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# ENDOGENOUS DEPOPULATION AND ECONOMIC GROWTH\*

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## ABSTRACT

*Fertility rates have declined dramatically across almost all high-income countries over the past decades. This has raised concerns about future economic prospects. Indeed, fully- and semi-endogenous growth models imply that a shrinking workforce would lead to declining income growth and perhaps even stagnation. We extend the previous analyses to explicitly incorporate an endogenous quantity/quality trade-off between fertility and human capital accumulation. This allows us to assess the extent to which a declining number of workers can be compensated by increasing education. Our analysis demonstrates that economic growth needs not necessarily to decline with a falling population. Under certain conditions, human capital investment can sustain technological progress and economic growth despite the demographic challenges we are facing.*

**JEL Classification:** J11, J13, O33, O41, I25

**Keywords:** Demographic Change; Fertility Decline; Economic Growth; Research and Development; Endogenous Fertility; Endogenous Education; Human Capital Accumulation.

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## 1. Introduction

Fertility rates have declined in almost all high-income countries in recent decades. Based on data from Bloom *et al.* (2024) and The United Nations (2022), Table 1 displays the total fertility rate (TFR)<sup>1</sup> for the year 1960 and for the year 2021 among OECD countries. We observe that the TFR was above the replacement rate of 2.1 –which is the rate that leads to a stable population in the long run– in most of the countries in 1960. Some countries such as Japan and Hungary exhibited slightly lower TFRs back then, but only marginally so. By contrast, in the year 2021, all OECD countries with the exception of Israel experienced TFRs below the replacement rate, with South Korea’s value being even as low as 0.88. Overall, this pattern implies that OECD countries will face population decline in the absence of high net immigration as a counterbalancing factor over the coming century. In some countries, population growth has already turned negative, such as in Eastern Europe and in Japan.

COUNTRY	TFR IN 1960	TFR IN 2021
Australia	3.45	1.60
Austria	2.70	1.47
Belgium	2.54	1.58
Canada	3.90	1.46
Chile	4.70	1.54
Colombia	6.74	1.72
Costa Rica	6.71	1.53
Czech Republic	2.10	1.70
Denmark	2.54	1.72
Estonia	1.98	1.68
Finland	2.72	1.39
France	2.73	1.79
Germany	2.38	1.53
Greece	2.31	1.37

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<sup>1</sup> The TFR is the number of children that a women can expect to have given her survival through the childbearing years and current age-specific fertility rates.

Hungary	2.02	1.58
Iceland	4.26	1.73
Ireland	3.78	1.77
Israel	4.05	2.98
Italy	2.38	1.28
Japan	2.01	1.30
Latvia	1.95	1.58
Lithuania	2.63	1.62
Luxembourg	2.29	1.39
Mexico	6.76	1.82
Netherlands	3.12	1.64
New Zealand	4.24	1.77
Norway	2.87	1.50
Poland	3.03	1.46
Portugal	3.16	1.36
Republic of Korea	5.95	0.88
Slovak Republic	3.04	1.57
Slovenia	2.19	1.63
Spain	2.78	1.28
Sweden	2.18	1.67
Switzerland	2.44	1.49
Turkey	6.38	1.89
United Kingdom	2.72	1.56
United States	3.55	1.66

**Table 1:** *TFR (in 1960 and in 2021) in OECD countries.*

The trend of falling fertility has led to many concerns ranging from geostrategic considerations –according to which some leaders think that the military might of their countries is diminished by population decline– to economic considerations according to which the decline of the workforce may have severe adverse consequences in terms of the unsustainability of social security and pension systems, reduced economic growth, and a decline in living standards more generally (see, for example, Bloom *et al.*, 2010, 2024;

Economist, 2011 for a description of the potential problems caused by low fertility, population aging, and population decline).

Most recently, the concerns have been voiced in an article by Jones (2022) in which he argues that population decline will lead to economic stagnation and low fertility will become so entrenched that the world will find itself converging to a situation of an “*Empty-Planet*” (Bricker and Ibbitson, 2019). This follows as a consequence of his semi-endogenous growth model (Jones, 1995) in which the growth rate of the number of people drives the growth rate of ideas in the long run. The rationale behind this relationship is that R&D becomes increasingly more complex because simpler ideas are developed first such that sustaining a constant exponential growth rate of ideas requires an exponentially growing number of researchers. Since the framework abstracts from education and the training of scientists, it is logically necessary that a slowdown of population growth implies fewer resources for R&D, and thus a slowdown of economic growth, too. Overall, in this framework, negative population growth sparks economic stagnation.

In contrast to the basic semi-endogenous growth model of Jones (1995), later contributions have emphasized the importance of human capital accumulation in this context (Galor and Weil, 2000; Galor, 2005 and 2011; Strulik *et al.*, 2013; Bucci and Prettnner, 2020)<sup>2</sup>. R&D is increasingly undertaken by highly educated scientists and engineers: To be able to expand the research frontier ultimately requires increasingly specialized training such that the major contributions of scientists nowadays occur much later in their lives than in the past (see also Jones, 2010; Jones and Weinberg, 2011). The training of scientists has become so important that the growth rate of the sheer number of workers is insufficient to propel technological progress and economic growth in a modern economy.

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<sup>2</sup> For a non-exhaustive list of contributions that analyze the effects of human capital accumulation on long-run economic growth, see Funke and Strulik (2000), Dalgaard and Kreiner (2001), Cervellati and Sunde (2005), Strulik (2005), Lagerlöf (2006), Bucci (2008), Schäfer (2014), Prettnner and Strulik (2016), d’Albis *et al.* (2018), Baldanzi *et al.* (2021), and Stokey (2021).

In this contribution, we extend the model proposed in the pathbreaking paper by Jones (2022) to include endogenous human capital accumulation alongside endogenous fertility decisions. Our objective in doing so is to answer the following three fundamental questions: What happens to economic growth and other endogenous variables (such as fertility, technological progress, and skill acquisition) when economic growth is driven either by the sole investment in human capital, or by the joint investment in human capital and R&D? Depending on its driving engine(s), what happens to economic growth when population growth becomes negative? And, more importantly, would it still be possible to recover sustained positive economic growth even in the presence of ongoing population decline? After answering these questions, our model will allow us to ultimately understand to which extent the crucial *Empty-Planet* result of Jones (2022) may hold when we take into account the fundamental role played by human capital in both production and R&D activities. Among other results, we show that some of the implications of Jones (2022) may be overturned in the presence of endogenous education and, thus, endogenous human capital accumulation, which interacts crucially with endogenous population growth. In particular, we show that in many settings, technological progress and economic growth do not decline when the population shrinks.

Our paper is closely related to the contributions by Feichtinger *et al.* (2023) and Strulik (2024). In both papers, the authors introduce human capital accumulation into the Jones (2022) model. In so doing, Strulik (2024) translates the model of Jones (2022) into discrete time and assumes decreasing returns to human capital in the learning technology. However, he allows for a positive feedback effect from technological progress to human capital accumulation in the sense that new ideas increase the productivity of education. This allows him to overcome the diminishing returns in human capital accumulation and get faster aggregate human capital growth whenever population growth decreases. Thus, per capita income growth may rise in the face of a shrinking population. In contrast to Strulik (2024), we augment the continuous time model of Jones (2022) directly by human capital and do not assume a positive feedback effect from technological progress to learning. Even without this feedback effect, we show that there are many situations in which

human capital accumulation can compensate for a shrinking population, particularly when there are class-size effects in education and when there is a cost of bringing the human capital of a newborn to the average level of the population.

The present contribution also shares similarities with Feichtinger *et al.* (2023) who augment the continuous time model of Jones (2022) to analyze the effects of human capital accumulation on economic prosperity. They show that even with decreasing returns to education and when the long-run solution of the model converges to that of Jones (2022), the transitional dynamics can last for a very long time and are associated with a rise of human capital accumulation and economic growth whenever population growth slows down. In contrast to our paper, Feichtinger *et al.* (2023) focus on the transition phase toward the long-run balanced growth equilibrium and they do not take into account both the role that class-size effects may potentially play in education investment, and the case of small economies not driving the research frontier.

Finally, our article is also closely related to recent works by Boikos *et al.* (2023), Bucci (2023), and Bucci *et al.* (2025). Boikos *et al.* (2023) include human capital accumulation in the Jones (2022) model and show that over the very long run the TFP-stagnation result (Bloom *et al.*, 2020) may be reconciled with the no-stagnation-result in GDP per capita, even in the presence of negative (and exogenous) population growth. Bucci (2023) also analyzes, within an R&D-based economy that invests in human capital, the conditions under which in the very long run a negative and exogenous population growth rate may be simultaneously conducive to positive growth rates of ideas, per capita income, and per capita human capital, respectively. Unlike these two contributions, in the current paper, population growth and human capital investment are both fully endogenous. Finally, Bucci *et al.* (2025) entirely abstract from R&D activities and focus solely on the role of human capital accumulation in determining a positive growth rate of real per capita GDP, even when the population growth rate (still an exogenous variable) becomes negative. In their paper, however, there is no endogenous quantity/quality trade-off between fertility (also an exogenous variable) and education.

The paper is structured as follows. In Section 2, we describe the model economy by taking seriously the role played by human capital. In Section 3, we solve the household maximization problem. Section 4 is devoted to the analysis and characterization of the competitive balanced growth path (BGP) equilibrium and contains the main results of the effects of population decline on human capital accumulation, technological progress, economic growth, and the allocation of time across different economic activities. Finally, we conclude and provide policy recommendations in Section 5.

## 2. The Model Economy

Building on Jones (2022), we consider an economy with perfectly competitive markets in which the production of homogeneous consumption goods ( $Y_t$ ) requires human capital ( $H_{Yt}$ ) according to the production function

$$Y_t = A_t^\Omega H_{Yt}, \quad \Omega \geq 0. \quad (1)$$

In Eq. (1),  $Y_t$  is aggregate real GDP (the *numéraire*),  $H_{Yt}$  is the fraction of available human capital ( $H_t$ ) employed in production activities,  $A_t$  refers to total factor productivity (more precisely, it represents the stock of ideas available at time  $t$ , which are developed in the R&D sector), and  $\Omega$  is a parameter. Production of goods occurs under constant returns to rival inputs –here skilled labor, rather than the number of people as in Jones (2022)– and (possibly) increasing returns to skilled labor *and* ideas together, with the degree of the returns being parameterized by  $\Omega$ .

Production factors are paid their marginal products implying that the wage rate is

$$w_{Yt} = \frac{\partial Y_t}{\partial H_{Yt}} = A_t^\Omega \equiv w_t,$$

so that the aggregate production function can be rewritten as:

$$Y_t = w_{Yt} H_{Yt} \quad \Leftrightarrow \quad \frac{Y_t}{H_{Yt}} = w_{Yt} = A_t^\Omega \equiv w_t.$$

Jointly, the last two equations suggest that all output is used to reward human capital (no profits are allowed), and that the marginal and average products of human capital do coincide.



Firms take the time path of the stock of ideas  $(A_t)$  as given by research output:

$$\dot{A}_t = \Gamma A_t^{1-\chi} H_t^{\Xi}, \quad \Gamma > 0, \quad 0 < \Xi \leq 1, \quad 1 - \chi \leq 1. \quad (2)$$

While  $\chi \geq 0$  governs the extent of the intertemporal knowledge spillovers arising from past ideas and measures how hard it is to find new ideas from the stock of knowledge already existing,<sup>3</sup>  $0 < \Xi \leq 1$  measures the depth of the duplication/congestion effect resulting from possibly overlapping research efforts,<sup>4</sup> and  $\Gamma$  is a productivity parameter. Unlike Jones (2022, p. 3495, Eq. (5)), the essence of our Eq. (2) is that ideas are not generated simply by the sheer number of people, but rather they result from the skills, or the human capital (a reproducible factor input), embodied in them. This is important because, as new technologies become increasingly more complex, it becomes close to impossible that un-educated workers advance the research frontier. We follow Jones (2022, p. 3498, Table 1) in assuming that households do not save but consume the whole amount of (final) output:

$$C_t = Y_t = A_t^{\alpha} H_{Yt}. \quad (3)$$

Let  $n_t$  be a representative agent's fertility rate at a generic time  $t$ , and  $d \geq 0$  her constant (and exogenous) death/mortality rate. In the absence of immigration, at any time  $t \geq 0$ , the growth rate of the size of the representative household/family  $(N_t)$  is endogenous and equal to

$$\frac{\dot{N}_t}{N_t} = (n_t - d) \gtrless 0, \quad N(0) = N_0 > 0, \quad (4)$$

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<sup>3</sup> The parameter  $\chi$  is an inverse measure of the '*standing-on-shoulders*' effect, following the statement by Isaac Newton that he was able to see further because he was standing on giant's shoulders, *i.e.*, his contributions built on previous scientific breakthroughs. Whenever  $\chi > 0$ , new ideas become harder to find.

<sup>4</sup> This is the so-called '*stepping-on-toes*' effect, according to which researchers may develop the same idea simultaneously, such that there could be duplication in R&D effort and a rise in the number of workers developing ideas would not translate one for one into a greater flow of ideas. When  $\Xi = 1$  and  $\chi = 0$ , the ideas production function (2) resembles that employed by Romer (1990) in his fully-endogenous growth model with strong scale effects (see also Jones, 2022, Eq. 2, p. 3493).

with  $n_t N_t$  and  $dN_t$  representing the total number of births and the total number of deaths per unit of time, respectively. While  $N_t$  is a state variable,  $n_t$  is a choice variable in our model. The representative household is endowed with one unit of time that may be allocated to:

- i. Raising children, using the fraction  $0 \leq l_t \leq 1$  of available time;
- ii. Producing consumption/final goods, using the fraction  $0 \leq u_t \leq 1$  of available time;
- iii. Acquiring ‘new’ human capital, using the fraction  $0 \leq v_t = (1 - u_t - l_t) \leq 1$  of available time.

In per capita terms, Eq. (3) can be recast as:

$$y_t = c_t = A_t^\alpha (u_t h_t) = w_t (u_t h_t), \quad (3')$$

with  $h_t \equiv \frac{H_t}{N_t}$ ,  $y_t \equiv \frac{Y_t}{N_t}$ , and  $c_t \equiv \frac{C_t}{N_t}$  representing per capita human capital, per capita real GDP, and per capita consumption, respectively, and  $u_t h_t = \frac{u_t H_t}{N_t} \equiv \frac{H_{Yt}}{N_t}$  referring to the fraction ( $u_t$ ) of the stock of human capital per capita ( $h_t$ ) devoted to the production of consumption goods ( $y_t$ ). We adopt the following specification for the child-rearing cost function in terms of parental time,  $l(n_t)$ :

$$l(n_t) = \tau n_t^\omega, \quad \tau \geq 0, \quad \omega \geq 0. \quad (5)$$

Under Eq. (5), which follows Yip and Zhang (1996, 1997) and Palivos (1995), one may easily find combinations of the parameters  $\tau$  and  $\omega$  for which it is simultaneously true that:

- $l(n_t)$  is strictly increasing in the fertility rate,  $n_t \geq 0$ , i.e.,  $l'(n_t) = \tau \omega n_t^{\omega-1} > 0$ ;
- $l(0) = 0$ ;
- $l''(n_t) = \tau \omega (\omega - 1) n_t^{\omega-2} \gtrless 0$ , meaning that the *marginal* time cost of taking care of children can be either increasing, or constant, or else decreasing in  $n_t$ .

Also notice that:

- When  $\tau = 0$ , rearing children implies no time costs, for any  $\omega \geq 0$  and  $n_t \geq 0$ ;

- When  $\omega = 1$  and  $\tau > 0$ , we end up with a cost function for child rearing being linear in  $n_t$  (exactly as in Yip and Zhang, 1997, p. 100).

Since time can be used to raise children, to produce consumption goods, or to acquire ‘new’ human capital, per capita human capital will evolve over time according to the following modified *Lucas (1988)-type equation*:

$$\dot{h}_t = \delta(1 - u_t - l_t)h_t \quad (6)$$

with  $\delta > 0$  measuring the efficiency of the individual human capital investment technology.<sup>5</sup> Lucas (1988) was the first to distinguish between ‘quantity’ (*i.e.*, size) and ‘quality’ (*i.e.*, human capital per capita) of the population within the context of endogenous growth theory. While he endogenized quality, he left quantity to follow an exogenous process. The main reason why we start from the basic Lucas’ (1988) description of the per capita human capital accumulation technology (Eq. 6) is that such specification has proved to be one of the most flexible in allowing for further possible theoretical extensions and advancements, even from a purely mathematical point of view. As an example, Boucekkine *et al.* (2013), by focusing on the relationship between population, human capital, and economic growth, have extended the Lucas model in the direction of scrutinizing the impact of population size and population growth not only on the growth rates, but also on the levels of human capital and income per capita. In more detail, they characterize the long-run level effects of demographic changes (along with the more conventional long-run growth effects) and study the transitional dynamics of their extended Lucas model in an attempt to distinguish between short-run (transitional dynamics) *vs.* long-run (balanced growth path) consequences of population growth and population size on economic performance.<sup>6</sup>

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<sup>5</sup> For the sake of simplicity, we abstract from any form of depreciation (or physical obsolescence) of human capital ( $h_t$ ) in the accumulation of ‘new’ human capital  $\left(\dot{h}_t\right)$ .

<sup>6</sup> In doing so they also rely on Boucekkine and Ruiz-Tamarit (2008), who were the first to produce analytical solutions of the Lucas–Uzawa model and to extract closed-form expressions corresponding to the optimal paths of this model’s variables in levels.

Generalizing Lucas (1988), where  $\delta$  is simply a parameter, we posit that it is, instead, a function of the population growth rate,  $\dot{N}_t / N_t$ , *i.e.*, we assume that

$$\delta \equiv \left[ \sigma - \varepsilon \left( \frac{\dot{N}_t}{N_t} \right) \right] = \left[ \sigma - \varepsilon (n_t - d) \right] > 0, \quad \sigma > 0, \quad \text{and} \quad \varepsilon \geq 0. \quad (7)$$

In this equation,  $\varepsilon$  introduces a *class size effect*, meaning that learning becomes less efficient when the class becomes larger and the teacher-pupil ratio is higher. Also notice that, when population growth is endogenous, the efficiency of the individual human capital investment technology,  $\delta$ , is endogenous, too. Evidently, when (i) The population remains stable over time

$\left( \text{i.e., } \dot{N}_t / N_t \equiv n_t - d = 0 \right)$ , or when (ii)  $\varepsilon = 0$ , or (iii) both, we are back to

the original *Lucas (1988)-type equation* of per capita human capital investment, with  $\delta \equiv \sigma > 0$ . While the *Lucas (1988)-equation* remains our benchmark, in the remainder of our analysis, the focus will be on situations in

which  $\varepsilon > 0$  and  $\frac{\dot{N}_t}{N_t} \equiv (n_t - d) \geq 0$ .

With  $\varepsilon > 0$  and with exogenous population growth  $\left( \dot{N}_t / N_t \right)$ , it is apparent from (7) that

$$\frac{\partial \delta}{\partial \left( \dot{N}_t / N_t \right)} = -\varepsilon < 0. \quad (7')$$

This derivative implies that, *ceteris paribus*, a negative correlation between the given rate of population growth and the efficiency with which any single individual augments her own stock of embodied knowledge,  $\delta$ , prevails. In other words, the presence of  $\varepsilon > 0$  in Eq. (7) allows a population declining at a given rate to yield, in principle, a better student's schooling performance. Allowing for this effect takes into account the empirical consensus of a positive, even though small, effect of a reduced class size on an individual learning achievement (see, among many others, Hattie, 2005; Kokkelenberg *et al.*, 2008; Konstantopoulos and Chung, 2009; Bandiera *et al.*, 2010; De Giorgi *et al.*,

2012; Kara *et al.*, 2021).<sup>7</sup> Hence, in our model, the magnitude of the *class-size effect* can ultimately be traced to the size of the parameter  $\varepsilon \geq 0$ .

In the light of what has been said above, combining Eqs. (6) and (7) finally leads to the following law of motion of per capita human capital:

$$\dot{h}_t = \underbrace{\left[ \sigma - \varepsilon (n_t - d) \right]}_{\equiv \delta > 0} (1 - u_t - l_t) h_t, \quad \sigma > 0, \quad \varepsilon \geq 0. \quad (8)$$

In addition to a possible class-size effect related to population growth ( $\varepsilon > 0$ , within the square brackets of Eq. (8)), and to the time cost of fertility ( $0 \leq l_t \leq 1$ ), we introduce a third generalization of the canonical *Lucas (1988) equation* for per capita human capital investment. We maintain that rearing *each single child* entails not only a *time-cost*, but also a *goods-cost*, in the form of expenditures (to be borne once and for all), equal to  $\eta_t$ , and proportional to the level of per capita human capital already attained, *i.e.*,

$$\eta_t = b h_t, \quad b \geq 0.^8$$

Since  $n_t N_t$  is the number of births per unit of time,  $\eta_t n_t N_t$  are the total expenditures on child rearing, and  $\eta_t n_t = (b n_t) h_t$  is the amount expended per capita. Thus, after taking into account the possible class-size effects resulting from the investment in education, and both the time- and goods- costs related to fertility, the law of motion of per capita human capital finally reads as:

$$\begin{aligned} \dot{h}_t &= \left[ \sigma - \varepsilon (n_t - d) \right] (1 - u_t - l_t) h_t - (b n_t) h_t, \\ \sigma &> 0; \quad \varepsilon \geq 0; \quad b \geq 0; \quad l(n_t) = \tau n_t^\omega \\ h(0) &= h_0 > 0. \end{aligned} \quad (9)$$

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<sup>7</sup> This strand of the literature also stresses that the impact of a class-size reduction is stronger for the very first years of schooling. According to Konstantopoulos and Chung (2009), however, longer periods in small classes produce higher increases in educational achievements in later grades for all types of students. For certain grades, in reading and science, they find that low achievers seem to benefit more from being in small classes for longer periods.

<sup>8</sup> Following Barro and Sala-i-Martin (2004, p. 412/413), one may interpret  $\eta$  ‘...as a large single outlay that represents the present value of expenditures for each child. ...More generally,  $\eta$  increases with the per capita quantities of human and physical capital...’. Notice that, in per capita terms, the cost of rearing children,  $\eta_t n_t = (b h_t) n_t$  is clearly increasing in  $n_t$ , for any  $b > 0$  and  $h_t > 0$ , and it will be equal to zero when either  $n_t = 0$  or  $b = 0$ , for any  $h_t > 0$ .

Notice that, written this way, the last term of Eq. (9), *i.e.*,  $(-bn_t)$ , suggests the presence of a sort of *dilution effect* in the per capita human capital accumulation technology: a value of  $b=1$  would imply that there is full human capital dilution due to the fact that any newborn enters the world completely uneducated (*i.e.*, with no human capital at all,  $h=0$ ). A value of  $b=0$ , instead, would mean that the human capital of the newborn is immediately equal to the average level of the economy's human capital at no cost. Indeed, the magnitude of  $b$  is going to play an important role within the subsequent analysis.

### 3. The Representative Household Problem

The economy is populated by many structurally identical households. Therefore, we may focus on the optimal behavior of a single, representative family with perfect foresight, and whose size coincides with that of the whole population  $(N_t)$ . Each member of the representative household is characterized by a *Millian-type* intertemporal utility function of the form:

$$U \equiv \int_0^{\infty} u(c_t; n_t) \cdot e^{-\rho t} dt = \int_0^{\infty} [\ln(c_t) + \phi \ln(n_t - d)] \cdot e^{-\rho t} dt, \quad \phi > 0, \quad \rho \equiv \bar{\rho} + d > 0 \quad (10)$$

where  $\phi > 0$  is the utility weight that any single individual attaches to the number of children (Prettner, 2013), and  $\rho \equiv \bar{\rho} + d > 0$  is the overall discount rate, according to which the pure time preference rate  $\bar{\rho}$  is augmented by the mortality rate  $d$ . Eq. (10) is similar to Eq. (9.54) in Barro and Sala-i-Martin (2004), with two major differences:

- The total size of the representative household,  $N_t$ , does not enter the instantaneous utility function of any single member of the family, a specification that delivers a more tractable mathematical solution (Barro and Sala-i-Martin, 2004, p. 412; Jones, 2001);<sup>9</sup>

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<sup>9</sup> This explains why we use a '*Millian-type*' intertemporal utility function. Jones (2022, p. 3505) also employs a specification for the flow utility, *i.e.*,  $u(c_t; N_t) = \log c_t + \varepsilon \log N_t$ , which includes  $N_t$ , rather than  $(n_t - d)$ , as we do in Eq. (10).

- Following, among others, Bucci and Prettner (2020), we adopt a logarithmic formulation for instantaneous utility, which serves the scope of making the problem more tractable analytically.<sup>10</sup>

Each individual chooses the number of children ( $n_t \geq 0$ ) and the fraction of her time endowment to be devoted to the production of consumption goods ( $0 \leq u_t \leq 1$ ) so as to maximize (10), subject to (3') and the laws of motion for  $N_t$  (4) and  $h_t$  (9). In doing so, agents take the dynamics of  $A_t$ , and, thus, the dynamics of wages  $w_t$ , as given. It is worth observing that, by choosing optimally  $n_t$  and  $u_t$ , the representative agent chooses at the same time the optimal amount of time to be spent both on raising children,  $l_t = \tau n_t^\omega$ , and on schooling investment,  $\nu_t = (1 - u_t - l_t)$ .

We assume a unitary household because we are interested in the macroeconomic effects of decreasing fertility and not in the microeconomic forces that cause low fertility in the first place. For contributions that model intrahousehold bargaining in much more detail and that derive important insights based on such a framework, see, for example, Gobbi (2018), Doepke and Kindermann (2019), and Doepke *et al.*, (2023).

Formally, the dynamic optimization problem of the representative agent is given by

$$\begin{aligned}
& \underset{\{c_t; n_t; u_t; N_t; h_t\}_0^\infty}{Max} \quad U = \int_0^\infty \left[ \ln(c_t) + \phi \ln(n_t - d) \right] \cdot e^{-\rho t} dt, & \rho \equiv \bar{\rho} + d > 0 \\
& s.t. : \\
& c_t = A_t^\Omega (u_t h_t) = y_t \\
& \dot{h}_t = \left[ \sigma - \varepsilon(n_t - d) \right] (1 - u_t - \tau n_t^\omega) h_t - (b n_t) h_t, & h(0) = h_0 > 0, \\
& \dot{N}_t = (n_t - d) N_t, & N(0) = N_0 > 0.
\end{aligned} \tag{11}$$

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<sup>10</sup> See also Barro and Sala-i-Martin (2004, p. 415).

#### 4. Balanced Growth Path (BGP) Equilibrium Analysis

We follow Yip and Zhang (1997, p. 102) in defining a *balanced growth path (BGP) equilibrium* in this model as follows.

**DEFINITION:** A balanced growth path (BGP) equilibrium is a collection of functions of time  $\{c_t, y_t, h_t, u_t, A_t, N_t, \text{ and } w_t\}$ , such that, in the very long run, they grow exponentially at constant (possibly different) rates, and  $n$  is constant.

From this definition and the law of motion of  $h_t$ , we immediately observe that along a BGP equilibrium, the time allocation  $\{l_t; u_t; v_t\}$  remains constant. This means that, at such a long-run equilibrium, the representative agent will optimally allocate a constant share of her own time endowment to raising children ( $l$ ), investing in education ( $v \equiv 1 - u - l$ ), and producing consumption goods ( $u$ ).<sup>11</sup> Proposition 1 summarizes our BGP equilibrium results.

**Proposition 1:**

Along a balanced growth path (BGP) equilibrium, the following results do hold.

$$g_N^* \equiv \left( \frac{\dot{N}_t}{N_t} \right)^* = (n^* - d) \gtrless 0, \quad (12)$$

$$l^* = \tau (n^*)^\omega, \quad \tau \geq 0, \quad \omega \geq 0, \quad (13)$$

$$g_h^* \equiv \left( \frac{\dot{h}_t}{h_t} \right)^* = [\sigma - \varepsilon (n^* - d)] (1 - u^* - l^*) - (bn^*), \quad (14)$$

$$u^* = \frac{\rho}{[\sigma - \varepsilon (n^* - d)]} > 0, \quad [\sigma - \varepsilon (n^* - d)] > 0, \quad \rho \equiv \bar{\rho} + d > 0, \quad (15)$$

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<sup>11</sup> In words,  $g_u^* = g_l^* = g_v^* = 0$ , where  $g_x^*$  refers to the growth rate ( $g$ ) of variable  $x$  along a BGP equilibrium.



$$g_A^* \equiv \left( \frac{\dot{A}_t}{A_t} \right)^* = \frac{\Xi}{\chi} (g_h^* + g_N^*), \quad (16)$$

$$g_y^* \equiv \left( \frac{\dot{y}_t}{y_t} \right)^* = g_c^* \equiv \left( \frac{\dot{c}_t}{c_t} \right)^* = \Omega g_A^* + g_h^* \quad (17)$$

with  $n^*$  being ultimately the solution to

$$\frac{\phi \rho}{(n^* - d)} = \varepsilon - \frac{\varepsilon \rho}{\left[ \sigma - \varepsilon (n^* - d) \right]} + (\sigma + \varepsilon d) \tau \omega (n^*)^{\omega-1} - \varepsilon \tau (1 + \omega) (n^*)^\omega + b. \quad (18)$$

**Proof:** See Online Appendix A.

At the BGP, the equilibrium value of the birth rate ( $n^*$ , Eq. 18), depends solely on the following parameters of the model:  $\phi$ ,  $d$ ,  $\bar{\rho}$  (which are utility-related parameters),  $\varepsilon$ ,  $\sigma$ ,  $\tau$ ,  $\omega$ , and  $b$  (which are technology-related parameters), whereas it does not depend on other technology-related parameters, such as  $\Omega$  (aggregate production function),  $\chi$  and  $\Xi$  (R&D production function). In turn, given  $n^*$ , Eqs. (12), (13) and (15) determine the BGP-equilibrium values of the population growth rate ( $g_N^*$ ), and of the shares of the fixed time endowment devoted to raising children ( $l^*$ ) and to producing consumption goods ( $u^*$ ). Finally, given  $n^*$ ,  $l^*$ , and  $u^*$ , our model allows obtaining the BGP-equilibrium values of the growth rate of per capita human capital ( $g_h^*$ , Eq. 14), the growth rate of ideas ( $g_A^*$ , Eq. 16), the growth rate of per capita real GDP ( $g_y^*$ , Eq. 17), along with the BGP-equilibrium value of the share of time endowment devoted to acquiring new human capital,  $\nu^* = (1 - u^* - l^*)$ . We observe that, while  $l^*$  depends in a direct manner on  $n^*$  (as a consequence of our assumptions in Eq. 5), the efficiency of human capital accumulation,  $\left[ \sigma - \varepsilon (n^* - d) \right]$ , and the share of time allocated to working,  $u^* = \frac{\rho}{\left[ \sigma - \varepsilon (n^* - d) \right]}$ , depend on  $n^*$  in a way that is mediated by the parameter measuring the class-size effect,  $\varepsilon$ , as clarified in the following proposition.

**Proposition 2:**

*In the absence of class-size effects ( $\varepsilon = 0$ ), both the efficiency of human capital accumulation and the share of time allocated to working are, at equilibrium, unresponsive to changes in population growth. With class-size effects,  $\varepsilon > 0$ , instead,  $u^*$  depends positively, whereas  $\left[\sigma - \varepsilon(n^* - d)\right]$  depends negatively on population growth,  $n^*$ .*

**Proof:** See Eq. (15).

The intuition is straightforward. When  $n^*$  increases, by assumption (Eq. 5) the amount of time spent on rearing children ( $l^*$ ) rises. In the presence of class-size effects ( $\varepsilon > 0$ ), a larger class-size (induced, *e.g.*, by a larger population) is associated, by definition, to poorer educational outcomes. This means that the efficiency of the human capital technology declines following a rise of  $n^*$ , which would ultimately lead agents to prefer working ( $u^*$ ) to educational activities ( $1 - u^* - l^*$ ), without any ambiguity. We are now ready to prove the first key result of the model.

**Proposition 3:**

*At a competitive BGP equilibrium with R&D activity and human capital accumulation, an endogenous quantity/quality trade-off between fertility and skill-acquisition exists.*

**Proof:** See Online Appendix B.

This result is important as it allows us to fill a gap in the recent path-breaking paper by Jones (2022).<sup>12</sup> To provide an overall picture of how our theoretical framework can be used and of the main results it can produce, we now simulate the model by giving realistic numbers to its parameters and

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<sup>12</sup> “...This setup excludes many other considerations that would be interesting to explore in the future such as human capital, physical capital, and a quantity-quality trade-off. We instead focus on the simplest model that allows us to highlight some important (and general) economic points.” (Jones, 2022, p. 3497).

obtaining the corresponding values of the endogenous variables (Eqs. (12)–(18)). Since we have a very general structure, different important cases can be analyzed and discussed separately in our model. We start from a straightforward replica of Jones (2022).

***Replication of Jones, 2022:*** *Only R&D activity, hence no human capital investment (therefore, no class-size effects)*

Abstracting from human capital investment and assuming that R&D activity is the sole engine of long-run economic growth, the model presented above can be modified slightly such that it is able to recover the same basic results of the competitive equilibrium of Jones (2022, pp. 3499-3500). These results can be summarized as follows.

***Proposition 4:***

*At a competitive BGP equilibrium with R&D activity and without human capital, economic growth turns out to be a monotonically positive function of population growth, which, indeed, can be negative.*

***Proof:*** See Online Appendix C.

We now depart from this baseline case and discuss other cases with various roles for human capital accumulation.

**CASE 1 (Benchmark):** *Human capital accumulation under endogenous population growth, but no R&D activity and no class-size effects in education,  $\varepsilon = 0$*

Our benchmark is represented by an economy without R&D activity in which human capital is the sole engine of economic growth. This could refer to the situation of those small countries that do not drive the worldwide technological frontier but still import technologies from abroad. Thus, in these countries, technological progress (determined within the country) is zero. In the benchmark case, we abstract from class-size effects in schooling ( $\varepsilon = 0$ ) and we use the baseline parameter values displayed in Table 2.

Parameters	Sources
$\phi = 0.0279$	This value is chosen so that the population growth rate fits to the data. Please note that estimates of this preference parameter are difficult to obtain so calibration is the best available option
$\rho = 0.025$ per year	This is not very different from the value (0.02) considered as ‘ <i>standard</i> ’ in the numerical treatment of the Uzawa–Lucas model by Barro and Sala-i-Martin (2004, p. 260)
$d = 0.012692$	This parameter leads to a life expectancy of approximately 81 years as many high-income countries are currently experiencing it (World Bank, 2025)
$\bar{\rho} = \rho - d = 0.012308$	As a consequence of the previous two parameter values
$\varepsilon = 0$	This implies the absence of class-size effects in per capita human capital investment in this baseline case
$\sigma = 0.038$	In the numerical exercises that Barro and Sala-i-Martin (2004, p. 260) perform on the basic Uzawa–Lucas model, they use a value of 0.11 for their parameter “ $B$ ” (our $\sigma$ ). This value boils down to 0.06, after explicit consideration of human capital depreciation
$\tau = 0.1$	This value is roughly in line with Strulik <i>et al.</i> (2013, p. 429), according to which rearing two children requires around 10% of an adult’s time endowment.
$\omega = 1$	Yip and Zhang, 1996 (p. 323), and 1997 (p. 100)
$\chi = 0.5$	This number is in line with the range of values used in the standard literature (see, for example, Grossmann <i>et al.</i> , 2013)
$\Xi = 0$	$g_A^* = 0$ when $\Xi = 0$ and $\chi \neq 0$
$\Omega = 0$	Human capital investment is the only engine of growth when $\Omega = 0$
$b$	This parameter is adjusted according to different simulation scenarios.

**Table 2:** *Parameter values in the benchmark case – Human capital accumulation under endogenous population growth, no R&D activity, and no class-size effects in education,  $\varepsilon = 0$ .*

To draw appropriate conclusions about the economic effects of changes in population growth within an economy that invests solely in human capital (and not in R&D), and where there are no class-size effects in schooling ( $\varepsilon = 0$ ), we use the parameter values of Table 2 to generate numerically the value of the endogenous variables of the model following a 10% rise in households’ preferences towards children ( $\phi$ ), for different values of  $b$ . The

results are displayed in Table 3. The first part of the table refers to the case  $\phi = 0.0279$  with lower population growth, and the second part of the table refers to the case  $\phi = 0.0307$  with higher population growth.<sup>13</sup>

$\phi = 0.0279$	$b = 0$	$b = 0.3$	$b = 0.7$	$b = 1$
$n^*$	0.196245	0.0149879	0.013683	0.0133869
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.183553	0.00229592	0.000991049	0.00069486
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.0122543	0.00844667	0.00336987	-0.00043773
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0	0	0	0
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.0122543	0.00844667	0.00336987	-0.00043773
$u^*$	0.657895	0.657895	0.657895	0.657895
$l^*$	0.0196245	0.00149879	0.0013683	0.00133869
$v^* = (1 - u^* - l^*)$	0.322481	0.340606	0.340737	0.340767
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.038	0.038	0.038	0.038
$\phi = 0.0307$	$b = 0$	$b = 0.3$	$b = 0.7$	$b = 1$
$n^*$	0.214666	0.0152183	0.0137825	0.0134566
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.201974	0.00252633	0.00109051	0.000764593
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.0121843	0.00837667	0.00329987	-0.00050773
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0	0	0	0
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.0121843	0.00837667	0.00329987	-0.00050773
$u^*$	0.657895	0.657895	0.657895	0.657895
$l^*$	0.0214666	0.00152183	0.00137825	0.00134566
$v^* = (1 - u^* - l^*)$	0.320639	0.340583	0.340727	0.340760
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.038	0.038	0.038	0.038

**Table 3:** Values of the endogenous variables of the model in the benchmark case – Human capital accumulation under endogenous population growth, no R&D, and no class-size effects in education,  $\varepsilon = 0$ , for two different levels of preferences towards children,  $\phi$ , and for different levels of the cost parameter,  $b$

<sup>13</sup> All numerical results are obtained using Wolfram Mathematica, version 14.0. The corresponding program can be obtained from the authors upon request.

We observe that the model is capable of producing realistic values for the endogenous variables, and it features a quantity-quality tradeoff effect: whenever the preference for children increases, fertility rises but human capital accumulation slows down. Thus, unlike in Lucas (1988), where the relation between (endogenous) economic growth and (exogenous) population growth along a competitive equilibrium BGP is monotonically positive, in our case the same relation turns negative under endogenous population growth ( $g_N^*$ ). Since human capital accumulation is the driving force of economic growth, this implies that a more pronouncedly negative population growth can sustain a higher (and positive) long-run growth rate of per capita GDP. This relation remains qualitatively stable across the different values of the parameter  $b \in [0,1]$ , although the perhaps most realistic values of endogenous variables are obtained when  $b=0.3$  (which is itself a realistic value according to Donni, 2015; see also Online Appendix E for a further discussion on this specific point). We summarize our insights for this case as follows.

**RESULT 1:** *In an economy in which fertility and education are endogenous and technologies are imported from abroad, human capital accumulation is the driving force of economic growth. A rise in population growth through a higher value of  $\phi$  slows down economic growth because it hampers human capital accumulation.*

When  $\varepsilon = 0$  and all else remains equal, following an increase in  $n^*$ , both  $[\sigma - \varepsilon(n^* - d)]$  and  $u^*$  remain constant. However,  $l^*$  increases when  $n^*$  rises (Eq. 13). Overall, the agents' schooling effort,  $(1 - u^* - l^*)$ , decreases. Ultimately, all this leads to a decline in human capital accumulation and therefore in the competitive equilibrium economic growth rate, as well.

To summarize, economies that are small and mainly import technologies from abroad need not necessarily expect a slowdown of the growth rate of per capita GDP when confronted with falling fertility (and, hence, with possibly negative population growth rates). We now add class-size effects to the case described above and obtain the following results.

**CASE 2:** *Human capital accumulation under endogenous population growth  
(without R&D activity, and with class-size effects in education,  $\varepsilon > 0$ )*

We use here the same parameter values as before (Table 2), with one major difference: we now explicitly allow for the presence of class-size effects in the attainment of human capital per capita,  $\varepsilon > 0$ . As already described, in our model the magnitude of these effects is ultimately linked to the size of the parameter  $\varepsilon$  and we follow the empirical literature in setting the effect to a positive but rather small number.

$\varepsilon = 0.1$	See, among many others, Hattie (2005); Kokkelenberg <i>et al.</i> (2008); Konstantopoulos and Chung (2009); Bandiera <i>et al.</i> (2010) <sup>14</sup> ; De Giorgi <i>et al.</i> (2012); Kara <i>et al.</i> (2021)
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Again, with these parameters we generate numerically the value of the endogenous variables of the model following a 10% increase in agents' preferences towards children ( $\phi$ ), for different values of  $b$ . The results are displayed in Table 4. Once more, the first part of the table refers to the model solution for  $\phi = 0.0279$  with lower population growth, and the second part of the table refers to the case  $\phi = 0.0307$  with higher population growth.

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<sup>14</sup> "...Our main results are as follows. First, the baseline within student and teacher estimates implies a negative and significant effect size of 0.108. This implies that if a student were to be reassigned from a class of average size to a class which was one standard deviation larger, the test score of the same student would fall by 0.108 of the within student standard deviation in test scores..." (Bandiera *et al.*, 2010, p. 1368).

$\phi = 0.0279$	$\mathbf{b} = 0$	$\mathbf{b} = 0.3$	$\mathbf{b} = 0.7$	$\mathbf{b} = 1$
$n^*$	0.033 <sup>15</sup>	0.0147588	0.0136375	0.0133641
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.0207115	0.00206677	0.00094550 6	0.000672125
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.0108088	0.00830991	0.0033075	-0.000482031
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0	0	0	0
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.0108088	0.00830991	0.0033075	-0.000482031
$u^*$	0.69582	0.661493	0.659536	0.65906
$l^*$	0.00334035	0.00147588	0.00136375	0.00133641
$v^* = (1 - u^* - l^*)$	0.30084	0.337032	0.3391	0.339603
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.0359288	0.0377933	0.0379054	0.0379328
$\phi = 0.0307$	$\mathbf{b} = 0$	$\mathbf{b} = 0.3$	$\mathbf{b} = 0.7$	$\mathbf{b} = 1$
$n^*$	0.0358426 <sup>16</sup>	0.0149665	0.0137324	0.0134316
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.0231506	0.00227447	0.00104042	0.00739588
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.010557	0.00822608	0.00323122	-0.000556487
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0	0	0	0
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.010557	0.00822608	0.00323122	-0.000556487
$u^*$	0.700576	0.661856	0.659701	0.659178
$l^*$	0.00358426	0.00149665	0.00137324	0.00134316
$v^* = (1 - u^* - l^*)$	0.29584	0.336647	0.338926	0.339479
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.0356849	0.0377726	0.037896	0.037926

**Table 4:** Values of the endogenous variables – Human capital accumulation under endogenous population growth, no R&D activity, and with class-size effects in

<sup>15</sup> Here and in the next columns, we focus solely on the lowest equilibrium value of  $n^*$  (in this case  $n_1^* = 0.033$ ) and neglect the highest ones. This has mainly one fundamental reason. Indeed, according to the World Bank (2025), the crude birth rate has reached a value of about 17.3 worldwide in 2020 and is projected to reach a value of 15.1 in 2040 and 14.0 in 2050. These insights will be useful also in Online Appendix E.

<sup>16</sup> See footnote 15.



education,  $\varepsilon > 0$ , for two different levels of preferences towards children,  $\phi$ , and for different levels of the cost parameter,  $b$ .

We summarize our insights for this case as follows.

**RESULT 2:** *Consider the presence of class-size effects ( $\varepsilon > 0$ ) in an economy that does not drive the research frontier, but imports technologies from abroad, and in which fertility and human capital accumulation are both endogenous. In such an economy:*

- *A reduction in population growth raises the efficiency of human capital accumulation,  $\left[\sigma - \varepsilon(n^* - d)\right]$ , whereas the share of time devoted to working activities,  $u^*$ , decreases (Proposition 2). Thus, individuals who decide to have fewer children, spend more time in education,  $(1 - u^* - l^*)$ ;*
- *When  $b$  gets sufficiently large (i.e.,  $b \rightarrow 1$ ), then ceteris paribus (due to the rising dilution effect of population inside the human capital accumulation equation), the growth rate of per capita human capital tends to become negative, despite the rise of effort in acquiring skills,  $(1 - u^* - l^*)$ .*

**RESULT 3:** *In the considered economy, increasing population growth through a higher  $\phi$  definitely slows down economic growth as it hampers human capital investment.*

From a direct comparison of Case 2 with the benchmark (Case 1), the following result is immediate.

**RESULT 4:** *Consider an economy that does not drive the research frontier, but imports technologies from abroad, and in which fertility and human capital accumulation are both endogenous. In the presence of larger class-size effects,  $\varepsilon > 0$  (such that even better educational outcomes are achieved when classes shrink), the increase of population growth (following an increase in the parameter that measures the preference towards children,  $\phi$ ) implies a more pronounced quantity/quality trade-off.*

Online Appendix D presents a simple numerical exercise with the objective of showing how this result ultimately comes out.

To summarize, in economies that are small and mainly import technologies from abroad, the presence of a larger class size effect implies a stronger quality-for-quantity substitution, such that a slowdown of population growth (driven by declining fertility) is followed by an even greater compensatory effect from human capital investment.

**CASE 3:** *Human capital accumulation and R&D activity as twin engines of long-term economic growth, under endogenous population growth and no class-size effects in education,  $\varepsilon = 0$ .*

We now consider a situation in which the two engines of growth, human capital accumulation and R&D, are simultaneously operative. Hence, we set the following additional parameter values.

$\Xi = 0.75$	Jones, 2022, p. 3510, Table 2
$\Omega = 1$	This normalization is convenient, “...so that $A$ has the units of total factor productivity” (Jones, 2022, p. 3508) in final output production, Eq. (1). With $\Omega = 1$ , we have: $g_y^* = g_c^* = g_A^* + g_h^*$

Again, we start from the baseline situation without class-size effects in schooling ( $\varepsilon = 0$ ). As before, we use the two parameters above, along with the other baseline parameters of Table 2, to simulate the values of the endogenous variables of the model following a 10% increase in the preference towards children ( $\phi$ ), with a varying  $b$ . The results are displayed in Table 5. As before, the first part of the table refers to the model solution for  $\phi = 0.0279$  and lower population growth, while the second part of the table refers to the case  $\phi = 0.0307$  and higher population growth.

$\phi = 0.0279$	$b = 0$	$b = 0.3$	$b = 0.7$	$b = 1$
$n^*$	0.196245	0.0149879	0.013683	0.0133869
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.183553	0.00229592	0.000991049	0.00069486
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.0122543	0.00844667	0.00336987	-0.00043773
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0.29371	0.0161139	0.00654138	0.000385695
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.305965	0.0245606	0.00991125	-0.000052035
$u^*$	0.657895	0.657895	0.657895	0.657895
$l^*$	0.0196245	0.00149879	0.0013683	0.00133869
$v^* = (1 - u^* - l^*)$	0.322481	0.340606	0.340737	0.340767
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.038	0.038	0.038	0.038
$\phi = 0.0307$	$b = 0$	$b = 0.3$	$b = 0.7$	$b = 1$
$n^*$	0.214666	0.0152183	0.0137825	0.0134566
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.201974	0.00252633	0.00109051	0.000764595
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.0121843	0.00837667	0.00329987	-0.00050773
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0.321237	0.0163545	0.00658557	0.000385297
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.333421	0.0247312	0.00988544	-0.000122432
$u^*$	0.657895	0.657895	0.657895	0.657895
$l^*$	0.0214666	0.00152183	0.00137825	0.00134566
$v^* = (1 - u^* - l^*)$	0.320639	0.340583	0.340727	0.34076
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.038	0.038	0.038	0.038

**Table 5:** Values of the endogenous variables – Human capital accumulation and R&D under endogenous population growth and no class-size effects in education,  $\varepsilon = 0$ , for two different levels of preferences towards children,  $\phi$ , and for different levels of the cost parameter,  $b$ .

We can summarize the main insights from this case as follows.

**RESULT 5:** *Consider an economy in which fertility and human capital accumulation are both endogenous and economic growth is driven by technological progress and human capital accumulation, together.*

- *A rise in fertility  $n^*$ , driven by an increase in the preference for children  $\phi$ , triggers a quantity/quality tradeoff between population growth  $g_N^*$  and human capital growth  $g_h^*$ , for any level of  $b$ . This, in turn, implies an indirect negative effect of population growth on economic growth  $g_y^*$ , through the human capital channel.*
- *However, the same increase in population growth has also a direct positive impact on technological progress,  $g_A^*$  (see Eq. 16). On the whole, the simultaneous increase of  $g_N^*$  and decrease of  $g_h^*$  delivers an ambiguous effect on  $g_A^*$ . The sign of the overall impact of population growth on technological progress is ultimately dependent upon the size of  $b$ . In the table, only when  $b$  becomes sufficiently large ( $b \rightarrow 1$ ), an increase in  $g_N^*$  ultimately leads to a decline in  $g_A^*$  without any ambiguity, meaning that the indirect negative effect of population growth on human capital growth (the human capital channel) prevails on the direct positive effect of population growth on technological progress, and this drives down the growth rate of per capita GDP  $g_y^*$ , as well.*
- *Though, since  $g_h^*$  (unlike  $g_A^*$ ) is decreasing in  $g_N^*$  for any level of  $b$ ,  $g_y^*$  will start decreasing in  $g_N^*$  earlier, i.e., for values of  $b$  lower than one, in the table  $b \rightarrow 0.7$ .*
- *Overall, the effect of population growth  $g_N^*$  on economic growth  $g_y^*$  is ambiguous – the higher the goods cost of children ( $b$ ) is, the more likely it becomes that the relation turns negative, contrary to Jones (2022).*

**CASE 4:** *Human capital accumulation and R&D activity as twin engines of long-term economic growth, under endogenous population growth and with class-size effects in education,  $\varepsilon > 0$ .*

Finally, we consider the most general case of an economy where the two engines of growth, human capital and R&D, are simultaneously operative, together with a class-size effect in education. In particular, we use the same parameters as in Case 3, but with  $\varepsilon = 0.1 > 0$ . As before, to draw appropriate conclusions about the economic effects of changes in population growth within an economy that invests in both human capital and R&D and where there are class-size effects in education, we generate numerically the values of the endogenous variables of the model following a 10% increase in agents' preferences towards children ( $\phi$ ) for different levels of  $b$ . The results are displayed in Table 6. The first part of the table refers again to the model solution for  $\phi = 0.0279$  and lower population growth, while the second part of the table refers to the case  $\phi = 0.0307$  and higher population growth.

$\phi = 0.0279$	$b = 0$	$b = 0.3$	$b = 0.7$	$b = 1$
$n^*$	0.0334035 <sup>17</sup>	0.0147588	0.0136375	0.0133641
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.0207115	0.00206677	0.000945506	0.000672125
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.0108088	0.00830991	0.0033075	-0.000482031
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0.0472806	0.015565	0.00637951	0.00028514
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.0580894	0.0238749	0.00968701	-0.000196891
$u^*$	0.69582	0.661493	0.659536	0.65906
$l^*$	0.00334035	0.00147588	0.00136375	0.00133641
$v^* = (1 - u^* - l^*)$	0.30084	0.337032	0.3391	0.339603
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.0359288	0.0377933	0.0379054	0.0379328

<sup>17</sup> See footnote 15.

$\phi = 0.0307$	$b = 0$	$b = 0.3$	$b = 0.7$	$b = 1$
$n^*$	0.0358426 <sup>18</sup>	0.0149665	0.0137324	0.0134316
$g_N^* \equiv \left( \dot{N}_t / N_t \right)^* = (n^* - d)$	0.0231506	0.00227447	0.00104042	0.000739588
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.010557	0.00822608	0.00323122	-0.000556487
$g_A^* \equiv \left( \dot{A}_t / A_t \right)^*$	0.0505614	0.0157508	0.00640747	0.000274651
$g_y^* \equiv \left( \dot{y}_t / y_t \right)^* = g_c^* \equiv \left( \dot{c}_t / c_t \right)^* = g_h^*$	0.0611185	0.0239769	0.00963869	-0.000281836
$u^*$	0.700576	0.661856	0.659701	0.659178
$l^*$	0.00358426	0.00149665	0.00137324	0.00134316
$v^* = (1 - u^* - l^*)$	0.29584	0.336647	0.338926	0.339479
$\left[ \sigma - \varepsilon (n^* - d) \right]$	0.0356849	0.0377726	0.037896	0.037926

**Table 6:** Values of the endogenous variables – Human capital accumulation and R&D under endogenous population growth and class-size effects in education,  $\varepsilon > 0$ , for two different levels of preferences towards children,  $\phi$ , and for different levels of the cost parameter,  $b$ .

Now, we can summarize the main insights from this case as follows.

**RESULT 6:** Consider an economy in which fertility and human capital accumulation are both endogenous and economic growth is driven by technological progress and human capital accumulation together.

- A rise in fertility  $n^*$ , driven by an increase in the preference for children  $\phi$ , triggers a quantity/quality tradeoff between population growth  $g_N^*$  and human capital growth  $g_h^*$ , for any level of  $b$ . This, in turn, implies an indirect negative effect of population growth on economic growth  $g_y^*$ , through the human capital channel.
- However, the same increase in population growth has also a direct positive impact on technological progress,  $g_A^*$  (see Eq. 16). On the whole, the simultaneous increase of  $g_N^*$  and decrease of  $g_h^*$  delivers an ambiguous effect

<sup>18</sup> See footnote 15.

on  $g_A^*$ . As in Case 3, the sign of the overall impact of population growth on technological progress is ultimately dependent upon the magnitude of parameter  $b$ . Again, we see that only when  $b$  is sufficiently large ( $b \rightarrow 1$  in the table), an increase in  $g_N^*$  ultimately leads to a decline in  $g_A^*$  without any ambiguity, meaning that the indirect negative effect of population growth on human capital growth (the human capital channel) prevails on the direct positive effect of population growth on technological progress, and this drives down the growth rate of per capita GDP  $g_y^*$ , as well.

- Once again, since  $g_h^*$  (unlike  $g_A^*$ ) is decreasing in  $g_N^*$  for any level of  $b$ ,  $g_y^*$  will start decreasing in  $g_N^*$  earlier, i.e., for values of  $b$  lower than one, in the table  $b \rightarrow 0.7$ .
- As in the previous Case 3, the effect of population growth  $g_N^*$  on economic growth  $g_y^*$  is therefore ambiguous – the higher the goods cost of children ( $b$ ) is, the more likely it becomes that the relation turns negative, contrary to Jones (2022).
- All in all, introducing explicitly class-size effects into the picture, i.e. setting  $\varepsilon > 0$ , does not qualitatively change the results already obtained in the previous Case 3, where  $\varepsilon = 0$ .<sup>19</sup>

However, since we know that a larger class-size effect contributes to reinforce, *ceteris paribus*, the quantity/quality trade-off (see Appendix D), we may infer that this last effect (namely, a larger  $\varepsilon$ ) reinforces the negative relation between population and economic growth rates.

To summarize, in modern economies that drive the research frontier, declining fertility, and thus lower (perhaps negative) population growth rates have ambiguous effects on economic growth. Whenever the goods-cost of fertility is sufficiently high (such that it is difficult to increase the human capital of newborns to the average human capital level of the whole economy), the direct positive effect of an increase in fertility (driven by a larger preference towards

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<sup>19</sup> Just compare Tables 5 and 6, and the corresponding Results 5 and 6, respectively.

children) on technological progress is overwhelmed by the quantity/quality trade-off, turning the population growth/economic growth nexus negative, contrary to Jones (2022).

All things considered, there are many different situations in which a slowdown of population growth (and even a population decline) may not necessarily be associated with a slowdown of economic growth. In particular, i) Countries that are small and do not drive the research frontier, may rather benefit in terms of economic growth when they face declining population growth or even a shrinking population; ii) For large modern countries that drive the research frontier, the economic growth effect of declining fertility is generally ambiguous. The more costly (in terms of goods) children are, the less likely it is that a slowdown in population growth has negative economic growth consequences.

## 5. Conclusions

We contribute to the debate on the economic consequences of population decline by extending the model of Jones (2022) to explicitly take into account the existence of an endogenous quality-quantity trade-off between fertility and education. Our findings suggest that the negative effects of population decline on economic growth, as they are predicted by standard semi-endogenous and fully-endogenous growth models, can be mitigated under certain conditions. First, we identify considerable heterogeneities across countries that need to be taken into account. Small economies that import technologies from abroad but do not drive the worldwide research frontier may benefit from lower (and perhaps negative) population growth if this comes with faster human capital investment. By contrast, large economies driving the research frontier may experience more ambiguous effects. In these countries, declining fertility could slow down technological progress unless it is mitigated by strong investments in education. Second, we show that the efficiency of schooling is important in this context. Whenever there is a class-size effect (by which education becomes more difficult when the education system is more crowded), the quantity/quality tradeoff is stronger and human capital accumulation can



more easily compensate for falling fertility rates. Finally, we show that the positive effect of falling fertility on economic growth is more pronounced whenever the child costs in terms of goods are sufficiently high.

The policy implications that follow from our results are that countries need not be overly concerned with falling fertility when human capital accumulation can compensate. This is most likely to be the case in small economies that mainly use technologies developed abroad and large economies in which the quantity/quality tradeoff effect is strong, *e.g.*, because of class-size effects. More concretely, investing in education and human capital accumulation and stimulating R&D could offset the negative effects of a slowdown in population growth and of population shrinking because these policies would ultimately reinforce the channels that our paper has identified as the ones through which compensatory effects to fertility decline operate (see also Lutz *et al.*, 2008; Minniti and Venturini, 2017).

In terms of future research, we believe that two possible avenues can be taken starting from the current contribution. The first is more empirically-oriented and would try to get a more precise (and point-wise) estimate of the parameter  $(b)$  that in the current framework measures the goods-cost of rearing children and that seems to play such a fundamental role in assessing the ultimate impact of a population growth slowdown on economic growth. The second is, instead, more theoretically-oriented and would try to assess the effects of introducing migration into the current model. Indeed, migration represents a ‘hot-issue’ not only in economics, but also in policy debates, especially in light of the fact that many countries do not experience population decline mainly through low fertility rates but because their inhabitants seem to face better economic prospects abroad (see also Blanc and Wacziarg, 2025). In this case, it may be particularly the young, well-educated, and most productive parts of the population who emigrate, which would have much more adverse effects on R&D and economic growth than falling fertility *per se*. Considering these forces within a theoretical model as the one proposed here would therefore be a challenging, though relevant, task in the future.

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## ONLINE APPENDICES

### ONLINE APPENDIX A: *Proof of Proposition 1*

The *Hamiltonian function*  $(J_t)$  related to the intertemporal optimization problem (11) in the main text reads as:

$$J_t = \left[ \ln(A_t^\omega u_t h_t) + \phi \ln(n_t - d) \right] \cdot e^{-\rho t} + \lambda_t \left\{ \left[ \sigma - \varepsilon(n_t - d) \right] (1 - u_t - \tau n_t^\omega) h_t - b n_t h_t \right\} + \mu_t (n_t - d) N_t$$

where  $\lambda_t$  and  $\mu_t$  are the co-state variables associated with the two state variables,  $h_t$ , and  $N_t$ , respectively. The (necessary) first-order conditions are:

$$\frac{\partial J_t}{\partial u_t} = 0 \quad \Leftrightarrow \quad \frac{e^{-\rho t}}{u_t} = \lambda_t h_t (\sigma - \varepsilon n_t + \varepsilon d), \quad (\text{A.1})$$

$$\begin{aligned} \frac{\partial J_t}{\partial n_t} = 0 \quad &\Leftrightarrow \\ \frac{\phi e^{-\rho t}}{(n_t - d)} + \mu_t N_t &= \lambda_t h_t \left[ \varepsilon - \varepsilon u_t + \sigma \tau \omega n_t^{\omega-1} - \varepsilon \tau (1 + \omega) n_t^\omega + \varepsilon d \tau \omega n_t^{\omega-1} + b \right], \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_t \quad &\Leftrightarrow \\ \frac{e^{-\rho t}}{\lambda_t h_t} + \left[ \sigma - \varepsilon n_t + \varepsilon d - \sigma u_t + \varepsilon n_t u_t - \varepsilon d u_t - \sigma \tau n_t^\omega + \varepsilon \tau n_t^{1+\omega} - \varepsilon d \tau n_t^\omega - b n_t \right] &= -\frac{\dot{\lambda}_t}{\lambda_t} \end{aligned} \quad (\text{A.3})$$

$$\frac{\partial J_t}{\partial N_t} = -\dot{\mu}_t \quad \Leftrightarrow \quad \frac{\dot{\mu}_t}{\mu_t} = -(n_t - d) \quad \Leftrightarrow \quad \frac{\dot{\mu}_t}{\mu_t} + \frac{\dot{N}_t}{N_t} = 0 \quad (\text{A.4})$$

along with the two transversality conditions:

$$\lim_{t \rightarrow +\infty} \lambda_t h_t = 0 \quad \text{and} \quad \lim_{t \rightarrow +\infty} \mu_t N_t = 0$$

and the two initial conditions:

$$h(0) = h_0 > 0 \quad \text{and} \quad N(0) = N_0 > 0.$$

According to Eq. (A.4),  $\mu_t N_t$  needs, at the optimum, to be constant at any time  $t \geq 0$ . Using the transversality condition,

$$\lim_{t \rightarrow +\infty} \mu_t N_t = 0,$$

it follows immediately that, at a long-run (or BGP) equilibrium, we must have

$$\mu_t N_t = 0, \quad \forall t \geq 0. \quad (A.5)$$

In the light of this result, along a BGP equilibrium, Eq. (A.2) can be recast as:

$$\lambda_t h_t = \frac{\phi e^{-\rho t}}{(n_t - d)} \cdot \frac{1}{\left[ \varepsilon - \varepsilon u_t + \sigma \tau \omega n_t^{\omega-1} - \varepsilon \tau (1 + \omega) n_t^\omega + \varepsilon d \tau \omega n_t^{\omega-1} + b \right]}. \quad (A.2')$$

From (A.1), we get

$$\lambda_t h_t = \frac{e^{-\rho t}}{u_t} \cdot \frac{1}{(\sigma - \varepsilon n_t + \varepsilon d)}. \quad (A.1')$$

Equalizing (A.2') and (A.1') delivers:

$$\frac{\phi}{(n_t - d)} = \frac{\left[ \varepsilon - \varepsilon u_t + \sigma \tau \omega n_t^{\omega-1} - \varepsilon \tau (1 + \omega) n_t^\omega + \varepsilon d \tau \omega n_t^{\omega-1} + b \right]}{u_t (\sigma - \varepsilon n_t + \varepsilon d)}. \quad (A.6)$$

Plugging (A.1') into (A.3) yields

$$-\frac{\dot{\lambda}_t}{\lambda_t} = \sigma - \varepsilon n_t + \varepsilon d - \sigma \tau n_t^\omega + \varepsilon \tau n_t^{1+\omega} - \varepsilon d \tau n_t^\omega - b n_t. \quad (A.3')$$

By using Eqs. (9) and (5) in the main text, (A.3') can ultimately be rewritten as:

$$-\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\dot{h}_t}{h_t} + \sigma u_t - \varepsilon n_t u_t + \varepsilon d u_t \quad (A.3'')$$

or, in a more compact form as:

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{h}_t}{h_t} = u_t (-\sigma + \varepsilon n_t - \varepsilon d). \quad (A.3''')$$

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<sup>20</sup> Recall that the transversality conditions are themselves (necessary) optimality conditions of the dynamic optimization problem. Along a BGP equilibrium,  $\dot{N}_t / N_t = (n - d)$  is constant by definition. By using (A.4), we observe that, along a BGP equilibrium, the transversality condition implies  $\lim_{t \rightarrow +\infty} \mu_t N_t = \lim_{t \rightarrow +\infty} \mu_0 e^{-(n-d)t} N_0 e^{(n-d)t} = \lim_{t \rightarrow +\infty} \mu_0 N_0 = \mu_0 N_0 = 0$ , where  $\mu_0$  and  $N_0$  are the given initial conditions for  $\mu_t$  and  $N_t$ , respectively. In other words, along a BGP equilibrium, the transversality condition requires that  $\mu_0 N_0 = 0$ , or that  $\mu_t N_t = 0$  at  $t = 0$ . Since, by (A.4),  $\mu_t N_t$  needs to be constant at any time  $t \geq 0$ , it immediately follows that this constant must be zero at a BGP equilibrium.



Along a BGP equilibrium,  $u$  remains constant at any time and  $n$  is constant by definition. Hence, the right-hand-side of Eq. (A.3''') is constant. This implies that we can rewrite Eq. (A.3''') as

$$\lambda_t h_t = e^{\Psi_t}, \quad (\text{A.7})$$

with  $\Psi \equiv u(-\sigma + \varepsilon n - \varepsilon d)$  being constant along a BGP equilibrium.

By employing the transversality condition

$$\lim_{t \rightarrow +\infty} \lambda_t h_t = \lim_{t \rightarrow +\infty} e^{\Psi_t} = 0,$$

one can immediately observe that, along a BGP equilibrium, this condition is trivially checked for any  $\Psi < 0$ , namely when (see Eq. (7) in the text):

$$\left[ \sigma - \varepsilon(n - d) \right] > 0, \quad \forall u \in (0, 1). \quad (\text{A.8})$$

From (A.1') we get

$$\ln \lambda_t + \ln h_t = -\rho t - \ln u + \ln \left[ \frac{1}{(\sigma - \varepsilon n + \varepsilon d)} \right]. \quad (\text{A.9})$$

Along a BGP equilibrium,  $u$  and  $n$  are constant, and  $\left[ \sigma - \varepsilon(n - d) \right] > 0$ .

Differentiating (A.9) with respect to time yields

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{h}_t}{h_t} = -\rho. \quad (\text{A.10})$$

Equalizing (A.10) and (A.3''') gives the BGP equilibrium value of  $u$  ( $u^*$ ) as a function of  $n^*$ :

$$u^* = \frac{\rho}{\left[ \sigma - \varepsilon(n^* - d) \right]} > 0, \quad \left[ \sigma - \varepsilon(n^* - d) \right] > 0, \quad \rho \equiv \bar{\rho} + d > 0, \quad (\text{A.11})$$

where  $n^*$  is the BGP equilibrium value of  $n$ . This value ( $n^*$ ) is the solution to the following equation, obtained by combining (A.6) and (A.11):

$$\frac{\phi \rho}{(n^* - d)} = \varepsilon - \frac{\varepsilon \rho}{\left[ \sigma - \varepsilon(n^* - d) \right]} + (\sigma + \varepsilon d) \tau \omega (n^*)^{\omega-1} - \varepsilon \tau (1 + \omega) (n^*)^\omega + b. \quad (\text{A.12})$$

Given  $n^*$  (from A.12) and  $u^*$  (from A.11), Eqs. (2)–(3')–(4)–(5)–(9) in the main text allow us to compute the BGP equilibrium values of:

$$g_N^* \equiv \left( \frac{\dot{N}_t}{N_t} \right)^* = (n^* - d) \gtrless 0,$$

$$l^* = \tau \left( n^* \right)^\omega, \quad \tau \geq 0, \quad \omega \geq 0,$$

$$g_h^* \equiv \left( \frac{\dot{h}_t}{h_t} \right)^* = \left[ \sigma - \varepsilon \left( n^* - d \right) \right] \left( 1 - u^* - l^* \right) - \left( b n^* \right),$$

$$g_y^* \equiv \left( \frac{\dot{y}_t}{y_t} \right)^* = g_c^* \equiv \left( \frac{\dot{c}_t}{c_t} \right)^* = \Omega g_A^* + g_h^*,$$

$$g_A^* \equiv \left( \frac{\dot{A}_t}{A_t} \right)^* = \frac{\Xi}{\chi} \left( g_h^* + g_N^* \right). \quad \blacksquare$$

## ONLINE APPENDIX B: *Proof of Proposition 3*

Using Eqs. (13)-(14)-(15) in the main text, the endogenous growth rate of per capita human capital can ultimately be written as:

$$g_h^* \equiv \left( \frac{\dot{h}_t}{h_t} \right)^* = \sigma - \varepsilon n^* + \varepsilon d - \rho - \tau \sigma \left( n^* \right)^\omega + \tau \varepsilon \left( n^* \right)^{1+\omega} - \tau \varepsilon d \left( n^* \right)^\omega - \left( b n^* \right).$$

To show the existence, at equilibrium, of an endogenous quantity/quality trade-off between fertility and human capital accumulation, we should ultimately be able to demonstrate that

$$\frac{\partial g_h^*}{\partial n^*} = -\varepsilon - \sigma \tau \omega \left( n^* \right)^{\omega-1} + \tau \varepsilon \left( 1 + \omega \right) \left( n^* \right)^\omega - \tau \omega \varepsilon d \left( n^* \right)^{\omega-1} - b < 0.$$

Just for the sake of clarity, as we do in the main text, we continue to keep here two different situations separate, *i.e.*,  $\varepsilon = 0$  and  $\varepsilon > 0$ .

$\varepsilon = 0$ :

When  $\varepsilon = 0$  it is immediate to verify that:

$$\frac{\partial g_h^*}{\partial n^*} = -\sigma \cdot \underbrace{\left[ \tau \omega \left( n^* \right)^{\omega-1} \right]}_{l'(n^*) > 0} - b < 0.$$

This means that an endogenous quantity/quality trade-off exists straightaway when  $\varepsilon = 0$ .

$\varepsilon > 0$ :

When  $\varepsilon > 0$ , a sufficient condition for

$$\begin{aligned} \frac{\partial g_h^*}{\partial n^*} &= -\varepsilon - \sigma\tau\omega(n^*)^{\omega-1} + \tau\varepsilon(1+\omega)(n^*)^\omega - \tau\omega\varepsilon d(n^*)^{\omega-1} - b \\ &= \underbrace{\varepsilon \left[ -1 + \tau(1+\omega)(n^*)^\omega - \tau\omega d(n^*)^{\omega-1} \right]}_{+} \underbrace{- \sigma \cdot \left[ \tau\omega(n^*)^{\omega-1} \right]}_{-} - b < 0 \end{aligned}$$

is to have

$$\left[ -1 + \tau(1+\omega)(n^*)^\omega - \tau\omega d(n^*)^{\omega-1} \right] < 0.$$

By using the same parameter values that we employ in all the numerical analyses performed in the main text,<sup>21</sup> the inequality above reads as:

$$n^* < \frac{1 + (0.1) \cdot (0.012692)}{(0.1) \cdot (2)} = 5.006346,$$

which is surely true given the fact that  $n^*$  is a rate. ■

## ONLINE APPENDIX C: *Proof of Proposition 4*

Here we modify our baseline theoretical model slightly such that we have R&D activity as the sole engine of long-run economic growth. At this aim, we follow Jones (2022, p. 3497 and Table 1) in assuming that the fertility rate ( $n_t \geq 0$ ) is a linear positive function of the fraction of time spent ‘*producing*’ children ( $\beta_t$ ), namely

$$n(\beta_t) = \alpha\beta_t, \quad \alpha > 0, \quad 0 \leq \beta_t \leq 1,$$

and that the population growth rate, the aggregate production function, and the law of motion of ideas are, respectively, provided by:

$$\begin{aligned} \dot{N}_t &= \left[ n(\beta_t) - d \right] \cdot N_t = (\alpha\beta_t - d) \cdot N_t, \\ Y_t &= A_t^\Omega (1 - \beta_t) N_t, \quad \Omega > 0, \\ \dot{A}_t &= \Gamma A_t^{1-\chi} N_t^\Xi, \quad \Gamma > 0, \quad 0 < \Xi \leq 1, \quad \chi > 0. \end{aligned}$$

---

<sup>21</sup> Namely,  $\tau = 0.1$ ,  $\omega = 1$ , and  $d = 0.012692$ , see Table 2 in the main text.

Notice that, if  $\beta_t$  is sufficiently small, the population growth rate is negative, see below. In per capita terms, the aggregate production function can be recast as

$$y_t = c_t = A_t^{\alpha} (1 - \beta_t),$$

where  $y_t \equiv \frac{Y_t}{N_t}$  and  $c_t \equiv \frac{C_t}{N_t}$  still represent per capita real GDP and per capita consumption, respectively. The dynamic optimization problem that the representative agent solves changes to

$$\begin{aligned} \underset{\{c_t, \beta_t, N_t\}_{t=0}^{\infty}}{\text{Max}} \quad U &= \int_0^{\infty} \left[ \ln(c_t) + \phi \ln(\alpha\beta_t - d) \right] \cdot e^{-\rho t} dt, & \rho &\equiv \bar{\rho} + d > 0 \\ \text{s.t. :} & & & \end{aligned} \quad (\text{C.1})$$

$$c_t = A_t^{\alpha} (1 - \beta_t)$$

$$\dot{N}_t = (\alpha\beta_t - d) N_t, \quad N(0) = N_0 > 0.$$

The corresponding *Hamiltonian function* ( $J_t$ ) is therefore

$$J_t = \left\{ \ln \left[ A_t^{\alpha} (1 - \beta_t) \right] + \phi \ln(\alpha\beta_t - d) \right\} \cdot e^{-\rho t} + \lambda_t (\alpha\beta_t - d) N_t,$$

where  $\lambda_t$  is the co-state variable associated with the state variable,  $N_t$ . The control variable is  $\beta_t$ . The (necessary) first order conditions now read as

$$\frac{\partial J_t}{\partial \beta_t} = 0 \quad \Leftrightarrow \quad e^{-\rho t} \left[ \frac{1}{1 - \beta_t} - \frac{\phi \alpha}{\alpha\beta_t - d} \right] = \alpha \lambda_t N_t, \quad (\text{C.2})$$

$$\frac{\partial J_t}{\partial N_t} = -\dot{\lambda}_t \quad \Leftrightarrow \quad -\frac{\dot{\lambda}_t}{\lambda_t} = (\alpha\beta_t - d), \quad (\text{C.3})$$

along with the transversality condition

$$\lim_{t \rightarrow +\infty} \lambda_t N_t = 0,$$

and the initial condition

$$N(0) = N_0 > 0.$$

Along a balanced growth path (BGP),  $n_t(\beta_t) = \alpha\beta_t$  is constant, which immediately implies that  $\beta$  is also constant. Accordingly, (C.2) leads to

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{N}_t}{N_t} = -\rho \quad \Leftrightarrow \quad \lambda_t N_t = e^{-\rho t}. \quad (\text{C.4})$$

Notice that, under (C.4), the transversality condition is trivially checked. Plugging (C.4) into (C.2) yields

$$\left[ \frac{1}{1-\beta} - \frac{\phi\alpha}{\alpha\beta-d} \right] = \alpha \Leftrightarrow \alpha^2\beta^2 - \alpha(-1-\phi+\alpha+d)\beta + [\alpha(d-\phi)-d] = 0. \quad (\text{C.5})$$

Eq. (C.5) can be solved immediately for  $\beta$ . This provides two possible solutions, namely:

$$\beta_1^* = \frac{(-1-\phi+\alpha+d) - \sqrt{(-1-\phi+\alpha+d)^2 - 4[\alpha(d-\phi)-d]}}{2\alpha}, \quad \text{and}$$

$$\beta_2^* = \frac{(-1-\phi+\alpha+d) + \sqrt{(-1-\phi+\alpha+d)^2 - 4[\alpha(d-\phi)-d]}}{2\alpha}.$$

A (sufficient) condition for  $\sqrt{(-1-\phi+\alpha+d)^2 - 4[\alpha(d-\phi)-d]} > 0$ , and therefore for having two distinct and real roots, is  $\alpha(d-\phi)-d < 0$ , which is equivalent to

$$d(1-\alpha) + \alpha\phi > 0.$$

This condition is trivially met for any

$$0 < \alpha < 1,^{22}$$

$$d \geq 0,$$

$$\phi > 0.$$

---

<sup>22</sup> From the definition of population growth,  $\dot{N}/N = [n(\beta) - d] = (\alpha\beta - d)$ , one immediately

obtains  $\alpha = \frac{\left(\frac{\dot{N}}{N}\right) + d}{\beta}$ . According to Jones (2022),  $\beta = 1/8$  (meaning that ‘...the typical person spends about one-eighth of his or her time endowment producing and raising children...’, p. 3509). With  $d = 0.012692$  (this is the death rate that we employ across all our

numerical exercises), it follows that  $\alpha = \frac{\left(\frac{\dot{N}}{N}\right) + 0.012692}{0.125}$ . Thus,  $0 < \alpha < 1$  for any

$-1.27\% < \dot{N}/N < 11.23\%$ . According to the United Nations (UN, 2024), medium variant forecasts), in 2060 the population growth rate  $\left(\frac{\dot{N}}{N}\right)$  of some selected countries will be as follows: France:  $-0.06\%$ ; Germany:  $-0.34\%$ ; Greece:  $-0.80\%$ ; Italy:  $-0.96\%$ ; Japan:  $-0.76\%$ ; UK:  $0.00\%$ ; US:  $+0.13\%$ . Taken singularly, all these numbers fall in the range specified above for  $\dot{N}/N$ . Moreover, their unweighted average is equal to about  $-0.4\%$ , a value that is not far from the one considered in Jones, 2022, p. 3510, Table 2 ( $-0.5\%$ ). Notice that, with this value  $\left(\frac{\dot{N}}{N} = -0.4\%\right)$ , we obtain  $0 < \alpha \cong 0.07 < 1$ .

In what follows, we take this parameterization for granted and, in particular, consider the case where  $\phi > d \geq 0$ , as in all the numerical exercises performed in the main text. It is possible to show that, for any  $0 < \alpha < 1$  and  $\phi > d \geq 0$ , hence  $d(1 - \alpha) + \alpha\phi > 0$ , we have

$$\beta_1^* < 0 \text{ and } \beta_2^* > 0.$$

Thus, we may focus solely on the positive root:

$$\beta_2^* = \frac{(-1 - \phi + \alpha + d) + \sqrt{(-1 - \phi + \alpha + d)^2 - 4[\alpha(d - \phi) - d]}}{2\alpha} > 0.$$

Notice that the equilibrium fertility rate, *i.e.*,

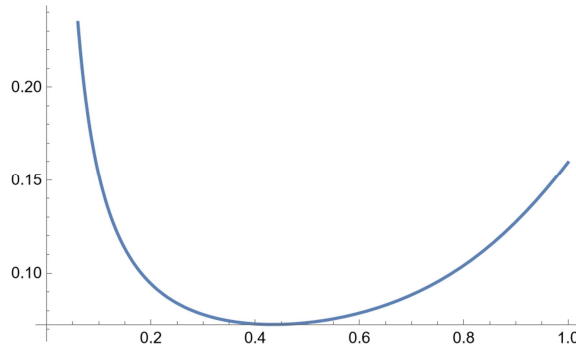
$$n(\beta_2^*) = \alpha\beta_2^*,$$

will be lower than one (as it should be from an empirical point of view) provided that the following restriction on the parameters is checked:

$$\phi(\alpha - 1) + (2 - \alpha)(d - 1) < 0.$$

This condition is immediately met for any  $0 < \alpha < 1$  and  $\phi > d \geq 0$  (see above), and remembering that  $d \leq 1$ , as  $d$  is a rate (the mortality/death rate).<sup>23</sup>

Just to give a simple graphical example, when  $\phi = 0.0279$  and  $d = 0.012692$  (as in the main text),  $\beta_2^*$  and  $\alpha \cdot \beta_2^*$ , behave, as a function of  $\alpha \in (0, 1)$ , respectively as:



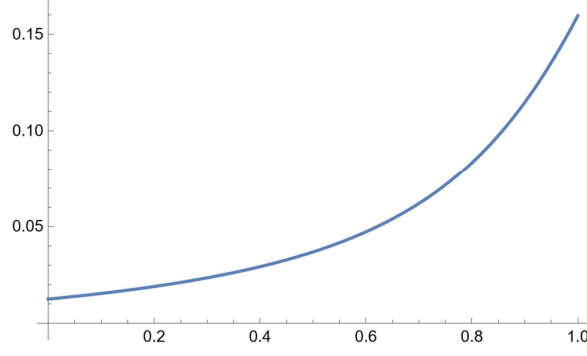
**Fig. C-1:**  $\beta_2^*$ , as a function of  $\alpha \in (0, 1)$ , with  $\phi = 0.0279$  and  $d = 0.012692$

and

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<sup>23</sup> With  $d = 0.012692$ ,  $\phi = 0.0279$  (these are the values that we employ across all our numerical exercises in the main text), and  $\alpha = 0.07$  (see previous footnote), we have

$\sqrt{(-1 - \phi + \alpha + d)^2 - 4[\alpha(d - \phi) - d]} \cong 0.97388 > 0$ ,  $\beta_2 \cong 0.2048 \in (0, 1)$ ,  $\alpha\beta_2 \cong 0.01434 \in (0, 1)$  and  $d(1 - \alpha) + \alpha\phi \cong 0.0138 > 0$ .



**Fig. C-2:**  $n(\beta_2^*) = \alpha\beta_2^*$ , as a function of  $\alpha \in (0,1)$ , with  $\phi = 0.0279$  and  $d = 0.012692$

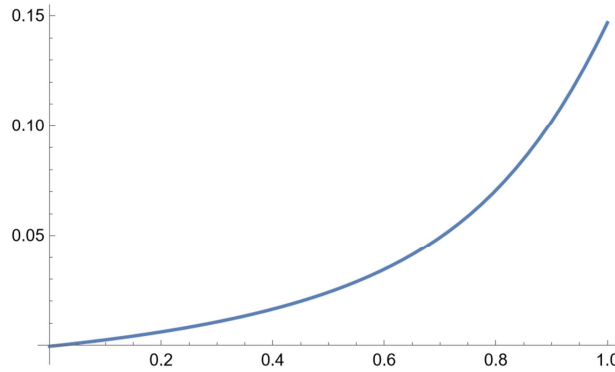
Finally, one can show that, at equilibrium, under the parameterization that we are using [*i.e.*,  $0 < \alpha < 1$  and  $\phi > d \geq 0$ ], the population growth rate,

$$\left( \frac{\dot{N}_t}{N_t} \right)^* = \alpha\beta_2^* - d, \text{ may be either positive or negative or equal to zero}$$

depending, respectively, on whether the parameter  $\alpha$  is larger or smaller or equal to the mortality rate,  $d$ . More precisely, we observe that

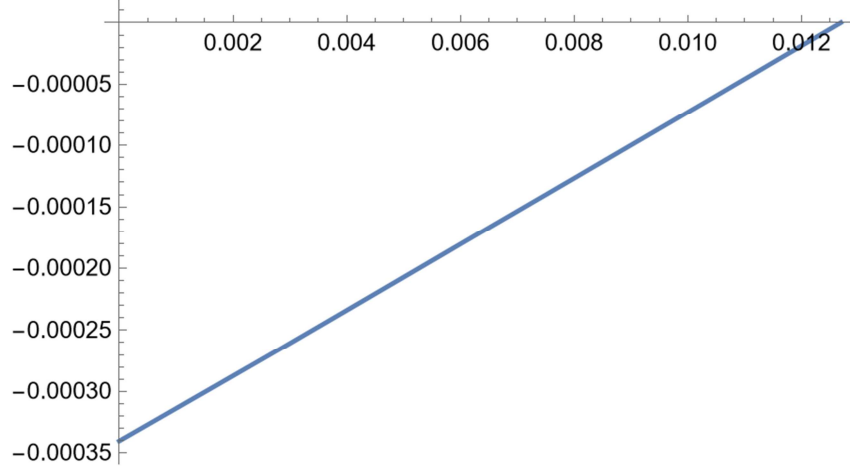
$$\begin{cases} \left( \frac{\dot{N}_t}{N_t} \right)^* > 0, & \text{if } \alpha > d \\ \left( \frac{\dot{N}_t}{N_t} \right)^* < 0, & \text{if } \alpha < d \\ \left( \frac{\dot{N}_t}{N_t} \right)^* = 0, & \text{if } \alpha = d \end{cases}$$

The plot of  $\left( \frac{\dot{N}_t}{N_t} \right)^* = \alpha\beta_2^* - d$  as a function of  $\alpha \in (0,1)$  with  $\phi = 0.0279$  and  $d = 0.012692$  is then as follows:



**Fig. C-3:**  $\left( \frac{\dot{N}_t}{N_t} \right)^* = \alpha\beta_2^* - d$ , as a function of  $\alpha \in (0,1)$ , with  $\phi = 0.0279$  and  $d = 0.012692$

As one can notice, the population growth rate is, indeed, negative for any  $0 < \alpha < d = 0.012692$ :



**Fig. C-4:**  $\left( \dot{N}_t / N_t \right)^* = \alpha \beta_2^* - d$ , as a function of  $\alpha \in (0; 0.012692)$ , with  $\phi = 0.0279$  and  $d = 0.012692$

This ultimately confirms (also graphically) that in the model of this appendix, a negative population growth rate can be supported as a competitive equilibrium outcome with endogenous fertility.

Along a BGP equilibrium, where growth rates are constant, it is easy to obtain

$$g_A^* \equiv \left( \frac{\dot{A}_t}{A_t} \right)^* = \frac{\Xi}{\chi} \left( \frac{\dot{N}_t}{N_t} \right)^* \quad \text{and} \quad g_y^* \equiv \left( \frac{\dot{y}_t}{y_t} \right)^* = g_c^* \equiv \left( \frac{\dot{c}_t}{c_t} \right)^* = \frac{\Omega \Xi}{\chi} \left( \frac{\dot{N}_t}{N_t} \right)^*.$$

Hence, at a competitive equilibrium, economic growth ( $g_y^*$ ) is always a positive function of population growth. In particular, given the ideas production function, the transition from a positive to a negative population growth rate implies that the competitive equilibrium with endogenous fertility features a growth rate of real GDP per capita that over time falls towards zero, so that output per person can ultimately converge to a stagnating level.



Therefore, the so-called ‘*empty planet result*’ can be supported as a competitive equilibrium outcome with endogenous fertility.<sup>24</sup> ■

#### ONLINE APPENDIX D: *The effect of an increase of $\varepsilon > 0$ on the extent of the quantity/quality trade-off (Result 4).*

To draw appropriate conclusions about the consequences (for the extent of the quantity/quality trade-off) of a rise of the degree of the class-size effect ( $\varepsilon > 0$ ) within an economy that invests solely in human capital (and not in R&D), here we use the same parameter values as in Table 2, with two major differences:

- We increase the parameter that measures the preference towards children ( $\phi$ ) from 0.0279 to 0.10;
- We increase  $\varepsilon > 0$  from 0.1 to 0.5.

In this exercise, we keep  $b = 0.3$ , as this seems to be the value that fits the data best across the numerical analyses performed in the main text. In addition, this value appears to be consistent with the results of Donni (2015) on the costs of children. Then we get the following numerical results.

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<sup>24</sup> As in Jones (2022, Fig. 6, p. 3511), during the transition towards the ‘*Empty Planet*’ solution, a constant negative rate of population growth may cause “*knowledge per person*” to increase over time. In this regard, define *knowledge per person* as:

$$x_t \equiv \left( \dot{A}_t / A_t \right)^{-1} = (1/\Gamma) \cdot (A_t^\chi / N_t^\varepsilon). \text{ From this definition, it follows that } \dot{x}_t = \chi - \varepsilon \cdot x_t \cdot \left( \frac{\dot{N}_t}{N_t} \right),$$

which is clearly positive for any  $\left( \frac{\dot{N}_t}{N_t} \right) < 0$ . In words, with a constant negative population

growth rate,  $x_t \equiv (1/\Gamma) \cdot (A_t^\chi / N_t^\varepsilon)$  increases over time. In Jones (2022, p. 3511)’s words: “...intuitively,  $A_t$  rises to an upper bound, while  $N_t$  is falling, which causes  $x_t$  to increase.

The *Empty Planet* steady state occurs as  $x_t$  goes to infinity, so that  $\dot{A}_t / A_t = 1/x_t$  falls to zero...”.

	A	B		C	D	
	<b>b = 0.3</b> <b><math>\varepsilon = 0.1</math></b> <b><math>\phi = 0.0279</math></b>	<b>b = 0.3</b> <b><math>\varepsilon = 0.1</math></b> <b><math>\phi = 0.10</math></b>		<b>b = 0.3</b> <b><math>\varepsilon = 0.5</math></b> <b><math>\phi = 0.0279</math></b>	<b>b = 0.3</b> <b><math>\varepsilon = 0.5</math></b> <b><math>\phi = 0.10</math></b>	
$n^*$	0.0147588	0.0201231	$\Delta n / n =$ +36.35%	0.014184	0.0182778	$\Delta n / n =$ +28.86%
$g_h^* \equiv \left( \dot{h}_t / h_t \right)^*$	0.00830991	0.00614498	$\Delta g_h / g_h =$ -26.05%	0.00794593	0.00465942	$\Delta g_h / g_h =$ -41.36%
			$\frac{\Delta g_h}{\Delta n} \cdot \frac{n}{g_h} =$ -0.72			$\frac{\Delta g_h}{\Delta n} \cdot \frac{n}{g_h} =$ -1.43

We observe that, as  $\phi$  increases, the population growth rate increases, whereas the growth rate of human capital decreases. This is the canonical quantity/quality tradeoff effect. More importantly, we observe that the elasticity of the growth rate of human capital with respect to changes in the growth rate of the population is smaller than 1 in absolute terms when  $\varepsilon = 0.1$  but greater than 1 in absolute terms when  $\varepsilon = 0.5$ . This implies that the quantity/quality tradeoff effect becomes more pronounced when the class-size effect rises.

As a final comment, note that introducing R&D as a second engine of economic growth (in addition to human capital) would not change the nature of the exercise performed above, as both  $n^*$  and  $g_h^*$  depend neither on  $\Xi$  nor on  $\Omega$ , see Eqs. (13), (14), (15), and (18) in the text.

## ONLINE APPENDIX E: *On the goods cost of rearing children ( $b$ )*.

In this Appendix we explain in more detail why using  $b = 0.3$  may be useful, also from an empirical/applied point of view. Indeed, as it is well-known, having an accurate direct estimate of this parameter may prove to be an extremely complicated task (Donni, 2015). Thus, consider Eq. (18) in the text:

$$\frac{\phi\rho}{(n^* - d)} = \varepsilon - \frac{\varepsilon\rho}{\left[\sigma - \varepsilon(n^* - d)\right]} + (\sigma + \varepsilon d)\tau\omega(n^*)^{\omega-1} - \varepsilon\tau(1 + \omega)(n^*)^\omega + b. \quad (\text{E.1})$$

This equation, if solved, would allow us to get the closed-form solution(s) for the endogenous value(s) of  $n^*$ . Starting from this equation, suppose that:

1.  $\varepsilon = 0$
2.  $\tau = 0$ .

The first assumption is employed very often in the numerical analyses performed throughout the main text and rules out the existence of class-size effects in education. The second assumption, instead, simply says that in this simplistic world there is no time-cost associated with child-rearing, but only a goods-cost ( $b$ ). With these two parameter values, (E.1) can immediately be recast as

$$\frac{\phi\rho}{(n^* - d)} = b \quad (\text{E.2})$$

Now, Eq. (E.2) has one straightforward closed-form solution, *i.e.*,

$$n^* = \frac{\phi\rho}{b} + d. \quad (\text{E.3})$$

Notice that this expression is very similar to the one obtained by Barro and Sala-Martin (2004, Section 9.2.2, Eq. 9.59, p. 416) in their continuous-time endogenous fertility model without human capital, and therefore without any quantity/quality trade-off. As in their model, we observe that:

- i. The fertility rate,  $n^*$ , varies one-to-one with the mortality rate,  $d$ ;
- ii. A higher value of  $b$  increases the goods-cost of child rearing and tends, accordingly, to reduce  $n^*$ ;
- iii. A higher value of  $\phi$  increases  $n^*$  by raising agents' preferences towards children;
- iv. A higher  $\rho$  deters investment (in  $h$ ), therefore raises  $n^*$  via the quantity/quality trade-off channel.<sup>25</sup>

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<sup>25</sup> Barro and Sala-i-Martin (2004) have no human capital, hence no quantity/quality trade-off in their model. This explains why in their framework a higher value of  $\rho$ , by just discouraging investment in  $N$ , tends to lower  $n^*$ .

Eq. (E.3) is now the central equation in our analysis. Using, in this equation, the usual parameter values (Table 2 in the text):

$$\begin{aligned}\phi &= 0.0279 \\ \rho &= 0.025 \\ d &= 0.012692,\end{aligned}$$

along with

$$b = 0.3$$

ultimately delivers

$$n^* = \frac{\phi\rho}{b} + d = 0.015, \tag{E.4}$$

which corresponds exactly to the crude birth rate (per 1,000 people) projected worldwide by the World Bank (2025) for year 2040.

This suggests that (under simplifying theoretical assumptions) using  $b = 0.3$ , along with other rather common parameter values (such as  $\phi = 0.0279$ ;  $\rho = 0.025$ ; and  $d = 0.012692$ ), would ultimately allow us to obtain an estimated fertility rate  $n^*$  coinciding exactly with the available medium/long-term forecast (referring to the year 2040) for this variable at the worldwide level.

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