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Re-Evaluating Sharpe Ratio in Hedge Fund Performance in Light of Liquidity Risk

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ABSTRACT

This paper demonstrates how the Sharpe Ratio can be modified by altering the measure of “total risk” in the denominator of the Sharpe Ratio (i.e., the standard deviation) to include liquidity risk, a major risk for investors in hedge funds that is missing from the standard Sharpe Ratio formulation. We refer to our liquidity-risk-adjusted performance ratio as the LRAPR. The results of our analysis of 1186 hedge funds alive in 2012–2020 show that funds with higher liquidity risk exhibit higher Sharpe Ratios and higher Alphas (as estimated in a 7-factor model that does not incorporate liquidity risk). We posit that analysts and investors should not necessarily take these higher Sharpe Ratios and higher Alphas as indications of fund superiority; what appears to be superior manager skill may rather be a compensation for bearing liquidity risk. Our LRAPR is a tool that analysts or investors could use to compare funds on a more equal footing, adjusting for differential liquidity risk across funds.

JEL Classification: G12; G23; C18

Keywords: liquidity risk, liquidity risk factor, serial correlation, Sharpe ratio, hedge fund performance.

1. INTRODUCTION

In finance, standard deviation is referred to as a measure of “total risk” in that it incorporates both the systematic risk and unique risk of an investment. To us, the moniker “total risk measure” is a bit of a misnomer, in that standard deviation does not capture liquidity risk. Previous researchers have recognized this deficiency, especially in case of measuring investment fund performance

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by the Sharpe ratio (Sharpe, 1966, 1994). They have pointed out that standard deviation may be understated and the Sharpe ratio therefore overstated in the presence of liquidity risk in a fund's portfolio. Lo (2002) has suggested methods for adjusting the Sharpe ratio to take liquidity risk into account, and Getmansky et al. (2004) have followed Lo (2002) in this by investigating hedge funds, for which liquidity is particularly important risk factor (C. Li et al., 2020; Siegmann & Stefanova, 2017).

In this study we propose an intuitive and simple modification to the Sharpe ratio that introduces into the calculation a proxy for liquidity risk, which is not directly captured in the standard deviation measure. This proxy is a serial correlation of fund returns. Our modification method results in a measure that we call a “liquidity-risk-adjusted performance ratio” (LRAPR). We calculate LRAPR for 1186 hedge funds alive in 2012–2020. Intuitively this measure should allow a better apples-to-apples comparison of funds exhibiting higher assumed liquidity risks with other funds where liquidity risk is absent or more muted. We find that in a certain group of higher-liquidity-risk funds in our fund universe, there is a strongly positive association between the level of liquidity risk and the fund Sharpe ratio. We also find that our LRAPR may be a more useful reward-for-risk measure than the Sharpe ratio in that the LRAPR seems to be independent of the fund's liquidity risk, so that differences in LRAPR across the funds may depend on more relevant forces, such as differential manager skill or exposure to more unusual risks that are harder to identify, measure, and intentionally incorporate into a diversified portfolio. To confirm our intuition, we calculate Alphas for all the hedge funds in the database using a 7-factor equilibrium model similar to that of Fung & Hsieh (2004). We find that higher-liquidity-risk funds have higher estimated Alphas in the model. This suggests that the extra Alpha for this group of funds may be a compensation for an eighth risk factor that is missing from the 7-factor model, namely, a liquidity risk factor.

We have organized our study as follows: In Section 2, we review the literature; in Section 3, we present the data and describe our methods, including our model and our liquidity risk factor; in Section 4, we present our findings; and in Section 5, we conclude.

2. LITERATURE REVIEW

There is a robust and historically important literature related to performance evaluation and measurement of hedge funds. Among many types of fund performance measures, two are the most commonly utilized. The first one is the Alpha coefficient proposed by Jensen (1969), developed by Fama & French (1993) and adjusted to the hedge fund industry by Fung and Hsieh (2004). Alpha measures the relationship between a fund return and a set of undiversifiable risk factors that influence this return. Because of its properties, this measure is a standard in academic research on hedge funds (e.g. Barras et al., 2010; Chen et al., 2016; Fung & Hsieh, 2001; Kosowski et al., 2007; Stulz, 2007). The second measure of particular popularity in practice is the reward-to-variability ratio of Sharpe (1966). The Sharpe ratio measures the relationship between the mean and the standard deviation of excess returns. It is one of the best-known and widely used metrics to measure and compare investment performance (Auer & Schuhmacher, 2013). For a long time it was treated as not appropriate for hedge funds because theoretically it was justified to be utilized in the case of normally distributed excess returns or quadratic investor preferences (Brooks & Kat, 2002; Mahdavi, 2004; Zakamouline & Koekebakker, 2009). More recent findings of Schuhmacher & Eling (2011, 2012) prove that the Sharpe ratio has a decision theoretic foundation even in the case of asymmetric or fat-tailed excess returns and thus it is applicable for the performance analysis of the hedge funds. Auer and Schuhmacher (2013) expand the analysis of the statistical properties of the Sharpe ratio and propose adequate testing that strengthen the theoretical plausibility of the Sharpe ratio as a hedge fund performance measure.

Sharpe ratio is a simple fund performance measure that can be easily interpreted by an ordinary investor. This is one of the reasons why it is widely and commonly used in practice to publish rankings of funds according to their performance. Such rankings serve then as a tool of investment advisors who help hedge fund clients to make investment decisions (Liang, 1999). However, the original version of the Sharpe ratio does not capture individual types of risks, especially liquidity risk that is of particular importance to hedge funds. Hedge funds are meaningful investors in markets of illiquid assets where they bear illiquidity risk as a major source of return, that becomes an “illiquidity premium.” This premium is investigated by researchers in two levels. In the present research we are focused on what we might call “micro level” liquidity risk at the level of the fund and the fund portfolio, that is, asset liquidity. Other sources of liquidity risk are “macro level” risks, and we have indeed seen such risks impact the liquidity of hedge funds at times of the market stress. The Global Financial Crisis comes to mind, of course, as does the “double whammy” in August 1998 of the Russian ruble crisis and the Long Term Capital Management disaster. These macro-level liquidity shocks tend to affect all hedge funds more or less at the same time.

The literature on liquidity risk at the “macro level” focuses on the effect of systemic liquidity shocks (that is, aggregate market-wide liquidity risk as an undiversifiable risk factor) on market microstructure aspects such as bid/offer spreads, trading volume, and price impact, as well as changes in funding. Several studies have found that systemic liquidity risk accounts for or explains a significant portion of fund Alpha (e.g., Gibson et al. (2013)) or that the large losses experienced during global liquidity shocks counteract the generally good performance of illiquid funds in calmer markets (Sadka (2010) and Sadka (2012)). Previously, Pastor and Stambaugh (2003) employ proxies for system liquidity risk, such as bid/offer spreads and trading volume for stocks, to rank and sort stocks to create a no-investment, long-short liquidity risk factor. They find that the aggregate liquidity risk measure helped to explain the cross-section of stock returns. Billio et al. (2011) find that some hedge fund strategies perform particularly poorly during bouts of financial distress due to funding problems and illiquid assets.

Another focus of the literature connected to the liquidity risk at the “macro level” is on well-organized markets for the underlying securities, such as stock exchanges or other exchanges where bids and offers are posted and where the price data is transparent and trading volumes are available. In such markets the researchers look for liquidity timing ability of hedge fund managers and they find it: Cao et al. (2013) on the equity market, B. Li et al. (2017) on the fixed-income market and Luo et al. (2017) and Ch. Li et al. (2020) on the foreign exchange market. Yet many of the hedge fund strategies deal in securities that do not trade on those markets. And virtually no such hedge fund discloses its portfolio holdings in enough detail to enable an analyst to assess the liquidity of the fund through examination of its holdings. One of the possibilities to assess liquidity risk at more “micro level”, at the level of the individual fund, is the model proposed by Lo (2002) who focuses on the Sharpe ratio. He finds the volatility of holding period returns, which serves as the risk measure, to be understated in the case of illiquid portfolio holdings. Lo (2002) establishes a fund’s correlation to its own one-month lagged returns as a proxy for liquidity risk and uses this approach as the basis for restating the Sharpe ratio. Lo (2002) adjusts the Sharpe ratio for liquidity risk through the process of annualizing Sharpe ratios typically calculated on the basis of monthly fund data. He recognizes that the usual method of multiplying by $\sqrt{12}$ to annualize data based upon monthly returns is not appropriate in situations where the returns are non-IID, serially correlated returns being one example of returns that violate the assumption of IID. The Lo (2002) factors essentially reduce traditionally calculated Sharpe ratios for funds with positive serial correlation and increase Sharpe ratios for funds with negative serial correlation.

Getmansky et al. (2004) extend Lo (2002), relying on the regression coefficient in an AR(1) serial correlation model to serve as a proxy for a fund’s liquidity risk. They point out that returns should be serially uncorrelated in an informationally efficient market. The presence of serially auto-correlated returns certain funds, then, they take as an implication of market inefficiency

and illiquidity. Getmansky et al. (2004) find that the presence of serial correlation in hedge fund returns can be caused by three factors: 1. the fund investment strategy and the nature of assets in the fund; 2. the method of month-end pricing; and 3. deliberate “smoothing” of returns by a fund manager. These factors bear directly on the liquidity of the fund’s underlying assets and therefore of the fund itself. Generally large cap equity funds should have low levels of serial correlation, because they are liquid and easy to price so the temptation of a fund manager to “smooth” his returns in that type of hedge funds is small. However, the strategies of small cap equity, distressed debt, PIPES or fixed income arbitrage tend to have high serial correlation. Therefore they have a greater risk of dislocation and a large negative performance surprise. For these types of funds, the standard deviation may understate the actual risk, and the Sharpe ratio may overstate the reward-for-risk tradeoff. To the extent that analysts rely heavily on the Sharpe ratio measure in these circumstances in their investment decision making, they may overestimate the diversification benefit of including such a fund in their portfolios.

Khandani and Lo (2011) apply the analysis of Getmansky et al. (2004) to both hedge funds and mutual funds, as well as artificially created portfolios of stocks. They rank and sort these various portfolios by the autocorrelation of monthly returns to create a no-investment, long-short liquidity risk factor. They find a positive relationship between elevated levels of serial correlation and funds with longer redemption periods (i.e., longer lock ups) as well as funds with investment strategies known to be more illiquid (such as small-cap stocks, emerging market stocks, and mortgage-backed bonds).

Our interest in this research is the fund liquidity risk at the “micro level”, where different funds are exposed to different levels of such risk at different times. We follow Lo (2002) and Getmansky et al. (2004) and use serial correlation of returns as a proxy of the liquidity risk and propose the liquidity risk adjusted performance ratio (LRAPR). By doing that we add a brick to the discussion on adjusting Sharpe ratio with the liquidity risk. Because of simplicity our measure could be easily used to rank hedge funds by the financial information systems and investment advisors who – then – could present them to the hedge fund investors.

3. DATA AND METHODOLOGY

We use the BarclayHedge database of “living” funds, with monthly returns through December 2020. Our first step is to eliminate duplicate funds². To do this, we took the monthly returns across all the funds in the database for the period July 2015 to June 2020, and we calculated the pairwise correlation coefficients. For any $R > 0.95$, we deemed this to be a duplicate fund. Per this procedure, we reduced the fund database from 5,000+ to 2,133 funds. Next, we decided on an analysis period of January 2012 through December 2020 as the period of the dynamic world capital market growth where the number on long “living” hedge funds was high and the reliable data on them available. We found there to be 1,186 funds for the January 2012 through December 2020 time period. For each fund for the nine-year period, we calculated the following metrics, using monthly holding period return data: CAGR; Standard Deviation; Risk-free rate; Sharpe Ratio³; AR(1) Beta – a one-period-lagged serial correlation measure; t-stat and p-value for the AR(1) Beta; and LRAPR – our liquidity-risk-adjusted performance ratio.

We follow Lo (2002) and Getmansky et al. (2004) and use serial correlation of fund returns as the proxy for liquidity risk. Lo (2002) measures serial correlation with the correlation coefficient.

² A duplicate fund is a fund that is an individual legal entity but it shares the same manager, the same strategy, and either the same or a very similar portfolio with another fund.

³ We took the returns for the US equity market benchmark index as well as the risk-free-rate data from the data library of David French (of Fama-French fame) found at the website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Getmansky et al. (2004) do that with the regression-estimated slope coefficient β_{iT} of a simple one period lagged autocorrelation AR(1) process model:

$$r_{it} = \alpha_{iT} + \beta_{iT} r_{it-1} + e_{it}, \quad \text{Eq. (1)}$$

where r_{it} – is a return of a fund i in time t , r_{it-1} is – is a return of a fund i in time $t-1$, α_{iT} , β_{iT} are coefficients and e_{it} is a standard error of Eq. (1). Both measures of serial correlation are essentially the same⁴. We modify the standard Sharpe ratio measure such that we divide the standard deviation in the denominator by 1 minus the regression-estimated coefficient for the AR(1) process serial correlation for a single fund. Our “liquidity-risk-adjusted performance ratio” (LRAPR) is calculated as follows:

$$LRAPR = \frac{R_i - R_f}{\sigma_i / (1 - \beta_{AR(1)i})}, \quad \text{Eq. (2)}$$

where $\beta_{AR(1)i}$ is the coefficient β_{iT} from Eq. (1). Further in this article we refer to it as the AR(1) Beta. Our method for adjusting the Sharpe ratio is simple, accessible to the analyst and easy to deploy for practical use. To give an example, consider two hedge funds from our universe for the period 2012 through 2020 (see Table 1).

Table 1

Example of Changed Rank Ordering of Funds under LRAPR versus standard Sharpe Ratio

Fund	Annual Return	Risk free rate	Ann Std Dev	Sharpe Ratio	AR(1) Beta	LRAPR
Hedge Fund A	11.85%	0.65%	8.67%	1.292	.257	.961
Hedge Fund B	7.31%	0.65%	4.94%	1.348	.319	.915

Source: own calculations.

Of note in this example is that the ordering of the funds has changed after the inclusion of the heightened liquidity risk in the reward-for-risk formulation. The fund with the higher Sharpe ratio actually records a lower LRAPR; this is due to its higher serial correlation and hence higher likely liquidity risk. When adjusted for the heightened possible liquidity risk, the reward-for-risk measures for both funds are re-stated at lower levels, and the rank ordering is reversed. We investigate whether this is the case in general by comparing the LRA performance ratio with the standard Sharpe ratio and with the Lo (2002) adjusted Sharpe ratio for each individual fund in the whole universe of our 1,186 hedge funds for the 9-year period 2012 through 2020.

Next, we ask a question: “Does a fund’s serial correlation that is a proxy for liquidity risk influence a fund’s Sharpe ratio?” Or stated somewhat differently, “Does a high Sharpe ratio evidence the fund’s superior performance relative to other funds, or is it reflective of compensation for bearing liquidity risk?” We regress in turn the fund LRAPR, standard Sharpe ratio and Lo (2002) adjusted Sharpe ratio on the fund AR(1)Beta to address the question. The regression outputs are presented for the 142 high AR(1)Beta funds and for the 1,186 fund universe as well as for the funds with positive and negative serial correlation – we do the latter in order to confirm our intuition that adjustment to the Sharpe ratio is best confined to funds

⁴ The formula for Beta is $\text{Beta} = \frac{\text{cov}(r_x, r_y)}{\sigma_x^2}$, and the formula for correlation is $R = \frac{\text{cov}(r_x, r_y)}{\sigma_x \sigma_y}$. The only difference is in the denominator, replacing σ_x with σ_y . Since x and y are essentially the same data (lagged one month, so that $n-1$ of the n data points are in common), the standard deviation of x is very close to the standard deviation of y , in cases where neither the first and nor last months of the returns time series is an extreme return.

with positive serial correlation, and that negative serial correlation funds harbor no particular liquidity risk.

Finally, we examine the association between the AR(1) Beta and a hedge fund Alpha. We are motivated by our findings that Sharpe ratios are overestimated due to exclusion of liquidity risk from the Sharpe ratio risk measure. We hypothesize that a fund Alpha is similarly overstated in the presence of heightened liquidity risk in the case where the Alpha is estimated in an equilibrium model that excludes liquidity risk as an identified and modeled risk factor – which describes just about every equilibrium model in use in hedge fund performance evaluation. We employ our own 7-factor equilibrium model – where liquidity risk is not among the risk factors – to estimate a fund Alpha for all of the 1,186 funds in the universe. We regressed the resulting fund Alpha on fund AR(1) Beta, to assess whether fund Alpha is driven by liquidity risk and is overstated in the presence of liquidity risk. Our 7-factor model is based on the Carhart 4-factor model (Carhart, 1997) and the extensive work of Fung and Hsieh over the many years, that resulted in their identification of useful risk factors (Fung & Hsieh, 2007). Our seven factors are: market risk, size, value, momentum, interest rate risk, credit spread risk, and emerging market equity risk, as represented in this model:

$$r_{it} - rf_t = \alpha_{iT} + \beta_{1iT}(RMRF)_t + \beta_{2iT}(SMB)_t + \beta_{3iT}(HML)_t + \beta_{4iT}(MOM)_t + \beta_{5iT}(10yrUTS)_t + \beta_{6iT}(10yrUTS - Baa Bonds)_t + \beta_{7iT}(EM Equity - R_f)_t + e_{it} \quad \text{Eq. (2)}$$

The results of all the steps of our research are presented in the next Section.

4. RESULTS

The average values of the standard Sharpe ratios, our LRA performance ratio and the Lo (2002) adjusted Sharpe ratio for the hedge funds in our universe are presented in Table 2. First, we focus on the 142 hedge funds where the AR(1) serial correlation coefficient is positive and significant at the 95% confidence level. The measures of AR(1)Beta for these 142 funds range from a low of 0.188 to a high of 0.878. Looking at the averages for those 142 hedge funds with higher assumed liquidity risk, we see a few noteworthy aspects. First, the hedge funds that have significant serial correlation on average have much higher Sharpe ratios than the average Sharpe ratio across the 1,186 fund universe (1.256 versus 0.699). Second, the 142 hedge funds with higher assumed levels of liquidity risk surrender much more of their Sharpe ratio in descending toward the LRA performance ratio, dropping 36,1% from 1.256 to 0.803; the average fund in the 1,186 fund universe, surrendered 7.7% of its Sharpe ratio in the process of incorporating liquidity risk into the reward-for-performance measure, from 0.699 to 0.645. Third, our LRA performance ratio gives about the same result as the Lo (2002) method, while being simpler to calculate and being easier to understand intuitively.

Table 2

Comparison of Sharpe Ratios and LRAPRs

	1	2	3	4	5	6
9-year period 2012–2020	Average AR(1) serial correlation coefficient	Average Sharpe Ratio value	Average LRAPR value	% decrease from Sharpe Ratio to LRAPR (2 – 3)	Average Lo (2002)-adjusted Sharpe ratio value	% decrease from Sharpe Ratio to Lo (2002)-adjusted (2 – 5)
142 Hedge Funds with positive AR(1)	0.284	1.256	0.803	-36.1%	0.904	-28.1%
1,186 Hedge Funds	0.048	0.699	0.645	-7.7%	0.652	-6.7%

Source: own calculations.

Additionally, whereas the Lo (2002) method allows for an increase in Sharpe ratio for a fund with negative one-period lagged correlation, we do not credit negative AR(1) funds with a higher Sharpe ratio simply for the fact that the fund's AR(1) measure is even lower than a level that already indicates “little or no” liquidity risk. There are interesting phenomena with negative serial correlation funds that merit attention. The AR(1) for the SP500 varies over time, but is sometimes in the range of about 0.10 to 0.15 for long periods of time. While in times of crisis there may be short-lived bouts of poor liquidity or illiquidity in the US large-cap equity market, in general, we would argue that we can classify the US large-cap equity market as “liquid” – and it is certainly liquid when considered in relation to managed funds, such as hedge funds. So, if a hedge fund has an AR(1) Beta measure that is on par with or smaller than the AR(1) Beta measure for the overall US large-cap equity market, we would not suggest that we can infer from the fund's serial correlation measure that fund harbors particular liquidity risk. So, if funds with AR(1) measures of serial correlation of 0.10 to 0.15 can be characterized as “little or no liquidity risk” funds, then what to make of funds with AR(1) measures of zero or -0.10 or -0.25. Is there such a thing as less liquidity risk than zero liquidity risk? We think not. We can consider funds with negative, near-zero, or only very modest levels of positive serial correlation to be funds characterized by mean reverting returns. Funds with negative measures for AR(1) tend to be in areas such as managed futures, global macro trading, etc., with high turnover and high exposure to high-liquidity non-equity and non-bond securities such as FX, futures, and other derivatives. Therefore, we make no adjustment to the Sharpe ratio in the case of funds where the serial correlation is negative.

The results for regressing the fund Sharpe ratio, LRAPR, Lo (2002) Sharpe ratio as well as 7-factor Alpha on the fund AR(1)Beta are summarized in Table 3.

Table 3

Results for Regressing Sharpe Ratio, LRAPR, 7-factor fund Alpha and Lo 2002 Sharpe Ratio on Fund AR(1)Beta

	Dependent Variable	Independent Variable	Universe	R ²	Regression Coefficient (loading)	p-value	Intercept	p-value
1	Sharpe Ratio	AR(1) Beta	142 high liquidity risk funds	.11	8.69	.0000	-1.21	.0462
2		AR(1) Beta	Entire universe 1,186 funds	.04	1.80	.0000	.61	.0000
3		AR(1) Beta	758 AR(1) Beta > 0 funds	.07	3.41	.0000	.35	.0000
4		AR(1) Beta	428 AR(1) Beta < 0 funds	.00	-.31	.4177	.59	.0000
5	LRAPR	AR(1) Beta	142 high liquidity risk funds	.01	1.44	.2389	.40	.2843
6		AR(1) Beta	Entire universe 1,186 funds	.00	.39	.0394	.63	.0000
7		AR(1) Beta	758 AR(1) Beta > 0 funds	.00	.67	.0490	.58	.0000
8		AR(1) Beta	428 AR(1) Beta < 0 funds	.00	-.31	.4177	.59	.0000
9	Lo (2002) Sharpe Ratio	AR(1) Beta	142 high liquidity risk funds	.04	3.35	.0148	-.05	.8969
10		AR(1) Beta	Entire universe 1,186 funds	.00	.47	.0186	.63	.0000
11		AR(1) Beta	758 AR(1) Beta > 0 funds	.02	1.33	.0002	.49	.0000
12		AR(1) Beta	428 AR(1) Beta < 0 funds	.01	-.99	.0172	.59	.0000
13	7-Factor Fund Alpha	AR(1) Beta	142 high liquidity risk funds	.03	.68	.0476	.14	.1731
14		AR(1) Beta	Entire universe 1,186 funds	.05	.76	.0000	.06	.0000
15		AR(1) Beta	758 AR(1) Beta > 0 funds	.06	.99	.0000	.03	.1606
16		AR(1) Beta	428 AR(1) Beta < 0 funds	.01	-.67	.0586	-.04	.0000

Source: own calculations.

These output suggest several findings. First (Table 3, row 1) for the 142 high-liquidity-risk funds in the universe, an increase in AR(1)Beta of, for instance, 0.10 (from, say, 0.45 to 0.55) is associated with an increase in Sharpe ratio of 0.86. And across the entire universe of 1,186 funds (Table 3, row 2), an increase in AR(1)Beta of 0.10 is associated with a 0.18 increase in Sharpe ratio. These findings are consistent with our intuition that standard deviation as a measure of “total risk” – and as the risk measure used in the standard Sharpe ratio – does not capture liquidity risk in that Sharpe ratios are sensitive to and positively related to increases in assumed fund liquidity risk.

Further, we performed similar regressions dividing the universe into two groups: positive serial correlation funds (Table 3, row 3) and negative serial correlation funds (Table 3, row 4). The results seem to confirm our intuition. For negative serial correlation funds, there is no particular association of the Sharpe ratio with the liquidity risk, as indicated by the high p-value for the slope coefficient and by the high level of Significance F for the model overall. As for the positive serial correlation funds (over 80% of which have AR(1) Betas that are not statistically significant at the 95% confidence level), a 0.10 increase in serial correlation is associated with a 0.34 increase in Sharpe ratio. And the model overall is significant.

Our expectation is that the modification to the Sharpe ratios that we performed when we transformed the fund Sharpe ratios into fund LRAPRs will result in a better measure of reward-for-risk than the original Sharpe ratios, which are overstated due to liquidity risk being absent from the Sharpe ratio risk measure. One area in which we can test if the LRAPR is an improvement

on the Sharpe ratio is to investigate whether or not the LRAPR is systematically and positively associated with liquidity risk, as the Sharpe ratio seems to be. If the LRAPR has successfully adjusted for fund liquidity risk, then we would expect to see little evidence (or at least less evidence) of a positive association between the LRAPR and liquidity risk. We also perform a similar analysis for the Lo (2002) modified Sharpe ratios. Repeating the foregoing regression analysis, substituting LRAPR (and then the Lo 2002 Sharpe ratios) for the Sharpe ratio we get the results presented in Table 3, rows 5 and 9.

For the 142 high liquidity risk funds, the estimated slope coefficient for the LRAPR regression has a p-value of 0.24 (lacking statistical significance), indicating that the LRAPR provides a reward-for-risk measure for hedge funds that accounts for differential liquidity risk as proxied by the serial correlation AR(1) Beta. In the case of the Lo (2002) Sharpe ratios, the estimated slope coefficient is statistically significant (p-value of 0.01), and a loading of 3.35, implying an increase in Lo (2002) Sharpe ratio of 0.33 for every 0.10 increase in AR(1) Beta. While this sensitivity to liquidity is less than the 8.69 slope coefficient in row 1 for the Sharpe ratio correlation, it still shows that variations in liquidity risk across the group of 142 funds explains a good portion of the differences in Lo (2002) Sharpe ratios.

Again, dividing the funds into two groups (positive and negative serial correlation) also shows that the LRAPR seems to be an improvement over the standard Sharpe Ratio, as well as over the Lo (2002) Sharpe ratios, in removing any dependence in the differences in performance ratios among the funds to the level of liquidity risk in those funds. Adding the Lo (2002) Sharpe ratios to this discussion (Table 3, rows 11 and 12), we can see that for the AR(1) Beta > 0 funds the Lo (2002) Sharpe ratios are statistically significantly and positively related to the level of liquidity risk in the funds' portfolios, with a 0.13 increase in Sharpe ratio associated with a 0.10 increase in AR(1) Beta.

As for the AR(1) Beta for the AR(1) Beta < 0 funds, as we have discussed above, we view such funds to be funds with mean reverting yields, not funds with assets that trade "sticky" and may be illiquid. These are likely funds with very liquid portfolios, in strategies such as global macro, managed futures, and the like. By contrast, Lo (2002) adjusts the Sharpe ratio upwards for these AR(1) Beta < 0 funds, resulting in the statistically significant and negative relationship between Lo (2002) Sharpe ratio and AR(1) Beta for this cohort. Finally, we examine the association between the AR(1) Beta and a hedge fund 7-factor Alpha. The finding for the 142 high liquidity risk funds indicates that at a 95% confidence level there seems to be a positive relationship between fund Alpha and the AR(1) Beta measure of fund liquidity risk. More specifically, we can interpret the relationship as follows: the regression slope shows that an increase in monthly Alpha of 0.6799% (or an annual Alpha of 8.16%) is associated with a 1.0 increase in serial correlation. On a more reasonable scale, we can say that a 0.10 increase in serial correlation is associated with a 0.82% increase in estimated annual Alpha in the 7-factor model. This seems to confirm our intuition that a significant portion of fund Alpha is likely compensation for bearing liquidity risk. For the entire universe of 1,186 hedge funds, the relationship is a statistically significant 0.92% increase in annual Alpha for an increase in serial correlation of 0.10.

Again, applying the analysis to positive and negative serial correlation funds as distinct groups gives the following results (Table 3 rows 15 and 16). For funds with positive serial correlation, and some assumed level of liquidity risk, it seems that an increase in AR(1) Beta of 0.10 is associated with an increase in annual Alpha of 1.19%. Stated differently, accounting for liquidity risk in fund performance evaluation may lead the analyst to reduce the evaluation of annual fund Alpha by 1.19% per every 0.10 of serial correlation (above some threshold that the analyst will have to decide upon) identified in the fund returns. Interestingly, again the negative serial correlation funds seem to exhibit a distinct lack of liquidity risk. In fact, the negative slope of -0.6710 seems to imply that funds that are strongly mean reverting (and this could include hedge

fund strategies such as global macro, managed futures, and the like) are adding value at the rate of 0.81% per annum for every increase of 0.10 of serial correlation in the negative direction.

We conclude this section on research findings with a summary comparison of the Sharpe ratios, Lo (2002) Sharpe ratios, and LRAPRs for the funds in our universe. We calculated each performance measure for each fund in the universe, and we present the findings aggregated by AR(1) Beta decile, in Table 4:

Table 4

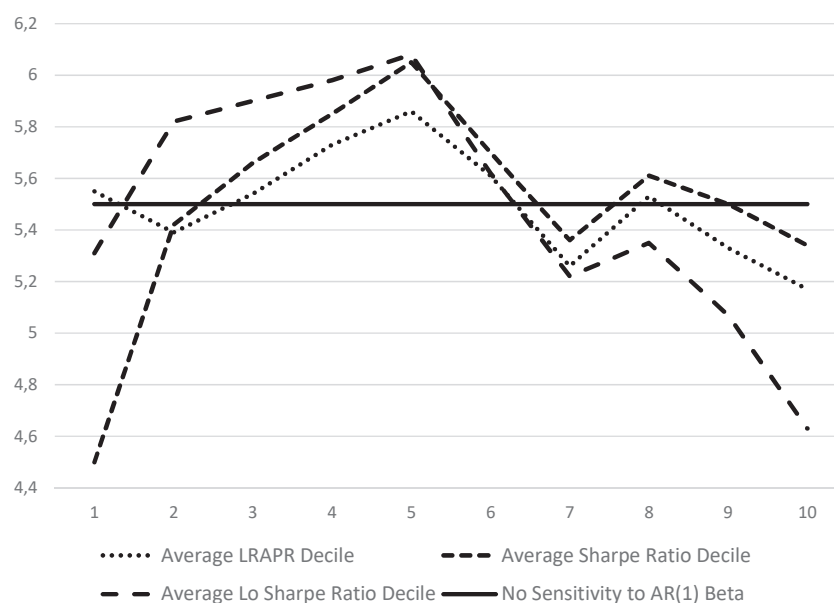
Sharpe ratios, Lo (2002) Sharpe ratios and LRAPR by AR(1)Beta Decile

AR(1) Beta Decile	Average AR(1) Beta	Average Sharpe Ratio Decile	Average Sharpe Ratio	Average Lo 2002 Sharpe ratio Decile	Average Lo 2002 Sharpe ratio	Average LRAPR Decile	Average LRAPR
1	.30	4.50	1.34	5.31	.94	5.55	.83
2	.17	5.42	.68	5.82	.58	5.39	.65
3	.11	5.66	.61	5.90	.55	5.54	.61
4	.08	5.85	.59	5.98	.55	5.73	.59
5	.05	6.05	.55	6.08	.52	5.86	.55
6	.02	5.70	.59	5.62	.58	5.61	.59
7	.00	5.36	.77	5.22	.77	5.26	.77
8	-.03	5.61	.62	5.35	.64	5.53	.62
9	-.07	5.50	.59	5.07	.63	5.33	.59
10	-.14	5.34	.65	4.63	.74	5.17	.65
Column Average		5.50		5.50		5.50	
Average of all Funds	.05		.70		.65		.64
Standard Deviation		.42		.46	.	.21	

We placed the deciles sorted by AR(1) Beta along the X-axis. Each decile contains 118 or 119 of the total 1,186 funds in the universe. The first decile, on the left side of the graph, contains the 119 funds with the largest estimates of AR(1) Beta. The tenth decile, on the right, contains the 118 funds with the lowest estimates of AR(1) Beta. For each decile by AR(1) Beta, we calculated three values. As an example, consider the short dashed line with a value of 4.5 for AR(1) Beta decile 1: the 119 funds on the top AR(1) Beta decile, on average, reside the middle of the fourth decile when we sorted the 1,186 funds by Sharpe ratio and placed them into deciles by Sharpe ratio. If there were no relationship at all between AR(1) Beta and the Sharpe ratio – that is a merely random association – then we would expect the Sharpe ratio decile average to be 5.50 in every AR(1) Beta decile. Displaying the decile averages graphically, we find what is seen in Graph 1.

Graph 1

Average Performance Ratio Decile for Three Different Performance Measures, across the Ten Deciles by AR(1) Beta



Source: own calculation.

The solid horizontal line at the vertical height of 5.50 represents no sensitivity to AR(1) Beta. We can see that the Sharpe Ratio and the Lo (2002) Sharpe ratio performance measures display some sensitivity to the AR(1) Beta. As shown in Table 4, the standard deviation of the decile averages are 0.42 and 0.46, respectively. For the LRAPR, we can see from the chart that the sensitivity to the liquidity risk proxy measure is much less, measured as 0.21 by standard deviation. The regression results in Table 3 tells a similar story. This asserts our belief that the LRAPR may provide the investor or analyst with a better apples-to-apples comparison of risk-reward performance across funds, in that the LRAPR does a better job than original Sharpe ratio of adjusting for varying liquidity risk among funds.

5. CONCLUSIONS

Ideally, a reward-for-risk measure should include all relevant risks, so that the risk is not understated and so that the reward-for-risk is not overstated. The original Sharpe Ratio relies on standard deviation as a measure of “total risk,” which measure does not incorporate potential liquidity risk at the fund level. We have borrowed from the literature a proxy measure for potential liquidity risk for hedge funds, and then applied and extended that in the direction of modifying the Sharpe Ratio to create a new liquidity-risk-adjusted performance ratio (LRAPR). Others have dealt with this topic before us, notably Lo (2002). We strived to improve on Lo (2002)’s formulation of a solution by applying it just to funds with liquidity risk (not all funds in the universe) and by proposing a simplified calculation method that yields similarly modified results when compared to the Sharpe ratio, but that is accessible to and implementable nowadays by just about any analyst or investor.

We find a positive and significant relationship between liquidity risk and Sharpe Ratio among funds with statistically significant levels of liquidity risk as measured in an AR(1) process. This indicates that funds with higher liquidity risk are rewarded with higher Sharpe Ratios, even though that higher reward may be the result of compensation for bearing liquidity risk and not from some other source, such as manager skill or some other factor. In contrast to this, we find no significant relationship between our LRAPR and liquidity risk, indicating that we might use

our LRAPR as a reward-for-risk measure that incorporates or corrects for differences in liquidity across funds.

Further, we find that funds with higher likely liquidity risk exhibit higher levels of fund Alpha as estimated in a 7-factor model that does not account for liquidity risk. This would seem to imply that some of the fund Alpha of higher liquidity risk funds is probably a compensation for bearing liquidity risk rather than returns due to the manager skill or some other source of return. Therefore, all the more important it is for the analyst or investor to have a method to take fund liquidity risk into account and to be able to evaluate fund performance on a more comparable basis. We are hopeful that analysts and investors will find our LRAPR to be accessible, implementable, and helpful in fund analysis: hedge funds, mutual funds or any other type of portfolios that are managed locally or globally but may be exposed to the liquidity risk.

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