilled type, so the equilibrium will not survive strong refinements such as D1 (Banks and Sobel, 1987) if skill signaling is an important factor for the decision maker. Similarly, in the separating equilibrium the unskilled type is admitting she is unskilled, so if the expected monetary loss from gambling is not too large she will deviate and mimic the skilled type.

³⁰We consider the average premium here but the risk premium for unskilled gamblers is also of interest. Since unskilled gamblers are the first to deviate from the both-gamble equilibrium, the risk premium for an unskilled gambler defines how generous the gamble has to be in order to sustain the equilibrium. Following the same steps as for the average premium it can be shown that the risk premium necessary to sustain the both-gamble equilibrium is smaller for a low probability gamble than a high probability gamble.

 $^{^{31}}$ We hold μ_r constant for high and low probability gambles to make the comparison. In practice observers may condition their beliefs on who gambles based on the odds of the gamble.

be satisfied strongly under reasonable beliefs. Since skilled decision makers are less likely to lose, divinity (Cho and Kreps, 1987) makes the understandable requirement that $\mu_r < \frac{1}{2}$, so the question is whether such beliefs can lead to negative risk premia for low probability gambles and positive risk premia gambles for high probability gambles. The answer is that such beliefs always exist. The third part of the proposition addresses loss aversion by considering gambles with an equal chance of winning or losing. Beliefs satisfying divinity which induce either positive or negative risk premia are shown to exist. Therefore either loss aversion or dare taking might result.

Proposition 2 With performance skill when the decision maker is informed, in the both-gamble equilibrium: (i) for all $p \in (0, \frac{1}{2})$ and $\mu_r \in (0, 1)$ the average risk premium is smaller for the low-probability gamble $(p, x_w; 1 - p, x_l)$ than for the high-probability gamble $(1 - p, x_w; p, x_l)$; (ii) for any $p \in (0, \frac{1}{2})$ there exists $\mu_r \in (0, \frac{1}{2})$ such that the average risk premium is negative for the low-probability gamble $(p, x_w; 1 - p, x_l)$ and positive for the high-probability gamble $(1 - p, x_w; p, x_l)$; (iii) there exists $\mu \in (0, \frac{1}{2})$ such that the average risk premium for the gamble $(\frac{1}{2}, x_w; \frac{1}{2}, x_l)$ is negative for $\mu_r \in (0, \mu)$ and positive for $\mu_r \in (\mu, 1)$.

Proof: (i) The risk premia for skilled and unskilled types are respectively,

$$\pi_s(p) = v(\mu_r) - \left(\Pr_p[x_w|s]v(\Pr_p[s|x_w]) + (1 - \Pr_p[x_w|s])v(\Pr_p[s|x_l])\right)$$
(15)

and

$$\pi_u(p) = v(\mu_r) - \left(\Pr_p[x_w|u]v(\Pr_p[s|x_w]) + (1 - \Pr_p[x_w|u])v(\Pr_p[s|x_l])\right). \tag{16}$$

Since $p_s + p_u = p$ the average risk premium is

$$\overline{\pi}(p) = \frac{\pi_s(p) + \pi_u(p)}{2}$$

$$= v(\mu_r) - \left(pv(\Pr_p[s|x_w]) + (1-p)v(\Pr_p[s|x_l])\right), \tag{17}$$

which is smaller for low p gambles than high p gambles if

$$pv(\Pr_{p}[s|x_{w}]) + (1-p)v(\Pr_{p}[s|x_{l}]) \ge (1-p)v(\Pr_{1-p}[s|x_{w}]) + pv(\Pr_{1-p}[s|x_{l}])$$
(18)

for $p \in (0, \frac{1}{2})$. This holds as shown in Proposition 1-i.

(ii) We are interested in $\mu_r \in (0, \frac{1}{2})$ such that for $p \in (0, \frac{1}{2})$, $\overline{\pi}(p) \le 0 \le \overline{\pi}(1-p)$ or, from (17),

$$pv(\Pr_{p}[s|x_{w}]) + (1-p)v(\Pr_{p}[s|x_{l}]) \ge v(\mu_{r}) \ge (1-p)v(\Pr_{1-p}[s|x_{w}]) + pv(\Pr_{1-p}[s|x_{l}]).$$
(19)

From (18) and the fact that $\pi(p) \leq \frac{1}{2}$, $\Pr_p[s|x_w] > 0$, and $\Pr_p[s|x_l] > 0$ for $p \in (0,1)$, such a μ_r exists since v' > 0.

(iii) From (17) and the fact that $0 < \Pr_{\frac{1}{2}}[s|x_w] < 1$ and $0 < \Pr_{\frac{1}{2}}[s|x_l] < 1$, such a μ exists since v' > 0. \blacksquare Since behavior depends on observer beliefs, multiple equilibria can result. If observers expect some groups to try to prove their skill levels and others not to, different behavior can result which confirms the beliefs.

For instance, it is documented that men take riskier investments than women do (Jianakopolis and Bernasek, 1998) and invest as if they are overconfident (Barber and Odean, 2001). Rather than the result of underlying differences in risk attitudes or confidence between men and women, this pattern might arise from different equilibrium beliefs among observers about how members of each group try to prove their abilities.

The existence of multiple equilibria also allows room for framing to affect decisions. In particular, if the framing of the question leads the decision maker to expect the receiver to have a negative impression of those who do not gamble, then risk premia will be low or even negative. This can result in the strong rather than weak four-fold pattern, and it can result in dare taking rather than loss aversion. For instance, children and teenagers often take risky actions as a dare to prove that they are not afraid. Even if they are not skilled at the risky activity they may prefer to take a chance than to confirm their inadequacy through refusing the dare.³²

3 Evaluation Skill

We now consider a complementary model where skilled decision makers are better at correctly inferring the odds of a gamble than unskilled decision makers. A gamble $(p, x_w; 1-p, x_l)$ offers x_w with average probability p and x_l with average probability 1-p where $x_w > x_l$ and $p \in (0,1)$. Again the decision maker's type is skilled s or unskilled u with equal probability. If a lottery is stated to have probability p of success then with equal chance the actual probability is either p_h or p_l where $p_h > p > p_l$. The decision maker knows the stated probability p and receives a signal $p \in \{H, L\}$ of the actual probability where $Pr[H|p_h] = Pr[L|p_l] = \alpha$ for skilled decision makers and $Pr[H|p_h] = Pr[L|p_l] = \beta$ for unskilled decision makers and $\frac{1}{2} \leq \beta < \alpha \leq 1$. We will again consider symmetric differences where $p_h = p + \varepsilon$ and $p_l = p - \varepsilon$ for some $\varepsilon \in (0, \min\{p, 1 - p\})$ and where, for all p, ε is the same for the gamble $(p, x_w; 1 - p, x_l)$ as it is for the gamble $(1 - p, x_w; p, x_l)$. As before, the utility function is U = Y + v(Pr[s]) where v' > 0, v'' < 0, v''' > 0.

For evaluation skill, we will restrict our attention to the case where the decision maker does not know whether she is skilled or unskilled.³³ Instead we will consider two different cases regarding observability of outcomes. In the first case the observer only learns the outcome if the observer takes the gamble, and in the second case the observer learns the outcome regardless.

Beginning with the first case, note that if the decision maker does not condition her choice to gamble on the signal θ , the observer learns nothing. So we restrict our attention to the separating equilibrium where a decision maker gambles if she receives the signal H and refuses to gamble if she receives the signal L. The

 $^{^{32}}$ In some cases they may also want to signal that they have low risk aversion regardless of their skill.

³³The results are not significantly affected if the decision maker knows her skill. Whether the decision maker has private information about her skill is very important for performance skill because unskilled types face pressure to mimic skilled types or be thought of as unskilled. With evaluation skill the distinction is less important because skilled decision makers do not face better odds for a given gamble, but are better able to evaluate a given gamble. Since skilled decision makers decide whether to take the gamble based on their private information, unskilled decision makers are not in a position to mimic them.

probability of winning with an H signal is

$$\Pr_{p}[x_{w}|H] = \Pr_{p}[s, x_{w}|H] + \Pr_{p}[u, x_{w}|H]$$

$$= \frac{1}{2}(\alpha p_{h} + (1 - \alpha)p_{l} + \beta p_{h} + (1 - \beta)p_{l})$$

$$= p + (\alpha + \beta - 1)\varepsilon$$
(20)

and similarly the probability of winning with an L signal is

$$\Pr_{p}[x_{w}|L] = 1 - p - (\alpha + \beta - 1)\varepsilon. \tag{21}$$

Believing that only decision makers with an H signal gamble, the observer estimates the decision maker's skill as

$$\Pr_{p}[s|x_{w}, H] = \frac{\Pr[s, x_{w}|H]}{\Pr[x_{w}|H]}$$

$$= \frac{1}{2} + \frac{\alpha - \beta}{2(p + (\alpha + \beta - 1)\varepsilon)}$$
(22)

if the gamble is successful and, similarly,

$$\Pr_{p}[s|x_{l}, H] = \frac{1}{2} - \frac{\alpha - \beta}{2(1 - p - (\alpha + \beta - 1)\varepsilon)}$$
(23)

if it is not.

Figure 4 shows observer skill estimates conditional on winning at the top and losing on the bottom, with the average estimate $\Pr_p[x_w|H] \Pr_p[s|x_w,H] + \Pr_p[x_l|H] \Pr_p[s|x_l,H] = \frac{1}{2}$ in the middle, when $\varepsilon = p(1-p)/2$ as before and $\alpha = 1$, $\beta = \frac{1}{2}$. As with performance skill, the downside risk under evaluation skill is clearly increasing in p. Note that the problem is even more severe in this case because the probability of being skilled conditional on losing drops sharply for large p. As p increases there is both higher downside risk, as represented by the downward shift of the two lines, and higher regular risk, as represented by the greater spread of the lines. The spread rises with p because informed decision makers only take the gamble when $\theta = H$ so the actual odds of the gamble winning conditional on it being taken are, on average, slightly better than p. This accentuates the likelihood that a decision maker who takes the gamble and loses is unskilled, and this effect is strongest for large p for the same reasons that losing in general is more embarrassing for large p.

Because of both the increased downside risk and the increased regular risk, gambling is particularly unattractive for high probability gambles. The first part of the following proposition shows that skill signaling implies lower risk premia on low probability gambles than on high probability gambles.³⁴ The second

 $^{^{34}}$ Note that this proof and the proof of the following proposition do not depend on the presence of regular risk as represented by the increasing spread between estimated skill for winning and for losing. Because of this spread, the risk premium will be strictly smaller for low p gambles even when v''' = 0.

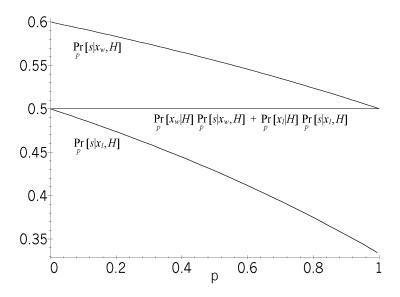


Figure 4: Evaluation Skill: Estimated skill from winning (top) and losing (bottom)

part of the proposition shows that the risk premia are always positive. Therefore the four-fold pattern is satisfied weakly rather than strongly and there is loss aversion rather than dare taking.

Proposition 3 With evaluation skill when the outcome is observed only if the gamble is taken, in the separating equilibrium: (i) for all $p \in (0, \frac{1}{2})$ the low-probability gamble $(p, x_w; 1 - p, x_l)$ has a smaller average risk premium than the high-probability gamble $(1 - p, x_w; p, x_l)$; (ii) for all $p \in (0, 1)$ the gamble $(p, x_w; 1 - p, x_l)$ has a positive average risk premium.

Proof: (i) First consider the case where the decision maker does not know her skill. The risk premium for a decision maker with a θ signal is

$$\pi_{\theta}(p) = v(\frac{1}{2}) - \left(\Pr_{p}[x_w|\theta]v(\Pr_{p}[s|x_w, H]) + \Pr_{p}[x_l|\theta]v(\Pr_{p}[s|x_l, H])\right). \tag{24}$$

Since $\Pr_p[x_w|H] + \Pr_p[x_w|L] = 2p$ and $\Pr_p[x_l|H] + \Pr_p[x_l|L] = 2(1-p)$, the average risk premium is

$$\frac{\pi_H(p) + \pi_L(p)}{2} = v(\frac{1}{2}) - \left(pv(\Pr_p[s|x_w, H]) + (1 - p)v(\Pr_p[s|x_l, H])\right). \tag{25}$$

So the low probability lottery has a lower average risk premium if

$$pv(\Pr_{p}[s|x_{w}, H]) + (1-p)v(\Pr_{p}[s|x_{l}, H])$$

$$\geq (1-p)v(\Pr_{1-p}[s|x_{w}, H]) + pv(\Pr_{1-p}[s|x_{l}, H]).$$
(26)

Following the same third-degree stochastic dominance argument as in Proposition 1, let F(z) be the distribution of estimated skill for the low probability lottery $(p, x_w; 1 - p, x_l)$ and G(z) be the distribution for the high probability lottery $(1 - p, x_w; p, x_l)$, so

$$F(z) = 0 \qquad \text{for } z < \Pr_p[s|x_l, H]$$

$$F(z) = 1 - p \quad \text{for } \Pr_p[s|x_l, H] \le z < \Pr_p[s|x_w, H]$$

$$F(z) = 1 \qquad \text{for } z \ge \Pr_p[s|x_w, H]$$

$$(27)$$

and

$$G(z) = 0 \quad \text{for } z < \Pr_{1-p}[s|x_l, H]$$

$$G(z) = p \quad \text{for } \Pr_{1-p}[s|x_l, H] \le z < \Pr_{1-p}[s|x_w, H]$$

$$G(z) = 1 \quad \text{for } z \ge \Pr_{1-p}[s|x_w, H].$$
(28)

First note that

$$p\Pr_{p}[s|x_{w}, H] + (1-p)\Pr_{p}[s|x_{l}, H] = (1-p)\Pr_{1-p}[s|x_{w}, H] + p\Pr_{1-p}[s|x_{l}, H]$$
(29)

so the two gambles offer the same expected skill. Therefore the inequality in (26) holds if

$$p\left(\Pr_{p}[s|x_{l},H] - \Pr_{1-p}[s|x_{l},H]\right) \ge 0,\tag{30}$$

$$2p\left(\Pr_{p}[s|x_{l}, H] - \Pr_{1-p}[s|x_{l}, H]\right) + p\left(\Pr_{1-p}[s|x_{w}, H] - \Pr_{p}[s|x_{l}, H]\right) \ge (1-p)\left(\Pr_{1-p}[s|x_{w}, H] - \Pr_{p}[s|x_{l}, H]\right), \tag{31}$$

$$3p\left(\Pr_{p}[s|x_{l},H] - \Pr_{1-p}[s|x_{l},H]\right) + 2p\left(\Pr_{1-p}[s|x_{w},H] - \Pr_{p}[s|x_{l},H]\right) + \Pr_{p}[s|x_{w},H] - \Pr_{1-p}[s|x_{w},H] \ge 2(1-p) \quad \left(\Pr_{1-p}[s|x_{w},H] - \Pr_{p}[s|x_{l},H]\right) + (1-p)\left(\Pr_{p}[s|x_{w},H] - \Pr_{1-p}[s|x_{w},H]\right). \tag{32}$$

Substituting, these three conditions simplify to

$$\frac{1}{2}p\varepsilon\frac{(\alpha-\beta)(1-2p)}{(1-p-(\alpha-\beta-1)\varepsilon)(p-(\alpha-\beta-1)\varepsilon)} \ge 0,$$
(33)

$$\varepsilon^{2} \frac{(\alpha - \beta)(1 - 2p)(\alpha + \beta - 1)}{(1 - p - (\alpha - \beta - 1)\varepsilon)(p - (\alpha - \beta - 1)\varepsilon)(1 - p + (\alpha - \beta - 1)\varepsilon)} \ge 0,$$
(34)

$$\varepsilon^{2} \frac{(\alpha - \beta)(1 - 2p)(\alpha + \beta - 1)(p + 2(\alpha + \beta - 1)\varepsilon)}{(1 - p - (\alpha - \beta - 1)\varepsilon)(p - (\alpha - \beta - 1)\varepsilon)(1 - p + (\alpha - \beta - 1)\varepsilon)(p + (\alpha - \beta - 1)\varepsilon)} \ge 0, \tag{35}$$

all of which hold strictly for $p < \frac{1}{2}$.

(ii) From (25) and the fact that $\Pr_p[x_w|\theta] \Pr_p[s|x_w, H] + \Pr_p[x_l|\theta] \Pr_p[s|x_l, H] = \frac{1}{2}$, this follows from v'' < 0.

We now broaden our analysis to consider the case where the outcome of a gamble is observed even when the gamble is refused. This relates to the idea of regret theory that not taking a gamble that wins is as frustrating as taking a gamble that loses. But from a skill signaling perspective, the key is not whether the decision maker learns the outcome but whether the observer learns the outcome. For performance skill such observation is irrelevant because skill is idiosyncratic so nothing can be learned about the decision maker's skill. But if evaluation skill is present, the success of a refused gamble reflects poorly on the decision maker, and the failure of a refused gamble reflects well.

For the same reasons as before we are interested in the separating equilibrium where a decision maker with an H signal gambles and a decision maker with an L signal does not. If the gamble is taken $\Pr_p[s|x_w, H]$ and $\Pr_p[s|x_l, H]$ are the same as already defined. If the gamble is refused the observer believes the decision maker has an L signal and therefore estimates the decision maker's skill as

$$\Pr_{p}[s|x_{w}, L] = \frac{\Pr_{p}[s, x_{w}|L]}{\Pr_{p}[x_{w}|L]}$$

$$= \frac{(1-\alpha)p_{h} + \alpha p_{l}}{(1-\alpha)p_{h} + \alpha p_{l} + (1-\beta)p_{h} + \beta p_{l}}$$

$$= \frac{1}{2} - \frac{\alpha - \beta}{2(p - (\alpha + \beta - 1)\varepsilon)}$$
(36)

if the gamble is successful and as

$$\Pr_{p}[s|x_{l},L] = \frac{1}{2} + \frac{\alpha - \beta}{2(1-p+(\alpha+\beta-1)\varepsilon)}$$
(37)

if it is not.

Note that the skill estimate from not taking a gamble $(p, x_w; 1-p, x_l)$ that wins is the same as the estimate from taking a gamble $(1-p, x_w; p, x_l)$ that loses, $\Pr_p[s|x_w, L] = \Pr_{1-p}[s|x_l, H]$, and that the skill estimate from taking a gamble $(p, x_w; 1-p, x_l)$ that wins is the same as the estimate from not taking a gamble $(1-p, x_w; p, x_l)$ that loses, $\Pr_p[s|x_w, H] = \Pr_{1-p}[s|x_l, L]$. Therefore the skill estimates from gambling are the same as in Figure 4, while the skill estimates from not gambling are the mirror image reflected around $p = \frac{1}{2}$. The expected value of taking the gamble or refusing it is the same, but the downside risk is much higher for refusing the gamble for $p < \frac{1}{2}$ and for taking the gamble for $p > \frac{1}{2}$. Note that $\Pr_p[s|x_w, H] - \Pr_p[s|x_l, H] < \Pr_p[s|x_l, L] - \Pr_p[s|x_w, L]$ for $p < \frac{1}{2}$ and the opposite for $p > \frac{1}{2}$, so there is also greater regular risk from refusing to gamble for low p gambles and from taking the gamble for high p gambles. As was the case in Figure 4, this asymmetry arises because informed decision makers take the gamble when $\theta = H$ and refuse it when $\theta = L$, implying the actual odds of the gamble conditional on it being taken are better than the odds conditional on it being refused. To take the gamble for high p and lose is therefore particularly embarrassing. And to refuse the gamble for low p and have the gamble win is similarly embarrassing because the decision maker probably refused the gamble because of an inaccurate signal.

Because of these three factors — the inability to escape an unfavorable skill estimate by refusing to gamble, the downside risk, and the regular risk — gambling is particularly attractive for low p gambles and unattractive

for high p gambles. The first part of the following proposition shows that skill signaling implies behavior that is observationally equivalent to overweighting of small probabilities and underweighting of large probabilities. The second part shows that neither loss aversion nor dare taking are relevant for gambles with an equal chance of success or failure.

Proposition 4 With evaluation skill when the outcome is observed even when the gamble is refused, in the separating equilibrium the average risk premium of a gamble $(p, x_w; 1-p, x_l)$ is (i) negative for $p < \frac{1}{2}$, positive for $p > \frac{1}{2}$, and (ii) zero for $p = \frac{1}{2}$.

Proof: (i) Decision maker does not know her skill. In the separating equilibrium the risk premium for a decision maker with signal θ is

$$\pi_{\theta}(p) = \left(\Pr_{p}[x_{w}|\theta]v(\Pr_{p}[s|x_{w},L]) + \Pr_{p}[x_{l}|\theta]v(\Pr_{p}[s|x_{l},L])\right) - \left(\Pr_{p}[x_{w}|\theta]v(\Pr_{p}[s|x_{w},H]) + \Pr_{p}[x_{l}|\theta]v(\Pr_{p}[s|x_{l},H])\right).$$
(38)

Since $\Pr_p[x_w|H] + \Pr_p[x_w|L] = 2p$ and $\Pr_p[x_l|H] + \Pr_p[x_l|L] = 2(1-p)$, and since $\Pr_p[s|x_w,L] = \Pr_{1-p}[s|x_l,H]$ and $\Pr_p[s|x_l,L] = \Pr_{1-p}[s|x_w,H]$, the average risk premium is

$$\frac{\pi_{H}(p) + \pi_{L}(p)}{2} = \left(pv(\Pr_{p}[s|x_{w}, L]) + (1 - p)v(\Pr_{p}[s|x_{l}, L]) \right) - \left(pv(\Pr_{p}[s|x_{w}, H]) + (1 - p)v(\Pr_{p}[s|x_{l}, H]) \right) \\
= \left(pv(\Pr_{p}[s|x_{l}, H]) + (1 - p)v(\Pr_{p}[s|x_{w}, H]) \right) - \left(pv(\Pr_{p}[s|x_{w}, H]) + (1 - p)v(\Pr_{p}[s|x_{l}, H]) \right) \\
= \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{w}, H]) \right) - \left(pv(\Pr_{p}[s|x_{w}, H]) + (1 - p)v(\Pr_{p}[s|x_{l}, H]) \right) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p}[s|x_{l}, H]) \right) + \left(pv(\Pr_{l-p}[s|x_{l}, H]) + (1 - p)v(\Pr_{l-p$$

From Proposition 3 this is negative for $p < \frac{1}{2}$. The problem is symmetric so the average risk premium is positive for $p > \frac{1}{2}$.

(ii) Follows directly from from (39). \blacksquare

Since the risk premium is, on average for H and L types, negative for $p \in (0, \frac{1}{2})$ and positive for $p \in (\frac{1}{2}, 1)$, from a probability weighting function perspective there is overweighting for $p \in (0, \frac{1}{2})$ and underweighting for $p \in (\frac{1}{2}, 1)$. Figure 5 shows the estimated weighting functions $w_{\theta}(p) = p - \pi_{\theta}(p)/(x_w - x_l)$ for types with an H signal (top) and L signal (bottom) for a (p, 10; 1 - p, 0) gamble where, as in previous examples, $v(x) = x^{1-a}/(1-a)$, the CRRA parameter a = 5, $\varepsilon = p(1-p)/2$ and $\alpha = 1$, $\beta = \frac{1}{2}$. For this case of evaluation skill where the observer learns the outcome of a refused gamble, omission of skill signaling from the estimated model leads to an estimated probability weighting function that closely follows that of prospect theory.

Regret theory also generates an estimated probability weighting function that is similar to prospect theory's. The distinguishing prediction of regret theory is that a decision maker will pay to avoid learning the outcome of a refused gamble (Bell, 1983). We find a similar result for skill signaling, except the decision maker prefers to keep the observer rather than herself in the dark. Consider the case where the decision maker does not know her skill. In the separating equilibrium if the decision maker does not gamble and the observer learns nothing, the decision maker receives utility $v(\frac{1}{2})$. But if the observer learns the outcome, a decision maker with signal θ receives utility $\Pr[x_w|\theta]v(\Pr[s|x_w,L]) + \Pr[x_l|\theta]v(\Pr[s|x_l,L])$. Since $\Pr[x_w|\theta] \Pr[s|x_w,L] + \Pr_p[x_l|\theta] \Pr_p[s|x_l,L] = \frac{1}{2}$, the no-observation case is preferred for v'' < 0.

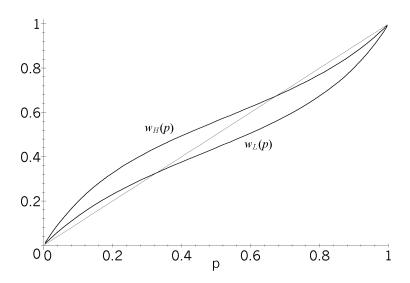


Figure 5: Evaluation Skill: Estimated probability weighting function given H and L signals

3.1 Extension: observer does not know if gamble is high or low probability

So far we have considered relatively straightforward information structures. Clearly more complicated situations are possible that are relevant for a more general understanding of skill signaling. For instance, as suggested in the discussion of regret theory above, the decision maker might be able to choose whether to report that a gamble is being taken. And it might be possible to not report unsuccessful gambles ex post. For instance, people often choose whether to inform friends and associates of gambling or stock market outcomes. If outcomes are self-reported then the absence of a report may be interpreted as a sign of failure, thereby giving an extra incentive to take risky actions. The presence of multiple gambles also changes the information structure substantially. Multiple gambles are important in the fallacy of large numbers (Samuelson, 1963), the house-money effect (Thaler and Johnson, 1990) and the disposition effect (Shefrin and Statman, 1995).

Of possible alternative information structures, of particular relevance to our analysis is the case where the only difference from the model so far is that the observer does not know the actual odds of the gamble. For instance a manager undertakes a project that the manager knows the difficulty of, but the manager's boss is in the dark. This problem has already been analyzed by Ross (1977) and Holmstrom (1982/1999), but it provides a simple and straightforward way to reverse most of the predictions of skill signaling so we revisit the problem here in order to make a clear comparison with our other results.

We consider the simplest situation where there are two possible gambles with equal chance, a low probability gamble $(p, x_w; 1-p, x_l)$ and a corresponding high probability gamble $(1-p, x_w; p, x_l)$ where $p \in (0, \frac{1}{2})$, and the observer does not know which gamble the decision maker faces. First consider performance skill. We examine

the case where the decision maker does not know her true skill level.³⁵ Even in this case multiple equilibria can arise since the odds of the gamble are private information held by the decision maker. For the sake of brevity, we will limit our attention to the equilibrium where both gambles are taken. Proofs for the separating equilibria and no-gamble equilibria are comparable. For both gambles to be taken, the risk premium for each must be sufficient to induce the decision maker to overcome their embarrassment aversion. We are interested in exactly how large these premia must be. Assuming that the observer expects both gambles to be taken,

$$\Pr[s|x_{w}] = \frac{\Pr[s, x_{w}]}{\Pr[s, x_{w}] + \Pr[u, x_{w}]} \\
= \frac{\frac{1}{2} \Pr_{p}[x_{w}|s] \Pr[s] + \frac{1}{2} \Pr_{1-p}[x_{w}|s] \Pr[s]}{\frac{1}{2} \Pr_{p}[x_{w}|s] \Pr[s] + \frac{1}{2} \Pr_{p}[x_{w}|u] \Pr[u] + \frac{1}{2} \Pr_{1-p}[x_{w}|u] \Pr[u]} \\
= \frac{1}{2} + \frac{\varepsilon}{4p(1-p)} \tag{40}$$

and similarly,

$$\Pr[s|x_l] = \frac{1}{2} - \frac{\varepsilon}{4p(1-p)}.\tag{41}$$

Since, conditional on the outcome, estimated skill is not affected by the gamble's probability of success clearly the low probability gamble has a higher risk premium. This is confirmed in the following proposition.

Proposition 5 With performance skill in the both-gamble equilibrium, for $p \in (0, \frac{1}{2})$ the low probability gamble $(p, x_w; 1-p, x_l)$ has a larger risk premium than the high probability gamble $(1-p, x_w; p, x_l)$ when the observer cannot distinguish between the gambles.

Proof: Since the decision maker does not know her own type and cannot condition her action on her skill, $\mu_r = \frac{1}{2}$. Therefore the risk premium is

$$\pi(p) = v(\frac{1}{2}) - (pv(\Pr[s|x_w]) + (1 - p)v(\Pr[s|x_l])). \tag{42}$$

Comparing,

$$\pi(p) - \pi(1-p) = ((1-p)v(\Pr[s|x_w]) + pv(\Pr[s|x_l])) - (pv(\Pr[s|x_w]) + (1-p)v(\Pr[s|x_l]))$$

$$= (1-2p)(v(\Pr[s|x_w]) - v(\Pr[s|x_l])) > 0$$
(43)

where the inequality follows since $p < \frac{1}{2}$, $\Pr[s|x_w] > \Pr[s|x_l]$, and v' > 0.

We now consider evaluation skill. To permit a more direct comparison with the results when p is known we will restrict attention to the simplest separating equilibrium where all those with a high signal gamble and all those with a low signal do not.³⁶ And for the same reasons as before, we also restrict attention to the case

³⁵If the decision maker knows her skill there is asymmetric information in two dimensions – the true odds of the gamble and the decision maker's skill – so the analysis is more complicated.

³⁶When the observer does not know the odds there are other separating equilibria of interest, such as the case where only those facing high odds gamble with a high signal, or only those facing low odds do not gamble with a low signal. The result is the same and the proofs are similar.

where the decision maker does not know her own skill. The probability of success given a high signal is

$$\Pr[x_{w}|H] = \Pr[s, x_{w}|H] + \Pr[u, x_{w}|H]
= \frac{1}{2} \Pr[s, x_{w}|H] + \frac{1}{2} \Pr[s, x_{w}|H] + \frac{1}{2} \Pr[u, x_{w}|H] + \frac{1}{2} \Pr[u, x_{w}|H]
= \frac{1}{2} + (\alpha + \beta - 1)\varepsilon$$
(44)

so the probability that a decision maker who took the gamble and won is skilled is

$$\Pr[s|x_w, H] = \frac{\Pr[s, x_w|H]}{\Pr[x_w|H]}$$

$$= \frac{\frac{1}{2}\Pr[s, x_w|H] + \frac{1}{2}\Pr_{1-p}[s, x_w|H]}{\Pr[x_w|H]}$$

$$= \frac{1}{2} + \frac{\alpha - \beta}{1 + 2(\alpha + \beta - 1)\varepsilon}$$
(45)

and similarly the probability that a decision maker who took the gamble and lost is skilled is,

$$\Pr[s|x_l, H] = \frac{1}{2} - \frac{\alpha - \beta}{1 + 2(\alpha + \beta - 1)\varepsilon}.$$
(46)

The following proposition confirms that higher probability gambles are favored.

Proposition 6 With evaluation skill in the separating equilibrium, for $p \in (0, \frac{1}{2})$ the low probability gamble $(p, x_w; 1-p, x_l)$ has a larger risk premium than the high probability gamble $(1-p, x_w; p, x_l)$ when the observer cannot distinguish between the gambles.

The decision maker does not know her own type, so $\mu_r = \frac{1}{2}$, and the risk premium is

$$\pi_{\theta}(p) = v(\frac{1}{2}) - \left(\Pr_{p}[x_{w}|\theta]v(\Pr[s|x_{w},H]) + (1 - \Pr_{p}[x_{w}|\theta])v(\Pr[s|x_{l},H])\right). \tag{47}$$

so

$$\pi_{\theta}(p) - \pi_{\theta}(1 - p) = \left(\Pr_{p}[x_{w}|\theta]v(\Pr[s|x_{w}, H]) + (1 - \Pr_{p}[x_{w}|\theta])v(\Pr[s|x_{l}, H]) \right) - \left(\Pr_{1-p}[x_{w}|\theta]v(\Pr[s|x_{w}, H]) + (1 - \Pr_{1-p}[x_{w}|\theta])v(\Pr[s|x_{l}, H]) \right)$$

$$= \left(\Pr_{1-p}[x_{w}|\theta] - \Pr_{p}[x_{w}|\theta] \right) \left(v(\Pr[s|x_{w}, H]) - v(\Pr[s|x_{l}, H]) \right) > 0$$
(48)

where the inequality follows since $\Pr_{1-p}[x_w|\theta] > \Pr_p[x_w|\theta]$ for $p < \frac{1}{2}$ and since $\Pr[s|x_w] > \Pr[s|x_l]$ and v' > 0.

4 Conclusion

Simple economic models are often poor predictors of human behavior. One approach to this failing is to investigate whether perceptual or cognitive biases lead to predictable deviations from rational behavior. A

different approach is to consider that many economic models are too simple to capture important strategic and information effects. In recent decades economists have developed richer models that show how a wide range of seemingly irrational behaviors may in fact be quite reasonable. For instance, the signaling literature shows how wasteful displays can paradoxically be individually rational, the folk theorem and reputation literatures show that cooperative behavior does not require altruism, the career concerns literature shows that people are often right to ignore the advice of economists and care about sunk costs, the information cascades literature shows how inefficient herding can arise, and the real options literature shows that refusal to discount the future according to standard economic formulas is wise under realistic information conditions. This paper shows that many of the seemingly irrational behaviors identified with prospect theory may be amenable to similar reanalysis. In particular, standard models from the career concerns literature appear to offer considerable insight into the key anomalies of probability weighting, loss aversion, and framing.

Determining whether decision makers are truly biased in their decisions or just act strategically to present a favorable impression of their skill is important theoretically and is also of practical interest to the many areas such as financial markets, contracting, political science, and the law where prospect theory has been applied. For instance, a number of papers have found that probability weighting and other predictions of prospect theory can help explain manager and investor behavior (see Shiller, 1999, and Mullainath and Thaler, 2000, for surveys). An implication of this literature is that principal-agent contracts should be adjusted to reflect predictable, biased choices by the agent. Our analysis of skill signaling supports the more standard career concerns perspective that contracts should instead be designed in full awareness of agent incentives to strategically choose their actions so as to manipulate performance information.

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