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REVIEW ARTICLE



An improved model accuracy for forecasting risk measures: application of ensemble methods

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ABSTRACT

Statistical-based predictions with extreme value theory improve the performance of the risk model not by choosing the model structure that is expected to predict the best but by developing a model whose results are a combination of models with different shapes. Using different ensemble algorithms to conglomerate the TBATS and the GEV distribution, we found that the stacking ensemble algorithm outperforms other ensembles hence the forecasting accuracy of risk measures is improved with the stacking ensemble algorithm. The risk estimates suggest that the returns on losses averaging 0.014 and 0.018 invested at 90 and 99 percent respectively, are riskier than the returns on gains. Backtesting results further revealed that all the risk measures are reliable, and the combined model is a good one for computing financial risk particularly in South Africa. At high confidence levels, all the risk measures seem to perform better than at lower confidence levels, as evidenced by higher probability values from backtesting using the Kupiec and Christoffersen test at 95 percent than at 99 percent levels of significance.

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1. Introduction

From the viewpoint of financial risk managers, a risk measure can be viewed as a map from the space of probability distributions to actual numbers. Risk management professionals can adjust their capital reserves against the downside risk by using risk measures to give banks and other financial institutions specific values for potential losses. Two prevalent measures of financial risk that rule modern financial regulation are value-at-risk (VaR) and expected shortfall (ES). In the worst-case scenario, at a specific confidence level, VaR gives banks and investment institutions a loss level (Lazar & Xue, 2020). Value-at-risk is not a cogent risk measure in the traditional sense and has inherent flaws because it ignores the shape and structure of the tail. As a result, following the financial crisis of 2007–2008, the BCBS (2014) herein referenced (Basel Committee on Banking Supervision) suggested switching from VaR, which had a confidence level of 99 percent,

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to ES, which has a confidence level of 97.5 percent. Given that its realisation must be lower than that of VaR, ES is the expected return on investment.

On the other hand, deriving a model that investors and/or stock markets will use to track, predict, and forecast their daily gains or losses in real time is therefore crucial. As a result, forecasting risk is a huge and active research area that has piqued the interest of many different academic disciplines, including finance, engineering, and statistics among others. Due to this, a significant amount of literature has focused on techniques that can generate reliable forecasts in a range of real-world scenarios (Hajirahimi & Khashei, 2019). The literature typically points to two main approaches: (1) developing and proposing new forecasting models, and increasing the accuracy of obtained results. (2) Combining different forecasting model types. This is because no single comprehensive model can simultaneously capture all of the patterns present in the data; therefore, hybridisation is usually used.

Therefore, the purpose of this study is to combine the TBATS with the generalised extreme value (GEV) distribution and compare the effectiveness of different ensemble methods in combining the two models. In this way, we aim to improve and, enhance the forecasting accuracy of risk measures, namely, Value-at-risk, expected shortfall, conditional tail expectation (CTE), and glue value-at-risk (Glue-VaR). Generally speaking, several models are estimated, and the most accurate model is chosen. Because of a few possible impacting factors such as sampling variance, model uncertainty, and structural changes, the final model chosen may not be the best for use in the future. With minimal additional work, the model selection problem is made easier by integrating various models. Secondly, time series data in the real world are rarely perfectly linear or non-linear; because both linear and non-linear patterns are frequently present. In this scenario, neither the TBATS nor the GEV distribution can be used effectively to model and forecast time series data specifically the stock market time series, since the TBATS is not designed to handle the tail behaviour of the distribution and extreme values. While the GEV distribution on the other hand, cannot handle both linear and non-linear patterns equally well on its own. For that reason, we find it interesting to us to accurately represent complex time series structures by merging the TBATS with the GEV distribution and see how the new combined model performs on a five day business financial time series exchange/Johannesburg stock exchange (FTSE/JSE) all-share index.

Thirdly, the forecasting literature virtually unanimously agrees that there is no single ideal strategy for every situation. This is mostly because real-world issues are frequently complicated, making it possible for distinct patterns to be captured differently by different models. Utilising the TBATS helps us to handle multiple seasonalities and irregular trends, making it suitable for us to capture the underlying dynamics of the data, while the generalised extreme value distribution on the other hand helps us to accurately capture extreme values in the tails of the distribution. The integration of this two models helps us to obtain more accurate estimates of tail risk, particularly in scenarios where extreme events are infrequent but significant; leveraging the strengths of both models to improve forecasting performance, particularly in situations where capturing extreme events accurately is crucial for risk management or decision-making.

With this proposed three-stage hybrid model, in the first stage we amalgamate the TBATS to obtain independently and identically distributed (i.i.d) residuals while at the same time modelling non-constant seasonal patterns over time. Instead, the model helps

us to adapt to changes in seasonality, making it suitable for time series data with irregular or evolving seasonal patterns. On the second stage, we extract the block maximas for the positive returns (i.e., the gains) and block minimas for negative returns (i.e., the losses) from the i.i.d residuals of the TBATS model and we fit the GEV distribution to model volatility, extreme tail losses, and gains of the South African stock market. In detail, we consider a generalised extreme value distribution with a specification that the extreme value sequence comes from the autoregressive and the moving average (ARMA) errors that capture temporal dependencies and autocorrelation in the time series. The dependence is captured by an appropriate temporal trend in the local and scale parameters of the GEV distribution. The TBATS-GEVD modelling approach is believed to perform better in forecasting the risk measures and it is suited to explain extremes better than the classical methods; where the classical methods such as Autoregressive integrated moving average (ARIMA) among others cannot capture the tail behaviour adequately because they neither assume a normally distributed nor even fatter tailed distributed (e.g., t) innovations as suggested by Calabrese and Giudici (2015). This study is the first empirical analysis that employs TBATS in conjunction with the GEV distribution to quantify the likelihood of future extreme daily losses and gains.

Extreme Value Theory (EVT) is one of the most useful techniques to predict extreme cases, which have a big effect but less occurrence probability. In this study, the Block Maxima and Minima (BM) approach, which is one of the main extreme value techniques, has been used. In this approach, there is no proper block size selection methodology (Özari et al., 2019). It has been observed that the block size selection for the estimation made in previous studies has been used randomly without relying on any assumption. It is necessary for new applications to find a certain block size that can be used for all of the data sets. Hence, the second main contribution of this study is the novelty of the application on how the optimal number of blocks to be fitted to the GEV distribution are selected. We propose a methodology to select the best block size for both maximal and minimal returns with a case study on the South African market all-share index which makes up the top 99 percent of the total pre-free-float market capitalisation of all listed companies on the Johannesburg Stock Exchange.

The rest of this study is organised as follows section 1.1 represents literature review. Section 2 is the methodology followed. While section 3 presents result and discussion from data analysis and finally section 4 presents the conclusion of the study.

1.1. Literature review

Conventional measures of risk in earnings based on historical standard deviation require long time-series data and are inadequate when the distribution of earnings deviates from normality. A methodology that is based on current fundamentals, and quantile regression to forecast risk reflected in the shape of the distribution for future earnings, was developed by Konstantinidi and Pope (2016). Even though VaR is widely used, many studies have thoroughly examined its drawbacks. When a loss distribution has fat tails, VaR is not only incoherent but also fails to accurately estimate the risk of the loss (Rockafellar & Uryasev, 2002), and therefore, this significantly undermines the reliability of this risk measure (Chen, 2018). Despite this, value-at-risk is still a popular risk indicator because it is so easy to compute and comprehend. In addition to demonstrating

the incoherence of VaR, Artzner et al. (1999) also introduced the expected shortfall and dubbed it the ideal risk indicator. Using coherent risk measure theory, Pflug (2000) further demonstrated that ES is a coherent risk measure. And, in contrast to VaR, which is more frequently used, expected shortfall is a risk measure that is sensitive to the shape of the tail of the distribution of returns on a portfolio.

To correctly estimate risk measures, a distribution that captures all extreme and stylised facts is required. Extreme or “tail” risk, as described in BCBS (2019), requires risk practitioners to fully understand it. It has been developed to use the extreme value theory where Fisher and Tippet (1928) and Pickands (1975) are credited for developing the field of EVT. By obtaining three asymptotic limits, Fisher and Tippet (1928) were able to describe the distribution of extreme values, assuming that the variables are independent. An argument for modelling extreme values using a generalised extreme value distribution can be attributed to Maritz and Munro (1967). Extreme value theory is a theory that measures and models extreme events (large fluctuations) (tails of statistical distributions), i.e., it is suitable for financial assets with extreme returns (very large fluctuations in returns); hence, it assumes independent and identically distributed observations. This assumption of i.i.d does not necessarily hold for financial time series data (Chen, 2023).

To correct this, McNeil et al. (2015) proposed a two-stage methodology in the form of a generalised autoregressive conditional heteroscedasticity (GARCH)-EVT model using five index returns in their illustrations. The first step was to capture the heteroscedasticity (non-constant variation or fluctuations) features by fitting the GARCH model. The second step was to apply the Generalised Pareto distribution (GPD) and the GEV distribution to the residuals extracted from a selected GARCH model. The advantages of the GARCH-EVT hybrid model are in its capacity to model the extreme tail (large fluctuations) behaviour using the EVT methods while also capturing conditional heteroscedasticity (changing variation) in the data through the GARCH framework.

By implementing a dynamic method for forecasting a 1-day-ahead VaR, which combines the GARCH models and EVT to examine the extreme behaviour of major economic stock indices before and during the outbreak of the COVID-19 pandemic, Omar et al. (2020) and Paul and Sharma (2018) were able to accurately calculate VaR by using EVT methods. Explicitly, these authors adopted the GPD via the Peaks-over-threshold (POT) method. Comprehensive in-sample volatility modelling was implemented with skewed student distribution assumptions, and the information criterions were used to establish their goodness of fit. Furthermore, the VaR quantiles were estimated by using the conditional EVT (C-EVT) framework to obtain out-of-sample VaR forecasting results. The combined GARCH and EVT model performed relatively well in estimating the risk for all stock indices. The backtesting results demonstrate that the exponential-GARCH skewed-student's-t and C-EVT models are the most appropriate techniques for better measuring and forecasting VaR in comparison with the conventional methods.

Other studies like those of Echaust and Just (2020) examined the ability of value-at-risk estimates when each estimate is made with an optimal choice of the tails of the distribution by the combination of GARCH-EVT. Here, 5 methods were applied to describe the tail, namely the distance-metric method with the mean absolute penalty function, the minimisation of the asymptotic mean squared error (AMSE) estimate, the path-stability algorithm, the fixed-quantile procedure, and the automated eyeball

method. The GARCH-EVT approach in combination with a novel algorithm to automatically determine the optimal threshold to model the tail distribution was proposed by Bruhn and Ernst (2022). Furthermore, individual market risks were aggregated with a t-student Copula to investigate possible diversification effects on a portfolio level (Hoffmann & Börner, 2020). The empirical analysis indicates that all examined cryptocurrencies show high volatility in their price movements, whereby Bitcoin acts as the most stable cryptocurrency. All returns distributions are heavy-tailed and subject to extreme tail risks.

To effectively capture clustering volatility in the presence of structural breaks (structural changes) and tail behavior, Makatjane et al. (2021) also developed a two-stage analysis. The first stage was to estimate a Markov-switching exponential GARCH model to obtain regime switching residuals that are i.i.d. In the second stage, these authors applied EVT to the upper regime residuals and estimated both the GPD and the GEV distribution. This was to complete the study by McNeil et al. (2015), Sahamkhadam et al. (2018) and Echaust and Just (2020) among others. To manage the risks of a portfolio made up of commodities, currency indices, and equity securities, Koliai (2016) presented the GARCH-EVT with an R-vine copula model. The effectiveness of the generalised Lambda distribution (GLD), the generalised Pareto distribution, and the generalised extreme value distribution were advocated by Huang et al. (2017) and these authors simulated daily VaR and ES for log-returns of platinum, gold, and silver prices; giving GPD and GLD better performance over GEV distribution.

The main approach to forecasting is the combination of several models (Hajirahimi & Khashei, 2019), which performs better than using just one model. Comparing the hybrid method to the single model, accurate performance can be produced (Büyüksahin & Ertekin, 2019). However, some studies like of Ibn Musah et al. (2018), for instance, aim to investigate the risks connected with the main Ghanaian stock exchange while combining EVT with artificial neural networks (ANNs). Recent research conducted by Ilyas et al. (2022) has proposed a new hybrid method, consisting of a fully modified Hodrick Prescott filter (FMHP) to improve prediction accuracy. This method consists of three main components: machine-learning-based prediction, novel features, and a noise-filtering technique. Moreover, the combination of generative adversarial networks (GANs) with extreme values proved to be more effective in modelling extreme values (Boulaguiem et al., 2022).

The use of an ensemble learning-based rolling window approach to investigate the advantage of combining multiple GARCH-type models with LSTM (herein referenced Long-short Term-memory) to forecast value at risk is advocated by Kakade et al. (2022). These authors use coverage tests loss functions for the financial crisis of 2008–2009 and the COVID-19 recession of 2020–21 to assess the model's performance on crude oil returns. When comparing the VaR forecasts of the hybrid models to the conventional and GARCH techniques, a notable improvement in quality and accuracy is noted. It is discovered that the most effective estimator of Value-at-Risk is the Filtered Historical Simulation technique.

Moreover, Barrera et al. (2022) proposed a non-asymptotic convergence analysis of a two-step approach to learn value-at-risk and expected shortfall in a nonparametric setting using Rademacher and Vapnik-Chervonenkis bounds. The approach of these authors for VaR is extended to the problem of learning at once multiple VaRs

corresponding to different quantile levels. This led to the development of effective learning systems based on least-squares regressions and neural network quantiles. Without access to the latter, a posteriori Monte Carlo (non-nested) method was employed to estimate distances to the ground-truth VaR and ES. Numerical experiments in a Gaussian toy model and a financial case study, where the goal is to determine a dynamic initial margin, were used to demonstrate this. These numerical tests indicate that, despite initially seeming counter-intuitive, learning several quantiles (multi- α (I), multi- α (II), or multi- α (III)) can aid in effectively targeting extreme quantiles than a typical single quantile learning strategy.

Recent research has established a flexible likelihood-based framework for the joint modelling of VaR and ES, based on the relationship between the quantile score function and the asymmetric Laplace density. Capturing the underlying combined dynamics of these two quantities is of great relevance in financial applications. To tackle this issue, Li et al. (2020) created a hybrid model that effectively captures the underlying dynamics of VaR and ES. The model is based on the asymmetric Laplace quasi-likelihood and uses the Long Short-Term Memory time series modelling technique from machine learning. This model is known as LSTM-AL. In the LSTM-AL model, these authors use the adaptive Markov chain Monte Carlo (MCMC) approach for Bayesian inference. Their empirical results show that the proposed LSTM-AL model has improved the VaR and ES forecasting accuracy over a range of well-established competing models.

An early warning system for extreme daily losses for financial markets is of crucial modelling system. A three-stage methodology was established by Makatjane and Moroke (2021) where in the first stage a seasonal Autoregressive integrated moving average (SARIMA) was estimated to filter the series to obtain i.i.d residuals. By using the Markov-chain Monte-Carlo approach, the Markov-switching exponential GARCH model coupled with generalised extreme value distribution was fitted to these i.i.d residuals and finally, the logistic model tree was time-honored to establish early warning signs. Alternatively, Makatjane and Tsoku (2022) overcome the dimensionality problem in forecasting VaR and ES uncertainty intervals in financial time series data by bootstrapping and backtesting density forecasts using Bayesian methods that are based on a weighted threshold and quantile of a continuously ranked probability score. These authors found that extension of this non-stationary distribution in literature is quite complicated since it requires specifications not only on how the usual Bayesian parameters change over time but also on those with bulk distribution components. This implies that the combination of a stochastic econometric model with extreme value theory procedures provides a robust basis necessary for the statistical backtesting and bootstrapping density predictions for VaR and ES.

2. Methodology

A five-day financial time series exchange Johannesburg stock exchange/All-share index (FTSE/JSE-ALSI) for the period of 4 January 2010 to 5 April 2024 is used in this study. This consists of 3646 observations. The use of a five-day (s) frequency is based on the fact that, on weekends and holidays, stock markets are closed. Therefore, the South African Stock Exchange is not an exception. No trading is happening on these days, hence the recorded market prices are from Monday(s) – Friday(s); except where the holiday arises

during the week. In this case, only four data points are available instead of five. To avoid any exchange rate fluctuations, the index is kept in its original currency; i.e., the FTSE/JSE-ALSI used in this study is kept in its ZAR; the South African currency.

Let X_t be a stock price index on the day t and X_{t-1} be a stock price index on the day $t - 1$. Letting r_t to be stock returns at time t ; Algieri and Leccadito (2020) showed that r_t can be modeled by

$$r_t = \mu_t + \varepsilon_t \quad (1)$$

where μ_t is a time-varying mean of a time series and ε_t is the error term that should be modeled by

$$\varepsilon_t = \eta_t \sigma_t. \quad (2)$$

In model (2), σ_t is a time-varying dynamics of a time series; while η_t is an i.i.d process.

2.1. Proposed TBATS—generalised extreme value distribution

The capacity of conventional seasonal and exponentially smooth models to handle dual-calendar, multi-seasonal, and noninteger seasonal time series is restricted (Sorokina et al., 2023). Several scholars have examined the challenge of handling intricate seasonal time series in this context, and to address this issue, the exponentially smooth model was a modified problem (Zhao & Zhang, 2022); hence the BATS model was introduced to address this complex time series pattern. The basic model form is expressed as $BATS(p, q, m_1, \dots, m_T)$ where B is the Box-Cox transformation for addressing heterogeneity, A is the ARMA error for addressing short-term dynamics, damping (if any) trends, and seasonal components, T and S are the trend and seasonal components of the time series respectively. In addition, p and q of the BATS model are the Autoregressive p and moving average q parameters respectively. Furthermore, m_1, \dots, m_T are the seasonal periods of the ARIMA model. The mathematical formula for the BATS model according to Munim (2022), is presented as follows

$$r_t^{(\omega)} = \begin{cases} \frac{r_t^\omega - 1}{\omega}, & \omega \neq 0 \\ r_t, & \omega = 0 \end{cases}. \quad (3)$$

The major objective of the time series prediction technique known as the TBATS is to forecast complex seasonal trends by using exponential smoothing. Trigonometric seasonal functions were used in place of the seasonal components to develop the TBATS model, which is a modified time series method based on the BATS model (Thayyib et al., 2023). Thus, the structure of this model is the initial T that signifies the “trigonometric” function. This is expressed as $TBATS(\omega, p, q, \varphi, \{m_1\} \dots, \{m_T, k_T\})$. Hence, Talkhi et al. (2024) showed that the mathematical formula for the TBATS model is as follows

2.2. Seasonal periods

$$r_t^{(\omega)} = l_{t-1} + \phi b_{t-1} + \sum_{i=1}^M S_{t-m_i}^{(i)} + d_t \quad (4)$$

where r_t is the closing stock price returns in the case of our study, $r_t^{(\omega)}$ represents Box-Cox transformation of r_t with the parameter ω , l_t is the local level in time t , b_t is the short-term trend in time t , ϕ is the trend damping parameter, $S_t^{(i)}$ denotes the i^{th} seasonal component at a time t , m_1, \dots, m_T refer to seasonal periods, d_t represents an ARMA (p,q) process.

2.3. The global and local trend

$$\begin{aligned} l_t &= l_{t-1} + \phi b_{t-1} + \alpha d_t \\ b_t &= (1 - \phi)b + \phi b_{t-1} + \beta d_t \end{aligned} \quad (5)$$

and according to Munim (2022), α, β and γ are smoothing parameters for $i = 1, \dots, T$. Hence, $\lambda_j^{(i)} = 2\pi j / m_i \cdot S_{j,t}^{(i)}$ represents the stochastic level of i^{th} seasonal component and $S_{j,t}^{*(i)}$ represents the change (or growth) in the stochastic level of i^{th} seasonal component over time.

2.4. ARMA error

$$d_t = \sum_{i=1}^p \phi_i d_{t-i} + \sum_{j=1}^q \theta_{t-j} \iota_t + \iota_t. \quad (6)$$

From ρ_1, \dots, ρ_T denote the seasonal period, level and trend of components of the time series can be denoted by l_t and Z_t and time t . The seasonal component can be denoted by S_t^i at time t , while d_t represents the $ARMA(p, q)$ component and ι_t is the white noise process. For seasonal data, the model (7) represents trigonometric exponential smoothing models

$$\begin{aligned} S_t^{(i)} &= \sum_{j=1}^{k_i} \alpha_{j,t}^{(i)} \cos[\psi_j^{(i)} t] \\ \alpha_{j,t}^{(i)} &= \alpha_{j,t-1}^{(i)} + k_1^{(i)} d_t \\ \beta_{j,t}^{(i)} &= \beta_{j,t-1}^{(i)} + k_2^{(i)} d_t \end{aligned} \quad (7)$$

where the smoothing parameters are $k_1^{(i)}$ and $k_2^{(i)}$. $\psi_j^{(i)} = 2\pi j / \rho_i$. This is an extended modified single source of error version of single seasonal multiple sources of error representation which was suggested by Hannan (1970).

2.5. Fourier seasonal term

Additionally, model (8) are seasonal patterns modelled by the fourier model. For a detailed explanation of the TBATS model estimation and model selection procedure, the reader is referred to De Livera et al. (2011).

$$\begin{aligned} S_t^{(i)} &= \sum_{j=1}^{k_i} S_{j,t}^{(i)} \\ S_{j,t}^{(i)} &= S_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + S_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ S_{j,t-1}^{*(i)} &= S_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + S_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned} \quad (8)$$

2.5.1. The generalised extreme value distribution via block maxima/minima

The generalised extreme value distribution is a probability distribution used in statistics to model the distribution of extreme values. It is particularly useful for modelling the distribution of the maximum or minimum of a set of random variables. The GEV distribution is often employed in fields such as hydrology, finance, and meteorology, where extreme events are of interest (Maposa et al., 2016).

To illustrate the GEV via block maxima/minim (BM), we now, let X_1, \dots, X_n represent independent and identically distributed residuals from the TBATS using equations (3)-(8). The maximum values (gains) are now computed by

$$M_n = \max[X_1, \dots, X_n], \quad (9)$$

while the minimum values (losses) are computed by

$$m_n = \min[X_1, \dots, X_n] = -M_n = -[\max[X_1, \dots, X_n]]. \quad (10)$$

Losses are negative returns which have been converted into positive values by multiplying by negative one (-1). M_n and m_n are the maximas and minimas over an n -observation period.

2.5.2. Choosing an optimal block sizes

The proposed methodology to choose the best block size includes the following eight steps.

Step 1: The data is chosen to operate and this is FTSE/JSE-ALSI for the methodology part. The ALSI is a market capitalisation-weighted index comprising all listed companies on the Johannesburg Stock Exchange. It serves as a broad measure of the performance of South African stocks. From this time, the All-share index represents 99 percent of the full market cap value of all eligible securities listed on the main board of the JSE. Unlike Özari et al. (2019) who used 10 percent of the generated data set (for the test part of the analysis), we only allocated the test dataset 25 percent. This was to provide a more robust estimate of the model's performance on unseen data. Furthermore, our aim was to reduce the risk of overestimating the model's accuracy due to variability in smaller testing sets.

Step 2: Data is separated into blocks with a minimum and maximum number determined. K is the block number. The FTSE/JSE-ALSI is separated into multiple blocks, with the first block having a numerical limit of five and continuing until block fifty-nine. This is designed to enable the risk measures to generate four or fewer exceptions over the course of ninety trading days.

Step 3: For each K i.e., the block size, a minimum/maximum set of values is generated by computing the minimum/maximum value of each block. In this step, a data set is created for $k = 5, 6, 7, \dots, 59$ and the minimum/maximum values of each block are taken separately for these data sets.

Step 4: For the general extremum value distribution, K data is set appropriately. Several tests are used to determine whether the over-distribution of all blocks is appropriate (such as the Shapiro-Wilk, Anderson Darling, and Kolmogorov Smirnov tests). It is decided which distribution is most suitable.

Step 5: The parameters of the optimal distribution for each K value are determined.

Step 6: For each K value, new variables are constructed by the parameters calculated in step 5 as the over-value distribution or the number of observations separated for testing with optimal distribution, and these variables are called predictors.

Step 7: The similarity between the test data and the k sets of estimated data is examined for each K . If at all feasible, it is ideal for these two sets of data to be equal. The degree of resemblance between these two data sets can be determined in a variety of ways. In this study, we opted to employ the Pearson correlation to check for similarity; unlike Özari et al. (2019) who used the absolute difference technique to check for similarity.

Step 8: The best block size is defined as the block with the highest correlation/similarity.

2.5.3. A stationary generalised extreme value distribution

The GEV distribution combines into a single expression the three possible limiting distributions that arise from the limit theorem of Fisher and Tippet (1928) on extreme values or maxima in sample data. The GEV distribution can be denoted as $GEV(\mu, \sigma, \xi)$ and according to Gagaza et al. (2019), this distribution is given by

$$G(x; \mu, \sigma, \xi) = \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{1/\xi} \right\}. \quad (11)$$

Nonetheless, distribution (11) is valid for $\{x : \mu - \sigma/\xi < x < \infty\}$ and its probability density function (PDF) concerning Chikobvu and Ndlovu (2023a) is given by

$$G_{\xi, \mu, \sigma}(x) = \frac{1}{\sigma} \left[1 + \xi(t) \left(\frac{x - \mu(t)}{\sigma(t)} \right) \right]^{-1/\xi} \exp \left\{ - \left[1 - \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}, \xi \neq 0. \quad (12)$$

$$\exp \left(- \frac{x - \mu}{\sigma} \right) \exp \left(- \exp \left(- \frac{x - \mu}{\sigma} \right) \right), \xi = 0$$

The vector $(\mu, \sigma, \xi)^T$ is the parameter estimates obtained by maximising the log-likelihood function $\ell = \ln \prod_{i=1}^k f(x : \mu, \sigma, \xi)$ in PDF (12); where, k denotes the number of block maxima or minima. The maximum likelihood estimate (MLE) method offers the advantage of estimating these three parameters simultaneously. Additionally, Hu and Scarrott (2018) revealed that when $\xi \neq 0$, the log-likelihood converges to

$$L_k = \ell(\mu, \sigma, \xi) k \ln \sigma \left(\frac{1}{\xi} + 1 \right) \sum_{i=1}^k \ln \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right] - \sum_{i=1}^k \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-1/\xi}. \quad (13)$$

To obtain the maximum likelihood estimates for μ , σ and ξ , a partial derivative of the log-likelihood L_k in the model (13) is applied and results in

$$L_k = 0 \quad L_k = \left(\frac{\partial L_k}{\partial \mu}, \frac{\partial L_k}{\partial \sigma}, \frac{\partial L_k}{\partial \xi} \right). \quad (14)$$

2.6. Risk measures

Risk measures are historical predictors of investment risk and volatility, and they also form an important component of the modern portfolio theory (MPT). The MPT is a standard financial and academic approach for evaluating a stock or a stock fund's performance compared with its benchmark index. The proposed risk measures for the FTSE/JSE-ALSI are discussed in this section.

2.6.1. Value-at-risk

The Value-at-Risk is estimated as

$$VaR_\alpha = \begin{cases} \hat{\mu} - \hat{\sigma}/\hat{\xi} \left\{ 1 - [-n \ln(1 - \alpha)]^{-\hat{\xi}} \right\}, & \hat{\xi} \neq 0 \\ \hat{\mu} - \hat{\sigma} \ln(1 - \alpha), & \hat{\xi} = 0 \end{cases} \quad (15)$$

2.6.2. Expected shortfall

Expected shortfall considers a loss beyond the VaR level and is shown to be sub-additive, while VaR disregards a loss beyond the percentile and is not sub-additive. The expected shortfall according to Rockafellar and Uryasev (2000) is the expected loss size given that VaR_α is exceeded. That is, $ES_\alpha = E[X|X > VaR_\alpha]$. Nevertheless, Nadarajah et al. (2014) provided a general computation of ES with a given probability π which is defined as

$$ES_\alpha = \frac{1}{\alpha} [E(XI\{X \leq VaR_\alpha(X)\}) + pVaR_\alpha(X) \Pr(X \leq VaR_\alpha(X))], \quad (16)$$

where $I(\bullet)$ denotes the risk indicator function.

2.6.3. Conditional tail expectation

The quantile risk measure assesses the worst case loss or gain, where worst case is defined as the event with a $1 - \alpha$ probability. Hardy (2006) discovered that one issue with the quantile risk measure is that it does not take into consideration what the loss or gain will be if the $1 - \alpha$, most pessimistic scenario occurs. The loss distribution above the quantile does not influence the risk measure. The Conditional tail expectation is chosen to address some of the problems with the quantile risk measure.

Adopting theorem 3 of Yang et al. (2015), the conditional tail expectation is computed by letting $\{X_1, \dots, X_n\}$ to be a real-valued independent random variable with the following density function $\{Y_1, \dots, Y_n\}$ and the row vector $\{\theta_1, \dots, \theta_n\}$ to be a nonnegative and nondegenerate zero random variable which is independent of $\{X_1, \dots, X_n\}$ but subjective on each other. If $Y_k \in L \cap D, E\theta_k^{\beta_k} < \infty, \beta_k > J_k^+$ and $\mathbb{P}(\theta_k X_k > x) = 0\mathbb{P}(\theta_k X_k > x) \quad k \in \{1, 2, 3, \dots, n\}$, the asymptotic conditional expectation of level q by Tang and Yuan (2014) is given by

$$CTE_q(S_n^\theta) = E(S_n^\theta | S_n^\theta > X_q) \quad (17)$$

where, $X_q = VaR_q(S_n^\theta) = \inf\{x \in \mathbb{R} : P(S_n^\theta < x) \geq q\}$. Model (17) has later been modified by Yang et al. (2015) to

$$CTE_q(S_n^\theta)x \underset{q \uparrow 1}{\sim} \alpha^{-(1-\alpha)} \left(\sum_{k=1}^n C_k E\theta(\theta_k^\alpha) \right)^{1/\alpha} VaR_q(x) \quad (18)$$

2.6.4. Glue value-at-risk

This is defined by a function with four parameters. Glue-VaR risk measurement can also be harmonised in a large range of contexts if the parameters have been standardised. This allows Glue-VaR to contain the risk of mild tagged losses that are between VaR and CTE. Given a confidence level α , the distortion function for Glue-VaR is

$$K_{\beta,\alpha}^{h_1,h_2}(u) = \begin{cases} \frac{h_1}{1-\beta} \cdot u, \\ h_1 + \frac{h_2-h_1}{\beta-\alpha} \cdot [u - (1-\beta)], \\ 1, \end{cases} \quad (19)$$

where, $\alpha, \beta \in [0, 1]$ so that $\alpha \leq \beta$, $h_1 \in [0, 1]$ and $h_2 \in [h_1, 1]$, β is the additional confidence parameter in addition to α . Belles-Sampera et al. (2014), showed that the shape of the Glue-VaR is determined by distorted survival probabilities h_1 and h_2 , while $1-\beta$ and $1-\alpha$ are levels respectively. Therefore, Glue-VaR is computed by

$$\text{Glue} - \text{VaR}_{\beta,\alpha}^{h_1,h_2}(X) = \omega_1 \cdot \text{CTE}_{\beta}(X) + \omega_2 \text{CTE}_{\alpha}(X) + \omega_3 \cdot \text{VaR}_{\alpha}(X) \quad (20)$$

where, ω_1, ω_2 and ω_3 are from the notation $\begin{cases} \omega_1 = h_1 - \frac{(h_2-h_1) \cdot (1-\beta), \alpha \neq \beta}{\beta-\alpha} \\ \omega_2 = \frac{h_2-h_1 \cdot (1-\alpha)}{\beta-\alpha}, \alpha \neq \beta \\ \omega_3 = 1 - \omega_1 - \omega_2 = 1 - h_2 \end{cases}$. The selection

of this metric is based on the Omega ratio, which is an effective measurement tool and also helps to divide the range of investment returns into two groups: the range of gains and the range of losses.

2.7. Backtesting

Backtesting is a statistical method that compares actual losses to appropriate risk measures on a systematic basis. For example, when the level of confidence in daily risk measurement, such as VaR, is 99 percent, an exception occurs once every 100 days. The backtesting process determines whether the frequency of exceptions over a specified period corresponds to the confidence level chosen (Nieppola, 2009). These types of testing are referred to as binding tests. They are straightforward tests to implement because there are no exceptions (Mwamba, 2012). Three commonly used methods for backtesting models in the literature are the Kupiec likelihood ratio (LR) test for unconditional coverage, which was established by Kupiec (1995), and the Christoffersen test for conditional coverage, which was developed by Christoffersen and Pelletier (2004) as an extension to the Kupiec test, and lastly the Basel II green zone which was modified by (BCBS, 2019).

2.7.1. Kupiec likelihood ratio test

The Kupiec unconditional coverage test by Kupiec (1995) is used to verify the model's suitability and efficacy in the calculation of VaR. Model adequacy is ensured using this method, which is preferred (Chan et al., 2022). There are other methods, like the Christoffersen conditional coverage test. This test is used for testing the clustering of model violations (Christoffersen & Pelletier, 2004). The unconditional coverage test by Kupiec assumes that, if the model is valid, the

proportion of risk measure estimate violations must be close to the corresponding tail probability level (Ndlovu & Chikobvu, 2023). The choice of this test in this study is based on its ability to evaluate whether the actual number of events (such as defaults) aligns with the predicted probabilities of those events occurring. In essence, the test is used to count the number of risk measure's exceptions, i.e., days (or holding periods of other length) when portfolio losses exceed the estimates of risk measure. The test also incorporates uncertainty by calculating confidence intervals around the predicted probabilities, which helps in assessing the reliability of the model's predictions.

Let x^α be the number of violations observed at level α , i.e., $r_t < VaR_\alpha$ (for long positions) or $r_t > VaR_\alpha$ (for short positions). The test procedure involves comparing the corresponding proportion of violation $\left[\frac{x^\alpha}{N}\right]$ to α . The $H_0 : E\left[\frac{x^\alpha}{N}\right] = p$ i.e., the expected proportion of violations is equal to α . Under H_0 , the Kupiec likelihood ratio test is given by

$$LR_{UC} = -2 \ln \left(\frac{p^{x^\alpha} (1-p)^{N-x^\alpha}}{\left(\frac{x^\alpha}{N}\right)^{x^\alpha} \left(1 - \frac{x^\alpha}{N}\right)^{N-x^\alpha}} \right) \sim \chi_1^2, \quad (21)$$

where, N is the total observations. If the observed probability value exceeds the calculated probability value, reject the null hypothesis and infer that the model is incorrect, meaning that the risk derived from the risk model, or both the VaR and the ES, is unreliable. This implies it can send incorrect signals to risk managers.

2.7.2. Christoffersen likelihood ratio

This test is also known as the Markov test and it examines the independence property, that is, the test examines if the probability of VaR violation on any given day depends on the outcome of the previous day. Hence we use this test to also account for both the unconditional distributional properties of returns and the dynamics of volatility clustering often observed in financial time series data. Furthermore, we use the Christoffersen likelihood ratio test to account for the presence of time-varying volatility in financial markets, which is crucial for accurately capturing the evolving risk dynamics.

By defining $p_{ij} = \frac{\eta_{ij}}{\sum_j \eta_{ij}}$ where p_{ij} presents a violation probability that occurs conditionally on state i at a time $t - 1$ such that $p_1 = \frac{\eta_{11}}{\eta_{11} + \eta_{21}}$, $p_2 = \frac{\eta_{22}}{\eta_{12} + \eta_{22}}$ and $p = \frac{\eta_{11} + \eta_{22}}{N}$, then the extended Kupiec LR test statistic is given by

$$LR_{cC} = -2 \ln \left(\frac{(1-p)\eta_{11} + \eta_{12} + p\eta_{21} + \eta_{22}}{(1-p)\eta_{11}p_1\eta_{21}(1-p_2)\eta_{12}p_2\eta_{22}} \right). \quad (22)$$

In line with Papastathopoulos and Tawn (2013), model (22) is an asymptotic chi-square distribution with one degree of freedom. Reject the null hypothesis if the observed probability value is greater than the calculated probability value.

2.7.3. The Basel III Green Zone

The Basel Committee on Banking Supervision required internal banking or financial market backtesting procedures in 2007. Financial institutions need to regulate

their capital requirements and this is done by backtesting to cover market risk due to their trading activities. The capital requirement size according to is calculated as

$$Cap = fact \times \max \left(VaR_t(0.01), \frac{1}{60} \sum_{i=1}^{59} VaR_{t-1}(0.01) \right) \quad (23)$$

where, *Fact* is the multiplication factor reported in Table 1. In other words, the required capital is equal to the multiplication factor that is multiplied by the highest value between today's 99 percent VaR and the mean of the last 60 days 99 percent VaR. The risk manager should automatically upgrade the multiplication factor from 3.0 to 4.0.

3. Results and Discussion

In this section of the study, we present the analysis and discussion of the results. These results are presented in tables and figures. Following Mushori and Chikobvu (2024), Chikobvu and Ndlovu (2023a), and Chinhamu et al. (2015), we analyse the gains and the losses separately because we want to model different risk profiles which leads us to understand the significance of a downside risk that is indicated by the negative returns (losses) and also understanding the tail behaviour of these losses which is crucial for risk management, particularly for setting capital reserves, insurance, and regulatory compliance. Moreover, positive returns (gains) highlight opportunities for substantial profits. Therefore, understanding the tail behaviour of gains can further help in devising investment strategies and optimisation of portfolios. It is worth noting that financial returns are asymmetric hence their distributions are often asymmetric, meaning that the tails on the left (losses) and the right (gains) can have different shapes and characteristics. Modelling them separately allows for a more accurate representation of their distinct behaviours. In addition, this approach helps to identify better and understand specific factors that drive positive and negative returns; where, negative returns might be driven by factors like market crashes, economic downturns, or unexpected negative news. However, positive returns might be driven by market booms, extraordinary company performance, or positive economic indicators. Finally, managing the risk of extreme losses is crucial for financial institutions, especially those involved in risk-sensitive activities like banking, insurance, and investment management. While on the other side, we want to identify and leverage the opportunities for extreme gains which are beneficial for investment strategies, hedge funds, and performance optimisation.

Figure 1 depicts a plot of returns for FTSE/JSE-ALSI. Observed in this figure are the relative dynamics of the FTSE/JSE-ALSI index for the specified period. It is observed that

Table 1. The Basel II zones and the exceptions based on a 250 intraday trading sample.

Zone	Confidence level			Multiplication factor
	90%	95%	99%	
Green	0–32	0–17	0–4	3
Yellow	33–43	18–26	5–9	3.40, 3.50, 3.65, 3.75 or 3.85
Red	≥ 44	≥ 27	≥ 10	4

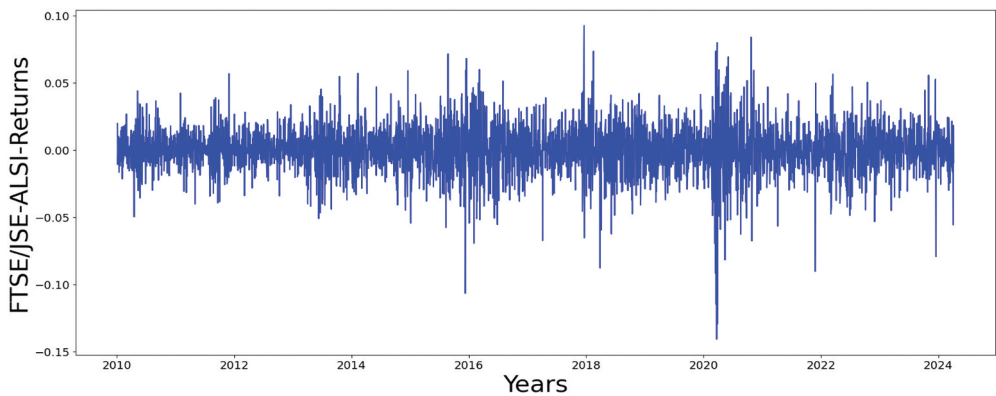


Figure 1. Five-day returns of FTSE/JSE-ALSI plot.

the mean of returns is not constant while the variance oscillates around high volatility (instability) and low volatility (tranquillity) with more extremes towards the end of the sample period. This implies that FTSE/JSE-ALSI have volatility clustering.

Regarding the marginal distribution, the quantile-quantile (Q-Q) and normal histogram plot in Figure 2 reveal a strong departure from linearity in the lower and upper tails of the distribution. This evidence is also seen in Table 2 where the reported kurtosis is greater than three and the skewness is less than zero indicating that returns on FTSE/JSE-ALSI are asymmetrical with negatively skewed innovations.

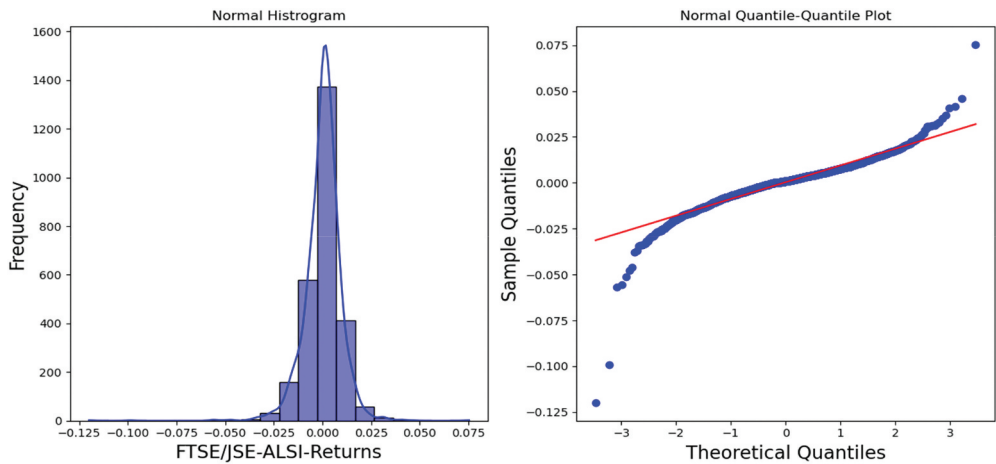


Figure 2. Normal histogram and normal quantile-quantile plots.

Table 2. Descriptive statistics for returns series.

	Mean	std deviation	Skewness	Kurtosis	J-B test	S-W test	K-S test
FTSE/JSE>Returns	0.000311	0.0096	−1.39970428	17.06483815	32932.373 (0.001)	0.898 (0.001)	0.484 (0.001)

Note: NB: values in () are probability values of the S-W and A-D respectively.

The normal histogram on the left panel of [Figure 2](#) further confirms this negative skewness which also possesses high kurtosis that is above three. Therefore, a conclusion is that FTSE/JSE-ALSI returns have fat tail and leptokurtic behaviour. Khan et al. (2021) in their study of extreme value theory and COVID-19 have found fat tail behaviour in NIFTY-50 they have used.

As evidenced in [Table 2](#), the deduction here is that the overall returns on FTSE/JSE-ALSI are increasing during the period in question. The mean of the average returns is very small compared to the standard deviation. Mokoena (2016) and Beytell (2016) in their studies reported the same results. All in all, one can infer that the mean of returns is somehow smaller than the standard deviation of the returns. The FTSE/JSE-ALSI returns considered revealed kurtosis that is above 3, indicating that the returns series are all leptokurtic. Another feature of the returns series is the presence of skewness; they are found to be negatively skewed, implying that the distribution of FTSE/JSE-ALSI returns is significantly fatter than the Gaussian distribution. This negative skewness indicates that the lower tail of the distribution is thicker than the upper tail and declines in returns are more common than their increases. The Jarque-Bera (J-B), Shapiro Wilk (S-W), and Kolmogorov – Smirnov (K-S) tests confirmed non-normality at a 5% level of significance. The three test statistics led to the rejection of the null hypothesis of normality, confirming the results reported in [Figure 2](#). According to Vee and Gonpot (2014) and Korkpoe and Junior (2018), stock markets are described by boom-bust cycles and such cycles feed into the volatility of the markets.

3.1. Trend analysis results

Trends are evident in financial time series data during the week, month, quarter, etc. On weekends and holidays, all exchanges are closed; therefore, no trading takes place. The degree of activity on weekdays is significantly high. Across FTSE/JSE-ALSI, the day-of-the-week impact is noticed in [Figure 3](#). Generally, market activity is lowest on Monday and highest on the last two working days of the week. To be precise, trading activity starts growing progressively from Monday to Friday. On weekends and holidays, there are no

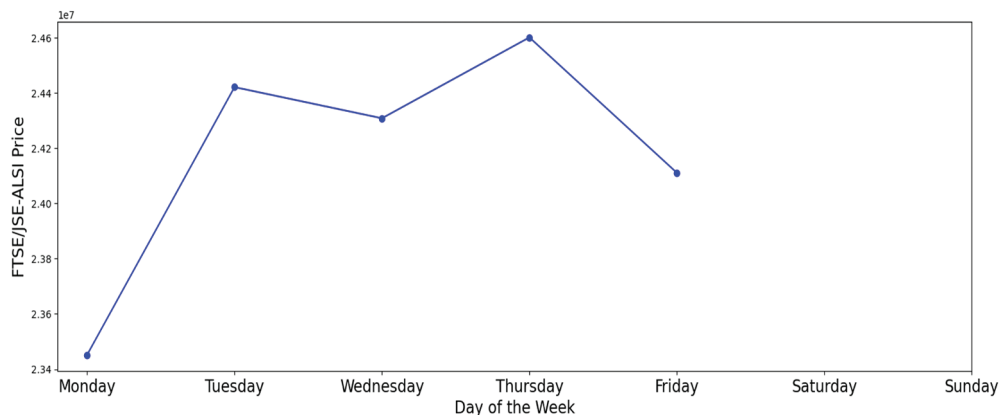


Figure 3. Day of the Week pattern.

trading activities, leading to low opening stock prices on Monday. The weekend effect (also called the Monday effect, the day-of-the-week impact, or the Monday seasonal) describes how stocks tend to perform better on Fridays than on Mondays (Strohsal et al., 2019). In general, over all the years, there has been an increasing trend in FTSE/JSE-ALSI prices and this observation is seen in Figure A1 in Appendix 1 for the whole sample period.

The Mann–Kendall test statistic and Sen’s slope estimator are used to analyse the long-term trends of the FTSE/JSE-ALSI. The outcome of the Mann–Kendall test results as reported in Table 3 and revealed that FTSE/JSE-ALSI has a significant monotonic increasing long-term trend because the value of $\tau > 0$. Sen’s slope value also shows significantly increasing magnitudes of trends, which correspond with the Mann–Kendall test results. The cause of this increase in the average returns for the other four days of the week is positive, while the average returns for Monday are noticeably negative, therefore, a significant positive trend is depicted in Table 3. According to Killian (2023) and Atsin and Ocran (2015), this increase is influenced by the demand that outstrips the supply. That is, more people want to buy the share price instead of selling it; and, consequently, the price rises. Otherwise, the price falls because supply is greater than demand. If more buyers move into the market, the demand grows, and share prices go up especially if there is limited supply. If supply and demand are just about equal, the share price is likely to move around in a narrow range for a while, until one of the factors outweighs the other (Khumalo, 2013). Other factors include macroeconomic factors such as interest rate changes, financial outlook, and inflation fluctuations. If the interest rate and inflation rate go down, and the economic outlook is in a good state, demand usually increases, and the share price is likely to rise (Killian, 2023).

3.2. *Tbats-stationary generalised extreme value distribution*

To begin the main analysis, a TBATS is trained with a ratio of 75% training and 25% validation sets. To account for stationarity in our returns series, we employ the augmented Dickey-Fuller (ADF) and Philips Peron (PP). The same procedure is done by Chinhamu et al. (2015). The PP test is carried out using the Bartlett Kernel spectral estimating method, while the ADF test is set to lag 0 using the Schwartz Information Criterion (SIC). Table 4 presents the results of the ADF and PP tests, which show that the unit root null hypothesis is not supported for any test. As a result, it is possible to regard the return series on FTSE/JSE-ALSI as stationary.

Table 3. Mann-Kendall test statistic and Sen’s slope estimator.

	M-K Test Statistic	Kendall’s Tau	p-value	Sen’s Slope
FTSE/JSE-ALSI	58.055	0.753	0.001	12.635

Table 4. Results for ADF and PP unit root tests for FTSE/JSE-ALSI return series.

Unit Root test	Test statistic	p-value
ADF Test	−13.696	0.001
Philips-Perron test	−45.459	0.001

Using the returns on FTSE/JSE-ALSI which are computed using model (1), a TBATS model is fitted to these returns. Assortment of the best TBATS model automatically is accomplished by using the *TBATS* function in Python for these returns. Employing the maximum likelihood estimation, the following parameter estimates are obtained: $\hat{\lambda} = 0.337515$, $\hat{\alpha} = 0.021854$ $\hat{\beta} = 0.960208$ the damping parameter estimate is $\hat{\phi} = -0.91980288$.

3.3. Selection of block minima/maxima

After fitting our TBATS model, we extract the residuals which are i.i.d. As a first step, 25% of the residual series is reserved for future testing. The time interval of the data set to be separated is from 21 August 2020 to 5 April 2024. In the second step, the data set consisting of 3646 observations is divided into blocks of different sizes. Unlike Chikobvu and Ndlovu (2023a) who extracted monthly period minima/maxima from the daily returns of the BTC/USD and ZAR/USD returns data, we determine the minimum block size as 52 (i.e., weekly) and the maximum block size as 59 because this is the trade-off problem between the bias and variance, and Figure 4 presents these results.

Table 5 shows the first two blocks resulting from partitioning the residuals into 52 blocks by date to provide a detailed overview of the block partitioning. After receiving all the blocks, the maximum and minimum values of the blocks are calculated as shown in Table 5, and different data sets of maximum and minimum values are obtained. Figure 4 further shows a graph of all minimum and maximum values obtained from 52 blocks. As shown in Figure 4, the two variables derived from the minimum and maximum waves behave in a similar structure for both positive returns and negative returns.

When a block is established, two random variables are calculated from the minimum and maximum values obtained from that block, and the characteristics of their distributions are recorded by determining how these variables are dispersed or best distributed. The block length ranges from 20 to 59 for negative returns and 18 to 59 for positive returns (owing to data sparsity), as determined by which block size is better explained or projected. The blocks allotted for the

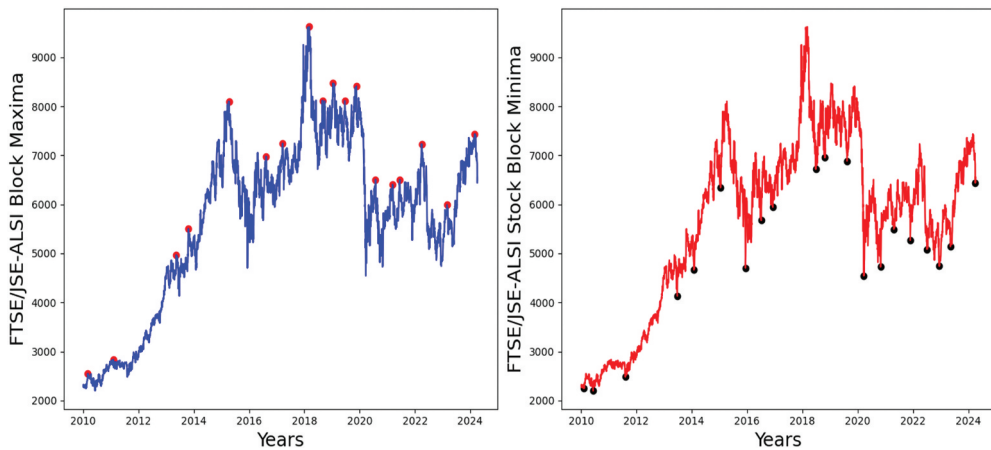


Figure 4. Block maxima (left panel) and block minima (right panel).

Table 5. FTSE/JSE-ALSI sample application.

Minima			Maxima		
Size	Date	Block	Size	Date	Block
1	08/02/2010	2246	1	04/03/2010	2546
2	08/62/2010	2200	2	01/02/2011	2835
3	08/08/2011	2481	3	16/05/2013	4967
4	24/06/2023	4133	4	22/10/2013	5500
5	30/01/2014	4666	5	10/04/2015	8101
6	16/01/2015	6343	6	15/08/2016	6979
7	11/12/2015	4703	7	20/03/2017	7242
8	06/07/2016	5674	8	20/03/2018	9620
9	06/12/2016	5944	9	28/08/2018	8119
10	27/06/2018	6717	10	15/01/2019	8469
11	26/10/2018	5952	11	20/06/2019	8109
12	14/08/2019	6886	12	20/11/2019	8408
13	23/03/2020	4544	13	29/07/2020	6505
14	30/10/2020	4727	14	12/03/2021	6405
15	23/04/2021	5496	15	15/06/2021	6500
16	26/11/2021	5270	16	30/02/2022	7232
17	06/07/2022	5086	17	01/03/2023	5989
18	15/12/2022	4740	18	01/03/2024	7433
19	11/05/2023	5141			
20	03/04/2024	6439			
Minimum		2200	Minimum		2546
Maximum		6886	Maximum		9620

residuals from the TBATS are used to construct the data blocks with the numbers 20, 21, and 22 for negative returns and 18,19 and 20 for positive returns. Aside from the maximum and maximum values of the generated block sizes, the column totals for the block sizes of each of the 212 negative returns data and the 210 positive returns data are calculated starting from 20 to 59 and 18 to 59, respectively. As a result, it is possible to match the best and worst outcomes among the chosen blocks. But otherwise, the outcomes show which blocks in this applied analysis are the worst and the best. We therefore found that the block size achieving the best prediction for the estimation method obtained from the maximum (minimum) values that is, positive returns is 15 with rank 3 and for the negative returns is 11 with rank 4. These results are reported in Table 6.

Since the best block sizes are obtained for both negative and positive returns, we now fit separately these gains (maxima) and losses (minima) to the GEV distribution and Table 7 shows the parameter estimates together with their corresponding standard errors (SE) for both gains and losses. It can be seen in Table 7 that for the negative returns (i.e., losses), the positive shape parameter is established, and the negative shape parameter is further established for positive returns (i.e., gains). This implies that negative returns or losses for the FTSE/JSE-ALSI are best modelled by a type II Fréchet-GEV distribution, while the gains are being modelled by a type III Weibull-GEV distribution. In a study comparing the riskiness of BitCoin/US Dollar and South African rand/US Dollar returns, Chikobvu and Ndlovu (2023a) fitted the GVE distribution to both the losses (negative returns) and gains (positive returns). The results of these authors portrayed a positive shape parameter for both returns, indicating a type II Fréchet-GEV which is a contrast to this study which found a mixture of family distributions for both gains and losses of FTSE/JSE-ALSI.

Table 6. Best and worst block size ranking maximum FTSE/JSE-ALSI.

Minimas			Maximas		
Ranking	Difference	Block size	Ranking	Difference	Block size
1	922	18	1	920	18
2	873	20	2	873	20
3	988	11**	3	977	22
4	889	15	4	889	15**
5	900	25	5	910	25
6	915	19	6	915	19
7	880	26	7	889	26
8	899	17	8	899	17
9	950	9	9	942	9
10	935	16	10	935	16
11	942	18	11	922	18
12	879	30	12	809	30
13	895	22	13	805	30
14	925	10	14	905	10
15	905	8	15	955	8
16	932	5	16	932	5
17	911	9	17	931	9
18	919	13	18	929	13
19	882	14			
20	879	12			

Table 7. Maximum likelihood estimates for the GEV distribution.

	Negative Returns (losses)	Positive Returns (Gains)
Extremes	11	15
$\hat{\xi}$	1.156	-0.266
$se(\hat{\xi})$	0.231	0.049
$\hat{\sigma}$	-0.033	0.024
$se(\hat{\sigma})$	0.028	0.084
$\hat{\mu}$	0.0208	0.0084
$se(\hat{\mu})$	0.031	0.015

3.3.1. Goodness of fit test

After model estimation, the goodness of fit (GoF) test is assessed. The Anderson-Darling and Shapiro–Wilk test for GoF tests are used and the results are presented in Table 8. Nonetheless, Stephens (1977) recommended these tests for the GoF for extreme value distributions, hence their use in this study. Chikobvu and Chifurira (2015) and Maposa et al. (2016) also used these tests for testing GoF for the GEV and GPD they estimated in their studies. The null hypothesis is that the returns following a GEV distribution are not rejected (p-values are not significant). Hence, the conclusion is that the FTSE/JSE-ALSI returns are modelled very well with the specified distribution. The estimated asymptotic laws can replace the distribution of extremes with the empirical distributions of the tails by these results. In extreme market conditions, it is thus possible to estimate the potential losses for stock and commodity indexes.

Table 8. Goodness of fit test for GEV distribution.

Test		Positive Returns	Negative Returns
Anderson Darling	Statistic	0.415	0.551
	P-Value	0.335	0.156
Shapiro Wilk	Statistic	0.062	0.074
	P-Value	0.362	0.247

3.3.2. Comparative analysis and combined forecast model using ensemble methods

The purpose of this section is to determine the model that best mimics the data and also produces fewer forecasts. This will help in assisting the maximum dispatching of the South African stock market. The three error metrics namely mean absolute error (MAE), mean square error (MSE), and mean forecast error (MFE) are used to measure the performance of each model and the results are summarised in Table 9. Some tentative conclusions are drawn from this table, which indicates that no model is superior to the other. For positive returns, MSE selects the best as the TBATS while for negative the selected model is GEV using the same MSE metric. However, MAE selects GEV as the best model for positive returns while MFE selects both models.

Because we found that no model is superior to the other, we therefore combine the two models and assess the performance of the combination. We determine the ideal weight to aggregate base learner predictions in such a way that the resulting ensemble minimises the total expected prediction error; hence, the aim is to find the most effective way to combine predictions. We, therefore, use the generalised ensemble method (GEM), stacking ensemble, blending, bagging, and boosting. These ensemble methods are chosen because they offer various advantages such as improved performance, reduced over-fitting, model robustness, and flexibility in model selection and combination, making them popular choices in machine learning for achieving better results. Each ensemble method has its strengths and it is chosen based on the specific characteristics of the problem and the data at hand.

Table 10 shows the average results of TBATS-GEV based on the MLE search method along with the mean squared error, mean absolute error, and mean forecast error made by each ensemble method. The superiority of the designed ensemble technique is seen by

Table 9. Performance Model selection criteria.

	MSE	MAE	MFE
Positive Returns			
TBATS	8.779	7.238	6.328
GEV	8.953	6.998	6.328
Negative Returns			
TBATS	8.879	7.028	6.879
GEV	8.053	7.028	6.128

Table 10. Average results and created ensembles on negative and positive returns.

		Ensemble Method	Positive Returns	Negative>Returns
TBATS-GEV	MSE	GEM	6.01	3.52
	MSE	Stacking	4.01	3.27
	MSE	Blending	5.31	3.45
	MSE	Bagging	6.15	4.87
	MSE	Boosting	7.28	6.88
TBATS-GEV	MAE	GEM	5.87	3.88
	MAE	Stacking	3.01	3.1
	MAE	Blending	7.31	5.45
	MAE	Bagging	8.15	7.87
	MAE	Boosting	7.13	6.19
TBATS-GEV	MFE	GEM	7.89	4.82
	MFE	Stacking	4.00	2.22
	MFE	Blending	5.31	5.01
	MFE	Bagging	7.25	4.87
	MFE	Boosting	7.28	6.88

comparing their forecasting errors. This demonstrates improvements in the stacking ensemble algorithm as compared to other ensemble algorithms. It is evident from Table 10 that the stacking ensemble outperformed other ensemble methods, by producing fewer forecasting errors; because, MSE, MAE, and MFE all advocate for the TBATS-GEV model that is combined using the stacking ensemble algorithm. We, therefore, proceed to estimate the market risk using TBATS-GEV combined with the stacking ensemble algorithm.

3.4. Risk estimates

Since the stacking ensemble algorithm has outperformed other ensemble algorithms, the four risk measures discussed in section 2 are computed using the TBAT-GEV distribution to evaluate the risk of losses and gains in the financial sector in South Africa for returns on five day FTSE/JSE-ALSI and the results are presented in Table 11. The computed values suggest that the losses (i.e., negative returns) are riskier than the gains (negative returns) since they have higher values of risk measures. For the gains, at 99%, the VaR value is 0.000850. This implies that there is a lower likelihood of significant losses; indicating that the risk of large negative returns is relatively low with a value of 0.85% for VaR at a 99% level of significance. The 99% level of significance suggests that lower risk is realised for the four estimated risk measures than at 90% and 95%. This is because, at a higher level of significance, less room for error is tolerated. These results are in contradiction of the one found by Chikobvu and Ndlovu (2023b); because the study by these authors had found the lower VaR estimate at 95% level.

It is worth noting that at 99% level, Glue-VaR is negative giving the value of -0.000012235 . This suggests that if one is focusing on the lower tail of the distribution of FTSE/JSE-ALSI returns, there is a high likelihood of investors incurring losses at this tail of the distribution. This negative value reflects a relatively high level of risk. It indicates that the investment has the potential for significant losses, especially in extreme market conditions or during events that negatively impact the investment. Yu et al. (2019) also revealed that a one-day change in the financial market's value would not decrease by more than 0.00123%. To take into account market liquidity constraints and Basel regulations, 5-day risk horizons in addition to the more typical 1-day horizon are considered. Comparing losses to gains, the results indicate that the prospects of potential extreme losses are greater than the prospects of potential extreme gains.

3.4.1. Evaluating the accuracy of risk measures

A backtesting procedure is used to evaluate the risk measures' accuracy, and the estimates are shown in Tables 12 and 13. While backtesting the estimated risk measures, a predicted loss is compared to the actual loss, observed the following day. An exceedance

Table 11. Computation risk measures.

P	Positive Returns				Negative Returns			
	VaR	ES	CTE	Glue-VaR	VaR	ES	CTE	Glue-VaR
0.9	0.001544	0.023454	0.023854	0.000630	0.001839	0.027366	0.027366	0.001110169
0.95	0.001230	0.023924	0.023924	0.000363	0.001471	0.027331	0.027331	0.000432336
0.99	0.000850	0.023967	0.023967	0.000134	0.000734	0.027366	0.027366	-0.000012350

Table 12. Backtesting risk measures.

p-values for Kupiec test					p-values for Christoffersen test			
	Risk Measure	Level	0.95	0.99	Risk Measure	Level	0.95	0.99
GEV	VaR	Gains	0.517	0.378	VaR	Gains	0.207	0.885
GEV	ES	Gains	0.919	0.877	ES	Gains	0.865	0.987
GEV	CTE	Gains	0.517	0.522	CTE	Gains	0.476	0.939
GEV	Glue-VaR	Gains	0.796	0.95	Glue-VaR	Gains	0.871	0.994
GEV	VaR	Losses	0.529	0.902	VaR	Losses	0.801	0.739
GEV	ES	Losses	0.668	0.872	ES	Losses	0.701	0.885
GEV	CTE	Losses	0.65	0.872	CTE	Losses	0.698	0.839
GEV	Glue-VaR	Losses	0.694	0.682	Glue-VaR	Losses	0.98	0.794

Table 13. Basel III zones backtest results.

	Zone	250 Days VaR	250 Days ES	250 Days CTE	250 Days Glue VaR
Gains	Green Zone	3	3	5	6
	Yellow Zone	2	2	1	3
	Red Zone	2	1	0	1
	average annual penalty	0.487	0.543	0.655	0.654
Losses	Green Zone	5	3	4	4
	Yellow Zone	3	1	1	3
	Red Zone	1	1	1	0
	average annual penalty	0.472	0.376	0.55	0.35

occurs when the actual loss exceeds the calculated risk measure. An expected number of exceedances over the VaR curve is predicted statistically for VaR. $VaR_{0.99}$, Rydell (2013) stressed that the loss exceeds that value 99 times out of 100, and one exceedance for every 100 observations is statistically expected. These exceedances are used as the reference. For this study, the combined TBATS-GEV is considered reliable if the number of exceedances when backtesting the model is within a 10% confidence interval from the statistically expected losses.

The null hypothesis is that the risk model is correctly specified and accurately predicts financial risk for FTSE/JSE-ALSI. Nevertheless, both the Kupiec and the Christoffersen tests suggest that the null hypothesis is not rejected and the conclusion is that, the four risk measures produced reliable, efficient, and unbiased risk estimates for both 95% and 99% confidence levels. For long-time periods such as more than ten years as is the case in this study, the combined model produces suitable risk estimates. According to Bee and Trapin (2018), these long periods are indicated by high probability values for losses and gains at all selected confidence levels of all the tests used.

The results of the regulatory backtest on the risk measures are presented in Table 13. The numbers in the table display how many times each risk measure in each zone has been backtested over the course of a year (250 trading days). The test is ranked as follows: (1) the green zone if there are 0 to 4 exceptions reported within a 250-trade day window; (2) the yellow zone if there are 5 to 9 exceptions; and (3) the red zone if there are more than 9 exceptions. All the risk measures fall into the yellow zone, which imposes a fine ranging from 0 to 85 times the risk measure to calculate the market risk charge.

Interestingly, the hybrids with 250 days of data performed uniformly throughout all the risk measures. This gives an annual average penalty that ranges from 20% to 59%. Generally, all the risk measures showed that the combined TBATS-GEV is a good

estimator for extreme losses and gains for FTSE/JSE-ALSI returns. At a 99% confidence level, the risk measures did not produce the most worrying results. Pigildin (2009) noted there is only a very small likelihood, which is less than 0.01%, that an accurate model with a correct coverage of 99% would produce as many as 10 or more exceptions. Then we found that the risk model had estimated extreme tail losses and gains with adequate accuracy.

3.5. Discussion

Point forecasts from the single models are combined using different ensemble methods. We first train the TBATS model to attain i.i.d residuals and use them to extract weekly block minimas and maximas that are fitted to the GEV distribution. The use of the stacking ensemble algorithm to combine the TBATS and the GEV distribution gave the best fit to the test data compared to other ensembles. Combining the models yielded the best forecasting results, in particular, the TBATS-GEV model using the stacking ensemble method is found to be more robust than the other used ensemble algorithms. In particular, the study is unique in terms of uniting traditional statistical models and extreme value theory in forecasting the four risk measures. Due to studying a specific sector in the economy, the approach taken in this study best simply treats the TBATS-GEV shape parameter. This has established a significant type II GEV known as a Fréchet family of distributions for the losses (i.e., negative returns) and a type III GEV known as a Weibull-GEV for the gains (i.e., positive returns). The findings from the study show that single forecast models can not accurately predict all share indexes for the South African stock market in the COVID-19 era. Combining forecasts using the stacking ensemble gives a perfect performance on the observed data. The Mann–Kendall test statistic and Sen's slope estimator also showed that there is a significant monotonic increasing long-term trend in this FTSE/JSE-ALSI.

When backtesting risk measures, no violations are met. All the backtesting tests accepted the null hypothesis, implying that computed risk measures are reliable and correct future uncertainty signals are given out by these risk measures to the risk managers. The high p-values indicate that the combined forecasting model used in the study is good and fits the FTSE/JSE-ALSI data set. The Kupiec test suggests that the TBATS-GEV with normally distributed errors is the best-fitting model for both the gains and losses on FTSE/JSE-ALSI. Christoffersen's test likelihood ratio test enables one to ascertain whether the output model violations are independent. For the gains and losses, the p-values are greater than 5%, leading to a conclusion that indeed the violations are independent of each other. Interestingly, Basel III results on the combined forecasts with 250 days of data performed uniformly throughout all the risk measures. Generally, all the risk measures showed that the TBATS-GEV is a good estimator for extreme losses and gains for FTSE/JSE-ALSI stock returns. Hence, the current study found that the proposed risk model had estimated extreme tail losses and gains with adequate accuracy. Nevertheless, the results obtained here are limited to the South African stock market. It would be interesting to see how the combination of the TBATS and GEV distribution would behave when combined with the ensemble algorithms used in this study while implementing the experiments on other stock market data such as the global stock market index SOXX, the FTSE, DAXE, NIKKEI, SP500, etc. This is because, for both negative returns and positive returns, the procedure for the selection of the block maxima/minima would behave differently as compared to the results obtained in this study. Moreover, the

behaviour of the ensemble algorithms used in this study is also believed they behave differently for a specific stock market, hence the behaviour of forecasting and prediction of the risk measures would be different.

4. Conclusion

The purpose of this study is to combine the TBATS and GEV distribution to improve the forecasting accuracy of risk measures and compare the riskiness of the losses and gains for the South African all-share index. The four estimated risk measures conclude that the losses are riskier than the gains especially when using Glue-VaR at 99% level. The combination of heavy tail error distribution to capture fat tails improved the estimation of the risk measures, and backtesting procedures confirmed this with high p-values above 0.2 that were obtained using the Kupiec's unconditional coverage test and Christoffersen test for both the gains and losses at 95% and 99% levels. Moreover, Basel III results indicated no violation of the risk measure for 250-day trading. This resulted in an annual average penalty that ranges from 20% to 59%. This information is useful to investors who need to fully appreciate the risk they are exposed to when they invest in FTSE/JSE-ALSI. In particular, when the market enters a turbulent time, investor encounters more losses than gains. Furthermore, these findings help risk managers to make adequate risk-based capital requirements more rational between the losses and gains. A piece of practical information for the South African stock market participants on their decision relating to this stock market is provided in this study. It further gives sound relevance in using the combined forecast to accurately model and predict the extreme gains and losses for JSE-ALSI. To other scholars, the study had shown that a financial time series more specifically JSE-ALSI can be modelled by TBATS-GEV distribution. However, it would be interesting to see what results would be found when modelling this time series with other combining methods and weights and compare the results with other stock markets datasets and also how the model will perform when compared with MS-GARCH model and other heavy tail distribut like the generalised hyperbolic distribution. Finally, the South African stock market and its participants can use the findings of this study to model and predict their stock prices on all share indexes.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendix

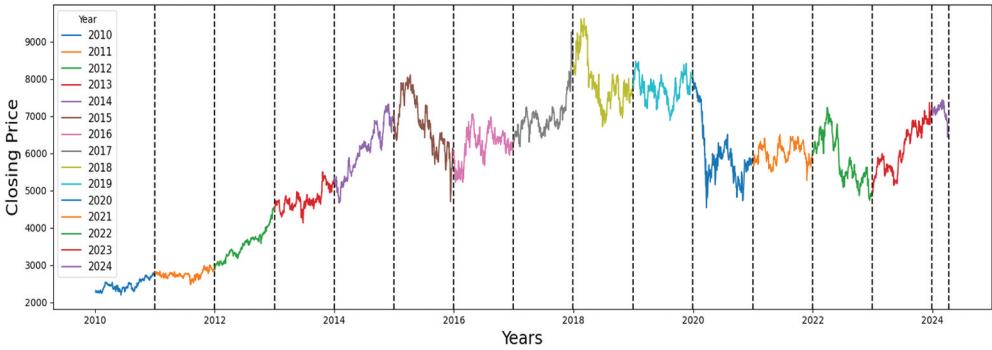


Figure A1. Annual trend for each year.