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Article

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Journal of Applied Economics

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University of CEMA, Buenos Aires

Suggested Citation: Tan, Yingxian; Wang, Yahui (2023) : Private investment and public stimulus: A bargaining model, Journal of Applied Economics, ISSN 1667-6726, Taylor & Francis, Abingdon, Vol. 26, Iss. 1, pp. 1-19,
<https://doi.org/10.1080/15140326.2022.2160897>

This Version is available at:

<https://hdl.handle.net/10419/314214>

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To cite this article: Yingxian Tan & Yahui Wang (2023) Private investment and public stimulus: a bargaining model, *Journal of Applied Economics*, 26:1, 2160897, DOI: [10.1080/15140326.2022.2160897](https://doi.org/10.1080/15140326.2022.2160897)

To link to this article: <https://doi.org/10.1080/15140326.2022.2160897>



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Published online: 12 Jan 2023.



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Private investment and public stimulus: a bargaining model

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ABSTRACT

Local governments often provide tax-subsidy programs to attract corporate investment. Using a game-theoretic real options model between a firm and a government, this paper aims to explore the interaction between the government's tax-subsidy policy and the firm's investment and financing decisions. The optimal incentive policies are derived for cooperative and non-cooperative bargaining settings between a government and a firm. We show that it is optimal for the government to offer a tax-subsidy combination in the cooperative setting. However, this is not true for the non-cooperative setting, in which the optimal policy is to only levy taxes with no investment subsidy. Whereas firms always have an incentive to rely on debt financing in the non-cooperative setting, firms are reluctant to issue debt in the cooperative setting. Finally, it is generally optimal for the government to collect taxes at a lower rate in the case of high risk high-tech enterprises.

ARTICLE HISTORY

Received 30 June 2020

Accepted 15 December 2022



KEYWORDS

Taxes; investment subsidy; non-cooperative game; cooperative game

1. Introduction

It is well known that local governments in many countries often provide tax rate and investment subsidy policy (tax-subsidy policy, henceforth) to attract foreign and domestic investors to invest in new projects. With different economic conditions and objectives, how governments make appropriate tax-subsidy policy is very significant. Several studies, such as Pennings (2000), Yu et al. (2007), Sarkar (2012), Barbosa et al. (2016), Azevedo et al. (2019) and Azevedo et al. (2021), have examined how governments set the tax-subsidy policy to stimulate firms to accelerate investment (e.g., immediate investment). To the best of our knowledge, however, no papers in the theoretical literature have explicitly analyzed the effects of cooperative and non-cooperative games on investment and financing decisions of a firm as well as the tax-subsidy policy of the government. This is the issue that we address in this paper.

There are two motivating reasons for doing this research. First, in reality, governments and firms usually reach cooperation agreements to invest jointly in some projects in certain areas, which they would not usually invest in and are implemented immediately by private firms. For example, in order to stimulate private firms to take part in large-scale infrastructure investments, public-private partnerships (PPPs) are very popular in

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China and some European countries.¹ Furthermore, the cooperative results in which a central planner desires to maximize social wealth can provide a benchmark for the government to implement a tax-subsidy policy under a non-cooperative scenario. Second, with the recovery of the world economy, it is very important for governments around the world to set an appropriate and effective tax-subsidy policy. Considering different bargaining settings, we provide theoretical guidance for the implementation of more effective tax-subsidy policy; otherwise, the government may either not obtain the desirable result or give up too much value.

This paper employs a game-theoretic real options model that explicitly incorporates the effects of cooperative and non-cooperative bargaining games on the interaction between the firm's investment and financing and government's tax-subsidy policy. To this end, we unify a workhorse real options model (e.g., Mauer and Sarkar (2005)) with the Nash bargaining model (e.g., Fan and Sundaresan (2000)). Specifically, we assume that a firm with a perpetual right to invest in a new project at any time by paying an irreversible investment cost, which is covered by government subsidies and by issuing equity and debt (or pure equity). We suppose that the firm makes investment and financing decisions, whereas the government decides on policy regarding tax rate and investment subsidy. We discuss two bargaining games: one is a cooperative game (setting) in which both players (firm and government) first jointly make decisions about investment, financing, tax rates and investment subsidies to maximize the total wealth as a social planner and then consider how to divide it, for which allocation proportion depends on their bargaining powers. The other is a non-cooperative game (setting) in which they independently determine their respective decisions to maximize their own benefits in a two-stage dynamic game of perfect information. Considering these items, we focus on how cooperative and non-cooperative games influence the firm's investment decision and the government's tax-subsidy policy.

Our framework allows us to answer the following questions. First, what type of tax-subsidy policy is optimally selected by the government in the cooperative setting, and is it consistent with the social planner's wishes? Second, in the non-cooperative setting, how does the government select tax-subsidy policy? Finally, how does the firm choose optimal investment and financing strategies in different bargaining games?

The model provides four novel results. First, we show that it is very important for the government to offer correct tax-subsidy policy, otherwise it may give up too much value. Second, we provide a closed-form expression for the relationship between the tax rate and investment subsidy to ensure maximizing both the total social wealth and the government's net benefits in the cooperative setting. By contrast, it is optimal for the government to only levy taxes with no investment subsidy in the non-cooperative setting, which is consistent with Lukas and Thiergart (2019). Third, the firm always has an incentive to issue debt for financing investment cost in the non-cooperative setting, but this is not true in the cooperative setting. In addition, the government should levy taxes at

¹Public-private partnerships (PPPs) are contractual agreements between the government and firm. The government and social private party establish a community relationship of "benefit sharing, risk sharing and whole-process cooperation" in the form of franchise agreement, which reduces the financial burden of the government and the investment risk of private firms. As shown in data reported by government in China, the PPPs project database had 12,553 projects with a total investment of 17.55 trillion yuan by the end of 2021, in which the industry with the largest number of PPPs is urban infrastructure with 3,039 projects and 4.5564 billion yuan.

a lower rate for high risk high-tech enterprises. Finally, compared with the first-best solution of cooperative setting, the non-cooperative setting leads to underinvestment.

The remainder of the paper is organized as follows. [Section 2](#) gives a brief overview of related literature. [Section 3](#) sets up the basic model. [Section 4](#) discusses model solution including investment decisions and tax-subsidy policy for unlevered and levered firms. [Section 5](#) concludes. The proofs of the propositions are relegated to the Appendix.

2. Literature review

The large body of literature related to our paper includes Pennings (2000), Pennings (2005), Yu et al. (2007), Danielova and Sarkar (2011), Sarkar (2012), Barbosa et al. (2016) and Azevedo et al. (2021), in which authors study how the government adopt tax-subsidy policies for stimulating investment. First, Pennings (2000) and Yu et al. (2007) employ the real options approach and investigate the effectiveness of tax reduction versus investment subsidy to attract private investment for firms with pure-equity financing. Their main conclusion is that direct investment subsidy dominate tax reduction as an investment incentive for the government. While Danielova and Sarkar (2011) argue that it is optimal to use a combination of tax reduction and investment subsidy for levered firms. This result is obtained by Sarkar (2012) when allowing for the government's discount rate to be different from that of the firm. Furthermore, Barbosa et al. (2016) investigate the investment stimulus problem in an extended real options model by considering some relevant macroeconomic factors (namely, different types of taxes, asymmetric investment multipliers, and public inefficiencies). They argue that the investment subsidy policy always dominates tax cuts to promote private investment. Azevedo et al. (2021) develop a real options model that examines the effect of government's subsidies and taxation policy on the timing and size of investments. Moreover, they discuss the impact of revenue-neutral incentive packages. In addition, some existing literature has also studied taxes from other perspective.² However, these papers have not explicitly analyzed tax-subsidy policy when firms and governments determine their respective decisions to maximize their own benefits in a two-stage dynamic game. Thus, our paper mainly focuses on the interaction between the government's tax-subsidy policy and the firm's investment and financing decisions in the different settings, cooperative and non-cooperative.

The most closely related literature to our paper is Pennings (2005) and Lukas and Thiergart (2019). Pennings (2005) considers how the host government maximizes net domestic benefits from foreign investments by using tax-subsidy policy under cooperative and non-cooperative games. He investigates the influence of tariffs on when exporting to a host country or when serving the market by indirect setting of grants. The main difference is as follows: One is that Pennings (2005) adopts a perspective of international finance and macroeconomics and examines the influence of tariffs on direct exporting and indirect serving. While our paper considers the interaction between the firm's investment and financing decisions and the government's tax-subsidy policy from

²For example, Foley et al. (2007) and Lopez-Gracia and Mestre-Barber (2011). The former analyzes the influence of the tax effect on the debt maturity structure of small and medium-sized enterprise (SME). The latter examines how taxes impact on the cash holding of firms.

a view of corporate finance and microeconomic; Another difference is that we consider the firm's financial structure (equity or debt financing) but he does not.

Lukas and Thiergart (2019) model a similar non-cooperative setting. Both our paper and Lukas and Thiergart (2019) investigate the effect of the government's investment subsidy on the firm's optimal investment and financing decisions by employing a game-theoretic real options model between a firm and a government. However, there are significant differences between them. First, our paper aims to explore the interaction between a government's tax-subsidy policy and a firm's investment and financing decisions. While Lukas and Thiergart (2019) analyze the effect of uncertainty and investment stimulus in the form of cash grants on optimal investment timing, financing and investment scaling, our paper examines the optimal tax rate and investment subsidy for the government but does not consider the firm's investment scaling. Lukas and Thiergart (2019) only explore optimal investment subsidy at a given corporate tax rate. Thus, they did not examine optimal corporate tax rate for the government. Naturally, the mutual interaction between tax rate and investment subsidy were not studied in Lukas and Thiergart (2019). Second, Lukas and Thiergart (2019) just consider a non-cooperative setting but our paper examines both cooperative and non-cooperative settings between a government and a firm. With respect to cooperative setting, such as public-private partnership (PPP) arrangements, the government and firm may reach cooperation agreement to invest jointly in some projects in certain areas, which would not usually invest in and are implemented immediately by private firms. Obviously, it is worth studying.

In addition, our paper is related to the literature that investigates corporate financing and investment decisions in a bargaining game model, see, e.g., Fan and Sundaresan (2000), Sundaresan and Wang (2007), Shibata and Tian (2010), Shibata and Nishihara (2015), Gan et al. (2016) and Antill and Grenadier (2019). The first four articles use cooperative bargaining games to consider a debt renegotiation problem when a firm is in financial distress, and the last paper considers it with guaranteed swaps between borrowers and insurers in financing difficulties. Our paper takes this cooperative bargaining idea into account private investment and public stimulus between the investor and the government. Additionally, our paper is closely related to a long line of research about real options that originated from Myers (1977). Recently, many papers have employed the real options approach to investigate the interaction between financing and investment under an exogenously given tax rate of government, such as Mauer and Sarkar (2005), Hackbarth and Mauer (2012), Sundaresan et al. (2015), Tan and Yang (2017) and Tan and Luo (2021). However, these studies do not consider tax-subsidy policy from the perspective of the government or how it impacts a firm's investment and financing decisions. Thus, our paper extends these studies by concentrating on the effects of different bargaining games on the interaction between the government's tax-subsidy policy and the firm's investment and financing decisions, and thus it complements the aforementioned analyses.

3. The model

This section presents a unified framework to analyze the interaction between the government's tax-subsidy policy and the firm's investment and financing decisions.

The novel feature of our model is that firm and government determine their respective decisions to maximize their own benefits in a two-stage dynamic game. We now start with the basic model assumptions.

3.1. Assumptions

Firm. Assume that a firm ³ with no assets in place has a perpetual option to invest in a new project at any time by paying an irreversible investment cost $I > 0$. The investment, once completed, produces a continuous stochastic earnings before interest and taxes (EBIT) X per unit time, which is described by the following geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dW_t^{\mathbb{Q}}, \quad (1)$$

where $W_t^{\mathbb{Q}}$ is a standard Wiener process under the risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$ with the filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions, the growth rate $\mu < r$ (for convergence) and volatility $\sigma > 0$ of the cash flow are constant, where r is the risk-free interest rate. Furthermore, it is assumed that the initial value of EBIT, $X_0 = x$, is sufficiently low such that the investment should not be undertaken immediately.

We assume that the investment cost I is financed by equity and straight bonds (or pure equity). The bonds have an infinite maturity and continuous coupon payment of c per unit time (determined optimally later) until default. The firm pays corporate income taxation at a constant tax rate $\tau \in (0, 1)$ to the government. Debt has tax shield, i.e., the coupon payment is tax-deductible. Moreover, suppose that the firm's bankruptcy is endogenously decided by shareholders who maximize their own value, as in standard tradeoff models of capital structure (e.g., Leland, 1994). Once default occurs, following Danielova and Sarkar (2011), the firm will be forced to liquidate their assets at fire-sale prices. The value of the firm's future cash flow is assigned to the bondholders, but a fraction, denoted by α , of the value will be lost due to discount (bankruptcy costs).

Government. Assume that the government maximizes its net benefits from the exercised project. To achieve this goal, the government should appropriately choose a tax-subsidy policy, namely, a combination of tax rate τ and investment subsidy $\delta \in [0, 1]$ (i.e., the fraction of the investment cost I). We discuss two bargaining games: one is a cooperative game in which both players (the government and the firm) first jointly make decisions about investment, financing, tax rates and investment subsidies to maximize the total wealth as a social planner desires, and then they consider how to divide it, where allocation depends on their respective bargaining power. The other is a non-cooperative game in which the firm and government independently determine their respective decisions to maximize their own benefits in a two-stage dynamic game. This game can solve by backwards induction: the firm first derives the optimal investment trigger for a given tax rate τ and investment subsidy δ , and then the government, knowing how the firm will react, chooses an optimal tax rate and subsidy level that maximizes its net benefits. Therefore, the government's policy can be completely characterized by the pair (τ, δ) .

³We assume that the firm is a representative corporate of an industry since the government often design tax-subsidy policy for some special industry, such as high-tech, new energy, etc.

3.2. Pricing of corporate claims and the value of waiting to invest

In this subsection, we provide the pricing of corporate contingent claims after investment and the value of waiting to invest for firm and government. Let $T_d = \inf\{t \geq 0 : X_t \leq x_d\}$ be the first passage time of EBIT, X , to reach the bankruptcy threshold, x_d , from above. To save space, following Leland (1994) and Dixit and Pindyck (1994), we summarize these solutions and values in the following proposition.

Proposition 3.1. *The value of an unlevered firm is given by*

$$V_u(x) = \frac{(1 - \tau)x}{r - \mu}, \quad (2)$$

and the value of tax stream arising from the project is

$$T_u(x) = \frac{\tau x}{r - \mu}. \quad (3)$$

Thus, the government's net benefit (or the government value) from the project is $G_u(x) = T_u(x) - \delta I$. By contrast, for all $x > x_d$, a levered firm's equity, $E(x)$, yielding a dividend flow of $(1 - \tau)(x - c)$ is worth

$$E(x) = (1 - \tau) \left[\left(\frac{x}{r - \mu} - \frac{c}{r} \right) - \left(\frac{x_d}{r - \mu} - \frac{c}{r} \right) \left(\frac{x}{x_d} \right)^{\gamma_-} \right]. \quad (4)$$

The total firm value for its cash flow $(1 - \tau)x$ is given by

$$V(x) = \frac{(1 - \tau)x}{r - \mu} + \frac{\tau c}{r} \left(1 - \left(\frac{x}{x_d} \right)^{\gamma_-} \right) - (\alpha - \tau) \frac{x_d}{r - \mu} \left(\frac{x}{x_d} \right)^{\gamma_-}, \quad (5)$$

The government's taxation revenue for its cash flow $\tau(x - c)$ can be written as⁴

$$T(x) = \frac{\tau x}{r - \mu} - \frac{\tau c}{r} \left(1 - \left(\frac{x}{x_d} \right)^{\gamma_-} \right) - \frac{\tau x_d}{r - \mu} \left(\frac{x}{x_d} \right)^{\gamma_-}. \quad (6)$$

Accordingly, the net benefit of the government is $G(x) = T(x) - \delta I$, where $\gamma_- = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$.

Shareholders select the default policy that maximize the value of their equity claim after investment, which is characterized by a smooth-pasting condition: $\frac{\partial E(x)}{\partial x} \big|_{x=x_d} = 0$, in that

$$x_d = \frac{\gamma_-}{\gamma_- - 1} \frac{r - \mu}{r} c. \quad (7)$$

Next, we provide the value of the option to invest for firm and government. When the investment is implemented, the government provides a fraction subsidy of investment costs, δI , and the firm undertakes the remaining investment costs $(1 - \delta)I$. Let us denote by x_i^u and x_i the investment threshold for an unlevered and levered firm, respectively.

⁴Following Danielova and Sarkar (2011), we assume that the firm will be forced to liquidate assets at fire sales prices and not continue operation when the firm default. The government could no longer obtain any taxation revenue stream after bankruptcy and thus it cannot obtain any liquidation value upon default, i.e., $T(x_d) = 0$.

Noting that the firm will obtain net surplus $V(X_{T_i}) - (1 - \delta)I$ at investment, the value of the option to invest, $F(x)$, for the levered firm is given by

$$F(x) = \mathbb{E} \left[\int_0^{T_i} e^{-rT_i} (V(X_{T_i}) - (1 - \delta)I) | X_0 = x \right], \quad (8)$$

where $T_i \equiv \inf\{t \geq 0 : X_t \geq x_i\}$ is a stopping time, i.e., the first passage time of X from below to investment threshold x_i . Following Dixit and Pindyck (1994), we have

$$F(x) = \{V(x_i) - (1 - \delta)I\} \left(\frac{x}{x_i}\right)^{\gamma_+}, \quad x < x_i. \quad (9)$$

where $\gamma_+ = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$.

Accordingly, noting that the government pays δI and gets in return benefits $T(x_i)$ at investment, the value of the government before the option is exercised, $F^g(x)$, is

$$F^g(x) = \{T(x_i) - \delta I\} \left(\frac{x}{x_i}\right)^{\gamma_+}, \quad x < x_i. \quad (10)$$

Similarly, for an unlevered firm, the value of the option to invest, $F_u(x)$, is defined as

$$F_u(x) = \mathbb{E} \left[\int_0^{T_i^u} e^{-rT_i^u} (V_u(X_{T_i^u}) - (1 - \delta)I) | X_0 = x \right] = \{V_u(x_i^u) - (1 - \delta)I\} \left(\frac{x}{x_i^u}\right)^{\gamma_+}, \quad (11)$$

and the value of the government before investment $F_u^g(x)$ is

$$F_u^g(x) = \{T_u(x_i^u) - \delta I\} \left(\frac{x}{x_i^u}\right)^{\gamma_+}, \quad (12)$$

where $T_i^u \equiv \inf\{t \geq 0 : X_t \geq x_i^u\}$ is a stopping time.

4. Model solution

Based on the explicit pricing formulas in [section 3.2](#), we now proceed to examine how bargaining game modes affect the firm's investment and financing decisions and the government's optimal tax-subsidy policy. First, we consider the benchmark case in which the bargaining game between the firm and government is cooperative.

4.1. Benchmark: cooperative solution

Similar to Pennings (2005), we first consider that both investment participants, i.e., the firm and government, cooperatively agree on decisions for investment, financing and tax-subsidy policy in a cooperative setting. In fact, it is analogous to the situation where an external central planner decides. Owing to internalizing the friction of tax-subsidy policy, we provide a first-best solution without any market friction in the cooperative setting.

In this case, the firm and government jointly maximize the total profits from the project and then divide it depending on their bargaining power.⁵ Let η and $1 - \eta$ respectively denote bargaining powers of the firm and government, where $\eta \in [0, 1]$. If a successful negotiation between them is realized, the firm's gain is $F(x)$, and the government's gain is $F^g(x)$; while not carried out, they all can't get anything. In cooperative bargaining game framework, similar to Pennings (2005), the optimal tax rate and lump-sum subsidy $\{\tau^*, \delta^*\}$, and optimal investment threshold and coupon payments $\{x_i^*, c^*\}$ is equivalent to solving the following optimization problems:

$$\{\tau^*, \delta^*, x_i^*, c^*\} = \arg \max_{\{\tau, \delta, x_i, c\}} \{[F(x)]^\eta [F^g(x)]^{1-\eta}\}, \quad (13)$$

where $F(x)$ and $F^g(x)$ are given by Equation (9) and Equation (10). Since optimal tax rate τ^* , lump-sum subsidy δ^* , investment threshold x_i^* and coupon payment c^* are jointly chosen at the same time to maximize the objective function $[F(x)]^\eta [F^g(x)]^{1-\eta}$, the envelope condition ensures that we do not need to consider the feedback effects between these variable $\{\tau^*, \delta^*, x_i^*, c^*\}$ when we get derivative with respect to them. Note that $F(x) = \{V(x_i) - (1 - \delta)I\} \left(\frac{x}{x_i}\right)^{y_+}$, $F^g(x) = \{T(x_i) - \delta I\} \left(\frac{x}{x_i}\right)^{y_+}$, and

$$\frac{\partial F(x)}{\partial \tau} = \frac{-1}{r - \mu} x_i + \frac{c}{r} \left(1 - \left(\frac{x_i}{x_d}\right)^{y_-}\right) + \frac{x_d}{r - \mu} \left(\frac{x_i}{x_d}\right)^{y_-} = -\frac{\partial F^g(x)}{\partial \tau}.$$

As to the government's maximization problem in (13) for how to select τ and δ , by the first order condition $\frac{\partial \{[F(x)]^\eta [F^g(x)]^{1-\eta}\}}{\partial \tau} = 0$ and $\frac{\partial \{[F(x)]^\eta [F^g(x)]^{1-\eta}\}}{\partial \delta} = 0$, we are easier to get $\eta F^g(x) = (1 - \eta)F(x)$. Moreover, we have $F(x) = \eta(F(x) + F^g(x))$ and $F^g(x) = (1 - \eta)(F(x) + F^g(x))$.

Obviously, the total wealth is $F(x) + F^g(x) = \left(\frac{x_i}{r - \mu} - \alpha \frac{x_d}{r - \mu} \left(\frac{x_i}{x_d}\right)^{y_-} - I\right) \left(\frac{x}{x_i}\right)^{y_+}$. Compared with the total wealth $F_u(x) + F_u^g(x) = \left(\frac{x_i}{r - \mu} - I\right) \left(\frac{x}{x_i}\right)^{y_+}$ for an unlevered (pure equity) firm, debt financing results in deadweight loss when bankruptcy occurs, i.e., $\alpha \frac{x_d}{r - \mu}$, and the value of expected loss at investment is $\alpha \frac{x_d}{r - \mu} \left(\frac{x_i}{x_d}\right)^{y_-}$. However, it would be absent for an unlevered firm. Because both the firm and government first jointly maximize the total wealth generated by the project and then divide it in the cooperative setting. Intuitively, this situation is not desirable and also not occur since the rational firm and government have no incentive to issue debt. Therefore, the following proposition summarizes the optimal investment threshold, tax rate and lump-sum subsidy, and the firm and government option values in the cooperative setting.

Proposition 4.1. *If the firm and the government jointly make investment and financing decisions and determine the tax-subsidy policy in the cooperative setting, the firm has no incentive to issue debt (or take a bank loan) to finance the exercise costs of the project.*

⁵For example, such as public-private partnerships (PPPs), the public and private party reach cooperation agreements to invest jointly in some projects in certain areas, which would not usually invest in and are implemented immediately by private firms (see Lukas and Thiergart (2019)). Thus the government would not just get a fixed proportion of cash flow (i.e., an exogenously specified tax rate τ), but the total benefits generated by exercising the new project should be shared between them.

Namely, the firm still prefers to finance the project with pure equity even if debt financing is possible. Hence, the optimal investment trigger, combination of tax rate and investment subsidy, and option values of the firm and government would be exactly the same as the ones for an unlevered firm. Thus the optimal investment trigger is

$$x_i^* = x_i^{u*} = \frac{\gamma_+}{\gamma_+ - 1}(r - \mu)I. \quad (14)$$

and the optimal tax-subsidy policy should satisfy the relationship about lump-sum subsidy $\delta^*(\tau)$ and tax rate τ , which can be written as⁶

$$\delta^*(\tau) = \delta_u^*(\tau) = \tau + \frac{\tau + \eta - 1}{\gamma_+ - 1}. \quad (15)$$

4.1.1. Furthermore, the option values of the firm and government are respectively given by

$$F^*(x) = F_u^*(x) = \eta \left(\frac{I}{\gamma_+ - 1} \right) \left(\frac{x}{x_i^{u*}} \right)^{\gamma_+}, \quad (16)$$

$$F^{g*}(x) = F_u^{g*}(x) = (1 - \eta) \left(\frac{I}{\gamma_+ - 1} \right) \left(\frac{x}{x_i^{u*}} \right)^{\gamma_+}, \quad (17)$$

where $F_u^*(x) + F_u^{g*}(x) = \left(\frac{I}{\gamma_+ - 1} \right) \left(\frac{x}{x_i^{u*}} \right)^{\gamma_+}$ is total social wealth from exercising the project, which is split by the firm and government on account of their own bargaining power η and $1 - \eta$.

Proof. See [Appendix A](#).

Proposition 4.1 indicates the outcome in the cooperative setting is optimal and efficient from a view of maximization social wealth since it can fully eliminate the inefficiency owing to market friction. Because the investment threshold x_i^* in the cooperative bargaining setting is exactly equal to $\frac{\gamma_+}{\gamma_+ - 1}(r - \mu)I$ in the complete market (i.e., without friction for $\tau = 0, \delta = 0$). Only if the government selects an appropriate combination of tax rate and investment subsidy as in (15), it can ensure maximizing both the total social wealth and the government's net benefits when the firm and government are engaged in the cooperative game. Meanwhile, the firm will choose the investment threshold as in (14). Therefore, we refer to this scenario as the first-best solution.

Moreover, the stronger bargaining power the firm has (i.e., a larger η), the more benefits they can obtain from exercising the project. In the extreme case, if the firm's bargaining power reach the maximum value $\eta = 1$, we discover that the firm will extract all the benefits and the government get nothing, $F_u^{g*}(x) = 0$. Namely, the government adopts a break-even tax-subsidy program, in which the value of all subsequent taxes

⁶The tax rate τ must take values in the interval, $[\frac{1-\eta}{\gamma_+}, \frac{\gamma_+-\eta}{\gamma_+}]$ to ensure $\delta^*(\tau) \in [0, 1]$.

levied from the firm's profits fully compensates for investment subsidy for exercising cost.

Why would the government be willing to adopt such the break-even tax-subsidy program even if they would not obtain any benefits from exercising the project? This may be because corporate investment has very significant and positive effect on economic growth and employment, such as reducing unemployment and promoting industrial development and technological progress, although these values are difficult to measure in our structure model. Based on this, the break-even tax-subsidy policy may be also worthwhile as long as the government can receive significant indirect or non-economic benefits from investment.

Finally, we find that Public-private partnerships (PPPs), as a way of the cooperative equilibrium, would be more likely to be viable when the government has weaker bargaining power. It means that the government will give up more benefits to increase the possibility of success for PPPs project.

4.2. Non-cooperative solution

This subsection examines the optimal combination of tax rate and lump-sum subsidy when both the firm and government maximize their own benefits in a two-stage dynamic game of perfect information.

To begin, we consider the firm's investment and financing decisions for a given tax-subsidy policy (τ, δ) . To maximize the option value before investment, following Mauer and Sarkar (2005), it is equivalent to selecting $\{x_i, c\}$ to maximize $F(x)$ given by Eq.(9). Namely, we solve the following optimization problem:

$$\{\bar{x}_i, \bar{c}\} = \arg \max_{\{x_i, c\}} \{F(x)\} = \arg \max_{\{x_i, c\}} \{V(x_i) - (1 - \delta)I\} \left(\frac{x}{x_i}\right)^{\gamma_+}. \quad (18)$$

Therefore, the following proposition presents the closed-form solutions for optimal investment and financing decisions as functions of tax-subsidy policy and other structural parameters in the model.

Proposition 4.2. *The firm's optimal investment threshold is given by $\bar{x}_i = hI$, the optimal coupon for debt can be written as $\bar{c} = \frac{\gamma_- - 1}{\gamma_-} \frac{r_-}{r_- - \mu} nhI$, and the optimal bankruptcy trigger is given by $x_d = nhI$, where $n = (1 - \frac{\alpha\gamma_-}{\tau})^{\frac{1}{\gamma_-}}$ and $h = \frac{\gamma_+(1-\delta)}{(\gamma_- - \gamma_+) \frac{r_- \gamma_- - 1}{r_- - \mu} + (\alpha - \tau) \frac{n}{r_- - \mu} (\frac{1}{n})^{\gamma_-} + (\gamma_+ - 1) \frac{1 - \tau}{r_- - \mu} + \frac{\tau n \gamma_+ - 1}{r_- - \mu} \frac{1}{\gamma_-}}$.*

Proof. See Appendix B.

Next, following Sarkar (2012), the government chooses the optimal combination vector $\{\tau^*, \delta^*\}$ that solves the following optimization problem:

$$\{\tau^*, \delta^*\} = \arg \max_{\{\tau, \delta\}} \{F^g(x)\} = \arg \max_{\{\tau, \delta\}} \{(T(x_i) - \delta I) \left(\frac{x}{x_i}\right)^{\gamma_+}\}. \quad (19)$$

Now we derive the optimal investment threshold \bar{x}_i and coupon payment \bar{c} by substituting the optimal tax-subsidy policy in (19) into proposition 4.2. Furthermore, we obtain the option values of the firm and government from (9) and (10) by substituting the above optimal values.

Solving the optimization problem (19) induces two non-linear algebraic equations, but they are rather tedious because both investment threshold \bar{x}_i and debt coupon \bar{c} in proposition 4.2 are functions of $\{\tau, \delta\}$ and the envelope theorem fails in this scenario. Therefore, instead of presenting the equations, we directly provide its numerical solution. For more details, please see [Appendix C](#).

4.2.1. Numerical results and analysis

In this subsection, based on the above theoretical derivation and previous discussions, we discuss the optimal tax rate and investment subsidy, investment threshold and leverage when investment costs are partially debt financing in the non-cooperative setting through numerical analysis.

Baseline parameter values. To make a reliable comparison, we borrow the parameter values by Sundaresan and Wang (2007), Hackbarth and Mauer (2012) and Sarkar (2012), and the baseline parameter values are selected as follows. The risk-free interest rate $r = 0.06$ and the bankruptcy cost $\alpha = 0.35$ are taken from Sundaresan and Wang (2007). The drift rate $\mu = 0.01$ and the volatility $\sigma = 0.25$ are rooted in Hackbarth and Mauer (2012). The current cash flow rate $x = 1$ and the exercise cost of the project $I = 10$ are come from Sarkar (2012). However, the computations are repeated with a wide range of parameter values to ensure robustness of results.

Basic results. With the above baseline parameter values, we have the following base results: the government selects the combination of $\tau^* = 0.31$ and $\delta^* = 0$ to maximize its own benefits; the firm chooses optimal coupon payments $\bar{c} = 1.5157$ and optimal investment threshold $\bar{x}_i = 1.3742$; the government value is $F^g(x) = 1.3539$, the firm value is $F(x) = 7.438$ and the total wealth is 8.7919, which is shown in the first panel of [Table 1](#). For first-best cooperative solution, i.e, a firm under the cooperative setting in [subsection 4.1](#), however, we know that the total wealth 10.156 is greater than 8.7919 and the optimal investment threshold $x_i^* = 1.1521$ is less than $\bar{x}_i = 1.3742$ in the non-

Table 1. The effect of parameter values on optimal tax-subsidy policy of the government and investment and financing decisions of the firm. All parameters take the following baseline parameter values unless stated otherwise: $r = 0.06, \mu = 0.01, \sigma = 0.25, \alpha = 0.35, I = 10, x = 1$.

	τ^*	δ^*	x_i^*	c^*	lev^*	$F(x)$	$F^g(x)$
base case	0.31	0	1.3742	1.5157	0.7707	7.438	1.3539
$\mu = 0$	0.283	0	1.4438	1.2998	0.7390	4.9791	1.0121
$\mu = 0.02$	0.34	0	1.3097	1.8361	0.8003	11.318	1.8366
$\sigma = 0.2$	0.346	0	1.1891	1.2851	0.8114	7.0724	1.411
$\sigma = 0.3$	0.285	0	1.5806	1.7937	0.7349	7.8887	1.3397
$\alpha = 0.25$	0.233	0	1.3071	1.479	0.7688	8.1256	1.0083
$\alpha = 0.45$	0.377	0	1.4431	1.5468	0.7713	6.8224	1.6681
$r = 0.05$	0.302	0	1.2135	1.417	0.7589	11.183	1.9251
$r = 0.07$	0.317	0	1.5312	1.623	0.7804	5.1427	0.9826
$I = 8$	0.310	0	1.0994	1.2126	0.7707	8.8259	1.6066
$I = 12$	0.310	0	1.649	1.8188	0.7707	6.4676	1.1773
$x = 0.8$	0.310	0	1.3742	1.5157	0.7707	5.0146	0.9128
$x = 1.2$	0.310	0	1.3742	1.5157	0.7707	10.265	1.8685

cooperative setting. These results are still established for other parameters as shown in Table 1. Therefore, the non-cooperative setting leads to underinvestment and a net loss from a welfare point of view. In addition, when its bargaining power η is lower than 0.7324, the firm has an incentive to choose non-cooperative solution because the value obtained by the firm (see as (16)) in the cooperative setting is less than that 7.438 in the non-cooperative setting.

Optimal tax-subsidy policy. In this paragraph, we examine the government's optimal combination of tax rate and investment subsidy with baseline parameter values. Figure 1 displays how the government's tax-subsidy policy affects the government's option value (the government's net benefits) for a firm in the non-cooperative setting. As shown in Figure 1, there is an optimal combination of tax-subsidy, $\{\tau^* = 0.31, \delta^* = 0\}$, to maximize the government's net benefits, which is $F^g(x) = 1.3539$. Otherwise, choosing a sub-optimal combination of tax rate and lump-sum subsidy, the government will give up a significant part of its benefits. For example, when the combination is tax rate $\tau = 0.31$ and investment subsidy $\delta = 0.15$, the government value is $F^g(x) = 0.3936$. Similarly, we have $F^g(x) = 1.306$ and $F^g(x) = 1.096$ for $\{\tau = 0.31, \delta = 0.01\}$ and $\{\tau = 0.31, \delta = 0.05\}$, respectively. And the government value is $F^g(x) = 1.1903$ for $\{\tau = 0.15, \delta = 0\}$ and $F^g(x) = 1.3074$ for $\{\tau = 0.45, \delta = 0\}$; Similarly, we have $F^g(x) = 1.352$ for $\{\tau = 0.29, \delta = 0\}$ and $F^g(x) = 1.353$ for $\{\tau = 0.33, \delta = 0\}$. That's to say, even if the government is able to provide an investment subsidy, it is optimal for the government to only levy taxes instead of providing a tax-subsidy policy, which is consistent with Lukas and Thiergart (2019). This result is also established for variation of parameters as shown in Table 1.

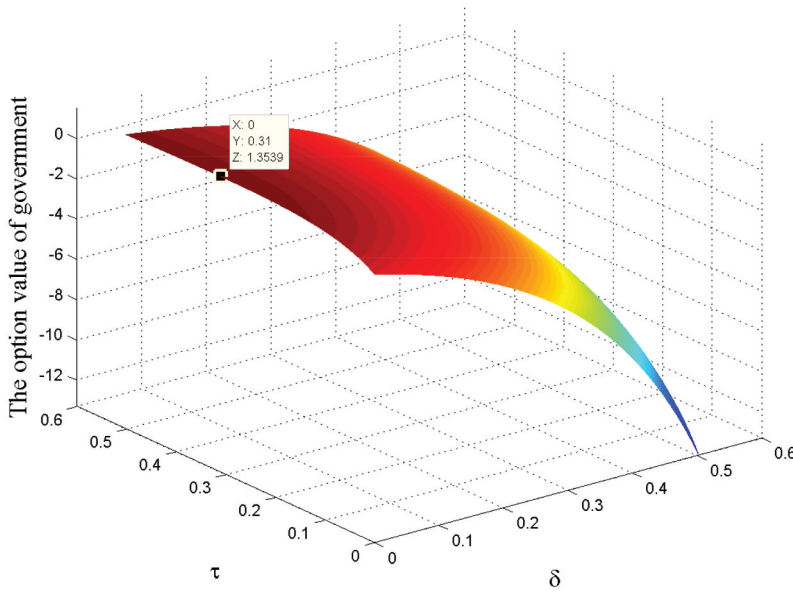


Figure 1. This figure displays the government's option value for different combination of tax and subsidy $\{\tau, \delta\}$ with baseline parameter values: $r = 0.06, \mu = 0.01, \sigma = 0.25, a = 0.35, l = 10, x = 1$.

The economic intuition that explains why the government has no incentive to offer investment subsidy is as follows. According to Lukas and Thiergart (2019), investment subsidy has two opposite effects on the government's net benefits. First, subsidies represent sunk cost for the government. Second, since subsidies affect investment timing (i.e., larger subsidies will stimulate the firm's earlier investment), they impact the discounting of the government's payoff. While the latter effect is positive (i.e., higher subsidies mitigate the effect of discounting), the former has a strictly negative influence on the government's payoff. For a low tax rate and a relatively large volatility, the negative effect dominates and the benefits of early investment for the government's payoff diminish. Namely, investment subsidy offered by the government leads to costs that outweigh the benefits. Therefore, the government has no incentive to offer an investment subsidy because it pays the full cost of accelerating the exercise of the project but shares the benefits with the firm.

It is well known that the firm always chooses an optimal investment threshold by Eq. (B.1) such that the marginal benefit is equal to the marginal cost. However, by investment subsidy, the government can collect taxes earlier but only obtain $\tau(x - c)$ of net increment cash flow. While the firm gets the remaining net increment $(1 - \tau)(x - c)$. Thus, the government's net increment of benefits is less than lump-sum grant cost for accelerating investment. Therefore, the government prefers to only levy taxes with no subsidy. Although Danielova and Sarkar (2011) and Sarkar (2012) argue that it might be optimal for the government to provide investment subsidy as well as charge a positive tax rate, they consider that the government stimulate the firm to invest at its desired investment level (e.g., immediate investment) for reducing unemployment and promoting development of an industry. However, without considering some special purpose for stimulating investment, the government selects optimal tax-subsidy policy to maximize its own benefits, which also exist in the real world. Thus, we provide an explanation why the government does not provide investment subsidy in some cases, which is consistent with Lukas and Thiergart (2019).

Furthermore, why the government's value first increases and then decreases with the tax rate, i.e., existing an optimal tax rate, is as follows. A larger corporate tax rate has two opposite effects on the government's net benefits. First, the levied taxation revenue of the government increases as the tax rate rises because the government obtains a greater proportion of the benefits from the project, which is the positive effect. Second, the negative effect is that a higher tax rate usually leads to a larger leverage of debt owing to its tax shield and thus enhances the likelihood of bankruptcy. Moreover, upon liquidation, the government has no longer tax revenue. For a low tax rate, the positive effect dominates, and for a high tax rate, the negative effect dominates. Therefore, there is an unique optimal tax rate to maximizing the government's net benefits.

Result 2. *If the firm and the government separately decide on investment and financing decisions, the tax rate and lump-sum subsidy policy to maximize their own benefits in the non-cooperative setting, it is generally optimal for the government to only levy taxes with no subsidy instead of implementing a tax-subsidy policy for a levered firm.*

Comparative static results. Table 1 shows the effects of various parameter values on the firm's optimal investment and financing decisions and the government's optimal tax-subsidy policy in the non-cooperative setting. Obviously, we still find that the

government prefers taxes to a tax-subsidy policy with a wide range of parameter values around the base case.

We mainly focus on the effects of the firm's characteristics on the government's optimal tax rate and the firm's investment trigger and leverage. First, as shown in Table 1, with the growth rate (μ) of the cash flow rising, the optimal government's tax rate and firm's leverage ratio increase, and the firm's investment threshold decrease. Moreover, their option values all increases with it. This is quite in agreement with intuition. The reason is as follows: a higher growth rate (μ) means the project is more attractive, and thus the firm has more incentive to invest earlier and a lower bankruptcy risk; while with respect to the choice of tax rate, the government trades off taxation revenue per unit of time and tax collection duration. Thus, the government choosing a larger tax rate shares a larger part of future profits. Also, not surprisingly, the leverage ratio and the option value of the firm and government are a increasing function of μ .

Second, as volatility σ increases, i.e., a larger business risk, the taxes of the government reduces. For example, as σ raises from 0.2 to 0.3, the optimal tax rate decreases from 34.6% to 28.5%. This implies that the government should implement a lower tax rate for high risk high-tech enterprises,⁷ which is consistent with empirical evidence. For instance, the general corporate effective tax rate is about 33% in China, but for high risk high-tech enterprises, they shall be levied at the reduced effective tax rate of 24%.⁸ Of course, more uncertainty discourages investment and reduces leverage of the firm. In fact, as shown in Table 1, the optimal exercising trigger of the project increases from 1.1891 to 1.5806 and the leverage ratio decreases from 0.8114 to 0.7349.

Third, a growth of bankruptcy loss rate α and risk-free interest r increase the effective tax rate and investment threshold. The reason is as follows. There are two effects for the firm's investment. As for an earlier investment, the firm obtains cash flow early, but faces a larger default risk. While for a relatively delayed investment, the firm reduces the default risk, but gets cash flow lately. Obviously, the firm chooses an optimal investment level by trading off the two effects. Therefore, under unfavorable conditions in which a higher bankruptcy cost α means a larger bankruptcy losses (i.e., a lower liquidation value) in default and a larger discount rate r makes the project less attractive, the firm has an incentive to choose a larger investment level to control investment risk (or reduce default disk). Correspondingly, the government compensates the loss of taxation by a relatively high tax rate. In addition, with respect to investment cost (I) and current cash flow (x), we find that the government's optimal tax rate and the firm's leverage ratio are independent of them.

All in all, from the above-mentioned analysis, we can draw the following conclusions: In the non-cooperative setting, the government's optimal policy is to only levy taxes, which is preferred to a tax-subsidy combination; moreover, the government should levy taxes at a relatively high rate for a firm with large growth rate, bankruptcy cost and risk-free interest, but at a relatively low rate for a firm with high risk.

⁷In general, high-tech enterprises are characterized by high risks, such as extremely high technical risks, market risks and financial risks. In addition, except for a very small number of successful high-tech enterprises that have good development potential and profitability, the vast majority of high-tech enterprises in China generally lack the ability of sustainable growth since they cannot effectively convert R&D input into output and the failure probability of R&D is extreme high. Thus the overall return rate of high-tech industry is not necessarily high in China.

⁸In China, the general enterprise income tax is 25%, but it is 15% for high risk high-tech enterprise. In addition, personal income tax for listed companies profit dividends is more than 10%.

5. Conclusions

In this paper, we examine the effects of cooperative and non-cooperative settings on investment and financing decisions of a firm in addition to the tax-subsidy policy of the government employing a game-theoretic real options model. The optimal tax-subsidy policies for different bargaining settings scenarios and financing ways are derived and discussed. Our findings can provide guidance for policy makers, and the main results are summarized as follows:

First, we provide a closed-form expression for the relationship between the tax rate and investment subsidy to ensure maximizing both the total social wealth and the government's net benefits when the firm and government are engaged in the cooperative game. However, in the non-cooperative game, the government's optimal policy is only levying taxes with no investment subsidy, which is consistent with Lukas and Thiergart (2019) and may explain why governments charge a positive tax rate on the profits from the project but not provide investment subsidy in general. Second, the firm may be reluctant to issue debt to partially finance the investment costs in the cooperative setting even if debt financing is possible. Whereas firms always have an incentive to debt financing in the non-cooperative setting. Moreover, it is generally optimal for the government to levy taxes at a low rate for high risk high-tech enterprises. Finally, compared with the first-best solution of cooperative setting, the non-cooperative setting results in underinvestment for the firm.

Throughout our analysis, we have assumed that the government had complete information about firms. Due to information asymmetry, however, the government usually does not know the mean growth rate of the firm's cash flow. Thus, an interesting question for future research is to extend our analysis and consider how partial information about the growth rate affects tax-subsidy policy of the government. Specifically, it may be assumed that the mean growth rate is an unobservable random variable that obeys a two-point distribution. The government may be assumed to be imperfectly informed about whether the expected return is high or low, and its beliefs continuously updated in a Bayesian fashion by observing the realizations of the cash flow process.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The research reported in this paper was supported by the National Natural Science Foundation of China (project Nos. 71901111); Social Science Planning Project of Jiangxi Province (19YJ38), Key Research Base Project of Humanities and Social Science of Jiangxi Province (JD19030), Graduate Innovation Special Fund Project of Jiangxi Province (YC2021-S380), Science and Technology Research Project of Jiangxi Province (GJJ200508), Natural Science Foundation of Jiangxi Province (20224BAB201004) and Education Science planning project of Jiangxi Province(22YB064).

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Appendices

Appendix A The proof of Proposition 4.1

We first prove the result that the firm does not issue debt in a cooperative setting. This is obvious because the central planner faces no trade-off: the interest tax shield disappears and only the bankruptcy costs remain. Naturally, it is optimal for the firm to not issue debt financing for investment costs in the cooperative setting, i.e., $c^* = 0$. Hence, the optimal investment trigger, combination of tax rate and investment subsidy, and option values of the firm and government would be exactly the same as that for an unlevered firm, which we derive as follows.

Since optimal tax rate τ_u^* , lump-sum subsidy δ_u^* and investment threshold x_i^{u*} are jointly chosen at the same time to maximize the objective function $[F_u(x)]^\eta [F_u^g(x)]^{1-\eta}$, the envelope condition ensures that we do not need to consider the feedback effects between the endogenous investment threshold x_i^{u*} and the combination $\{\tau_u^*, \delta_u^*\}$ when we get derivative with respect to them. Noting that $F_u(x) = \left\{ \frac{(1-\tau)x_i^u}{r-\mu} - (1-\delta)I \right\} \left(\frac{x}{x_i^u} \right)^{\gamma_+}$ and $F_u^g(x) = \left\{ \frac{\tau x_i^u}{r-\mu} - \delta I \right\} \left(\frac{x}{x_i^u} \right)^{\gamma_+}$, in order to maximize $[F_u(x)]^\eta [F_u^g(x)]^{1-\eta}$ with respect to τ and δ , through the first order condition $\frac{\partial \{ [F_u(x)]^\eta [F_u^g(x)]^{1-\eta} \}}{\partial \tau} = 0$ and $\frac{\partial \{ [F_u(x)]^\eta [F_u^g(x)]^{1-\eta} \}}{\partial \delta} = 0$, it is not hard to derive $\eta F_u^g(x) = (1-\eta)F_u(x)$. Hence, the optimal vector $\{\tau_u^*, \delta_u^*\}$ should satisfy the following two equations

$$\begin{cases} F_u(x) = \eta F_u(x) + (1-\eta)F_u(x) = \eta(F_u(x) + F_u^g(x)) \\ F_u^g(x) = \eta F_u^g(x) + (1-\eta)F_u^g(x) = (1-\eta)(F_u(x) + F_u^g(x)). \end{cases} \quad (\text{A.1})$$

Next, maximizing $[F_u(x)]^\eta [F_u^g(x)]^{1-\eta}$ in regard to x_i^u , through first order condition and the equation $\eta F_u^g(x) = (1-\eta)F_u(x)$, we have

$$\frac{\partial \{F_u(x) + F_u^g(x)\}}{\partial x_i^u} = 0.$$

Solving it, we easily derive $x_i^{u*} = \frac{\gamma_+}{\gamma_+ - 1}(r - \mu)I$. Substituting $x_i^u = x_i^{u*}$ into the first (or second) equation(A.1), we have $\delta_u^*(\tau) = \frac{\gamma_+}{\gamma_+ - 1}\tau - \frac{1-\eta}{\gamma_+ - 1} = \tau + \frac{\tau + \eta - 1}{\gamma_+ - 1}$. Naturally, taking $x_i^u = x_i^{u*}$ and $\delta = \delta_u^*(\tau)$ into (11) and (12), we respectively obtain the option value of the firm and government (16) and (17).

In fact, to maximize $[F(x)]^\eta [F^g(x)]^{1-\eta}$ in regard to x_i in (13), by first order condition, we have $(\gamma_+ - \gamma_-)\alpha \frac{x_d}{r-\mu} \left(\frac{x_d}{x_i} \right)^{\gamma_-} + \gamma_+ I = (\gamma_+ - 1) \frac{x_i}{r-\mu}$. From the above, we know the firm does not issue debt and then without bankruptcy, i.e., $\alpha = 0$, and thus obtain by simple calculation, $x_i^* = \frac{\gamma_+}{\gamma_+ - 1}(r - \mu)I$, which is also just the optimal investment trigger x_i^{u*} .

Appendix B The proof of Proposition 4.2

To maximize the option value $F(x)$ before investment in (18), it must satisfy the following smooth-pasting conditions at the investment threshold \bar{x}_i :

$$\frac{\partial F(x)}{\partial x} \Big|_{x=\bar{x}_i} = \frac{\partial V(x)}{\partial x} \Big|_{x=\bar{x}_i}. \quad (\text{B.1})$$

Thus the investment threshold \bar{x}_i is a solution of the following algebraic equation:

$$\left(\frac{\gamma_-}{\gamma_+} - 1 \right) \left[\frac{\tau c}{r} \left(\frac{1}{x_d} \right)^{\gamma_-} + (\alpha - \tau) \frac{x_d}{r-\mu} \left(\frac{1}{x_d} \right)^{\gamma_-} \right] (\bar{x}_i)^{\gamma_-} + \left(1 - \frac{1}{\gamma_+} \right) \frac{1-\tau}{r-\mu} \bar{x}_i + \frac{\tau c}{r} - (1-\delta)I = 0. \quad (\text{B.2})$$

With respect to optimal coupon payment \bar{c} of debt in (18), using the first-order condition, $\frac{\partial F(x)}{\partial \bar{c}}|_{c=\bar{c}} = 0$, and applying the envelope theorem, it is easy to derive that \bar{c} can be explicitly given by

$$\bar{c} = \frac{\gamma_- - 1}{\gamma_-} \frac{r}{r - \mu} \left(1 - \frac{\alpha\gamma_-}{\tau}\right)^{\frac{1}{\gamma_-}} \bar{x}_i. \quad (\text{B.3})$$

Let $n = \left(1 - \frac{\alpha\gamma_-}{\tau}\right)^{\frac{1}{\gamma_-}}$, we have $c^* = \frac{\gamma_- - 1}{\gamma_-} \frac{r}{r - \mu} n \bar{x}_i$ and $x_d = n \bar{x}_i$. Substituting them into (B.2) and simplifying, we easily derive optimal investment $\bar{x}_i = hI$, optimal debt coupon rate $\bar{c} = \frac{\gamma_- - 1}{\gamma_-} \frac{r}{r - \mu} nhI$, and optimal bankruptcy trigger $x_d = nhI$, where

$$h = \frac{\gamma_+(1-\delta)}{(\gamma_- - \gamma_+) \left(\frac{r\mu}{r-\mu} \frac{\gamma_- - 1}{\gamma_-} + (\alpha - \tau) \frac{\mu}{r-\mu} \left(\frac{1}{n} \right)^{\gamma_-} + (\gamma_+ - 1) \frac{1-\tau}{r-\mu} + \frac{r\mu\gamma_+ - 1}{r-\mu} \frac{1}{\gamma_-}}.$$

Appendix C Numerical method

To determine the government's optimal corporate tax rate τ^* and investment subsidy δ^* for the levered firm in non-cooperative bargaining setting, we should know in advance the decision variables, including the investment threshold \bar{x}_i , the default threshold x_d and coupon payment \bar{c} , which are some functions for a given τ and δ . All functions are closed-form expressions given in the proposition 4.2. By taking these into account, the net benefits of the government $F^g(x)$ is a function of the two variables of τ and δ in addition to the current cash flow level x . Therefore, the algorithm is shown as follows: First, for some given τ and δ , we get the optimal coupon payment \bar{c} , investment threshold \bar{x}_i and default threshold x_d . Second, plotting the government's net benefits $F^g(x)$ against τ and δ , we jointly choose optimal corporate tax rate τ^* and lump-sum subsidy δ^* that make the highest value $F^g(x)$. Finally, substituting optimal combination $\{\tau^*, \delta^*\}$ into the expressions of \bar{x}_i and \bar{c} , we are able to obtain optimal investment threshold and coupon payment of debt, and accordingly get the optimal firm and government payoffs.