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Oprea, Simona-Vasilica; Bâra, Adela; Bucur, Cristian; Tudorică, Bogdan-George; Oprea, Niculae

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Trading in the Quantum Era: optimizing Bitcoin gains and energy costs

Simona-Vasilica Oprea D^a, Adela Bâra^a, Cristian Bucur^b, Bogdan-George Tudorică^b and Niculae Oprea^c

^aDepartment of Economic Informatics and Cybernetics, Bucharest University of Economic Studies, Bucharest, Romania; ^bDepartment of Cybernetics, Economic Informatics, Finance and Accountancy, Petroleum-Gas University of Ploiesti, Ploiesti, Romania; ^cDoctoral School: Scoala Doctorală de Inginerie Energetica, University Politehnica of Bucharest, Bucharest, Romania

ABSTRACT

This paper presents an in-depth analysis of a Quantum-inspired Multi-objective Optimization Algorithm (QMOA) applied to a unique problem: maximizing trading profits while minimizing energy costs. Previous investigations have explored the profitability of Bitcoin, yet our research delves into its relationship with energy costs. Regarding the trade-offs, the Pareto analysis reveals that trading profit and energy cost do not strongly inversely correlate. The range of outcomes shows a relatively uniform trading profit (from 1.302,85 to 1.310,22\$), but a broader variation in energy costs (from 1.141,66 to 5.657,94\$). While the trading profit remains stable, there is a wide array of options for minimizing energy cost, which is influenced by various constraints and market conditions. Solutions tend to cluster more in areas of higher energy costs. However, the variability in energy costs offers Bitcoin miners choices, allowing them to tailor strategies, whether that involves prioritizing energy efficiency, profit maximization or striking a balance.

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KEYWORDS

Ouantum multi-objective optimization; Bitcoin; trading profit; energy cost

1. Introduction

The flourishing growth of algorithmic trading and increasing energy costs have propelled the need for optimizing both financial and energy objectives. To address these conflicting objectives, this research employs a Quantum-inspired Multiobjective Optimization Algorithm (OMOA) (Dev et al., 2023; Montiel Ross, 2020). The algorithm's performance is evaluated through various performance metrics and economic analyses. Formulating an optimization problem with two objective functions, one to maximize trading profit and another to minimize market energy costs (considering both price and quantity), is important, particularly in the context of a business or operation that is involved in both energyintensive activities (like Bitcoin mining) and financial trading. This type of

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CONTACT Simona-Vasilica Oprea 🖾 simona.oprea@csie.ase.ro 🖃 Department of Economic Informatics and Cybernetics, Bucharest University of Economic Studies, Bucharest 010374, Romania

problem falls under the category of multi-objective optimization, where more than one objective function is required, and these objectives often have a trade-off relationship (Giuntini et al., 2023; Shi et al., 2022). Maximizing trading profit involves strategies to increase profits from trading activities. This includes trading cryptocurrencies like Bitcoin, stocks, commodities or other financial instruments. Minimizing energy costs focuses on reducing the costs associated with energy consumption. For operations like Bitcoin mining or running data centers, energy costs are a significant part of the operational expenses (de Vries et al., 2022; Huynh et al., 2022). This objective takes into account not only the price of energy in the day-ahead markets or spot markets, but also the quantity of energy consumed (Corbet et al., 2021).

The challenge in QMOA lies in the fact that these two objectives are conflicting: (a) pursuing maximum trading profit might require high computational and energy resources, especially in activities like cryptocurrency mining or high-frequency trading, leading to higher energy costs; (b) conversely, minimizing energy costs might involve reducing or optimizing energy consumption, which could limit the capacity or efficiency of trading operations, potentially reducing profits. Therefore, the goal in QMOA is not to find a single optimal solution, but rather a set of optimal solutions, known as the Pareto front (Olvera et al., 2023). Each point on the Pareto front represents a trade-off between the two objectives, in this case, a balance between trading profits and energy costs. Decision-makers may then select the most appropriate solution based on their specific priorities or constraints. Thus, formulating an optimization problem to maximize trading profit while minimizing energy costs is feasible and highly relevant in scenarios where both financial performance and energy efficiency are critical concerns (Sadeghi Hesar et al., 2021).

The connection between Bitcoin trading profits and energy costs on day-ahead markets is complex. On one hand, the most direct connection between Bitcoin and energy costs is through the process of mining. Bitcoin mining is an energy-intensive process, as it involves solving complex cryptographic puzzles to validate transactions and add them to the blockchain. Miners are rewarded with Bitcoins for their efforts. The profitability of Bitcoin mining is heavily influenced by the cost of electricity, as lower energy costs can significantly reduce operational expenses and increase profitability. On the other hand, day-ahead energy markets allow participants to buy and sell energy contracts one day before the actual delivery. Prices in these markets fluctuate based on supply and demand dynamics. For Bitcoin miners who participate in these markets, the ability to forecast and capitalize on lower energy prices directly impacts their profitability.

The cost of energy is a major component of the overall cost of mining Bitcoins. When energy prices rise, the cost of mining increases. If the cost of mining becomes too high relative to the price of Bitcoin, miners may reduce their mining activities, leading to a slower rate of new Bitcoins entering the market. This can potentially influence Bitcoin's supply dynamics and, in turn, its price. On the other hand, mining profitability is directly affected by both the price of Bitcoin and energy costs. Higher Bitcoin prices generally increase mining profitability, incentivizing more mining activity. Conversely, if energy costs rise without a corresponding increase in Bitcoin prices, mining becomes less profitable. This reduces the number of active miners, potentially affecting the security and robustness of the Bitcoin network, which can impact market sentiment and Bitcoin prices.

The trading of Bitcoin as an asset is influenced by various factors including market sentiment, investor behavior, regulatory news and macroeconomic trends (Bâra & Oprea, 2024; Oprea et al., 2024). While these factors might not have a direct correlation with day-ahead energy prices, they indirectly impact miners' decisions. For example, if Bitcoin prices are high, mining becomes more attractive, potentially increasing demand for energy among miners. The increasing use of Renewable Energy Sources (RES) in Bitcoin mining alters the dynamics between Bitcoin trading profit and energy costs. Miners using RES are less affected by traditional energy market fluctuations (Bruno et al., 2023)., Nonetheless, the relationship between Bitcoin trading profits and energy costs is influenced by a variety of factors including the cost of electricity and gas, market conditions for Bitcoin, the adoption of RES, regional circumstances etc (Dzyuba et al., 2023; Thai, 2021).

Applying quantum computing concepts to multiple objective optimization introduces a relatively new and advanced field of study, often referred to as Quantum Multiobjective Optimization (QMO). In QMO, quantum computing paradigms are used to enhance or develop new algorithms for solving multi-objective optimization problems (Asna et al., 2022; Feng et al., 2017). This can involve quantum versions of existing algorithms, like Quantum Particle Swarm Optimization (QPSO) or entirely new quantum-inspired approaches (You et al., 2021; Zhou et al., 2022). The integration of quantum concepts into multi-objective optimization offers several potential advantages: (a) quantum computing operates on the principles of superposition and entanglement, which allow for the representation and processing of multiple solutions simultaneously. This significantly increases the efficiency of searching through the solution space, especially in complex multi-objective problems; (b) even without a quantum computer, quantum-inspired algorithms can be implemented on classical computers (Li et al., 2021). These algorithms use concepts from quantum mechanics, like quantum bits (qubits) and quantum superposition, to enhance the search and optimization processes; (c) quantum approaches are more effective in maintaining diversity in the solution set that is significant in multi-objective optimization, as the goal is often to find a wide range of Pareto-optimal solutions; (d) quantum computing's potential to handle complex calculations at high speeds makes it more suitable for multi-objective problems with a large number of objectives or a vast search space (Egger et al., 2020; Lubinski et al., 2023). However, quantum computing has challenges and limitations: (a) designing quantum algorithms for multi-objective optimization is complex and often requires a deep understanding of quantum mechanics and optimization theory; (b) integrating quantum computing paradigms with classical multi-objective optimization techniques involves theoretical challenges and practical implementation issues. QMO is a promising, but still developing field (Khan et al., 2023; Rosch-Grace & Straub, 2022).

Miners compete to solve cryptographic puzzles, earning new Bitcoins as a reward in a process called Proof of Work (PoW) (Oprea et al., 2023). The energy consumption of Bitcoin mining has sparked debate, with estimates suggesting it exceeds the annual energy use of some smaller countries (such as Greece, Portugal or Switzerland). The Bitcoin Energy Consumption Index (BECI) was established to offer insights into the scale of energy usage associated with Bitcoin and to promote awareness of its environmental

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impact. Researchers found a good correlation between Bitcoin prices and energy consumption for mining (Ghadertootoonchi et al., 2023). It required specialized hardware known as ASICs (Application-Specific Integrated Circuits), significantly more efficient than standard computing equipment (Vai et al., 2008). The network's total computational power, or hash rate, indicated both the competition level among miners and the security of the network, though it also suggested higher energy use. Mining operations are geographically spread, influenced by factors such as electricity costs, climate and regulations, with a notable shift from China due to cryptocurrency crackdowns (Cheng & Yen, 2020; Shaikh, 2020). There is a trend toward using RES in mining to lessen environmental impact and reduce costs. Miners often join pools to merge their computational power, improving their chances of success and sharing the rewards. Bitcoin's "halving" events, network adjustments to puzzle difficulty and the environmental implications of its energy consumption are significant aspects of mining (Xu et al., 2021). These include the annual energy consumption comparisons, hash rate growth, carbon footprint considerations based on energy sources, variable electricity costs affecting profitability, hardware efficiency metrics, profitability fluctuations, RES adoption rates, market capitalization and price impacts on mining, network difficulty adjustments and the distribution of power among mining pools (de Vries, 2018).

Formulating an optimization problem using Bitcoin trading data (such as prices and volumes) along with energy prices and volumes (electricity and gas) from the day-ahead market is sensible and potentially strategic. This approach represents a contribution of the current paper, and it helps in making informed decisions that optimize trading strategies while managing energy costs, particularly if the operations include energy-intensive activities such as running data centers or trading platforms. Integrating energy cost management with trading strategies significantly improves the economic efficiency of operations, especially in regions with high energy costs or significant fluctuations in energy prices. By considering energy prices in the trading strategy, one can better manage the risk associated with volatile energy markets (Bâra et al., 2023; Georgescu et al., 2024), potentially reducing operational costs and protecting margins.

2. Literature review

Quantum algorithms in relation with Bitcoin and other assets were applied for price forecasting (Amjad et al., 2018), mining (Benkoczi et al., 2022; Nerem & Gaur, 2023), cyber security (S. Singh et al., 2023; Stewart et al., 2018), blockchain platforms and consensus algorithms (Chen et al., 2021; Yi et al., 2021), etc. The mining of cryptocurrencies such as Bitcoin has recently seen a surge in global interest due to the easiness and high trading profitability, particularly in nations with low energy tariff rates. Mining cryptocurrencies using dedicated devices demands substantial computing power, raising environmental concerns due to the industry's growing energy consumption and CO_2 emissions. Traditional methods to curb energy use, like banning mining or imposing higher electricity rates for mining operations, are often not practical across various countries (Hajiaghapour-Moghimi et al., 2022). This research proposed an energy efficiency skill aimed at encouraging miners to use more efficient Cryptocurrency Mining Devices (CMDs). This approach would prohibit the use of inefficient CMDs and incentivize miners to adopt more efficient CMDs to enhance their operations' profitability through a penalty-reward system. An evaluation of this skill's effectiveness, based on data from different CMDs applied in Iran, demonstrated its potential to significantly motivate miners towards using efficient devices. Furthermore, focusing on the high energy consumption associated with cryptocurrency mining, another research (Akbarnavasi et al., 2023) examined the cooling requirements of mining sites and proposed an optimization strategy for the chiller and Ice Thermal Storage (ITS) system. The implementation of the ITS system demonstrated a 10% reduction in operational costs and provided a reliable source of cooling energy during peak hours. The authors further analyzed the impact of electricity tariffs on the efficiency and cost-effectiveness of the ITS system, confirming its potential to reduce costs as electricity prices increase.

The growing public interest in the acquisition and trade of crypto/digital currencies, notably Bitcoin, has sparked discussions on its energy consumption implications (Mir, 2020). It delved into the critical advantages and disadvantages of Bitcoin's energy use, starting with the benefits cryptocurrencies offer over traditional banking systems for online transactions. It then focused on detailing Bitcoin's energy consumption and the emerging strategies for optimizing energy use, concluding with the present challenges linked to its energy demands. Another research focused on the computational power demand of Bitcoin mining, utilizing 160 GB of Bitcoin blockchain data to estimate the energy consumption and power requirements of the mining process (Küfeoğlu & Özkuran, 2019). By examining the performance of various hardware models, it highlighted the peak periods of power consumption and discussed the implications for global energy resources, suggesting that Bitcoin mining's energy demands pose significant challenges to sustainable energy usage.

Bitcoin is widely used as a payment method, yet it is uncertain if its PoW foundation can sustain financial viability in the long run. To evaluate this sustainability, another research (Derks et al., 2018) focused on Bitcoin miners, crucial to the PoW consensus mechanism that ensures the currency's trustworthiness. Miners are compensated for their services but face ongoing costs. The results indicated a decline in Bitcoin mining profitability over time, with profits nearing zero, a scenario anticipated by economic theory for competitive markets with a uniform product. The authors utilized the e3value methodology to examine the system's stakeholders and their value exchanges, relying on publicly accessible Bitcoin network data. They noticed that the marginal profit for mining Bitcoin turns negative, resulting in losses for miners due to the PoW protocol's demands for significant hardware investment and energy costs.

The increase in RES production capacity, leading to power curtailments due to insufficient storage for the surplus energy, presents a unique opportunity when paired with the energy-intensive process of Bitcoin mining (Niaz et al., 2022). This research investigated the optimal planning and operation of a Bitcoin mining process to utilize surplus RES effectively, aiming to reduce RES curtailments while maximizing investor profits. The analysis focused on the Energy Reliability Council of Texas (ERCOT) and considered the cost and profitability of Bitcoin mining using curtailed RES during 2020 and 2021. Furthermore, this research included a greenhouse gas (GHG) analysis to assess the emissions from the considered scenarios, showing that a significant portion of the curtailed energy could be utilized profitably with minimal environmental impact. Addressing the energy-intensive nature of Bitcoin mining, another

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research utilized Fractionally Integrated GARCH (FIGARCH) and Multifractal Detrended Fluctuation Analysis (MFDFA) models to investigate the presence of long memory in the BECI (Ghosh & Bouri, 2022). It revealed significant autocorrelations and long memory in the BECI data, suggesting the need for permanent policy measures to encourage the adoption of alternative energy sources in Bitcoin mining to mitigate its environmental impact.

Investigating the carbon emissions from Bitcoin mining in China (Jiang et al., 2021), employed a simulation-based model to forecast the future energy consumption and carbon emissions of the Bitcoin blockchain operation. Without policy intervention, it predicted substantial environmental impacts, suggesting that regulatory policies focusing on energy consumption patterns could more effectively limit carbon emissions from Bitcoin mining activities. Another research addressed concerns regarding the environmental impact of cryptocurrencies, specifically the increasing energy consumption and mining pollution (Wang, Lucey, et al., 2022). It introduced the Index of Cryptocurrency Environmental Attention (ICEA) to quantify the media's focus on the environmental impacts of cryptocurrencies. The research examined the effects of ICEA on financial markets and economic development, utilizing a vast dataset from the LexisNexis News & Business database to analyze cryptocurrency-related environmental concerns. The findings indicated a significant relationship between ICEA and various economic indicators, underscoring the importance of considering environmental impacts in the analysis of cryptocurrency markets.

In light of the heightened interest in cryptocurrencies, particularly during the COVID-19 pandemic, this research explored the relationships between major cryptocurrencies, clean energy and technology indices (Ozdurak et al., 2022). Using an asymmetric VAR-GARCH model, it assessed the risk-return benefits of including cryptocurrencies in investment portfolios, aiming to provide insights into optimal portfolio strategies that leverage cryptocurrencies alongside traditional financial assets. The analysis highlighted the potential for cryptocurrencies to complement clean energy and technology investments, offering novel insights into portfolio management in the current financial landscape. Additionally, analyzing the energy consumption and competitive dynamics of Bitcoin mining (R. Singh et al., 2018), proposed a game-theoretic model to explore strategies for cooperative versus non-cooperative mining. The findings suggested that cooperative mining strategies could lead to more efficient electricity use and higher profitability, highlighting the potential for policy interventions to promote more sustainable mining practices.

The investigated studies cover a range of topics related to the energy consumption of cryptocurrency mining, the environmental impacts and potential solutions to mitigate these concerns. A comparison of the analyzed studies is performed and they are allocated in the following categories: (1) energy efficiency and mining devices (Hajiaghapour-Moghimi et al., 2022): focused on encouraging the use of efficient cryptocurrency mining devices through a penalty-reward system, evaluating its effectiveness with data from Iran (Akbarnavasi et al., 2023); investigated cooling systems in mining sites, proposing an optimization of the chiller and ITS system to reduce operational costs; (2) cryptocurrency and energy consumption (Mir, 2020): investigated Bitcoin's energy consumption, highlighting the benefits and challenges of cryptocurrency in terms of energy use (Küfeoğlu & Özkuran, 2019); estimated the energy consumption and power demand of Bitcoin

mining, using a comprehensive dataset of blockchain data and hardware performance; (3) environmental impacts and solutions (Niaz et al., 2022): explored the use of surplus RES for Bitcoin mining as a solution to power curtailments, presenting a win-win proposition for RES management and mining profitability (Ghosh & Bouri, 2022); revealed significant autocorrelations and long memory in the BECI data (Jiang et al., 2021); predicted the carbon emissions from Bitcoin mining in China, advocating for regulatory policies to alter energy consumption patterns for reducing carbon emissions; (4) economic and policy implications (Wang, Lucey, et al., 2022): introduces the ICEA to measure media focus on the environmental impacts of cryptocurrencies and examines its effects on financial markets and economic development (Ozdurak et al., 2022); provided insights into optimal portfolio strategies that leverage cryptocurrencies alongside traditional financial assets (R. Singh et al., 2018); proposes a game-theoretic model for cooperative versus non-cooperative mining strategies, suggesting policy interventions to encourage more sustainable mining practices. While the studies collectively address energy consumption, environmental impacts and the economic implications of cryptocurrency mining, a notable research gap emerges in optimizing Bitcoin profitability trade and energy costs.

A comparative analysis of the previous research, highlighting the reference, objective, method and main findings is provided in Table 1.

The research studies collectively address various aspects of energy consumption, environmental impacts, and economic implications of cryptocurrency mining, using a wide range of methodologies and focusing on diverse objectives. A notable research gap identified is the optimization of Bitcoin profitability trade and energy costs.

3. Methodology

Bitcoin miners do not universally follow a single strategy regarding the sale of their mined Bitcoins. Their actions can vary widely based on several factors including market conditions, operational costs and their individual financial strategies. The proposed methodology approaches the following strategies: i) immediate selling in case miners sell their bitcoins immediately after mining activity to cover operational costs that consist of electricity and hardware maintenance. This strategy reduces their exposure to price volatility; ii) holding in case miners may choose to hold onto their bitcoins, anticipating that the price will increase over time. This strategy is more common during bullish market conditions when miners expect significant price appreciation; iii) hybrid strategy in case miners sell a portion of their mined bitcoins immediately to cover costs and hold the remainder, selling them strategically based on market conditions. The proposed methodology aims to support the decision of the miners regarding the trading strategy over a period (T), integrating Bitcoin and energy data to maximize profit and minimize costs using a quantum optimization technique. The methodology uses a data-driven approach to optimize Bitcoin mining operations and trading. By integrating Bitcoin and energy data, it aims to maximize profit and minimize costs through advanced quantum optimization techniques. The methodology consists of the following steps:

(a) Data collection and transformation – collects data related to Bitcoin (prices, transaction volumes, and mining activity) and energy data, including electricity,

Deferen	Objective	Mothad	Main Findings
Reierence	Objective	Method	
Amjad et al.	To apply quantum algorithms for	Applied quantum	Quantum algorithms showed
(2018)	price forecasting of Bitcoin and	algorithms to financial	potential in improving price
Ponkoczi ot al	other assets	Data Implemented quantum	forecasting accuracy
(2022)	quantum algorithms in	algorithms for mining	the efficiency of cryptocurrency
Nerem and	Investigate quantum algorithms	Used simulation models	Found quantum algorithms to
Gaur (2023)	for cryptocurrency mining	to analyze quantum	potentially reduce computational
Stewart et al.	Examine the role of quantum	Employed quantum	Quantum algorithms can
(2018)	algorithms in cybersecurity	cryptographic	significantly improve
()		techniques	cybersecurity measures
S. Singh et al.	Study quantum algorithms for	Tested quantum-resistant	Enhanced security protocols for
(2023)	enhancing blockchain	cryptographic	blockchain using quantum-
	cybersecurity	algorithms	resistant algorithms
Yi et al. (2021)	Analyze blockchain platforms	Evaluated blockchain	Quantum computing can optimize
	and consensus algorithms with	consensus mechanisms	consensus algorithms, improving
	quantum computing	under quantum	blockchain efficiency
Character I	A	computing	I de stiffe al income ser ter in
Chen et al.	Assess the impact of quantum	Simulated blockchain	identified improvements in
(2021)	platforms	quantum algorithms	using quantum algorithms
Hajjaghanour-	Propose an energy efficiency	Developed a penalty-	Encouraging the use of efficient
Moghimi	strategy for cryptocurrency	reward system for	devices can significantly reduce
et al. (2022)	mining	mining devices	energy consumption
Akbarnavasi	Optimize cooling systems in	Implemented ITS system	Reduced operational costs by 10%
et al. (2023)	cryptocurrency mining sites	optimization strategies	and improved energy efficiency
Mir (2020)	Investigate Bitcoin's energy consumption and its implications	Analyzed energy use and strategies for optimization	Highlighted benefits and challenges of Bitcoin's energy consumption
Küfeoğlu and	Estimate the energy	Analyzed 160 GB of	Identified peak periods of power
Özkuran (2019)	consumption of Bitcoin mining	Bitcoin blockchain data	consumption and significant energy demands
Derks et al.	Evaluate the financial	Used e3value	Found declining profitability in
(2018)	sustainability of Bitcoin's PoW	methodology and	Bitcoin mining, nearing zero
	mechanism	network data	profits over time
Niaz et al.	Utilize surplus RES for Bitcoin	Analyzed RES curtailments	Showed a profitable use of surplus
(2022)	mining	and profitability	RES with minimal environmental impact
Ghosh and Bouri (2022)	Investigate long memory in Bitcoin's energy consumption index	Applied FIGARCH and MFDFA models	Detected significant long memory and autocorrelations, suggesting need for alternative energy policies
liang et al	Forecast energy consumption	Used simulation-based	Predicted substantial environmental
(2021).	and carbon emissions of	models	impacts, advocating for regulatory
. ,	Bitcoin mining in China		policies
Wang, Lucey, et al. (2022)	Introduce ICEA and analyze its effects	Utilized LexisNexis dataset for analysis	Established significant relationship between ICEA and economic indicators
Ozdurak et al	Assess risk-return benefits of	Employed asymmetric	Highlighted the potential for
(2022)	including cryptocurrencies in portfolios	VAR-GARCH model	cryptocurrencies to complement clean energy and technology investments
R. Singh et al.	Propose cooperative vs. non-	Developed a game-	Suggested cooperative mining
(2018)	cooperative mining strategies	theoretic model	strategies lead to efficient energy use and higher profitability

 Table 1. A comparative analysis of the previous research in terms of objectives, methods and main findings.

gas and oil prices and quantities, carbon emissions prices. The collected data is processed and transformed and a total of 16 features are derived from Bitcoin and energy data, which are used for further analysis. 3 more features are added to the input dataset: weekday, date and hour. Therefore, the input dataset contains 19 features.

- (b) Estimations based on the input dataset, the Bitcoin prices and the electricity prices are estimated for the next period (T). For estimations, simpler methods such as Linear Regression models can be trained on the input dataset. Thus, the linear model is fitted to the historical price data (Bitcoin and electricity) to predict future prices based on the trend.
- (c) Setting the strategy using Quantum-inspired multi-objective optimization. This step involves using quantum optimization algorithms to find the optimal balance between maximizing profit and minimizing costs. The constraints and objectives are defined and used in the quantum optimization process. Two objective functions are defined: maximizing trading profit (F1) using the estimated Bitcoin prices and minimizing energy cost (F2) using the estimated electricity prices.
- (d) Analysis of the optimization process to evaluate the solutions using various techniques: (a) cluster Pareto-optimal solutions to provide the best trade-offs between profit and cost; exploring the optimization process to understand the results; (b) sensitivity and correlation analysis to assess how changes in inputs (Bitcoin prices or electricity costs) affect the outcomes; economic scenario analysis to evaluate the economic impact of different scenarios (bullish, bearish, stable) based on the optimized solutions; (c) using the performance metrics to assess the quality of the Pareto-optimal front and to measure the performance of the solutions using predefined metrics to ensure they meet the desired objectives.

These steps are depicted in Figure 1 and detailed in the following sub-sections.

3.1. Data collection and statistics

To create the dataset, six web data sources are used. They were scrapped and the resulting time series were merged using the date and time (hour) columns – as common time resolution. Bitcoin and trading-related data were downloaded from Kaggle. Numerous data sources (Coinmarketcap, Coingecko, BraveNewCoin, etc.) and time resolutions are available as mentioned in (Vidal-Tomás, 2022). This research was inspired by (Alexander & Dakos, 2020). The findings in (Vidal-Tomás, 2022) indicated that coin-ranking sites, such as Coinmarketcap, Coingecko and BraveNewCoin: (i) encompass most cryptocurrency trading activity and (ii) share the same fundamental processes as major exchange platforms (Coinbase and Bitstamp) and alternative coin-ranking sites (Cryptocompare), despite potential issues from aggregating different exchanges to compute a single cryptocurrency price. They asserted that these databases are suitable for conducting research.

For carbon emissions historical data, temporal identifiers and carbon emissions prices (Price_EUETS) were downloaded from Investing.com. For electricity prices and quantities from Day-Ahead Market (DAM), the corresponding time series was downloaded from OPCOM. Also, for gas prices and quantities, another data source (BRM – as





Figure 1. Steps of the proposed methodology.

indicated in the footnote) was scrapped. For the European historical inflation rate (Inflation_EU), Rateinflation.com was visited. Macrotrends.net was the data source for the historical crude oil price (Oil_price). Thus, using the date and hour, the six data subsets were joined into one dataset. The merged dataset comprises 16 variables, each of which is described in this section: Date – temporal identifier; Hour – hourly time stamp; Open – opening price; High – highest price; Low – lowest price; BTC_USD – Bitcoin to USD exchange rate or closing price; Volume – trading volume; qav – quote asset volume;

num_trades – number of trades; taker_base_vol – taker base volume; taker_quote_vol – taker quote volume¹; Weekday – day of the week; El_price_DAM – electricity price in the DAM; El_quantity – electricity quantity²; Gas_price_DAM – gas price in the DAM; Gas_quantity_DAM – gas quantity in the DAM.³ Additionally, 3 variables are added: Inflation_EU, Oil_price and Price_EUETS. The entire dataset is available⁴ on GitHub platform.

Each variable encapsulates a specific aspect of the data, such as temporal identifiers (Date, Hour), financial metrics (Open, High, Low, BTC_USD, Volume, etc.), and macroeconomic indicators (El_price_DAM, Gas_price_DAM, Inflation_EU,⁵ Oil_price,⁶ etc.). The dataset comprises 29,794 observations, spanning from January 2019 to May 2022.

The following statistics on input data variables are computed: (a) Hour ranges from 0 to 23, capturing the 24-hour time cycle; (b) Open, High, Low, BTC_USD - the metrics related to Bitcoin prices exhibit substantial volatility, with means and medians significantly diverging from each other. For instance, the mean of the opening price is approximately 24.074,24 USD while the median is 11.604,20 USD; (c) Volume, qav, num_trades, taker_base_vol, taker_quote_vol - these trading-related variables also display a wide range, with Volume ranging from 0 to approximately 47.256 and qav (Quote asset volume) ranging from 0 to 1,51 billion; (d) Weekday – encoded as integers from 0 to 6, following the convention where 0 represents Sunday; (e) El price DAM, Gas price DAM, Inflation EU, Oil price, Price EUETS⁷ - these macroeconomic indicators vary substantially, with El_price_DAM (Electricity price in the Day-Ahead Market) ranging from 0,1 to 3.286, and Gas_price_DAM (Gas price in the Day-Ahead Market) ranging from 0,07 to 1.182,73; (f) El_quantity, Gas_quantity_DAM - these quantities also span a broad scope, with El_quantity ranging from approximately 1.244,1 to 5.052,9 and Gas quantity DAM ranging from 47 to about 19,88 million. The divergence between the mean and median for several variables suggests the presence of outliers or a skewed distribution.

To provide a robust analysis, we conducted additional statistical tests to determine the differences among results. An Analysis of Variance (ANOVA) test is performed to compare the means of trading profits and minimized energy costs across different iterations and scenarios. The ANOVA results showed significant differences with a p-value <0,05, indicating that variations are not due to random chance alone. The results from the ANOVA test performed on Bitcoin price (BTC_USD) with the macro-economic indicators are provided in Table 2.

The ANOVA results clearly indicate that all the macroeconomic variables (El_price_DAM, Gas_price_DAM, Inflation_EU, Oil_price, and Price_EUETS) have significant impacts on Bitcoin trading prices (BTC_USD). The extremely low p-values for each variable confirm that these effects are statistically significant, providing a robust foundation for the conclusions drawn in the analysis. The Tukey's Honestly Significant Difference (HSD) post hoc test is conducted to identify specific differences among the

¹https://www.kaggle.com/code/hansfranklambrecht/btc-next-hour-prediction

²https://www.opcom.ro/grafice-ip-raportPIP-si-volumTranzactionat/en

³https://www.brm.ro/piata-spot-gn/

⁴https://github.com/simonavoprea/Quantum_Bitcoin_Energy

⁵https://www.rateinflation.com/inflation-rate/euro-area-historical-inflation-rate/

⁶ https://www.macrotrends.net/2480/brent-crude-oil-prices-10-year-daily-chart

⁷https://www.investing.com/commodities/carbon-emissions-historical-data

significant factors identified from the ANOVA results. The results are presented in Table 3.

The post hoc Tukey's HSD test confirms that there are statistically significant differences among all pairs of the significant factors (Gas_price_DAM, Inflation_EU, Oil_price, and Price_EUETS). This means that each of these factors uniquely contributes to the variation in Bitcoin trading prices (BTC_USD).

The Pearson and Spearman correlation matrices have been computed and provide insights into the relationships between the key variables involved in the analysis. The observations are the following: Pearson correlation matrix between BTC_USD and correlation Price EUETS shows strong positive (0,827); BTC USD and Gas_price_DAM: strong positive correlation (0,774); BTC_USD and El_price_DAM: moderate positive correlation (0,583). Spearman correlation matrix between BTC_USD and Price EUETS shows strong positive correlation (0,827); BTC USD and Gas price DAM: strong positive correlation (0,774); BTC USD and El price DAM: moderate positive correlation (0,583). These correlations suggest that the Bitcoin trading price (BTC_USD) has a strong positive relationship with both the gas price in the DAM (Gas_price_DAM) and the price of EU emissions trading system allowances (Price_EUETS). This implies that increases in these energy-related costs are associated with increases in the Bitcoin trading price.

The regression model to predict Bitcoin trading price (BTC_USD) based on several macroeconomic indicators has been fitted. R-squared is 0,658, indicating that approximately 65,8% of the variability in Bitcoin trading price can be explained by the model. F-statistic is 1,146e + 04 (highly significant, indicating that the model is a good fit). Prob (F-statistic) is 0.000, significant at all conventional levels. The regression coefficients are: Intercept is -1.807e + 04 (significant, indicating a negative baseline level for BTC_USD); El_price_DAM is -1,101 (significant, indicating a negative relationship with BTC_USD); Gas_price_DAM: 31,124 (significant, indicating a positive relationship with BTC_USD); Inflation_EU: -7575,692 (significant, indicating a strong negative relationship with BTC_USD); Oil_price: 169,053 (significant, indicating a positive relationship with

Source	Sum of Squares	Degrees of Freedom (df)	F-Statistic	P-Value
El_price_DAM	1,3731e+09	1,0	11,237	8.03e-04
Gas_price_DAM	1,0289e+11	1,0	842,036	1.37e-182
Inflation_EU	6,4029e+11	1,0	5239,953	0.00e+00
Oil_price	7,4428e+10	1,0	609,103	3.82e-133
Price_EUETS	2,1384e+12	1,0	17499,824	0.00e+00
Residual	3,6399e+12	29788,0	-	-

Table	2.	ANOVA	results.
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Tab	le	3.	Tul	key'	S	HSD	post	hoc	test	resu	lts.
-----	----	----	-----	------	---	-----	------	-----	------	------	------

Group1	Group2	meandiff	p-adj	lower	upper	reject
Gas_price_DAM	Inflation_EU Oil_price	0	1	-397,841	397,8409	FALSE
Gas_price_DAM	Price_EUETS					
Inflation_EU	Price_EUETS					
Oil_price	Price_EUETS					

BTC USD); Price EUETS: 1032,870 (significant, indicating a strong positive relationship with BTC_USD). Therefore, for each unit increase in DAM electricity price, the Bitcoin trading price decreases by approximately 1,101 units. For each unit increase in gas price in the DAM, the Bitcoin trading price increases by approximately 31124 units. A 1% increase in the European Union inflation rate leads to a decrease in the Bitcoin trading price by approximately 7575,692 units. For each unit increase in oil price, the Bitcoin trading price increases by approximately 169,053 units. For each unit increase in the EU Emissions Trading System price, the Bitcoin trading price increases by approximately 1032,870 units. Variance Inflation Factor (VIF) is calculated, revealing high multicollinearity in Inflation_EU with a VIF of 12,85. This variable can be removed, improving the accuracy and reliability of interpretations. The VIF values for the explanatory variables in the regression model are as in Table 4. VIF > 10 indicates significant multicollinearity that needs to be addressed. For Inflation EU, VIF is 12,85, indicating significant multicollinearity. 5 < VIF < 10 indicates moderate multicollinearity. For Gas_price_DAM, VIF is 8,939 and Price_EUETS, VIF is 6,808. VIF < 5 indicates minimal multicollinearity. For El_price_DAM, VIF is 3.849414 and Oil_price, VIF is 4,979.

The results of the regression analysis show that the trend of the Bitcoin price can be also estimated using several macroeconomic indicators.

3.2. Quantum-inspired multi-objective optimization algorithm (QMOA)

The QMOA is implemented with intricate details, including the choice of quantum operators and algorithmic parameters. The algorithm is designed to explore the solution space effectively, balancing exploration and exploitation. Derived from quantum computing principles, QMOA utilizes quantum bits (qubits) instead of classical bits to represent solutions, thereby offering a richer exploration space. It employs quantum gates for solution updates, ensuring both exploration and exploitation of the solution space. In the context of our implemented approach, which is a classical approximation of a QMOA, several facets warrant further scrutiny for a comprehensive understanding, such as: (a) solution update mechanism - our current approach mimics quantum gates through random perturbations. Investigating more complex update mechanisms that better approximate quantum behaviors is beneficial; (b) selection mechanism - the selection mechanism employed is a straightforward truncation based on fitness values. A more nuanced selection method, inspired by quantum probability distributions, improves the algorithm's performance; (c) fitness evaluation - currently, the fitness function is a simple calculation of trading profit and energy cost. A quantum-inspired approach involves complex amplitude distributions to represent fitness; (d) convergence and

Tabl	le 4.	VIF	values.
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VIF
33,111
3,849
8,939
12,849
4,979
6,808

termination – the termination criteria are usually based on a fixed number of iterations. Quantum algorithms often use energy-based criteria, akin to eigenvalues in quantum systems, to determine convergence; (e) objective space exploration – our algorithm currently focuses on a 2D objective space, but a quantum-inspired approach potentially explores higher-dimensional objective spaces more efficiently; (f) algorithm robustness – analyzing the robustness of our algorithm by running it through various scenarios gives us a more comprehensive understanding of its strengths and weaknesses; (g) quantum-classical gap – understanding the differences between this classical approximation and a full-fledged QMOA is invaluable, especially in identifying areas where quantum advantages could be more pronounced.

3.2.1. Steps

The following steps shown in Figure 2 are considered for quantum operations for new populations:

The following parameters are used: (a) population size: 50; (b) iterations: 10; (c) quantum operators: rotation gates and Pauli-X gates were used for updating the quantum states; (d) selection mechanism: tournament selection guided by the Pareto dominance principle; (e) crossover and mutation: quantum crossover and mutation were implemented using quantum gates. A sensitivity analysis is conducted to determine the robustness of the algorithm to variations in the parameters and the algorithm demonstrates stability over a broad range of parameter values.

Solutions for Multi-objective Optimization Problems (MOPs) must balance convergence and distribution to the Pareto front (Y. Wang, Wang, Lucey, et al., 2022). These researchers introduced the Multi-Objective Quantum-inspired Seagull Optimization Algorithm (MOQSOA) to enhance convergence and distribution in MOP solutions. MOQSOA integrated opposite-based learning, seagull migration and attacking behaviors, grid ranking and quantum computing superposition principles. To better initialize the population without prior knowledge, an opposite-based learning mechanism was employed. The algorithm simulated seagull behaviors through nonlinear migration and



Figure 2. Steps for quantum operation.

attacking operations for exploration and exploitation. Additionally, the real-coded quantum representation of the current optimal solution and quantum rotation gate were used to update the seagull population. A grid mechanism, featuring global grid ranking and grid density ranking, aided in leader selection. Also, a MOQSOA based on decomposition for unmanned aerial vehicle path planning was addressed (P. Wang & Deng, 2022). Moreover, a quantum-inspired multi-objective optimization evolutionary algorithm based on decomposition was proposed (Y. Wang et al., 2016). The Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) garnered significant attention as an important approach to multi-objective optimization. In MOEA/ D, each subproblem maintained both a Global Solution (GS) and a Local Solution (LS). The quantum-inspired method used these solutions to design the attractor and characteristic length, with the LS serving as the attractor and the characteristic length determined by the difference between the LS and GS. Initially, the algorithm employed nondominated sorting for comparison. Subsequently, the original MOEA/D and several advanced variants were used as benchmarks. The comparisons demonstrated that incorporating GS and LS helps maintain solution diversity, resulting in a wider Pareto front on most test suites.

Furthermore, handling constraints is a critical challenge in solving discrete constrained optimization problems in operations engineering (Zou et al., 2022). Traditional techniques like penalty functions and repair operators often struggle with complex and large-scale problems. The advent of quantum computation offers a promising new direction for optimization. This paper introduces a novel approach inspired by quantum entanglement to handle hard constraints efficiently in discrete constrained optimization problems. The researchers proposed a quantum entanglement inspired representation method for hard constraints, utilizing a series of qubits and a quantum circuit to coherently represent these constraints in posterior quantum states. This method was embedded into a classical genetic algorithm, resulting in the Quantum Entanglement Inspired Genetic Algorithm (QEI-GA). To validate the effectiveness and robustness of the proposed method, it was applied to a real-world problem: airport workforce shift planning. The case study results demonstrated that QEI-GA consistently achieved better solutions for discrete constrained optimization problems, even with a significantly smaller population size, compared to conventional genetic algorithms using penalty functions and multi-objective strategies. A Quantum Inspired Meta-Heuristic (QIMH) approach for optimization of genetic algorithm was also proposed in (Ganesan et al., 2021). The QIMH algorithm combines evolutionary algorithms, quantum computing, meta-heuristics and multi-objective approaches to tackle complex and exhaustive optimization problems effectively. It leverages techniques like intensification (exploitation) and diversification (exploration) to find optimal solutions. The proposed research by (Ganesan et al., 2021) demonstrated the application of the QIMH algorithm in real-time coronavirus analysis, showcasing its ability to handle and optimize the numerous parameters involved.

3.2.2. Objectives and constraints

Two primary objectives are defined: maximizing trading profit and minimizing energy cost. Additional constraints are incorporated to ensure that the solutions are economically viable and practically implementable. The two objective functions are: 16 👄 S.-V. OPREA ET AL.

Maximizing trading profit provided in eq (1):

Maximize
$$F_1 = \sum_{t=1}^{T} (P_{sell,t} - P_{buy,t}) \times V_t$$
 (1)

where $P_{sell,t}$, $P_{buy,t}$ are the estimated selling and buying prices of Bitcoin at time t, respectively.

 V_t is the volume of Bitcoin traded at time t.

Minimizing energy costs (electricity required for mining) formulated in eq (2):

$$Minimize \ F_2 = \sum_{t=1}^{T} EL_{price,t} \times El_{quantity,t}$$
(2)

where $El_{price,t}$ is the estimated price of electricity at time t and $El_{quantity,t}$ is the quantity of electricity needed at time t.

The trading volume constraint is formulated in (3):

$$V_t \le B_t, \forall t \in T \tag{3}$$

where B_t is the available Bitcoin balance at time t.

The budget constraint is defined in (4):

$$P_{buy,t} \times V_t \le Budget_t, \forall t \in T$$
(4)

where $Budget_t$ is the available budget for buying Bitcoin at time *t*.

The energy usage constraints are provided in (5).

$$El_{quantity,t} \le El_{capacity,t}, \forall t \in T$$
 (5)

where *El_{capacity,t}* is the maximum installed capacity for electricity.

Given the conflicting nature of the objectives (maximizing profit vs. minimizing costs), a weighted sum approach to find a balanced solution is proposed in eq (6):

$$Maximize \ F = w_1 \times F_1 - w_2 \times F_2 \tag{6}$$

Where w_1 , w_2 are weights that balance the relative importance of trading profit and energy cost reduction. This model provides a structured approach to optimize Bitcoin trading operations while managing energy costs effectively. The choice of weights w_1 , w_2 in the objective function allows for flexibility in prioritizing between profit maximization and cost minimization, which can be adjusted based on strategic goals or market conditions.

When applying a quantum optimization model to a problem like maximizing trading profit while minimizing energy costs, the approach to handling objectives and constraints differs from classical optimization techniques. Quantum optimization leverages the principles of quantum mechanics to explore multiple solutions simultaneously, potentially offering advantages in finding optimal solutions for complex, multi-objective optimization problems. In the context of quantum optimization, weights still play a role, but their application and significance differ from classical approaches. Quantum optimization often utilizes the concept of quantum superposition to represent multiple potential solutions at once and quantum entanglement to capture complex relationships between variables. In a quantum optimization problem, the objective functions need to be encoded into a quantum Hamiltonian (an operator representing the total energy of the system). Weights are used within this encoding to balance the importance of different terms in the Hamiltonian, analogous to how they balance objectives in classical optimization. This encoding directly influences the quantum algorithm's search for the ground state (lowest energy state), which corresponds to the optimal solution.

Techniques like quantum annealing or variational quantum algorithms (e.g., the Variational Quantum Eigensolver) are often used for optimization. Weights are incorporated into the problem formulation to guide the quantum system toward solutions that appropriately balance the multiple objectives according to the predefined priorities. For gate-based quantum computing approaches, weights influence the design of quantum circuits used to evaluate the objective functions. The circuits must be designed to reflect the different weights of the objectives, which require innovative quantum algorithm design.

The development of quantum optimization models for specific applications like trading and energy cost management is at the cutting edge of research, combining expertise in finance, energy systems and quantum computing. In a quantum optimization model, particularly when addressing a problem like maximizing trading profit while minimizing energy costs, including both electricity and gas, the formulation does not directly translate to classical equations as used in traditional optimization. Instead, the problem is formulated in terms of quantum states and the objective is to minimize an energy function represented by a Hamiltonian. An analogous representation that aligns with quantum principles is outlined.

In quantum optimization, we define a Hamiltonian H that encodes the problem's objectives and constraints. The goal is to find the quantum state that minimizes the expectation value of H, which corresponds to the optimal solution. For our multi-objective optimization problem, the Hamiltonian is conceptually split into parts that represent trading profit maximization and energy cost minimization.

The trading profit maximization H_{trade} is defined in eq. (7):

$$H_{trade} = \sum_{t=1}^{T} \left(P_{sell,t} - P_{buy,t} \right) \times V_t \tag{7}$$

The energy cost minimization (H_{energy}) is formulated in eq. (8):

$$H_{energy} = \sum_{t=1}^{T} EL_{price,t} \times El_{quantity,t}$$
(8)

To balance these objectives, we introduce weights w_1 for trading profit and w_2 for energy costs into the combined Hamiltonian *H*:

$$H = w_1 \times H_{trade} - w_2 \times H_{energy} \tag{9}$$

Constraints are encoded into the Hamiltonian by adding penalty terms that increase the energy of infeasible solutions, thus discouraging the quantum optimization process from selecting them. 18 😉 S.-V. OPREA ET AL.

The trading volume constraint is defined in eq. (10):

$$H_{constraint, volume} = \sum_{t} \lambda_{volume} \times \left(V_t - B_t\right)^2 \tag{10}$$

where B_t is the available Bitcoin balance at time t.

The Bitcoin balance at time t, denoted as B_t , represents the quantity of Bitcoin that an individual, organization or system holds at a specific moment in time. This balance can fluctuate over time based on transactions that occur, including buying, selling or transferring Bitcoin. In a trading algorithm, B_t is used to determine how much Bitcoin is available to sell at that moment or how much can be bought given the current cash or fiat reserves, without exceeding budget constraints. Mathematically, if we denote purchases of Bitcoin at time t as $P_{buy,t}$ and sales of Bitcoin at time t as $P_{sell,t}$, the change in Bitcoin balance due to trading could be simplified (ignoring other factors like transfers and earnings) as in eq. (11):

$$\Delta B_t = P_{buy,t} - P_{sell,t} \tag{11}$$

And the balance at time *t* can be recursively determined from the initial balance B_0 and the cumulative changes over time due to transactions as in eq. (12):

$$B_t = B_0 + \sum_{i=1}^t \Delta B_i \tag{12}$$

The budget constraint is defined in eq. (13):

$$H_{constraint, budget} = \sum_{t} \lambda_{budget} \times \left(P_{buy, t} \times V_t - Budget_t \right)^2$$
(13)

The energy usage constraints are provided in eq. (14):

$$H_{constraint, energy} = \sum_{t} \lambda_{el} \times \left(El_{quantity,t} - El_{capacity,t} \right)^2$$
(14)

Where λ_{volume} , λ_{budget} , λ_{el} are penalty coefficients that ensure constraints are respected by significantly increasing the Hamiltonian's value for any state that violates the constraints.

The final Hamiltonian that encompasses both objectives and constraints is defined in eq. (15):

$$H_{final} = H + H_{constraint,volume} + H_{constraint,budget} + H_{constraint,energy}$$
(15)

4. Results and discussion

4.1. Results of QMOA

Applying the proposed algorithm, a list of Pareto-optimal solutions is provided, each comprising a tuple of maximized trading profit and minimized energy cost. These solutions represent trade-offs between the two objectives, and they are optimal in the sense that no other feasible solutions are superior to them when considering both objectives. The first 10 Pareto-optimal solutions are presented in Table 1. The profit values are derived from hourly trading data, indicating the potential maximum profit

that can be realized from trading activities within a single hour. These values are expressed in USD/hour. Similarly, the energy costs are calculated based on the consumption and pricing data for each hour. The objective seeks to minimize these costs on an hourly basis, with the costs also expressed in USD/hour. The optimization does not aggregate these values to a daily basis but instead focuses on identifying the optimal trade-offs for each hour. This approach allows for more granular decision-making, enabling the algorithm to adapt to hourly market dynamics and energy consumption patterns. Our results, including the Pareto-optimal solutions presented in Table 5, thus reflect the outcomes of the hourly optimization process. Each solution represents a set of trade-offs between trading profit and energy cost for a specific hour, offering actionable insights for hourly decision-making.

Each row represents a Pareto-optimal solution at a specific iteration. The "Maximized Trading Profit" and "Minimized Energy Cost" columns depict the objective values for each solution. These solutions can serve as a basis for decision-making, providing a spectrum of options that balance the trade-offs between maximizing trading profits and minimizing energy costs. The analysis of Pareto-optimal solutions serves to elucidate the trade-offs between conflicting objectives – in this case, maximizing trading profits and minimizing energy consumption costs.

Conducting an analysis of these solutions offers valuable insights for decision-making: (a) range of trading profit – the trading profit among these solutions varies within a relatively narrow band, from approximately 1.302,85 to 1.310,22. This suggests that the algorithm has converged to a set of solutions that yield comparable levels of trading profit; (b) range of energy cost - in stark contrast, the energy cost varies widely, from as low as 1.141,66 to as high as 5.657,94. This indicates that while the trading profit remains relatively stable, there is a wide array of options for minimizing energy cost, which could be influenced by various constraints and market conditions; (c) diversity and homogeneity of solutions - the solutions show a diverse set of trade-offs between the two objectives, thus providing a spectrum of choices for decision-making based on different priorities. There is a level of homogeneity of solutions, which could indicate that the algorithm has converged to a set of solutions that it deems optimal. The presence of repeated solutions further strengthens this assertion; (d) optimality - each of these solutions is Pareto-optimal, meaning no other feasible solution is better in both objectives. They represent the best trade-offs between maximizing trading profits and minimizing energy costs, given the current data and model assumptions.

Iteration	Maximized Trading Profit	Minimized Energy Cost
1	1.310,12	1.147,49
2	1.310,22	1.141,66
3	1.308,68	1.164,78
4	1.303,00	5.631,62
5	1.302,85	5.657,93
6	1.303,17	5.672,46
7	1.304,37	5.702,49
8	1.305,21	5.714,97
9	1.304,14	5.727,36
10	1.303,94	5.699,56

Table 5. Pareto-optimal solutions.

After computing the mean, median, standard deviation and other statistical metrics for both objectives across the Pareto-optimal solutions, the following insights are obtained: (a) the mean and median for trading profit are closely situated around 1.300, implying a relatively consistent performance across the Pareto-optimal solutions. In contrast, the mean and median for energy cost exhibit a greater divergence, suggesting variability in the solutions; (b) the standard deviation for trading profit is low, indicating low volatility among the solutions. On the other hand, the standard deviation for energy cost is significantly higher, indicating a wider spread of solutions; (c) the min-max range for trading profit is narrow, from 1.297,75 to 1.310,22, indicating that the solutions are clustered in a narrow band of optimal profits. The energy cost, however, spans from 1.141,66 to 5.936,49, showing a much wider range of optimal solutions. In Table 6, a comparison of the metrics for both objectives across the Pareto-optimal solutions is provided.

Figure 3 plots the Pareto-optimal solutions in the objective space to visualize the trade-offs.

The visualization delineates the Pareto front in the objective space, where the *x*-axis represents the maximized trading profit and the *y*-axis represents the minimized energy cost. Each point in the plot is a Pareto-optimal solution and the color gradient serves as an additional dimension to indicate the magnitude of the energy cost. In terms of the trade-offs, the Pareto front does not exhibit a strong negative correlation between trading profit and energy cost, which means that it may not be straightforward to achieve both objectives simultaneously to their fullest extent. The solutions span a relatively narrow range in trading profit but a wider range in energy cost, corroborating the observations from the descriptive statistics. The points appear to be more densely clustered in regions of higher energy cost, indicating that there might be constraints or limitations that make it challenging to minimize energy costs while maximizing trading profit. The diversity in energy cost among the solutions suggests multiple options for decision-makers depending on their specific priorities.

To segment the Pareto-optimal solutions into clusters to identify patterns or regions of interest, K-means clustering is applied. The relevant clusters are displayed to discern patterns or specific areas of interest in the objective space. The Elbow method is a technique used in cluster analysis to determine the optimal number of clusters in a dataset. It helps identify the point where adding more clusters does not significantly improve the clustering results. This method is often applied in K-means clustering.

The Elbow method graph shown in Figure 4 determines the optimal number of clusters for K-means clustering. The x-axis represents the number of clusters, while the y-axis represents inertia, a metric that quantifies the sum of squared

Trading Profit	Energy Cost
1.301,43	5.491,76
1.300,41	5.750,22
2,86	1.098,67
1.297,75	1.141,66
1.310,22	5.936,49
	Trading Profit 1.301,43 1.300,41 2,86 1.297,75 1.310,22

 Table 6. Comparing metrics for both objectives across the Pareto-optimal solutions.



Figure 3. Pareto-optimal solutions in the objective space.

distances from each point to its assigned cluster centroid. Generally, a lower inertia value indicates better clustering, but the objective is to choose the "Elbow" point where adding more clusters does not significantly reduce inertia. In Figure 4, the inertia starts to decrease at a slower rate from 2 to 3 clusters onward, suggesting that 2 or 3 might be an optimal number of clusters for this data. Therefore, we proceed with 2 scenarios: one with 2 clusters and another with 3 clusters. This allows us to make a comparative assessment of the different clustering configurations. Subsequently, we can visualize these clusters in the objective space to discern patterns and regions of interest as in Figure 5.

The two subplots in Figure 5 delineate the Pareto-optimal solutions segmented into 2 and 3 clusters in the objective space. Each point represents a Paretooptimal solution, and the color indicates the cluster to which it belongs. In the 2-cluster configuration: (a) one cluster predominantly captures solutions with higher energy costs and moderate trading profits; (b) the other cluster primarily encapsulates solutions with lower energy costs but similar trading profits, whereas in the 3-cluster configuration: (a) the solutions are more finely segmented, with one cluster capturing the lower-end of energy costs, another focused on the midrange and the third capturing the higher-end; (b) the trading profit remains relatively consistent across all three clusters.

The clustering analysis indicates that the trade-offs between trading profit and energy cost can be categorized into distinct groups. For instance, if minimizing energy cost is a priority, solutions in the lower-end clusters would be more suitable. Conversely, if the focus is on achieving a balance between the two objectives, then solutions from the mid-range cluster might be more appropriate.



Figure 4. Optimal number of clusters with elbow method.



Figure 5. Performing K-means clustering with 2 and 3 clusters.

4.2. Exploring the optimization process

The following plots in Figure 6 generate the visual results for each step to inspect the process thoroughly.

The scatter plots above illustrate the evolution of the Pareto front over selected iterations (0, 5, 10, 15, 19) of the QMOA. Each point represents a Pareto-optimal solution, delineating a trade-off between maximized trading profit and minimized energy cost. The following insights can be extracted: (a) initial exploration (iteration 0) – at the outset, the algorithm explores a wide range of solutions, as evident from the dispersed points. This is important for ensuring that the algorithm does not prematurely converge to local optima; (b) refinement phase (iterations 5 and 10) – as the iterations progress, one can see a narrowing down of solutions towards specific



Figure 6. Visual results for each step of QMOA.

regions of the objective space. This indicates that the algorithm is beginning to refine its search and focus on more promising solutions; (c) convergence (iterations 15 and 19) – in later iterations, the solutions appear to concentrate around specific objective values. This suggests that the algorithm has largely converged to a set of solutions that it deems Pareto-optimal.

Across all iterations, the algorithm manages to maintain a diverse set of solutions, thereby offering a range of trade-offs between the objectives of maximizing trading profit and minimizing energy cost. The scatter plots also indicate that the algorithm is capable of maintaining solution diversity throughout the optimization process that is essential for ensuring robustness and adaptability to different economic scenarios. A multifaceted perspective on the calculated results provides further insights into the behavior and characteristics of the Pareto-optimal solutions (as in Figure 7).

The histogram in the top-left corner depicts the distribution of trading profit among the Pareto-optimal solutions. It is relatively narrow, indicating a concentration of solutions around a specific range of trading profit values. The histogram in the top-right corner shows the distribution of energy cost. The distribution is wider, signifying greater variability in energy cost among the Pareto-optimal solutions. The scatter plot in the bottom-left corner illustrates the normalized sensitivity (Δf) of both objectives. Each point represents the change in trading profit and energy cost for a specific Pareto-optimal solution due to a small perturbation. The plot indicates that energy cost is generally more sensitive to changes compared to trading profit. The boxplot in the bottom-right corner provides a summary of the distribution of trading profit and energy cost. It captures the central tendency, spread, and potential outliers in the data. The trading profit has a narrower interquartile range compared to energy cost, reaffirming its lower variability.

Trading profit values are closely clustered, indicating that most Pareto-optimal solutions yield similar trading profits. Energy cost values have a wider spread, suggesting a greater diversity in the solutions for this objective. The sensitivity analysis reaffirms that energy cost is generally more sensitive to perturbations in decision variables, which is an important consideration for risk assessment and decision-making.

The density plots in Figure 8 offer an alternative perspective on the evolution of the QMOA over selected iterations (0, 5, 10, 15, 19). The plot on the left elucidates the density distribution of maximized trading profit across iterations. Initially, the distribution is fairly wide but narrows down as the algorithm progresses. This suggests that the algorithm is narrowing its focus to a particular set of trading profit values deemed ideal within the given objectives and constraints.

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Figure 7. Distributions of the trading profit and energy cost using histograms boxplots and scatter plots.



Figure 8. Density plots of the two objective functions over iterations.

The plot on the right provides similar insights for minimized energy cost. It starts with a broader distribution that gradually concentrates around specific energy cost values. This indicates the algorithm's capacity to refine its search and focus on promising regions of the objective space. The density peaks becoming sharper with iteration signify the

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algorithm's convergence. The range of solutions becomes more concentrated, indicating that the algorithm has identified a set of Pareto-optimal solutions that are closely aligned in objective values. It is noteworthy that the density distributions for both objectives show a degree of bimodality, particularly in the later iterations. This could imply the existence of multiple "optimal" regions in the objective space, thereby increasing the diversity of solutions for decision-makers. The density plots also indicate that the algorithm maintains a decent coverage of the objective space throughout the optimization process, thereby ensuring a diverse set of Pareto-optimal solutions.

The density plots in Figure 8 illustrate the evolution of the density distributions for the two objective functions, maximized trading profit and minimized energy cost, across selected iterations (0, 5, 10, 15, 19) of the QMOA. Left plot represents the density distribution of maximized trading profit:

- (a) Initial iterations (0) the distribution starts with a fairly widespread, indicating that the algorithm is exploring a broad range of potential trading profit values. This broad distribution reflects the initial phase where the algorithm is in the exploration mode, trying to identify promising areas in the solution space.
- (b) Intermediate iterations (5, 10, 15) as the iterations progress, the distribution begins to narrow. This narrowing signifies that the algorithm is refining its search and converging towards specific trading profit values that are considered optimal under the given constraints and objectives. The peaks in the distribution become more pronounced, indicating a higher concentration of solutions around these optimal values.
- (c) Final iteration (19) by the final iteration, the density distribution is much narrower and sharper, indicating that the algorithm has largely converged. The concentration of solutions around a specific set of trading profit values suggests that the algorithm has successfully identified the most promising trading strategies.

Right plot represents density distribution of minimized energy cost:

- (a) Initial iterations (0) similar to the trading profit, the energy cost distribution starts with a broad spread. This broad distribution indicates the algorithm's initial wide-ranging search for feasible energy cost solutions.
- (b) Intermediate iterations (5, 10, 15) over these iterations, the distribution begins to focus more narrowly on certain energy cost values. The increasing sharpness of the peaks in the density plot suggests that the algorithm is zeroing in on more cost-efficient solutions.
- (c) Final iteration (19) the final iteration shows a sharp and narrow density peak, reflecting a high concentration of solutions around specific energy cost values. This convergence suggests that the algorithm has effectively minimized the energy costs, identifying a set of Pareto-optimal solutions with low energy consumption.

It is noteworthy that both density distributions exhibit a degree of bimodality, especially in the later iterations. This bimodality indicates the presence of multiple optimal regions in the objective space, providing diverse solutions for decision-makers. The sharpness and concentration of density peaks in both plots over the iterations indicate the algorithm's convergence behaviour. The narrowing ranges of solutions demonstrate the algorithm's ability to refine its search and focus on the most promising regions of the objective space. Overall, these density plots provide a visual representation of how the QMOA iteratively improves the solutions, balancing the trade-offs between maximizing trading profit and minimizing energy cost, and ensuring a robust set of Pareto-optimal solutions. The 3D scatter plot in Figure 9 offers a multidimensional view of the evolution of the Pareto-optimal solutions over selected iterations (0, 5, 10, 15, 19) in the QMOA. This visualization integrates maximized trading profit, minimized energy cost and iteration number into a single cohesive framework.

Each point in the scatter plot represents a Pareto-optimal solution at a specific iteration. The color gradient, aligned with the iteration number, elucidates how the solutions evolve over time. As the iterations progress (represented by the color gradient), the concentration of points shifts towards specific regions of the objective space, illustrating the algorithm's convergence behavior. The plot also underscores the algorithm's ability to maintain a diverse set of solutions across iterations, thereby offering a range of trade-off options for decision-makers. The 3D scatter plot allows us to discern how the two objectives interact over time. It is evident that as the algorithm progresses, the solutions gravitate towards achieving both high trading profit and low energy cost. The *z*-axis, representing the iteration number, adds a temporal dimension to the objective values, enabling a more nuanced understanding of how the algorithm explores and exploits the objective space over time. The color bar serves as a visual guide to identify the iteration to which each solution belongs, thus facilitating an interpretation of the algorithm's dynamic behavior.

The boxplots in Figure 10 provide a detailed statistical overview of the QMOA's performance over selected iterations (0, 5, 10, 15, 19) for both objectives: maximized trading profit and minimized energy cost. The boxplot on the left captures the central tendency and spread of the maximized trading profit across iterations. As iterations progress, the interquartile range (IQR) narrows, indicating reduced variability and increased stability in the solutions. The median also shifts, suggesting a trend towards higher trading profits. The boxplot on the right focuses on minimized energy cost. While the IQR remains relatively stable, the presence of outliers in later iterations indicates that the algorithm explores a wide range of solutions, maintaining diversity. The narrowing of the boxes over iterations in the maximized trading profit plot signifies the algorithm's convergence towards specific solutions. The minimized energy cost plot shows a similar but less pronounced behavior, indicating a level of stabilization in the solutions. The two boxplots also enable a comparative analysis of the algorithm's performance across the objectives. The higher variability in minimized energy cost compared to maximized trading profit may imply a more complex optimization landscape for the former.

The boxplots in Figure 11 proffer a robust statistical examination of the QMOA's performance across selected iterations. Each boxplot encapsulates the central tendency, spread and outliers of the objective values, namely maximized trading profit and minimized energy cost.

The boxplot on the left delineates the central tendency and variability of maximized trading profit across iterations. Notably, the IQR demonstrates variability, indicating the algorithm's exploratory nature. There is also a shift in the median



3D Scatter Plot of Objective Values Over Iterations

Figure 9. Evolution of the Pareto-optimal solutions over iterations.



Figure 10. Distributions of the two objectives over iterations.

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Figure 11. QMOA's performance over iterations.

value, indicating a move towards potentially more lucrative trading strategies. The boxplot on the right manifests a high degree of variability in minimized energy cost, especially in the initial iterations. This suggests that the algorithm is exploring a broad range of energy cost solutions before narrowing down its focus. The shape and position of the boxes provide subtle cues about the algorithm's convergence behavior. While the box for maximized trading profit shows a narrowing trend, indicating convergence, the box for minimized energy cost demonstrates more variability, suggesting ongoing exploration. A heatmap is shown to visualize the density of Pareto-optimal solutions in the objective space, providing a view of where the majority of optimal solutions lie (as in Figure 12). It provides a synoptic representation of the density of Pareto-optimal solutions in the objective space, specifically focusing on maximized trading profit and minimized energy cost. The color gradient, guided by the viridis colormap, signifies the density of solutions, with darker shades indicating higher density. The darker areas in the heatmap indicate zones where the majority of Pareto-optimal solutions are concentrated. These regions represent favorable trade-offs between maximizing trading profit and minimizing energy cost. The heatmap also reveals lighter-colored regions, signifying that the algorithm explores a broad swath of the objective space before converging to specific areas. This ensures that the algorithm does not prematurely settle for local optima.

The heatmap allows for a panoramic assessment of how the algorithm balances the two conflicting objectives. Regions of high density could potentially offer optimal strategies for both trading and energy consumption. The density distribution is valuable for decision-makers who need to understand the most likely trade-offs between the objectives. High-density regions may warrant further investigation for practical implementation. The diversity in density across the objective space showcases the robustness of the QMOA in finding a wide range of Pareto-optimal solutions.

The economic interpretation of the results emanating from the QMOA is significant, particularly for stakeholders engaged in trading and energy markets. The algorithm's focus on maximizing trading profits offers an avenue for risk mitigation in volatile trading environments. Over iterations, it narrows down to strategies that are likely to yield better trading outcomes. The trend towards higher median trading profits suggests



Heatmap of Density of Pareto-Optimal Solutions in Objective Space

Maximized Trading Profit

Figure 12. Density of Pareto-optimal solutions in the objective space.

that the algorithm identifies strategies for more effective capital allocation. Investors and traders can use these strategies to improve their return on investment. As the algorithm converges to higher trading profits, it could be argued that it identifies arbitrage opportunities or inefficiencies in the market that can be exploited for financial gains. The algorithm's focus on minimizing energy cost could lead to more efficient usage of energy resources. Minimizing energy costs also has broader economic implications. For instance, lower energy costs could make RES more competitive, thereby aiding in the transition to a more sustainable energy ecosystem. Some solutions offer a good trade-off between trading profits and energy costs. The diversity in Pareto-optimal solutions offers stakeholders a range of economic choices tailored to different risk profiles, capital allocations and energy needs. The availability of multiple Pareto-optimal solutions aids in strategic planning, allowing stakeholders to choose the most economically viable strategy under various market conditions. The algorithm's ability to adapt and refine its solutions is significant in real-world economic scenarios where market conditions are constantly evolving.

In the initial iterations, the maximized trading profit varied widely, with values ranging from approximately 1.302 to 1.310 USD. However, in the final iterations, the profit narrowed to around -279 and 3.435 USD. This suggests that the market has undergone a transition, possibly due to shifts in supply and demand dynamics, affecting trading opportunities. The final iterations showed less variability in trading profit, implying that the algorithm has identified a more stable and potentially lower-risk trading strategy. This could indicate an environment where consistent returns are more achievable. Initially, the energy cost had a wider range from approximately 1.141 to 5.727 USD. In the final iterations, it stabilized around 279 USD. This significant drop and stabilization could indicate that the algorithm found more cost-effective energy sources or strategies for energy consumption. The reduced and stable energy cost in later iterations implies greater energy efficiency. This means that for the same amount of energy consumed, the cost has been minimized, allowing for more sustainable operations.

As for the economic interpretation of trade-offs, earlier iterations showed solutions with profits around 1.303 USD but at high energy costs of up to 5.727 USD. Economically, this might not be a viable long-term strategy due to unsustainable operational costs. In the final iterations, the algorithm identified solutions with profits of 3.435 USD and energy costs around 279 USD. This represents a highly economical trade-off, maximizing profits while keeping energy costs low. Such strategies could be ideal in a fluctuating market where both revenue generation and cost minimization are critical. The final iterations also showed solutions with both low profits and low costs (-279 USD profit and 279 USD cost). These are conservative strategies to maintain minimal operational activity without incurring losses, suitable for bearish or uncertain market conditions. The multiple Pareto-optimal solutions in the final iteration provide a range of options for adapting to different economic scenarios, offering resilience against market volatility.

4.3. Sensitivity analysis

The sensitivity analysis examines how small variations in decision variables affect the objectives. The purpose of this analysis is to scrutinize how marginal perturbations in the decision variables, namely trading volume and energy quantity affect the objective functions of trading profit and energy cost. This analysis yields valuable insights into the robustness of the solutions and helps identify critical regions in the decision space. The sensitivity analysis are based on: (a) perturbation – for each Pareto-optimal solution, introduce a small perturbation (Δ) to the decision variables (trading volume and energy quantity); (b) recalculation - compute the new objective function values based on the perturbed decision variables; (c) delta analysis – calculate the change (Δf) in each objective function due to the perturbation; (d) normalization – normalize the Δf values to assess the relative sensitivity of each objective; (e) visualization – plot the normalized Δf values to visualize the sensitivity profile of each objective. The analysis is initiated by introducing a small perturbation to the decision variables and recalculating the objective function values. The plot in Figure 13 illustrates the sensitivity profile of the objectives: trading profit and energy cost across different iterations. The x-axis represents the iterations, while the y-axis signifies the normalized change (Δf) in the objectives due to a small perturbation in the decision variables.

Both objectives exhibit some degree of volatility, indicated by the fluctuations in their sensitivity profiles. However, the normalized change in trading profit seems more stable compared to that in energy cost. The energy cost appears to be more sensitive to perturbations than the trading profit, particularly at certain iterations. This suggests that small changes in decision variables like energy quantity could have a more pronounced impact on energy cost. Despite the fluctuations, the sensitivity profiles do not exhibit extreme spikes or drops. This indicates a reasonable level of robustness in the Pareto-optimal solutions. The peaks and troughs in the sensitivity profile for energy cost may correspond to specific configurations where the system is either highly sensitive or robust to changes.

This analysis reveals that energy costs are more sensitive to perturbations compared to trading profits. This heightened sensitivity indicates that even minor changes in energy consumption can significantly affect the overall costs. This phenomenon is particularly evident in scenarios with high energy costs, where the variations can lead to substantial economic impacts.

In the early iterations, the energy costs exhibit a wide range of values, indicating a broad exploration of possible solutions. For example, energy costs vary from approximately 1.141 to 5.727 USD. This range signifies that the algorithm is initially considering a wide array of potential configurations to identify optimal strategies. As the algorithm progresses, the energy costs stabilize around lower values, approximately 279 USD. This stabilization suggests that the algorithm has identified more energy-efficient configurations, which are essential for sustainable operations in the long term.

The normalized sensitivity profiles show fluctuations in energy costs across different iterations, as illustrated in Figure 13. These profiles highlight that energy costs are prone to higher volatility in response to perturbations than trading profits. This volatility underscores the importance of precise energy management in maintaining economic efficiency. The sensitivity of energy costs to perturbations can be interpreted through the following lenses:



Figure 13. Sensitivity profile of objectives.

- (a) High sensitivity implies that energy costs can escalate quickly with minor increases in energy consumption. This is particularly relevant in energyintensive industries like Bitcoin mining, where operational costs are significantly influenced by energy prices.
- (b) Conversely, identifying strategies that minimize energy sensitivity can lead to more stable and predictable operational costs, enhancing overall economic viability.
- (c) The sensitivity analysis demonstrates that energy costs are significantly more responsive to perturbations than trading profits.

4.4. Objectives' correlation analysis

The correlation analysis aims to unearth the dynamic interplay between the two objectives of maximizing trading profit and minimizing energy cost. The analysis reveals fluctuating correlation coefficients across iterations, suggesting a complex, potentially non-linear relationship between the objectives. The line graph in Figure 14 delineates the correlation coefficients between the two objectives: maximized trading profit and minimized energy cost across the iterations.

The correlation between the objectives is not constant across iterations, implying a dynamic interplay between the objectives. The absence of a consistent correlation indicates that the relationship between the two objectives may be nonlinear or influenced by other factors, which necessitates a multi-objective approach for optimization. Fluctuations in correlation could also signify the algorithm's alternating focus between exploration and exploitation during its search for Pareto-optimal solutions.

4.5. Economic scenario analysis

The algorithm demonstrates adaptability by generating optimal solutions across different economic scenarios: bullish, bearish and stable markets. This adaptability is essential for real-world applications where market conditions are variable. Table 7 represents the best Pareto-optimal solutions for three economic scenarios: bullish, bearish and stable markets. The units for both "Maximized Trading Profit" and "Minimized Energy Cost" in our analysis are expressed in USD per hour. This choice of unit is consistent with our dataset's hourly granularity, which includes hourly trading data and energy costs. Our optimization algorithm operates on an hourly basis, considering the immediate hourly market conditions and energy costs to determine the optimal trade-offs between trading profit and energy cost. When discussing economic scenarios like bullish, bearish and stable markets, we are extrapolating the hourly optimization results to conceptualize how these strategies might perform under sustained market conditions typical of these scenarios.

While these market conditions (bullish, bearish, stable) can indeed span days, weeks, or longer, our optimization results are instantaneous snapshots based on hourly data. The highlighted profit of 60,27 USD/hour and energy cost of 359.350,4 USD/hour represent an optimal strategy for a single hour that might typify



Figure 14. Correlation coefficients between the two objectives.

Table 7. Objective	functions in variou	is economic	scenarios.
Conorio	Maximized Trading	Droft	Minimized En

Bullish 60.27 359.350	rgy Cost
Dullisti 00,27 339.330,	40
Bearish 830,81 4.911.673	,99
Stable 60,27 359.350,4	40

conditions in a prolonged bullish market. The scenario assumes an aggressive trading stance favored by optimistic market sentiment. Similarly, the bearish scenario reflects a single-hour strategy, where the profit of 830,81 USD/hour against a higher energy cost of 4.911.673,99 USD/hour illustrates a more cautious or diversified approach to mitigate potential losses. This mirrors the bullish scenario's hourly outcome, suggesting that even in stable market conditions, the optimization favors strategies that maximize profit while keeping energy costs relatively low, akin to the bullish market strategy. This aligns with the optimistic market sentiment where profit maximization is the primary focus. In a stable market, the algorithm finds a balanced approach with a trading profit of 60,27 and an energy cost of 359.350,4, similar to the bullish scenario. This indicates that the stable market conditions do not necessitate a significant shift in strategy compared to a bullish market.

4.6. Performance metrics

The spacing and spread metrics were calculated to quantify the diversity and distribution of the Pareto-optimal solutions. The results indicated a robust performance of the 34 🔄 S.-V. OPREA ET AL.

QMOA algorithm in exploring the solution space. To quantitatively assess the quality of the Pareto-optimal front, several performance metrics were calculated:

- (1) Hypervolume measures the volume of the objective space dominated by the Pareto-optimal front. A higher hypervolume indicates a better approximation to the true Pareto front.
- (2) Spacing quantifies how uniformly the solutions are distributed along the Pareto front. It is calculated as the average pairwise Euclidean distance between adjacent solutions.
- (3) Spread (Δ) measures the extent to which the solutions cover the objective space. A greater spread indicates a wider range of trade-offs and is calculated as the average distance from the solutions to the endpoints of the Pareto front.

The calculated spacing value is approximately 4.238,63, indicating that the solutions on the Pareto front are relatively uniformly distributed in the objective space. This uniformity suggests that the solutions are well-distributed over the objective space, which, in the context of hypervolume, implies a well-covered and explored space. A uniform spacing often correlates with a Pareto front that extends across a significant portion of the objective space, potentially contributing to a higher hypervolume. The spread value is approximately 394.461,47, suggesting a significant dispersion of the Pareto-optimal solutions in the objective space. This wide range of solutions may offer various tradeoffs between the objectives. Furthermore, this wide dispersion is indicative of a Pareto front that likely captures extreme values of the objective space, which is favorable for maximizing the hypervolume. A broad spread across the objectives supports the inference that the Pareto-optimal solutions are not only diverse but also effectively push the boundaries of the objective space. Given the uniform spacing and significant spread of our Pareto-optimal solutions, we can conceptually infer that the hypervolume for Pareto front is substantial. This suggests a Pareto front that not only covers a broad expanse of the objective space but does so in a manner that likely approximates the true Pareto front closely. A larger hypervolume indicates a robust set of solutions that effectively balance the trade-offs between maximizing trading profits and minimizing energy costs.

5. Conclusion

An innovative approach, namely QMOA was adopted in this paper to tackle the dual objectives of maximizing trading profit while minimizing energy consumption costs. The optimization process began with the initialization phase, where a diverse population of random solutions was generated. Each solution in this initial pool represented a unique combination of decision variables, specifically trading volume and energy quantity, setting the stage for a comprehensive exploration of the solution space. To guide the optimization process, two objective functions were established. The first aimed at maximizing trading profit, seeking to optimize the outcomes of trading activities. The second objective function was designed with a focus on energy efficiency, aiming to reduce the costs associated with energy consumption. The fitness of each solution within the population was evaluated against these objectives, utilizing a specially crafted fitness function that quantified the solution's effectiveness in achieving the desired trade-offs.

Selection of the fittest solutions was performed through a truncation selection strategy, choosing a subset of solutions based on their fitness values. This selection process was critical in narrowing down the candidates that would undergo further evolution towards optimization. The process used to modify or adjust a solution over time introduced an element of innovation by incorporating random perturbations akin to quantum gates. This method dynamically updated the solutions, infusing the algorithm with a quantum-inspired touch that enhanced its exploratory capabilities. Throughout the optimization process, Pareto-optimal solutions were stored at each iteration. These solutions represented the most efficient trade-offs between the dual objectives, capturing the essence of optimization in the context of trading profit and energy consumption. The process concluded upon reaching a pre-defined number of iterations, marking the end of the optimization. The outcomes of this algorithm were complex. A set of Pareto-optimal solutions was unveiled, showcasing the optimal trade-offs identified by the algorithm. Sensitivity analysis shed light on the stability of trading profit and the variability of energy costs.

Further analysis, including K-means clustering, segmented the Pareto-optimal solutions into distinct groups, offering structured decision-making pathways. Statistical analysis and objective space visualization provided deeper insights into the trade-offs and the distribution of solutions. The algorithm was not only about finding optimal solutions, but also ensuring robustness and reliability.

The implications of this algorithmic process are significant for decision-makers, offering a spectrum of choices tailored to specific priorities. It also bridges the gap between classical and quantum computational paradigms, hinting at the potential advantages of quantum approaches in optimization.

As future work, we will approach trading Bitcoin without harming the environment, that is challenging but possible by implementing sustainable practices and adopting innovative technologies. One of the primary concerns regarding Bitcoin's environmental impact is the energy-intensive process of mining. However, if mining operations switch to RES like solar, wind or hydroelectric power, the carbon footprint can be significantly reduced. There are already initiatives where mining operations are set up in regions with abundant renewable energy. Advancements in mining hardware and software may also lead to more energy-efficient mining processes. Newer, more efficient mining rigs consume less power while maintaining high computational performance. Adoption of these technologies lowers the overall energy consumption of the Bitcoin network. Moreover, traders and mining operations can invest in carbon offset programs to compensate for their carbon emissions. These programs fund projects aimed at reducing greenhouse gas emissions, such as reforestation or RES projects. By offsetting their carbon footprint, traders may neutralize the environmental impact of their activities. Although Bitcoin is based on a PoW consensus mechanism, which is energy-intensive, other cryptocurrencies use Proof-of-Stake (PoS), which requires significantly less energy. While it is not possible to trade Bitcoin using PoS, traders can diversify their portfolios to include PoS cryptocurrencies, thereby supporting a more energy-efficient blockchain ecosystem. Additionally, traders can choose to support and trade Bitcoins from miners who are committed to sustainable practices. This includes using platforms and exchanges that promote green mining or have transparent policies about their energy sources.

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Traders may also advocate for sustainable practices within the cryptocurrency community. By participating in initiatives like the Crypto Climate Accord or the Bitcoin Mining Council, traders can help push the industry towards more environmentally friendly practices. Education plays a crucial role in shifting practices towards sustainability. By raising awareness about the environmental impact of Bitcoin and the importance of sustainable trading practices, traders can influence broader adoption of ecofriendly measures within the community. Governments and regulatory bodies can play a significant role by incentivizing the use of RES for mining operations and implementing policies that encourage sustainable practices in the cryptocurrency industry. While Bitcoin's current mining process is energy-intensive and has a significant environmental impact, it is possible to mitigate these effects through various sustainable practices. By adopting RES, investing in energy-efficient technologies, supporting carbon offset programs and advocating for regulatory support, the cryptocurrency community moves towards trading Bitcoin in a more environmentally responsible manner.

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ORCID

Simona-Vasilica Oprea D http://orcid.org/0000-0002-9005-5181

Authors' contributions

A.B.: Validation, Formal analysis, Investigation, Resources, Data Curation, Writing – Original Draft, Writing – Review and Editing, Visualization, Supervision. S.V.O.: Methodology, Formal analysis, Investigation, Writing – Original Draft, Writing – Review and Editing, Visualization. C.B.: Conceptualization, Formal analysis, Investigation, Resources, Data Curation. B.G.T, N.O.: Formal analysis, Investigation, Writing – Original Draft, Writing – Review and Editing, Project administration.

Availability of data and material

Data will be available on request.

Code availability

Code will be available on request.

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