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Education Versus Infrastructure: Is There a Trade-off in an Open Economy with Perfect Capital Mobility?

Michał Konopczyńskio

ABSTRACT

We present an open economy growth model incorporating various elements of fiscal policy, including government expenditure on education and public capital (infrastructure), budget deficit, internal and external public debt, public consumption, and four tax rates. This detailed description of fiscal policy allows for a systematic study of the relationship between fiscal policy and economic growth. We derive the balanced growth path and analyze its properties, including existence, uniqueness, and stability. The theoretical results are supported by numerical simulations for Poland, with the model calibrated based on data from the years 2010 to 2019. In the baseline scenario, the GDP growth rate converges to 3.98%. However, through appropriate adjustments to fiscal policy, economic growth can be significantly accelerated. Notably, increasing spending on both education and public infrastructure proves to be the most effective way to permanently boost economic growth, even if it requires raising taxes on consumption or increasing public debt.

KEY WORDS: economic growth, human capital, public capital, public debt, infrastructure.

JEL Classification: E62, F43, H52, H6.

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Introduction

Within economic growth theory, two factors of production have recently garnered significant attention: human capital, primarily determined by education, and public capital, which is influenced mainly by public infrastructure investments. The literature on human capital is already well-established. In contrast, public capital remains a topic of discussion due to controversies not only in theoretical models but also in its definition and statistical measurement. Nonetheless, many researchers have incorporated public capital into mathematical models of economic growth using one of two alternative approaches: the stock approach or the flow approach. In the stock approach, public capital is viewed as a stock accumulated over time that depreciates at a certain rate. On the other hand, the flow approach treats public capital as a stream of services provided by the government that cannot be stored or accumulated. In reality, both approaches have evident counterparts. For instance, public infrastructure, such

Correspondence concerning this article should be addressed to: **Michał Konopczyński**, Poznań University of Economics and Business al. Niepodległości 10, 61-875 Poznań, Poland. E-mail: michal.konopczynski@ue.poznan.pl as roads, seaports, and bridges, represents a tangible stock, while areas like law and justice, national security, and healthcare are (at least in part) flows directly dependent on the annual budget and cannot be accumulated.

Investments in public infrastructure have played a crucial role in accelerating the impressive economic growth of the Polish economy in recent years. Notably, Poland has experienced substantial investments in various aspects of public infrastructure, including transport, public buildings, and utilities. Projects such as roads and highways, railways, airports, schools, universities, hospitals, water and sewage systems, and more have been undertaken, with a significant portion of the funding coming from the EU Cohesion Fund. Kollias and Paleologou (2013) argue that "a range of (...) economic activities gain from such public spending. In particular, improved transport infrastructure reduces effective distances between different poles of economic activity, between centres of production and consumption, and reduces road congestion bringing about lower travel times and costs

for both enterprises and passengers. Increased trade is a strong stimulus of growth." The positive impacts of enhancing public infrastructure in Poland extend beyond domestic and international trade. The country's improved infrastructure has also been attracting more Foreign Direct Investment (FDI), adding another crucial factor to its overall economic growth.

In Poland, the majority of improvements have been focused on material infrastructure. As a result, the stock approach has been chosen for our research, as it is also more commonly used in the literature. Notable early contributions in this area include works by Aschauer (1989), Barro (1990), Futagami et al. (1993), and Easterly and Rebelo (1993), as well as contributions by Turnovsky (1997, 2004), Fisher and Turnovsky (1998), and Dasgupta (1999).

Let's briefly review some recent developments that are relevant to our research. Chen (2007) introduces public infrastructure as a substitute for private capital, without distinguishing human capital as a separate production factor. In Chen's model, the government behaves passively, collecting both labor income taxes and capital income taxes and then spending the total amount of taxation to accumulate public infrastructure stock. Only income taxes are analyzed in this model.

Marrero and Novales (2007) consider public consumption expenditure and infrastructure investment but make the peculiar assumption that both public and private capital fully depreciate each period. This assumption makes their model challenging to interpret and practically impossible to verify empirically. Similar to Chen (2007), they assume a balanced government budget and do not include human capital in their analysis.

In contrast, the model constructed by Dhont and Heylen (2009) incorporates human capital in a closed economy setting, and it encompasses various details of fiscal policy, such as taxes, public consumption, and productive government spending. This spending includes investments in education, active labor market programs, research and development (R&D) expenditures, and public investment. All these expenses are aggregated and accumulated as human capital, which becomes one of the key factors of production in their model.

Several researchers have integrated both the stock and flow approach into a single model. For instance, Agénor and Yilmaz (2011) incorporate infrastructure (stock) and healthcare (flow) in their endogenous growth model. Similarly, Bucci and Bo (2012) present a model where public capital serves as both input to the production of the final product (flow) and as a component that accumulates to increase its own supply in the future (stock). They highlight that the share of public investment in GDP plays a critical role in determining the long-run growth rate. However, their model is based on a few simplifying assumptions, such as a single flat tax (presumably because they only include government expenditure as a share of GDP), no depreciation, and no consideration of human capital.

Escobar-Posada and Monteiro (2015) propose a two-sector model that focuses on the accumulation of physical and human capital, where public goods contribute to both productive capital and utilityenhancing services. They analyze how the level and composition of government spending impact growth and welfare, and they determine the optimal levels that maximize growth and welfare.

A more recent paper by Zhang et al. (2016) also combines both the stock and flow approach. They derive the optimal tax structure and demonstrate that it is equivalent to the optimal public spending composition.

These studies demonstrate the complexity and significance of considering both stock and flow aspects when analyzing the role of public capital and its effects on economic growth and welfare. By incorporating various elements of fiscal policy, these models provide valuable insights into the dynamics of economic growth and the importance of well-structured public investments.

It is important to acknowledge that much of the public capital literature is based on closed economy models, where there is no consideration of foreign trade and financial flows between countries, foreign debt, or assets. While this approach may be suitable for larger countries that are relatively closed, it may not fully capture the complexities of small economies, such as most European countries, which have more significant interactions with the global economy.

Another major simplification found in many papers is the assumption of a permanently balanced government budget (zero deficit and debt). This tradition is closely related to the closed economy assumption, where the concept of Ricardian equivalence holds, suggesting that budget deficits are neutral for the longrun growth rate. However, in an open economy, this assumption may not hold true, as both the deficit level and the structure of public debt (foreign vs. domestic) can have implications for long-term growth and welfare. For instance, Turnovsky (2002) demonstrates that with a proportional tax on capital, Ricardian equivalence does not hold. Nevertheless, fiscal policy remains largely neutral in his model, as an increase in the tax on capital reduces the growth rate of capital while leaving the growth rate of consumption unaffected.

In contrast, Konopczyński (2014b) examines the implications of the size and structure of the budget deficit in an open economy with perfect capital mobility. Additionally, Konopczyński (2018b) extends the model to incorporate imperfect capital mobility. Both studies show that disregarding the government deficit and public debt in an open economy is not correct. The size of the deficit and the structure of public debt do have an impact on long-term growth and welfare.

In summary, it is essential to consider the unique characteristics of open economies, particularly smaller ones with significant international interactions, and to move away from the simplifying assumption of a permanently balanced government budget when studying the role of public capital and fiscal policy in fostering economic growth and welfare.

It should be realized that the public capital literature is almost entirely based on closed economy models: there is no foreign trade and financial flows between countries, no foreign debt and assets, etc. This approach is perhaps acceptable for large countries that are relatively closed. However, in our opinion it is not suitable for small economies, such as most European countries. In addition, most papers suffer from another major simplification: the assumption of a permanently balanced government budget (zero deficit and debt). In fact, this unfortunate tradition is closely related to the assumption of a closed economy, because in closed economy models, the so-called Ricardian equivalence holds, and therefore the budget deficit is neutral for the long-run growth rate. In an open economy, this may not be true: both the deficit level and the structure of public debt (foreign vs. domestic) can affect long-term growth and welfare. For example, Turnovsky (2002) shows that with the proportional tax on capital Ricardian equivalence does not hold. Nevertheless, in his model fiscal policy remains largely neutral, because "an increase in the tax on capital reduces the growth rate of capital, but leaves the growth rate of consumption unaffected". In contrast, Konopczyński (2014b) studies the implications of the size and structure of the budget deficit in an open economy with perfect capital mobility, and Konopczyński (2018b) extends the model to the conditions of imperfect capital mobility. Both papers prove that the disregard for the government deficit and public debt in an open economy is incorrect: both the size of the deficit and the structure of public debt do have an impact on long-term growth and welfare.

The main goal of this paper is to deviate from these two assumptions commonly made in existing literature to develop a growth model that aligns more closely with reality. In our model, both the public and private sectors have the capacity to borrow and lend (invest) abroad. The government holds the ability to actively influence the economy by employing a wide range of fiscal policy tools, encompassing both revenue and expenditure aspects. Notably, the public deficit is financed by a combination of domestic and foreign debt, without regard to the external debt of the private sector. The model incorporates four types of taxes: labor tax, capital tax, consumption tax, and interest on government bonds. Furthermore, public expenditures are divided into four categories: public consumption, education, public infrastructure, and financial transfers to the private sector. This model is well-suited for examining the effects of alterations in fiscal policy, making it a valuable tool to investigate various scenarios. As an example of its applicability, we present several simulations for Poland, showcasing how this model can be utilized to assess the outcomes of different fiscal policy changes in a specific context.

The paper is structured as follows: Section 1 presents the model, while Section 2 derives a balanced growth path and examines its mathematical properties, including existence, uniqueness, and stability. In Section 3, two special cases are presented to provide an intuitive understanding of the model. Section 4 calibrates the model for Poland using statistical data from 2010 to 2019. Moving on to Section 5, the baseline scenario is provided, and in Section 6, selected scenarios of increased spending on education and public capital are presented, along with a discussion of the trade-off between them. Section 7 looks at the remaining parameters of fiscal policy. In Section 8, simultaneous changes to 4 fiscal parameters are proposed, and the results are evaluated. This section also includes a sensitivity analysis for the most uncertain parameter of the model. Finally, Section 9 summarizes the main results and offers some hints for future research, while mathematical proofs are included in the appendix.

1. THE MODEL

1.1. Output and Factors of Production

The output of the representative firm is described by the following production function:

$$Y_i = F(K_i, L_i) = AK_i^{\alpha} (hL_i)^{\beta} P^{\varepsilon} \text{ with } \alpha + \beta = 1, \quad \alpha, \beta, \varepsilon > 0, \quad A > 0,$$
(1)

where κ_i denotes the stock of physical capital, L_i repre-

tion of the entire economy is:

sents raw labor, h is the average stock of human capital per worker: h = H/L, and P represents the flow of public services provided by government. Notice that Eq. (1) implies constant returns to scale with respect to private inputs (K, H), whereas public services P are complementary to private inputs: an increase in P raises their marginal products. Obviously, the aggregate produc-

$$Y = AK^{\alpha}(hL)^{\beta}P^{\varepsilon} = AK^{\alpha}H^{\beta}P^{\varepsilon}, \qquad (2)$$

where *K* is the aggregate stock of capital and *L* is the labor supply in the country. We assume that labor supply is determined exogenously, so it can be normalized to unity: L = 1. Therefore, Eq. (2) can also be treated as the production function per capita.

Following the standard approach in the literature initiated by Barro and Martin (1992), we assume that *P* (the flow) is a linear function of the amount of public capital κ_p (the stock) per unit of GDP, i.e.

$$P = x K_p / Y, \qquad x = const. > 0.$$
(3)

Without losing generality, parameter *x* can be normalized to 1, because after substituting equation (3) into (1) or (2) and separating x^e from $(K_r/Y)^e$, we can combine *A* with x^e and replace it with a new constant. Therefore, we set x = 1.

Firms maximize profits in perfectly competitive markets, which implies that labor and capital are paid their marginal products. It follows that

$$\forall t \quad w_{\kappa} = \alpha Y/K$$
, (4)

$$\forall t \quad w_L = \beta Y / L = \beta Y . \tag{5}$$

Therefore, $\forall t \ w_{K}K + w_{L}L = Y$. The accumulation equations are:

$$\partial K / \partial t = I_K - \delta_K K$$
, $0 < \delta_K < 1$, (6)

$$\partial H / \partial t = I_H - \delta_H H$$
, $0 < \delta_H < 1$, (7)

$$\partial K_p / \partial t = G_K - \delta_p K_p$$
, $0 < \delta_p < 1$, (8)

where $\delta_{i_{\ell}}$ denote the depreciation rates, and G_{κ} is the government's investment in public capital. The investment in private capital *K* is obviously financed by the

private sector. Meanwhile, investment in human capital is financed exclusively by the government, with the linear production function of human capital:

$$I_H = eG_E$$
, $0 < e < 1$, (9)

where G_E represents public expenditures on education.

1.2. The Public Sector (The Government)

The tax revenue of the government in real terms is:

$$T = \tau_L w_L L + \tau_K w_K K + \tau_C C + \tau_D r D_D, \qquad (10)$$

where τ_L , τ_K , τ_C , τ_D are the average tax rates on wages, capital income, consumption, and interest on government bonds purchased by domestic lenders, respectively and *r* is the real interest rate. The deficit of the public sector is by definition, in real terms, equal to: J = G + rD - T, where *G* is total government spending and *D* is total public debt. We assume that the budget deficit is fixed to GDP, i.e., $J = \xi Y$, where $\xi = const > 0$ is a decision parameter. Therefore, government spending follows the budgetary rule:

$$G = T - rD + \xi Y. \qquad (11)$$

The deficit is financed by government bonds, which causes an increase in public debt according to the equation: $\partial D / \partial t = \xi Y$. A predetermined percentage (ω) of bonds is sold to foreigners and the rest is purchased by domestic investors, i.e.

$$\partial D_F / \partial t = \omega \xi Y$$
, (12)

$$\partial D_D / \partial t = (1 - \omega)\xi Y,$$
 (13)

where D_D and D_F represent the government's domestic and foreign debt, respectively. Government spending has four components:

$$G = G_T + G_E + G_C + G_K, \qquad (14)$$

where G_r represents direct transfers to the private sector (pensions, various benefits, social assistance, etc.), and G_c is public consumption. For the sake of simplicity, we assume that public consumption is proportional to private consumption, whereas investment in education and public capital are predetermined percentages of GDP:

$$G_c = \sigma_c C, \qquad 0 < \sigma_c < 1, \tag{15}$$

$$G_E = \gamma_E Y, \qquad 0 < \gamma_E < 1, \tag{16}$$

$$G_{\kappa} = \gamma_P Y$$
, where $0 < \gamma_P < 1$. (17)

Note that the deficit rule $(J = \xi Y)$ implies that transfers G_r serve as the balancing item. For example, if the government decides to spend more on either education, or public capital, or public consumption (with everything else unchanged), it must reduce cash transfers to the private sector accordingly.

1.3. The private sector

The preferences of a representative household are expressed by the following standard intertemporal utility function:

$$U = \int_{0}^{\infty} \frac{1}{\gamma} (cg_{c}^{\kappa})^{\gamma} e^{-\rho t} dt, \quad \rho > 0,$$
 (18)

where *c* stands for private consumption and g_c is public consumption (both per capita, but since labor is normalized to unity, i.e. L=1, we can also apply absolute values: *C* and G_c). The elasticity of substitution between both types of consumption is expressed by $\kappa > 0$. The fraction $1/(1-\gamma)$ is equal to the intertemporal elasticity of substitution. We assume that $\gamma < 0$, which is justified by empirical research; see, e.g., Turnovsky (2009), p. 177.

The private sector receives income from labor and capital, interest on domestic public debt, returns on foreign assets *B*, and government transfers. It is important to note that net foreign assets *B* may very well be negative, which means a negative net international investment position (NIIP) of the private sector.

The real disposable income after tax is defined as follows:

$$Y_{d} = (1 - \tau_{L})w_{L}L + (1 - \tau_{K})w_{K}K + (1 - \tau_{D})rD_{D} + rB + G_{T}.$$
(19)

This income is used for consumption and investment, as well as for the purchase of government bonds. Any difference is covered by (net) lending/borrowing to/from abroad. Therefore, the instantaneous budget constraint (in real terms) is expressed as follows: $Y_a = C(1+\tau_c)+I_{\kappa}+\partial D_{\rho}/\partial t+\partial B/\partial t$. Substituting Eq. (25) and rearranging yields: $\partial B/\partial t = Y_a - C(1+\tau_c)-I_{\kappa} - (1-\omega)\xi Y$. Using Eqs. (19), (4), and (5) the budget constraint can be written as follows:

$$\begin{aligned} \partial B / \partial t &= (1 - \tau_L) \beta Y + (1 - \tau_K) \alpha Y + (1 - \tau_D) r D_D \\ &+ (r - n) B + G_T - C(1 + \tau_C) - I_K - (1 - \omega) \xi Y. \end{aligned}$$

It is worth emphasizing that a representative agent treats all fiscal variables as exogenous, practically assuming that their individual impact on the entire market is negligible. In particular, when making decisions, the representative agent respects the budget constraint (20) treating G_r and D_p as exogenous constants.

The private sector selects the flows (trajectories) of consumption and investment to maximize the utility level expressed by Eq. (18), subject to the budgetary constraint (20). The initial values of the variables (endowments) are given by

 B_0 , $K_0 > 0$, $_0 \ge 0$, $D_{F0} \ge 0$, $D_{D0} \ge 0$ with $D_{F0} + D_{D0} = D_0$.

The following fiscal variables are considered exogenous by the individual decision-maker: G_T , G_C , D_D , D_F .

2. The Balanced Growth Path (BGP)

2.1. The Solution of the Optimization Problem

The current value hamiltonian is:

$$H_{c} = \frac{1}{\gamma} \left(C G_{c}^{\kappa} \right)^{\gamma} + \lambda_{1} \cdot \left[(1 - \tau_{L}) \beta Y + (1 - \tau_{K}) \alpha Y + (1 - \tau_{D}) r D_{D} + (r - n) B \right]$$

+
$$G_{T} - C (1 + \tau_{c}) - I_{K} - (1 - \omega) \xi Y] \cdot$$

+
$$\lambda_{2} \cdot \left[I_{K} - (n + \delta_{K}) K \right] + \lambda_{2} \cdot \left[I_{H} - (n + \delta_{H}) H \right].$$
(20)

The optimal solution must meet the following (necessary and sufficient) conditions:

$$\forall t \ \partial H_c / \partial C = 0$$
, (a)

$$\forall t \quad \partial H_c / \partial I_K = 0, \qquad (b)$$

$$\partial \lambda_{\rm l} / \partial t = -\partial H_c / \partial B + \lambda_{\rm l} \rho , \qquad (c)$$

$$\partial \lambda_2 / \partial t = -\partial H_c / \partial K + \lambda_2 \rho$$
, (d)

$$\lim_{t \to \infty} e^{-\rho t} \lambda_1(t) B(t) = 0, \qquad (e)$$

$$\lim_{t \to \infty} e^{-\rho t} \lambda_2(t) K(t) = 0$$
 (1)

Condition (a) can be written as

$$\lambda_1(1+\tau_C) = C^{\gamma-1}G_C^{\kappa\gamma}, \quad (22)$$

that is, the shadow price of wealth (in the form of bonds), adjusted for consumption tax must be (for all t) equal to the marginal utility of private consumption. Log-differentiating this equation with respect to t yields:

$$\hat{\lambda}_{1} = (\gamma - 1)\hat{C} + \kappa\gamma\hat{G}_{C}$$
⁽²³⁾

Eq. (15) implies that private and public consumption grow at the same rate, say ψ . Thus $\hat{G}_c = \hat{C} = \psi$. Condition (*c*) can be written as:

$$\hat{\lambda}_{1} = \frac{\partial \lambda_{1} / \partial t}{\lambda} = \rho - r \cdot$$
(24)

Substituting Eq. (24) into Eq. (23), and using $\hat{G}_c = \hat{C} = \psi$ we get:

$$\psi = \frac{\partial C / \partial t}{C} = \frac{r - \rho}{1 - (1 + \kappa)\gamma} = const.$$
(25)

Thus, the optimal trajectory of private consumption is

$$C(t) = C_0 e^{\psi t}, \tag{26}$$

Log-differentiating the production function (2) gives:

$$\varphi_Y = \hat{Y} = \alpha \hat{K} + \beta \hat{H} + \varepsilon \hat{P} = \alpha \varphi_K + \beta \varphi_H + \varepsilon \varphi_P.$$
 (27)

In what follows, it is convenient to use certain variables expressed as shares of GDP. We denote these shares with small (lowercase) letters, e.g., k = K/Y, c = C/Y, $k_p = K_p/Y$, etc.

Dividing both sides of Eq. (7) by H and substituting Eqs. (9) and (16) yields:

$$\varphi_{\mu} = \hat{H} = e \gamma_{\mu} / h - \delta_{\mu}. \qquad (28)$$

Also, note that dividing both sides of Eq. (2) by *r* gives $_{Ak^{\alpha}h^{\beta}P^{e}=1}$, i.e. $_{Ak^{\alpha}h^{\beta}k_{p}^{e}=1}(_{x=1})$ which implies that the three types of capital k, h and k_{p} are always (not only in the steady state, but always) linked by the following nonlinear relationship:

$$\frac{1}{k_p^{el\beta}} = A^{\nu\beta} k^{el\beta} h.$$
⁽²⁹⁾

Condition (b) boils down to the equality:

$$\lambda_2 = \lambda_1$$
, (30)

which means that the shadow price of both types of capital (domestic K and net foreign assets B) must be the same: optimal allocation of resources requires no possibility of gaining (in terms of utility) on shifts between these forms of capital.

Considering Eqs. (24) and (30), Eq. (d) can be written as: $-\partial \lambda_2 / \partial t = \lambda_1 W_2 \frac{\partial Y}{\partial K} - \lambda_2 (\rho + \delta_K)$ where

$$W_2 = [1 - \tau_k \alpha - \tau_L \beta - (1 - \omega)\xi] = const.$$
 (31)

Dividing both sides by λ_2 and using Eqs. (24) and (30) together with Eq. (1), after some manipulation we get:

$$k = \frac{\alpha W_2}{r + \delta_\kappa}.$$
(32)

Eq. (32) has an obvious economic interpretation. It can be written as $(r+\delta_{\kappa})K = W_2\alpha Y$, which means that at any moment, the rental rate of capital (compensation of capital owners) is equal to the "technological" share of capital in GDP adjusted for taxes. Eq. (32) also reveals some important implicit assumptions and limitations of the model. On the one hand, for a given (constant) set of fiscal parameters we have $W_2 = const$. On the other hand, we assume a constant, exogenously determined

interest rate (perfect capital mobility in the case of a small open economy). Therefore, at any moment of time, k = const. According to Eq. (32), the ratio of capital to GDP (k) at any moment of time is determined by fiscal and technological parameters embedded in the expression W_2 , together with the real interest rate augmented for the depreciation of capital (i.e. the gross rate of return on capital).

Importantly, any change in these parameters implies an *immediate* adjustment in k. For example, an increase in tax rates on labor or capital reduces the value of k (*ceteris paribus*). An immediate adjustment to this new, lower value of k requires an *instant outflow* of capital to other countries. Obviously, in reality such adjustments are never immediate, they take time. However, the model (as a simplified picture of reality) assumes *instant* reactions of capital owners, which require no barriers to international movement of capital of any kind, including physical resources.

Therefore, we must be very careful about the interpretation of any results delivered by simulations: it is safe to apply comparative statics (comparing various steady states, i.e. balanced growth paths), but any simulations of adjustments (transitory dynamics) will certainly be unrealistic – instantaneous shifts of capital (especially physical resources) simply cannot happen in the real world.

Dividing both sides of Eq. (8) by K_p and using Eq. (17), we get the growth rate of K_p :

$$\varphi_{kp} = \hat{K}_p = \frac{\gamma_p}{r} - \delta_p \,. \tag{33}$$

Now, let us use the definition of h = H/Y. Taking time derivative yields: $\partial h/\partial t = (\partial H/\partial t)/Y - \varphi_{tt}h$, which can be written as:

$$\partial h / \partial t = (\varphi_H - \varphi_Y)h, \qquad (34)$$

Analogously,

$$\partial k_p / \partial t = (\varphi_{KP} - \varphi_Y)k_p$$
. (35)

Note that all growth rates φ_i ($i = Y, K, H, K_p$) are functions of h and k_p . Therefore, Eqs. (34) and (35) form a system of two differential (nonlinear, autonomous) equations with the following structure:

$$\begin{bmatrix} \frac{\partial h}{\partial t} \\ \frac{\partial k_p}{\partial t} \end{bmatrix} = \begin{bmatrix} f^3(h, k_p) \\ f^4(h, k_p) \end{bmatrix}.$$
(36)

2.2. Existence, Uniqueness, Stability and Transversality Conditions

Let the upper bar denote the steady state. The steady state is defined as the set of values of $\frac{1}{h}$, $\frac{1}{k_p}$ that satisfies equations (36) and is stationary with time. Obviously, the steady state can be found by equating the right-hand sides of equations (36) with zeros. If follows immediately that along the steady-state path all growth rates are equal:

$$\overline{\varphi}_{Y} = \overline{\varphi}_{K} = \overline{\varphi}_{H} = \overline{\varphi}_{KP} = \overline{\varphi}$$
. (37)

Therefore, the steady state is conventionally referred to as **the balanced growth path (BGP)** while the growth rate $\overline{\varphi}$ is called **the balanced growth rate (BGR)**. Let us now analyze the properties of the steady state. First, note that this system of equations can be reduced to a single differential equation. Eq. (32) implies that at any moment of time (not only along the balanced growth path) $\varphi_k = \varphi_Y$. The following definition: $P = K_P/Y$ implies that $\varphi_p = \varphi_{KP} - \varphi_Y$. Substituting these formulae into (27) yields $(1+\varepsilon)\varphi_Y = \alpha\varphi_Y + \beta\varphi_H + \varepsilon\varphi_{KP}$, which can be written as

$$\varphi_{Y} = \frac{\beta}{\beta + \varepsilon} \varphi_{H} + \frac{\varepsilon}{\beta + \varepsilon} \varphi_{KP} \,. \tag{38}$$

Substituting (38) into (34) and rearranging yields:

$$\partial h / \partial t = \frac{\varepsilon}{\beta + \varepsilon} (\varphi_{H} + \varphi_{KP}) h.$$
(39)

Substituting (28) and (33) yields:

Figure 1

The Steady State

$$\partial h / \partial t = \frac{\varepsilon}{\beta + \varepsilon} \left(e \gamma_E + \left(\delta_P - \delta_H \right) h - \gamma_P \frac{h}{k_P} \right), \qquad (40)$$

Using (29) and (32) allows to write the right-hand side as a function of h only:

$$\partial h / \partial t = \frac{\varepsilon}{\beta + \varepsilon} \left(e \gamma_E + \left(\delta_P - \delta_H \right) h - \gamma_P A^{\gamma_E} \left(\frac{\alpha W_2}{r + \delta_K} \right)^{\frac{\alpha}{2}} h^{\frac{\beta + \varepsilon}{\varepsilon}} \right), \tag{41}$$

For convenience, let us write down Eq. (41) as follows: $\partial h / \partial t = \frac{\varepsilon}{B + \varepsilon} (f^{1}(h) - f^{2}(h)),$ (42)

 $f^{1}(h) = \frac{\varepsilon}{\beta + \varepsilon} \left(e\gamma_{E} + \left(\delta_{P} - \delta_{H} \right) h \right), \quad \text{and}$

$$f^{2}(h) = \frac{\varepsilon}{\beta + \varepsilon} \gamma_{p} A^{\gamma_{\varepsilon}} \left(\frac{\alpha W_{2}}{r + \delta_{K}} \right)^{\gamma_{\varepsilon}} h^{\frac{\beta + \varepsilon}{\varepsilon}}.$$

For any given (constant) set of parameters, $f^{i}(h)$ is a linear function with positive slope and intercept, whereas $f^{2}(h)$ is an exponential, strictly convex function, because $\frac{\beta+\varepsilon}{\varepsilon}>1$. These functions are illustrated in Fig. 1. Obviously, for any set of parameters of the model, these curves intersect precisely once, so **the steady state always exists and is unique**.

In order to calculate the balanced growth path, one must find the intersection of these functions, i.e., find the solution of the following equation:

$$f^{1}(h) = f^{2}(h).$$
(43)



Note that this equation can only be solved numerically, which is feasible when all parameters are replaced with actual numbers. In other words, an analytical formula for the steady-state value of \overline{h} (and, consequently, for the BGR) does not exist. Once the steady-state value of \overline{h} is numerically cal-

culated, the steady-state values of all other variables can be calculated as well. In particular, the BGR follows from Eq. (28):

$$\overline{\varphi} = \overline{\varphi}_{Y} = \overline{\varphi}_{K} = \overline{\varphi}_{H} = e\gamma_{E}/\overline{h} - \delta_{H}. \qquad (44)$$

The steady-state level of public capital (in relation to GDP) can be derived from Eq. (29):

$$\bar{k}_{p} = A^{-l_{\ell}\epsilon} \left(\frac{\alpha W_{2}}{r + \delta} \right)^{-a/\epsilon} \bar{h}^{-\beta/\epsilon}. \quad (45)$$

From Eqs. (12) and (13) it follows that: $\hat{d}_F = \hat{D}_F - \hat{Y} = \omega \xi/d_F - \varphi_Y$, $\hat{d}_D = (1 - \omega)\xi/d_D - \varphi_Y$. Since $\partial d_F / \partial t = \hat{d}_F \cdot d_F$ and $\partial d_D / \partial t = \hat{d}_D \cdot d_D$, we have:

$$\partial d_F / \partial t = -\phi_Y d_F + \omega \xi$$
, (46)

$$\partial d_D / \partial t = \varphi_Y d_D + (1 - \omega) \xi$$
(47)

Therefore, the steady-state debt-to-GDP ratios are:

$$\overline{d}_F = \frac{\omega\xi}{\overline{\phi}}, \ \overline{d}_D = \frac{(1-\omega)\xi}{\overline{\phi}}$$
(48)

The steady state is globally asymptotically stable, which does not even require a formal proof. It becomes obvious, once we graph the function $\partial h / \partial t$ as a function of h, given by Eq. (41).

The transversality conditions (e) and (f) are analyzed in Appendix A. In short, they determine initial consumption per capita and impose the upper cap on the rate of growth of consumption per capita.

Finally, it is worth noting that the system of equations (36) includes all fiscal parameters with the notable exception of τ_c and τ_b . Therefore, two of the four tax rates are neutral for the model: not only for the balanced growth path, but also for the transitory dynamics. This property of the model results from the adopted set of strict fiscal rules. Obviously, these two specific tax rates may cease to be neutral, if the government follows different rules or abandons them altogether.

3. Special Cases

3.1. What if $\varepsilon = \beta$?

If the elasticity of output with respect to public capital happens to be equal to the share of labor in GDP, then Eq. (41) can be written as follows:

$$\partial h / \partial t = \frac{1}{2} \left(e \gamma_E + \left(\delta_P - \delta_H \right) h - \gamma_P \mathcal{A}^{1/p} \left(\frac{\alpha W_2}{r + \delta_K} \right)^{0/p} h^2 \right).$$
(49)

Note that $\frac{\partial h}{\partial t}(h)$ is a quadratic function:

Figure 2

Global Asymptotic Stability



$$\begin{split} \partial h/\partial t &= a_i h^2 + a_2 h + a_3, \quad \text{with} \quad a_i &= -\frac{1}{2} \gamma_\mu A^{\nu / \rho} \left(\frac{\alpha W_2}{r + \delta_\kappa} \right)^{\nu / \rho} < 0, \\ a_2 &= \frac{1}{2} \left(\delta_\rho - \delta_\mu \right), \; a_3 &= \frac{1}{2} e \gamma_E > 0. \end{split}$$

The graph of the function $\frac{\partial h}{\partial t}(h)$ is a parabola with one negative and one positive root, the latter being the steady-state (globally asymptotically stable) value of \overline{h} :

$$\overline{h} = \frac{-a_2 - \sqrt{\Delta}}{2a_1} > 0, \text{ where } \Delta = \sqrt{a_2^2 - 4a_1a_3}.$$
(50)

Substituting (50) into Eq. (44) yields the analytical formula for the BGR. Let us write it down in the expanded form to reveal all parameters that influence the BGR:

$$\overline{\varphi} = \frac{\delta_p - \delta_{ll}}{2} + \sqrt{\frac{(\delta_p - \delta_{ll})^2}{4} + e\gamma_E \gamma_p A' \int_{\theta}^{\theta} \left(\frac{\alpha \left[1 - \tau_K \alpha - \tau_L \beta - (1 - \omega)\xi\right]}{r + \delta_K}\right)^{\frac{\theta}{2}}} \quad (51)$$

Partial derivatives $\partial \bar{\varphi} / \partial (.)$ with respect to all fiscal parameters have unambiguous signs. Therefore, an increase in τ_{κ} , τ_{L} or ξ reduces the BGR, whereas an increase in γ_{E} , γ_{p} or ω raises the BGR.

Finally, let us assume for a moment that $\gamma_E + \gamma_P = \gamma = const.$, which means that the government devotes certain amount of resources (γ percent of GDP) to both types of investment together. It's straightforward to verify

that in order to maximize the BGR, these resources should be split equally, i.e. γ_E should be equal to γ_P .

3.2. What if $\delta_{P} = \delta_{H}$?

If public capital happens to depreciate at the same rate as human capital, then Eq. (41) can be written as follows:

$$\partial h / \partial t = \frac{\varepsilon}{\beta + \varepsilon} \left(e \gamma_E - \gamma_P A^{V_{\varepsilon}} \left(\frac{\alpha W_2}{r + \delta_K} \right)^{\alpha_{\varepsilon}} h^{\frac{\beta + \varepsilon}{\varepsilon}} \right), \quad (52)$$

The steady-state (globally asymptotically stable) value of \overline{h} :

$$\overline{h} = \left(\frac{e\gamma_E}{\gamma_p}\right)^{\frac{e}{\beta+e}} A^{\frac{-1}{\beta+e}} \left(\frac{aW_2}{r+\delta_K}\right)^{\frac{-a}{\beta+e}}.$$
(53)

Substituting (53) into Eq. (44) yields:

$$\overline{\varphi} = A^{\frac{1}{\beta+\varepsilon}} \left(\frac{\alpha W_2}{r+\delta_\kappa} \right)^{\frac{\beta}{\beta+\varepsilon}} \left(e\gamma_\varepsilon \right)^{\frac{\beta}{\beta+\varepsilon}} \gamma_\rho^{\frac{\varepsilon}{\beta+\varepsilon}} - \delta_H \cdot$$
(54)

Let us consider the same experiment as above, i.e. let us assume that $\gamma_E + \gamma_P = \gamma = const.$, which means that the government devotes γ percent of GDP to both types of investment. It's easy to show that in order to maximize the BGR, these resources should be split according to the following (necessary and sufficient) condition:

$$\frac{\gamma_P}{\gamma_E} = \frac{\varepsilon}{\beta}.$$
(55)

The optimal distribution of these resources in this particular case depends solely on the ratio of elasticities of



Figure 3 Public Capital (Left Scale) and Public Investment (Right Scale) in Poland; % of GDP

Source: "IMF Investment and Capital Stock Dataset", by International Monetary Fund (2019). Available at http://www.imf. org/external/np/fad/publicinvestment/

production with respect to public capital and human capital.

4. CALIBRATION FOR POLAND

Figure 3 shows the stock of public capital in Poland and total government investment in public capital (gross fixed capital formation) as a percentage of GDP.

Poland's public capital stock has generally been declining over the past three decades: from around 55% of GDP to around 45%. There are indications that EU convergence funds have allowed Poland to accelerate investment in public infrastructure since around 2005, but there are also significant fluctuations in the rate of public investment that are puzzling. For our purposes, as reference values, we will assume that the initial level of public capital is 44.1% of GDP and public capital investment is γ_p =3.67% of GDP. Both numbers are arithmetic means from the last available decade of data, i.e., 2008–2017.

Table 1 summarizes the next part of the calibration procedure. It contains a set of parameters and initial values (endowments) with a brief short explanation. In general, these values are based on the statistics of the last decade, i.e. the period 2010–2019, which we consider to be the "starting point" (endowment). The data comes from the Eurostat database, the National Bank of Poland, the Central Statistical Office of Poland, the Kiel Institute for the World Economy, and selected empirical literature on OECD countries.

To complete the procedure, we need to calibrate the real rate of return on capital (r). For that purpose we shall use equation (32), which can be transformed to the following: $r = \alpha W_2 / k - \delta_K$, with $W_2 = [1 - \tau_L \beta - \tau_K \alpha - (1 - \omega) \xi]$. All parameters in this formula have already been calibrated (table 1), except for k. The statistical value of Y/K is difficult to estimate for Poland - Konopczyński (2018a) pointed out important problems. In short, the statistics available for Poland reflect only a fraction of the total capital, namely the "gross value of fixed assets". This problem occurs in the databases of Eurostat and the IMF. Therefore, following Konopczyński (2018a), we will start with the average ratio from the Kiel database, that is Y/K = 1/3. Obviously, we need to change this number a bit because we distinguish between public and private capital. Subtracting public capital (44.1% of GDP) yields: $Y/K = 1/(3 - K_p/Y) = 1/(3 - 0.441) = 0.3908$. Substituting this value into Eq. (32) yields r = 4.6764%.

Let us do some growth accounting. The average GDP growth rate in Poland in 2010–2019 was 3.63%. Note that

$$\hat{K} = \frac{\partial K / \partial t}{K} = \frac{I_K}{K} - \delta_K = \frac{I_K}{Y} \frac{Y}{K} - \delta_K \,. \tag{56}$$

According to Eurostat, gross fixed capital formation in Poland in 2010–2019 averaged 19.16% of GDP. So Eq. (56) gives $\hat{K} = 0.1916/3 - 0.055 = 1.99\%$. Substituting (17) to (8) yields $\hat{K}_p = \frac{\gamma_p}{K_p/Y} - \delta_p$. Using statistical data, we obtain:

$$\hat{K}_{p} = \frac{3.67\%}{44.1\%} - 4.0\% = 4.32\%.$$
(57)

This implies that the flow of public services grew at a moderate pace:

$$\hat{P} = \hat{K}_{p} - \hat{Y} = 4.32\% - 3.63\% = 0.69\%$$
 (58)

Using the production function $Y = AK^{\alpha}H^{\beta}P^{\epsilon}$, we can estimate the growth rate of human capital:

$$\hat{H} = \frac{\hat{Y} - \alpha \hat{K} - \varepsilon \hat{P}}{\beta} = \frac{3.63\% - 1/3 \cdot 1.99\% - 2/3 \cdot 0.69\%}{2/3} = 3.76\%$$

These figures mean that in 2009–2018, economic growth in Poland was driven by two factors: the rapid accumulation of human capital coupled with an even faster accumulation of public capital. Private capital also grew, but not so fast. The impressive increase in human capital in Poland is a well-known "stylized fact" confirmed by the sharp increase in the number of graduates, doctoral students, etc. Additionally, a significant qualitative and quantitative improvement in public infrastructure is noticeable. However, it should be noted that the Polish economy is not yet on the balanced growth path (BGP).

For simulations, it is necessary to determine the value of the total factor productivity *A*. Substituting (9) and (16) into Eq. (7) and dividing both sides by *H* gives $\hat{H} = e\gamma_{\mu} \gamma/H - \delta_{\mu}$. Therefore,

$$Y/H = \frac{\hat{H} + \delta_H}{e\gamma_E} , \qquad (59)$$

Meanwhile, dividing both sides of the production function $Y = AK^{\alpha}H^{\beta}P^{\alpha}$ by *H* and *K*, respectively, yields:

$$\frac{Y}{H} = A \left(\frac{K}{H}\right)^a p^a,\tag{60}$$

$$\frac{Y}{K} = A \left(\frac{K}{H}\right)^{-\rho} P^{\epsilon} \,. \tag{61}$$

It follows from Eq. (61) that $\frac{K}{H} = \left(A\frac{K}{Y}\right)^{\nu/\theta} P^{\prime/\theta}$. Using this together with (60) and (3) in Eq. (59) gives:

$$\frac{\hat{H} + \delta_H}{\gamma_E} = e A^{\eta/\rho} \left(\frac{K}{Y} \right)^{\eta/\rho} \left(\frac{K_P}{Y} \right)^{\eta/\rho}, \tag{62}$$

Education Versus Infrastructure: Is There a Trade-off

Table 3

The Main Part of Calibration

Parameters & endow-	w- Sources of data and explanations					
ments						
	Technology					
$\alpha = 1/3,$	Review of empirical literature: Mankiw, Romer, Weil (1992), Bernanke, Gurkaynak (2001), Willman (200					
$\beta = 2/3$,	Balistreri et.al. (2003) and studies focusing on Poland: Cichy (2008) and Growiec (2012). We assume $s =$ following Barro and Martin (2004), p. 220					
$s - \beta$						
c - p						
	There is no reliable data on the depression of the productive conital in Deland. One reason is rapid aconom					
	ic transformation of the last 2-3 decades which resulted in a buge amount of obsolete machinery. infrastruc-					
$\delta_{K} = 5.5\%$	ture, etc., inherited from the centrally "planned" economy. In various research studies on OECD countries.					
$\delta = 1.5\%$	physical capital depreciation ranges from about 3.5% to 7.5%. We decided to take the midpoint. The rate					
$v_{H} = 1.570$	of human capital depreciation is borrowed from Manuelli, Seshadri (2005) and Arrazola, de Hevia (2004).					
	There are no statistics on the rate of depreciation of public capital in Poland. Therefore we follow the method-					
$\delta_P = 4\%$	ology used by the IMF (2019), which in turn is adopted from Gupta et. al. (2014). They argue that "country-					
	specific depreciation rates (\ldots) are likely to increase with income assuming that the share of assets with					
	a shorter life spans (such as technology assets) rises with income levels". In particular, they assume that					
	public capital in middle-income countries depreciates at a rate of 3.52% per year compared to 4.55% in high-					
	income countries at. Poland is somewhere in between these two groups, so we set $\delta_p = 4\%$.					
	Ine utility function The mean of values estimated by Turnovsky (1909) and (2004). Dark Dilipnopoulos (2004). Dhont Heylen (2009)					
$\kappa = 0.27$	The mean of values estimated by furnovsky (1999) and (2004), 1 ark, 1 mippopoulos (2004), Dilont, Treyen (2007).					
	The meta-analysis by Nijkamp, Percoco (2006) from 42 previous studies and the European Commission (2002).					
$\rho = 0.04$						
$\gamma = -1$	A comprehensive meta-analysis by Havranek et.al. (2013) from 169 previous studies.					
	<i>Fiscal policy</i>					
$\sigma_c = 30.25\%$	According to Eurostat, in 2010–2019 public consumption as a snare of GDP averaged 18.00%, while private consumption averaged 59.7% of GDP. Therefore, $\sigma_c = 18.06\%/59.7\% = 30.25\%$					
	consumption averaged 59.7 % of GD1. Therefore, $-c = 10.00 / (79.7 / 0 - 50.25 / 0.00)$					
x = 5.23%	Public expenditure on education (as a percentage of GDP) in 2010–2019.					
$\gamma_E = 5.2376$						
	Average public sector deficit in 2010–2019 (according to Eurostat methodology).					
$\xi = 3.12\%$						
	Average share of foreign debt in public debt in 2010 – 2019.					
$\omega = 0.487$						
$\tau_{\scriptscriptstyle K} = 22.50\%, \ \tau_{\scriptscriptstyle L} = 19.38\%,$	Calibrated to be consistent with the tax revenue statistics (shares of GDP; 2009-2018 averages - latest avail-					
$\tau_D=19\%$, $\tau_c=19.79\%$	able data) published by Eurostat.					
	The initial capital stack per capita is act arbitrarily (as a summine). 200 is commined by the initial capital stack per capital is act arbitrarily (as a summined by a su					
$k_0 = 300$	The initial capital stock per capital is set atomating (as a numerative); souris convenient decause if gives $v_{c} = 100$ and therefore the initial values of all other variables are identical to their nercentages of CDD					
$b_0 = -55.6\%$	$y_0 = 100$, and therefore the initial values of all other variables are identical to their percentages of GDP. Statistical data for Poland published by the National Bank of Poland (NRD), not international investment					
$d_{F0} = 25.2\%$	position (NIIP) of the private sector and public sector; mean values in 2010–2019.					
$d_{D0} = 26.5\%$	The difference between public debt (mean value in 2010–2019, i.e., 51.7%) and the d_{F0} = 25.2%.					

T

Almost all parameters and ratios in Eq. (62) have already been calibrated; there are only 2 "unknowns": *e* and *A*. Note that they are bound together in Eq. (62). Thus, there is no way to calibrate these values within our model, i.e. without resorting to other models or econometric evidence. Therefore, we borrow A=0.7555 from Konopczyński (2018a), who obtained this number in a similar, although simplified model (with public capital, but without human capital) with an analogous production function. It follows from Eq. (62) that e=2.1707. Using this value in Eq. (59) gives Y/H = 0.4633. Finally, we need to calculate the endowments: initial values

of *K*, *H* and K_p . For convenience, let us normalize the initial value of GDP to 100, i.e. let Y(0) = 100. Given the above initial ratios of Y/K = 1/3, Y/H = 0.4633, and $K_p/Y = 0.441$ we obtain: K(0) = 255.9, H(0) = 215.9, $K_p(0) = 44.1$.

5. The Baseline Scenario

The baseline scenario is defined as a simple continuation of the 2010–2019 reference period without any parameter changes. Using the procedure outlined in section 2 we numerically calculated the BGR in the baseline scenario: $\bar{\varphi}$ =3.98%, which is 0.35 percentage points *higher* than the average growth rate in the reference period. The human capital index actually *decreases* over time to 207% of GDP (recall that the initial value was 216%). The public capital stock *grows* from the initial 44.1% of GDP to 46.0%. Debt-to-GDP ratios converge to the following values: $\bar{d}_r = 38.1\%$, $\bar{d}_p = 40.2\%$, $\bar{d} = 78.3\%$.

The next two sections contain selected simulations of changes in fiscal policy.

6. Education and Public Capital

Table 2 presents four alternative scenarios:

• E1 and E2: public expenditure on education is permanently increased by 1 or 2 pp of GDP (at the expense of G_r , i.e. financial transfers to the private sector)

• E1 P1 and P2: expenditure on public capital is permanently increased by 1 or 2 pp of GDP (at the expense of G_r)

All other parameters of fiscal policy remain the same as in the baseline scenario.

Table 3

|--|

	В	E1	E2	P1	P2
The BGR and	Baseline scenario	Increase in spending on education		Increase in spending on public capital	
structural indica- tors (%)	$\gamma_{\scriptscriptstyle E} = 5.23\%$ $\gamma_{\scriptscriptstyle P} = 3.67\%$	by 1 pp of GDP: $\gamma_E = 6.23\%$	by 2 pp of GDP: $\gamma_E = 7.23\%$	by 1 pp of GDP: $\gamma_P = 4.67\%$	by 2 pp of GDP: $\gamma_p = 5.67\%$
\overline{arphi}	3.98	4.58	5.13	4.82	5.57
\overline{k}	255.9	255.9	255.9	255.9	255.9
\overline{k}_{P}	46.0	42.8	40.2	53.0	59.3
\overline{h}	207.1	222.5	236.8	179.7	160.6
$\overline{d}_{\scriptscriptstyle D}$	40.2	35.0	31.2	33.2	28.7
$\overline{d}_{_F}$	38.1	33.2	29.6	31.5	27.3
\overline{d}	78.3	68.2	60.8	64.8	56.0



Figure 4

The BGR as a Function of the "Shift Parameter" from Education to Public Capital

The E1 and E2 simulations confirm that human capital will remain very important for economic growth in Poland: increasing public expenditure on education by 1 pp of GDP raises the BGR by 0.6 percentage points. The allocation of an additional 2 percentage points of GDP to education raises the GDP growth rate to over 5%. It is noteworthy that both E1 and E2 scenarios are also beneficial for public debt, which converges to 68% or 61% of GDP, respectively, instead of 78%.

The P1 and P2 scenarios show that additional investment in public capital can also significantly stimulate economic growth. Increasing γ_{P} by 1 pp of GDP raises the BGR to 4.8%. Adding another percentage point of GDP increases the BGR to almost 5.6%. Clearly, public infrastructure in Poland needs more resources. It should be noted that in both scenarios (P1 and P2) public capital increases over time from the starting level of 44.1% of GDP to as much as 53% in scenario P1, and 59% in scenario P2. In scenario P2, Poland would catch up with more developed countries, where this ratio is in the range of 60–80%. For example, according to the IMF, in 2017 (last year of data) it was 56% in Canada, 60% in Austria, 63% in the United States, 64% in Italy, 75% in France, 77% in Finland, 85% in Denmark, and 120% in Japan (an outlier?). Last but not least, comparing these four scenarios leads to a somewhat surprising conclusion: although both education and public capital require more resources, the latter is clearly more important. Therefore, if the government considers spending a lump sum on education or public infrastructure, it is better to choose the latter. It also means that it is worth transferring acertain part of public funds from education to public capital. However, there must be some limits. To shed more light on this, consider an experiment: transfer a certain amount (say x% of GDP) from education to public infrastructure, keeping the total amount invested in both types of assets constant. More specifically, we assume that γ_E increases by x% (percentage points of GDP) whereas γ_{p} decreases by x% compared to the baseline scenario. Figure 4 shows the results. The BGR reaches its maximum (4.09%), if the government shifts around 0.8% of GDP from education to public investment. Obviously, this scenario has no negative welfare implications - not even a temporary welfare cost - the trajectories of public consumption and cash transfers (e.g., pensions) are at least equal to their baseline counterparts over the entire time horizon. Nevertheless, it is crucial to remember that the benefits of such a *shift* in public expenditures are tiny compared to the significant impact of an *increase* in education expenditures or investment in public infrastructure, or both.

7. Other Parameters of Fiscal Policy

Finally, let us analyze the remaining parameters of fiscal policy one be one. Table 3 summarizes the results of simulations. It contains the values of selected variables along the balanced growth path. In each scenario (each row in the table) we changed the value of only one parameter, assuming that all other parameters retain their base values. Recall that two out of the four tax rates (τ_c and τ_b) are neutral for the balanced growth path, so they are not included in the table.

In the reference period (2010-2019), the public deficit amounted to 3.12% of GDP. Given the relatively modest value of the BGR in the baseline scenario, a straightforward continuation of such a policy inflates the public debt up to 78.3% of GDP, which violates the constitutional limit of 60%. For this reason, cutting budget deficit is recommended. However, even in the extreme case of a permanently balanced budget (zero deficit) there is little influence on the rate of growth: the BGR is only 0.03 pp higher than in the baseline scenario. Given the significant benefits of additional investment in education and infrastructure (as discussed in the previous section), it makes sense to consider the exact opposite: it probably makes sense for the government to actually borrow more provided that these loans are wisely invested in education and public capital.

The financing structure of public debt has some influence on economic growth: the greater the share of foreigners in the public debt, the higher the BGR. To maximize the rate of growth of GDP, the government should not borrow from domestic lenders. It should be noted that, however appealing and sensible this proposal may be, its practical application is virtually impossible given the free mobility of capital between countries.

Although the reduction in taxes on labor and capital slightly accelerates economic growth, the effect is relatively small. Nevertheless, it turns out that if the government needs to collect additional revenue, it should do so by raising consumption taxes, which are neutral to economic growth. It comes as the result of the strict fiscal rules applied in the model: the government deficit, expenditure on education and investment in public capital are fixed to GDP, whereas public consumption is tied to private consumption. Therefore, reducing tax revenues forces the government to appropriately reduce financial transfers to the private sector: pensions, etc. As a result, the total disposable income of households is almost unchanged, and the balanced growth path remains largely the same.

Finally, given the strict fiscal rules built into the model, the volume of public consumption is irrelevant for the BGR and debt indicators. It affects private consumption only as individuals try to compensate for any changes in public consumption. Reducing public consumption induces households to increase private consumption expenditure, and vice versa. This may have minor welfare implications, but GDP growth rate remains unchanged.

8. What Should We Do About Fiscal Policy?

On the one hand, increasing public deficit or income taxes reduces the rate of growth, but the negative effect is rather small. On the other hand, increasing spending on education and public capital has very significant positive impact on economic growth. Therefore, the message of this paper is simple and clear: spend as much as possible on both education and public infrastructure. If the resources of the government are limited - and they may be especially during the period of transition to the new balanced growth path it's worth to raise taxes (particularly on consumption) or increase public borrowing, provided that these additional funds are invested in education and public infrastructure. Table 4 presents three such scenarios which illustrate the significance of education and public capital.

Given our calibration results, the optimal strategy for Poland is to devote the same amount of resources to education and public capital (note that $e_{E=B}$, so we have a special case described in section 3.1). However, this conclusion hinges on the value of the elasticity of production with respect to public capital (). An empirical value of this parameter in Poland is basically unknown, due to the lack of reliable econometric research coupled with difficulties related to the measurement of the stock of public capital. Therefore, in our simulations, we have simply assumed that e = B, following other researchers. This requires verification and should be the subject to intense future research because policy recommendations depend on this assumption. In short, if ε turns out to be higher, the government should devote even more resources to public capital - perhaps at the cost of education. To the con-

Table 4

Increased Investment in Education and Public Capital Financed by Foreign Debt

	the BGR	public capital in-	human capital	public debt
		dex index		
	\overline{arphi}	\overline{k}_{P}	\overline{h}	\overline{d}
The baseline scenario $\xi = 3.12\% \qquad \omega = 48.7\%$ $\gamma_E = 5.23\%, \ \gamma_P = 3.67\%$	3.98%	46.0%	207.1%	78.3%
Scenario A $ \begin{split} \xi &= 3.0\% \omega = 100\% \\ \gamma_E &= 5.0\% \ , \ \gamma_P = 5.0\% \end{split} $	4.94%	55.9%	168.5%	60.7%
Scenario B $\xi = 3.5\% \qquad \omega = 100\%$ $\gamma_E = 6.0\%, \ \gamma_P = 6.0\%$	6.44%	57.5%	164.0%	54.3%
Scenario C $\xi = 4.0\% \omega = 100\%$ $\gamma_E = 7.0\%, \ \gamma_P = 7.0\%$	7.95%	58.6%	160.8%	50.3%

trary, if ε is, in fact, smaller, then investing more in education rather than in public capital probably makes sense. Fig. 5 is an illustration of this sensitivity: it contains the same information as fig. 3, but for three different values of ε . The middle line corresponds to our original calibration. Note that within the range of this sensitivity analysis our "qualitative" conclusion doesn't change: it is worth to transfer some resources (but not too much) from education to public capital.

9. Conclusions and Discussion

We have presented an open economy growth model with many parameters of fiscal policy: government spending on education and public capital, the size of the budget deficit, public debt financed domestically and by foreign lenders, public consumption, and four types of taxes. Including so many interdependent tools of fiscal policy in one model leads to some interesting theoretical conclusions. First, the economy is converging towards a balanced growth path (BGP) which is unique, and globally asymptotically stable. Due to the complexity of the model, the growth rate along the BGP (the balanced growth rate, BGR) can only be calculated numerically. Nonetheless, we have proved that two out of the four tax rates (on consumption and interest on government bonds held by domestic lenders) are neutral for the BGP and the transitory dynamics. In contrast, all other parameters of fiscal policy do matter: any change in their values changes not only the BGP, but also the transitory dynamics. Unfortunately, due to the relative complexity of the model, these relationships cannot be determined by standard analytical methods - the only viable tool is numerical simulations. This is not true only in special cases that we discuss in section 3 in order to build an intuitive understanding of the model.

Figure 5

The BGR as a Function of the "Shift Parameter" from Education to Public Capital – Sensitivity to ε



We examined Poland as an example. First, we calibrated the model on the basis of statistical data from 2010–2019. We found that during this period economic growth in Poland was driven mainly by two factors: fast accumulation of human capital (growing at 3.76% annually) coupled with even faster growth of public capital (4.32%). Obviously, private productive capital has also been growing, but not as rapidly.

Secondly, if all parameters remain unchanged (the baseline scenario), the GDP growth rate in Poland will converge to 3.98% which is slightly higher than in the reference period of 2010-2019 when the annual GDP growth rate amounted to 3.6%. Meanwhile, public debt will increase from recent 55% to 78% of GDP. The best way to permanently accelerate economic growth is to increase spending on both education and public infrastructure - even if this requires raising taxes on consumption or increasing public debt. For instance, increasing expenditure on education by 1 pp of GDP raises the GDP growth rate by 0.6 percentage points, while doing the same with expenditure on public infrastructure has an even greater effect of 0.84 percentage points. Clearly, both factors of production require more resources.

These results suggest that in Poland, there exists a specific trade-off between education expenditure and investment in public capital: a slight shift of resources

from the former to the latter would indeed slightly accelerate growth. Nonetheless, it should be borne in mind that the benefits of such a *shift* are **negligible** compared to the huge effect of an *increase* in either education expenditure, investment in public capital, or preferably both – as seen in scenarios A, B and C in section 8.

Reducing public deficit or income taxes would also contribute to accelerating growth, but the effects are relatively smaller. Therefore, it may be worth considering the opposite approach (though it could be politically challenging): allowing the government to borrow more, on the condition that these additional funds are directed towards education and/or public infrastructure. Implementing such a policy would require stringent rules and regulations regarding public debt and expenditures to prevent misuse of the funds for increasing public consumption or social transfers, which could be tempting for many politicians, especially during periods leading up to public elections.

As for the empirical aspect of the paper, it is important to acknowledge that the model neglects certain factors that have undeniably had, and continue to have, a significant impact on the economy of Poland. First, the substantial capital inflows in the form of foreign direct investments (FDI), portfolio investments, and EU convergence funds. Second, the technology diffusion from more developed countries. Third, the substantial migration from Poland, mainly to other EU countries, partly offset by temporary workers, primarily from Ukraine. These empirical factors have undoubtedly influenced the economy over the past 2-3 decades. However, it is crucial to recognize that they are of a temporary nature, and while they have influenced the past and possibly the present, their importance is likely to diminish in the future.

We agree that the model presented in this article offers a detailed analysis of fiscal policy while maintaining a relatively general modeling framework. As a result, it can be applied to many small countries worldwide, particularly in Europe and East Asia. However, it is important to note that the assumption of perfect capital mobility, a fundamental aspect of the model, may not hold true in many countries, especially in larger economies. In such cases, significant modifications to the model would be necessary, replacing the ideal assumption of perfect capital mobility with alternative, more realistic assumptions to accurately capture the economic dynamics of those countries.

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Footnotes

1. Even the United States is now deeply interconnected with the rest of the world. For example, the net international investment position (NIIP) of the U.S. is currently at minus 65% of GDP, whereas in 2000, it was close to minus 10%.

2. In fact, public services and public capital are usually inseparable. To illustrate, consider the provision of healthcare, which requires not only the stock of infrastructure such as hospitals, ambulances, and medical equipment, but also the flow of expenditures on wages for medical staff, medicines, electricity, utilities, and more. The same applies to various other public services, including national security (military sector), law enforcement, airports, bridges, roads, etc. All of these services may experience congestion (with the possible exception of the military sector), as reflected in Eq. (3). 3. This assumption is supported by empirical evidence from Poland and many other countries where private spending on education is very small compared to public expenditure. For further reference, see Konopczyński (2014a).

4. Note that many researchers assign the government the role of a "benevolent social planner" and solve the problem of dynamic optimization of government spending on education, public capital and public consumption. My approach is somewhat pessimistic, because I don't think governments behave like that. I deliberately chose to model government as someone who follows certain simple, perhaps naive rules. I think this actually mirrors the behavior of many governments: each year they decide the annual budget. It usually turns out that they fix certain expenditures to GDP. For example, in Poland the government plans (or at least announes) to increase public spending on healthcare from the current level (around 5.5% of GDP) to 7% over the next 3 years.

5. Note that $_{D_p}$ is not a decision variable faced by households: it is the government at any time that decides about the level of public debt (both domestic and foreign) according to rules (12) - (13). To this end, the private sector behaves passively, accepting any government decisions and gladly buying the bonds that the government supplies. Waiving this assumption would significantly increase the complexity of the model. We intend to consider such a modification of the model in the future.

6. Throughout the paper, hats over the variables denote growth rates, e.g. $\hat{c}_{=(\partial C/\partial t)/C}$, etc.

7. Turnovsky (2009) applies such method in chapter 4, only instead of shares in GDP he applies ratios to capital. Our approach is slightly different.

8. It does exist in some special cases that are quite illustrative, so we will take care of them in the next section.
9. There are serious problems with measuring public capital even in OECD countries. Some of these problems were discussed by Konopczyński (2018a). For our purposes, we use the IMF Investment and Capital Stock Dataset 2019, released in August 2019 (IMF, 2019). Despite using the well-established Perpetual Inventory Method, IMF statisticians come up with data

that appears to be of limited reliability. It's enough to mention just one suspicious symptom: the enormous, often counterintuitive differences between countries (even within a subset of 'advanced economies'). For example, in 2017, according to the IMF, public capital amounted to 38% of GDP in Argentina, 39% in Ireland, 43.5% in Germany, 46% in the United Kingdom, 59% in Mexico, 70% in both Sweden and Iran, 86% in Taiwan, 120% in Japan, and last but not least, an incredible 165% in China. Overall, many rich countries have very low statistics, whereas some developing countries have very high statistics, which calls into question the reliability of the data. Moreover, in most OECD countries, the ratio of public capital to GDP does not necessarily increase over time: it has been stable or declining over the past two decades. This is observed even in the most developed countries. There are two possible explanations: either public capital in developed countries is declining (perhaps underinvested, as suggested by Dobbs et.al. 2013), or it is gradually losing importance for economic growth, or both. This is an intriguing research question for future research.

10. Database on Capital Stocks in OECD Countries constructed by Kiel Institute for the World Economy.

Appendix

A. Transversality conditions and constraints on consumption.

The transversality condition (e) determines the initial value of consumption. To demonstrate this, we need to solve the budget constraint (20) along the BGP. The fiscal rules (11) and (14) imply that $G_T = T + \xi Y - rD - G_C - G_E - G_K$. Substituting this into Eq. (20) gives:

$$\frac{\partial B}{\partial t} = (1 - \tau_L)w_L + (1 - \tau_K)w_K K + (1 - \tau_D)rD_D + rB + + T - rD - G_C - G_E - G_K - C(1 + \tau_C) - I_K + \omega\xi Y.$$
(A1)

It follows from Eq. (10) that $t = \tau_L w_L + \tau_K w_K K + \tau_D r D_D + \tau_C C$, so Eq. (A1) can be reduced to: $\partial B / \partial t = w + w_K K + r D_D + r B - r D - G_C - G_E - G_K - C - I_K + \omega \xi Y$. Recall that $w + w_K K = Y$ and $D - D_D = D_F$. Therefore, the budget constraint takes the following form:

$$\partial B / \partial t = (1 + \omega \xi) Y - C - G_C - G_E - G_K - I_K + rB - rD_F.$$
 (A2)

Substituting fiscal rules (15), (16) and (17) into Eq. (A2) yields:

$$\partial B / \partial t = (1 + \omega \xi - \gamma_E - \gamma_P) Y - (1 + \sigma_C) C - I_K + rB - rD_F.$$
(A3)

Eq. (6) implies that $I_{\kappa}/K = \varphi_{\kappa} + \delta_{\kappa}$, whereas Eq. (32) implies that at any moment of time (not only along the balanced growth path) $\varphi_{\kappa} = \varphi_{\gamma}$. Therefore Eq. (A3) can be written as:

$$\partial B / \partial t = (1 + \omega \xi - \gamma_E - \gamma_P)Y - (1 + \sigma_C)C - (\varphi_Y + \delta_K)K + rB - rD_F$$
. (A4)

Recall that $C(t) = C_0 \cdot e^{\psi t}$, where $\psi = \frac{r-\rho}{1-(1+\kappa)\gamma} = const$. However, this is the only simple element of Eq. (A4). All the other trajectories on the right-hand side of this equation are far more complex. For example, since the GDP rate of growth is not constant along the transitory path, the trajectory of output can only be written in a very general form: $Y(t) = Y_0 e^{\phi_1(t)/4t}$. It follows from Eq. (32) that k = const. which implies that for all t, $\varphi_k = \varphi_r$. Therefore the trajectory of capital has the following form:

$$K(t) = K_0 e^{\vartheta}$$
 (A5)

Substituting such trajectories into (A4) leads to an equation that cannot be solved analytically. Therefore, let us consider the economy which is on the BGP from the very beginning (loosely speaking, we may think about the economy which has already fully converged towards the BGP, and we start the calculations at the right moment of time). Thus, we will substitute what follows: $K(t) = K_0 e^{pt}$, $Y(t) = Y_0 e^{pt}$.

[Note that K_0 represents initial endowment, whereas Y_0 is calculated as follows. By definition, k = K/Y, so

Y = K/k, For $t = t_0$ we have $Y_0 = K_0/k_0$. However, we have assumed that the economy is on the BGP from $t = t_0$, therefore $Y_0 = K_0/\overline{k}$, where \overline{k} is the capital-to-GDP ratio on the BGP]

Now, let us determine the trajectory $D_F(t)$ along the <u>BGP</u>. It follows from Eq. (12) that

$$\partial D_F / \partial t = \omega \xi Y = \omega \xi Y_0 e^{\overline{\phi} t}$$
. (A6)

The general solution of this equation is:

$$D_F(t) = s_3 + \frac{\omega \xi Y_0}{\overline{\alpha}} e^{\overline{\rho} t} , \qquad (A7)$$

where the unknown constant s_1 is a function of the initial foreign debt: $D_F(t=0) = D_{F_0}$. Substituting D_{F_0} into Eq. (A7) yields:

$$s_3 = D_{F_0} - \frac{\omega \xi Y_0}{\overline{\varphi}}.$$
 (A8)

To find the analytical form of the trajectory B(t) along the BGP, we need to substitute (A7) together with $K(t) = K_0 e^{\bar{n}t}$, $Y(t) = Y_0 e^{\bar{n}t}$ and $C(t) = C_0 e^{\mu t}$ into Eq. (A4). After rearrangement we get:

$$\partial B / \partial t = rB + \upsilon e^{\overline{\vartheta} t} - (1 + \sigma_c) C_0 e^{\vartheta t} - rs_3, \qquad (A9)$$

where

$$\upsilon = \left(1 + \omega\xi - \gamma_E - \gamma_P - \frac{r\omega\xi}{\overline{\varphi}}\right) Y_0 - (\overline{\varphi} + \delta_K) K_0.$$
(A10)

The general solution of Eq. (A9) takes the form:

$$B(t) = Se^{r_{t}} - \frac{\upsilon}{r - \overline{\varphi}} e^{\overline{\varphi} t} + \frac{C_{0}(1 + \sigma_{c})}{r - \psi} e^{\psi t} + s_{3}, \qquad (A11)$$

where the unknown constant *s* is a function of the initial value $B(t=0) = B_0$. Substituting B_0 into Eq. (A11) yields:

$$S = B_0 - D_{F_0} + \frac{\upsilon}{r - \overline{\varphi}} - \frac{C_0(1 + \sigma_c)}{r - \psi} + \frac{\omega \xi Y_0}{\overline{\varphi}}.$$
 (A12)

Eq. (24) implies that the trajectory $\lambda_1(t)$ is:

$$\lambda_{1}(t) = \lambda_{1}(0)e^{(\rho-r)t}. \tag{A13}$$

Substituting (A13) and (A11) with (A12) into the transversality condition (e) gives:

$$\lambda_{i}(0) \cdot \lim_{t \to \infty} \left\{ S - \frac{\upsilon}{r - \overline{\varphi}} e^{(-r + \overline{\varphi})t} + \frac{C_{0}(1 + \sigma_{c})}{r - \psi} e^{(-r + \psi)t} + s_{3} e^{-rt} \right\} = 0, \quad (A14)$$

which is satisfied if, and only if, the following three conditions are met:

$$S = 0$$
, (A15)

- r > -, (A16)
- $r > \psi$ (A17)

which means that the interest rate must simply be sufficiently high. Importantly, Eq. (A15) determines the initial amount of consumption:

$$C_{0} = \left(B_{0} - D_{F0} + \frac{\upsilon}{r - \overline{\varphi}} + \frac{\omega \xi Y_{0}}{\overline{\varphi}}\right) \frac{r - \psi}{1 + \sigma_{c}} \cdot$$
(A18)

Therefore, the trajectory of foreign assets can be written as follows:

$$B(t) = \frac{C_0(1+\sigma_c)}{r-\psi} e^{yt} - \frac{\upsilon}{r-\overline{\varphi}} e^{\overline{\varphi} t} + s_3,$$
 (A19)

or, equivalently,

$$B(t) = \left(B_0 - D_{F0} + \frac{\upsilon}{r - \overline{\varphi}} + \frac{\omega \xi Y_0}{\overline{\varphi}}\right) e^{\psi t} - \frac{\upsilon}{r - \overline{\varphi}} e^{\overline{\varphi} t} + \left(D_{F0} - \frac{\omega \xi Y_0}{\overline{\varphi}}\right), \quad (A20)$$

Let us now analyze the second transversality condition (f). Using Eq. (30) ($\lambda_2 = \lambda_1$) allows to write down Eq. (f) as follows

$$\lim_{t \to \infty} e^{-\rho t} \lambda_1(t) K(t) = 0.$$
(A21)

The trajectory of capital is given by Eq. (A5). Similarly to the first transversality condition (and for similar reasons), we assume that the economy is on the BGP from t=0, so $K(t) = K_0 e^{\frac{\pi}{2}t}$. This formula together with Eq. (A5) allows to write condition (A21) as follows:

$\lambda_1(0)K_0 \cdot \lim_{t \to \infty} \left\{ e^{(\overline{\varphi} - r)t} \right\} = 0,$

which is satisfied if, and only if, $r > \overline{\varphi}$, which is identical to Eq. (A16). Therefore, if the first transversality condition (e) is satisfied, the second condition is satisfied as well.

T