

Hamaguchi, Yoshihiro

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Effects of a policy mix of uniform or unilateral environmental tax and trade liberalization on economic welfare

Yoshihiro Hamaguchi 

Department of Economics, Faculty of Economics and Business Administration, Kyoto University of Advanced Science, Kyoto, Kyoto-shi, Japan

ABSTRACT

To address global warming, the EU introduced the Carbon Border Adjustment Mechanism in 2023, raising concerns about its potential negative effects on GDP and trade volumes in the Global South, including China, India, and African countries. The Border Tax Adjustment (BTA) is perceived as a discriminatory environmental tax by firms, impacting pollution levels by changing productivity through entry and exit. Employing an R&D-based growth model with heterogeneous firms, I examine the impact of a discriminatory environmental tax and trade liberalization on welfare. Results indicate that under the BTA, tax hikes on exporters improve welfare, while the welfare effect of trade liberalization varies based on tax rates. Ideally, abolishing the BTA improves, while its maintenance necessitates cautious consideration, as liberalization can enhance welfare when excessive trade liberalization is avoided. A thorough examination of the pros and cons of the BTA policy is imperative.

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1. Introduction

1.1. Motivation

To address carbon leakage, the EU implemented the Carbon Border Adjustment Mechanism (CBAM) in April 2023, following extensive debate (Wettstad, 2024). However, concerns have emerged regarding its cost-effectiveness and potential adverse effects on international trade. Notably, Eicke et al. (2021) argues that the impact on the Global South has been neglected. Simulation analyses indicate that China and Turkey are expected to experience a decrease in GDP owing to the EU CBAM (Acar et al., 2022; Chen, 2023). Furthermore, changes regarding trade may shift the burden of climate policy to developing countries (Cosbey et al., 2019). The challenge of promoting decarbonization in some parts of the Global South could exacerbate trade frictions if GDP and trade volumes decline because of the EU's CBAM. Hence, doubts have been raised about the effectiveness of the Border Tax Adjustment (BTA). For instance, Jakob et al. (2013) suggests that the

CONTACT Yoshihiro Hamaguchi  hamaguchi.yoshihiro@kuas.ac.jp; prestidigitateur@gmail.com 

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EU's BTA increases the leakage rate in China. When examining this policy issue, it is essential to consider the new trade theory proposed by Melitz (2003), as Balistreri and Rutherford (2012) demonstrates that the leakage rate is higher in the trade structure in Melitz (2003) than in others. Overall, while the BTA serves as the policy instrument of CBAM as a carbon tax, it necessitates analysis concerning trade liberalization.

This study utilizes the heterogeneous firms model of Melitz (2003) to analyze the welfare effects of BTA and trade policies, clarifying their interconnectedness. I present a new policy variable that extends previous studies on the impact of environmental policies on firm entry and exit (Andersen, 2018; Anouliès, 2017; Cherniwchan & Najjar, 2022; Konishi & Tarui, 2015), the influence of trade policies on firm productivity (Kreickemeier & Richter, 2014; LaPlue, 2019), and the relationship between these policies and technology (Coria & Kyriakopoulou, 2018; Cui, 2017). Environmental and trade policies contribute to fostering innovation for sustainable development. Therefore, unlike previous studies, I explore the relationship with innovation using the R&D-based growth model proposed by Gustafsson and Segerstrom (2010). This study elucidates the dynamic effects of BTA policies in the new trade theory of Melitz (2003) in the recent literature.

This framework enables us to assess the welfare effects of BTA and trade liberalization regarding productivity and pollution, offering valuable insights for policymakers. Additionally, I explore how trade liberalization influences the welfare effects of BTA. Our findings suggest that abolishing BTA is advisable, as trade liberalization consistently enhances welfare. However, even under a BTA, moderate trade liberalization can improve welfare. Therefore, policymakers must carefully balance environmental and trade policies. This study aligns with SDGs 8, 9, and 13 by examining how BTAs addressing global climate change affect industry productivity and innovation. Notably, fossil fuels, a significant source of greenhouse gas emissions, are not explicitly considered as a production factor in this study. Therefore, caution is warranted when generalizing the results and policy recommendations.

1.2. Literature review

The impact of environmental policy and trade liberalization on pollution emissions in international trade has long been debated, with conflicting results in the literature. Recent attention has focused on firm heterogeneity, as demonstrated by Cherniwchan et al. (2017), Kreickemeier and Richter (2014), Levy and Dinopoulos (2016), and LaPlue (2019), indicating that trade liberalization enhances productivity and environmental quality. Such studies as Forslid et al. (2018) reveal that environmental taxes decrease pollution intensity for exporting firms when pollution removal is considered, while trade liberalization has varying effects on aggregate emissions across countries. Environmental policies generally prompt firms to exit (Andersen, 2018; Cherniwchan & Najjar, 2022). However, in Konishi and Tarui (2015) and Anouliès (2017), the policy does not affect entry-exit. The effect of environmental policies on firm entry and exit depends on industry-specific regulatory discrimination. Notably, Richter et al. (2021) finds that such policies reduce domestic pollution and negative leakage. Additionally, studies have examined the relationship between pollution abatement investment and total pollution emissions (Cao et al., 2016; Tang et al., 2014). Incorporating innovation into

these frameworks reveals a close relationship with the Porter hypothesis. Applying the R&D-based growth model, Hamaguchi (2023) identify environmental tax evasion and corruption as determinants of the Porter hypothesis and the pollution haven hypothesis. While these previous studies have discussed the impact of environmental policies and trade liberalization on pollution and environmental quality through productivity, output, and industry composition, the impact of BTA policies remains unexplored.

As noted by Grubb et al. (2022), no evidence supporting the occurrence of leakage due to climate policies exists. The effect of BTA policies remains subject to theoretical debate, with Keen and Kotsogiannis (2014) and Böhringer et al. (2014) suggesting uniform optimal emissions prices across industries, while Fischer and Fox (2012) contends that full border adjustments are invariably inefficient. The relationship between BTA policies and technology is analyzed by Gerlagh and Kuik (2014) and Helm and Schmidt (2015). Concerning welfare effects, studies generally report positive outcomes (Böhringer et al., 2017; Eichner & Pethig, 2015; Tsakiris et al., 2019), predominantly in perfectly competitive markets, with Yomogida and Tarui (2013) analyzing imperfect competition. Notably, minimal analyses exist on BTA policies within the heterogeneous firm model proposed by Melitz (2003), although such analysis is crucial. This is because, under BTA policies, where productivity is higher among exporting firms, discriminatory tax rates affect productivity and pollution emissions through the firm entry and exit dynamics. Therefore, the reallocation effect in Cherniwchan et al. (2017) may yield divergent results from previous studies. This study bridges these gaps by utilizing the heterogeneous firm model proposed by Melitz (2003).

Theoretical model results find empirical validation in various studies. Dechezleprêtre and Sato (2017) suggests that environmental policies diminish firm productivity in the short term, as concluded with limited statistical significance. Controlling for the effect of firms' productivity on output, Tang et al. (2015) notes a negative influence of productivity on pollution. Supporting Porter's hypothesis, Yang et al. (2021) observes a non-monotonic relationship between environmental policy and firm productivity. China's Key Air Pollution Control Zone policy in Jiang et al. (2023), improves the quality of exporting firms. In a heterogeneous firm model, Dardati and Sayagili (2020) demonstrates diverse welfare costs of emission reductions contingent on emission allowance allocation methods. Considering the model by Melitz (2003), Cherniwchan et al. (2017) emphasizes the significance of the pollution haven effect while disputing the pollution haven hypothesis' significance. They assert that while trade liberalization reduces firm and industry pollution emissions, precise mechanisms warrant further elucidation. In the U.S., Shapiro and Walker (2018) attributes air pollution reduction to environmental tax hikes, with marginal effects on firm productivity. Contrary to the strong Porter hypothesis, Rubashkina et al. (2015) fails to support that strict environmental policies enhance firm productivity in Europe. Additionally, Hong et al. (2024) argues that lowering trade barriers may increase pollution emissions. However, evidence suggests that trade liberalization can mitigate pollution. The empirical effects of trade liberalization on aggregate emissions through entry-exit and productivity, merit further scrutiny and clarification through model analysis.

Since the CBAM will not be fully implemented until 2023, data analysis relies on Input-Output analysis and the Computable General Equilibrium Model. With the accumulation of empirical data, the empirical analysis of BTA policy will progress in the future. In doing

so, the theoretical analysis in this study can provide a mechanism for future empirical studies of the impact of BTA on aggregate pollution through firm productivity. As for the theoretical analysis, it not only presents the link between BTAs and trade liberalization in a heterogeneous firm model but also complements the analysis of Hamaguchi (2023, 2024) with a full welfare analysis. These are the contributions of this study.

The rest of the paper is organized as follows. An overview of the present model is provided in Section 2. In Section 3, the dynamic general equilibrium model is described and the equilibrium is derived. Section 4 presents the results of the comparative statics analysis of each policy and explains the intuition. Section 5 concludes the paper.

2. Methods

I expand the R&D-based growth model of heterogenous firms developed by Gustafsson and Segerstrom (2010), featuring discriminatory environmental tax and a unit input factor. There are two symmetric countries with a representative household supplying labor, a manufacturing sector competing monopolistically, and an innovation sector creating knowledge via R&D activities. The first country of the present model is called North, and the other is South.

Each firm emits pollution as a byproduct of production and can reduce emissions by substituting labor for pollution inputs. Moreover, the lower the unit input requirement of its production, the lower the pollution level emitted by firms, following Hamaguchi (2023, 2024). The government of the country imposes environmental taxes on firms entering the local and foreign markets to internalize the negative externality of pollution. Here, the government imposes a unilateral environmental tax or uniform environmental tax between exporting and local sectors. Non-transboundary pollution emitted by firms disturbs households in each country.

A fixed number of households live in each country. Each member of a household is endowed with one unit of labor, inelastically supplying this labor to sectors to obtain wage income. The size of each household is measured by the number of members. The population size grows exponentially at $n > 0$, which is the exogenous population growth rate. Following this population structure, $L_t = L_0 e^{nt}$ is the labor supply in each country. Perfect competition prevails in the labor market of each country. Labor supplied by each member of a household is employed to produce varieties manufactured by firms or invent new ones in the R&D sector. Thus, the labor market is represented by $L_t = L_{P,t} + L_{I,t}$, where $L_{P,t}$ is the total labor employed during production, and $L_{I,t}$ is the total labor employed to invent new varieties. Considering both countries as symmetrical and labor as the numeraire, $w = 1$ holds in the present model.

In this model, lower transportation costs for exporting firms are captured as trade liberalization. Aggregate pollution is influenced by household expenditure, the ratio of local firms to exporters, and the transfer of factors of production from local firms to exporters. Aggregate consumption also depends on this transfer. Considering that this study relies on a semi-endogenous growth model, economic growth is not affected by these policies. However, they affect consumption through innovation challenges. Welfare, comprising consumption and pollution, is affected by environmental taxes and trade liberalization, altering firms' entry and average productivity.

As a result, the average productivity of the entire industry changes via these policies. These policies affect consumption and pollution through three primary effects, shaping welfare effects.

3. Theory

3.1. Household

Each household, characterized by identical preferences, lives indefinitely and maximizes discounted lifetime utility. The intertemporal utility of the representative household is:

$$U_t = \int_0^\infty e^{-(\rho-n)t} (\log M_t - \eta_D \log D_t) dt, \quad (1)$$

where M_t , D_t , $\rho > n$, and $\eta_D > 0$ represent the consumption of manufacturing goods, total emissions in the North, subjective discount rate, and weight of the utility devoted to the total emissions, respectively. Following Dixit and Stiglitz (1977), the composite of the differentiated goods takes the following C.E.S. function:

$$M_t = \left[\int_0^{m_t^e} x_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where $x_t(\omega)$ is the variety ω manufactured by the firm, m_t^e is the total number of varieties produced domestically and exported internationally by firms, and $\sigma > 1$ is the elasticity of substitution between varieties.

Solving the static optimization problem yields the following usual demand function for a representative variety:

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma}}{\int_0^{m_t^e} p_t(\omega)^{1-\sigma} d\omega} e_t, \quad (3)$$

where e_t is the household expenditure, and $p_t(\omega)$ is the price of variety ω manufactured by the firm. The aggregate price index for the varieties of manufactured goods is as follows.

$$P_t = \left(\int_0^{m_t^e} p_t(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \quad (4)$$

where $c_t \equiv e_t/P_t$ is defined as the real consumption expenditure by using (4). Taking prices and expenditure as given, dynamic optimization requires the following Euler equation: $\dot{e}_t/e_t = r_t - \rho$. Here, e_t in (3) is replaced by economy-wide household's expenditure $E_t \equiv e_t L_t$, and then, $\dot{E}_t/E_t = r_t - (\rho - n)$ holds.

3.2. R&D sector

Firms in the R&D sector employ researchers with $b_{I,t}$ units of labor to create knowledge. When $\phi < 1$ is the degree of knowledge spillover and $\lambda \in [0, 1]$ is the degree of global spillover, the knowledge spillover in (Jones, 1995) is

$$b_{I,t} = \frac{1}{(m_{L,t} + \lambda m_{F,t})^\phi} = \frac{1}{(1 + \lambda)^\phi m_t^\phi}, \quad (5)$$

which is assumed as symmetric equilibrium ($m_{L,t} = m_{F,t} = m_t$), where $m_{L,t}$, $m_{F,t}$, and m_t represent the number of varieties manufactured in the local, foreign, and world markets, respectively. Here, $\phi > 0$ implies standing-on-the-shoulders effect, while $\phi < 0$ implies fishing-out effect.

Firms in the R&D sector must create F_I units of knowledge to invent a new variety or adapt one to market-specific regulations, implying that the cost of inventing a new variety leads to $b_{I,t}F_I$. Thus, a firm must create F_L units of knowledge by paying $b_{I,t}F_L$ to sell a new variety in the local market, and F_E units of knowledge by paying $b_{I,t}F_E$ to sell a new variety in the foreign market.

The unit input requirement a associated with its production is unknown to a firm before inventing a new variety. A firm inventing it for the first time acquires a by drawing a from a probability density function $g(a)$ with support $[0, \bar{a}]$ and corresponding cumulative distribution function $G(a)$. After the firm finds out the a , it is unchanging over time to the unit input requirement of the firm associated with producing its variety. I assume the probability distribution of unit input requirements under the Pareto distribution:

$$G(a) = \int_0^a g(a) da = \left(\frac{a}{\bar{a}}\right)^k, \quad a \in [0, \bar{a}],$$

where the distribution's shape and its scale change according to k and \bar{a} .

The heterogeneity in a shapes three types of firms, according to a : non-producing, local, and exporting firms. The a_L shows the unit input requirements at which the firm is indifferent between incurring the fixed cost $b_{I,t}F_L$ of selling in the local market and immediately shutting down production. The firm drawing a under $a_{L,t} < a < \bar{a}$ becomes non-producing by exiting the local market. Further, a_E represents the unit input requirements at which the firm is indifferent between selling in the local market and incurring the additional fixed cost $b_{I,t}F_E$ to export its variety. The firm drawing a under $a_{E,t} < a < a_{L,t}$ becomes local by entering the local market while the firm drawing a under $a < a_{E,t}$ becomes exporting by entering the foreign market. In summary, the threshold values for a is $0 < a_{E,t} < a_{L,t} < \bar{a}$.

3.3. Manufacturing sector

Although firms emit pollution as a byproduct of production, they can reduce emissions by substituting labor for pollution inputs. As in Copeland and Taylor (1994), the following joint production function is assumed.

$$x_{i,t}(\omega) = \begin{cases} \left(\frac{\hat{\delta}}{a(\omega)}\right) d_{i,t}^\beta(\omega) l_{i,t}^{1-\beta}(\omega) & \text{if } d_{i,t}(\omega) < \psi l_{i,t}(\omega), \quad i = L, E \\ \left(\frac{\hat{\delta}}{a(\omega)}\right) A l_{i,t}(\omega), & \text{otherwise, } i = L, E, \end{cases}$$

where $l_{i,t}(\omega)$ and $d_{i,t}(\omega)$ represent the labor and pollution inputted by local and exporting firms manufacturing a variety ω , respectively. Following Hamaguchi (2023, 2024), $\hat{\delta} > 0$ is

the industry-wide productivity parameter and $a(\omega)$ is the firm specific productivity. Here, $\psi > 0$ is the bound on the substitution possibility between labor and pollution inputs, where $A = \psi^\beta$ holds as in Copeland and Taylor (1994). The variable cost function of local and exporting firms consists of a variable component as $C_{i,t}(\omega) = l_{i,t}(\omega) + t_i d_{i,t}(\omega)$ for $i = L, E$, where $t_i \in (0, 1)$ for $i = L, E$ states environmental tax imposed on local and exporting firms.

A firm's cost minimization problem, with technology and factor prices, yields the following variable input demand for both pollution and labor:

$$\begin{aligned} d_{i,t}(\omega) &= \left[\frac{1}{t_i} \left(\frac{\beta}{1-\beta} \right) \right]^{1-\beta} \frac{a(\omega) x_{i,t}(\omega)}{\hat{\delta}}, \quad i = L, E, \\ l_{i,t}(\omega) &= \left[\frac{1}{t_i} \left(\frac{\beta}{1-\beta} \right) \right]^{-\beta} \frac{a(\omega) x_{i,t}(\omega)}{\hat{\delta}}, \quad i = L, E, \end{aligned} \quad (6)$$

which are used to derive $C_{i,t}(\omega) = (a(\omega) x_{i,t}(\omega) / \delta_i) (t_i^\beta / [\beta^\beta (1-\beta)^{1-\beta}])$ for $i = L, E$. Note that $d_{i,t}(\omega) / x_{i,t}(\omega)$ in (6) presents the firm's specific pollution intensity and increasing environmental tax rate reduces the pollution intensity via a technology effect. Assuming $\beta^\beta (1-\beta)^{1-\beta} = \varepsilon$ for $1/2 \leq \varepsilon < 1$ and defining $\delta \equiv \varepsilon \hat{\delta}$ yields $C_{i,t}(\omega) = (t_i^\beta a(\omega) x_{i,t}(\omega)) / \delta_i$ for $i = L, E$ as in Hamaguchi (2023, 2024).

Each local firm drawing $a \leq a_{L,t}$ from the common density function $g(a)$, maximizes its profit by taking demand (3) and other firms' prices as given

$$\pi_{L,t}(\omega) = \max_{p_{L,t}(\omega)} \left(p_{L,t}(\omega) - t_L^\beta \frac{a(\omega)}{\delta} \right) x_{L,t}(\omega),$$

where $x_{L,t}(\omega)$ is the variety ω manufactured by the local firm, and $p_{L,t}(\omega)$ is the price of this variety ω . The first-order condition of profit maximization yields the following price increase over marginal cost $a(\omega)$ and environmental regulation cost t_L^β / δ :

$$p_{L,t}(\omega) = \frac{\sigma}{\sigma-1} \left(\frac{t_L^\beta}{\delta} \right) a(\omega).$$

Substituting for the price, the optimal profit of a local firm is:

$$\pi_{L,t}(\omega) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\delta p_t}{t_L^\beta a(\omega)} \right)^{\sigma-1} E_t. \quad (7)$$

Similarly, each exporting firm drawing $a \leq a_{E,t}$ from the common density function $g(a)$, maximizes their profit by taking demand (3) and other firms' prices as given

$$\pi_{E,t}(\omega) = \max_{p_{E,t}(\omega)} \left(p_{E,t}(\omega) - \tau t_E^\beta \frac{a(\omega)}{\delta} \right) x_{E,t}(\omega),$$

where $x_{E,t}(\omega)$ is the variety ω manufactured by the exporting firm and $p_{E,t}(\omega)$ is the price of this variety ω . Here, an iceberg cost $\tau > 1$ represents the variety of τ unit ships reaching their destination, as in (Samuelson, 1954). The first-order condition of profit maximization creates the following price increase over marginal cost $a(\omega)$ and environmental regulation cost t_E^β / δ :

$$p_{E,t}(\omega) = \frac{\sigma}{\sigma - 1} \left(\frac{t_E^\beta}{\delta} \right) \tau a(\omega),$$

implying that $p_E(\omega)$ is higher than $p_L(\omega)$ via trade cost and the environment tax. Substituting for the price, the optimal profit of an exporting firm is

$$\pi_{E,t}(\omega) = \frac{\theta(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\delta P_t}{t_E^\beta a(\omega)} \right)^{\sigma-1} E_t, \quad (8)$$

where $\theta \equiv (1/\tau^{\sigma-1})$ is a measure of trade liberalization, as the economy transfers from autarky to a free trade system, according to θ .

3.4. Market entry and incentives for innovation

As in [Appendix A](#), the local market condition is

$$\frac{\frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\delta P_t}{t_L^\beta a_{L,t}} \right)^{\sigma-1} E_t}{r_t - \frac{b_{L,t}}{b_{L,t}}} = b_{L,t} F_L, \quad (9)$$

as well as the following foreign market entry conditions:

$$\frac{\frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\delta P_t}{t_E^\beta a_{E,t}} \right)^{\sigma-1} E_t}{r_t - \frac{b_{L,t}}{b_{L,t}}} = \frac{b_{L,t} F_E}{\theta}. \quad (10)$$

The case of firms not exporting at all ($a_{L,t} > a_{E,t}$) holds, assuming that the costs of entering the foreign market are higher than the local market. This implies that $F_E > F_L$ is valid. Using (9) and (10) under this parameter condition yields the following productivity gap:

$$\frac{a_{L,t}}{a_{E,t}} = \left(\frac{t_E}{t_L} \right)^\beta \left(\frac{F_E}{\theta F_L} \right)^{\frac{1}{\sigma-1}} > 1, \quad \forall \theta \in (0, 1), \quad (11)$$

which implies the following. Firms enter the local market by drawing $a_{E,t} < a \leq a_{L,t}$, and enter the foreign market by drawing $a \leq a_{E,t}$. Firms cannot enter either market by drawing $a_{L,t} < a < \bar{a}$. Notably, the environmental tax does not affect the productivity gap under a uniform environmental tax of $t_L = t_E$, as in Konishi and Tarui (2015). This shows that the unilateral environmental tax has a crucial role in the distribution of firm productivity.

As in [Appendix A](#), a free entry condition equalizes the ex-ante expected discounted profits to the ex-ante expected fixed costs of developing a new variety via the following:

$$\frac{\frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} P_t^{\sigma-1} E_t \Delta_t}{r_t - \frac{b_{L,t}}{b_{L,t}}} = b_{L,t} \bar{F}_t, \quad (12)$$

where the weighted average of firms' productivity Δ_t is defined as:

$$\Delta_t \equiv \left(\frac{\delta}{t_L^\beta} \right)^{\sigma-1} \int_{a_{E,t}}^{a_{L,t}} \left(\frac{1}{a^{\sigma-1}} \right) \frac{g(a)}{G(a_{L,t})} da + \theta \left(\frac{\delta}{t_E^\beta} \right)^{\sigma-1} \int_0^{a_{E,t}} \left(\frac{1}{a^{\sigma-1}} \right) \frac{g(a)}{G(a_{L,t})} da, \quad (13)$$

and \bar{F}_t is defined as:

$$\bar{F}_t \equiv F_I \frac{1}{G(a_{L,t})} + F_L + F_E \frac{G(a_{E,t})}{G(a_{L,t})},$$

which is further rewritten using the properties of the Pareto distribution as:

$$\bar{F}_t = F_I \left(\frac{\bar{a}}{a_{L,t}} \right)^k + F_L + F_E \left(\frac{a_{E,t}}{a_{L,t}} \right)^k. \quad (14)$$

which, (14) represents the ex-ante fixed expected cost of developing various created knowledge per unit. See [Appendix A](#) for more details.

Firms in the R&D sector create new varieties by employing labor and contributing to successful innovation. The flow of new varieties is developed by the following mechanism:

$$\dot{m}_t = \frac{L_{I,t}}{b_{I,t} \bar{F}}, \quad (15)$$

where $L_{I,t} = \sum_i l_{I,it}$ represents total labor input for innovation in the economy.

3.5. Steady state

In this section, I derive equilibrium focusing on symmetric equilibrium. See [Appendix B](#) for the detailed derivation process, as only the main results are described. In the balanced growth path (BGP) equilibrium, the Euler equation implies that $r = \rho$ is valid. Moreover, (12) and (14) show that \bar{F} , a_L , and a_E become constant in BGP equilibrium when $a_{L,t} = a_L$ and $a_{E,t} = a_E$ for all t hold. There exists a unique symmetric steady-state equilibrium.

Defining $g \equiv \dot{m}_t / m_t$ as the growth rate of innovation yields

$$g \equiv \frac{\dot{m}_t}{m_t} = \frac{L_{I,t} (1 + \lambda)^\phi}{\bar{F} m_t^{1-\phi}} = \frac{n}{1 - \phi}, \quad (16)$$

where $\phi < 1$ is assumed to guarantee a positive and finite economic growth rate and the scale effect in innovation is ruled out, as in Jones (1995). The aggregate price index yield

$$P_t = \left(\frac{\sigma}{\sigma - 1} \right) \left(\frac{1}{m_t \Delta} \right)^{\frac{1}{\sigma-1}}, \quad (17)$$

where m_t is the total number of varieties developed for the country. Relative R&D difficulty is defined as

$$z_t \equiv \frac{m_t^{-\phi}}{\frac{L_t}{m_t}} = \frac{m_t^{1-\phi}}{L_t}, \quad (18)$$

which states that relative R&D difficulty is measured by a proportion of the degree of absolute R&D difficulty $m_t^{-\phi}$ to the size of market share corresponding to each variety L_t/m_t . Furthermore, logarithmic-differentiating (18) with (16) yields $\dot{z}_t/z_t = (1 - \phi)g - n = 0$, where z becomes constant in BGP equilibrium.

I derive the labor required by the economy in BGP equilibrium. The total labor input for manufacturing production in the economy is

$$L_{P,t} = \int_{a_E}^{a_L} \frac{t_L^\beta a(\omega)}{\delta} x_{L,t}(\omega) m_t \frac{g(a)}{G(a_L)} da + \int_0^{a_E} \frac{\tau t_E^\beta a(\omega)}{\delta} x_{E,t}(\omega) m_t \frac{g(a)}{G(a_L)} da, \quad (19)$$

where $L_{P,t} = \sum_i l_{P,it} = [(\sigma - 1)/\sigma]E_t$ holds. Environmental tax revenue becomes

$$H_t = \int_{a_E}^{a_L} t_L d_{L,t}(\omega) m_t \frac{g(a)}{G(a_L)} da + \int_0^{a_E} \tau t_E d_{E,t}(\omega) m_t \frac{g(a)}{G(a_L)} da,$$

where the government transfers the tax revenue to a household as lump sum.¹ Here, $H_t = [(\sigma - 1)/\sigma]E_t$ holds. Using H_t , $L_{P,t}$, and $L_t = L_{P,t} + L_{I,t}$ yields the following.

$$E_t = L_t + \frac{E_t}{\sigma} - L_{I,t}, \quad (20)$$

implying that aggregate expenditure is composed of aggregate labor income L_t , aggregate profit income E_t/σ , and the wage paid in the R&D sector $L_{I,t}$. Finally, using (5), (15), and (18), the following total labor used in the R&D sector in BGP equilibrium is derived:

$$L_{I,t} = \frac{\bar{F}zL_t}{(1 + \lambda)^\phi} g. \quad (21)$$

Considering that a firm draws a from the Pareto distribution, (13) is rewritten as the following constant Δ in BGP equilibrium:

$$\Delta = \frac{\frac{1}{a_L^{\sigma-1}}}{1 - \frac{\sigma-1}{k}} \left[\left(\frac{\delta}{t_L^\beta} \right)^{\sigma-1} + \theta \left(\frac{\delta}{t_E^\beta} \right)^{\sigma-1} \left(\frac{a_L}{a_E} \right)^{\sigma-1-k} \right],$$

which is further rewritten as

$$\Delta = \frac{\gamma}{\gamma - 1} \left(\frac{1}{a_L^{\sigma-1}} \right) \left(\frac{\delta}{t_L^\beta} \right)^{\sigma-1} (1 + \tilde{\Omega}), \text{ for } \tilde{\Omega} \equiv \left(\frac{t_L}{t_E} \right)^{\beta k} \Omega, \quad (22)$$

where (11), $\gamma \equiv k/(\sigma - 1) > 1$, $T \equiv F_E/F_L$, and $\Omega \equiv \theta^\gamma T^{1-\gamma}$ are used to derive (22). Here, $\gamma > 1$ is defined to guarantee a finite Δ , and Ω is defined as the degree of openness, which changes according to F_E and τ . Moreover, $\Omega \in [0, 1]$ holds under (11), and firms not exporting at all on $\Omega = 0$: Infinite τ and F_E/F_L lead to $\Omega = 0$, while $\tau = 1$ and $F_E = F_L$ lead to $\Omega = 1$. Here, Ω in $\tilde{\Omega}$ implies a cleansing effect of trade

¹In the BTA literature, previous studies consider that the government refunds exporting firms the environmental tax revenue of importing firms to reduce the tax burden via (Böhringer et al., 2017; Jakob et al., 2013; Keen & Kotsogiannis, 2014). However, analyzing the present model is more complicated after introducing tax refunds into the model. Thus, consider that the government imposes a lower environment tax rate on exporting firms than on local ones as BTA to simplify the analysis using the present model, following Sanctuary (2018), who considers that the government reduces the environmental tax rate imposed on exporting firms.

liberalization in Gustafsson and Segerstrom (2010), which is further enhanced by a differentiated environmental tax policy of $(t_L/t_E)^{\beta k}$. Discriminatory environmental taxes with different tax rates across sectors influence firms' entry and exit behaviors through this effect.

Here, \bar{F} is represented as

$$\bar{F} = \frac{\gamma F_L}{\gamma - 1} (1 + \tilde{\Omega}), \quad (23)$$

where \bar{F} depends on Ω . Then,

$$b_{I,t}\bar{F} = \frac{\bar{F}}{(1 + \lambda)^\phi m_t^\phi}.$$

The aggregate expenditure growing over time is composed of the sum of labor income and the dividends from the ownership of firms as follows:

$$E_t = L_t \left(1 + \frac{(\rho - n)\bar{F}z}{(1 + \lambda)^\phi} \right). \quad (24)$$

Using (11), (14), and (23), the following cut-off unit input requirement for selling in the local market in BGP equilibrium is obtained:

$$a_L = \bar{a} \left[\frac{F_I(\gamma - 1)}{F_L(1 + \tilde{\Omega})} \right]^{\frac{1}{k}}, \quad (25)$$

where a_L decreases according to an increase in $\tilde{\Omega}$ via trade liberalization. This is the cleansing effect of trade liberalization. In addition, unilateral environmental taxes have a further impact on a_L . Increased environment taxes on local firms make it more difficult for them to bear the costs of environmental regulation. At this time, the least productive firms in the local market exit the local market. Hence, a_L falls as t_L rises. However, higher environment taxes on exporting firms make them more heavily burdened with environmental regulatory costs. This implies that the burden of such costs on local firms is lighter than on exporters. At this time, less productive firms enter the local market, and a_L rises in response to a rise in t_E .

Furthermore, combining (11) with (25) yields the following cut-off unit input requirement for exporting in the foreign market in BGP equilibrium:

$$a_E = \bar{a} \left[\frac{F_I(\gamma - 1)}{F_E[(1/\tilde{\Omega}) + 1]} \right]^{\frac{1}{k}}, \quad (26)$$

where a_E increases according to an increase in $\tilde{\Omega}$ via trade liberalization). Unilateral environmental taxes have a further impact on a_E . The higher environment tax on local firms makes these firms bear a heavier burden of environmental regulatory costs. This implies that such a burden of environmental regulatory costs on exporting firms is relatively lighter than on local firms. At this time, the most productive local firms enter foreign markets, and a_E rises in response to a rise in t_L . However, environmental tax increases on exporting firms make them bear a heavier burden of environmental

regulatory costs. At this time, the least productive firms in the foreign market exit the foreign market and enter the local market. Hence, a_E falls as t_E rises.

Substituting (25) for (22) yields the following average productivity of firms:

$$\Delta = \frac{\gamma}{\gamma - 1} \left[\frac{F_L}{\bar{a}F_I(\gamma - 1)} \right]^{\frac{1}{\gamma}} \left(\frac{\delta}{t_L^\beta} \right)^{\sigma-1} (1 + \tilde{\Omega})^{1+\frac{1}{\gamma}}, \quad (27)$$

where trade and environmental policies affect the average productivity of firms indirectly through the cleansing effect of trade liberalization and directly through environment taxes on local firms.

The relative R&D difficulty in BGP equilibrium is

$$z = \frac{(\gamma - 1)(1 + \lambda)^\phi}{\gamma \hat{g} F_L (1 + \tilde{\Omega})}, \text{ for } \hat{g} \equiv (\sigma - 1)\rho + \sigma\phi g + n, \quad (28)$$

where \hat{g} is the term associated with economic growth rate and $z > 0$ holds for all $\phi < 1$ as in (Gustafsson & Segerstrom, 2010). Here, $E_t/(P_t L_t)$ is defined as a measure of productivity in the present model, following Gustafsson and Segerstrom (2010), p. I derive $M_t = C_t = c_t L_t = E_t/P_t$ to evaluate household welfare.

Substituting (17) and (24) for $c_t = E_t/(P_t L_t)$, the consumption is rewritten as

$$c_t = \left(1 + \frac{(\rho - n)\bar{F}z}{(1 + \lambda)^\phi} \right) \left(\frac{\sigma - 1}{\sigma} \right) (m_t \Delta)^{\frac{1}{\sigma-1}},$$

which is further rewritten with (23) and (28) as

$$c_t = \frac{C_t}{L_t} = \left(\frac{\sigma - 1}{\sigma} \right) \left(1 + \frac{\rho - n}{\hat{g}} \right) \left(\frac{L_t}{F_L \hat{g}} \right)^{\frac{1}{\sigma-1}} \left(\frac{\delta}{t_L^\beta a_L} \right), \quad (29)$$

where $\phi = 0$ is assumed to make comparative statics clearer in subsequent analyses.² Here, (29) implies that environment taxes influence productivity via a_L .

The pollution emitted by a firm manufacturing a variety ω for the local and foreign markets as

$$\begin{aligned} d_{L,t}(\omega) &= \frac{\delta^{\sigma-1}}{t_L^{\beta(\sigma-1)+1}} \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{1}{a(\omega)} \right)^{\sigma-1} \frac{E_t}{m_t \Delta}, \\ \tau d_{E,t}(\omega) &= \frac{\theta \delta^{\sigma-1}}{t_E^{\beta(\sigma-1)+1}} \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{1}{a(\omega)} \right)^{\sigma-1} \frac{E_t}{m_t \Delta}, \end{aligned} \quad (30)$$

Summing the total pollution emitted by firms yields total pollution in the BGP equilibrium as

$$D_t = \int_{a_E}^{a_L} d_{L,t}(\omega) m_t \frac{g(a)}{G(a_L)} da + \int_0^{a_E} \tau d_{E,t}(\omega) m_t \frac{g(a)}{G(a_L)} da,$$

²Under $\phi < 1$, consumption in (29) is a more complex equation, which makes comparative statics more complex. Thus, $\phi = 0$ is assumed to obtain clearer results and implications. This assumption implies that a standing-on-the-shoulders effect is in equilibrium with a fishing-out effect. Kruse-Andersen (2023) empirically supports the semi-endogenous

implying that total pollution in the economy increases according to the unit input requirement of a firm with a variety ω for selling in the local market and exporting in the foreign market. Previous studies argue that improved firm productivity reduces pollution emission (Kreickemeier & Richter, 2014; LaPlue, 2019; Tang et al., 2014, 2015). Thus, considering that improved productivity via reduced a leads to decreased pollution emitted by firms, and substituting (30) for the above D_t , total pollution in the economy is obtained as³

$$D_t = \left[E_t \left(\frac{\sigma - 1}{\sigma} \right) \right] \frac{1 + \frac{t_L}{t_E} \tilde{\Omega}}{t_L (1 + \tilde{\Omega})} \quad (31)$$

This is composed of the first term with scale effect and the numerator in the second term with composition effect (Copeland & Taylor, 2004; Grossman & Krueger, 1993), as well as the denominator in the second term with reallocation effect (Cherniwchan et al., 2017; Kreickemeier & Richter, 2014). The first term implies that an expanded production scale leads to increased total pollution. The numerator in the second term shows that an increase in environment tax on local firms decreases total pollution according to changes in firms' composition. Finally, the denominator in the second term shows that total pollution reduces according to the improved average firm productivity via reallocating resources within the industry from local firms with lower productivity to exporting firms with higher ones. The first term is not dependent on environment or trade policies because the R&D-based growth model is analyzed without a scale effect, demonstrating that the economic growth rate is not proportional to the level of R&D employment. Therefore, these policies have no effect on total pollution within scale effect via changes in economic growth rate. However, the second term is dependent on the environment and trade policies. An increase in the environment tax on exporting firms increases total pollution by the reallocation effect, while the policy decreases total pollution by the composition effect. By contrast, trade liberalization and an increase in environment tax on local firms decrease total pollution by reallocation effect and these policies increase total pollution by the composition effect. These results are generated by a cleansing effect of trade liberalization. Thus, environmental policies have an ambiguous impact on total pollution via these two effects.

Aggregate pollution levels emitted by firms in the local and foreign markets are defined, respectively, as

$$D_{L,t} = \int_{a_E}^{a_L} d_{L,t}(\omega) m_t \frac{g(a)}{G(a_L)} da, \quad D_{E,t} = \int_0^{a_E} \tau d_{E,t}(\omega) m_t \frac{g(a)}{G(a_L)} da,$$

implying that each aggregate pollution increases according to the unit input requirement of a firm with a variety ω for selling in the local market and exporting in the foreign one. The following aggregate pollution levels are emitted by firms in the local and foreign markets, respectively:

variety, not the full endogenous variety in this analysis, where $\phi < 1$, redefined with some parameters, also includes the case of $\phi = 0$.

³Hamaguchi (2023, 2024) discard some of the composition effects by defining aggregate pollution emissions in efficiency units, whereas in (31) all effects are taken into account.

$$D_{L,t} = \left[E_t \left(\frac{\sigma - 1}{\sigma} \right) \right] \frac{\frac{1}{t_L}}{1 + \tilde{\Omega}}, \quad D_{E,t} = \left[E_t \left(\frac{\sigma - 1}{\sigma} \right) \right] \frac{\frac{\tilde{\Omega}}{t_E}}{1 + \tilde{\Omega}}, \quad (32)$$

This is composed of the first term with scale effect, the second with composition effect, and the third with reallocation effect, as in (31). Using $D_{L,t}$ and $D_{E,t}$, the proportion of the aggregated pollution emitted by firms in the local market to that of the foreign one is obtained as follows:

$$\frac{D_E}{D_L} = \left(\frac{t_L}{t_E} \right)^{1+\beta k} \Omega, \quad (33)$$

where D_E/D_L is dependent only on the reallocation effect.

The measure of welfare in the present model is (1). Substituting (25) for (29) plus (31) for (1), the following is obtained.

$$U = \frac{1}{\rho - n} \log \left[\left(1 + \frac{\rho - n}{\hat{g}} \right) \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{1}{F_L \hat{g}} \right)^{\frac{1}{\sigma-1}} \left(\frac{\delta}{t_L^\beta} \right) \left[\frac{F_L(1 + \tilde{\Omega})}{F_I(\gamma - 1)} \right]^{\frac{1}{k}} \right] \\ - \frac{\eta_D}{\rho - n} \log \left[\left(1 + \frac{\rho - n}{\hat{g}} \right) \frac{1 + \frac{t_L}{t_E} \tilde{\Omega}}{t_L(1 + \tilde{\Omega})} \right] + \left[\frac{\sigma - \eta_D(\sigma - 1)}{(\sigma - 1)(\rho - n)} \right] L_0, \quad (34)$$

where the first term of the RHS in (34) represents the improvement in welfare due to increased consumption, the second represents the worsening of welfare due to increased pollution, and the third represents the effect of population expansion on welfare. In the semi-endogenous growth model, the economic growth rate depends only on the population growth rate and the discount rate. Hence, in the present model, trade and environmental policies cannot affect welfare through growth effects.

4. Results

4.1. Industries structure and difficulty of technology

The impact of trade environment policies on the entry and exit behavior of firms can be summarized in the following proposition.

Proposition 4.1 (Effects of policies on firms' entry and exit)

i) $t_L \uparrow$ leads to $a_L \downarrow$ and $a_E \uparrow$. ii) $t_E \uparrow$ leads to $a_L \uparrow$ and $a_E \downarrow$. iii) $\Omega \uparrow$ leads to $a_L \downarrow$ and $a_E \uparrow$.

Proof. See Appendix C.

Trade and environmental policies affect firms' entry and exit behavior through a cleansing effect of trade liberalization, represented by Ω . The $t_L \uparrow$ reduces the profit margins of local firms. The least productive local firms then exit the local market because they can no longer afford the fixed costs of innovation. Likewise, the most productive local firms enter the foreign market because their environmental policies make the regulatory burden on exporters relatively light. The $t_E \uparrow$ reduces their profits. The least productive exporters then exit the foreign market and enter the local market, as they can no longer afford the fixed costs for the foreign market. Yet, the least productive firms enter the local market because their environmental policies make the burden of

regulatory costs relatively light for local firms. As in (Melitz, 2003), trade liberalization increases the profits of exporting firms, so the most productive local firms enter foreign markets while the least productive exit the local market.

The entry and exit behavior of firms also changes the average productivity of firms. The $\Omega \uparrow$ usually improves their average productivity but may worsen it, depending on the shape of the distribution of productivity. Besides, discriminatory environment taxes also affect average productivity through the cleansing effect of trade liberalization. This result is summarized in the following proposition.

Proposition 4.2 (Effects of environmental and trade policies on productivity, innovation, and consumption)

i) $\Omega \uparrow$ leads to $\Delta \uparrow$, $z \downarrow$, and $c \uparrow$. ii) $t_E \uparrow$ leads to $\Delta \downarrow$, $z \uparrow$, and $c \downarrow$. iii) There is a U-shaped relationship between t_L and Δ , a negative relationship between t_L and z , and an inversed U-shaped relationship between t_L and c .

Proof. see [Appendix D](#).

The $\Omega \uparrow$ leads to the $\Delta \uparrow$ because productive local firms enter foreign markets while unproductive ones exit the market. However, many of the least productive firms enter the local market due to the higher environment tax on exporting firms. Therefore, the environment tax hike leads to a deterioration in average firm productivity.

Here, (27) concerning t_L , yields the following: The t_L affects the Δ through its direct effect, in addition to the cleansing effect of Ω . Due to this additional effect, this environment policy has a non-monotonic effect on Δ . Differentiating (22) with respect to t_L yields the following:

$$\frac{d\Delta}{dt_L} \gtrless 0 \Leftrightarrow t_L \gtrless \left(\frac{1}{\gamma}\right)^{\frac{1}{\beta k}} \frac{t_E}{\Omega^{\frac{1}{\beta k}}} \equiv \tilde{t}_L, \quad (35)$$

where \tilde{t}_L is the threshold for whether an environmental tax on local firms improves or worsens the average productivity of firms, $d\tilde{t}_L/dt_E > 0$ and $d\tilde{t}_L/d\Omega < 0$ holds. Under $t_L < \tilde{t}_L$, when the direct effect dominates the cleansing effect of trade liberalization, $t_L \uparrow$ worsen Δ because they encourage productive local firms more to enter foreign markets. However, when the cleansing effect of trade liberalization dominates the direct effect under $t_L \geq \tilde{t}_L$, further $t_L \uparrow$ improves it because they encourage less productive local firms to exit. Here, $t_E \uparrow$ ($\Omega \uparrow$) increase the area in which $t_L \uparrow$ worsen (improve) Δ . This is because the tax increase encourages less (the least) productive firms to enter (exit) the local market.

Effects of these policies on the relative R&D difficulty and consumption are proved in [Appendix D](#). Through a cleansing effect of trade liberalization, $t_L \uparrow$ and $\Omega \uparrow$ encourage the most productive local firms to enter foreign market. Hence, local firms have to pay fixed costs for additional innovation. As stated in (15), this increase in fixed costs reduces the speed of development of new varieties. Then, $t_L \uparrow$ and $\Omega \uparrow$ leads to $z \downarrow$ via the decrease in variety on (18). However, $t_E \uparrow$ encourages the least productive exporters to enter the local market from foreign markets. Hence, local firms do not have to pay fixed costs for additional innovation. As stated in (15), this reduction in fixed costs increases the speed of development of new varieties. Then, $t_E \uparrow$ leads to $z \downarrow$ via the increase in variety on (18). Thus, $\Omega \uparrow$ expands consumption while $t_E \uparrow$ reduces it. Additionally, $t_L \uparrow$ expands consumption on $t_E \geq t_L$ while $t_L \uparrow$ reduces it on $t_E < t_L$.

In (29), consumption increases in response to an increase in variety weighted by average productivity as $m_t \Delta_t (t_L^\beta a_L)$. Because $\Omega \uparrow$ encourages the exit of the least productive local firms, $m_t \Delta_t$ rises. This expansion in production increases consumption. However, $t_E \uparrow$ encourages the entry of unproductive firms, so $m_t \Delta_t$ declines. This contraction in production decreases consumption. The $t_L \uparrow$ affects consumption through $m_t \Delta_t$ in the same way as trade liberalization does. Differentiating (29) by t_L yields

$$\frac{dc}{dt_L} = \frac{\beta}{t_L} - \frac{\beta \tilde{\Omega}}{t_E(1 + \tilde{\Omega})} \gtrless 0,$$

where the first term on the right-hand side implies that the improvement in average productivity due to the cleansing effect of trade liberalization has the effect of increasing consumption, while the second term implies that the deterioration in average productivity due to its direct effect has the effect of reducing consumption. The threshold is obtained by rewriting $dc/dt_L \gtrless 0$ as follows.

$$1 + \Omega \left(1 - \frac{t_L}{t_E}\right) \left(\frac{t_L}{t_E}\right)^{\beta k} > 0, \quad (36)$$

for $t_E \geq t_L$ and $dc/dt_L > 0$ holds. Under the pollution havens effect, $t_L \uparrow$ leads to $c \uparrow$. Conversely, under BTA at $t_L > t_E$, the policy leads to $c \downarrow$ under the following parameter conditions:

$$1 > \Omega^{\frac{1}{k}} \left(\frac{t_L}{t_E} - 1\right)^{\frac{1}{k}} \left(\frac{t_L}{t_E}\right)^{\beta},$$

for $t_L > t_E$ and $dc/dt_L < 0$ holds.⁴

4.2. Pollution

The impact of trade and environmental policies on pollution emissions and economic welfare is investigated. Here, two types of unilateral environment tax policies are considered. On the one hand, the government in the home country imposes a higher environmental tax rate on exporting firms than local ones as it intends to prevent firms from exporting polluting goods to foreign countries with relatively loose environmental regulations. This tax policy implies that $t_E > t_L$ is associated with the pollution havens effect as in Zeng and Zhao (2009). Conversely, the home country's government imposes lower environment taxes on exporting firms as it intends to reduce the tax burden of firms exporting polluting goods in foreign countries with relatively loose environmental regulations. This policy implies that $t_E < t_L$ is associated with BTA policy, which is a border rebate for exports as in Fischer and Fox (2012). Moreover, Sanctuary (2018) considers that the government reduces the environment tax rate on exporting firms according to the environment tax rate on importing firms. Thus, in the present model, the tax rate difference between local and foreign firms on $t_E < t_L$ measures a degree of

⁴Environmental taxes on local firms have an ambiguous effect on consumption, but only monotonous effects are proven due to the difficulty of analytical proof.

BTA policy. When the government repeals the BTA policy, a uniform environment tax of $t_E = t_L$ is imposed on firms.

The impact of those policies on aggregate pollution is summarized in the following proposition.

Proposition 4.3 (Effects of environment tax and trade liberalization on pollution emissions)

i) $\Omega \uparrow$ leads to $D \uparrow$ in $t_L \geq t_E$ or the policy leads to $D \downarrow$ in $t_L < t_E$. ii) There is a U-shaped relationship between t_L and D or a U-shaped relationship between t_E and D .

Proof. See Appendix E.

The $\Omega \uparrow$ encourages the most productive local firms to enter export markets. At this time, output in the export sector increases while output in the local sector decreases. The increase in aggregate pollution associated with increased production in the export sector implies a composition effect, while the reduction in aggregate pollution associated with reduced production in the local sector implies a reallocation effect. The effect of Ω on D depends on the extent of discriminatory environment tax on local and exporting firms. In $t_L \geq t_E$, $\Omega \uparrow$ leads to $D \downarrow$ because the reallocation effect dominates the composition effect. Conversely, in $t_L < t_E$, $\Omega \uparrow$ leads to $D \uparrow$ because the composition effect dominates the reallocation effect. In $t = t_L = t_E$, $\Omega \uparrow$ does not affect D because the two effects cancel each other out. This result implies that the policy mix of environmental and trade policy plays an important role.

Differentiating () by t_L , the following is obtained.

$$\frac{dD}{dt_L} = \frac{E_t[(\sigma - 1)/\sigma]}{t_L(1 + \tilde{\Omega})^2} \left(\frac{t_L}{t_E} - 1 \right) \frac{\partial \tilde{\Omega}}{\partial t_L} \gtrless 0,$$

for $t_L \gtrless t_E$. The $t_L \uparrow$ on D depends on the composition effect and the reallocation effect. The $t_L \uparrow$ encourages the exit of the least productive local firms and the entry of the most productive local firms into export markets. The entry of new local firms then expands production in the export sector, which increases aggregate pollution through the composition effect. Conversely, the exit of the most productive local firms reduces aggregate pollution through the reallocation effect, as production in the local sector is reduced. The effect of $t_L \uparrow$ on D also depends on the extent of discriminatory environmental taxes. In $t_L < t_E$, the reallocation effect dominates the composition effect, so $t_L \uparrow$ leads to $D \downarrow$. However, in $t_L \geq t_E$, $t_L \uparrow$ leads to $D \uparrow$ because the composition effect dominates the reallocation effect. In $t_L = t_E$, $t_L \uparrow$ does not affect D because the two effects cancel each other out.

The $t_E \uparrow$ has similar effects similar to that on local firms. Differentiating (31) by t_E yields

$$\frac{dD}{dt_E} = \frac{E_t[(\sigma - 1)/\sigma]}{t_L(1 + \tilde{\Omega})^2} \left(\frac{t_L}{t_E} - 1 \right) \frac{\partial \tilde{\Omega}}{\partial t_E} \gtrless 0,$$

$$t_L \gtrless t_E.$$

The impact of $t_E \uparrow$ on D also depends on the composition and reallocation effects. The $t_E \uparrow$ encourages the least productive exporters and firms to enter the

local market. The exit of unproductive exporters then reduces aggregate pollution through the composition effect, as production in the export sector is reduced. However, the entry of unproductive firms into the local market increases aggregate pollution through the reallocation effect, as production in the local sector expands. The effect of $t_E \uparrow$ on D also depends on the extent of discriminatory environmental taxes. In $t_L \geq t_E$, $t_E \uparrow$ leads to $D \downarrow$ because the composition effect dominates the reallocation effect. In $t_L < t_E$, the reallocation effect dominates the composition effect, so $t_E \uparrow$ leads to $D \uparrow$. In $t_L = t_E$, $t_E \uparrow$ does not affect D because the two effects cancel each other out.

Here, the threshold for the impact of $t_L \uparrow$ on Δ and D can be aggregated to a single threshold as $t_E \gtrless \tilde{t}_L$. (35) and $t_L \gtrless t_E$, this threshold can be rewritten as

$$\Omega \gtrless \frac{1}{\gamma} \equiv \bar{\Omega},$$

where $\bar{\Omega}$ implies a threshold for the effect of $t_L \uparrow$ on Δ and D . The results are summarized in the following propositions.

Proposition 4.4 (Relationship between firms' average productivity and total pollution emissions)

i) Under $\tilde{t}_L \geq t_E$, $t_L \uparrow$ leads to a trade-off between firms' average productivity and total pollution such that $\Delta \downarrow$, $D \downarrow$ in $t_E > t_L$ or $\Delta \uparrow$, $D \uparrow$ in $t_L > \tilde{t}_L$, a lose-lose relationship between them such that $\Delta \downarrow$, $D \uparrow$ in $\tilde{t}_L > t_L > t_E$. ii) under $\tilde{t}_L < t_E$, $t_L \uparrow$ leads to a trade-off between firms' average productivity and total pollution such that $\Delta \downarrow$, $D \downarrow$ in $\tilde{t}_L > t_L$ or $\Delta \uparrow$, $D \uparrow$ in $t_L > t_E$, a win-win relationship between them such that $\Delta \uparrow$, $D \downarrow$ in $t_E > t_L > \tilde{t}_L$.

Under $\tilde{t}_L \geq t_E$, it is difficult to both improve productivity and reduce pollution emissions through environmental tax increases for local firms. In particular, under $\tilde{t}_L > t_L > t_E$, the higher environmental tax will increase pollution emissions through lower average productivity. Here, trade liberalization changes $\bar{\Omega} \geq \Omega$ to $\Omega > \bar{\Omega}$, thereby changing $\tilde{t}_L \geq t_E$ to $\tilde{t}_L < t_E$. Environmental tax cuts for exporting companies also change $\tilde{t}_L \geq t_E$ to $\tilde{t}_L < t_E$. At this time, environmental tax increases for local firms can both improve productivity and reduce pollution emissions.

4.3. Welfare

Environment policy influences welfare via consumption and total pollution. The welfare effect becomes monotonous under a specific region of environmental tax rate. This result is summarised as follows.

Proposition 4.5 (Effects of environmental tax on welfare)

i) Under $t_L \geq t_E$, $t_E \uparrow$ worsens welfare. ii) $t_L \uparrow$ improves welfare under $t_E \geq t_L$, while $t_L \uparrow$ worsens welfare under $t_E < t_L$.

Differentiating (34) with respect to t_E results in

$$\frac{dU}{dt_E} = \frac{1}{(\rho - n)(1 + \tilde{\Omega})} \left[\frac{1}{k} + \eta_D \left(1 - \frac{(1 + \tilde{\Omega})}{1 + (t_L/t_E)\tilde{\Omega}} \right) \right] \frac{\partial \tilde{\Omega}}{\partial t_E} < 0,$$

for $t_L \geq t_E$ because $1 > [(1 + \tilde{\Omega})]/[1 + (t_L/t_E)\tilde{\Omega}]$ holds under $t_L \geq t_E$. The $t_E \uparrow$ worsens welfare because they reduce consumption based on expenditure. Furthermore, $t_E \uparrow$ leads

to $D \downarrow$ through the composition effect, while $t_E \uparrow$ leads to $D \uparrow$ through the reallocation effect. Therefore, the welfare effect of $t_E \uparrow$ through aggregate pollution is ambiguous. Under $t_L \geq t_E$, the welfare deterioration through the reallocation effect dominates the welfare improvement through the composition effect. The $t_E \uparrow$ results in worsening welfare.

I now rewrite (1) as:

$$U = \frac{1}{\rho - n} \log \hat{C} - \frac{\eta_D}{\rho - n} \log \hat{D} + \left[\frac{\sigma - \eta_D(\sigma - 1)}{(\sigma - 1)(\rho - n)} \right] L_0,$$

where \hat{C} is the term on consumption and \hat{D} is the term on pollution. Differentiating this rewritten welfare for t_L , the following is obtained.

$$\frac{dU}{dt_L} = \frac{1}{\rho - n} \frac{\partial \hat{C}}{\partial t_L} - \frac{\eta_D}{\rho - n} \frac{\partial \hat{D}}{\partial t_L},$$

where the first bracket on the right-hand side represents the welfare effects of t_L through consumption, while the second bracket represents the welfare effects of t_L through pollution.⁵ Under the pollution havens effect at $t_E \geq t_L$, when $t_L \uparrow$ leads to $\Delta \uparrow$, the reallocation effect leads to $c \uparrow$ and $D \downarrow$. The policy results in improving welfare in $dU/dt_L > 0$. By contrast, under BTA at $t_E < t_L$, when $t_L \uparrow$ leads to $\Delta \downarrow$, the composition effect leads to $c \downarrow$ and $D \uparrow$. The policy results in worsening welfare in $dU/dt_L < 0$.

Trade policy has ambiguous effects on welfare via c and D . Differentiating (34) with respect to Ω yields

$$\frac{dU}{d\Omega} = \frac{1}{(\rho - n)(1 + \tilde{\Omega})} \left[\left(\frac{1}{k} + \eta_D \right) - \frac{\eta_D(1 + \tilde{\Omega})}{1 + (t_L/t_E)\tilde{\Omega}} \right] \frac{\partial \tilde{\Omega}}{\partial \Omega} \gtrless 0,$$

where the welfare effects of $\Omega \uparrow$ depend on the cleansing effect of trade liberalization on c and D . The first term on its right-hand side represents the welfare improvement from $c \uparrow$ due to $\Omega \uparrow$ at $(1/k)$ and $D \downarrow$ due to the reallocation effect at η_D , while the second term represents the welfare deterioration from $D \uparrow$ due to the composition effect in η_D . The threshold for whether trade liberalization improves or worsens welfare can be obtained by rewriting its $dU/d\Omega$ as

$$\frac{dU}{d\Omega} \gtrless 0 \Leftrightarrow \hat{\Omega} \equiv \left(\frac{t_E}{t_L} \right)^{\beta k} \left[\frac{\frac{1}{k\eta_D} - 1}{1 - \frac{t_L}{k\eta_D t_E}} \right] \gtrless \Omega,$$

where $\hat{\Omega} > 0$ holds for $(t_L/t_E) < k\eta_D < 1$ with $t_E > t_L$ or $1 < k\eta_D < (t_L/t_E)$ with $t_E < t_L$. Then, there is an inversed U-shape relationship between trade liberalization and welfare. This result is summarised as follows.

Proposition 4.6 (Effects of trade liberalization on welfare)

Under specified parameter conditions, there is an inversed U-shaped relationship between Ω and U via $c \uparrow$ and $D \uparrow$.

Here, $\hat{\Omega}$ decreases with δ_E , k and η_D , while it increases with δ_L . A larger k implies that there are many firms in the economy with lower a , due to the narrower hem of the

⁵Environmental taxes on local firms may have a non-monotonic effect on welfare, but due to the difficulty of analytical proof, only monotonic effects are shown.

distribution on productivity. At this time, productivity in the export sector as a whole improves, leading to $\hat{\Omega} \downarrow$ in an economy where consumers are more sensitive to negative externalities due to pollution. This decline implies an expansion of the area of welfare deteriorating effects of Ω . By contrast, an improvement in local sector productivity leads to $\hat{\Omega} \uparrow$, implying that the area of welfare-improving effects of trade liberalization expands. Based on the discussion so far, under $(t_L/t_E) < k\eta_D < 1$ with $t_E > t_L$, $d\Delta/d\Omega > 0$ for $k > \sigma - 1$, $dD/d\Omega < 0$, $dU/d\Omega \geq 0$ for $\hat{\Omega} \geq \Omega$ holds. Therefore, under $k > \sigma - 1$ and $\hat{\Omega} > \Omega$, trade liberalization can further reduce polluting emissions through improved average firm productivity, and consequently improve welfare.

Finally, I discuss the case where the government abolishes a unilateral environmental tax and adopts a uniform one with $t = t_E = t_L$. In this case, the uniform tax does not affect a_L/a_E and therefore does not affect the entry and exit behavior of firms (See (11)). In (22), $\tilde{\Omega} = (\delta_E/\delta_L)^k \Omega$, so the cleaning effect of a uniform environmental tax on trade liberalization disappears. In other words, $t \uparrow$ ceases to affect the fixed costs of R&D, a relative R&D difficulty, and household expenditure. However, due to the direct effect of $t \uparrow$ on Δ , $t = t_E = t_L \uparrow$ leads to $\Delta \downarrow$. This output contraction due to $\Delta \downarrow$ leads to $c \downarrow$ and $D \downarrow$. The welfare effects of a uniform environmental tax and its trade liberalization at $t = t_E = t_L$ can be obtained by differentiating (34) concerning t as follows:

$$\frac{dU}{dt} = \frac{1}{t} \left(\frac{\eta_D - \beta}{\rho - n} \right) \geq 0, \quad \frac{dU}{d\tilde{\Omega}} = \frac{1}{k(\rho - n)(1 + \tilde{\Omega})} \frac{\partial \tilde{\Omega}}{\partial \Omega} > 0,$$

where the η_D in the first term of the right-hand side is the welfare-improving effect of $D \downarrow$, while the β in its second term is the welfare-worsening effect of $c \downarrow$. For $\eta_D \geq \beta$, $t \uparrow$ monotonically improves welfare due to $D \downarrow$, while for $\eta_D < \beta$, $t \uparrow$ monotonically worsens welfare due to $c \downarrow$. Trade liberalization does not affect this welfare effect; only the economic structure, which is composed of preference for pollution and the labor pollution ratio of firms, determines the welfare effect. In addition, under $t = t_E = t_L$, the composition and reallocation effects on D cancel out and the pollution-reducing effects of $\Omega \uparrow$ disappear. At this time, trade liberalization monotonically improves welfare through increased consumption.

Under the BTA policy of $t_L \geq t_E$ and a unilateral environmental tax of $t = t_E = t_L$, $t_E \uparrow$ and $t_L \uparrow$ monotonically worsen welfare in certain parameter conditions. Hence, in an economy with a high preference for pollution emissions in $\eta_D \geq \beta$, welfare can be improved by abolishing the BTA policy and introducing a uniform environmental tax, though at the cost of losing the pollution-reducing effects of trade policy. However, when the pollution haven effect of $t_L < t_E$ with a unilateral environmental tax, welfare can be monotonically improved by $t \uparrow$. Hence, in $\eta_D < \beta$, in economies with a low preference for pollution emissions, welfare can be improved by introducing a BTA policy as a unilateral environmental tax. At this time, $\Omega \uparrow$ has a non-monotonic effect on welfare, and environment policy can affect its threshold. However, if a unilateral environmental tax of $t = t_E = t_L$ is introduced, the impact of t on the threshold disappears, and only through changes in the structure of the economy can the welfare-improving effects of $\Omega \uparrow$ be made viable. In this case, introducing a unilateral environmental tax and adjusting its tax rate would improve welfare through trade policy.

This study's robustness is briefly confirmed by comparing it to the analyses of Hamaguchi (2023) and Hamaguchi (2024) regarding the endogenization of environmental policies and a full endogenous growth model. The observed improvements in average productivity due to trade liberalization and the non-monotonic relationship between environmental taxes and productivity are consistent across all analyses, indicating robust policy effects on average productivity. However, this study diverges from Hamaguchi (2023) and Hamaguchi (2024) regarding the effect of trade liberalization on aggregate pollution, which varies depending on the environmental tax rate. This discrepancy may stem from discarding some composition effects in both cases to assess aggregate pollution of the efficiency units. Moreover, in Hamaguchi (2024), emission quotas constrain pollution in the export sector, thus reinforcing the composition effects of trade liberalization on exports. Therefore, the definition of the pollution emission function is critically crucial in analyzing the effects of trade and environmental policies on aggregate pollution.

In Hamaguchi (2023), only the non-monotonicity of welfare effects arising from trade and environmental policies is discussed, as the growth effects of the policies complicate welfare analysis. Conversely, in Hamaguchi (2024), the welfare analysis is simplified capping aggregate pollution through emission quotas, revealing the non-monotonic welfare effects of environmental policy and the welfare-enhancing effects of trade liberalization under dual regulation. This contrasts with this study's findings, where the welfare effects of environmental policy depend on tax rates and exhibit non-monotonic trends with trade liberalization. Hence, the endogenization of trade environmental policy alters its welfare effects. In an endogenous growth model with heterogeneous firms, it is essential to consider the origin of differences in pollution changes and welfare effects.

5. Conclusion

Employing an R&D-type growth model with heterogeneous firms, this study analyzes the impact of BTA policies on the environment and trade dynamics through industrial structures. The interaction between trade liberalization and environmental taxes on local firms and exporters yields various effects on productivity and pollution, leading to a complex trade-off. Environmental taxes generally diminish welfare, but under the BTA, welfare improves only with increased environmental taxes on exporters, with the welfare effects of trade liberalization contingent on these tax rates. However, abolishing the BTA policy enhances welfare through trade liberalization and environmental tax adjustments, influenced by the economic structure. These results have several policy implications.

This study recommends policymakers to adopt a balanced mix of environmental and trade policies for sustainable development. While trade liberalization improves innovation and industry productivity, its impact on pollution varies depending on the environmental tax rates. Under the BTA, trade liberalization increases pollution, leading to decreased welfare with excessive liberalization. However, by increasing environmental taxes on local firms, trade liberalization can transform the relationship between productivity and pollution from negative to positive. Abolishing the BTA policy would render trade liberalization neutral on pollution, resulting in continuous welfare improvement.

Therefore, governments promoting trade liberalization should eliminate the BTA and introduce uniform environmental tax rates to optimize welfare outcomes.

This environmental tax hike may not improve welfare unless the country prioritizes environmental concerns, demonstrating a high preference for pollution and a low ratio of polluting inputs to production. Conversely, lowering tax rates for exporters, rather than local firms, can trigger a pollution haven effect. While trade liberalization can mitigate pollution, excessive liberalization diminishes welfare. However, if local firms' overall productivity improves, liberalization is more likely to improve welfare. Moreover, abolishing the BTA can transform the loose-loose relationship between productivity and pollution into a win-win relationship. However, while this enhances welfare, it may also lead to leakage to other countries through expanded export sectors. Trading powers should weigh the pros and cons of BTA policies. Conversely, less developed countries aiming to become trading nations may enhance their welfare through moderate trade liberalization. Continuous international policy discussions on BTAs remain imperative.

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Notes on contributor

Yoshihiro Hamaguchi (Ph.d) is a junior associate professor at the Department of Economics, the Faculty of Economics and Business Administration, Kyoto University of Advanced Economics. He has some teaching experience such as a research assistant at the Faculty of Management, Osaka Seikei University from 2017 to 2020 and a Full-time lecturer at Kyoto College of Economics from 2020 to 2023. His research interests include macroeconomics, environmental economics, political economy, international economics, and tourism economics. His research focuses on the mechanisms and policy instruments of sustainable development based on the theory of endogenous economic growth. The results of his research have been published in refereed international journals such as *Transport Policy*, *Economic Policy and Analysis*, and the *Journal of Macroeconomics*. Additionally, he has received awards for the number of citations he received.

ORCID

Yoshihiro Hamaguchi  <http://orcid.org/0000-0001-7068-0759>

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Appendix A. Market entry and incentives for innovation

This section provides more detail on decision-making concerning innovation.

A.1. No arbitrage condition

Local and exporting firms' profits determine whether they enter local or foreign markets. The value of a firm with either of the two threshold values $a = a_{i,t}$ for $i = L, E$ on the stock market is represented as the present discounted value of all future profits associated with draw $a = a_{i,t}$:

$V_{i,t}(a_{i,t}) = \int_t^\infty e^{-r(o-t)} \pi(a_{i,o}) do$, where r_t state the risk-free market interest rate. Taking the time derivative of the firm value yields the following no-arbitrage condition: $\pi_{i,t}(a_{i,t})dt + \dot{V}_{i,t}(a_{i,t})dt = r_t V_{i,t}(a_{i,t})dt$ for $i = L, E$, which is rewritten as follows:

$$V_{i,t}(a_{i,t}) = \frac{\pi_{i,t}(a_{i,t})}{r_t - \frac{\dot{V}_{i,t}(a_{i,t})}{V_{i,t}(a_{i,t})}}, \quad i = L, E,$$

assuming that firms escape from the exogenous death rate, as in (Gustafsson & Segerstrom, 2010). The firms obtaining threshold values $a_{L,t}$ and $a_{E,t}$ by drawing $a = a_{i,t}$ for $i = L, E$, become indifferent between entering and exiting the local and foreign markets, respectively. This implies that the costs of entering the market are equal to the benefits of entering it, as in: $V_{i,t}(a_{i,t}) = b_{i,t}F_i$ for $i = L, E$. Using these two equations, (7), and (8), the local and foreign markets entry conditions are derived as (9) and (10) respectively.

A.2. Incentives for innovation

Local and foreign market entry conditions determine the incentives to invent new products. Perfect competition prevails in the R&D sector, so that any firm freely entering the market can develop a new variety. This shows that the ex-ante expected benefit of developing a new product is equal to the cost of developing, represented by the following equation.

$$\int_{a_{E,t}}^{a_{L,t}} \left[\frac{\pi_{L,t}(a)}{r_t - \frac{\dot{b}_{L,t}}{b_{L,t}}} - b_{L,t}F_L \right] dG(a) + \int_0^{a_{E,t}} \left[\frac{\pi_{E,t}(a)}{r_t - \frac{\dot{b}_{E,t}}{b_{E,t}}} - b_{E,t}F_E \right] dG(a) = b_{L,t}F_L,$$

where $G(a)$ represents the Pareto cumulative distribution function, and a potential entering firm draws a from $G(a)$. Using this equation, (7), and (8), a free entry condition equalises ex-ante expected discounted profits to ex-ante expected fixed costs of developing a new variety as in (12).

A.3. Interpretation of (14)

Here, (14) represents the ex-ante fixed expected cost of developing various created knowledge per unit. Firms in the manufacturing sector that want to enter the market pay fixed costs for innovation to firms in the R&D sector. At this time, the firms still do not observe their own productivity. Therefore, the firm does not know whether the variety it purchases from the R&D sector firms will generate sufficient profits. This implies that this fixed cost is an-ex-ante expected cost. A firm pays an expected cost per unit of new knowledge of $b_L F_L$ and acquires a new variety. It then observes that its productivity is a in $a_{L,t} < a < \bar{a}$. It is unable to pay its fixed costs and therefore exits its local market and abandons the acquisition of a new variety; $1/G(a_{L,t}) = (\bar{a}/a_{L,t})^k$ is the number of trails for developing a profitable variety. Another firm seeking to enter the local market pays the expected cost for the local market per $b_L F_L$ unit of

new knowledge and acquires the new variety needed to enter the local market. The firm then observes that its productivity is a in $a_{E,t} < a < a_{L,t}$. It pays the fixed costs and enters the local market. A firm that observes that its productivity is a at $a < a_{E,t}$ pays an additional cost of $b_I F_E$ units, which is the expected cost for the foreign market per new knowledge. It then enters the foreign market from the local market but gains enough profit to pay the additional fixed cost: $(a_{E,t}/a_{L,t})^k = G(a_{E,t})/G(a_{L,t})$ is the likelihood of a firm developing a variety profitable enough to export after entering the local market.

Appendix B. Derivations of steady state

This section details the derivations of the steady state. I define $g \equiv \dot{m}_t/m_t$ as the growth rate of innovation. Substituting (15) for (5) yields (16).

Notably, the constant a_L and a_E lead to $\Delta_t = \Delta$ for all t in (13). Moreover, (4) is rewritten as follows:

$$P_t^{1-\sigma} = \int_0^{m_t^e} p(\omega)^{1-\sigma} d\omega = \int_{a_E}^{a_L} p_L(a)^{1-\sigma} m_{L,t} \frac{g(a)}{G(a_L)} da + \int_0^{a_E} p_E(a)^{1-\sigma} m_{E,t} \frac{g(a)}{G(a_L)} da,$$

where $m_{L,t}$ and $m_{E,t}$ are the number of locally and foreign developed varieties, respectively, and $g(a)/G(a_L)$ states the steady state density function conditional on market entry. Substituting the price margins $p_L(a)$ and $p_E(a)$ for the aggregate price index, the aggregate price index is rewritten as (17).

In BGP equilibrium, the household's expenditure becomes constant as $r = \rho$ holds in the Euler equation. Logarithmic differentiating (5) with respect to time leads to $\dot{b}_{I,t}/b_{I,t} = -\phi g$, indicating the capital gain. Substituting (5) for (9) yields

$$\frac{\left(\frac{\delta}{t_{L,t}^\beta a_L}\right)^{\sigma-1} \frac{E_t}{\Delta\sigma}}{r + \phi g} = \frac{F_L z L_t}{(1 + \lambda)^\phi}, \quad (\text{B1})$$

and substituting (5) for (10) yields

$$\frac{\left(\frac{\delta}{t_{E,t}^\beta a_E}\right)^{\sigma-1} \frac{E_t}{\Delta\sigma}}{r + \phi g} = \frac{F_E z L_t}{\theta(1 + \lambda)^\phi}, \quad (\text{B2})$$

where the LHS of (37) and (38) represent the discounted benefits of a firm with $a = a_L$ and $a = a_E$, and the RHS shows the entry cost of local or foreign markets, respectively. Using (5), (17), (18), and $\dot{b}_{I,t}/b_{I,t} = -\phi g$, I rewrite (12) as the following incentives for inventing new varieties in BGP equilibrium:

$$\frac{\frac{E_t}{\sigma}}{r + \phi g} = \frac{\bar{F} z L_t}{(1 + \lambda)^\phi}, \quad (\text{B3})$$

indicating that the discounted benefits of inventing a profitable variety in the LHS of (39) is equal to the expected costs of invention and market entry in the RHS of (39).

A firm with a variety ω employs $(t_{L,t}^\beta/\delta)a(\omega)x_{L,t}(\omega)$ units of labor to sell the product in the local market. Substituting (17) and $p_L(\omega)$ for (3), yields

$$\frac{t_L^\beta}{\delta} a(\omega) x_{L,t}(\omega) = \left(\frac{\sigma-1}{\sigma} \right) \left(\frac{\delta}{t_L^\beta a(\omega)} \right)^{\sigma-1} \frac{E_t}{m_t \Delta}. \quad (\text{B4})$$

Additionally, a firm with a variety ω employs $(t_E^\beta/\delta) \tau a(\omega) x_{E,t}(\omega)$ units of labor to export the variety in the foreign market. Substituting (17) and $p_E(\omega)$ for (3) creates

$$\frac{t_E^\beta}{\delta} \tau a(\omega) x_{E,t}(\omega) = \theta \left(\frac{\sigma-1}{\sigma} \right) \left(\frac{\delta}{t_E^\beta a(\omega)} \right)^{\sigma-1} \frac{E_t}{m_t \Delta}. \quad (\text{B5})$$

Summing the labor employed by firms yields the total labor used in the manufacturing sector in BGP equilibrium as (19).

Using (13), (7), and (8) yields the following aggregate profit income.

$$\int_{a_E}^{a_L} \pi_L m_t \frac{g(a)}{G(a_L)} da + \int_0^{a_E} \pi_E m_t \frac{g(a)}{G(a_L)} da = \frac{E_t}{\sigma}.$$

Substituting (16) and E_t/σ in (39) for (20) yields the aggregate expenditure in BGP equilibrium as (24). Using (39), (23), and (24) yields the relative R&D difficulty in BGP equilibrium as (28).

Appendix C. Proof of proposition 4.1

Differentiating $\tilde{\Omega}$ in (22) with respect to t_L , t_E , and Ω yields

$$\frac{d\tilde{\Omega}}{dt_L} = \frac{\beta k \tilde{\Omega}}{t_L} > 0, \quad \frac{d\tilde{\Omega}}{dt_E} = -\frac{\beta k \tilde{\Omega}}{t_E} < 0, \quad \frac{d\tilde{\Omega}}{d\Omega} = \frac{\tilde{\Omega}}{\Omega} > 0, \quad (\text{C1})$$

which are frequently used within this appendix for other calculations. Differentiating a_L in (25) with respect to t_L , t_E , and Ω yields

$$\frac{da_L}{dt_L} = -\frac{a_L}{k(1+\tilde{\Omega})} \frac{\partial \tilde{\Omega}}{\partial t_L} < 0, \quad \frac{da_L}{dt_E} = \frac{a_L}{k(1+\tilde{\Omega})} \frac{\partial \tilde{\Omega}}{\partial t_E} > 0, \quad \frac{da_L}{d\Omega} = -\frac{a_L}{k(1+\tilde{\Omega})} \frac{\partial \tilde{\Omega}}{\partial \Omega} < 0, \quad (\text{C2})$$

which (42) are used to derive. Differentiating a_L in (26) with respect to t_L , t_E , and Ω yields

$$\begin{aligned} \frac{da_E}{dt_L} &= \frac{a_E}{k} \left[\frac{1/\tilde{\Omega}^2}{(1/\tilde{\Omega}) + 1} \right] \frac{\partial \tilde{\Omega}}{\partial t_L} > 0, \\ \frac{da_E}{dt_E} &= \frac{a_E}{k} \left[\frac{1/\tilde{\Omega}^2}{(1/\tilde{\Omega}) + 1} \right] \frac{\partial \tilde{\Omega}}{\partial t_E} < 0, \quad \frac{da_E}{d\Omega} = \frac{a_E}{k} \left[\frac{1/\tilde{\Omega}^2}{(1/\tilde{\Omega}) + 1} \right] \frac{\partial \tilde{\Omega}}{\partial \Omega} > 0, \end{aligned} \quad (\text{C3})$$

which (C1) are used to derive.

Appendix D. Proof of proposition 4.2

Differentiating (27) with respect to t_L , t_E , and Ω yields

$$\frac{d\Delta}{d\Omega} = \left(1 + \frac{1}{\gamma} \right) \left(\frac{\Delta}{1+\tilde{\Omega}} \right) \frac{\partial \tilde{\Omega}}{\partial \Omega} > 0, \quad \frac{d\Delta}{dt_E} = \left(1 + \frac{1}{\gamma} \right) \left(\frac{\Delta}{1+\tilde{\Omega}} \right) \frac{\partial \tilde{\Omega}}{\partial t_E} < 0,$$

and

$$\frac{d\Delta}{dt_L} = -\frac{\beta\Delta(\sigma-1)}{t_L} + \left(1 + \frac{1}{\gamma}\right) \left(\frac{\Delta}{1+\tilde{\Omega}}\right) \frac{\partial\tilde{\Omega}}{\partial t_L},$$

which is rewritten as

$$\frac{d\Delta}{dt_L} \gtrless 0 \Leftrightarrow \left(1 + \frac{1}{\gamma}\right) \left(\frac{\Delta}{1+\tilde{\Omega}}\right) \frac{\beta k \tilde{\Omega}}{t_L} \gtrless \frac{\beta\Delta(\sigma-1)}{t_L},$$

which is further rewritten as

$$t_L \gtrless \left(\frac{1}{\gamma}\right)^{\frac{1}{\beta k}} \frac{t_E}{\tilde{\Omega}^{\frac{1}{\beta k}}} \equiv \tilde{t}_L, \quad (D1)$$

where $d\tilde{t}_L/dt_E > 0$ and $d\tilde{t}_L/d\Omega < 0$ holds.

Differentiating (23) and (28) with respect to t_L , t_E , and Ω yields

$$\frac{dz}{dt_L} = -\frac{z}{1+\tilde{\Omega}} \frac{\partial\tilde{\Omega}}{\partial t_L} < 0, \quad \frac{dz}{dt_E} = -\frac{z}{1+\tilde{\Omega}} \frac{\partial\tilde{\Omega}}{\partial t_E} > 0, \quad \frac{dz}{d\Omega} = -\frac{z}{1+\tilde{\Omega}} \frac{\partial\tilde{\Omega}}{\partial \Omega} < 0,$$

and

$$\frac{d\bar{F}}{dt_E} = \frac{\gamma F_L}{\gamma-1} \frac{\partial\tilde{\Omega}}{\partial t_E} < 0, \quad \frac{d\bar{F}}{dt_L} = \frac{\gamma F_L}{\gamma-1} \frac{\partial\tilde{\Omega}}{\partial t_L} > 0, \quad \frac{d\bar{F}}{d\Omega} = \frac{\gamma F_L}{\gamma-1} \frac{\partial\tilde{\Omega}}{\partial \Omega} > 0,$$

which (C1) are used to derive.

Differentiating (29) with respect to t_L , t_E , and Ω yields

$$\frac{dc}{dt_L} = -\frac{\beta c}{t_L} - \frac{c}{a_L} \frac{\partial a_L}{\partial t_L} \gtrless 0, \quad \frac{dc}{dt_E} = -\frac{c}{a_L} \frac{\partial a_L}{\partial t_E} < 0, \quad \frac{dc}{d\Omega} = -\frac{c}{a_L} \frac{\partial a_L}{\partial \Omega} > 0,$$

which (C2) are used to derive.

Appendix E. Proof of propositions 4.3

Differentiating (31) by Ω , I obtain

$$\frac{dD}{d\Omega} = \frac{E_t[(\sigma-1)/\sigma]}{t_L(1+\tilde{\Omega})^2} \left(\frac{t_L}{t_E} - 1\right) \frac{\partial\tilde{\Omega}}{\partial \Omega} \gtrless 0,$$

$$t_L \gtrless t_E.$$

Differentiating (31) by t_L , I obtain

$$\frac{dD}{dt_L} = \frac{E_t[(\sigma-1)/\sigma]}{t_L(1+\tilde{\Omega})^2} \left(\frac{t_L}{t_E} - 1\right) \frac{\partial\tilde{\Omega}}{\partial t_L} \gtrless 0,$$

$$t_L \gtrless t_E.$$

Differentiating (31) by t_E , I obtain

$$\frac{dD}{dt_E} = \frac{E_t[(\sigma-1)/\sigma]}{t_L(1+\tilde{\Omega})^2} \left(\frac{t_L}{t_E} - 1\right) \frac{\partial\tilde{\Omega}}{\partial t_E} \gtrless 0,$$

$$t_L \gtrless t_E.$$

Differentiating (32) with respect to t_L , t_E , and Ω , the following is obtained:

$$\begin{aligned} \frac{dD_L}{dt_L} &= -\frac{D_L}{t_L} - \frac{D_L}{1 + \tilde{\Omega}} \frac{\partial \tilde{\Omega}}{\partial t_L} < 0, \quad \frac{dD_L}{dt_E} = -\frac{D_L}{1 + \tilde{\Omega}} \frac{\partial \tilde{\Omega}}{\partial t_E} > 0, \quad \frac{dD_L}{d\Omega} = -\frac{D_L}{1 + \tilde{\Omega}} \frac{\partial \tilde{\Omega}}{\partial \Omega} > 0, \\ \frac{dD_E}{dt_L} &= \frac{D_E}{\tilde{\Omega}^2 [(1 + \tilde{\Omega})]} \frac{\partial \tilde{\Omega}}{\partial t_L} > 0, \quad \frac{dD_E}{dt_E} = \frac{D_E}{\tilde{\Omega}^2 [(1 + \tilde{\Omega})]} \frac{\partial \tilde{\Omega}}{\partial t_E} - \frac{D_E}{t_E} < 0, \quad \frac{dD_E}{d\Omega} = \frac{D_E}{\tilde{\Omega}^2 [(1 + \tilde{\Omega})]} \frac{\partial \tilde{\Omega}}{\partial \Omega} > 0, \end{aligned}$$

which (C1) are used to derive. Differentiating (33) with respect to t_L , t_E , and Ω , the following is obtained.

$$\begin{aligned} \frac{d}{dt_L} \left(\frac{D_{E,t}}{D_{L,t}} \right) &= \frac{1 + \beta k}{t_L} \left(\frac{D_{E,t}}{D_{L,t}} \right) > 0, \quad \frac{d}{dt_E} \left(\frac{D_{E,t}}{D_{L,t}} \right) = -\frac{1 + \beta k}{t_L} \left(\frac{D_{E,t}}{D_{L,t}} \right) < 0, \\ \frac{d}{d\Omega} \left(\frac{D_{E,t}}{D_{L,t}} \right) &= \left(\frac{\delta_E}{\delta_L} \right) > 0. \end{aligned}$$