ECONSTOR Make Your Publications Visible.

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Dierkes, Stefan; Siepelmeyer, David

Article

Material flow cost accounting with multiple inefficiency factors and recycling

Schmalenbach Journal of Business Research (SBUR)

Provided in Cooperation with: Schmalenbach-Gesellschaft für Betriebswirtschaft e.V.

Suggested Citation: Dierkes, Stefan; Siepelmeyer, David (2025) : Material flow cost accounting with multiple inefficiency factors and recycling, Schmalenbach Journal of Business Research (SBUR), ISSN 2366-6153, Springer, Heidelberg, Vol. 77, Iss. 1, pp. 57-93, https://doi.org/10.1007/s41471-024-00197-z

This Version is available at: https://hdl.handle.net/10419/313716

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.



WWW.ECONSTOR.EU

https://creativecommons.org/licenses/by/4.0/

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



ORIGINAL ARTICLE



Material Flow Cost Accounting with Multiple Inefficiency Factors and Recycling

Stefan Dierkes · David Siepelmeyer

Received: 30 March 2023 / Accepted: 16 October 2024 / Published online: 11 December 2024 \circledcirc The Author(s) 2024

Abstract Sustainability management requires differentiated information on the environmental and economic consequences of material demand's increasing and decreasing factors. Material flow cost accounting fails to provide this information due to the common undifferentiated determination of realized costs of material and product losses and, therefore, the lack of a production theoretical basis for cost planning. We develop a material flow model considering the impacts of waste and reject as material demand increasing factors and reworking and recycling as material demand decreasing factors at the company, quantity center, and product unit levels. This enables designing a material flow cost accounting system with specific cost information on the different factors at each level. We also analyze the consequences of the material distribution key and possible alternatives for allocating costs in material flow cost accounting. Finally, we discuss further development opportunities for this accounting system.

Keywords Material flow cost accounting · Material flow model · Reworking · Recycling · Resource efficiency · Sustainability management

JEL-Classification Q56

Stefan Dierkes

David Siepelmeyer DSV Global Transportation and Logistics, Schlachte 15–18, 28195 Bremen, Germany E-Mail: david-siepelmeyer@gmx.de

The article is dedicated to Prof. Dr. h.c. Dr. h.c. Josef Kloock, who passed away on May 29th, 2023. His outstanding research in production theory and cost accounting is the theoretical basis for this article.

Chair of Finance and Control, Georg-August University, Platz der Göttinger Sieben 3, 37073 Göttingen, Germany E-Mail: stefan.dierkes@wiwi.uni-goettingen.de

1 Introduction

The increasing scarcity of materials and the negative environmental impact of corporate manufacturing have drawn public attention to companies' environmental performance. Hence, stakeholders and political regulations, such as sustainability reporting and the EU Taxonomy (Christensen et al. 2021; European Commission 2020), put pressure on the management of companies to align their business activities with the environment (Hahn 2022; Sanders and Wood 2019). Therefore, sustainability management attempts to integrate environmental and social requirements into business activities essential for companies' future existence and success. Consequently, companies strive to improve resource efficiency and introduce different recycling measures to establish closed material cycles. Analyzing these measures' ecological and economic impacts requires detailed information on the quantity and composition of incoming and outgoing material flows with their monetary consequences (Aguilar Esteva et al. 2021; Prosman et al. 2017; Schmidt 2005). In addition, reducing carbon emissions caused by the material and energy flows is essential to achieve carbon netzero production processes (Greenhouse Gas Protocol 2015; Intergovernmental Panel on Climate Change 2023). This results in the need for suitable carbon accounting systems to control carbon emissions at the company, cost center, and product unit level (Gibassier and Schaltegger 2015; Penman 2024; Reichelstein 2023).

A potentially suitable environmental cost accounting system for this purpose is material flow cost accounting (MFCA). It measures the quantities and costs of the material flows inside a company and determines the costs of products separately from those of material and product losses (Kawalla et al. 2018; Kitada et al. 2022; Nishimura et al. 2021). However, MFCA does not differentiate between the effects of the influencing factors. These factors include, on the one hand, material demand increasing inefficiency factors, especially waste and reject. On the other hand, sustainability managers seek environmental protection measures to decrease material demand. These measures include recycling and reworking, two of the most critical factors in reducing inefficiencies. Despite the necessity of differentiated information about the effects of the material increasing and decreasing factors for sustainability management (Hahn 2022), most MFCA concepts do not distinguish between the different causes of material demand, preventing a differentiated analysis of the environmental and economic effects (ISO 2011; Schmidt and Nakajima 2013; Wan et al. 2015). Such analyses require information about the efficient and inefficient material demand and cost at the company, quantity center, and product levels, and, therefore, a production theoretical foundation. Consequently, MFCA is unsuitable for predicting a company's material demand depending on influencing factors. In addition, these shortcomings prevent an extension of MFCA for differentiated planning of carbon emissions caused by the material and energy flows, which could be embedded into a carbon accounting system (May and Günther 2020; Nertinger 2014; Penman 2024; Reichelstein 2023). Furthermore, despite MFCA's focus on material costs, all costs are allocated to the products and material losses based on a material distribution key using the allocation base material demand. This results in unclear consequences for cost allocation and difficulties in integrating MFCA into other cost accounting systems, as the material distribution key is not used in other established cost accounting systems, such as direct costing or activity-based costing (Günther et al. 2015; Wagner 2015).

To overcome these theoretical and practical shortcomings, we design an MFCA system that provides sustainability management with detailed cost information regarding a company's increasing and decreasing material demand factors. We consider waste and rejects as material demand increasing factors, and reworking and recycling as material demand reducing factors linked to environmental protection measures in a material flow model. We analyze the creation and reduction of the material and product losses at the company level; the level of different quantity centers in manufacturing, recycling, and disposal; and the product unit level. Furthermore, we develop an MFCA system to determine the corresponding costs at all three levels, providing a theoretical basis for incorporating carbon emissions caused by material and energy flows. At the product unit level, we present a flexible calculation of product unit costs for various purposes in sustainability management. Finally, we analyze the consequences and possible alternatives of the material distribution key for cost allocation in MFCA.

The paper proceeds as follows: In Chap. 2, we review the MFCA literature. In Chap. 3, we design a material flow model to determine a company's material demand and analyze the corresponding material flows. Additionally, we illustrate the material flow model with an example. Chapter 4 treats the development of an MFCA system to budget the quantity center costs and product unit costs. Chapter 5 concludes the paper with the main results and suggestions for future research.

2 Literature Review

MFCA is one of the most promising environmental cost accounting systems; it separates the costs of the products from those of the material and product losses (Behnami et al. 2019; ISO 2011; Yagi and Kokubu 2019). The costs of products include all the costs for materials that make up a physical part of a company's intended products. In contrast, the costs of the material and product losses encompass all costs that can be directly or indirectly traced back to creating and treating unintended co-products. Despite its features and potential benefits for sustainability management, MFCA is rarely implemented in corporate practice (Bux and Amicarelli 2022; Dekamin and Barmaki 2019). Kokubu and Kitada (2015) analyzed the reasons for the limited application. They identified multiple facilitating and complicating factors that influence a company's successful introduction of this environmental cost accounting tool, such as team cooperation, lack of technical knowledge, and training. However, MFCA's limited use is not only because of company-specific factors but also some general factors.

First, MFCA uses terms, such as cost categories, quantity centers, or the material distribution key, that are uncommon in other cost accounting systems (Nakajima 2004; Nishitani et al. 2022). To inform about the main cost effects at each level of the cost accounting system, MFCA differentiates between the four cost categories: material, energy, system, and waste management costs. Therefore, the well-known cost types, such as material costs, wages, depreciation, and other costs, are aggregated into the four cost categories (Bux and Amicarelli 2022; ISO 2011; Kawalla et al. 2018). All primary material and energy cost types are allocated to the material and energy costs. The other remaining cost types are assigned according to their particular use in the transformation processes to system and waste management costs. The waste management costs consist only of the costs directly related to the treatment, transportation, and reduction of the material and product losses, whereas system costs cannot be clearly attributed to the products or the material and product losses (May and Günther 2020; Nishimura et al. 2021; Schmidt 2005). Quantity centers refer to single transformation processes, so cost centers usually encompass multiple quantity centers, e.g., manufacture, reworking, or disposal quantity centers (Dierkes and Siepelmeyer 2019; Günther et al. 2017; ISO 2011). Additionally, MFCA uses a material distribution key for cost allocation depending on the material demand, which differs from allocation bases in other cost accounting systems (Günther et al. 2015; Wagner 2015).

Second, MFCA is only implemented on a project basis in small companies with comparatively simple production processes (Günther et al. 2015; ISO 2011; Schrack 2016). Accordingly, it is unclear how it can be applied in large corporations with complex production processes. Third, the focus is primarily on the costs of the material and product losses, while the related causes remain unclear (ISO 2011; Schmidt et al. 2015). However, environmental cost accounting systems should identify the causes of these losses as starting points for forecasting and improving a company's economic and environmental performance. Thus, MFCA requires a production theory-based material flow model as known from other cost accounting systems and recycling management.

Kloock (1969) provided a general production theory foundation for cost accounting systems. Production theory analyzes the relationship between the input factors and the output of firms and is used in direct costing, one of the well-established cost accounting systems in practice. However, it can also be used in other cost accounting systems, such as activity-based costing (Dierkes 1998; Lengsfeld 1999). Kloock and Schiller (1997) explain the structure of direct costing and activity-based costing using a production theory-based material flow model for cost budgeting. In the field of environmental cost accounting, some systems also use a production-theoretical foundation. Keilus (1993) develops an environmental cost accounting system for determining product unit costs considering waste, rejects, reworking, and recycling. Letmathe (1998) categorizes the environmental impact that results from corporate manufacturing and develops a production model for their quantitative measurement. He calculates the costs of the environmental impacts and shows the general integration of these costs into a cost accounting system. However, in all these environmental cost accounting systems, the differentiated costs of the material and product losses are not disclosed at either the cost center or product unit levels.

We also found some approaches using material flow models in recycling. Spengler et al. (1997) provide a formal model for planning regarding the dismantled and recycled components from demolition waste and byproducts from the steel industry, and Schmidt (2005) uses a material flow model to predict such flows in complex production processes for an e-waste recycling industry company. Nevertheless, these models do not provide differentiated information on the causes of material and

product losses and are not systematically embedded in a cost accounting system. Therefore, these approaches are insufficient for a differentiated cost forecast in a cost accounting system.

Only Dierkes and Siepelmeyer (2019) have used a material flow model to develop a forward-looking MFCA system. Their system comprises waste and reject as inefficiency factors; they analyze the effects on material demand at quantity center and product unit levels, as well as the costs of the products and the material and product losses. However, they do not consider the effects of reworking and recycling as material demand reducing factors, with recycling, in particular, becoming increasingly crucial for sustainability management. In addition, their basic material flow model is unsuitable for integrating reworking and recycling because it requires a more differentiated separation of the material demands resulting from the inefficiency factors.

3 Material Flow Model

3.1 Overview of the Corporate Transformation Process

A manufacturing company converts materials and intermediate products into products in multi-stage transformation processes associated with creating material and product losses (Aguilar Esteva et al. 2021; Bhimani et al. 2019; Datar and Rajan 2018). While the products are sold to customers, the material and product losses have no economic value for the customers and have negative environmental impacts. The physical transformation processes are executed in quantity centers, with the creation and reduction of material and product losses mainly emerging in manufacturing, recycling, and disposal quantity centers. Therefore, we focus our analysis on the material flows in these quantity centers. Figure 1 gives an overview of the structure of the transformation processe.

Manufacturing quantity centers produce a company's products. Their input includes materials and intermediate products; the output consists of the provisional material and product waste as well as the provisional production yield and rejects. Waste refers to all untransformed materials and intermediate products in a manufacturing process that do not become a physical part of the product (Keilus 1993; Kloock and Schiller 1997; Letmathe 1998). The provisional production yield consists of faultlessly manufactured products, whereas the provisional rejects are products not meeting the predefined quality requirements. Some rejects with minor product defects can be reworked to meet the defined quality requirement associated with the additional use of materials and intermediate products (Krüger 1959; Kilger et al. 2012). We assume that the product defects are immediately detected after manufacturing and are eliminated in the same manufacturing quantity center. To keep our material flow model simple, we do not refer to separate reworking quantity centers, which could be integrated into the material flow model without problems. By adding the reworked products and the provisional production yield, we obtain the final production yield that can be used for sale or as input into other manufacturing quantity centers. The rejects with severe product defects are denoted as final rejects, which



Fig. 1 Corporate transformation process

belong to a manufacturing quantity center's material and product losses, alongside the provisional material and product waste (Chompu-inwai et al. 2015; Schmidt 2005; Schrack 2016).

The provisional material and product waste, as well as the final rejects, cannot be directly reused in the manufacturing quantity centers. Companies implement recycling processes to recover some of these materials and intermediate products. We only consider a company's internal recycling and assume a separate **recycling quantity center** for each material and product loss type. Here, we must consider the additional recycling-related material and product demand (Keilus 1993; Kilger et al. 2012; Schmidt 2005). The output of the recycling quantity centers consists of the recovered materials and products re-entering the transformation processes as well as the material and product losses.

The material and product losses are discarded at separate material and product disposal quantity centers. The input of a **disposal quantity center** includes an additional disposal-related material demand plus the material or product losses. The discarded materials and products are the output of the disposal quantity centers.

3.2 Development of a Material Flow Model without Inefficiency Factors

Efficient material demand is characterized by the absence of material and product losses in the transformation process (Dierkes and Siepelmeyer 2019; ISO 2011). It represents a company's demand for materials and products directly related to producing the intended products. Accordingly, it is unnecessary to include the effects of waste and rejects as inefficiency factors and reworking and recycling as inefficiency-reducing factors. Consequently, we only consider the material demand of the manufacturing quantity centers in a material flow model to determine the efficient material demand.

In manufacturing quantity centers enter m = 1, ..., M materials for manufacturing j = 1, ..., J products, whereby each manufacturing quantity center produces only one product. Therefore, j can be used not only as a product but also as a quantity center index. The direct production coefficient $a_{m,j}^{rm'}$ represents the efficient demand for material m used to manufacture one unit of product j.¹ In addition to this primary material demand, manufacturing quantity centers require intermediate products as a secondary material demand (Dörner 1984; Keilus 1993; Kloock and Schiller 1997). The direct production coefficient $a_{j,k}^{rp'}$ indicates the efficient demand for intermediate product j required for manufacturing one unit of product k, where k = 1, ..., J is another product and quantity center index. A quantity center's primary and secondary material demand depends on the sales volume x_{a_j} of product j. Thus, we can calculate the efficient material and product demand rm'_m and rp'_j as follows (Boons 1998; Fandel et al. 2009; Kloock 1999; Kloock and Schiller 1997):²

$$rm'_{m} = \sum_{k=1}^{J} a^{rm'}_{m,k} \cdot rp'_{k}$$
(1)

$$rp'_{j} = \sum_{k=1}^{J} a^{rp\prime}_{j,k} \cdot rp'_{k} + xa_{j}$$
⁽²⁾

We refrain from additionally integrating inventory changes of materials and products as well as sales volumes of materials into the equation system to keep the following analysis as simple and understandable as possible.³ We transform the equation system into a matrix notation to solve for the material and product demand. The symmetrical matrix \underline{A}' represents the efficient direct production coefficients matrix with the dimension M + J times M + J. In (3), the column vectors \underline{xa} and \underline{r}' denote the sales volume and the efficient demand for materials and products (Fandel et al. 2009; Kloock and Schiller 1997):

$$\underline{\mathbf{r}}' = \underline{\mathbf{A}}' \cdot \underline{\mathbf{r}}' + \underline{\mathbf{x}}\underline{\mathbf{a}} \tag{3}$$

¹ In direct costing, the term net production coefficients is used for the efficient direct production coefficients (Kilger et al. 2012).

² A list of the symbols is provided in Appendix A.

³ For a possible integration, see, e.g., Dierkes and Siepelmeyer (2019).

Using the unity matrix \underline{E} to solve for the vector of the efficient material demand, we also obtain the matrix of the efficient total production coefficients \underline{B}' :

$$\underline{\mathbf{r}}' = \left(\underline{\mathbf{E}} - \underline{\mathbf{A}}'\right)^{-1} \cdot \underline{\mathbf{x}} = \underline{\mathbf{B}}' \cdot \underline{\mathbf{x}}$$
(4)

In the following, we have to incorporate the inefficiency and inefficiency-reducing factors in the material flow model, resulting in material and product losses and savings.

3.3 Development of a Material Flow Model with Inefficiency and Inefficiencyreducing Factors

The reference point in MFCA for the differentiation between efficient and inefficient material demand is an inefficient-free production process.⁴ One option for including the inefficiency factors of waste and reject into the material flow model is to add the corresponding effects to the efficient direct production coefficients (Dierkes and Siepelmeyer 2019; Keilus 1993). Regarding waste, this procedure is known from direct costing involving determination of gross production coefficients as the sum of the net production coefficients and the waste per product units (Kilger et al. 2012). However, with the integration of reworking and recycling as inefficiency-reducing factors, this procedure becomes too complex and unsuitable for separate disclosure of the material demand increasing and reducing effects. Therefore, we need a more differentiated material flow model providing information on the material flows of the single inefficiency factors and environmental protection measures.

The starting point for developing the material flow model is the efficient material demand coefficients $a_{m,k}^{rm\prime}$ multiplied with the product demand rp_k . The material demand as input of a manufacturing quantity center is influenced by the inefficiency factor waste and the inefficiency-reducing factor recycling, as illustrated in Fig. 1. Therefore, the quantity of provisional material waste rwm_m increases the demand for material m, whereas the quantity of recycled materials rcm_m of a recycling quantity center has a reducing impact on the primary material demand. This results in the material demand rm_m:

$$rm_{m} = \sum_{k=1}^{J} a_{m,k}^{rm'} \cdot rp_{k} + \underbrace{rwm_{m}}_{\text{provisional}} - \underbrace{rcm_{m}}_{\text{recycled materials}}$$
(5)

Furthermore, the inefficiency factors waste and reject as well as inefficiencyreducing factors reworking and recycling, have effects on the product demand rp_j . On the one hand, the quantities of the provisional product waste rwp_j and provisional rejects rv_j in a manufacturing quantity center increase the product demand. On the

⁴ In direct costing, the waste- and reject-related material demand is undifferentiated included in the budgeted material demand per unit, if the material demand is unavoidable in the production process (Kilger et al. 2012).

other hand, the quantities of the reworked products rn_j and recycled products rcp_j have a product demand reducing effect. Therefore, we obtain the product demand under consideration of the sales volume xa_j :

$$rp_{j} = \sum_{k=1}^{J} a_{j,k}^{rp'} \cdot rp_{k} + \underbrace{rwp_{j}}_{\substack{\text{provisional}\\ \text{product waste}}} + \underbrace{rv_{j}}_{\substack{\text{provisional}\\ \text{rejects}}} - \underbrace{rn_{j}}_{\substack{\text{reworked}\\ \text{products}}} - \underbrace{rcp_{j}}_{\substack{\text{recycled}\\ \text{products}}} + xa_{j}$$
(6)

To calculate the quantities of the provisional material and product waste, we determine the waste-related direct production coefficients $\alpha_{m,j}^{rm}$ and $\alpha_{k,j}^{rp}$ representing the waste-related demand for material m and intermediate product k caused by the production of one unit of product j (Dierkes and Siepelmeyer 2019; Dörner 1984; Keilus 1993):⁵

$$rwm_{m} = \sum_{k=1}^{J} \alpha_{m,k}^{rm} \cdot rp_{k}$$
⁽⁷⁾

$$\operatorname{rwp}_{j} = \sum_{k=1}^{J} \alpha_{j,k}^{\mathrm{rp}} \cdot \operatorname{rp}_{k} \tag{8}$$

Rejects have a wide range of product defects differing in frequency and scope. We assume an average reject rate β_j representing the proportion of the production quantity with slight to severe product defects. Considering the product demand rp_j, we obtain the quantity of the provisional rejects rv_j (Dierkes and Siepelmeyer 2019; Keilus 1993; Kilger et al. 2012):

$$\mathbf{rv}_{\mathbf{j}} = \beta_{\mathbf{j}} \cdot \mathbf{rp}_{\mathbf{j}} \tag{9}$$

For calculating the quantity of reworked products rn_j , we use the reworking rate τ_j as the proportion of reworked products to provisional rejects. The remaining final rejects ra_j cannot be reworked for technological or economic reasons and are forwarded to the recycling quantity centers (Kilger et al. 2012; Letmathe 1998; Schmidt 2005):⁶

$$\mathbf{rn}_{\mathbf{j}} = \tau_{\mathbf{j}} \cdot \mathbf{rv}_{\mathbf{j}} \tag{10}$$

$$\mathbf{ra}_{\mathbf{j}} = (1 - \tau_{\mathbf{j}}) \cdot \mathbf{rv}_{\mathbf{j}} \tag{11}$$

The provisional material and product waste rwm_m and rwp_j as well as the final reject ra_j are treated in the recycling quantity centers in two steps. In the first step, the provisional product waste and final rejects are decomposed to the value-added stage, where their bound materials and products are reusable. The direct production coefficients $a_{m,i}^{rp'}$ and $a_{k,i}^{rp'}$ represent the quantity of material m and intermediate product k

⁵ The causes of waste can be manifold; see Kilger et al. (2012).

⁶ The reworking-related materials and product demand can be additionally considered in (5) and (6). To keep the equations simple, we take this additional demand at quantity center level into account.

that are recovered from a single decomposition stage of product j, whereby the direct product coefficients are deducted from the products' parts lists and decomposition structure. Summing the quantities of the recovered material m and product k from all decomposition stages, we obtain the recycling-related direct production coefficients $a_{m,j}^{rn,\lambda}$ and $a_{j,k}^{rp,\lambda}$. These coefficients determine the quantity of reusable materials and products from a unit of product k. Accordingly, we can calculate the quantities of the reusable material m and product j, rum_m and rup_j from the provisional product waste rwp_k and the final rejects ra_k:

$$\operatorname{rum}_{m} = \sum_{k=1}^{J} a_{m,k}^{\mathrm{rm},\lambda} \cdot (\operatorname{rwp}_{k} + \operatorname{ra}_{k})$$
(12)

$$\operatorname{rup}_{j} = \sum_{k=1}^{J} a_{j,k}^{\operatorname{rp},\lambda} \cdot (\operatorname{rwp}_{k} + \operatorname{ra}_{k})$$
(13)

In the second step, reusable materials and products as well as material waste are recycled. The recycling rates λ_m^{rm} and λ_j^{rp} determine the share of material m and product j recycled from the reusable materials, the provisional material waste, and the reusable products (Keilus 1993; Letmathe 1998). The remaining material and product losses are discarded in disposal quantity centers, as shown in Fig. 1. We can calculate the quantities of the recycled materials and products rcm_m and rcp_j as well as the disposed materials and products rvm_m and rvp_j depending on the recycling rates:⁷

$$\operatorname{rcm}_{\mathrm{m}} = \lambda_{\mathrm{m}}^{\mathrm{rm}} \cdot (\operatorname{rum}_{\mathrm{m}} + \operatorname{rwm}_{\mathrm{m}}) \tag{14}$$

$$\operatorname{rcp}_{j} = \lambda_{j}^{\operatorname{rp}} \cdot \operatorname{rup}_{j} \tag{15}$$

$$\operatorname{rvm}_{\mathrm{m}} = (1 - \lambda_{\mathrm{m}}^{\mathrm{rm}}) \cdot (\operatorname{rum}_{\mathrm{m}} + \operatorname{rwm}_{\mathrm{m}}) \tag{16}$$

$$\operatorname{rvp}_{j} = (1 - \lambda_{j}^{\operatorname{rp}}) \cdot \operatorname{rup}_{j} \tag{17}$$

The equation system with the Eqs. 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17 can be transformed into a matrix notation. The symmetrical matrix <u>A</u> represents the matrix of the direct production coefficients and possesses the dimension $5 \cdot M + 8 \cdot J$ times $5 \cdot M + 8 \cdot J$:

$$\underline{\mathbf{r}} = \underline{\mathbf{A}} \cdot \underline{\mathbf{r}} + \underline{\mathbf{x}}\underline{\mathbf{a}} \tag{18}$$

If we solve for the vector of the demand vector \underline{r} , we obtain the matrix of the total production coefficients \underline{B} :

$$\underline{\mathbf{r}} = (\underline{\mathbf{E}} - \underline{\mathbf{A}})^{-1} \cdot \underline{\mathbf{x}} = \underline{\mathbf{B}} \cdot \underline{\mathbf{x}} a$$
(19)

⁷ We plan the additional recycling- and disposal-related material and product demand at quantity center level, like the one for reworking.

Changes in inventories for the provisional rejects, reworked products, or recycled materials and products can be integrated by adding a corresponding vector to (18). However, according to the determination of the efficient material demand, we forgo their integration into the material flow model. The material flow model can be extended to analyze the carbon emissions. In this case, we have to consider the different energy types in addition to the materials. With the knowledge of the CO_2 emission factors of the material and energy types, differentiated in Scope 1, 2, and 3 CO_2 emissions, we can determine and analyze the CO_2 emissions at the company, quantity center, and product levels.

Using the material flow models from this and the last section, we can determine a company's efficient material demand and total material demand. Hence, we obtain the inefficient material demand v_m of material m by subtracting the efficient material demand rm[']_m from the material demand rm^m (Dierkes and Siepelmeyer 2019):

$$v_{\rm m} = r m_{\rm m} - r m_{\rm m}^{\prime} \tag{20}$$

However, to determine the isolated impact of waste, reject, reworking, and recycling in the quantity centers on material demand, we must analyze the incoming and outgoing material flows of the quantity centers.

3.4 Analysis of the Material Demand at the Quantity Center Level

To identify the effects of a quantity center on material and product losses, we subdivide its material demand into three categories: efficient, inefficient, and inefficiencyreducing material demand. In the first step, we use product-oriented input-output tables disclosing the demand for materials and intermediate products as it is common in other cost accounting systems, such as direct costing. However, in MFCA, the focus is on the material flows regardless of their value-added stage. Therefore, in the second step, we convert the product-oriented input-output tables into materialoriented ones to determine the entering and leaving material flows of a quantity center. According to the sequence of the corporate transformation processes shown in Fig. 1, we begin by analyzing the input and output of the manufacturing quantity centers.

The input of a *manufacturing quantity center* consists of materials denoted as primary demand $rm_{m,j}^p$ and the obtained intermediate products $rp_{k,j}^s$ representing the secondary demand. The primary demand is determined by multiplying the sum of the efficient and waste-related direct production coefficients from (5) and (7) with the production quantity of product j. For the calculation of the secondary demand, we assume that the inefficiency factors occur only in the analyzed quantity center, whereas the processes in all other quantity centers are efficient (Dierkes and Siepelmeyer 2019). Consequently, we can determine the material and product losses caused by the transformation processes of a quantity center. The manufacturing quantity center's output j consists of products and rejects as well as the provisional material and product waste. The quantities of the products and rejects can be directly taken from the material flow model and further subdivided into the sales volume, intermediate products, provisional rejects, reworked products, and final rejects, as

			• • • •	
Input		Ou	tput	
Materials		Pro	oducts	
$m = 1, \ldots, M$	$rm_{m,j}^p$		Sales volume	xaj
Intermediate products	·	+	Intermediate products	$(rp_j - ra_j) - xa_j$
$k = 1, \ldots, J$	$rp_{k,j}^{s}$	=	Final production yield	rp _j – ra _j
		Rej	iects	
			Provisional rejects	rvj
		_	Reworked products	rnj
		=	Final rejects	raj
		Pro	ovisional waste	
			Material $m = 1, \dots, M$	rwm _{m,j}
			Products $k = 1, \ldots, J$	rwp _{k,j}

Table 1 Product-oriented input-output table of manufacturing quantity center j

illustrated in Table 1. The quantities of the provisional material and product waste must be calculated separately by multiplying the waste-related direct production coefficients from (7) and (8) with the production quantity of the manufacturing quantity center j.⁸

To calculate the material flow of material m of a manufacturing quantity center, we transform the product-oriented input-output table into a material-oriented one. The primary material demand $rm_{m,j}^{p}$ of material m is already known from Table 1. The secondary material demand $rm_{m,j}^{s}$ of manufacturing quantity j can be calculated by multiplying the quantity of the obtained intermediate product $rp_{k,j}^{s}$ with the efficient total material demand coefficient $b_{m,k}^{rm'}$. Thus, we can determine the input $rm_{m,j}$ of material m into manufacturing quantity center j (Dierkes and Siepelmeyer 2019):

$$rm_{m,j} = rm_{m,j}^{p} + rm_{m,j}^{s} = \underbrace{\left(a_{m,j}^{rm'} + \alpha_{m,j}^{rm}\right) \cdot rp_{j}}_{primary material demand} + \underbrace{\sum_{k=1}^{J} b_{m,k}^{rm'} \cdot \underbrace{\left(a_{k,j}^{rp'} + \alpha_{k,j}^{rp}\right) \cdot rp_{j}}_{rp_{k,j}^{s}}_{secondary material demand}$$
(21)

On the output-side of a manufacturing quantity center, we subdivide the demand of material m into an efficient, inefficient, and inefficiency-reducing material demand. We start with the calculation of the efficient material demand $rm'_{m,j}$ related to the faultlessly manufactured products and thus no waste or reject:

$$rm'_{m,j} = \underbrace{a_{m,j}^{rm'} \cdot (rp_j - rv_j)}_{primary material demand} + \underbrace{\sum_{k=1}^{J} b_{m,k}^{rm'} \cdot a_{k,j}^{rp'} \cdot (rp_j - rv_j)}_{secondary material demand}$$
(22)

⁸ If the additional reworking-related material and product demand are integrated into the material flow model, the product-oriented input-output table can be extended in this respect.

If we subtract (22) from (21), we receive the inefficient material demand $v_{m,j}$ consisting of the provisional material and product waste as well as the provisional rejects. To reveal the impacts of the reworking activities on the material demand, we replace the provisional rejects rv_j by the sum of the final rejects ra_j and reworked products rn_j . Thus, we can separately disclose the effects of waste and final reject $v_{m,j}^{\alpha}$ and $v_{m,j}^{\beta}$ as well as for reworking $v_{m,j}^{\tau}$, denoted as inefficiency-reducing material demand:

$$\begin{aligned} \mathbf{v}_{m,j} &= \underbrace{\alpha_{m,j}^{rm} \cdot \mathbf{rp}_{j}}_{\text{provisional material waste}} + \underbrace{\sum_{k=1}^{J} \mathbf{b}_{m,k}^{rm'} \cdot \alpha_{k,j}^{rp} \cdot \mathbf{rp}_{j}}_{\text{provisional product waste}} \\ &+ \underbrace{\mathbf{a}_{m,j}^{rm'} \cdot (\mathbf{ra}_{j} + \mathbf{rn}_{j}) + \sum_{k=1}^{J} \mathbf{b}_{m,k}^{rm'} \cdot \mathbf{a}_{k,j}^{rp'} \cdot (\mathbf{ra}_{j} + \mathbf{rn}_{j})}_{\text{provisional rejects}} \end{aligned}$$

$$\begin{aligned} &= \underbrace{\alpha_{m,j}^{rm} \cdot \mathbf{rp}_{j} + \sum_{k=1}^{J} \mathbf{b}_{m,k}^{rm'} \cdot \mathbf{rwp}_{k,j}}_{\text{waste-related material loss}} + \underbrace{\mathbf{a}_{m,j}^{rm'} \cdot \mathbf{rn}_{j} + \sum_{k=1}^{J} \mathbf{b}_{m,k}^{rm'} \cdot \mathbf{rn}_{j}}_{\text{final reject-related material loss}} \end{aligned}$$

$$\begin{aligned} &+ \underbrace{\mathbf{a}_{m,j}^{rm'} \cdot \mathbf{rn}_{j} + \sum_{k=1}^{J} \mathbf{b}_{m,k}^{rm'} \cdot \mathbf{a}_{k,j}^{rp'} \cdot \mathbf{rn}_{j}}_{\text{reworking-related material savings}} \end{aligned}$$

$$\begin{aligned} &= \mathbf{v}_{m,j}^{\alpha} + \mathbf{v}_{m,j}^{\beta} + \mathbf{v}_{m,j}^{\tau} \end{aligned}$$

$$\begin{aligned} \end{aligned}$$

With Eqs. 21, 22 and 23, we can calculate for the manufacturing quantity center a material-oriented input-output table for material m, as illustrated in Table 2.

The manufacturing quantity centers' losses are forwarded to the recycling quantity centers. The input of the *product recycling quantity center j* includes the provisional product waste rwp_j and the final reject ra_j of product j from all manufacturing quantity centers. The quantities of these inputs can be derived from the material flow model. The output of the product recycling center consists of recycled materials and products $rcm_{m,j}$ and $rcp_{k,j}$ as well as the material and product losses $rvm_{m,j}$ and $rvp_{k,j}$. These

Input		Output	
Primary material demand	rm ^p _{m,j}	Efficient material demand	
Secondary material demand	rm ^s _{m,j}	+ Provisional production yield	rm' _{m,j}
		Inefficiency-reducing material demand	
		+ Reworked products	$v_{m,i}^{\tau}$
		Inefficient material demand	
		+ Final rejects	$v_{m,i}^{\beta}$
		+ Provisional waste	$v_{m,j}^{\alpha}$
Material demand	rm _{m,j}	Material demand	rm _{m,j}

Table 2 Material-oriented input-output table of manufacturing quantity center j

Input		Output	
Provisional product waste	rwp _j	Recycled	
Final rejects	raj	Products $k = 1, \dots, J$	rcp _{k,j}
		Materials $m = 1, \dots, M$	rcm _{m,j}
		Losses	
		Products $k = 1, \ldots, J$	rvp _{k,j}
		Materials $m = 1, \dots, M$	rvm _{m,j}

Table 3 Product-oriented input-output table of product recycling quantity center j

quantities can be calculated using the recycling-related production coefficients $a_{m,j}^{rm,\lambda}$ and $a_{k,j}^{rp,\lambda}$ as well as the recycling rates λ_m^{rm} and λ_k^{rp} :

$$\operatorname{rcm}_{m,j} = \lambda_m^{\mathrm{rm}} \cdot a_{m,j}^{\mathrm{rm},\lambda} \cdot (\operatorname{rwp}_j + \operatorname{ra}_j)$$
(24)

$$\operatorname{rcp}_{k,j} = \lambda_k^{\operatorname{rp}} \cdot a_{k,j}^{\operatorname{rp},\lambda} \cdot (\operatorname{rwp}_j + \operatorname{ra}_j)$$
(25)

$$\operatorname{rvm}_{m,j} = (1 - \lambda_m^{rm}) \cdot a_{m,j}^{rm,\lambda} \cdot (\operatorname{rwp}_j + ra_j)$$
⁽²⁶⁾

$$\operatorname{rvp}_{k,j} = (1 - \lambda_k^{\operatorname{rp}}) \cdot a_{k,j}^{\operatorname{rp},\lambda} \cdot (\operatorname{rwp}_j + \operatorname{ra}_j)$$
(27)

The inputs and outputs of a product recycling quantity center can be summarized in a product-oriented input-output table, illustrated in Table 3.⁹

To transform the product-oriented input-output-tables into material-oriented ones, we must derive the corresponding material demand of the input and output. To calculate the input of material m of the recycling center of a product, we multiply the quantities of the provisional product waste and final rejects with the efficient total material demand coefficient $b_{m,j}^{rm'}$. The product recycling quantity center's output includes the inefficiency-reducing and inefficient material demand. The recycled products and materials belong to the inefficiency-reducing material demands of the recycled products and the loss of products are calculated by multiplying (25) and (27) with the efficient total material demand coefficient $b_{m,k}^{rm'}$. The material demand corresponding to recycled materials and material losses are already presented in Table 3. This leads to the material-oriented input-output table of a product recycling quantity center for a material, illustrated in Table 4.

The input of *material recycling quantity center m* includes only the material waste rwm_m. Its output can be subdivided with the recycling rate λ_m^{rm} into the quantities of the recycled material and the material losses, representing the inefficiency-reducing and inefficient material demand, illustrated in Table 5.¹⁰

⁹ The product-oriented input-output table can be expanded if the additional recycling-related material and product demand are integrated into the material flow model.

¹⁰ The additional recycling-related material and product demand can also be integrated into the materialoriented input-output table of the material recycling quantity center.

Input	Output	
Secondary material demand	Inefficiency-reducing material demand	
	+ Recycled products k = 1,, J	
	+ Recycled materials	
	Inefficient material demand	
	+ Loss products k=1,, J	
	+ Loss materials	
Material demand	Material demand	

 Table 4
 Material-oriented input-output table of product recycling quantity center

 Table 5
 Material-oriented input-output table of a material recycling quantity center

Input	Output
Secondary material demand	Inefficiency-reducing material demand
	+ Recycled material
	Inefficient material demand
	+ Loss material
Material demand	Material demand

Next, the material and product losses are discarded in separate *disposal quantity centers* with no further transformation processes. Therefore, we can turn to determining the material demand of product units.

3.5 Determination of the Material Demand at the Product Unit Level

To calculate the material demand of a product unit, we must allocate the material demand from the manufacturing, recycling, and disposal quantity centers to the product units. We start with allocating the efficient, inefficient, and inefficiency-reducing material demand of the manufacturing quantity centers. We can determine the efficient total material demand coefficient already known from Sect. 3.2 by dividing the efficient material demand of *manufacturing quantity center* **j** by its provisional production yield (Dierkes and Siepelmeyer 2019):

$$b_{m,j}^{rm'} = \frac{rm'_{m,j}}{rp_j - rv_j}$$
(28)

In a multi-stage transformation process, the production of product j occurs not only in manufacturing quantity center j, but also in *other manufacturing quantity centers* whose intermediate products go into product j. Therefore, we calculate a separate allocation rate for each manufacturing quantity center for waste, rejects, and reworking. The provisional waste-related allocation rate $ar_{m,j}^{\alpha}$ is calculated by dividing the waste-related material demand of a manufacturing quantity center by its production quantity. To provide separate information at the product unit level on the reject-related material losses and the reworking-related material savings, we calculate with $ar_{m,j}^{\beta+\tau}$ and $ar_{m,j}^{\tau}$, a provisional reject- and reworking-related allocation rate. The reject-related allocation rate represents the quantity of the provisional reject-related material losses without reworking. It is determined by dividing the sum of the final reject- and reworking-related material demand from (23) by the production quantity. In contrast, the reworking-related allocation rate discloses the material savings related to the reworking activities of a manufacturing quantity center. This allocation rate is calculated by dividing the reworking-related material demand of a manufacturing quantity center by its production quantity:

$$ar_{m,j}^{\alpha} = \frac{v_{m,j}^{\alpha}}{rp_{j}} \quad ar_{m,j}^{\beta+\tau} = \frac{v_{m,j}^{\beta} + v_{m,j}^{\tau}}{rp_{j}} \quad ar_{m,j}^{\tau} = \frac{v_{m,j}^{\tau}}{rp_{j}}$$
(29)

To allocate material demand of the *product recycling quantity centers* to the product units, we must determine the recycled quantity of material m per unit of product j. Here, we must consider three components: First, the rejects of manufacturing quantity center j are recycled in the product recycling quantity center j. The recycled material demand can be calculated based on the recycling-related direct production coefficients $a_{m,j}^{m,\lambda}$ and $a_{k,j}^{rp,\lambda}$, the efficient total material demand coefficient $b_{m,k}^{rm'}$, and the recycling rates λ_m^{rm} and λ_k^{rp} . Second, the provisional product waste caused by the production of product j is recycled in product recycling quantity center j and products. Third, the material waste m of manufacturing quantity of the recycled materials is determined by multiplying the recycling rate of material material waste of manufacturing quantity center j. If we sum up the three components of the recovered materials and products, we obtain with $v_{m,j}^{\lambda}$ the recycling-related material demand of manufacturing quantity center j, where symbol h is another product and quantity center index with h=1,..., J:

$$v_{m,j}^{\lambda} = \underbrace{\lambda_{m}^{rm} \cdot a_{m,j}^{rm,\lambda} \cdot ra_{j} + \sum_{k=1}^{J} b_{m,k}^{rm'} \cdot \lambda_{k}^{rp} \cdot a_{k,j}^{rp,\lambda} \cdot ra_{j}}_{\text{product recycling quantity center j}} + \underbrace{\sum_{k=1}^{J} \lambda_{m}^{rm} \cdot a_{m,k}^{rm,\lambda} \cdot rwp_{k,j} + \sum_{k=1}^{J} \sum_{h=1}^{J} b_{m,h}^{rm'} \cdot \lambda_{h}^{rp} \cdot a_{h,k}^{\lambda} \cdot rwp_{k,j}}_{\text{product recycling quantity centers } k=1,...,J}$$
(30)
$$+ \underbrace{\lambda_{m}^{rm} \cdot rwm_{m,j}}_{\text{material recycling quantity center m}}$$

Dividing the recycling-related material demand by the production quantity of manufacturing quantity center j yields a recycling-related allocation rate $ar_{m,j}^{\lambda}$, which can be further disaggregated in separate allocation rates according to (30):

$$ar_{m,j}^{\lambda} = \frac{v_{m,j}^{\lambda}}{rp_{i}}$$
(31)

After calculating the waste-, reject-, reworking-, and recycling-related allocation rates, we can determine the material demand of a product unit. Taking the production relationships among the manufacturing quantity centers into account, we multiply the allocation rates with the total demand coefficients $b_{k,j}^{rp}$ from the total production matrix to receive the total material demand $b_{m,i}^{rm}$ that we already know from (19):

$$b_{m,j}^{rm} = b_{m,j}^{rm'} + \sum_{k=1}^{J} ar_{m,k}^{\alpha} \cdot b_{k,j}^{rp} + \sum_{k=1}^{J} ar_{m,k}^{\beta+\tau} \cdot b_{k,j}^{rp} - \sum_{k=1}^{J} ar_{m,k}^{\tau} \cdot b_{k,j}^{rp}$$

$$= \underbrace{b_{m,j}^{rm'}}_{\text{material demand}} + \underbrace{b_{m,j}^{rm,\alpha}}_{\text{provisional waste-related}} + \underbrace{b_{m,j}^{rm,\beta+\tau}}_{\text{provisional reject-related}} - \underbrace{b_{m,j}^{rm,\tau}}_{\text{final reject-related}} + \underbrace{b_{m,j}^{rm,\beta+\tau}}_{\text{final reject-related}} - \underbrace{b_{m,j}^{rm,\lambda}}_{\text{final reject-related}} + \underbrace{b_{m,j}^{rm,\lambda}}_{\text{recycling-related}} + \underbrace{b_{m,$$

According to (32), we can disaggregate the total material demand per product unit into the efficient material demand $b_{m,j}^{rm'}$, the impacts of waste and provisional reject $b_{m,j}^{rm,\alpha}$ and $b_{m,j}^{rm,\beta+\tau}$ as material demand increasing factors as well as reworking and recycling $b_{m,j}^{rm,\tau}$ and $b_{m,j}^{rm,\lambda}$ as the environmental protection measures' material demand reducing factors.

3.6 Example

The use of the developed material flow model is illustrated by a simple multi-stage transformation process with one material (m=1), two manufacturing quantity centers with one product each (j=1,2), and three recycling quantity centers: one for the material and two for the products. As the disposal quantity centers are unproblematic regarding the material demand, we do not take them into account in the example. The direct production coefficients and the sales volumes are given in Fig. 2. As inefficiency factors, we assume waste-related direct production coefficients for the material $\alpha_{1,1}^{\rm rm} = 0.1$ and $\alpha_{1,2}^{\rm rm} = 0.2$ and reject rates of the products $\beta_1 = 0.05$ and $\beta_2 = 0.01$. The environmental protection measures include reworking and recycling. The reworking rates of products 1 and 2 are $\tau_1 = 0.3$ and $\tau_2 = 0.2$. In the recycling process, the final reject of product 1 is decomposed into two units of material according to the product 2, we assume that 40% is decomposed into product 1 and the material, and 60% is completely decomposed into the material. Considering the material demand coefficients, this leads to recycling-related direct production



Fig. 2 Transformation process of the example

coefficients $a_{1,2}^{rm,\lambda} = 3.4$ for the material and $a_{1,2}^{rp,\lambda} = 0.8$ for product $1.^{11}$ The recycling rates of the material and product 1 are assumed to be $\lambda_1^{rm} = 0.2$ and $\lambda_1^{rp} = 0.3.^{12}$ All information on the transformation process of the example is summarized in Fig. 2.

The efficient material demand and product demand can be determined with Eqs. 1 and 2: $rp'_2 = 2000$, $rp'_1 = 2 \cdot 2000 + 1000 = 5000$ and $rm'_1 = 2 \cdot 5000 + 1 \cdot 2000 =$ 12,000. Determining the efficient material and product demand using the matrix of the efficient total production coefficients is included in Appendix B. Furthermore, the equation system, the direct demand matrix, and the total demand matrix to determine the material demand and production units of products 1 and 2 are provided in Appendices C and D. The material demand amounts to $rm_1 = 13,093.18$. Subtracting the efficient material demand from this material demand determines the inefficient material demand $v_1 = 1093.18$. According to Sect. 2.4, the efficient and inefficient material demand can be assigned to the quantity centers with a differentiated reporting of the impacts of waste, reject, reworking, and recycling. Tables 6

¹¹ The decomposition of one unit of the final reject of product 2 yields under consideration of the efficient total demand coefficient for the material to $0.6 \cdot 5 = 3$ units of the material. The other 0.4 units of product 2 are decomposed in $0.4 \cdot 2 = 0.8$ units of product 1 and $0.4 \cdot 1 = 0.4$ units of the material. As result, we obtain the recycling-related direct production coefficients 3 + 0.4 = 3.4 for the material and 0.8 for product 1.

 $^{^{12}}$ The recycling rate for product 2 is not necessary because every unit of the final reject of product 2 is decomposed into the material and product 1.

and 7 illustrate the material-oriented input-output tables of the manufacturing quantity centers 1 and 2. The product-oriented input-output-tables of the manufacturing centers and the product- and material-oriented input-output-tables of the recycling centers are listed in Appendix E.

The differentiated determination of material demand at the quantity center level enables the corresponding differentiation at the product unit level. We obtain the efficient total material demand coefficients of the two products according to Eq. 28: $b_{1,1}^{mr} = \frac{9900.45}{5210.76-260.54} = 2$ and $b_{1,2}^{mr} = \frac{9979.84}{2016.13-20.16} = 5$. The inefficient material demand for waste and reject as well as the inefficiency-reducing material demand for reworking and recycling can be assigned to the product units with the allocation rates according to Eq. 29. The allocation rates are listed in Table 8.

Using the total demand coefficients of the total material demand coefficients matrix enables, using Eq. 32, the differentiated determination of the material demand on the product unit level provided in Table 9.

For example, we obtain the waste-related inefficient material demands per product unit by using the provisional waste-related allocation rates and the total demand coefficients of the products in the total demand coefficient matrix in Appendix D: $1.0363 \cdot 0.1 = 0.1036$ for product 1 and $2.0872 \cdot 0.1 + 1.0081 \cdot 0.2 = 0.4103$ for product 2.

The example illustrates the usefulness of the material flow model for determining the material demand at the company, quantity center, and product unit levels. In

Input		Output	
Primary material demand	10,942.60	Efficient material demand	
Secondary material demand	0	+ Provisional production yield	9900.45
		Inefficiency-reducing material demand	
		+ Reworked products	156.32
		Inefficient material demand	
		+ Final rejects	364.75
		+ Provisional waste	521.08
Material demand	10,942.60	Material demand	10,942.60

Table 6 Material-oriented input-output table of manufacturing quantity center 1

r 2

Input		Output	
Primary material demand	2419.35	Efficient material demand	
Secondary material demand	8064.52	+ Provisional production yield	9979.84
		Inefficiency-reducing material demand	
		+ Reworked products	20.16
		Inefficient material demand	
		+ Final rejects	80.65
		+ Provisional waste	403.87
Material demand	10,483.87	Material demand	10,483.87

Table 8	Waste-, reject-	, reworking-,	and recycling-related	allocation rates
---------	-----------------	---------------	-----------------------	------------------

Manufacturing quan- tity center k	1	2
Provisional waste-relat	ed allocation rates	
$\operatorname{ar}_{1,k}^{\alpha} = \frac{v_{1,k}^{\alpha}}{rp_k}$	$\operatorname{ar}_{1,1}^{\alpha} = \frac{521.08}{5210.76} = 0.1$	$\operatorname{ar}_{1,2}^{\alpha} = \frac{403.23}{2016.13} = 0.2$
Reject-related allocatio	on rates	
$\operatorname{ar}_{1,k}^{\beta+\tau} = \frac{v_{1,k}^{\beta} + v_{1,k}^{\tau}}{rp_k}$	$\operatorname{ar}_{1,1}^{\beta+\tau} = \frac{156.32 + 364.75}{5210.76} = 0.1$	$\operatorname{ar}_{1,2}^{\beta+\tau} = \frac{20.16 + 80.65}{2016.13} = 0.05$
Reworking-related allo	cation rates	
$\operatorname{ar}_{1,k}^{\tau} = \frac{v_{1,k}^{\tau}}{r p_k}$	$\operatorname{ar}_{1,1}^{\tau} = \frac{156.32}{5510.76} = 0.03$	$\operatorname{ar}_{1,2}^{\tau} = \frac{20.16}{5510.76} = 0.01$
Recycling-related alloc	ation rates	
$\operatorname{ar}_{1,k}^{\lambda} = \frac{\operatorname{v}_{1,k}^{\lambda}}{\operatorname{rp}_{k}}$	$ar_{1,1}^{\lambda} = \frac{104.22 + 72.95}{5210.76} = \frac{177.17}{5710.76} = 0.0340$	$ar_{1,2}^{\lambda} = \frac{80.65 + 18.71}{2016.13} = \frac{99.35}{2016.13} = 0.0493$

For the determination of the recycling-related material demand in the calculation of the recycling-related calculation rates see the material-oriented input-output tables of the recycling quantity centers in Appendix E.

Product j	1	2
Efficient material demand per product unit	2.0000	5.0000
Inefficient material demand per product unit		
Waste	0.1036	0.4103
Reject	0.1036	0.2591
Sum	0.2073	0.6695
Inefficiency-decreasing material demand per product unit		
Reworking	0.0311	0.0727
Recycling	0.0352	0.1206
Sum	0.0663	0.1933
Material demand per product unit (resp. total material de- mand coefficient)	2.1409	5.4761

 Table 9
 Differentiated calculation of the material demand per product unit

practical application, the material flow model must be implemented in an IT system with an automatized solving procedure. Based on the material flow model, we can now develop an MFCA system.

4 Development of the Material Flow Cost Accounting System

4.1 Assumptions and Structure of the Material Flow Cost Accounting System

The MFCA system should provide sustainability management with differentiated information on product costs as well as the cost of material and product losses. We make the following basic assumptions in our MFCA system:

• Cost type, cost or quantity center, and cost unit accounting

- The common cost accounting systems consist of a cost type, cost center, and cost unit accounting. We assume the same structure for the MFCA system to facilitate its integration into other cost accounting systems and the organization of companies. Cost centers usually include more than one quantity center (Dierkes and Siepelmeyer 2019; Günther et al. 2017; ISO 2011).
- *Full cost accounting system* MFCA is primarily described as a full cost accounting system (e.g., Chompu-inwai et al. 2015; Dekamin and Barmaki 2019; May and Günther 2020). Therefore, we also design a full cost accounting system and do not differentiate between variable and fixed costs.¹³
- Determination of the costs of the products as well as material and product losses The MFCA focuses on determining the costs of the products as well as the costs of the material and product losses (Schmidt and Nakajima 2013; Schmidt 2015; Schrack 2016). To provide management with differentiated information on a company's environmental impacts and environmental protection measures, we distinguish between the costs of the product-, waste-, reject-, reworking-, recycling-, and disposal-related material flows at the quantity center and product unit levels.
- *Cost planning at the quantity center level* We plan the costs of each cost type at the quantity center level, although cost budgeting has hardly been discussed in MFCA. For cost planning, we can resort to the procedures known from other cost accounting systems (Bhimani et al. 2019; Coenenberg et al. 2016; Datar and Rajan 2018; Ewert et al. 2023). If cost budgeting is not possible at the quantity center level for technical or economic reasons, it should be done at the cost center level. In this case, the cost center costs must be subdivided among the quantity centers based on suitable allocation criteria.
- Aggregation of the cost types into cost categories The different cost types of a quantity center, such as material costs, wages, depreciation, and other costs, are aggregated into four cost categories: material, energy, system, and waste management costs (Bux and Amicarelli 2022; ISO 2011; Kawalla et al. 2018).
- Application of the material distribution key The material distribution key is used in MFCA to subdivide the energy and system costs on the basis of the material quantities between the products and the material and product losses (Behnami et al. 2019; Günther et al. 2017; Schmidt and Nakajima 2013). Accordingly, we use this allocation criterion in our more differentiated MFCA system for each allocation of costs to material flows and thus also, for example, to allocate waste management costs to different material flows.
- Allocation of the efficient costs between the manufacturing quantity centers In our material flow model, we do not allocate the materials between the quantity centers relating to the material and product losses. Accordingly, we assign only the products' costs among the manufacturing quantity centers (Günther et al. 2015; Ho et al. 2021; ISO 2011). The costs of the provisional waste and the final reject, as

¹³ For MFCA as a marginal cost accounting system, see Dierkes and Siepelmeyer (2019).

well as the ones for recycling and disposal, are assigned from the quantity centers to the product units.

Based on the assumptions, we obtain the structure of the MFCA system illustrated in Fig. 3.

After presenting the central assumptions and structure of the MFCA system, we explain the single cost budgeting steps in the next section.

4.2 Quantity Center Accounting

The costs of a *manufacturing quantity center* are divided into the costs of the four categories material, energy, system, and waste management costs, which can be further subdivided into primary and secondary costs (Dierkes and Siepelmeyer 2019). Using the material flow model and the material distribution key, these costs can be allocated to products as well as to the material and product losses. The product costs can be separated into the costs of provisional production yield and reworked products. Accordingly, the costs of the material and product losses can be divided into the cost of final rejects and costs of provisional waste. Furthermore, we can split the costs of provisional waste into the costs of provisional material and intermediate product waste, with the material costs including only the primary and secondary costs, respectively. Additionally, we must consider the costs for the reworkingrelated material demand (Keilus 1993; Letmathe 1998), which we did not include in the material flow model. The waste management costs consist only of primary costs because no inefficient costs are allocated between the manufacturing quantity centers. The primary energy, system, and waste management costs are assigned with the material distribution key (MDK) to the material flows (Ho et al. 2021; ISO 2011; Kawalla et al. 2018). The cost planning in a manufacturing quantity center can be



 calculation of the product unit costs differentiated by the costs of the products and the costs of the material and product losses

Fig. 3 Structure of the material flow cost accounting system

summarized in a cost calculation scheme, shown in Fig. 4 For allocating the costs to the product units, we use the allocation bases according to the material flow model in Sect. 3.5. Therefore, the provisional production yield costs are allocated based on the faultlessly manufactured products $rp_j - rv_j$. The manufacturing costs of the other elements are distributed based on the production volume rp_i .

The costs of the *product recycling quantity centers* are structured into the costs of their outgoing material flows and the four cost categories. The material costs can be divided into primary and secondary costs. Energy, system, and waste management costs include only primary costs, because we do not allocate inefficient costs other than material costs from the manufacturing quantity centers to the recycling quantity centers. The product recycling quantity centers' costs are subdivided among the recycled products and materials as well as the recycling-related materials and the material and product losses. We separate the costs of the recycling-related materials from the costs of recycled products and materials to determine the opposing economic effects of the recycling measures at the quantity center and product unit levels, as it is common in environmental cost accounting (Diaz et al. 2022; Keilus 1993; Letmathe 1998). The costs of the material flows are allocated based on the production quantities of the manufacturing quantity centers to the product units according to the material flow model in Sect. 3.5. The result of the cost planning in a product recycling quantity center is summarized in Fig. 5.

As the cost calculation schemes of the material recycling quantity centers and those of the product and material disposing quantity centers are structured accordingly, we will not go into greater detail here. As a result, we have all the necessary cost rates of the quantity centers for cost unit accounting.

4.3 Cost Unit Accounting

In cost unit accounting, we calculate the product unit costs consisting of the efficient, inefficient, and inefficiency-reducing costs of the manufacturing, recycling, and disposal quantity centers. We develop a flexible designed product unit cost calculation scheme to allocate waste, rejects, reworking, and recycling costs for singleand multi-dimensional analyses to provide sustainability management with decisionuseful cost information.

To determine the product unit costs, we must allocate the costs of the different material flows from all manufacturing, recycling, and disposal quantity centers to the product units (Dierkes and Siepelmeyer 2019). We must multiply the cost rate of each material flow of the quantity centers by the corresponding total material demand coefficient and sum up the cost amounts over all material flows, cost categories, and quantity centers. Thus, we obtain the calculation scheme for the product unit costs listed in Table 10, with the manufacturing, recycling, and disposal costs per product unit. Each of the three elements can be further subdivided. The manufacturing quantity center costs per product unit can be split into the efficient, waste-, reject-, and reworking-related manufacturing costs as well as the reworking-related manufacturing costs per product unit. The recycling costs per product unit can be disaggregated into the recycling-related material cost savings per product unit, the recycling-related material costs increase per product unit, and the loss-

Cos	at categories	-	Product		-	Mater	ial and product	losses		Sum
		Provisional	Reworked	Sum	Reworking-	Final rejects	Provision	al waste	Sum	
		production	product		related ma-		Materials	Products		
		yield			terials					
Ma	terial costs (1)									
	Primary costs									
+	Secondary costs									
Ш	Sum									
Em	rgy costs (2)	↦	→		→	→	→	→	MDK	
	Primary costs									Plan
+	Secondary costs									
Ш	Sum									
Sys	tem costs (3)	•	→		→	→	→	→	MDK	
	Primary costs									Plan
+	Secondary costs									
	Sum									
Wa	ste management cos	ts (4)			\	→	→	→	MDK	
	Primary costs									Plan
+	Secondary costs									
Ш	Sum									
Ma	nufacturing costs (1)	()+(2)+(3)+(4)								
	Primary costs									
+	Secondary costs									
Ш	Sum									
Co	st allocation base	$rp_j - rv_j$	rp_j		rp j	rp_j	rp_j	rp_j		
Cos	it rate									

Fig. 4 Cost calculation scheme of a manufacturing quantity center

Co	st categories	Recy	vcled	Losses				
		Products	Materials	Recy-	Products	Materials	Sum	
				cling-re-				
				lated ma-				
				terials				
M	aterial costs (1)							
	Primary costs							
+	Secondary							
	costs							
=	Sum							
En	ergy costs (2)	↓	+	+	+	¥	MDK	
	Primary costs							Plan
Sy	stem costs (3)	↓	+	+	+	+	MDK	
	Primary costs							Plan
W	aste management o	costs (4)		↓	+	+	MDK	
	Primary costs							Plan
Re	cycling costs (1)+((2)+(3)+(4)						
	Primary costs							
+	Secondary							
	costs							
=	Sum							
Co	st allocation	rp.	rp.	rp.	rp.	rp.		
ba	se	- r [,] j	- r [,] j	- P j	- P J	- r' j		
Co	st rate							

Fig. 5 Cost calculation scheme of the product recycling quantity center

related costs per product unit. To calculate these costs, we assign the costs of the material flows from all product and material recycling quantity centers to the product unit. To determine the disposal costs per product unit of a product, we allocate the disposal costs of the material and product losses in the disposal quantity centers caused by a product unit. As the disposal quantity centers include only one material or product, we forego further separation in the following calculation scheme of the product unit costs.

Furthermore, the product unit costs can be disaggregated according to other analysis criteria, such as cost categories, quantity centers, primary and secondary costs, efficient, inefficient, and inefficiency-reducing costs, materials and products, and

Table 10	Cost calculation scheme of the product unit costs
+	Efficient manufacturing costs per product unit
+	Waste-related manufacturing costs per product unit
+	Reject-related manufacturing costs per product unit
-	Reworking-related manufacturing cost savings per product unit
+	Reworking-related manufacturing cost increase per product unit
=	Manufacturing costs per product unit (1)
-	Recycling-related material cost savings per product unit
+	Recycling-related cost increase per product unit
+	Loss-related costs per product unit
=	Recycling costs per product unit (2)
	Disposal costs per product unit (3)
=	Product unit costs $(1) + (2) + (3)$

material types. This variety of analysis dimensions allows for adjusting product unit costs to the sustainability management's information needs. In additional multidimensional analyses, we can combine the analysis dimensions, for example, the criteria cost categories and quantity centers, which provide insights into the cost structure of the quantity centers and their cost contributions to the product unit costs (Bhimani et al. 2019; Coenenberg et al. 2016; Datar and Rajan 2018). Moreover, changing the analysis criteria sequence provides additional insights into the product unit costs.

4.4 Effects and Alternatives to the Use of the Material Distribution Key

The material distribution key is used in MFCA to distribute the energy and system costs between the products and the material and product losses (Bux and Amicarelli 2022; Ho et al. 2021; Kawalla et al. 2018). Accordingly, products are charged with higher costs the higher their material demand, which can be interpreted as penalty costs for material consumption. Consequently, quantity center owners or product managers are incentivized to reduce the material demand of the product, especially the material and product losses. This is sensible from a sustainability perspective, as a company's efforts are directed towards reducing material demand, but this procedure does not take sufficiently into account the cost drivers for system, energy, and waste management costs (ISO 2011; Wagner 2015).

If MFCA is used to provide decision-useful information, the material distribution key often leads to an unjustified allocation of costs to products as well as material and product losses. In addition, the use of the material distribution key results in problems in integrating MFCA into other cost accounting systems, since they usually allocate the costs according to the principle of cost causation or demand (Coenenberg et al. 2016; Datar and Rajan 2018; Kilger et al. 2012). To provide decision-useful information and to increase the connectivity of MFCA with more established cost accounting systems, we should use these allocation criteria to allocate costs to the products and the material and product losses. Compared to the material distribution key, these allocation criteria are more closely related to the production processes of a quantity center, such as the production quantities, manufacturing minutes, or number of processes (Bhimani et al. 2019; Guan et al. 2009; Kilger et al. 2012). Moreover, the focus on the corporate transformation processes leads to the idea of an activity-based expansion of MFCA. In this case, the costs in quantity centers are allocated based on the activities among the products as well as the material and product losses, as it is known in activity-based costing (Cooper and Kaplan 1988; Jing and Songqing 2011; Schweitzer et al. 2016). This results in even more precise cost allocation, but one must weigh the related benefits against the additional information costs.

5 Conclusion

Sustainability management requires differentiated cost information about the ecological and economic consequences of a company's transformation processes. As it separately determines the costs of the products and those of the material and product losses, one suitable cost accounting system for this purpose is MFCA (Kitada et al. 2022; Nishitani et al. 2022; Wagner 2015). However, MFCA has paid less attention to different causes of material and product losses and related costs. Accordingly, the consequences of waste and reject as inefficiency factors, and reworking and recycling as inefficiency-reducing factors, remain unclear.

We developed an MFCA system for budgeting the costs of the products and the costs of the material and product losses. We designed a production theory-based material flow model considering the effects of waste and rejects as inefficiency factors, as well as reworking and recycling as inefficiency-reducing factors, on a company's material demand for any transformation process. Moreover, we analyzed the material flows of a company's manufacturing, recycling, and disposal quantity centers. We allocated the material demands from the quantity centers to the product units and differentiated between the material increasing and decreasing effects. Based on the material flow model, we presented the assumptions and structure of an MFCA as a full cost accounting system. Its main characteristic is the possibility to analyze the impacts of waste, rejects, reworking, and recycling on product costs and the costs of material and product losses at the company, quantity center, and product unit levels. Therefore, MFCA provides sustainability management with relevant cost information; but it is important to consider the consequences of using the material distribution key, which results in cost charges according to the material demand. Although this leads to a desirable incentive to reduce material losses from a sustainability perspective, it does not necessarily correspond to cost causation. For this reason, we discussed alternative cost allocations resulting in improved decisionuseful cost information.

The presented MFCA system can be expanded in several ways. We have excluded inventory changes, sales volumes of materials as well as recycling-, and disposalrelated additional material demand from the material flow model, which can be integrated into the model. Furthermore, further inefficiency factors, such as throughput speed, human error, or material quality can be included (Kilger et al. 2012; Schmidt 2015). In addition, the cost accounting system can be expanded from a single company to a product's entire value chain, which thereby helps to measure the productand material loss-related costs at each value-added stage considering company's external recycling (Günther et al. 2015; Schmidt et al. 2015; Schrack 2016). The external costs of the business activities can also be included in the MFCA system to provide sustainability management information on the costs of all environmental impacts of a company's products as well as material and product losses. Finally, the material flow model provides the theoretical basis for a differentiated analysis of CO₂ emissions caused by the material and energy flows. Therefore, it would be of special interest to further develop the material flow model for analyzing Scope 1, 2, and 3 CO₂ emissions of a firm.

6 Appendix A: List of symbols

a ^{rm} ′	efficient direct demand of material m per unit of product j
$a_{j,k}^{rp\prime}$	efficient direct demand of product j per unit of product k
$a_{m,j}^{rm,\lambda}$	recycling-related direct production coefficient of material m per unit of product i
$a_{j,k}^{rp,\lambda}$	recycling-related direct production coefficient of product j per unit of product k
$ar_{m,i}^{\alpha}$	waste-related allocation rate of material m for product j
$ar_{m,j}^{\beta+\tau}$	provisional reject-related allocation rate of material m for product j
$ar_{m,j}^{\tau}$	reworking-related allocation rate of material m for product j
$\mathrm{ar}_{\mathrm{m,j}}^{\lambda}$	recycling-related allocation rate of material m for product j
b ^{rm} ′ m,j	efficient total demand of material m per unit of product j
$b_{m,j}^{rm, \alpha}$	provisional waste-related demand of material m per unit of product j
$b_{m,j}^{\mathrm{rm},eta+ au}$	provisional reject-related demand of material m per unit of product j
$\mathbf{b}_{\mathrm{m,j}}^{\mathrm{rm, au}}$	reworking-related savings of material m per unit of product j
$b_{m,j}^{\mathrm{rm},\lambda}$ r a_j	recycling-related savings of material m per unit of product j final reject of product j
rcm _m	recycled materials of material m
rcm _{m,j}	recycled materials of material m caused by product j
rcpj	recycled products of product j
rcp _{k,j}	recycled products of product k caused by product j
rm _m	efficient material demand of material m
rm _m	material demand of material m
rm _{m,j}	material demand of material m caused by product j
rnj	reworked products of product j
rp'	efficient product demand product j
rpj	product demand of product j
$rp_{k,j}$	product demand of product k caused by product j
rum _m	reusable materials of material m
rup _j	reusable products of product j
rvj	provisional rejects of product j
rvm _m	disposed materials of material m
rvm _{m,j}	disposed materials of material m caused by product j
rvpj	disposed products of product j
rvp _{k,j}	disposed products of product k caused by product j
rwm _m	provisional waste of material m
rwm _{m,j}	provisional waste of material m caused by product j
rwpj	provisional waste of product j
rwp _{k,j}	provisional waste of product k caused by product j

Vm	inefficient material demand of material m
V _{m,j}	inefficient material demand of material m caused by product j
v _{m,j}	waste-related material loss of material m caused by product j
$v_{m,j}^{\beta}$	final reject-related material loss of material m caused by product j
$v_{m,j}^{\tau}$	reworking-related material savings of material m caused by product j
$v_{m,i}^{\lambda}$	recycling-related material demand of material m caused by product j
xaj	sales volume of product j
$\alpha_{\mathrm{m,k}}^{\mathrm{rm}}$	waste-related demand of material m per unit of product k
$\alpha_{j,k}^{\mathrm{rp}}$	waste-related demand of product j per unit of product k
β_{i}	reject rate of product j
$\lambda_{\rm m}^{\rm rm}$	recycling rate of material m
λ_{i}^{rp}	recycling rate of product j
τ_{i}	reworking rate of product i
v J	Ten offining face of product j

Indices and Subscripts

/	efficient demand
h,j,k	product indices; $h, j, k = 1,, J$
m	material index; m=1,, M
р	primary demand
rm	related to materials
rp	related to products
S	secondary demand
α	related to waste
β	related to reject
λ	related to recycling
τ	related to reworking

7 Appendix B: Determination of the Efficient Material and Product Demand with the Matrix of the Total Production Coefficients

• Efficient direct production coefficients matrix

$$\underline{\mathbf{A}}' = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

• Efficient total production coefficients matrix

$$\underline{\mathbf{B}}' = \begin{pmatrix} 1 & 2 & 5\\ 0 & 1 & 2\\ 0 & 0 & 1 \end{pmatrix}$$

• Efficient material and product demand

$$\underline{\mathbf{r}}' = \begin{pmatrix} \mathbf{rm}_1 \\ \mathbf{rp}_1 \\ \mathbf{rp}_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1000 \\ 2000 \end{pmatrix} = \begin{pmatrix} 12,000 \\ 5000 \\ 2000 \end{pmatrix}$$

8 Appendix C: Equation System of the Material Flow Model

Material demand

$$rm_1 = 2 \cdot rp_1 + 1 \cdot rp_2 + rwm_1 - rcm_1$$

Product demand

 $rp_1 = 2 \cdot rp_2 + rwp_1 + rv_1 - rn_1 - rcp_1 + 1000$ $rp_2 = rv_2 - rn_2 + 2000$

• Material waste (no product waste in the example)

 $rwm_1 = 0.1 \cdot rp_1 + 0.2 \cdot rp_2$

Provisional reject

 $\begin{aligned} rv_1 &= 0.05 \cdot rp_1 \\ rv_2 &= 0.01 \cdot rp_2 \end{aligned}$

Reworked products

 $\begin{aligned} rn_1 &= 0.3 \cdot rv_1 \\ rn_2 &= 0.2 \cdot rv_2 \end{aligned}$

• Final reject

 $ra_1 = rv_1 - rn_1$ $ra_2 = rv_2 - rn_2$

• Reusable material and products (no reusable units of product 2 in the example)

 $rum_1 = 2 \cdot ra_1 + 3.4 \cdot ra_2$ $rup_1 = 0.8 \cdot ra_2$

• Recycled materials and products

 $rcm_1 = 0.2 \cdot rwm_1 + 0.2 \cdot rum_1$ $rcp_1 = 0.3 \cdot rup_1$

• Final waste material and product

 $\begin{aligned} \mathrm{rvm}_1 &= (1-0.2) \cdot \mathrm{rvm}_1 + (1-0.2) \cdot \mathrm{rum}_1 = 0.8 \cdot \mathrm{rvm}_1 + 0.8 \cdot \mathrm{rum}_1 \\ \mathrm{rvp}_1 &= (1-0.3) \cdot \mathrm{rup}_1 = 0.7 \cdot \mathrm{rup}_1 \end{aligned}$

9 Appendix D: Determination of the Material and Product Demand with the Matrix of the Total Production Coefficients

• Direct production coefficient matrix

	(0	2	1	1	0	0	0	0	0	0	0	0	-1	0	0	0 \
	0	0	2	0	1	0	$^{-1}$	0	0	0	0	0	0	$^{-1}$	0	0
	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0
	0	0.1	0.2	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0.3	0	0	0	0	0	0	0	0	0	0	0
۸ <u> </u>	0	0	0	0	0	0.2	0	0	0	0	0	0	0	0	0	0
$\underline{\mathbf{A}} =$	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	2	3.4	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0.8	0	0	0	0	0	0
	0	0	0	0.2	0	0	0	0	0	0	0.2	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0.3	0	0	0	0
	0	0	0	0.8	0	0	0	0	0	0	0.8	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0.7	0	0	0	0 /

The dimension of the symmetrical matrix is 16 times 16 lower than $5 \cdot M + 8 \cdot J = 5 \cdot 1 + 8 \cdot 2 = 21$, because we have no waste of both products, no reusable product 2, no recycled product 2, and no waste of product 2 in the example.

• Total production coefficient matrix

	1	2.1409	5.4761	0.8	1.2187	3.4258	-1.7409	-4.2823		
	0	1.0363	2.0872	0	0.7254	1.4708	-1.0363	-1.8385		
	0	0	1.0081	0	0	0.8065	0	-1.0081		
	0	0.1036	0.4103	1	0.0725	0.3084	-0.1036	-0.3855		
	0	0.0518	0.1044	0	1.0363	0.0735	-0.0518	-0.0919		
	0	0	0.0101	0	0	1.0081	0	-0.0101		
	0	0.0155	0.0313	0	0.3109	0.0221	0.9845	-0.0276		
р_	0	0	0.0020	0	0	0.2016	0	0.9980		
<u></u>	0	0.0363	0.0731	0	0.7254	0.0515	-1.0363	-0.0643	•••	•
	0	0	0.0081	0	0	0.8065	0	-1.0081		
	0	0.0725	0.1735	0	1.4508	2.8449	-2.0725	-3.5561		
	0	0	0.0065	0	0	0.6452	0	-0.8065		
	0	0.0352	0.1168	0.2.	0.3047	0.6307	-0.4352	-0.7883		
	0	0	0.0019	0	0	0.1935	0	-0.2419		
	0	0.1409	0.4671	0.8	1.2187	2.5226	-1.7409	-3.1533		
	0/	0	0.0045	0	0	0.4516	0	-0.5645		
			-0.4	-1.19	38 -0.2	-0.642	3 -1	-2.1409	0	0
			0	-0.24	87 0	-0.310	90	-1.0363	0	0
			0	0	0	0	0	0	0	0
			0	-0.02	49 0	-0.031	1 0	-0.1036	0	0
			0	-0.01	24 0	-0.015	5 0	-0.0518	0	0
			0	0	0	0	0	0	0	0
			0	-0.00	37 0	-0.004	70	-0.0155	0	0
			0	0	0	0	0	0	0	0
			1	-0.00	87 0	-0010	90	-0.0363	0	0
			0	1	0	0	0	0	0	0
			2	3.382	26 1	-0.021	8 0	-0.0725	0	0
			0	0.800	0 0	1	0	0	0	0
			0.4	0.671	5 0.2	-0.010	6 1	-0.0352	0	0
			0	0.240	0 0	0.3000) ()	1	0	0
			1.6	2.686	62 0.8	-0.042	3 0	-0.1409	1	0
			0	0.560	0 0	0.7	0	0	0	1/

• Determination of the demand vector

	(rm_1)		(0)		(13,093.18)	
	rp ₁		1000		5210.76	
	rp ₂		2000		2016.76	
	rwm1		0		924.30	
	rv ₁		0		260.54	
	rv ₂		0		20.16	
	rn ₁			0		78.16
r	rn ₂	_ D	0	_	4.03	
<u>1</u> —	ra ₁	$= \overline{\mathbf{p}}$.	0	_	182.38	
	ra ₂		0		16.13	
	rum ₁		0		419.59	
	rup ₁		0		12.90	
	rcm1		0		268.78	
	rcp ₁		0		3.87	
	rvm ₁		0		1075.12	
	rvp ₁		\ 0 <i>)</i>		9.03	

10 Appendix E: Product- and Material-oriented Input-output Tables of the Quantity Centers

Table E.1	Product-oriented	input-output	it table of	f manufacturing	quantity	center	1
-----------	------------------	--------------	-------------	-----------------	----------	--------	---

Input		Output					
Material	10,942.60	Products					
Intermediate products	0.00	Sales volume	1000.00				
		+ Intermediate products	4028.39				
		= Final production yield	5028.39				
		Rejects					
		Provisional rejects	260.54				
		 Reworked products 	78.16				
		= Final rejects	182.54				
		Provisional waste					
		Material	521.08				
		Products	0.00				

Table E.2	Product-oriented	input-output	table of	manufacturing	quantity	center 2
-----------	------------------	--------------	----------	---------------	----------	----------

Input		Output					
Material	2419.35	Products					
Intermediate product 1	4032.26	Sales volume	2000.00				
		+ Intermediate products	0.00				
		= Final production yield	2000.00				
		Rejects					
		Provisional rejects	20.16				
		 Reworked products 	4.03				
		= Final rejects	16.13				
		Provisional waste					
		Material	403.23				
		Products	0.00				

Table E.3 Material-oriented input-output table of material recycling quarterial	ntity center
---	--------------

Input		Output Inefficiency-reducing material demand		
Secondary material demand				
From product 1	521.08	Recycled material from product 1	104.22	
From product 2	403.23	+ Recycled material from product 2	80.65	
		= Recycled material	185.86	
		Inefficient material demand		
		+ Loss material	739.44	
Material demand	924.30	Material demand	924.30	

Input		Output	
Provisional product waste	0.00	Recycled	
Final reject product 1	182.38	Products	0.00
		Material	72.95
		Losses	
		Products	0.00
		Material	291.80

Table E.4 Product-oriented input-output table of recycling product quantity center 1

Table E.5 Material-oriented input-output table of recycling product quantity center 1

Input		Output		
Secondary material demand	364.75	Inefficiency-reducing material demand		
		+ Recycled products	0.00	
		+ Recycled materials	72.95	
		Inefficient material demand		
		+ Loss products	0.00	
		+ Loss materials	291.80	
Material demand	364.75	Material demand	364.75	

Table E.6 Product-oriented input-output table of recycling product quantity center 2

Input		Output	
Provisional product waste	0.00	Recycled	
Final reject product 2	16.13	Product 1	3.87
		Material	10.97
	Losses		
		Product 1	9.03
		Material	43.87

Table E.7	Material-oriented	input-output ta	able of recycling	product quantity center 2
-----------	-------------------	-----------------	-------------------	---------------------------

Input		Output		
Secondary material demand	80.65	Inefficiency-reducing material demand		
		Recycled material product 1	7.74	
		+ Recycled material	10.97	
		= Sum	18.71	
		Inefficient material demand		
		+ Loss product 1	18.06	
		+ Loss material	43.87	
Material demand	80.65	Material demand	80.65	

Conflict of interest S. Dierkes and D. Siepelmeyer declare that they have no competing interests.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

- Aguilar Esteva, L.C.A., A. Kasliwal, M.S. Kinzler, H.C. Kim, and G.A. Keoleian. 2021. Circular economy framework for automobiles—Closing energy and material loops. *Journal of Industrial Ecology* 25:877–889. https://doi.org/10.1111/jiec.13088.
- Behnami, A., K.Z. Benis, M. Shakerkhatibi, E. Fatehifar, S. Derafshi, and M.M. Chavoshbashi. 2019. Integrating data reconciliation into material flow cost accounting: The case of a petrochemical wastewater treatment plant. *Journal of Cleaner Production* 218:616–628. https://doi.org/10.1016/j.jclepro.2019. 01.218.
- Bhimani, A., C.T. Horngren, S.M. Datar, and M.V. Rajan. 2019. Management & Cost Accounting, 7th edn., Upper Saddle River: Pearson.
- Boons, A.N.A.M. 1998. Product costing for complex manufacturing systems. *International Journal of Production Economics* 55:241–255. https://doi.org/10.1016/S0925-5273(98)00064-4.
- Bux, C., and V. Amicarelli. 2022. Material flow cost accounting (MFCA) to enhance environmental entrepreneurship in the meat sector: Challenges and opportunities. *Journal of Environmental Management* 313:115001. https://doi.org/10.1016/j.jenvman.2022.115001.
- Chompu-inwai, R., B. Jaimjit, and P. Premsuriyanunt. 2015. A combination of material flow cost accounting and design of experiments techniques in an SME: The case of a wood products manufacturing company in northern Thailand. *Journal of Cleaner Production* 108:1352–1364. https://doi.org/10. 1016/j.jclepro.2014.08.039.
- Coenenberg, A.G., T.M. Fischer, and T. Günther. 2016. Kostenrechnung und Kostenanalyse, 9th edn., Stuttgart: Schäffer-Poeschel.
- Cooper, R., and R.S. Kaplan. 1988. Measure costs right: make the right decisions. *Harvard Business Review* 66:96–103.
- Christensen, H.B., L. Hail, and C. Leuz. 2021. Mandatory CSR and sustainability reporting: economic analysis and literature review. *Review of Accounting Studies* 26:1176–1248. https://doi.org/10.1007/ s11142-021-09609-5.
- Datar, S.M., and M.V. Rajan. 2018. Horngren's Cost Accounting—A Managerial Emphasis, 16th edn., Harlow: Pearson.
- Dekamin, M., and M. Barmaki. 2019. Implementation of material flow cost accounting (MFCA) in soybean production. *Journal of Cleaner Production* 210:459–465. https://doi.org/10.1016/j.jclepro.2018.11. 057.
- Diaz, M.A.H., A. Scouse, and S.S. Kelly. 2022. Environmental full cost accounting of alternative materials used for railroad ties: treated-wood and concrete case study. *Journal of Cleaner Production* 364:132536. https://doi.org/10.1016/j.jclepro.2022.132536.
- Dierkes, S. 1998. Planung und Kontrolle von Prozeßkosten. Wiesbaden: Springer Gabler.
- Dierkes, S., and D. Siepelmeyer. 2019. Production and cost theory-based material flow cost accounting. *Journal of Cleaner Production* 235:483–492. https://doi.org/10.1016/j.jclepro.2019.06.212.
- Dörner, E. 1984. Plankostenrechnung aus Produktionstheoretischer Sicht. Bergisch Gladbach: Josef Eul.
- Ewert, R., A. Wagenhofer, and A. Rohlfinger-Bastian. 2023. Interne Unternehmensrechnung, 9th edn., Berlin: Springer Gabler.
- European Commission. 2020. Technical report—taxonomy: final report of the technical expert group on sustainable finance. https://finance.ec.europa.eu/system/files/2020-03/200309-sustainable-finance-teg-final-report-taxonomy_en.pdf. Accessed 15 Jan 2024.
- Fandel, G., A. Fey, B. Heuft, and T. Pitz. 2009. Kostenrechnung, 3rd edn., Berlin: Springer Gabler.

- Gibassier, D., and S. Schaltegger. 2015. Carbon management accounting and reporting in practice. Sustainability Accounting, Management and Policy Journal 6:340–365. https://doi.org/10.1108/SAMPJ-02-2015-0014.
- Greenhouse Gas Protocol. 2015. A corporate accounting and reporting standard, revised edition. World resources institute. https://ghgprotocol.org/sites/default/files/standards/ghg-protocol-revised.pdf. Accessed 5 Jun 2024.
- Guan, L., D.R. Hansen, and M.M. Mowen. 2009. Cost management, 6th edn., Mason: South-western Cengage learning.
- Günther, E., C. Jasch, M. Schmidt, B. Wagner, and P.P. Ilg. 2015. Material flow cost accounting—Looking back and ahead. *Journal of Cleaner Production* 108:1249–1254. https://doi.org/10.1016/j.jclepro. 2015.10.018.
- Günther, E., R. Rieckhof, M. Walz, and D. Schrack. 2017. Material flow cost accounting in the light of traditional cost accounting. Unweltwirtschaftsforum 25:5–14. https://doi.org/10.1007/s00550-017-0446-7.
- Hahn, R. 2022. Sustainability management—global perspectives on concepts, instruments, and stakeholders. Fellbach.
- Ho, J.Y., D.K.S. Ng, Y.K. Wan, and V. Andiappan. 2021. Synthesis of wastewater treatment plant based on minimal waste generation cost: a material flow cost accounting (MFCA) approach. *Process Safety* and Environmental Protection 148:559–578. https://doi.org/10.1016/j.psep.2020.10.013.
- Intergovernmental Panel on Climate Change (2023). Climate Change 2023. Synthesis Report: Summary for Policymakers. https://www.ipcc.ch/report/ar6/syr/downloads/report/IPCC_AR6_SYR_SPM.pdf.
- ISO. 2011. Environmental management—material flow cost accounting—general framework (ISO 14051:2011), German and English version EN ISO 14051:2011. Berlin: Beuth.
- Jing, H., and L. Songqing. 2011. The research of environmental costs based on activity based costs. Procedia Environmental Science 10:147–151. https://doi.org/10.1016/j.proenv.2011.09.026.
- Kawalla, C., W. Berkel, R. Kawalla, M. Höck, and M. Ligarski. 2018. Material flow cost accounting analysis of twin-roll casting magnesium strips. *Proceedia Manufacturing* 15:193–200. https://doi.org/10. 1016/j.promfg.2018.07.194.
- Keilus, M. 1993. Produktions- und Kostentheoretische Grundlagen einer Umweltplankostenrechnung. Bergisch Gladbach: Josef Eul.
- Kilger, W., J.R. Pampel, and K. Vikas. 2012. Flexible Plankostenrechnung und Deckungsbeitragsrechnung, 13th edn., Wiesbaden: Springer Gabler.
- Kitada, H., T. Tennojiya, J. Kim, and A. Higashida. 2022. Management practice of material flow cost accounting and its discontinuance. *Cleaner Environmental System* 6:100089. https://doi.org/10.1016/ j.cesys.2022.100089.
- Kloock, J. 1969. Betriebswirtschaftliche Input-Output-Modelle. Wiesbaden: Gabler.
- Kloock, J. 1999. Arbeitsunterlagen zum Studienfach Unternehmensrechnung und Revision Teil 1: Operatives Erfolgsmanagement, 13th edn., Köln.
- Kloock, J., and U. Schiller. 1997. Marginal costing: Cost budgeting and cost variance analysis. Management Accounting Research 8:299–323. https://doi.org/10.1006/mare.1996.0048.
- Kokubu, K., and H. Kitada. 2015. Material flow cost accounting and existing management perspectives. *Journal of Cleaner Production* 108:1279–1288. https://doi.org/10.1016/j.jclepro.2014.08.037.
- Krüger, G. 1959. Erfassung und Verrechnung von Ausschuss. Frankfurt am Main: Franz Nowack.
- Lengsfeld, S. 1999. Kostenkontrolle und Kostenänderungspotentiale. Wiesbaden: Springer Gabler.
- Letmathe, P. 1998. Umweltbezogene Kostenrechnung. München: Vahlen.
- May, N., and E. Günther. 2020. Shared benefit by Material Flow Cost Accounting in the food supply chain – The case of berry pomace as upcycled by-product of a black currant juice production. *Journal of Cleaner Production* 245:1–17. https://doi.org/10.1016/j.jclepro.2019.118946.
- Nakajima, M. 2004. On the differences between material flow cost accounting and traditional cost accounting—In reply to the questions and misunderstandings on material flow cost accounting. *Kansai* University review of business and commerce 6:1–20.
- Nertinger, S. 2014. Carbon and Material Flow Cost Accounting: Ein integrierter Ansatz im Kontext nachhaltigen Erfolgs und Wirtschaftens. Wiesbaden: Springer Gabler.
- Nishimura, S., T. Ohtsuki, N. Goto, and K. Hanaki. 2021. Technical-knowledge-integrated material flow cost accounting model for energy reduction in industrial wastewater treatment. *Cleaner Environmen*tal System 3:100043. https://doi.org/10.1016/j.cesys.2021.100043.
- Nishitani, K., K. Kokubu, Q. Wu, H. Kitada, E. Günther, and T. Günther. 2022. Material flow cost accounting (MFCA) for the circular economy: An empirical study of the triadic relationship between

MFCA, environmental performance, and the economic performance of Japanese companies. *Journal of Environmental Management* 303:114219. https://doi.org/10.1016/j.jenvman.2021.114219.

- Penman, S.H. 2024. Accounting for Carbon. SSRN Electronic Journal https://doi.org/10.2139/ssrn. 4721974.
- Prosman, E.J., B.V. Waehrens, and G. Liotta. 2017. Closing global material loops: Initial insights into firmlevel challenges. *Journal of Industrial Ecology* 21:641–650. https://doi.org/10.1111/jiec.12535.
- Reichelstein, S. 2023. Carbon emission statements: balance sheets and flow statements. SSRN Electronic Journal https://doi.org/10.2139/ssrn.4672099.
- Sanders, N.R., and J.D. Wood. 2019. Foundations of Sustainable Business—Theory, Function, and Strategy, 2nd edn., Hoboken.
- Schmidt, A., U. Götze, and R. Sygulla. 2015. Extending the scope of material flow cost accounting-methodical refinements and use case. *Journal of Cleaner Production* 108:1320–1332. https:// doi.org/10.1016/j.jclepro.2014.10.039.
- Schmidt, M. 2005. A production-theory-based framework for analysing recycling systems in the e-waste sector. *Environmental Impact Assessment Review* 25:505–524. https://doi.org/10.1016/j.eiar.2005.04. 008.
- Schmidt, M. 2015. The interpretation and extension of material flow cost accounting (MFCA) in the context of environmental material flow analysis. *Journal of Cleaner Production* 108:1310–1319. https://doi. org/10.1016/j.jclepro.2014.11.038.
- Schmidt, M., and M. Nakajima. 2013. Material flow cost accounting as an approach to improve resource efficiency in manufacturing companies. *Resources* 2:358–369. https://doi.org/10.3390/ resources2030358.
- Schrack, D. 2016. Nachhaltigkeitsorientierte Materialflusskostenrechnung Anwendung in Lieferketten, der Abfallwirtschaft und Integration Externer Effekte. Wiesbaden: Springer Gabler.
- Schweitzer, M., H.U. Küpper, G. Friedl, C. Hofmann, and B. Pedell. 2016. Systeme der Kosten- und Erlösrechnung, 11th edn., München: Vahlen.
- Spengler, T., H. Püchert, T. Penkuhn, and O. Rentz. 1997. Environmental integrated production and recycling management. *European Journal of Operational Research* 97:308–326. https://doi.org/10.1016/ S0377-2217(96)00200-7.
- Wagner, B. 2015. A report on the origins of material flow cost accounting (MFCA) research activities. Journal of Cleaner Production 108:1255–1261. https://doi.org/10.1016/j.jclepro.2015.10.020.
- Wan, Y.K., R.T.L. Ng, D.K.S. Ng, and R.R. Tan. 2015. Material flow cost accounting (MFCA)-based approach for prioritisation of waste recovery. *Journal of Cleaner Production* 107:602–614. https:// doi.org/10.1016/j.jclepro.2015.05.024.
- Yagi, M., and K. Kokubu. 2019. Waste decomposition analysis in Japanese manufacturing sectors for material flow cost accounting. *Journal of Cleaner Production* 224:823–837. https://doi.org/10.1016/ j.jclepro.2019.03.196.
- Zaharudin, Z.A., A. Brint, and A. Genovese. 2022. A multi-period model for reorganising urban household waste recycling networks. *Socio-Economic Planning Sciences* 84:101396. https://doi.org/10.1016/j. seps.2022.101396.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.