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Article

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Long-term relationship of KGHM share prices and the market value of high grade copper

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ABSTRACT

The aim of the article is to try to explain the long-term price volatility of KGHM shares. Therefore the paper presents the relationship between KGHM stock prices and High Grade copper prices. The empirical part of the paper uses econometric cointegration analysis. Based on the estimated models, the thesis of the existence of a long-run relationship between the studied variables was confirmed. Within the framework of econometric analyses, Johansen and Engle-Granger procedures and the Granger test of causality were applied. The study was conducted using monthly data covering quotes from August 2012 to April 2021. In the end, both modeling procedures used led the researcher to convergent conclusions. Moreover very similar values of long-run equilibrium parameter estimates were obtained for both methods. Thus, on the basis, it is necessary to confirm the main hypothesis formulated at the beginning of the study, that is, in the case of a company operating within the KGHM Polska Miedź mining industry, the quotation of its shares is long-term dependent on the quotation of HG copper prices.

JEL classification: G17, C01, C5, G1

Keywords: causality, KGHM, Johansen procedure, Engle-Granger procedure, cointegration.

1. INTRODUCTION

The main aim of the article is to try to explain the long-term equilibrium of KGHM share prices listed on the Warsaw Stock Exchange (WSE). At the outset of the analysis carried out, it was assumed that the quotation of KGHM shares may depend on the quotation of copper, an important raw material exploited by the company. KGHM is unique among top copper producers in generating majority of its revenues from copper ore mining and copper processing. Therefore the relationship between prices of copper and share prices might be strong. It is worth noting that other top copper producers (BHP, Rio Tinto, Glencore) are more diversified mining companies. In the case of higher diversification the company's revenues might be dependant on prices of various commodities. The main hypothesis of the conducted study boils down to the statement that, in the case of KGHM Polska Miedź, a company operating within the mining industry, the quotation of its shares depends on the quotation of HG (High Grade) copper prices in the long term. Moreover, the course of the time series of both $KGHM_t$ and HG_t variables seen in Figure 1 (included in the methodology and data section) allows us to expect that

the series are characterized by common long-term trends. Due to the statistical properties of the time series of KGHM stock quotes and copper prices, it is insufficient to use the quantitative correlation coefficient or classical regression analysis in their analysis. In the case of time series analysis, ignoring the issue of stationarity leads to the risk of the appearance of so-called sham regressions (Granger and Newbold, 1974). Econometric analysis for non-stationary time series requires the use of non-classical techniques (Majsterek, 2014; Domanski and Pruska, 2000). Hence, realization of the stated aim of the study requires a recourse to methods of analyzing financial time series and stochastic processes, which are the object of interest of the so-called new econometrics. In the indicated area, it becomes possible to identify common stochastic trends, long-term equilibrium relations and mechanisms of short-term adjustments, the knowledge of which is economically important in the context of the formulated objective of the study.

2. LITERATURE REVIEW

In the scientific literature, there have been attempts to analyze price quotations of copper and KGHM shares (Fijałkowski, 2011) for daily data of the Warsaw Stock Exchange from September 15, 2010 to March 15, 2011. The cited example covered a short period of time and was based on the estimation of a classical regression model. There are many valuable publications regarding the relationship between the stock and commodity markets. In most cases, researchers deal with links between stock and commodity markets from a volatility perspective (Creti et al., 2013; Kang et al., 2016; Vardar et al., 2018). Especially, a study about prices of shares of Southern Copper Corporation should be mentioned here (Zevallos and Carpio, 2015). It is worth noting that the cited studies concern stock and commodity markets volatility. It is an important objective of literature review to identify the gap in the existing literature. Therefore the article concerns a cointegration analysis between variables from the stock and commodity markets. The author developed a hypothesis about long terms relationship existing between $KGHM_t$ and HG_t . In this case the cointegration analysis let the author identify a common long-term trend despite of short-term volatility. Financial time series are often characterized by non-stationarity and require the adoption of procedures coinciding with the so-called new econometrics (Charemza and Deadman, 1997). For the reason, it seems interesting to carry out a study that is subject to the rigors of non-stationary data analysis and, at the same time, covers a sufficiently wide time period so that conclusions can be generalized to the long-term horizon. It should be added that cointegration analysis is more common in macroeconomic studies (Karp et al., 2013; Kusideł, 2000), as well as in the analysis of stock market index correlations (Assidenou, 2011). An example of this type of analysis for financial data is the study of the relationship between stock market indices, exchange rates (Gędek, 2013; Buszkowska, 2014). In the publications there is a typical problem of volatility clustering for stock market data, where the so-called effect “volatility clustering” is one of the accentuated properties of financial time series. A cointegration analysis for non-aggregated series is a far less common phenomenon, although also found in scientific literature (Baghestani, 1991). A review of the publications leads to the conclusion that it is more common for such models to be used in the analysis of commodity prices quoted on commodity markets. It is especially true for energy commodities such as liquid fuels and crude oil (Gędek, 2017; Socha, 2013; Erdos, 2012; Asche et al., 2003). Within the precious metals market, publications provide many interesting analyses of long-term relationships conducted from an investment perspective. Hence, they focus on the analysis of the cointegration of gold (Mamcarz, 2018), silver (Kasprzak-Czelej, 2016) and their relationship with the aforementioned energy commodities (Sindhu, 2013; Kasprzak-Czelej, 2018). In the English-language literature, researchers point to the existence of a long-term relationship between the prices of the raw materials (Lucey and Tully, 2006; Pierdzioch et al., 2015). It is worth noting that the researchers’ results may not be fully comparable, due to

different methodology and reference to different countries and research periods (Mamcarz, 2018). In addition, the result of the analysis may be an apparent regression phenomenon if the property of stationarity and cointegration is ignored (Demidova-Menzel and Heidorn, 2007; Sindhu, 2013). The problem does not occur in the analysis of return series due to their zero degree of integration.

3. METHODOLOGY AND DATA

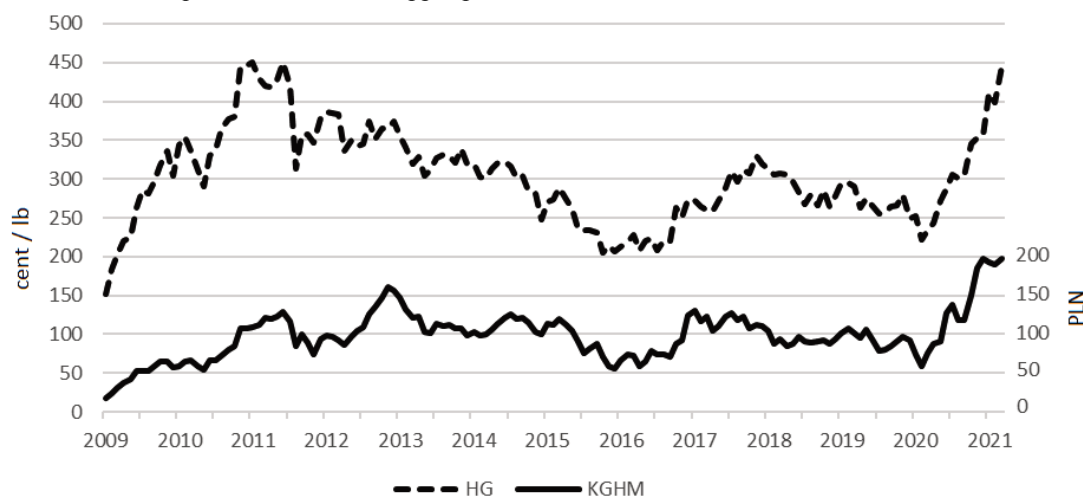
To the author's best knowledge and the literature review conducted, in this case cointegration modelling might provide an estimation of the long-term equilibrium of variables mentioned. The study was conducted using monthly data covering quotes from August 2012 to April 2021. The data used in the empirical part of the article has a monthly frequency. Designations of the variables specific to the time series under study are as follows:

$KGHM_t$ – monthly quotation of KGHM shares during period t on the Warsaw Stock Exchange

HG_t – monthly quotation of copper type HG in period t (COMEX exchange, cent per lb).

Figure 1

Time series of KGHM quotations and HG copper prices at levels



Source: own elaboration.

The initial modeling step when analyzing time-series data is to determine the degree of integration. In this case, popular tests such as ADF (Augmented Dickey-Fuller test), PP (Phillips-Perron test) or KPSS (Kwiatkowski-Phillips-Schmidt-Shin test) can be used. The Augmented Dickey-Fuller (ADF) test was based on LS (the Least Squares method) estimation of the auxiliary regression, the basic form of which in the version without a constant and deterministic trend can be written as (Maddala, 2006):

$$\Delta y_t = \delta y_{t-1} + \sum_{i=1}^k \gamma_i \Delta y_{t-i} + \varepsilon_t, \quad \text{Eq. (3.1)}$$

where: δ and γ – parameters estimated by the Least Squares method, k – number of lags, Δy_{t-1} – value of the series of first differences of the studied phenomenon in period $t-1$, ε_t – random component.

The null hypothesis of the ADF test assumes non-stationarity of the y_t series caused by the presence of a unit root. While, the alternative hypothesis of the ADF test assumes stationarity of the time series under study.

The analysis also referred to the Granger causality test. Causality in the Granger sense is written as $x \rightarrow y$ and involves testing whether the variable x can be removed from that part of the VAR (Vector Autoregression) model (Sims, 1980), which describes y (Granger, 1969). Dependency testing, in the Granger sense, allows one to determine whether the highlighted variable is the cause of changes in the value of the other variable. The approach assumes that variable x is the cause of changes in variable y if the values of variable y can be more accurately predicted by using past values of variable x than without taking them into account. Thus, the Granger test was developed based on a system of equations:

$$\begin{aligned} y_t &= c_1 + \sum_{i=1}^k \beta_{1i} y_{t-i} + \sum_{j=1}^k \beta_{1j} x_{t-j} + u_{1t} \\ x_t &= c_2 + \sum_{i=1}^k \beta_{2i} x_{t-i} + \sum_{j=1}^k \beta_{2j} y_{t-j} + u_{2t}, \end{aligned} \quad \text{Eq. (3.2)}$$

where: y_t – values of variable y in the current period, x_t – values of variable x in the current period, β – structural parameters of the model, u_t – random component of the model.

An interesting issue in this case is causality testing for non-stationary or cointegrated variables. Quite an easy way to deal with this is to use the procedure proposed by Toda and Yamamoto (1995). The procedure is based on the VAR model despite of non-stationarity or cointegration variables. In addition, the cointegration Engle-Granger procedure allows the determination of a single cointegration vector that is consistent with economic theory. It uses a regression equation $\ln y_t = \beta \cdot \ln x_t + c + \xi_t$, which describes the equilibrium relationship between the x and y variables (Charemza and Deadman, 1997). The procedure can be used for variables integrated to the first degree (Enders, 1995). Preliminary estimation of the equilibrium relationship $\ln y_t = \beta \cdot \ln x_t + c + \xi_t$ by the least squares method is used to learn the value of the residual component. Only diagnosing the stationarity of the residual component ξ_t determines the presence of a cointegrating vector. Such a vector describes the long-term relationship between variables. In addition, based on Granger's representation theorem for cointegrated variables, it is possible to estimate a short-run ECM (error correction model) of the form: $\Delta \ln y_t = \alpha \cdot \Delta \ln x_t - \gamma \cdot \xi_{t-1} + c$.

The critical discussion of the chosen method should be included here. The Engle-Granger procedure limits the researcher to the possibility of knowing one cointegrating vector the most (Gajda, 2004). Moreover, it will only be the cointegrating vector that corresponds to the a priori accepted economic theory (Welfe, 1998), which was the basis for the division of the variables under study into exogenous and endogenous.

The Engle-Granger method is an interesting tool for studying time series cointegration, but it has several shortcomings:

1. The procedure requires a two-stage estimation. First, the residuals resulting from the long-run equilibrium relationship are generated, and then the same residuals become the basis for the second regression. Thus, if the regression of the long-run relationship has been subject to an error then this error will be transferred to the second stage of the procedure.
2. Estimating the long-run equilibrium relationship requires making a theoretical assumption about which of the variables under study is the dependent variable and which is the independent variable. Particularly in the case of a limited sample size, this can lead to the paradox that for a pair of specific variables, the long-run relationship will be confirmed or not depending on the scenario of defining the dependent and independent variable. According to asymptotic theory, the problem will not occur with sufficiently large samples, although the practice of economic research often struggles with the phenomenon of limited sample size.

- Testing for cointegration seems clear in the case of a pair of variables. In this case, the long-run equilibrium relationship can be unambiguously determined or the absence of cointegration can be found. In a situation where the number of explanatory variables is greater than 1, the whole analysis gets complicated. If the model contains » n « variables, there can be from 0 to $(n-1)$ linearly independent cointegrating vectors. Meanwhile, the Engle-Granger procedure does not even offer a test of the number of such vectors, much less determines them.

The properties of the Engle-Granger procedure and its limitations make it a good idea to verify the cointegration test also on the basis of the Johansen procedure (Johansen, 1988). It is an approach commonly used when modeling multiple cointegrated variables. Johansen's approach, together with VAR and Vector Error Correction Model (VECM) models, allows cointegration to be analyzed multidimensionally. As mentioned earlier, while Engle-Granger's approach is based on the assumptions of economic theory and allows the determination of a maximum of one cointegrating vector, Johansen's procedure allows the testing of a total number of such vectors. In addition, in the Johansen approach it is possible to determine all cointegrating vectors. The Johansen approach uses vector autoregressive VAR models.

A VAR model without boundary conditions and deterministic components is written in a compact form using the equation (Sims, 1980):

$$Y_t = \sum_{i=1}^k A_i Y_{t-i} + E_t, \quad \text{Eq. (3.3)}$$

where: Y_t – is a vector containing n variables of the model; A_i – is a matrix of parameters with lagged variables; E_t – is a vector of random components.

Using the so-called cointegrating transformation, the VAR model can be brought to the form of the VECM model (Brooks, 2012):

$$\Delta Y_t = \sum_{i=1}^k \Gamma_i \Delta Y_{t-i} + \Pi Y_{t-k} + U_t, \quad \text{Eq. (3.4)}$$

where: $\sum_{i=1}^k \Gamma_i \Delta Y_{t-i}$ – is the short-run part of the model (deviations from the long-run path); ΠY_{t-k} – is the long-term part of the model containing long-term trends; U_t – is a vector of random components.

If there are n variables in the vector Y_t and the row of the matrix Π is equal to $0 < r < n$ then the matrix Π can be decomposed as follows:

$$\Pi = \alpha \beta^T, \quad \text{Eq. (3.5)}$$

where: α – adjustment matrix, β – matrix of long-term trends.

The matrix β consists of cointegrating vectors that, when normalized, can be interpreted as long-term parameters. Assuming that the vector Y_t consists of n variables then the matrix Π will be of dimension $(n \times n)$, so its row can be equal to at most n . Based on the Granger's Representation Theorem (Engle and Granger, 1987; Johansen, 1991) it can be assumed that:

- If the row of the matrix Π is equal to n then all of the studied variables included in Y_t are stationary. Then it is advisable to estimate the traditional VAR model for the levels of the variables. In addition, if the row of the matrix Π is equal 0 then the traditional VAR model for

variable increments is recommended. All components of the matrix Π are equal 0 therefore the representation in the form of error correction does not exist.

2. If the row of the matrix Π is equal to $0 < r < n$ then there exists a representation Π such that $\Pi = \alpha\beta^T$ within which the matrices α and β have dimension $(n \times r)$. Then the matrix β is a cointegration matrix that has the property that $\beta Y_t \sim I(0)$, when $Y_t \sim I(0)$. In this case, a VECM model is built, which is a transformation of the VAR model through the use of a cointegrating transformation. Within a VAR model with n variables, there can be at most $r = n - 1$ cointegrating vectors.

An important practical issue in this case is determining the appropriate number of delays for VAR or VECM models. One possibility in this regard seems to be the use of lag length tests (Kusideł, 2000), which, however, do not always lead to clear conclusions. Often, a more effective technique for determining lags k is to assess stability of the estimates of the adjustment coefficients included in the matrix of α . Then, the smallest possible value of lag k is taken, which provides similar values for the coefficients of the matrix α . For VAR models, the lag k should be adequate for the length of the response (i.e., adjustment) to the deviation from the long-run trajectory according to the principle of the error correction mechanism (ECM). Theoretically, the adjustments are assumed to occur over a relatively short period. However, lags k should be large enough to eliminate possible autocorrelation of the model's random components. Thus, in general, long-term trends should not depend on lags k in the model, but already short-term adjustments will depend on k . In the study presented here, lags of 1 to 2 were assumed for the VECM model. It should be noted that any reduction in the dimension of the estimated model has a beneficial effect on reducing the number of its parameters required for estimation. The critical discussion of the Johansen approach focuses on the application of the VAR / VECM model. In this case every variable is specified as endogenous and some researchers point out the non-theoretical approach.

4. RESULTS

4.1. Evaluation of the stationarity of the studied time series

In verifying the stationarity of the time series, it turned out that both series are non-stationary in levels, but have stationary first differences. The property should be described as their integration on first differences. Thus, the application of the differential filter made it possible to transform both non-stationary processes into stationary series. Table 1 contains the results of ADF tests for the levels and differences of the KGHM and HG copper quotation series.

Table 1

Stationarity test of KGHM and HG copper quotations on levels and increments

	KGHM		HG	
	t-Statistic	Probability	t-Statistic	Probability
ADF level	-1.653772	0.4517	-1.224534	0.6616
Test (ADF level) critical values 5% level	-2.889474		-2.889753	
ADF 1st difference	-8.035865	0.0000	-5.415778	0.0000
Test (ADF 1st difference) critical values 5% level	-2.889474		-2.889753	

Source: own elaboration based on test checks and critical values obtained in Eviews program.

It is worth noting that although Table 1 contains the critical values of the unit root tests for a significance level of 0.05, analogous conclusions were obtained even for a much lower significance level of 0.01. Both time series are integrated on the first differences.

4.2. Testing the causality of KGHM and HG copper quotes in the Granger sense

Granger tests include levels of the variables in the VAR model according to the Toda and Yamamoto procedure (1995) with the Wald test statistic. In this case the Wald test is asymptotically chi-square distributed. The procedure is based on the VAR model despite of variables non-stationarity or cointegration. The null hypothesis of the Granger causality test assumes the absence of causality, in which case all β parameters are equal to zero. The alternative hypothesis, on the other hand, assumes that if causality is present then some β_i or β_j parameters are different from zero.

Table 2

Granger causality test for variables $KGHM_t$ and HG_t

Null Hypothesis:	Obs	Chi-sq	Prob.
KGHM does not Granger Cause HG	102	4.8009	0.1870
HG does not Granger Cause KGHM		12.8149	0.0051

Source: own elaboration based on a causality test conducted in Eviews software.

The results of the test contained in Table 2 indicate the existence of significant causality in the Granger sense of the changes in the levels of HG copper quotations relative to the changes in the levels of KGHM stock quotations. It is indicated by the relatively high value of the F statistic = 12.8149 and the low value of the empirical significance level (Probability = 0.0051), as a consequence of which the null hypothesis of causality in the Granger sense test should be rejected. The opposite relationship is not substantiated, that is, changes in the KGHM's stock price are not the cause of changes in the HG copper price. In this case, there is no basis for rejecting the null hypothesis of the Granger test performed (Probability = 0.1870).

4.3. Study of the long-term relationship of KGHM and HG copper quotations

Table 3

Long-term relationship model for natural logarithms of $KGHM_t$ and HG_t variables

Included observations: 106				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LN(HG)	1.367883	0.080279	17.03912	0.0000
C	-3.100294	0.454077	-6.827685	0.0000
R-squared	0.736263	Mean dependent var		4.633575
Adjusted R-squared	0.733727	S.D. dependent var		0.260688
S.E. of regression	0.134520	Akaike info criterion		-1.155527
Sum squared resid	1.881933	Schwarz criterion		-1.105274
Log likelihood	63.24293	F-statistic		290.3317

Source: own development in Eviews software.

The model in Table 3 shows a correct fit to the empirical data. The adjusted coefficient of determination in this case was more than 0.73. It means that about 73 percent of the volatility of KGHM stock prices was described by the volatility of HG copper prices. In the estimated model, both the long-run relationship parameter and the constant appear to be significantly different from zero, but it should be remembered that the indications of Student's t-test or F-test can be misleading due to the non-stationarity of the time series under study at levels. The preliminary form of the long-run relationship equation should be written as:

$$\ln(KGHM_t) = 1.367883 * \ln(HG_t) - 3.100294. \quad \text{Eq. (4.1)}$$

Thus, potentially, a 1 percent increase in the HG copper quotation results in an average increase in the KGHM stock price quotation of 1.367883 percent over the long term, assuming *ceteris paribus*. Whether the indicated relationship is true can only be determined by testing for stationarity at the levels of the residual component implied by the estimated model.

Table 4

Stationarity test of the residual component from long-run relationship model at levels

	Residuals	
	t-Statistic	Probability
ADF level	−3.905480	0.0028
Test (ADF level) critical values 5% level	−2.889200	

Source: own elaboration based on test check and critical value obtained in Eviews program.

Table 4 confirms the stationarity of the tested residual component at levels because the value of the ADF test (−3.905480) is less than the critical value for the significance level of 0.05. Thus, the modeled time series is characterized by a long-run relationship called cointegration. Moreover, according to Granger's representation theorem, there should be a short-run relationship in the case with an error correction mechanism.

As can be seen in Table 5 for the studied set of variables, the short-run relationship with the error-correction mechanism is also true. The ECM model has a moderate fit to the empirical data (i.e., the adjusted coefficient of determination was 0.429852). It should be noted, however, that this is a model estimated for incremental variables, where this type of model does not usually

Table 5

ECM short-run relationship model for logarithm increments of KGHM_t and HG_t variables

Included observations: 105 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LN(HG))	1.231513	0.144465	8.524654	0.0000
Z(−1)	−0.235451	0.062487	−3.768016	0.0003
R-squared	0.435334	Mean dependent var		0.006065
Adjusted R-squared	0.429852	S.D. dependent var		0.112544
S.E. of regression	0.084980	Akaike info criterion		−2.073933
Sum squared resid	0.743828	Schwarz criterion		−2.023381
Log likelihood	110.8815	F-statistic		79.40865

Source: own development in Eviews software.

have high coefficients of fit to empirical data. In this case, about 43 percent of the variation in the incremental natural logarithm of KGHM's stock price was explained by the model built, namely the variation in the incremental natural logarithm of HG copper prices and the one measurement period lagged residual component derived from the long-run relationship. The coefficient of short-run elasticity in the case was 1.231513. On the other hand, the adjustment parameter has a correct value less than zero and at the same time less than unity in absolute value. Thus, the studied system has the ability to return to equilibrium. The short-run model with the error correction mechanism in the final version was estimated without a constant value. Hence, the short-run relationship was written as:

$$\Delta \ln(KGHM_t) = 1.231513 * \Delta \ln(HG_t) - 0.235451 * z_{t-1}, \quad \text{Eq. (4.2)}$$

where: z_{t-1} – the value of the residual component in period $t-1$ from the long-run relationship model.

Based on the estimated model, it can be concluded that a 1 percentage point increase in the growth rate of the HG copper quotation results in an average increase in the growth rate of the KGHM stock price of 1.231513 percentage points, in the short term, assuming *ceteris paribus*. In addition, the adjustment parameter of -0.235451 suggests that more than 23% of the previous period's imbalance will be removed in the current period as a result of the ECM's error correction mechanism.

Table 6 contains results of the Johansen cointegration test, which confirms the conclusions of the Engle-Granger procedure. Thus, for the studied variables $KGHM_t$ and HG_t there is a long-run relationship described by a single cointegrating vector. The Johansen cointegration test proceeds sequentially that is: in the first stage, the value of the trace test (Trace Statistic) is 15.79454, which is greater than the critical value (15.49471). This means that the number of cointegration vectors is greater than 0. In stage two, the trace test (Trace Statistic) is 2.657169 which is less than the critical value (3.841466). Hence, the number of cointegration vectors equals one.

Table 6

Johansen test of the number of cointegration vectors for the variables $\ln(KGHM_t)$ and $\ln(HG_t)$

Series: LN(KGHM) LN(HG)				
Lags interval (in first differences): 1 to 2				
Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.119748	15.79454	15.49471	0.0451
At most 1	0.025468	2.657169	3.841466	0.1031
Trace test indicates 1 cointegrating eqn(s) at the 0.05 level				
* denotes rejection of the hypothesis at the 0.05 level				
** MacKinnon-Haug-Michelis (1999) p-values				

Source: own development in Eviews software.

Thus, the results of the Johansen test indicate the existence of a long-term relationship between KGHM stock prices and HG copper prices. Hence, it can be assumed that the Engle-Granger procedure carried out earlier for the study also led to significant conclusions. The next step in the analysis will be to see whether the two cointegration test procedures yield similar estimates of the coefficient of long-run elasticity.

Based on the VECM model and the $\Pi = \alpha\beta^T$ matrix, a cointegration vector was estimated to describe the long-term relationship between the $KGHM_t$ and HG_t variables. Its parameter after normalization is included in Table 7.

Table 7

Long-term relationship between $KGHM_t$ and HG_t variables based on the Johansen procedure

Normalized cointegrating coefficients (standard error in parentheses)	
$\text{Ln}(KGHM)$	$\text{Ln}(HG)$
1.000000	−1.318959
	(0.21062)

Source: own compilation based on calculations in Eviews software.

The analytical form of the cointegrating relationship can be expressed by the equation (Syczewska, 1999; Majsterek, 2014):

$$\ln(KGHM_t) = 1.318959 * \ln(HG_t). \quad \text{Eq. (4.3)}$$

Based on the estimated equation, it can be concluded that a 1 percent increase in the quoted price of HG copper results in an average increase of 1.318959 percent in KGHM's stock price over the long term, assuming *ceteris paribus* (Gajda, 2004; Borkowski et al., 2007). The impact of HG copper quotations on KGHM stock prices should be considered significant in the long term.

At the same time, recall that the estimate of the coefficient of the long-run relationship obtained from the two-stage estimation according to the Engle-Granger procedure was 1.367883. Thus, not only the signs, but also the values of the obtained parameters of long-run elasticity turned out to be similar (1.318959 and 1.367883). A slightly higher parameter value was recorded for the Engle-Granger procedure relative to the Johansen procedure. Referring to the theoretical assumptions of the study and the literature review, it should be noted that the long-term equilibrium of KGHM share prices and the HG copper is a unique case of the relationship dependence between the stock and commodity markets. Most publications cited in the literature review section concern the study of volatility (Creti et al., 2013; Kang et al., 2016; Vardar et al., 2018; Zevallos and Carpio, 2015). As it has been mentioned before the problem of volatility clustering for stock market data, is one of the accentuated properties of financial time series. From the cointegration point of view short-term deviations are really important too, therefore the research is in compliance with papers cited in the literature review. In this case short-term deviations from the long-term equilibrium are vital to identify the error correction mechanism. It is worth noting that the mentioned before publication by Fijałkowski (2011) contains quite different research results. This is understandable because of quite different methodology and periods analyzed. Both studies were focused on the examination of the dependence between KGHM share and copper prices. However, it should be noted that the time series modelling requires procedures coinciding with the so-called new econometrics (Charemza and Deadman, 1997). The aim of the article was to show the application of such procedures in relation to time series of KGHM shares and copper prices. In this context, it seems that this intention has been achieved.

5. CONSLUSIONS

Concluding the conducted study of the cointegration relationship for the $KGHM_t$ and HG_t variables, it should be emphasized that financial data including stock quotes are most often non-stationary. Their analysis is hampered by the presence of stochastic trends and high volatility.

Under the conditions, techniques for modeling integrated variables, such as the Engle-Granger and Johansen procedure, seem to be helpful. Although they are characterized by slightly different numerical properties, the example of the article shows that they can lead to very similar conclusions. Let's point out that the analysis presented here, by design, concerned only two variables $KGHM_t$ and HG_t therefore Johansen's multivariate method did not present its full capabilities. At the same time, a rather large set of stock quotes covering almost a decade made it possible to capture the common trajectories of the studied time series. Of course, the Engle-Granger method used in the first instance has several limitations mentioned in the body of the article, so its use should be confirmed by a more advanced method (the Johansen procedure). This is what happened in this case, where in the end very similar values of long-run equilibrium parameter estimates were obtained for both methods. Thus, on this basis, it is necessary to confirm the main hypothesis formulated at the beginning of the study, that is, in the case of KGHM Polska Miedź operating within the mining industry, the quotation of its shares is long-term dependent on the quotation of HG copper prices. Limitations of the study conducted include the inclusion of only two variables in the analysis. However, given the fairly unambiguous results on common long-term trends, expansion of the presented models with new variables will become a direction for future research. The purpose of the paper is to try to explain the long-term price volatility of KGHM shares. In the context, it seems that the intended purpose has been achieved. The main findings are related to the obtained cointegration vector that describes the long-term dependence between the variables examined. According to the research results there are quite sensitive values of KGHM share prices to changes of copper prices. This can be said based on the value of the long-term parameter around 1.3. The significance and relevance of the results are related to many areas with economic consequences. The presented modeling results can be an inspiration for investors and experts in company valuation. In addition, the analysis of highly volatile stock market data can contribute to a better understanding of processes on the WSE. In addition, the conducted empirical study enables making careful comparisons with the previous publications in the literature of time series cointegrating of the stock and commodity markets.

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