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Network as information: endogenous network in coordination game

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ABSTRACT

We investigate the concept of endogenous information leadership within the financial market. Prior to coordinating their investments, investors establish a star network at a certain cost. The central node of this star network functions as an information leader, collecting and sharing information with all network members, thereby enhancing the chances of successful coordination. We delve deeper into the process of selecting the information leader by introducing a name market, inspired by Tadelis (1999), to identify individuals with the highest leadership potential to assume this role. The interplay between leadership selection through the name market and successful coordination creates a reinforcing loop. The emergence of contagion is attributed to leadership dynamics. Furthermore, we identify an equilibrium where leadership alternates. Our findings provide new insights into the regulation of social media platforms.

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KEYWORDS

Information leader: endogenous network; name market; contagion

1. Introduction

The financial industry utilises social networks as a means of communication, sharing information that can influence traders through the sentiments, feelings, and opinions expressed by others. This can lead to immediate and widespread emotional reactions in the market. One prominent example is Elon Musk, the CEO of Tesla, who has frequently tweeted about Dogecoin, including statements about SpaceX (his rocket company) putting a "doge on the moon." Following his tweets, the price of Dogecoin has surged by 15% almost instantly.¹ In another famous case, the Reddit community led an army of amateur investors to snap up shares of GameStop, driving the price from 18 dollars to 483 dollars, merely in a month, outperforming many institutional investors who short heavily on GameStop.²

We derive an important observation from both cases. An information leader often emerges in social media outlets to mobilise collective investment. He/she disseminates his/her information to every participant and fuels the market sentiments. Why do

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¹In a later live show SNL (Saturday Night Live), Dogecoin lost more than a third of its price on Sunday after Elon Musk called it a "hustle" during his guest-host spot.

²A campaign framed as pitting Main Street versus Wall Street.

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investors subscribe to the same people for information sources? How does information leadership affect financial panic? How does financial panic become contagious through information leadership? Despite the burgeoning literature on social interaction and the financial markets such as Ozsoylev and Walden (2011) and Han and Yang (2013), there remains a lack of understanding regarding the formation of endogenous information networks during financial frenzies. This study seeks to address this gap by introducing a theoretical framework that highlights the formation of endogenous networks.

In the model, investors participate in a collective investment venture where their final payoff will be determined by their aggregate action and a fundamental shock. Investors might acquire information about the shock from any other investors by a positive linking cost. We find an equilibrium that all investors link to one particular investor to form a star network. The centre of the star network, acting as the information leader, aggregates and transmits information to linked investors. We further introduce a name market in the sense of Tadelis (1999) to address the leadership selection problem. Only investors with exceptional communication skills are willing to offer the highest price for a name, positioning themselves as the front-runner who serves as the focal point for the formation of a star network. This process of leadership selection creates a contagious effect through a feedback loop between leadership selection and the effectiveness of coordination. Stronger leadership results in greater coordination success, leading to the recognition of a standout name for selecting superior leadership in subsequent rounds of network formation. We call this contagion equilibrium.

We also study leadership competitions where two prominent investors vie for the role of leader. Our findings reveal an equilibrium in which these two investors take turns serving as the leader. For instance, if an investment led by Elon Musk experiences a setback, the investors collectively look to Mark Zuckerberg for the subsequent round of network building. Mark Zuckerberg remains in the leadership position as long as he can effectively rally coordinated investments. Essentially, when coordination falters, the mantle of informational leadership shifts. We call this alternation equilibrium.

Returning to the leadership formation theory, the most close to our paper is Akerlof and Holden (2016) who study the information leadership in a global game framework. In their paper, the information network also takes a star shape, and its centre player exerts efforts to disseminate public information throughout the network before the second-stage coordination game.³ A distinctive aspect of our research lies in the incorporation of information within the network's size. Upon receiving the signal, only investors with favourable signals demonstrate a willingness to incur costs associated with joining the network. Consequently, the central hub of the star network can derive the aggregate state by calculating the number of connections. Subsequently, this central player transmits the information to all interconnected investors, ensuring that it is indeed optimal for investors with positive signals to establish the costly network connections. In essence, investors' decisions regarding network connections serve as a mechanism for aggregating information, thereby instilling the information itself into the network structure. In short, *investors' linking decisions aggregate information, and the information is the network itself.*

³In Akerlof and Holden (2016), the centre of a star network, also bids for a critical asset. However, their bidding process does not select leadership. In our paper, the investor with the best communication skills pays the highest for the name to become the centre of the star network.

To illustrate the point that *the information is the network itself*, let us consider the GameStop case. As the GameStop stock price surges, investors turn to Elon Musk, likely due to his prominent and active Twitter presence, to gauge GameStop's business prospects. Elon Musk analyses the frequency of the term "GameStop" appearing in his recent tweets and deduces that the heightened discussion indicates significant business potential. Consequently, he disseminates a public message to all his Twitter followers, inciting a frenzy of investment activity. Here, the volume of social media discussions reflects valuable insights into GameStop's underlying fundamentals.

Finally, we would like to compare our model with the traditional research on cheap talk. A typical cheap talk model necessitates a public device for transmitting information to coordinate agents, whereas our study delves into the emergence of a public device through strategic network investments. Only once the investors' information network is established can they access inexpensive information; otherwise, investors situated far apart are unable to access the same information freely. For instance, during the GameStop frenzy, investors scattered globally struggled to reach a consensus on a shared public device for information dissemination. In such scenarios, our network formation process could come into play to address the coordination challenge. Essentially, our theory focuses on the investment in information channels preceding the cheap talk strategy.

1.1. Model preview

The exact model proceeds as follows. Our benchmark model is a two-stage game. There is a group of agents who are ex-ante identical, facing a coordination project, such as a bank run in the second stage. To complete this project, at least half of the agents should invest (not withdraw). Investing comes with a cost, while the payoff from a successfully completed project is uncertain. Each agent receives individual signals regarding the potential return of the project, based on the prevailing state of the world: in a good (bad) state, more positive (negative) signals are generated.

At the first stage, each agent makes a decision on whom to connect with based on their private signal. This act of linking serves as an investment in creating an information channel to facilitate the flow of information. The cost of establishing each connection is a fixed amount K > 0, referred to as the linking cost. Linked agents gauge the overall level of uncertainty by observing the network size, and subsequently share information within the network at no additional cost.

During the second stage, the coordination game commences, where each agent decides whether to invest or withdraw based on the information received. If the majority of agents choose to invest, those who do so will receive a return based on the overall state of the world once the project is completed.

We find a separating equilibrium where solely agents possessing a favourable signal are motivated to incur the connecting expense to contact a specific agent, who assumes the central role in a star network. This central agent functions as the informational leader, deducing the aggregate state from the network's scale. A substantial network size signifies a good state. Agents holding favourable signals are prepared to invest if they are informed of the message "The state is favourable." We then extend the baseline model by introducing leadership heterogeneity to address leadership selection. Our approach involves introducing a naming mechanism inspired by Tadelis (1999) to identify the most competent leaders. Within an Overlapping Generations (OLG) framework, a seasoned member at the core of the information network offers their name. Each period, if a leader successfully guides a profitable collective investment, they earn a favorable reputation. Young individuals within the group then compete for this name, with those possessing superior communication abilities being more inclined to bid higher, as it is more cost-effective for them to establish a positive reputation as a leader. Individuals with strong communication skills stand to gain the most from leadership roles. Notably, the name serves as an indicator of the leader's type and serves as a focal point for the establishment of the network.

Through the mechanism of the name market, a reinforcing loop exists between the selection of leadership and the success of coordination. This loop operates in the following manner: success in coordination results in the establishment of a positive reputation (a good name), which in turn attracts capable leaders. These leaders then contribute to further success in coordination. This interplay creates a feedback loop where the success of coordination and the quality of leadership reinforce each other, ultimately leading to the emergence of collective actions within cohorts.

Our theoretical framework departs from existing theories such as emotional contagion (as discussed in Yukl (2013)) or the contagion theory based on information linkage (as proposed by Chen and Suen (2016)). In our model, we observe a phenomenon where success spreads through a network united under a common name. The contagious effect of leadership manifests itself in the form of a shared identity or reputation rather than traditional emotional or informational channels.

Furthermore, our analysis reveals the presence of an alternating equilibrium scenario, wherein two leaders engage in competition to guide and influence investment decisions. This dynamic interplay adds another layer of complexity to the name market dynamics and the role of leadership in driving collective action.

Finally, our model considers the linking cost as a strategic tool for social media regulation. Surprisingly, a relatively low linking cost can lead to the absence of a separating equilibrium and hinder coordination, as even agents with inferior signals are motivated to connect to the core network. Consequently, the network fails to aggregate valuable information across various states. In the presence of a reputation market, a higher linking cost within the contagion equilibrium offers greater incentives to the information providers until the point where agents with reliable signals are disinclined to establish links. These observations enable us to determine the optimal linking cost within the contagion equilibrium. Conversely, in an alternate equilibrium, there is no necessity for incentivization, and the optimal linking cost aligns with the minimum sustainable expense. Therefore, effective social media regulation varies according to the distinct equilibria present.

1.2. Related literature

Our research paper contributes to the growing body of literature on information leadership, following the works of scholars such as Dessein (2002), Dewan and Myatt (2008) and Akerlof and Holden (2016). Specifically, we align with Akerlof and Holden (2016), who presents a theory of information leadership at the core of our study. In our paper, we introduce a linking cost to the formation process and address the issue of leadership selection. We draw inspiration from the motivations outlined in Ozsoylev and Walden (2011) and Han and Yang (2013), emphasising a costly network formation process that underpins effective leadership practices, a topic previously explored by researchers including Bloch and Dutta (2009), Bala and Goyal (2000), Galeotti and Goyal (2010), and Galeotti et al. (2006).

Our paper also complements the research on the information policy such as Chwe (1999), Dessein (2002), Majumdar and Mukand (2008), and Little (2016). Specifically, we explore the influence of information communication technology on economic activities, such as political protests, as demonstrated in the study by Little (2016). While his paper delves into a combination of a global game and a beauty contest game, our research relies on a simpler coordination setup to underscore the network formation process. Consequently, our model enables us to investigate the linking cost as a compelling policy tool, a facet not previously addressed in the information leadership literature.

Our paper sheds light on the role of leader selection in driving contagion, while also shedding light on the political contagion theory put forward by Drazen (2000). According to his work, crises diminish the perceived value of belonging to a government "club" and exacerbate the commitment issue faced by individual nations. Our paper introduces a novel contagion theory centred around the selection of information leaders. According to our theory, the successful investment in GameStop enhanced the reputation of those chosen as information leaders for investing in other correlated stocks. This led to trading frenzies spreading throughout the entire market.

Finally, our paper is related to Mailath and Samuelson (2001), Hermalin (1998), Tadelis (1999), Tadelis (2002) and Wang (2011) by introducing a name market to screen agents with different leadership. The functioning of our name market mirrors that of previous research. Nonetheless, what sets our work apart is our exploration of the contagion effect stemming from the name market, a novel aspect of our contribution.

The rest of the paper is organized as follows. Section 2 presents the basic network formation model without the name market. Section 3 introduces the name market and discusses how it acts as a coordination device and increases investment efficiency through leadership selection. Section 4 discusses the optimal linking cost policy and its implications for regime information control. Section five concludes. We relegate most proofs to the Appendix.

2. Basic model

Consider a discrete-time OLG model with infinite horizons. Every period, one cohort is born and lives for two periods, when they are young and old, respectively. There is one kind of player: investors.⁴ Investors are of measure one and indexed over a continuum [0, 1]. They are risk-neutral and consumed at the end of the second period of life.

At the beginning of the investors' second period, the project suffers a fundamental shock. Each investor has the option to invest in the bank (risky option) or withdraw with

⁴They could also be ex ante identical speculators or protesters by different interpretations.

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a safe return c > 0. $I \in \{0, 1\}$ is the action of agents, I = 1(0) means invest (withdraw). The project yields a return at the end of the game that depends on both the bank's underlying fundamentals (*w*) and the proportion of agents who stay (*l*). $w \in \{G, B\}$. We say the state is good if w = G; bad if w = B. $Pr(w = G) = Pr(w = B) = \frac{1}{2}$, and *w* is drawn IID over time.

The project yields a return 0 if $l < \frac{1}{2}$, because the coordination fails⁵; the project yields $v_H > c$ if $l > \frac{1}{2}$ and w = G; $v_L < c$ if $l > \frac{1}{2}$ and w = B. In GameStop investment, retail investors need to beat institutional investors who hold large short positions. So coordination among retail investors becomes really important.

The utility of investors is summarized as follows: When investor does not invest, hence I = 0, the utility of investor $U_A = 0$; When investor invests, hence I = 1, U_A is summarized by the following matrix:

	$l < \frac{1}{2}$	$l > \frac{1}{2}$
w = G	- <i>c</i>	<i>v_H</i> – с
w = B	-c	$v_L - c$

Before the investment, investors receive private signals $S \in \{S_g, S_b\}$. If w = G, a proportion q of investors will receive signals S_g while remaining 1 - q investors receive signals S_b . If w = B, a proportion q of investors will receive signal S_b , while 1 - q investors receive S_g . Therefore, $q > \frac{1}{2}$ is the informativeness of our signal structure. We call S_g (S_b) the good (bad) signal hereafter. We keep a common pessimistic prior: $qv_H + (1 - q)v_L < c$ to highlight the necessity⁶ of the information network formation.

The timing of the game in each period is as follows: New cohort is born; Nature draws the aggregate state w; Old investors receive signal S based on w; Old investors decide whether to invest, I = 0 or 1; Payoff is realised.

2.1. Equilibrium without network

Throughout the paper, our solution concept is perfect Bayesian equilibrium (PBE). No one will invest if the expected return is lower than the cost. Therefore, the coordination game admits a unique equilibrium. We summarise the results in our first proposition:

Proposition 1 Without an information network, there is a unique equilibrium: for each cohort

- (1) No investor investment: I(i) = I = 0;
- (2) Aggregate Investment: $Prob(l > \frac{1}{2}) = 0$.

Proof: Even with $l \ge \frac{1}{2}$, no one would invest. So, everyone chooses I = 0. Q.E.D.

⁵For example, the bank investment requires a substantial fixed cost to kick off.

⁶All our results hold without this assumption. Without it, there will be two equilibria: all invest or no one invests. All our arguments, as well as the conclusions, apply to the no-invest equilibrium.

The proposition suggests that even with w = G, investors will miss the investment opportunity because of the information friction. In the following subsection, we demonstrate that an information network arises to overcome such information friction.

2.2. Network formation

We now introduce a network formation game. Before investing and after receiving the private signal, investors can pay a positive linking cost of K > 0 to form a link with any other agents and receive information from such network investment freely.⁷

There are two points worth mentioning. First, an investor has the option to connect with multiple other agents, provided that they are willing to incur the cost of K for each connection. Second, though receiving information is free, there exists a cost $m \sim F(m)$ on [0, M], distributed across the population for any agent to send information through the network. Here, F(m) denotes the Cumulative Distribution Function (CDF) of m, which captures the heterogeneity of leadership ability. For example, the investor needs to deliver a public speech to spread certain information, while some people are good at it, others are not. M is the upper bound, indicating the investor would take tremendous effort (great cost) to spread information. We let $M > v_H - c$, so there is always a positive measure of investors unwilling to send information even when w = G. Finally, m is private information. The introduction of private m raises the leadership selection issue as we shall see in the next section.

The timing of events goes as follows: investors receive signal S; investors choose whether to pay a cost K for each link to any other agent; investor decides whether to pay $m \sim F(m)$ to send any information through his/her network connections; investors receive sent information through the network freely; investors decide to invest or not, I = 0 or 1; Payoff is realised based on the aggregate investment l and the state w.

2.3. Separating equilibrium with network formation

Before formalising any proposition, we focus on a few key insights. Firstly, the investor will establish a single connection to the centre of a star network. Secondly, the centre of the information network will deduce the overall state based on the size of their network only if investors with positive signals decide to link. Lastly, investors with positive signals are more inclined to make connections as they tend to hold a more optimistic view of the aggregate state. We now define the lower bound and upper bound of *K*: $\underline{K}_{NN} = F(v_H - c)(1 - q)(v_H - c)$; $\overline{K}_{NN} = F(v_H - c)q(v_H - c)$, where the subscript "NN" refers to the case with no name market. We have the following proposition:

Proposition 2 When $\overline{K}_{NN} > K > K_{NN}$, there is an endogenous information network:

(1) A random investor is selected to be the centre of an information network;

⁷K captures social media regulation controlled by the government and is the critical parameter throughout the paper. It controls whether a network can emerge and how frequently to coordinate. We will detail its influence in Section 4 when discussing policy control.

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 - (2) Every investor with a good signal is willing to link to the centre; every investor with a bad signal is unwilling to do so;
 - (3) The centre of this star network infers the aggregate state w from the size of the network:
 - (4) The centre sends information w = G if and only if $m \le v_H c$; he/she will not send any information if w = B or $m > v_H - c$;
 - (5) investors choose to invest if and only if receiving information w = G.

The fact that only investors with good signals are willing to link underpins the information network formation. The insight of our model is that the source of information is the network itself. The network is a result of an endogenous separating strategy; therefore, it is of different sizes in different states. The desire for information generates the supply of information through networking.

Proposition 2 focuses on the equilibrium in which K sustains a separating equilibrium. Another interesting result revealed in Proposition 2 is the relationship between the linking cost (K) and aggregate investment (l). We have the following lemma:

There is a non-monotonic relationship between the linking cost K and aggre-Lemma 1 gate investment success possibility $Pr(l > \frac{1}{2})$

- (1) If $F(v_H c)(1 q)(v_H c) > K$, the investment success possibility $Pr(l > \frac{1}{2}) = 0$;
- (2) If $F(v_H c)(1 q)(v_H c) \le K \le F(v_H c)q(v_H c)$, the investment success possibility $Pr(l \ge \frac{1}{2}) = \frac{F(v_H - c)}{2}$; (3) Else, $F(v_H - c)q(v_H - c) < K$, the investment success possibility $Pr(l \ge \frac{1}{2}) = 0$.

Lemma 1 addresses the impact of information technology on coordination activities. A common belief is that advanced information technology, such as Social Network Service (SNS), can enhance coordination. However, we introduce a simple model to demonstrate why this may not always hold true. When $K \leq F(v_H - c)(1 - q)(v_H - c)$, both investors with positive and negative signals are eager to connect to the central node. Nevertheless, in such scenarios where both types of investors are keen to connect, the central node fails to deduce the overall state due to the network size always remaining at 1, regardless of the value of w. Recognising this, no one opts to connect to the central node, leading to the collapse of the information network. Consequently, while superior information technology may decrease K, it can impede information aggregation, disrupt the formation of an information network, and ultimately limit investment.

The last comment of this subsection is that there is a coordination game behind the leadership. In other words, the network formation game serves as a coordination game among investors with positive signals. A investor with a positive signal is only willing to establish connections if all other investors with positive signals are also willing to connect. Otherwise, the centre will remain uninformed. Therefore, our model provides some validation for the theory put forth by Schelling (1960) regarding leadership formation: leadership is essentially a coordination game. Currently, the centre is chosen randomly, but this only addresses a portion of our motivation. We will delve into the topic of leadership selection in the following section by introducing a name market.

2.4. Leadership competition

Given Proposition 2, one could immediately imagine that there could be a core-periphery network with many centre players rather than one. For example, we could divide agents into N groups, so each group has $\frac{1}{N}$ agents. Agents choose a centre within the same group, so we have N centre agents. However, the fundamental structure is still a star network as we present in the formal proposition and has no substantial effect on any economic implication.

Nevertheless, we are going to discuss a more interesting case. We assume there are two high-profile investors with reputations but differing in their inference ability. When state w = G, one investor is able to infer the aggregate state from the size of the social network with correctness probability $\tau_G \in (0, 1)$. Another investor is able to infer the good state with correctness probability $\tau_B \in (0, 1)$. $\tau_G > \tau_B$.

This behaviour assumption captures the fact that investors might miss the investment opportunities even presented with positive evidence. Lastly, we let two high-profile investors choose the linking cost: K_G and K_B respectively. The leader investors derive utility from a higher linking⁸ cost. Proposition 3 follows.

Proposition 3 There are two endogenous information networks:

- (1) Type τ_G chooses $K_G = F(v_H c)q\tau_G(v_H c)$;
- (2) Type τ_B chooses $K_B = F(v_H c)q\tau_B(v_H c);$
- (3) $K_G > K_B;$
- (4) Every investor with a good signal randomly selects type τ_G and τ_B to connect;
- (5) Everything else follows Proposition 2;

Proposition 3 suggests that potential leadership charges a different price in a Bertrand competition in the networking market. In fact, we could allow many more potential leaders to compete in the networking market, but the basic message remains: high-ability leadership charges a higher price. In subsection 3.3, we shall see that leadership competition gives rise to more interesting dynamic patterns of collective investment.

2.5. Other extensions

In this subsection, we discuss the robustness of the model:

1. Two state, two signal information structure

We streamline the information structure by categorising signals as either positive (indicating profitable investment) or negative (indicating unprofitable investment) when multiple signals are available. This two-signal framework is applicable even when there are more than two states, as we can classify states into profitable and non-profitable categories. As long as the signals vary consistently across different states, the information leader can infer the overall state based on the varying network sizes. Our findings remain reliable even when dealing with more intricate information structures.⁹

⁸For example, the linking cost is a membership fee for an investment club. ⁹See appendix B for results with continuous states and signal error space.

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2. Heterogeneous linking cost

In our formal model, all agents are assumed to share the same linking $\cot K$. However, in actuality, people vary in their geographical distance, leading to different linking costs. Despite this variation, if the distribution of linking costs is known to all, the central authority can still deduce the overall status based on this information. There exists a threshold for the linking cost below which even agents with unreliable signals will opt to link. In the equilibrium state, the central authority calculates this threshold strategically and deduces the aggregate condition from the network's size.

3. Costly to receive

Receiving the information, even through the established social network, may require some time or money. In theory, this can be seen as raising the cost of linking, which in turn does not impact most of our conclusions.

2.6. Other interpretation

Despite the motivation of investment frenzy in the financial market, our theory sheds light on a range of observed economic patterns.

- (1) Speculation Attack. One important issue in international finance and central bank policy discussion is whether transparency helps to defend against a speculation attack in which a group of speculators decide to abandon currency at the expense of interest return. We show that an information club might emerge to share the information and finally jeopardize the regime. Famous speculators like Soros might be the focal point of the network and would share their information through his social network. A prestigious player with proven leadership increases the chance of regime change as in Corsetti et al. (2010).¹⁰ The underlying mechanism is an endogenous information network.
- (2) Public Protest. Protests might be profoundly shaped by modern information technology like SNS. Today people are flooded with information. Therefore, it is much easier to organize a public protest. Nevertheless, our story highlights some adverse effects of lowered networking costs: failure of information aggregation.
- (3) FDI. The most common interpretation of our investment game is FDI. Some prestigious investors will be selected as the leader formally. For example, Coca-Cola entered many foreign markets like China, as the first wave of American foreign investment.

3. Name and contagion

In the baseline model, the centre of the network is randomly selected. However, the selection of leadership holds significance for various reasons: the leader's competence is crucial for the success of coordination efforts; the establishment of power rotation is necessary to reduce social instability, and we are keen on exploring any strategies that could impact the process of leadership selection. Within this straightforward OLG framework, we introduce an inter-generation

¹⁰In Corsetti et al. (2010), the presence of a great player heightens the strategic concern of each play in a global game.

name market¹¹ to determine the most suitable leader. Additionally, this section aims to demonstrate a correlation between leadership selection and the spread of collective investment contagion. We describe contagion as an outcome of a positive feedback loop between leadership selection and successful coordination towards the conclusion of this section.

Recall the agent's communication skills, which is modelled as the cost to send a message, differs across the population as $m \sim F(m)$. We want agents with the best communication skills, hence m = 0, to be the centre of the information network,¹² which is the ultimate purpose of leadership selection. To achieve this, we formally propose a name mechanism, an idea that comes from Tadelis (1999).

3.1. Name market

In every period, a name may be supplied by an old centre of the information network. We now call the centre of the network as (information) leader. In every period, if the leader leads a profitable return on the collective investment, that is v_H , then the leader has a good name; otherwise, he/she has a bad name.¹³

We are looking for such a separating equilibrium in which only young agents with m = 0 acquire the good name through the name market, and no one will purchase the bad name. Why do young agents purchase a good name?

Because in the equilibrium, only the agent with a good name will be selected as the centre of the network, or rather, the leader. As a result, agents purchase a good name to save the linking cost K and sell the good name as the old leader in the next period, because only the leader can sell his/her name. In summary, linking cost K plus resale revenue minus the information send costs m shall be the bid for a good name.

So, who shall submit the highest bid to win the good name? The answer is the agent with the best communication skills because they have the least sending cost m = 0. ¹⁴ For any *m*, the expected benefit of being the leader is K + q(N - m), where *N* denotes the name price on the equilibrium. Given the same *K* and *N*, this benefit is maximised when m = 0. An agent with m = 0 wins the bid by the highest bid $K + q \times N$.

To summarise the name market, the leader will supply a good name if there is a successful investment v_H under his/her leadership; young agents with good signals and m = 0 will submit the highest bid and acquire the good name. That is the name demand. Finally, we define $K_N = (1 - q)(v_H - c)$ and $\overline{K}_N = F(v_H - c + \frac{K}{1-q})q(v_H - c)$, where the subscript "N "refers to with a name market case. We now obtain the following proposition:

¹¹We focus on the model in Section 2 throughout this paper. Nevertheless, the screen function of the name market can be quite general. In a model like Akerlof and Holden (2016), we can also allow agents to differ in their ability to disseminate public information, and then the most capable person will also arise to buy the good name to be the mover and shaker in their framework. The bottom line is the conclusion that the name market can select leadership because contagion does not rely on our information leader theory and deserves more broad interpretation.

¹²The least cost does not necessarily have to be 0. It is only for technical convenience.

¹³An information leader can have a bad name for two reasons: first, coordination failure: $l < \frac{1}{2^{\prime}}$ second, investment loss: $v_L - c < 0$. In the equilibrium, the bad name price is zero.

¹⁴Notice agents with a good signal will always outbid agents with a bad signal because they are optimistic about the resale possibility; hence, q > 1 - q.

Proposition 4 When $\underline{K}_N < K < \overline{K}_N$, there is a separating equilibrium;

- 1. When there is a good name
 - (a) Young agents with m = 0 and good signals bid highest for the good name: $b = \frac{K}{1-q}$
 - (b) Good name is allocated to the highest bidder;
 - (c) Good name owner becomes the centre of the information network;
 - (d) Separating network linking strategy as before;
 - (e) The centre as an information leader always sends a message if w = G.
- 2. When there is a bad name, a random young agent is the informational leader. (a) Random information leader sends a message if and only if $m < v_H - c + N$;
- 3. Equilibrium Good Name Price: $N = \frac{K}{1-a}$.

There could be many other benefits of being a leader, such as more financial resources or better public perception. We do not specifically account for these factors in our analysis; instead, we prioritise the linking cost as the primary determinant of the name price. Although we could incorporate these additional factors into our model, it would likely result in a higher name price. However, such modifications would not alter our findings or provide any novel perspectives. Our preference is to examine the name market under the most basic conditions. We will now explore the impacts of the name market.

The name market enhances the leadership selection. Agents with m = 0 will always send a message whenever w = G. For the social optimal, we expect agents with m = 0 to become the leader. This is indeed the case in the name market since agents with m = 0 will outbid any other agents.

The name market increases the communication incentive. A leader with a good name will send a message whenever w = G. However, for a random leader, he/she will consider the benefits and costs: when w = G, sending a message costs m; The benefits would be investment profit $v_H - c$ plus the name resale revenue N. The message is sent if and only if $m < v_H - c + N$. Given F(m), the chance of sending message when w = G is: $\Pr(m < v_H - c + N | w = G) = F(v_H - c + N | w = G)$. Name market increases the chance of sending a message: $F(v_H - c + N)$; the higher the name price N, the higher the incentive to send w = G.

The name market changes the range of K. Both the lower bound and upper bound of K shift upward.¹⁵ With the name market, the expected profit of both good and bad signals increases due to leadership selection and communication incentives. For the information network to exist, K should be large enough to deter agents with bad signals. When the information leader has a good name, the expected profit of agents with bad signals will be $(1-q)(v_H - c)$. This lower bound shifts upward because of the selection effect. For the upper bound, K should be lower than the expected profits of agents with good signals regardless there is a good name or not. Since the profit is lower with a random leader, the upper bound is determined by the expected profit with a random leader: $K < F(v_H - c + N)q(v_H - c)$. Because of the incentive effect of the name market, the upper bound also shifts upward.

¹⁵Whether the range expands or shrinks, depends on the exact functional form of F(m).

The following lemma summarises the influences of the name market:

Lemma 2 Name enhances the selection of a good leader and the performance of the random leader;

- (1) Leadership Selection: $Pr(m = 0) = \frac{F(\nu_H c + \frac{K}{1-q})}{1 + F(\nu_H c + \frac{K}{1-q})};$
- (2) The Investment Success Possibility Under a Good Name Leadership:

$$Pr(l > \frac{1}{2} | w = G) = Pr(w = G) = \frac{1}{2};$$

(3) The Investment Success Possibility Under a Random Leadership:

$$Pr(l > \frac{1}{2} | w = G) = F\left(v_H - c + \frac{K}{1 - q}\right)/2$$

- (a) increases with the linking cost K;
- (b) *increases with the transparency q*;
- (c) increases with the investment profit $v_H c$.

A higher name price increases the random leader's communication possibility. The name price is $N = \frac{K}{1-q}$: when the linking cost K increases, the benefit of avoiding linking cost increases, and so does the name price; when the informativeness q increases, the estimated name resale possibility increases because agents with good signals will make more optimistic inference of the state w, so does the name price. Through the channel of the name price, both K and q increase the investment success probability.

The last thing we want to mention about the name system is that "*it is* a separating equilibrium conditional on a separating equilibrium." If $(1-q)(v_H - c) > K$, agents with bad signals will also connect, and the network will collapse. In this case, there is no reason to buy a name to become the centre of the network. The name market can operate if and only if the network formation is possible. That is why we still need the constraints for the linking cost: $(1-q)(v_H - c) < K < F(v_H - c + \frac{K}{1-q})q(v_H - c)$.

We treat communication skills as a critical ingredient of leadership in this paper. There might be some other heterogeneity across agents that affects the leader's performance. The name market can still take effect to screen the most capable type, provided a good type has a relative advantage in generating a good name.

3.2. Contagion by leadership

A perplexing phenomenon frequently witnessed in the realm of collective action is known as contagion. This entails a cascading effect where a surge in successful coordination triggers numerous subsequent occurrences. To illustrate, during the Asian Financial Crisis, the currency turmoil originating in Thailand propagated to several other Asian nations. Similarly, within the realm of Dogecoin investments, a single profitable 14 🕒 B. WANG AND S. ZHENG

investment can catalyze a cascade of investments across various types of digital currencies. While previous scholars have attributed contagion to factors such as inter-region exposure or information dissemination, our model introduces an alternative explanation.

Proposition 5 Contagion by Leadership:

- On the separating equilibrium with an active name market, we have
- (1) $Pr(Success|Last Success) = \frac{1}{2}$;
- (2) $Pr(Success|Last Failure) = \frac{F(V_H c + \frac{K}{1-q})}{2};$
- (3) Contagion: $\frac{1}{2} > \frac{F(V_H c + \frac{K}{1-q})}{2}$.

By Proposition 5, we call this *contagion equilibrium*. Because of the leadership selection function of the name market, one collective success generates a good name and a good name screens a good leader. Throughout time, we see positive feedback between the inter-temporal investments. Consequently, an outside observer observes that a series of successes take place under the same name. Fundamentally, it is the selection effect of the name market that leads to a series of successes. Contagion by name is essentially contagion by leadership.

3.3. Leadership alternation

One concern about contagion equilibrium is that failure of investment ends with a random selection of leadership, which is hardly the case in reality. In this subsection, we consider two leadership¹⁶ candidates: Musk and Zuckerberg.

Notice, both "Musk" and "Zuckerberg" are merely names with the ownership changing in the name market. The novel part, compared to the contagion equilibrium is that once failed, every investor coordinates on another name, rather than the failed name, for the next round of network formation. WLOG, we let "Musk" be the current information leader. We have the following proposition:

Proposition 6 (Alternation Equilibrium) When $K_N < K < \overline{K}_N$, there is a separating equilibrium;

- 1. When "Musk" obtains a good name
 - (a) Young agents with m = 0 and good signals bid highest for the good name: $b = \frac{K}{1-a}$
 - (b) Good name is allocated to the highest bidder;
 - (c) Good name owner becomes the centre of the information network;
 - (d) Separating network linking strategy as before;
 - (e) The centre as an information leader always sends a message if w = G.
- 2. When "Musk" obtains a bad name, young agents with m = 0 and good signals bid highest for "Zuckerberg". Everything else is the same as above.

¹⁶Our results easily apply to an environment with more than two candidates.



Figure 1. Leadership alternation.

See Figure 1 for a diagram illustration of *alternation equilibrium*. Because there is always a name screening the leadership, we conclude

Lemma 3 Welfare and investment volume is higher in the alternation equilibrium than in the contagion equilibrium.

4. Social media regulation

In this section, we focus on the linking cost K for policy discussion. In Akerlof and Holden (2016), the network formation is free, while in this section, we treat the positive linking cost K as a regulation on the social network service. We start with contagion equilibrium first.

Before we lay out the proposition, we formally define the welfare of agents, and summarise the influences of *K* in our theoretical framework.

	$l < \frac{1}{2}$	$l > \frac{1}{2}$
w = G	-c	$v_H - c$
w = B	-c	$v_L - c$

Definition: For agents on [0, 1], aggregate welfare is $AW = E(\int U_A(i)di)$, where $U_A(i)$

is the agent's risk neutral utility function defined in Section 2. We rephrase here: when agent does not invest, hence I = 0, $U_A = 0$; when agent invests, hence I = 1, the utility of agent U_A is defined by the following matrix:

The integral of the utility function seems intricate to solve. However, we show the welfare maximisation problem on the equilibrium can be largely simplified:

Lemma 4 Welfare Maximization: On the contagion separating equilibrium with the name market, maximising AW is equivalent to maximizing $F(v_H - c + \frac{K}{1-a})$.

Lemma 4 suggests we only need to maximise the random leader's communication possibility. We are ready to study the influences of K on AW.

First, $(1 - q)(v_H - c) < K$. The left-hand side is the expected payoff of a costly link for the agents with bad signals when there is a good-name leader. For the separating equilibrium of network formation to exist, agents with bad signals shall not connect even with a good name leader, so K should be larger than this payoff. That is the lower bound.

Secondly, $K \leq F(v_H - c + \frac{K}{1-q})q(v_H - c) = \overline{K}_N$. The right-hand side is the expected payoff of a costly link for agents with good signals when there is a random leader. For the same reason, agents with good signals must be willing to connect even with a random leader. That is the upper bound.

Eventually, *K* should maximise $F(v_H - c + \frac{K}{1-q})$ as in lemma 4. The higher *K*, the higher the possibility for a random leader to send the message: $F(v_H - c + \frac{K}{1-q})$. We still have a natural boundary for such incentive effect, that is $v_H - c + \frac{K}{1-q} \leq M$. We let K_M denote this natural bound, where $K_M = (1 - q)(M - v_H + c)$.

The following proposition concludes an optimal linking cost by taking into account all three considerations:

Proposition 7 The optimal linking cost K_c^* for the aggregate welfare in the contagion equilibrium is determined by:

$$K_c^* = \min(\overline{K}_N, K_M)$$

Proposition 7 implies that a long-run optimal information control should be some intermediate linking cost: high enough to deter the pessimistic investors, while low enough to accommodate the optimistic investors. The exact optimal cost differs with some exogenous parameters.

Proposition 8 The optimal linking cost K_c^* weakly:

- (1) increases with the transparency q;
- (2) increases with the profit $v_H c$;
- (3) increases with the average communication cost M.

Proposition 8 provides a series of policy recommendations for a government:

Lemma 5 The government should set up a high linking cost K when

- (1) the local economic condition is transparent;
- (2) the expected investment profit is high;
- (3) the communication skills of investors are low.

The underlying rationale behind these policy recommendations is as follows: Proposition 8 posits the optimal linking cost should hit the upper bound of K in the separating equilibrium. When the investment is profitable, the upper bound is high, so the optimal linking cost should be high; When q is high, agents with good signals make more optimistic inferences of the aggregate state, so the upper bound is also high; lastly, M directly increases the upper bound.

4.1. Regulation on alternation equilibrium

In the *alternation equilibrium*, there is always a name to screen the leadership with the highest communication skill. Consequently, there is no need to consider the incentive effect since the leadership always sends the message if and only if w = G. Proposition 9 follows.

Proposition 9 The optimal linking cost K_a^* in the alternation equilibrium is:

$$K_a^* = (1 - q)(v_H - c)$$
(1)

That is, the optimal cost is the minimal sustainable cost. The social planner only needs to guarantee the name market operates. We summarise our conclusions on social media regulation in Table 1:

5. Conclusion

In this paper, we confirm Schelling's (1960) doubt that the leadership itself might be a coordination process. By introducing a pre-coordination network formation game, we show agents form a star network for information aggregation and transmission.

The essence of the network in this paper is that the network itself is information. The centre of the star network can infer the aggregate state by counting the number of links, given the separating linking strategies of agents. We then introduce the name market to address the leadership selection issue. In the equilibrium, agents with the best communication skill are willing to pay the highest price for a name, to become the centre of the network. Overall, the contribution is two-fold: we propose a new theory on information leadership; we address the leadership selection problem by the name market.

The main idea presented in this paper is that the network can be viewed as a form of information in itself. By observing the number of links within the star network and considering the connecting strategies of the agents, the central node can infer the aggregate state. To resolve the leadership selection dilemma, we introduce the concept of a "name market". In this market, agents with superior communication abilities are willing to pay a premium to acquire a unique name, aiming to become the centre of the star network. This work makes a dual contribution: it puts forth a novel theory regarding information leadership, and it tackles the leadership selection issue through the innovative idea of the name market.

The model addresses various practical challenges related to how individuals coordinate their actions and exchange information in economic endeavours. It offers an explanation for the presence of prominent speculators such as Soros or powerful business conglomerates in orchestrating group investments. Additionally, our framework can

Endogenous Network Formation with Name Market		
Contagion Equilibrium	Incentive Effect	$K_c^* = K_M$
	Willingness to Connect	$K_c^* = \overline{K}_N$
Alternation Equilibrium	Minimum Sustainable Cost	$\tilde{K_a^*} = (1-q)(v_H - c)$

Table 1. Optimal regulation of social media.

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elucidate how leadership plays a role in collective contagion. The success of coordination efforts and the choice of leaders form a mutually reinforcing cycle. Lastly, our model presents fresh perspectives on the regulation of social media, emphasising the adverse impact of low connectivity costs on the consolidation of information.

The coordination game is simplified to emphasise the process of network formation. Subsequent studies could explore more intricate coordination scenarios like voting and platform pricing, offering fresh perspectives. For instance, how does platform pricing influence the internal network development process, or how does network formation impact the informativeness of the financial market? We leave these topics for future research.

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Appendix

A Proofs of Propositions

A.1.Proof of Proposition 2

Proof: We first solve the agent's linking decision, then discuss the centre's information inference, and finally consider the information sending decision.

For agents with good signals, linking to the centre costs K, and the payoff depends on his/her inference. The good signal means with probability q, w = G. Conditional w = G, with probability $F(v_H - c)$, centre will send w = G to every agent. Hence, the expected investment success possibility is $F(v_H - c)q$; The success profit is $v_H - c$. So the payoff of such a link to the centre is $F(v_H - c)q(v_H - c)$.

The payoff of a link to the centre is $F(v_H - c)q(v_H - c)$, for agents with good signal. So when $F(v_H - c)q(v_H - c) < K$, the linking cost is so high, even the agents with good signals are unwilling to connect. The expected return from information can not compensate for networking cost; so we have $F(v_H - c)q(v_H - c) > K$.

The payoff of link to the centre is $F(v_H - c)(1 - q)(v_H - c)$, for agents with bad signals. So when $K < F(v_H - c)(1 - q)(v_H - c)$, the linking cost is so low, the agents with a bad signal are willing to link to the centre; However, since both types are willing to connect, information aggregation fails; So we must have $K < F(v_H - c)(1 - q)(v_H - c)$.

If and only if $F(v_H - c)q(v_H - c) > K > F(v_H - c)(1 - q)(v_H - c)$, we have separating strategies of agents with different signals. Given the separating strategy, the size of the network varies across different states. That is when w = G, the centre has q links; when w = B, the centre has 1 - q links. Because of that, the centre can infer the aggregate state.

The next step shall be the information-sending decisions. So far, the centre is randomly chosen, so w = G will be sent if and only if $m < v_H - c$. The possibility of investment success is $F(v_H - c)$.

Now every agent with bad signals receives no information, and they are unwilling to invest at the very first. For agents with good signal: if there is no information, then $Pr(w = G|No \ Infor) < q$, so no investment; if there is information w = G, then he/she knows for sure profit $v_H - c$, and a fraction $q > \frac{1}{2}$ will invest; he/she will optimally invest. Q.E.D.

Proof of Proposition 4

Proof: Agents with good signals and m = 0 compute the expected payoff as the leader: K + qN. They will submit the total surplus since there is a Bertrand competition among such agents, so N = K + qN. Solving the equation, we have $N = \frac{K}{1-q}$. Therefore, a leader with a good name sends the information w = G since sending costs m = 0; while there is a bad name, the agent can choose a random leader at best. A random leader has incentive $v_H - c + N$ to send such a message: investment profit $v_H - c$; and name resale profit N.

The networking part is the same as the previous proposition except for the boundaries of *K*. Now we have a good name leader or random leader:

When there is a good name leader, the expected payoff of a link to the centre for agents with the bad signal is: $(1 - q)(v_H - c)$;

When there is a random leader, the expected payoff of a link to the centre for agents with bad signal is: $F(v_H - c + N)(1 - q)(v_H - c)$;

For any type of leader, agents with bad signals should be unwilling to connect, so $K > (1-q)(v_H - c) > F(v_H - c + N)(1-q)(v_H - c)$.

Compute the payoff for agents with good signals and any type of leader, agents with good signals should be willing to connect, we have

$$K < F(v_H - c + N)q(v_H - c) < q(v_H - c).$$

We now have both lower bounds and upper bounds of K. Q.E.D.

Proof of Lemma 2

Proof: Let x denote the stationary possibility of good name. Then we have

$$(x \quad 1-x) \left(\frac{\frac{1}{2}}{\frac{F(\nu_H - c + \frac{K}{1-q})}{2}} \quad \frac{\frac{1}{2}}{\frac{2 - F(\nu_H - c + \frac{K}{1-q})}{2}} \right) = (x \quad 1-x);$$

Solving the equation, we have $x = \frac{F(v_H - c + \frac{K}{1-q})}{1 + F(v_H - c + \frac{K}{1-q})}$. The other part of the lemma is simple; *Q.E.D.*

Proof of Lemma 5

Proof: Last period success generates a good name, then a good name selects a leader with m = 0. For the next period, as long as w = G, there will be an investment success. Since $Pr(w = G) = \frac{1}{2}$, we have first item; while failure leads to a random leader, we have second item by lemma 2. The third item is trivial since any $F(\cdot) \leq 1$. Q.E.D.

Proof of Lemma 4

Proof: We define agent's aggregate welfare: $F = l \times (\Pr(G) \Pr(l > \frac{1}{2}|G)(\nu_H - c) + \Pr(B) \Pr(l > \frac{1}{2}|B)(\nu_H - c) + \Pr(l < \frac{1}{2})(-c)).$ As we shall see on the network formation separating equilibrium, only agents with good signal will invest if w = G. So in real computation, we only need to compute the first part $l \times \Pr(G) \Pr(l \geq \frac{1}{2}|G)(\nu_H - c).$ We could further reduce it to $F = \frac{q}{2} \Pr(l > \frac{1}{2} | G)(v_H - c)$. Since q and $(v_H - c)$ are given, we only need to consider the investment possibility when the state is good: $\Pr(l > \frac{1}{2}|G)$. By lemma 2, on the separating equilibrium, when leader has a good name, $\Pr(l > \frac{1}{2}|G) = \frac{1}{2}$; When leader has a random name, $\Pr(l \ge \frac{1}{2}|G) = F(v_H - c + \frac{K}{1-q})$. The chance of good name leader is $\frac{F(v_H - c + \frac{K}{1-q})}{1 + F(v_H - c + \frac{K}{1-q})}$. So $AW = \frac{F(\nu_H - c + \frac{K}{1-q})}{1 + F(\nu_H - c + \frac{K}{1-q})} \times \frac{1}{2} + \frac{1}{1 + F(\nu_H - c + \frac{K}{1-q})} \times F(\nu_H - c + \frac{K}{1-q}) = \frac{F(\nu_H - c + \frac{K}{1-q})}{1 + F(\nu_H - c + \frac{K}{1-q})} \times \frac{3}{2}; \text{ obviously,}$ maximise AW is equivalent to maximise $F(v_H - c + \frac{K}{1-a})$. Q.E.D.

Proof of Proposition 7

Proof: We want to solve $Max \ F(v_H - c + \frac{K}{1-q})$, with the good signal constraint that $K \leq F(v_H + \frac{K}{1-q})q(v_H - c)$ and boundary constraint $v_H - c + \frac{K}{1-q} \leq M$. The optimised function is monotonically increasing in K, so the solution is determined by the two upper bounds: $K \leq F(v_H + \frac{K}{1-q})q(v_H - c)$ and $v_H - c + \frac{K}{1-q} \leq M$. Q.E.D.

B Information Leadership with Continuous States and Errors

In this appendix, we show our main result holds in the presence of continuous states and signal error. We let the return of successful investment be drawn from some distribution $\theta \sim f(\theta)$. Every agent receives a signal $s = \theta + \varepsilon_i$, while ε_i is distributed across agents by $g(\varepsilon)$ and independent of θ . We have the following results: agents with signal $s > s^*$ will choose to link to the centre. By doing so, the centre infers aggregate state by rationally expecting the network size, which is determined by $F(\theta > s^* - E(\varepsilon))$. We define $L(\theta) = E(\theta | \theta \ge c)$, then on equilibrium s^* is defined as: $E(L(\theta)|s^*) = K$, where c is the investment cost, and K is the linking cost.