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Investment timing and quantity under ambiguity and business cycles

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ABSTRACT

We extend a dynamic investment model that captures the conjoint effect of ambiguity and the business cycle on the investment threshold and endogenous investment quantity choice. This paper focuses on investment strategies under the combined effects of ambiguity and business cycles. We reveal through quantitative results that the risk effect and ambiguity effect have opposite effects on optimal investment threshold and optimal investment quantity, and the risk effect dominates the ambiguity effect. The transfer intensity coefficient from a boom period to a recession period and the risk effect are opposite effects, and the transfer intensity coefficient effect dominates the risk effect. Moreover, the transfer intensity coefficient from a boom period to a recession period has a synergistic effect with the ambiguity effect. Meanwhile, the transfer intensity coefficient from recession to boom is the opposite effect of risk and a synergistic effect with ambiguity.

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KEYWORDS
Ambiguity; business cycle; investment timing; endogenous quantity

1. Introduction

Firms are under the influence of the macro-environment of the business cycle, and the business cycle directly affects their investment strategies (Arnold et al., 2013; Chen & Manso, 2017), while ambiguity has always existed in the process of firms' operations, and ambiguity has an important impact on firms' investment threshold and investment quantity (Nishimura & Ozaki, 2007; Sarkar, 2021). The demand ambiguity that firms face after investment should be included as a key factor in the investment model when firms make investments. This requires the integration of both the business cycle effect outside the firm and the ambiguity effect faced by the firm after the investment on the investment decision, and these two influences exist simultaneously in the investment decision process and have yet to be investigated. Therefore, in this paper, a dynamic investment model with business cycles and ambiguity is developed to investigate the effects of ambiguity and business cycles on optimal investment timing and endogenous investment quantity.

The business cycle is a key macroeconomic factor that influences firms' investment decisions. Guo (2001) and Guo et al. (2005) propose an investment model under the business cycle. Jeon and Nishihara (2015), Chen and Manso (2017), and Arnold et al.

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(2018) examine business cycle risk and financing decisions on investment. Luo and Yang (2017) consider the impact of different financing methods on investment decisions. Begenau and Salomao (2019) examined how business cycles affect financing methods in investment. Ilut and Schneider (2014) use a medium-scale DSGE model to test the effects of agents' dislike of risk, ambiguity, and shocks to confidence in the business cycle. Altug et al. (2020) use the smooth ambiguity model to quantitatively analyze the cyclical behavior under the business cycle of ambiguity-averse consumers and investment irreversibility. Nevertheless, the existing literature on the business cycle does not consider the issue of endogenous investment quantity and demand ambiguity after investment.

After investment, firms will face ambiguity in the future market, which will inevitably lead to ambiguity in demand after investment. This important issue should be included in the firm's investment model when making investment decisions. Nishimura and Ozaki (2007) examine the impact of ambiguity on the value of the investment opportunity. Thijssen (2011) and Balter et al. (2021) analyze the effects of ambiguity on investment decisions. Agliardi et al. (2016) and Viviani et al. (2018) discuss the effects of ambiguity on investment timing. Kim (2021) examines how ambiguity influences risk management and capital structure. Jang et al. (2021) and Delaney (2022) discuss the optimal investment strategy of an ambiguity-averse investor. Furthermore, Xiao et al. (2013) and Zheng et al. (2021) study the effects of demand ambiguity on capacity choices. Asano and Shibata (2011), Niu et al. (2019), and Song and Chen (2023) investigate ambiguity's impact on the firm's optimal investment threshold and capacity choice. However, the existing literature on investment under ambiguity has not taken the business cycle into account.

As far as we know, there are no papers that investigate the joint effect of the business cycle and demand ambiguity on investment threshold and endogenous investment quantity. This paper extends ambiguity and business cycles to the same investment model and investigates investment decisions under the combined effect of ambiguity and business cycles. Our quantitative results illustrate the following conclusions: First, under business cycles and ambiguity, the optimal investment thresholds and optimal endogenous investment quantities are both countercyclical. Second, the risk effect and ambiguity effect have opposite effects on optimal investment thresholds and optimal endogenous investment quantities. Third, in terms of effects on investment thresholds and investment quantities, the transfer intensity coefficient from boom to recession has a synergistic effect with risk and the opposite effect with ambiguity. Finally, the transfer intensity coefficient from recession to boom is the opposite effect of risk on optimal investment thresholds and optimal endogenous investment quantities and the synergistic effect of ambiguity on optimal investment thresholds and optimal endogenous investment quantities.

The remainder of the paper is structurally organized as follows. Section 2 describes the model setup. Section 3 presents the model. Section 4 provides the quantitative results and analysis. Section 5 concludes the paper.

2. Model setup

Following Guo et al. (2005) and Hackbarth et al. (2006), we can divide the business cycle state s into a boom (B) period and a recession (R) period, i.e., $s \in (B, R)$, so that the state

of the business cycle can be characterized by a two-state continuous Markov chain, and the transfer coefficient of the Markov chain state transition from a boom (B) period to a recession (R) period is λ_B. The transfer coefficient from a recession (R) period to a boom (B) period is λ_R.

It is assumed that the firm has assets in place and growth options for investment. The cash flow generated from the assets in place of the firm before the investment is x_t. The cash flow x_t follows the following geometric Brownian motion:

$$\frac{dx_t}{x_t} = \mu_s dt + \sigma dz_t, \ \ s \in (B, R), \tag{1}$$

where z_t is a standard Brownian motion on the probability space $(\Omega, \mathcal{F}, \mathrm{P})$, and μ_s is the drift parameter. The cash flow increases immediately from x_t to $\theta_s x_t$ after the investment. Following Sarkar (2021), the investment cost is $I(\theta_s) = \varphi\theta_s^{\eta}$, $s \in (B, R)$ and φ is the unit cost coefficient, η is the return parameter of the investment cost, and θ_s is the endogenous investment quantity.

The future market is full of uncertainty, which makes the invested firms face market uncertainty. These uncertainties should be fully reflected in the firm's investment decision-making model. Only such investment decisions that contain uncertainty are reasonable and appropriate. Following Nishimura and Ozaki (2007), we describe market uncertainty by characterizing the cash ambiguity of firms. Ambiguity is a series of measurement methods that replace the original single probability measure, that is, transforming from a probability measure to a series of probability measures. This can ensure that the ambiguity of the characterization is more accurate and reasonable. According to Girsanov's theory, the transformation of the measure is

$$dz_t^{\vartheta} = dz_t - \vartheta_t dt, \tag{2}$$

where z_t^{ϑ} is a standard Brownian motion. Under the series of probability measures P^{ϑ}, $\vartheta \in [-k, k]$ $(k>0)$ the cash flow follows

$$\frac{dx_t}{x_t} = \left(\mu_s + \sigma\vartheta_t\right)dt + \sigma dz_t^{\vartheta}, s \in (B, R). \tag{3}$$

Under ambiguity, the cash flow of the firm is considered under the lowest or worst scenario in this series of probability measures. Following Nishimura and Ozaki (2007), there exists a unique minimizing value $\vartheta_t = -k$. Equation 3 can then be rewritten as follows:

$$\frac{dx_t}{x_t} = \left(\mu_s - \sigma k\right)dt + \sigma dz_t^{k}, \ s \in (B, R), \tag{4}$$

where $\sigma>0$ is the volatility, k is the degree of ambiguity, and $\mu_s < r_s$, r_s is the risk-free interest rate. Obviously, when $k = 0$, there is no ambiguity.

3. The model

The main content of this section is to determine the firm's value before and after investment, as well as to quantify and select the optimal investment thresholds and endogenous investment quantities. It adopts the maxi-min criterion under business

cycles and ambiguity for pricing. First, the firm's value after the investment is priced, and then the firm's value before the investment is priced. A backward induction approach is used to determine the investment thresholds and investment quantities.

3.1. Firm value after investment

After the investment, the firm has two states from the perspective of cash flow: normal operation or bankruptcy. Under the corresponding business cycle, after investment, the given bankruptcy thresholds are x_B^d and x_R^d, and $x_B^d < x_R^d$. The specific method for determining the bankruptcy thresholds after investment will be provided later. The cash flow of the invested firm can be divided into cash flow intervals based on the bankruptcy thresholds. And then price the firm's value after investing in the cash flow range.

Following Hackbarth et al. (2006) and Nishimura and Ozaki (2007), we price the value of corporate securities by the maxi-min criterion under ambiguity.

The value of the firm $V_s(x)$ after investment meets the following equation:-

$$V_s(x) = \max_{\varepsilon} \min_{\vartheta} E_t\left[\int_0^{\infty} e^{-r(\varepsilon - t)}(\theta_s x_t)dt | x_0 = x\right], s \in (B, R), \tag{5}$$

where $E_t[\cdot]$ represents the expected operational value, and ε indicates the bankruptcy time point after investment.

Applying Ito's lemma, given x_B^d and x_R^d, the value of the firm $V_s(x)$ is given as follows:
For $x > x_R^d$,

$$(r_B + \lambda_B)V_B(x) = \min_{\vartheta \in [-k,k]} \theta_B x + (\mu_B - \sigma\vartheta)xV_B'(x) + \frac{1}{2}\sigma^2 x^2 V_B''(x) + \lambda_B V_R(x), \tag{6}$$

$$(r_R + \lambda_R)V_R(x) = \min_{\vartheta \in [-k,k]} \theta_R x + (\mu_R - \sigma\vartheta)xE_R'(x) + \frac{1}{2}\sigma^2 x^2 E_R''(x) + \lambda_R V_B(x); \tag{7}$$

For $x_B^d < x \leq x_R^d$,

$$(r_B + \lambda_B)E_B(x) = \min_{\vartheta \in [-k,k]} \theta_B x + (\mu_B - \sigma k)xE_B'(x) + \frac{1}{2}\sigma^2 x^2 V_B''(x), \tag{8}$$

The value of the firm after the investment is taken as the lowest or worst value in a series of probability measures. Existence and uniqueness of solutions for minimizing values $\vartheta = -k$. Applying Ito's lemma, under given bankruptcy thresholds x_B^d and x_R^d, the value of the firm $V_s(x)$ after investment satisfies the following form:
For $x > x_R^d$,

$$(r_B + \lambda_B)V_B(x) = \theta_B x + (\mu_B - \sigma k)xV_B'(x) + \frac{1}{2}\sigma^2 x^2 V_B''(x) + \lambda_B V_R(x), \tag{9}$$

$$(r_R + \lambda_R)V_R(x) = \theta_R x + (\mu_R - \sigma k)xV_R'(x) + \frac{1}{2}\sigma^2 x^2 V_R''(x) + \lambda_R V_B(x); \tag{10}$$

For $x_B^d < x \le x_R^d$,

$$(r_B + \lambda_B)V_B(x) = \theta_B x + (\mu_B - \sigma k)xV_B'(x) + \frac{1}{2}\sigma^2 x^2 V_B''(x) + \lambda_B \alpha_R I(\theta_R), \tag{11}$$

$$V_R(x) = \alpha_R I(\theta_R); \tag{12}$$

For $x \le x_B^d$,

$$V_B(x) = \alpha_B I(\theta_B), \tag{13}$$

$$V_R(x) = \alpha_R I(\theta_R). \tag{14}$$

Thus, we can obtain the following proposition 1.

Proposition 1. Under the business cycle and ambiguity, the firm value after the investment is $V_s(x)$.

$$V_B(x) = \begin{cases} \alpha_B I(\theta_B), x \le x_B^d, \\ C_1 x^{\beta_1} + C_2 x^{\beta_2} + \frac{\theta_B x}{m} + \frac{\lambda_B \alpha_R I(\theta_R)}{r_B + \lambda_B}, x_B^d < x \le x_R^d \\ A_1 x^{r_1} + A_2 x^{r_2} + \frac{\theta_R \lambda_B + \theta_B n}{M} x, x_R^d < x \end{cases} \tag{15}$$

$$V_R(x) = \begin{cases} \alpha_R I(\theta_R), & x \le x_R^d, \\ H_1 x^{r_1} + H_2 x^{r_2} + \frac{\theta_B \lambda_R + \theta_R m}{M} x, & x_R^d < x. \end{cases} \tag{16}$$

The firm's bankruptcy value is $\alpha_B I(\theta_B)$ and $\alpha_R I(\theta_R)$ in the boom period and the recession period, respectively. The coefficientsA_1, A_2, C_1, C_2, H_1, H_2, M, n, m and proof are given in the Appendix. After investment, a firm can only be in two states: bankruptcy or normal operation. When the business cycle is in a recession period, if the firm's cash flow reaches x_R^d, that is, $x \le x_R^d$, the firm will choose to go into bankruptcy. If the firm's cash flow is higher than x_R^d, that is, $x_R^d < x$, the firm will continue its operations. When the business cycle is in a boom period, if the firm's cash flow reaches x_B^d, that is, $x \le x_B^d$, the firm will choose to go into bankruptcy. When the firm's cash flow is higher than x_B^d but lower than x_R^d, that is, $x_B^d < x \le x_R^d$, the firm has already chosen to go into bankruptcy during the recession period but will operate normally during the boom period. If the firm's cash flow is higher than x_B^d, the firm will continue its operations. When the firm's cash flow falls within the range $x_B^d < x \le x_R^d$, the firm may go into bankruptcy due to the transition from the boom period to the recession period of the business cycle, which demonstrates the significant impact of business cycle transitions on the firm's operations. Meanwhile, it can be intuitively seen that it is correct and reasonable to include the investment quantity in the investment model.

Under the business cycle and ambiguity, the bankruptcy thresholds x_B^d and x_R^d after investment are the solutions of the following equations:

$$\beta_1 C_1 \left(x_B^d\right)^{\beta_1} + \beta_2 C_2 \left(x_B^d\right)^{\beta_2} + \frac{\theta_B}{m} x_B^d = 0, \tag{17}$$

$$r_1 H_1 \left(x_R^d\right)^{r_1} + r_2 H_2 \left(x_R^d\right)^{r_2} + \frac{\theta_B \lambda_R + \theta_R m}{M} x_R^d = 0. \tag{18}$$

This is based on maximizing the value of the firm after the investment. Equation 17 and Equation 18 give the bankruptcy thresholds after investment in the boom and recession periods, respectively.

3.2. Firm value before investment

Before investing, the investment can be postponed until the cash flow reaches the investment thresholds. Under the given investment thresholds x_B^u and x_R^u, when the firm's cash flow reaches investment thresholds of the corresponding business cycle, the firm will immediately make investments. The firm's cash flow before investment can be based on the investment thresholds x_B^u and x_R^u divided by cash flow intervals.

Similarly, we price the firm value before the investment using the maxi-min criterion under ambiguity. Applying Ito's lemma and given x_B^u and x_R^u, under the business cycle and ambiguity, the value of the firm $F_s(x)$ before the investment is given as follows:

For $x_t < x_B^u$,

$$(r_B + \lambda_B) F_B(x) = x + \left(\mu_B - \sigma k\right) x F_B{}'(x) + \frac{1}{2}\sigma^2 x^2 F_B{}''(x) + \lambda_B F_R(x), \tag{19}$$

$$(r_R + \lambda_R) F_R(x) = x + \left(\mu_R - \sigma k\right) x F_R{}'(x) + \frac{1}{2}\sigma^2 x^2 F_R{}''(x) + \lambda_R F_B(x); \tag{20}$$

For $x_B^u \le x_t < x_R^u$,

$$(r_R + \lambda_R) F_R(x) = x + \left(\mu_R - \sigma k\right) x F_R{}'(x) + \frac{1}{2}\sigma^2 x^2 F_R{}''(x) + \lambda_R F_B(x), \tag{21}$$

$$F_B(x) = V_B(x) - I(\theta_B); \tag{22}$$

For $x_R^u \le x_t$,

$$F_B(x) = V_B(x) - I(\theta_B), \tag{23}$$

$$F_R(x) = V_R(x) - I(\theta_R). \tag{24}$$

Using the same proof method as the firm value after the investment, the following properties 2 can be obtained.

Proposition 2. Under the business cycle and ambiguity, the firm value before the investment is $F_s(x)$.

$$F_B(x) = \begin{cases} A_3 x^{r_3} + A_4 x^{r_4} + \frac{\lambda_B + n}{M} x, & x < x_B^u, \\ V_B(x) - I(\theta_B), & x_B^u \le x; \end{cases} \tag{25}$$

$$F_R(x) = \begin{cases} H_3 x^{r_3} + H_4 x^{r_4} + \frac{\lambda_R + m}{M} x, & x < x_B^u, \\ C_3 x^{\beta_3} + C_4 x^{\beta_4} + A_1 x^{r_1} + A_2 x^{r_2} + \frac{x}{n} - \frac{\lambda_R I(\theta_B)}{r_R + \lambda_R} \\ \quad + \frac{\theta_R \lambda_B + \theta_B n}{Mn} \lambda_R x, & x_B^u \le x < x_R^u, \\ V_R(x) - I(\theta_R), & x_R^u \le x. \end{cases} \tag{26}$$

Proof. The remainder proof is like the Appendix. The firm value before investment includes the value of growth options. Before investment, the firm has only two states: investment and normal operation. Here, bankruptcy before investment is not considered. When the business cycle is in a boom period, if the firm's cash flow reaches x_B^u, that is, when $x_B^u \le x$, the firm will choose to invest. When the firm's cash flow is lower than x_B^u, that is, when $x < x_B^u$, the firm will operate normally. When the business cycle is in a recession period, if the firm's cash flow is below x_R^u, that is, when $x < x_R^u$, the firm will continue to operate. However, if the firm's cash flow reaches x_R^u, that is, when $x_R^u \le x$, the firm will immediately proceed with the investment. When the firm's cash flow falls within the range $x_B^u \le x < x_R^u$, the firm will continue its operations in a recession period. When a firm's cash flow is within the range $x_B^u \le x < x_R^u$, the firm may invest due to the transition of the business cycle from a recession period to a boom period. This indicates that changes in the business cycle directly affect a firm's investment timing decisions and demonstrates the necessity of incorporating the business cycle into a firm's investment model.

3.3. Optimal investment thresholds and investment quantities

Following Guo et al. (2005) and Arnold et al. (2013), the following optimal investment thresholds and endogenous investment quantities can be obtained.

Proposition 3. Under the business cycle and ambiguity, the optimal investment thresholds x_B^u and x_R^u are the solutions of the following equations:

$$A_1 r_1 \left(x_B^u\right)^{r_1} + A_2 r_2 \left(x_B^u\right)^{r_2} + \frac{\theta_R \lambda_B + \theta_B n}{M} x_B^u = A_3 r_3 \left(x_B^u\right)^{r_3} + A_4 r_4 \left(x_B^u\right)^{r_4} + \frac{(\lambda_B + n) x_B^u}{M}, \tag{27}$$

$$\begin{aligned} &H_1 r_1 \left(x_R^u\right)^{r_1} + H_2 r_2 \left(x_R^u\right)^{r_2} + \frac{\theta_B \lambda_R + \theta_R m}{M} x_R^u - \frac{\theta_R \lambda_B + \theta_B n}{Mn} \lambda_R x_R^u \\ &\quad = C_3 \beta_3 \left(x_R^u\right)^{\beta_3} + C_4 \beta_4 \left(x_R^u\right)^{\beta_4} + A_1 r_1 \left(x_R^u\right)^{r_1} + A_2 r_2 \left(x_R^u\right)^{r_2} + \frac{x_R^u}{n}; \end{aligned} \tag{28}$$

And optimal investment quantities θ_B and θ_R are the solutions of the following equations:

$$\frac{\theta_B n}{M} x_B^u = \eta \varphi {\theta_B}^{\eta}, \tag{29}$$

$$\frac{\theta_R m}{M} x_R^u = \eta \varphi {\theta_R}^{\eta}. \tag{30}$$

The optimal investment thresholds obtained from Equation 27 and Equation 28 are based on maximizing the value of the firm after investment during the boom period and recession period. Equation 29 and Equation 30 are based on maximizing the value of the firm after investment to obtain the optimal endogenous investment quantities during the boom period and recession period. Obviously, the optimal investment thresholds and optimal investment quantities are the result of a combination of the business cycle and demand ambiguity after the investment.

4. Quantitative results

In this section, we numerically simulated and quantified the effects of some key parameters on the optimal investment threshold and endogenous investment quantity. The parameter values in the model are as follows: Following Jeon and Nishihara (2015), the drift rate of a boom period $\mu_B = 0.03$ and a recession period $\mu_R = 0.02$. We use the transition intensity from a boom period to a recession period $\lambda_B = 0.1$ and a recession period to a boom period $\lambda_R = 0.15$ as suggested in the literature (e.g., Guo et al., 2005; Hackbarth et al., 2006). According to Hackbarth et al. (2006) and Luo and Yang (2017), the risk-free rate of interest in the boom period $r_B = 0.06$, and the recession period $r_R = 0.05$. As in Sarkar (2021), volatility $\sigma = 0.15$, the recovery rate in the boom period $\alpha_B = 0.5$ and the recession period $\alpha_R = 0.4$. Following Sarkar (2021) and Song and Chen (2023), the degree of ambiguity $\kappa = 0.05$, the unit cost coefficient $\varphi = 5$, the return parameter of the investment cost $\eta = 2.1$.

Nishimura and Ozaki (2007) show that the volatility of cash flow is the risk effect of firm operations. Figure 1 shows that an increase in risk (volatility) leads to an increase in investment thresholds and investment quantities, which is in line with Viviani et al. (2018) and Sarkar (2021). However, as the degree of ambiguity increases, both the investment thresholds and investment quantities decrease. This is a result of the firm's reaction to ambiguity. Regarding the effect on the investment thresholds and quantities, the risk effect has the opposite effect of the ambiguity effect, and the risk effect dominates the ambiguity effect. Specifically, the increase in risk will lead to higher optimal investment thresholds, delayed investment timing, and larger optimal investment quantities,

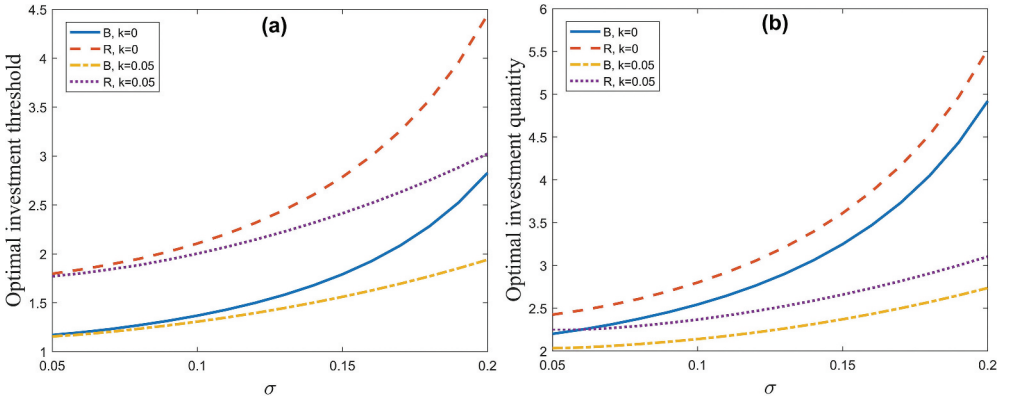

Figure 1. (a,b) The effect of σ and κ on the investment thresholds and quantities.

which are not conducive to investment. Conversely, an increase in the degree of ambiguity will reduce the optimal investment thresholds, accelerate investment, and decrease the optimal investment quantities. Moreover, the optimal investment thresholds and optimal investment quantities are countercyclical.

As shown in Figure 2, the optimal investment thresholds and optimal investment quantities in the recession period are greater than in the boom period. This result is in line with Arnold et al. (2013). The optimal investment threshold is a convex function of ambiguity, and the greater the volatility, the higher the optimal investment thresholds and optimal investment quantities. The risk effect and ambiguity effect work in opposite directions. Meanwhile, the investment quantities decrease as the degree of ambiguity increases. These results are consistent with Niu et al. (2019) and Song and Chen (2023). It is a result of the combined effect of the ambiguity effect and the risk effect, with the ambiguity effect being the dominant one.

As depicted in Figure 3, both the optimal investment thresholds and optimal investment quantities decrease with an increase in the transition intensity coefficient λ_B, indicating a negative correlation between the optimal investment thresholds, optimal

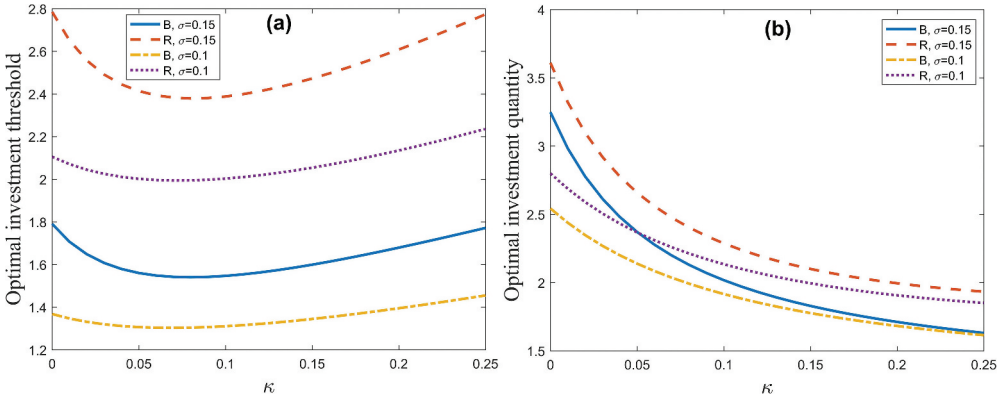

Figure 2. (a,b) The effect of κ and σ on the investment thresholds and quantities.

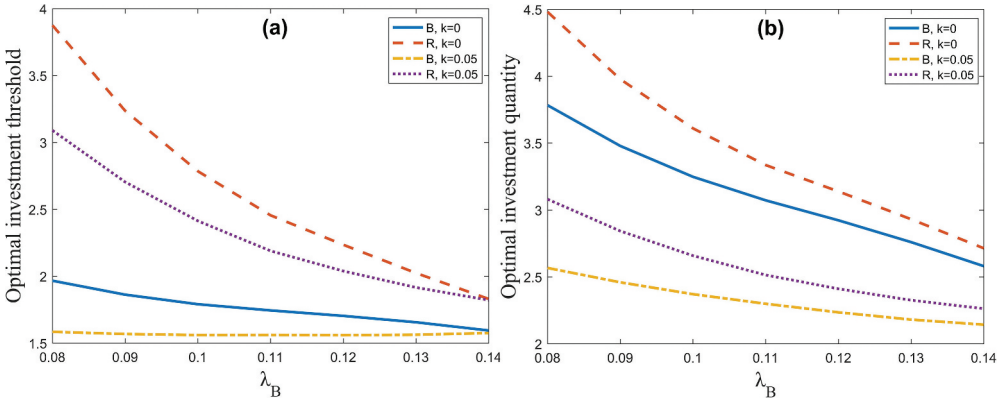

Figure 3. (a,b) The effect of λ_B and κ on the investment thresholds and quantities.

investment quantities, and the transition intensity coefficient λ_B. Meanwhile, an increase in ambiguity also leads to a decrease in both the optimal investment thresholds and optimal investment quantities. Moreover, the optimal investment thresholds and optimal investment quantities in the boom period are smaller than those in the recession period. There is a synergistic effect between the transition intensity coefficient λ_B and ambiguity. An increase in the transition intensity coefficient λ_B increases the probability of being in a recession period during the business cycle, leading to reduced investment for firms during the recession period.

Figure 4 demonstrates that both the optimal investment thresholds and optimal investment quantities are countercyclical. Similarly, an increase in the transition intensity coefficient λ_B leads to a decrease in the optimal investment thresholds and optimal investment quantities. However, an increase in risk results in an increase in both the optimal investment thresholds and optimal investment quantities. In other words, the transition intensity coefficient λ_B and the risk effect have opposite effects, with the transition intensity coefficient λ_B playing a dominant role in influencing the optimal investment thresholds and optimal investment quantities.

As shown in Figure 5, the transition intensity coefficient λ_R positively correlates with the optimal investment thresholds and optimal investment quantities. Specifically, an increase in the transition intensity coefficient λ_R results in an increase in both the optimal investment thresholds and optimal investment quantities, while an increase in ambiguity leads to a decrease in both. When considering the impact on the optimal investment thresholds and optimal investment quantities, the transition intensity coefficient λ_R from the recession period to the boom period has the opposite effect of ambiguity. The transition intensity coefficient λ_R dominates this effect. As the transition intensity coefficient λ_R from the recession period to the boom period increases, the probability of a business cycle being in a boom period also increases, and firms tend to make larger investments during such boom periods.

As can be seen from Figures 3 to 6, increases in the transition intensity coefficients λ_R and λ_B have opposite effects on the optimal investment thresholds and optimal investment quantities. As shown in Figure 6, the optimal investment thresholds and optimal investment quantities in the recession period are greater than in the boom period. An

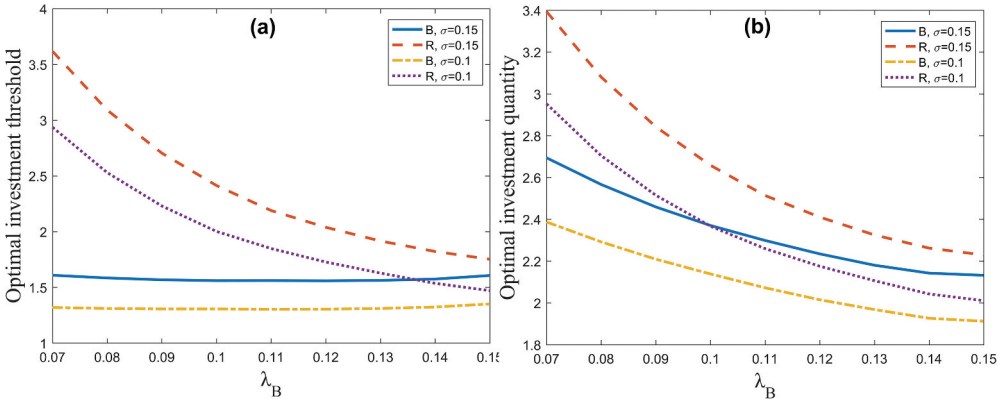

Figure 4. (a,b) The effect of λ_B and σ on the investment thresholds and quantities.

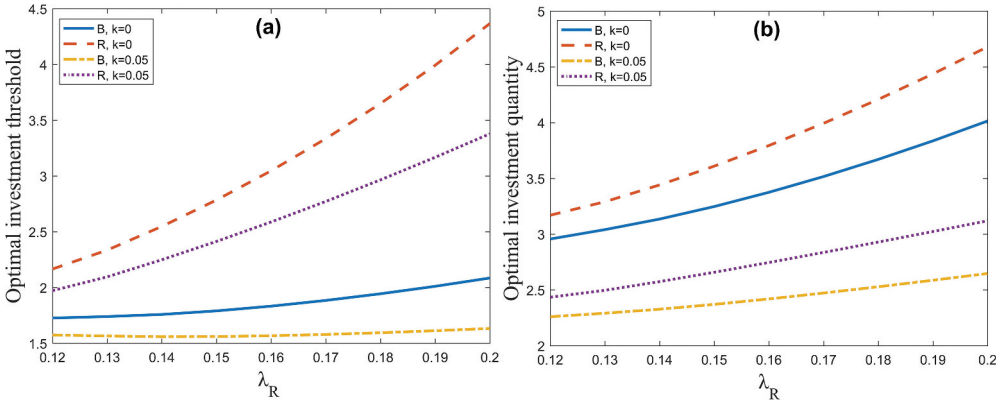

Figure 5. (a,b) The effect of λ_R and κ on the investment thresholds and quantities.

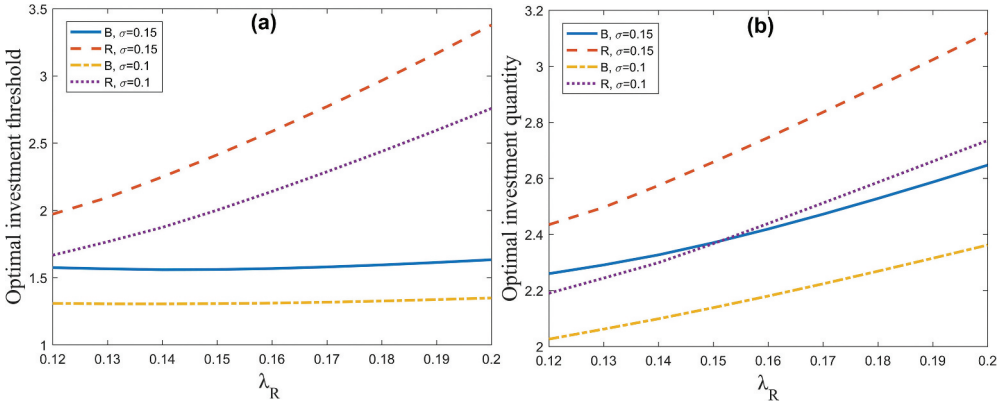

Figure 6. (a,b) The effect of λ_R and σ on the investment thresholds and quantities.

increase in the transition intensity coefficient λ_R leads to an increase in both the optimal investment thresholds and optimal investment quantities. At the same time, an increase in risk also increases both the optimal investment thresholds and optimal investment quantities. The effects of the transition intensity coefficient λ_R and the risk effect are synergistic.

5. Conclusion

We extend a dynamic investment model to examine the ambiguity effect and the effect of the business cycle on investment decisions. The model captures the interaction effect of ambiguity, risk, and the business cycle on the optimal investment thresholds and endogenous investment quantities. The numerical simulation results clearly indicate that the increase in risk leads to an increase in optimal investment thresholds and optimal investment quantities. As the degree of ambiguity increases, the optimal investment thresholds and optimal investment quantities decrease. The optimal investment thresholds and optimal investment quantities are both countercyclical. The transfer intensity

coefficient from boom to recession and ambiguity has a synergistic effect, while this transfer intensity coefficient exhibits an opposite impact with the risk effect. However, the transition intensity coefficient from recession to boom has an opposing effect with the ambiguity effect, while this transfer intensity coefficient exhibits a synergistic effect with the risk effect. Obviously, the joint effect of different influencing factors differs from the effect of a single factor on investment thresholds and quantities. The combined effect of ambiguity, risk, and the business cycle is more suitable as a basis for firm investment decisions.

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Appendix

We assume $A_1 = g_1 H_1, A_2 = g_2 H_2$. And γ_1, γ_2 are the two negative roots, γ_3, γ_4 are the positive root of the following equation:-

$$\left((\mu_B - \sigma k)\gamma + \frac{1}{2}\sigma^2\gamma(\gamma-1) - \lambda_B - r_B\right)\left((\mu_R - \sigma k)\gamma + \frac{1}{2}\sigma^2\gamma(\gamma-1) - \lambda_R - r_R\right) = \lambda_B\lambda_R, \tag{A1}$$

$$\gamma_1 = \tfrac{1}{2} - \tfrac{(\mu_B-\sigma k)}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{(\mu_B-\sigma k)}{\sigma^2}\right)^2 + \tfrac{2(r_B+\lambda_B)}{\sigma^2}},\ \gamma_2 = \tfrac{1}{2} - \tfrac{(\mu_B-\sigma k)}{\sigma^2} - \sqrt{\left(\tfrac{1}{2} - \tfrac{(\mu_B-\sigma k)}{\sigma^2}\right)^2 + \tfrac{2(r_B+\lambda_B)}{\sigma^2}}$$

$$\gamma_3 = \tfrac{1}{2} - \tfrac{(\mu_R-\sigma k)}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{(\mu_R-\sigma k)}{\sigma^2}\right)^2 + \tfrac{2(r_R+\lambda_R)}{\sigma^2}},\ \gamma_4 = \tfrac{1}{2} - \tfrac{(\mu_R-\sigma k)}{\sigma^2} - \sqrt{\left(\tfrac{1}{2} - \tfrac{(\mu_R-\sigma k)}{\sigma^2}\right)^2 + \tfrac{2(r_R+\lambda_R)}{\sigma^2}}.$$

And definition g_1, g_2, g_3, g_4, as follows:

$$g_1 := \lambda_B/\left(r_B + \lambda_B - (\mu_B - \sigma k)\gamma_1 - \tfrac{1}{2}\sigma^2\gamma_1(\gamma_1 - 1)\right),$$
$$g_2 := \lambda_B/\left(r_B + \lambda_B - (\mu_B - \sigma k)\gamma_2 - \tfrac{1}{2}\sigma^2\gamma_2(\gamma_2 - 1)\right),$$
$$g_3 := \lambda_B/\left(r_B + \lambda_B - (\mu_B - \sigma k)\gamma_3 - \tfrac{1}{2}\sigma^2\gamma_3(\gamma_3 - 1)\right),$$
$$g_4 := \lambda_B/\left(r_B + \lambda_B - (\mu_B - \sigma k)\gamma_4 - \tfrac{1}{2}\sigma^2\gamma_4(\gamma_4 - 1)\right).$$

In addition, the value of the firm $V_s(x)$ is related to the following smooth-conditions and the boundary conditions as follows:-

$$lim_{x\to x_R^d+} V_B(x) = lim_{x\to x_R^d-} V_B(x), \tag{A2}$$

$$lim_{x\to x_R^d+} \frac{\partial V_B(x)}{\partial x} = lim_{x\to x_R^d-} \frac{\partial V_B(x)}{\partial x}, \tag{A3}$$

$$lim_{x\to x_B^d-} V_B(x) = \alpha_B I(\theta_B), \tag{A4}$$

$$lim_{x\to x_R^d-} V_R(x) = \alpha_R I(\theta_R). \tag{A5}$$

From smooth-conditions and the boundary conditions, the $(H_1, H_2, C_1, C_2)^T$ is the solution of the system of linear equations: $M_1(H_1, H_2, C_1, C_2)^T = N_1$, and

$$M_1 = \begin{bmatrix} g_1\left(x_R^d\right)^{r_1} & g_2\left(x_R^d\right)^{r_2} & -\left(x_R^d\right)^{\beta_1} & -\left(x_R^d\right)^{\beta_2} \\ g_1 r_1\left(x_R^d\right)^{r_1} & g_2 r_2\left(x_R^d\right)^{r_2} & -\beta_1\left(x_R^d\right)^{\beta_1} & -\beta_2\left(x_R^d\right)^{\beta_2} \\ 0 & 0 & \left(x_R^d\right)^{\beta_1} & \left(x_R^d\right)^{\beta_2} \\ \left(x_B^d\right)^{r_1} & \left(x_B^d\right)^{r_2} & 0 & 0 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} \frac{\theta_B x_R^d}{m} + \frac{\lambda_B\alpha_R I(\theta_R)}{r_R+\lambda_B} - \frac{\theta_R\lambda_B+\theta_B n}{M} x_R^d \\ \frac{\theta_B}{m} x_R^d - \frac{\theta_R\lambda_B+\theta_B n}{M} x_R^d \\ \alpha_B I(\theta_B) - \frac{\theta_B}{m} x_B^d - \frac{\lambda_B\alpha_R I(\theta_R)}{r_R+\lambda_B} \\ \alpha_R I(\theta_R) - \frac{\theta_B\lambda_R+\theta_R m}{M} x_R^d \end{bmatrix}.$$

We definition $m = r_B + \lambda_B - (\mu_B - \sigma k)$, $n = r_R + \lambda_R - (\mu_R - \sigma k)$, $M = \left(r_B + \lambda_B - (\mu_B - \sigma k)\right)\left(r_R + \lambda_R - (\mu_R - \sigma k)\right) - \lambda_B\lambda_R$. From the equation $M_1(H_1, H_2, C_1, C_2)^T = N_1$, it is possible to determine the coefficients of proposition 1. Hence, the proposition 1 have proved.