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REGULAR ARTICLE



Can the evolution of joint savings agreements counter the effect of higher costs of migration on its intensity?

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Abstract

In this paper we consider a population of would-be migrants in a developing country. To begin with, this population is divided into two sets: those who save by themselves to pay for the cost of their migration, and those who pool their savings with the savings of another would-be migrant to pay for the cost. Saving jointly brings forward the timing of migration: funds needed to pay for the migration of one of the co-savers can be accumulated more quickly, enabling him, using his higher income at destination than at origin, to speed up the migration of his co-saver. However, people may hesitate to save jointly for fear that a co-saver who is the first to migrate might fail to keep his part of the agreement. We show that an increase in the cost of migration stimulates the formation of co-financing, joint-saving arrangements that enable would-be migrants to cushion the impact of the increase. The evolution of joint-saving arrangements can create a time window during which the intensity of migration need not decrease: no fewer people (and conceivably even more of them) will migrate during a time interval that follows the increase in the cost. This prediction is at variance with the canonical economic model of migration according to which if migration is costlier, then there will be less of it.

Keywords: Cost of migration; Saving for migration schemes; Evolution of joint saving agreements; Proximity in social space; Revised perception of the risk of joint saving; The intensity of migration

JEL Classification: D01; D71; D81; D83; D84; D91; F22; G41; O15

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This paper is dedicated to the memory of Luigi Orsenigo, an inspiring colleague, insightful scholar, and dear friend. Luigi was a stimulating and dedicated Co-Editor of this journal.

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1. Introduction

According to the canonical economic model of migration, if migration is costlier, there will be less of it. (Early examples of this model include the widely cited articles by Sjaastad, 1962, and Todaro, 1969.) In migration research and in migration policy formation, this notion has become the conventional wisdom, mainly because of its intuitive appeal. In this paper we show that an increase in the cost of migration can result in intensification of migration: we say that intensification occurs when more people migrate during a time interval that follows the increase in cost than would have migrated during the same time interval had there been no increase in cost. The reason for this outcome is that an increase in the cost of migration can trigger changes in the financial and social circumstances designed to enable would-be migrants to save enough to pay for the cost of their migration. Specifically, as explained below and modeled in the next two sections, the increase in cost can shift the line of demarcation between the set of lone savers and the set of joint savers in favor of the latter. Because saving jointly speeds up the accumulation of funds to pay for the cost of migration as compared to saving alone, the number of migrants during a time interval that follows the increase in the cost need not decrease, and may even increase.

In a review of immigration in American history, Abramitzky and Boustan (2017) remark that in the nineteenth century "[o]nce migrant communities were established in US cities and rural areas, many prospective migrants were able to travel on prepaid tickets financed by friends or family" (p. 1314). In evaluating the role of costs in shaping migration patterns, Abramitzky and Boustan note that those costs "need not imply that the poor are priced out of migration because of a lack of credit or financing for their journey. Both in the past and the present, there is evidence that immigrant networks can alleviate such financial constraints" (p. 1325).

Ilahi and Jafarey (1999) report that in Pakistan informal contracts agreed between migrants and their extended families, whereby the latter finance the migrants' travel abroad: about 58% of the migrants borrow from their extended family, with the amounts borrowed covering, on average, nearly half of the cost of migration. Borrowing from the extended family is more common among migrants of rural origin, who face higher costs of migration and are on average poorer, than among migrants of urban origin. Akkoyunlu and Siliverstovs (2013) provide evidence that a higher cost of migration from Turkey to Germany encourages the conclusion of informal financial contracts between would-be migrants and their extended families to pay for the cost of migration, and that the remittances that the migrants send back are likely to be used to finance subsequent migration by other family members. Genicot and Senesky (2004) report that Mexican migrants whose travel to the US was arranged by "coyotes" (migrant smugglers) were more likely to have received financial support from relatives and friends than Mexicans who set off to the US on their own. A higher cost of migration (arising from paying a "coyote") appears to have been linked to reliance on an extended financial support network. Indeed, Mexico-to-US migration, where an increase in border patrols made migration more difficult and hence more expensive, but possibly resulting in higher flows, could serve as a case study.

Texts on migrants' remittances have particularly acknowledged and documented that would-be migrants are helped by their families in obtaining the funds needed to pay for migration, and that once they have migrated and landed gainful employment they share their destination earnings with their families by means of remittances. (The articles on the reasons for sending remittances by Lucas and Stark, 1985, and by Stark and Lucas, 1988, have inspired a large empirical literature that has yielded insights about the motives for sending remittances and about the roles that the earnings of migrants and the incomes of their families play in determining the incidence of remittances and the sums remitted.) The line of reasoning advanced in this paper is distinct. We model the behavior of a would-be migrant who enters the "game" with own savings - these can be savings accumulated alone or jointly with his family - yet still faces a period of waiting in order to amass the required funds. Cooperation with another would-be migrant who faces a similar constraint is a strategy that goes farther than reliance on the support that might be provided by their own family. This perspective is similar to a setting in which a person who seeks insurance, while already covered by some level of self insurance, can gain from an exchange of insurance promises with another, independent self-insurer. And as mentioned below in footnote 7, the perspective has features reminiscent of Rotating Savings and Credit Associations.

We consider a setting in which people who seek to migrate are financially constrained, so that prior to migrating they have to collect the funds needed to pay for the cost of migration and initial settlement in a country of destination where incomes are higher than at origin.¹ We assume that a would-be migrant can do this either by accumulating the required funds himself, "lone financing," or by cooperating with another individual, "joint financing." In lone financing, the financier and the migrant are one and the same, and the raising of funds to pay for migration precedes and is completed prior to migration. In joint financing, migration and the financing to pay for migration are intertwined: migration begins when sufficient funds are amassed to allow one of the joint savers to set off, and co-financing by the migrant, who lands a job in the country of destination, helps secure the funds needed to facilitate the migration of the co-saver who has yet to migrate. There are advantages and disadvantages to each method. Lone financing is free from the possibility of others reneging, but it takes longer than (successful) joint financing. On the other hand, while joint financing speeds up the accumulation of funds, it is subject to the possibility that the co-saver who departs first might fail to support the migration of the co-saver who has yet to migrate.²

What incentive does a migrant have not to renege? What measures are available to an individual, who contributed to the savings pool but did not end up as the first-to-go, to effectively dissuade the co-saver who has already left from reneging? If they use means that help cement joint saving, the perceived risk involved in

¹Bryan et al. (2014) find that even in the case of internal seasonal migration, the cost of travel, food, and other incidentals during the trip poses a barrier to migration.

²Our interest in this paper is in the position of the line of demarcation between the two types of savers. We abstract from other forms of finance for migration: either they do not exist, or are far too costly / far too risky. Turning to loan sharks who might be willing to advance the funds needed to facilitate migration might be worse than giving up migration altogether.

joint financing can be moderated, and this form of financing will be attractive. Conversely, when such means are not available, lone saving will be more appealing. A standard menu of responses to the preceding two questions includes social deterrents, reputational concerns, and repeat transactions, with obvious linkages between the three. Compliance can be strengthened by applying social pressure, for example in the form of sanctions such as ostracizing the miscreant migrant and his family. The option of sanctioning will be effective when the migrant is close in social space to a co-saver who has yet to migrate, but will not have teeth when the contracting parties are distant in social space. Furthermore, sanctioning will be more effective when the migrant wants to keep open the option of return migration, regardless of whether return is imposed or voluntary, and regardless whether return is temporary or permanent. In the "grand" scheme of things, this implies that migration is not a final event, the last act in a sequence of moves; rather, it is a stage in a process, part of a broader, lasting, and dynamic relationship. (This discussion implies that although altruism can support compliance, if it is absent or fails, there are still available means to press for adherence.)

Let the opening configuration be such that the population of would-be migrants is divided into two sets: those who save enough by themselves to pay for the cost of their migration, and those who pair with others to jointly pool savings to pay for the cost. The first set consists of people who accepted the time required for lone saving or who, while preferring to save jointly, did not find people in sufficiently close proximity in social space to make low risk co-saving arrangements feasible. Let the cost of migration increase. Then, lone savers will be less hostile to entering a joint financing arrangement with people who are farther in social space if the risk arising from participation with them in joint financing is more acceptable than the delay in migration caused by the time required to provide for lone financing. As a result, in a time window following an increase in the cost of migration, the incidence of migration need not be less.

As a back-of-envelope illustration of such an evolution of the financial environment, suppose that at time zero there are four individuals at origin who seek to amass the funds needed for them to migrate. Two individuals are lone savers, the other two save together. Suppose that monthly income is 2, that the cost of migration is 12, that individuals can set aside all but one unit of their monthly income, and that income at the migration destination is twice the income at origin. The pattern of migration will then be as follows. At the end of month six, one individual out of the two who save together will migrate. Assuming the migrant sticks to the terms of the joint savings agreement, three months later the other individual will migrate. And after twelve months, the two lone savers will migrate. Now let the cost of migration rise from 12 to 14 and suppose that, consequently, the two lone savers shift to joint saving; saving alone as before would lead to too long a postponement. Then, after seven months two individuals will migrate, and three and a half months later the remaining two individuals will migrate. If we look at the time window of the first seven months, then we will see that prior to the increase in cost, migration would have been by one individual, and that following the increase, migration is by two individuals. This is the intensification alluded to above. Figure 1 which presents these configurations is

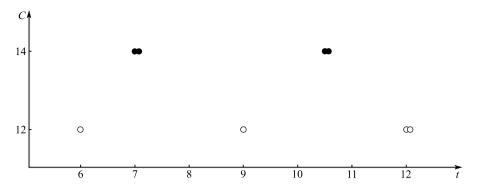


Figure 1. The timing of migration in months, t, of four individuals: when the cost of migration, C, is 12 (in which case, two individuals are lone savers, and the other two individuals save together); and when the cost of migration is 14 (in which case, two pairs of individuals who save together are formed). Light circles represent a migrant when the cost of migration is 12, dark circles represent a migrant when the cost of migration is 14.

drawn under the assumption of perfect compliance by the individuals who migrate first.³

In Section 2 we construct an intertemporal utility model to investigate the possibility that an individual enters a co-saving agreement with another individual to save together the sum needed to pay for the cost of migration and initial settlement in the country of destination, thereby speeding up migration. In Section 3 we present our two main results. First, we show that the propensity to enter a co-saving agreement which carries the risk of a co-saver defaulting increases with the cost of migration. The reason for the evolution of such an arrangement is that when the cost becomes higher, people choose the lesser of two evils: joint saving which could be risky, and postponed timing of migration if saving alone. When it comes to the risk that a co-saver will renege, a stronger desire to save jointly lowers the bar of acceptable social affinity of co-savers. Second, we formulate a condition under which the dynamics (time pattern) of the migration outflow will be such that there will be a post cost-increase period during which the incidence of migration will not be lower when the cost of migration is higher. In Section 4 we list tentative empirical implications. Section 5 concludes. In the Mathematical Addendum we present a detailed protocol for solving the utility-maximization problem of individuals who save jointly. That procedure yields the parameters that we use in the streamlined analysis undertaken in the main text of the paper.

2. Formal modeling

In a population of would-be migrants, let the normalized income, y(t), of a member of this population be given by

³Intensification of migration in the time window of the first seven months will occur even when one or two of the individuals who migrate first under the two joint saving agreements fail to comply and the betrayed individuals fail to enforce compliance.

$$y(t) = \begin{cases} 2 & \text{when working in the home country,} \\ \alpha + \beta & \text{when working in the destination country} \end{cases}$$

where time, *t*, measured in months, is taken to be continuous. We assume that the income of an individual is divided into two parts: the part needed to meet the essential cost of living, denoted by l(t), and the remainder, referred to henceforth as the spare income, which can be set aside as savings or spent on non-essential consumption. When in the home country, the monthly essential cost of living is l(t) = 1, and the monthly spare income is 1. When in the destination country, the monthly essential cost of living is $\alpha > 1$. The savings of an individual at time *t* are denoted by s(t). In addition, we assume a zero rate of interest on savings.

The individual's instantaneous utility function is u(x(t)) = x(t) + 1, where x(t) = y(t) - s(t) - l(t) is the individual's non-essential consumption at time *t*. Resorting to this representation assumes that covering the essential needs of living yields the same level of utility (which is equal to 1) in both countries. The utility of the individual can be increased by spending the spare income x(t) on consumption.

Let the intertemporal preferences of the individual be expressed by a continuous discount term $e^{-\delta t}$, where $\delta > 0$ is the discount factor. And let the expected length of the working life of an individual be *T* months. Then, the lifetime utility of an individual is

$$U(x(t)) = \int_{0}^{T} e^{-\delta t} u(x(t)) dt.$$

Suppose that the cost of migration which, for example, includes the fees paid to brokers and the expenses associated with the initial settlement in the destination country, is equal to C > 0. To render migration feasible, in all the scenarios analyzed below we assume that C < 2T/3,⁴ and that α is greater than some critical value $\alpha_0 > 1$: $\alpha > \frac{e^{\delta T} - 1}{e^{\delta(T - 3C/2)} - 1} \equiv \alpha_0$.⁵

As a benchmark for comparing joint financing with lone financing, we consider first lone financing.

Saving alone

Consider an individual who at month t = 0 starts to save to pay for his migration. As shown in Lemma M2 in the Mathematical Addendum, it is optimal for an individual to save his spare income of 1 every month. Because the individual's savings need

⁴Consider an individual whose length of working life is *T* months. Then, this condition implies that the cost of migration is not too high to prevent an individual who after co-saving for C/2 months was betrayed and left to save on his own for a period of *C* months: C/2 + C < T. We revisit this condition in the discussion that follows the proof of Lemma M1 in the Mathematical Addendum.

⁵The condition on α being greater than the critical value α_0 , which renders migration a viable option under lone saving and under joint financing, is derived formally in the Mathematical Addendum (consult Lemma M1).

to build up to meet the cost of migration, *C*, the number of months of savings is $T_A = C$ (the subscript *A* stands for alone). Thus, the lifetime utility of an individual who saves alone at the said rate of 1 per month (during which time his non-essential consumption is nil), and who then migrates (during which time his non-essential consumption is α) is

$$U_{A} = \int_{0}^{T_{A}} e^{-\delta t} u(0) dt + \int_{T_{A}}^{T} e^{-\delta t} u(\alpha) dt.$$
(1)

Saving jointly

Consider now an arrangement between two individuals who save together to meet the cost of migration. In all relevant respects other than for the distribution of the levels of affinity to others, which are individual-specific and are characterized below, the individuals are similar to each other. This implies symmetry and invites randomness in the selection of roles. We model the joint saving arrangement as follows. The individuals agree to save the maximum amount available to them in the home country (1 per month each), and they entrust the accumulated funds to a trustworthy third party (for example, the funds are kept safe by a village elder). Once the individuals save enough between them to pay for the migration of one of them, which happens after C/2 months of joint saving, they toss a coin to select the one who will migrate first; henceforth we refer to this individual as the winner of the draw, and to the other individual as the loser of the draw.^{6,7} The winner of the draw migrates. Using his higher income in the destination country, which allows for greater savings than at origin, he helps the loser of the draw who stayed behind to reach the destination country as fast as possible. After the departure of the winner of the draw, the individuals continue to save the maximum amounts possible: the loser of the draw continues to save 1 per month in the home country, and the winner of the draw saves α per month in the destination country.^{8,9}

To reinforce our argument, we add the assumption that the income to be obtained in the destination country is high enough (namely $\alpha > \alpha_0$) so that if the co-saving agreement is annulled after the winner of the draw migrates, the cheated loser of the draw will still find it attractive to save for migration, starting to do so all over again from scratch (at the maximum rate of 1 per month), but this time without seeking

⁶The parking of the savings with a trusted third party assures the winner of the draw that once realizing the outcome of the draw, the loser of the draw will not be able to opt out with all his savings intact, which would have been possible had he kept his savings for himself.

⁷This mechanism of joint saving resembles a ROSCA (Rotating Savings and Credit Association); consult Geertz (1962), Ardener (1964), and Besley et al. (1993).

⁸In Lemmas M3 through M5 in the Mathematical Addendum we show that these saving rates maximize the expected utilities of the individuals who save jointly.

⁹In the scheme described in this paragraph, the winner of the draw will contribute more to the common pot of savings than the loser of the draw. However, in terms of the sacrifice that each of the two individuals makes rather than in terms of the financial contributions that each of the individuals makes, such a saving program is fair.

to strike a new co-saving agreement with yet another individual ("once bitten, twice shy").¹⁰

In a population of would-be migrants which is of finite discrete size N that is not too small, we next characterize and measure the risk involved in a time-phased co-saving agreement and the link between this risk, the cost of migration, and the propensity to enter a two-person co-saving agreement aimed at facilitating migration. We relate the severity of the risk to the distance in social space. To quantify the risk, we characterize the proximity in social space between a pair of any would-be migrants by a single number between zero and one, which measures the personal bond between the individuals. Thus, for individual j (j = 1, 2, ..., N), the values of the levels of the affinity towards individuals i = 1, 2, ..., N are given by a sequence $P^{j} = (p_{1}^{j}, p_{2}^{j}, ..., p_{N}^{j})$, where $0 \le p_{i}^{j} \le 1$ for $i = 1, 2, ..., N^{11}$ In terms of the p_i^j values, we can think of individual j as if he were positioned at some point, surrounded by a sequence of circles of increasing radii, such that a radius is inversely proportional to p_i^{\prime} . Naturally, members of individual *i*'s closest family will be characterized by the highest p_i^{j} 's, thus occupying the most inner circle, members of the extended family of j by somewhat lower p_i^{j} 's, thus occupy the next, wider circle, friends of individual *i* by lower still p_i^{j} 's, occupying the third outward circle, and so on. Taking affinity to be mutual, we assume symmetry in the p_i^j values, that is, $p_i^j = p_i^j$. To map the affinity values onto the risk involved in a co-saving agreement, we assume that the probability that individual j assigns to the likelihood of individual *i* honoring the agreement after individual *i* emerges as the winner of the random draw of who will be the first to migrate - a probability that we term the trust parameter between j and i - is p_i^{j} .¹²

We now assemble the building blocks needed to construct the expected utility function of an honest individual j (that is, of an individual who is planning to keep his part of the agreement if he emerges as the winner of the draw), who co-saves with individual i.

First, individual *j* has a 50 percent chance of winning the draw, in which case he departs after $T_W = C/2$ months (where subscript *W* stands for winner). This individual sends back the maximum available amount of α per month which, when combined with the savings of the individual who stayed behind (1 per month), allows

the latter to take the migration journey after an additional $\frac{1}{1+\alpha}C$ months, namely

¹⁰That a cheated would-be migrant will next time go it alone could be reasoned in yet another way, namely from the "supply side" rather than from the "demand side," as follows. An individual who was cheated is likely to have a stronger temptation to make up for the lost time by cheating a co-saver should he have one. Illuminating evidence is provided by Houser et al. (2012) to the effect that an individual who was treated unfairly in one encounter is more likely to cheat in a subsequent encounter with another person. Alempaki et al. (2019) present intriguing findings in a similar vein. Assuming that other would-be migrants are aware of the fact that an individual was cheated (and that he is likely to be vengeful), they will be reluctant to enter a co-saving agreement with him. Thus, it will be hard for a cheated individual to find a co-saver.

¹¹For sake of notational consistency, the affinity of individual j "towards himself" is $p_i^j = 1$.

¹²There is an obvious variability in the likelihood of reneging caused by variability in the degree of social connectedness among co-savers. We do not need to include other contributing factors to that variability, even though, if such factors were to be added, that could accentuate it (operate in the same direction) as does social distance. In a laboratory experiment, Hermann and Ostermaier (2018) find that a reduction of social distance is likely to promote honesty in social interactions.

 $T_L = \frac{1}{2}C + \frac{1}{1+\alpha}C = \frac{3+\alpha}{2(1+\alpha)}C$ months after striking the co-saving agreement. (The subscript *L* stands for loser.) From then on, the two individuals can enjoy spending their income in the destination country as they please. Such a realization of the arrangement yields utility to an honest individual of

$$U_{W}^{H} = \int_{0}^{T_{W}} e^{-\delta t} u(0) dt + \int_{T_{W}}^{T_{L}} e^{-\delta t} u(0) dt + \int_{T_{L}}^{T} e^{-\delta t} u(\alpha) dt$$

$$= \int_{0}^{T_{L}} e^{-\delta t} u(0) dt + \int_{T_{L}}^{T} e^{-\delta t} u(\alpha) dt,$$
(2)

where U_W^H stands for the utility of an honest winner. (The superscript *H* stands for honest.)

Second, individual *j* has a 50 percent chance of losing the draw, in which case his utility will depend on the behavior of the winner of the draw who, we recall, is assumed to fulfill the agreement with probability p_i^j . If individual *i* does not renege, the utility of individual *j* will be

$$U_{L} = \int_{0}^{T_{W}} e^{-\delta t} u(0) dt + \int_{T_{W}}^{T_{L}} e^{-\delta t} u(0) dt + \int_{T_{L}}^{T} e^{-\delta t} u(\alpha) dt$$

$$= \int_{0}^{T_{L}} e^{-\delta t} u(0) dt + \int_{T_{L}}^{T} e^{-\delta t} u(\alpha) dt = U_{W}^{H},$$
(3)

where U_L stands for the utility of a loser of the draw whose co-saver is honest.

Third, with probability $1 - p_i^j$ individual *i* reneges, in which case individual *j*'s utility is

$$U_{Ch} = \int_{0}^{T_{W}} e^{-\delta t} u(0) dt + \int_{T_{W}}^{T_{Ch}} e^{-\delta t} u(0) dt + \int_{T_{Ch}}^{T} e^{-\delta t} u(\alpha) dt$$

$$= \int_{0}^{T_{Ch}} e^{-\delta t} u(0) dt + \int_{T_{Ch}}^{T} e^{-\delta t} u(\alpha) dt,$$
(4)

where U_{Ch} stands for the utility of a loser of the draw whose co-saver behaves dishonestly, and where $T_{Ch} = C/2 + C = 3C/2$ is the point in time at which the cheated individual can take the journey after saving alone from scratch. (The subscript *Ch* stands for cheated.)¹³

¹³We assume that the winner of the draw either reneges, failing to remit from the moment he arrives at the destination country (because his gain from reneging is then at its highest), or that he sticks to the agreement all the way up to the migration of the loser of the draw.

Joining together the preceding three building blocks, the expected utility of an honest would-be migrant j is

$$EU^{H} = \frac{1}{2}U^{H}_{W} + \frac{1}{2}\left[p^{j}_{i}U_{L} + \left(1 - p^{j}_{i}\right)U_{Ch}\right] = \frac{1 + p^{j}_{i}}{2}U^{H}_{W} + \frac{1 - p^{j}_{i}}{2}U_{Ch}, \quad (5)$$

where the third part of (5) follows from the middle part of (5) because from (2) and (3), $U_L = U_W^H$.

In an analogous manner, we formulate the expected utility of a "dishonest" would-be migrant j from striking a co-saving agreement with individual i, which is

$$EU^{D} = \frac{1}{2}U^{D}_{W} + \frac{1}{2}\left[p^{j}_{i}U_{L} + \left(1 - p^{j}_{i}\right)U_{Ch}\right]$$
(6)

(the superscript D stands for dishonest), and where

$$U_W^D = \int_0^{T_W} e^{-\delta t} u(0) dt + \int_{T_W}^T e^{-\delta t} u(\alpha) dt$$

is the utility of a dishonest winner, namely the utility of an individual who wins the draw, uses the savings of his co-saver to reach the destination country soonest (after $T_W = C/2$ months), and thereafter keeps for himself the higher income that he gets there.

To assess the inclination of individual j to enter a co-saving agreement, we look at the difference between the expected utility from co-saving (this utility is measured by EU^{H} , the expected utility of an honest would-be migrant, as given by (5)) and the utility from saving alone (this utility is U_A as given in (1)). We express this difference as a function of the trust parameter, p_i^{j} , and of the cost of migration, C:

$$\Delta U(p_i^j, C) = EU^H(p_i^j, C) - U_A(C).$$
⁽⁷⁾

It turns out that for a given pair (p_i^j, C) , the sign of $\Delta U(p_i^j, C)$ determines whether individual *j*, no matter if honest or not, will strike a co-saving agreement with individual *i*: if $\Delta U(p_i^j, C) > 0$, then individual *j* will strike an agreement, whereas if $\Delta U(p_i^j, C) < 0$, he will save alone. For an honest individual *j*, the reasoning is trivial. For a dishonest individual *j*, we have that $EU^D(p_i^j, C) > EU^H(p_i^j, C)$ for any p_i^j and *C*; therefore, $EU^D(p_i^j, C) > U_A(C)$ whenever $\Delta U(p_i^j, C) > 0$. Additionally, there exists a range of p_i^j for which $EU^D(p_i^j, C) > U_A(C)$ even though $\Delta U(p_i^j, C) < 0$. However, the willingness of individual *j* to strike a co-saving agreement with individual *i* when $\Delta U(p_i^j, C) < 0$ constitutes a signal of bad (dishonest) intentions of individual *j*. Therefore, noting that as a measure of mutual affinity p_i^j is known to both individual *i* and individual *j*, rational individuals (honest and dishonest alike) will not be keen to form co-saving agreements for a p_i^j for which $\Delta U(p_i^j, C) < 0$.

3. An increase in the cost of migration and the evolving propensity to form joint saving agreements

To determine the relationship between the propensity to strike a co-saving agreement and the cost of migration, we inquire how a marginal increase in this cost influences the range of the levels of p_i^j that render co-saving agreements desirable, namely that result in $\Delta U(p_i^j, C) > 0$. To this end, we treat the p_i^j in (7) as a continuous variable, and we refer to this variable as p. We denote by $p_0(C)$ the critical level of the trust parameter, expressed as a function of the cost of migration, such that for any $p > p_0(C)$ it holds that $\Delta U(p, C) > 0$. We can then formulate and prove the following proposition.

Proposition 1. Let the initial cost of migration be C_1 . Assuming that a marginal increase in the cost of migration from C_1 does not overturn the decision to migrate, the critical level of the trust parameter $p_0(C)$ is a non-increasing function of C in the neighborhood of C_1 . Moreover, if $\Delta U(0, C_1) < 0$, then $p_0(C)$ is a decreasing function of C in the neighborhood of C_1 .

Proof. Looking at the middle part of (5), we note that (because, obviously, $U_L > U_{Ch}$) the derivative of EU^H with respect to p_i^j is strictly positive and, thus, so is the derivative of $\Delta U(p, C)$ (as per (7)) with respect to p for any C. On comparing (2) and (1), we see that $\Delta U(1, C) = U_W^H(C) - U_A(C) > 0$ for any C. Because the sign of $\Delta U(0, C_1) = \frac{1}{2}U_W^H(C_1) + \frac{1}{2}U_{Ch}(C_1) - U_A(C_1)$ can be any, there are two cases to consider.

When $\Delta U(0, C_1) > 0$, an individual is willing to cooperate with any individual regardless of that individual's level of trust. A (marginal) increase of the cost from C_1 does not interfere with this inclination, namely in the neighborhood of C_1 , $p_0(C)$ is a non-increasing function of C.

When $\Delta U(0, C_1) < 0$, $p_0(C)$ in the neighborhood of C_1 can be characterized as the level at which $\Delta U(p_0(C), C) = 0$, that is,

$$p_0(C) = \frac{2U_A(C) - U_W^H(C) - U_{Ch}(C)}{U_W^H(C) - U_{Ch}(C)},$$

which, on taking the integrals in the expressions $U_A(C)$ (as per (1)), $U_W^H(C)$ (as per (2)), and $U_{Ch}(C)$ (as per (4)), and on performing several algebraic steps, yields

$$p_0(C) = 1 - \frac{2\left[1 - e^{-\frac{(\alpha-1)}{2(1+\alpha)}\delta C}\right]}{1 - e^{-\frac{\alpha}{1+\alpha}\delta C}}.$$

Taking the derivative of this expression of $p_0(C)$ with respect to C and evaluating the derivative at C_1 yields

$$p'_{0}(C) = -\frac{\delta e^{\frac{\delta C_{1}}{2}} \left[1 + \alpha + (\alpha - 1)e^{\frac{\alpha}{1+\alpha}\delta C_{1}} - 2\alpha e^{\frac{\alpha - 1}{2(1+\alpha)}\delta C_{1}} \right]}{(1+\alpha) \left(e^{\frac{\alpha}{1+\alpha}\delta C_{1}} - 1 \right)^{2}}.$$
(8)

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Because

$$-\frac{\delta e^{\frac{\delta C_1}{2}}}{(1+\alpha)\left(e^{\frac{\alpha}{1+\alpha}\delta C_1}-1\right)^2}<0.$$

we will be able to determine the sign of (8) once the sign of the term inside the square brackets in (8) is known. We denote this term by

$$R(\alpha, C_1, \delta) = 1 + \alpha + (\alpha - 1)e^{\frac{\alpha}{1+\alpha}\delta C_1} - 2\alpha e^{\frac{\alpha-1}{2(1+\alpha)}\delta C_1}.$$

Let $z = \delta C_1 > 0$, and let $S(\alpha, z) \equiv 1 + \alpha + (\alpha - 1)e^{\frac{\alpha}{1+\alpha}z} - 2\alpha e^{\frac{\alpha-1}{2(1+\alpha)}z}$. Then
$$\bigvee \lim_{\alpha > 1} \sum_{z \to 0} S(\alpha, z) = 0,$$
(9)

and

$$\bigvee_{\alpha>1,z>0} \frac{\partial S(\alpha,z)}{\partial z} = \frac{(\alpha-1)\alpha e^{\frac{\alpha-1}{2(1+\alpha)^{z}} \left(e^{\frac{z}{2}}-1\right)}}{1+\alpha} > 0.$$
(10)

From (9) and (10) it follows that the function $S(\alpha, z)$ is positive for every $\alpha > 1$ and z > 0, and that the function $R(\alpha, C_1, \delta)$ is positive for every $\alpha > 1$, $\delta > 0$, and $C_1 > 0$. Therefore,

$$p_0'(C_1) = -\frac{\delta e^{\frac{\delta C_1}{2}} R(\alpha, C_1, \delta)}{(1+\alpha) \left(e^{\frac{\alpha}{1+\alpha}\delta C_1} - 1\right)^2} < 0,$$

for every $\alpha > 1$, $\delta > 0$, and $C_1 > 0$, which leads us to conclude that in the neighborhood of C_1 , $p_0(C)$ is a decreasing function of C. Q.E.D.

Proposition 1 implies that after the cost of migration increases (but not by enough to overturn the decision to migrate),¹⁴ a would-be migrant will be in favor of entering a co-saving agreement with another would-be migrant who is farther away in social space (positioned at a farther out trust circle). If so, then as the cost becomes higher, more individuals will be predisposed to enter co-saving agreements to facilitate their migration.

At first sight, the lesser stringent stance described might appear counterintuitive: after all, as the cost of migration increases, the financial penalty incurred when a co-saver fails to keep his side of the agreement is heavier. However, as the cost of migration increases, the gain from co-saving can outweigh the possible loss: because individuals discount future consumption ($\delta > 0$), a gain realized earlier due to co-saving can overshadow the possible pain to be sustained in the more distant future.

¹⁴The increase in the cost of migration will not overturn the decision to migrate as long as following the increase in the cost of migration, the inequality $\alpha > \frac{e^{\delta T} - 1}{e^{\delta (T - 3C/2)} - 1}$ continues to hold.

We have implicitly assumed that the increase in the cost of migration does not imply or invite re-evaluation of the trust parameters that an individual attributes to his potential co-savers. Namely individual j, who accords a trust parameter p_i^j to individual i when the cost of migration is C_1 , will keep this evaluation of i when the cost increases to C_2 ; the longer period of amassing the required funds in the case of increased cost of migration will not render a given co-saver riskier. A reason for that is that when an individual selects a co-saver, the individual bases his choice on an established bond (mutual affinity in social space), not on a characteristic of a passing event (the prevailing cost of migration). Thus, individual j need not formulate his assessment of the likelihood of a potential co-saver sticking in the future to a deal as a function of the associated amount; he bases the assessment on the circle in social space occupied by the candidate co-saver.

Drawing on Proposition 1, we next show that following an increase in the cost of migration, there is a time window during which the intensity of migration will not be lower when the cost of migration is higher.

Proposition 2. Let there be a marginal increase in the cost of migration from C_1 to C_2 such that this increase does not overturn the decision to migrate, and such that $C_2 < \frac{3+\alpha}{1+\alpha}C_1$. Then, in the course of time span $\mathbf{T} = [0, C_2/2]$, the intensity of migration under cost C_2 will not be lower than the intensity of migration under cost C_1 .

Proof. Under cost C_1 , co-saving agreements will be formed among individuals with a trust parameter of at least $p_0(C_1)$. Let there be N_1 such individuals. The manner of the selection of individuals into pairs notwithstanding, let the number of pairs formed when the cost is C_1 be M_1 . Then, there will be $M_1/2$ individuals (winners of the draws in co-saving pairings), each of whom migrates after $T_W^1 = C_1/2$ months.

Let the cost of migration increase from C_1 to C_2 , where the increase does not overturn the decision to migrate. Drawing on Proposition 1, then under cost C_2 co-saving agreements will be formed among N_2 individuals with a trust parameter of at least $p_0(C_2) \le p_0(C_1)$. Therefore, $N_2 \ge N_1$. Under any plausible manner of the selection of individuals into pairs, the number of pairs M_2 formed among $N_2 \ge N_1$ individuals under cost C_2 will not be lower than under cost $C_1: M_2 \ge M_1$. Then, after $T_W^2 = C_2/2$ months, $M_2/2 \ge M_1/2$ individuals will migrate. Additionally, the losers of the draws in co-saving pairs formed under cost C_1 will migrate at the earliest after $T_L^1 = \frac{3+\alpha}{2(1+\alpha)}C_1 > T_W^2$ months, which follows from the assumption of the proposition that $C_2 < \frac{3+\alpha}{1+\alpha}C_1$. Thus, during time span $\mathbf{T} = [0, C_2/2]$, the intensity of migration under cost C_2 will not be lower than the intensity of migration under cost C_1 . Q.E.D.

4. Examples of empirical implications

Our model gives rise to implications that can be tested. By way of illustration, we list two.

First, suppose that there are two communities: a tightly-knit community A, and community B where social links between members are loose; it is not difficult to imagine that communities can and do differ in their "trust capital" or "social bonds capital." To begin with, we will observe an earlier participation in migration in community A than in community B. The reason is that individuals in community A are more likely to enter migration-facilitating joint saving agreements than individuals in community B. However, as Proposition 1 reveals, when the cost of migration increases, we can expect that individuals in community B will find entering joint saving agreements to expedite migration more attractive than continuing to save alone. Then, an increase in the cost of migration could narrow the difference in the timing of migration between communities that are dissimilar in terms of their "social bonds capital."

Second, we have in place a cost-based explanation for the emergence of co-saving agreements: high costs invite increased collaboration which, in turn and inter alia, assumes the form of established migrants subsidizing / supporting the migration of other members of their home community. Other things held constant, the higher the cost of migration, the higher the prevalence of co-saving, and the higher the incidence of subsidization / remittances. An intriguing testable prediction is that remittances to a community which responds to a rising cost of migration by higher incidence of co-saving will be higher when the cost of migration increases.

5. Discussion and conclusions

We have studied how financial cooperation between would-be migrants could accelerate costly journeys to a country where incomes higher than at origin can be enjoyed. The mutual financing of the cost of migration allows would-be migrants to avoid the need to take out expensive loans from loan sharks or pawn-brokers (if loan-taking is at all possible), or become a prey to smuggling organizations and traffickers.¹⁵ We have shown that when the risk involved in entering a co-saving agreement is taken into account, the propensity to enter an agreement depends positively on the cost of migration. An increase in this cost may not be followed by a slow-down in migration. And a possible intensification of migration is not caused by the expectation of an even higher cost in the future, but rather by a shift of the line of demarcation between the set of lone savers and the set of joint savers in favor of the latter.

In the analysis undertaken in this paper we have (implicitly) assumed that in terms of productivity and chances of finding employment at destination, the individuals who contemplate migrating are homogenous. Seemingly, in an "asymmetrical" environment with relatively low-skilled would-be migrants and relatively high-skilled would-be migrants, if mixed pairs were to form, a rational choice would be to forfeit the random selection of the first-to-go migrant and instead to let the relatively high-skilled individual migrate first. However, this is only "apparently" so because

¹⁵According to Djajić and Vinogradova (2014), the interest rate on a loan a migrant takes from smuggling organizations can reach 60 percent per annum.

when skills heterogeneity is introduced, there is a good chance that a high-skilled individual will gain little by pooling his savings with a low-skilled individual. Consequently, we can expect a pairing of similar-by-skill would-be migrants, with the random draw process retained. If matching by skill type is not possible and a mixed match is considered better than no match then, because it is likely that the random selection of the first-to-go will be replaced by an agreement that the high-skilled individual will leave first, the entire ex ante risk involved in striking the joint financing agreement will be borne by the low-skilled individual. If the affinity of this individual to the high-skilled individual is close enough, then the risk taken might not be too high to negate the appeal of an asymmetrical pairing.

In our analysis, we have based our definition of the "trust parameter" between the would-be migrants on the concepts of proximity in social space and affinity. A possible alternative perspective, under which our main result will hold, is to base the evaluation of the trustworthiness of a potential co-saver on the latter's known and well-established record. An example borrowed from the US financial scene can be used to illustrate. In the US, the best possible credit (FICO) score is 850. Superimposing the US setting on our migration scenario, suppose that when the cost of migration is low, individual *j* might prefer to save alone rather than to save together with another individual because that individual's score is 700, which measured as a ratio of 850 is 0.82. Nor will individual *j* want to co-save with yet another potential co-saver whose credit score is 600, which measured as a ratio is 0.71. These measures, which are based on past record and a history of honoring financial commitments, serve as individual j's "yardsticks." When the cost of migration increases, individual j gives a second consideration to co-saving with someone else; and when the cost is becoming higher still, individual i might even consider co-saving with the "0.71 individual." The numbers 850/850 = 1, 700/850 = 0.82, and 600/850 = 0.71 serve as probabilities that an individual will be a trustworthy collaborator in a pending saving scheme.

We have analyzed the difference between joint saving and lone saving under the assumption that joint saving is undertaken by two individuals. We took this track because we were of the opinion that this comparison nicely encapsulates the advantages and disadvantages of joint saving as opposed to lone saving, and because doing so was analytically manageable. A question could nonetheless be raised whether the qualitative conclusions drawn from that comparison will hold if more than two individuals were to team up to co-save: will not co-saving by, say, three individuals expedite migration by more than co-saving by two individuals? In response, we note that an increase in the number of co-savers is not an ideal means to expediting the accumulation of funds needed to facilitate migration. In Stark and Jakubek (2013) we studied the optimal size of a joint saving scheme in the context of the formation of a migration network, and we showed that this size is limited: even though adding another individual to the scheme can expedite the migration of co-savers, it is also the case that enlargement of the group of co-savers involves recruitment of people who are farther away in social space. Thus, for a given cost of migration, the risk involved in a bigger saving scheme can fast overshadow the potential gain from speeding-up migration. Under what conditions individuals will be willing to bear the associated increased risk when the cost of migration is increasing

calls for a full-scale analysis of the optimal number of co-savers as a function of the cost of migration, an intriguing subject for future inquiry.

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Mathematical Addendum

In this Addendum we show that the saving rates presented in the main text of the paper are the solutions of utility maximization problems of, respectively, a single individual who saves alone, and of two individuals who save jointly, and we provide strict conditions regarding the relationship between the parameters of the model $(C, T, \alpha, \text{ and } \delta)$ which render migration a viable option under the alternative schemes of saving.

Case 1: Saving alone

Consider an individual who at the beginning of month t = 0 elects to save alone in order to finance his migration. To concentrate on essentials, we assume that in each month the individual saves a constant amount out of his spare income, say an amount $s(t) \equiv s_A$. In order to meet the cost of migration, C, saving at this rate requires $T_A(s_A) \equiv C/s_A$ months of saving, where subscript A stands for alone. We normalize at 1 the maximum monthly amount available to a single individual for saving after covering the essential costs of living. To render migration possible, we assume throughout that C < 2T/3. Because the period of saving cannot possibly be longer than the length of a working life, we have that $s_A \in [C/T, 1]$. Thus, the lifetime utility of an individual who saves alone at the rate of s_A per month and then migrates is

$$U_A(s_A) \equiv \int_0^{T_A(s_A)} e^{-\delta t} u(1-s_A) dt + \int_{T_A(s_A)}^T e^{-\delta t} u(\alpha) dt,$$

where $u(\cdot)$ is defined in Section 2 of the main text of the paper. We denote by s_A^* the saving rate that maximizes $U_A(s_A)$.

To derive a condition that renders migration a viable option for an individual who saves alone, we denote by U_H the utility of an individual who spends his entire working life in his home country,

$$U_H \equiv \int_0^T e^{-\delta t} u(1) dt.$$
 (M1)

In the following lemma we present a requirement that needs to be fulfilled regarding a constellation of the parameters C, T, α , and δ that render migration a viable option for an individual who saves alone. We formulate the requirement as a condition on the parameter α because in the subsequent analysis this formulation will be the most straightforward one to use.

Lemma M1. If

$$\alpha > \frac{e^{\delta T} - 1}{e^{\delta (T - 3C/2)} - 1} \equiv \alpha_0, \tag{M2}$$

then there exists $s_A \in [C/T, 1]$ such that $U_A(s_A) > U_H$; that is, it is desirable for an individual to choose to save alone and migrate, rather than to spend his working life in his home country.

Proof. Let $s_A = 1$. Then, the lifetime utility of an individual who saves the amount of 1 per month in order to be able to migrate is

$$U_A(1) = rac{1}{\delta} \left(1 - e^{-\delta C}
ight) + rac{1+lpha}{\delta} \left(e^{-\delta C} - e^{-\delta T}
ight).$$

The requirement that migration after saving at the rate $s_A = 1$ is a gainful option for an individual is equivalent to the condition $U_A(1) - U_H > 0$. We have that

$$U_{A}(1) - U_{H} = \frac{1}{\delta} \left(1 - e^{-\delta C} \right) + \frac{1 + \alpha}{\delta} \left(e^{-\delta C} - e^{-\delta T} \right) - \frac{2}{\delta} \left(1 - e^{-\delta T} \right)$$

$$= \frac{1}{\delta} \left[\alpha e^{-\delta C} - (\alpha - 1) e^{-\delta T} - 1 \right] > \frac{1}{\delta} \left[\alpha e^{-3\delta C/2} - (\alpha - 1) e^{-\delta T} - 1 \right],$$
(M3)

from which it follows that $U_A(1) - U_H > 0$ if $\frac{1}{\delta} \left[\alpha e^{-3\delta C/2} - (\alpha - 1)e^{-\delta T} - 1 \right] > 0$, and for which to hold it is sufficient that

$$\alpha > \frac{e^{\delta T} - 1}{e^{\delta (T - 3C/2)} - 1}.$$

Q.E.D.

Comment: from (M3) it is seen that the condition (M2) on α is sufficient, but not necessary, for migration to constitute a viable option for a single individual. (In a similar vein, in the case of a single individual, the requirement that C < 2T/3 is sufficient, but not necessary.) However, as will be clarified in Lemma M4 below, conditions (M2) and C < 2T/3 are sufficient also for an individual who was cheated in a co-saving agreement to find it desirable to start saving alone from scratch. Thus, in order not to introduce assumptions superfluously, throughout our analysis we have chosen (M2) as the single assumption on α , and C < 2T/3 as the single assumption on C.

In the next lemma we derive the optimal saving rate of a single individual who on his own accumulates the funds that are needed to facilitate the migration journey.

Lemma M2. If condition (M2) is met, then an optimizing individual chooses a saving rate that is equal to his spare income, namely $s_A^* = 1$.

Proof. We consider the following maximization problem:

$$\max_{s_A \in [C/T, 1]} \left\{ U_A(s_A) \right\} = \max_{s_A \in [C/T, 1]} \left\{ \frac{2 - s_A}{\delta} \left(1 - e^{-\frac{\delta C}{s_A}} \right) + \frac{1 + \alpha}{\delta} \left(e^{-\frac{\delta C}{s_A}} - e^{-\delta T} \right) \right\},$$

assuming that $\alpha > \alpha_0$ (defined in (M2)). Differentiating this objective function with respect to s_A yields

$$U_A'(s_A) = \frac{e^{-\frac{\delta C}{s_A}} \left[s_A^2 \left(1 - e^{\frac{\delta C}{s_A}} \right) + \delta C \left(\alpha + s_A - 1 \right) \right]}{\delta s_A^2}.$$
 (M4)

Because s_A , δ , C > 0, the sign of $U'_A(s_A)$ is the same as the sign of the expression in square brackets in (M4). We denote $F(s_A, \alpha, \delta, C) \equiv s_A^2 \left(1 - e^{\frac{\delta C}{\delta_A}}\right) + \delta C \left(\alpha + s_A - 1\right)$.

First, we note that

$$\frac{\partial F}{\partial \alpha}(s_A, \alpha, \delta, C) = \delta C > 0. \tag{M5}$$

In addition,

$$F(1, \alpha, \delta, C) = 1 - e^{\delta C} + \alpha \delta C.$$

Using (M5) and (M2), we get that

$$F(1,\alpha,\delta,C) > F(1,\alpha_0,\delta,C).$$

Treating α_0 in (M2) as a function of T, we get that

$$lpha_0'(T)=-rac{e^{\delta(T+C)}ig(e^{\delta C}-1ig)\delta}{ig(e^{\delta T}-e^{\delta C}ig)^2}<0,$$

and that

$$\lim_{T\to\infty}\alpha_0(T)=e^{\delta C}\equiv\alpha_\infty.$$

Using (M5), we get that

$$\frac{dF(1,\alpha_0(T),\delta,C)}{dT} = \alpha'_0(T)\frac{\partial F}{\partial \alpha}(1,\alpha_0(T),\delta,C) < 0,$$

and, thus,

$$F(1, \alpha_0, \delta, C) > \lim_{T \to \infty} F(1, \alpha_0(T), \delta, C) = F(1, \alpha_\infty, \delta, C)$$
$$= e^{\delta C} \left[e^{-\delta C} + \delta C - 1 \right] > 0,$$

where the inequality in this last expression is due to the property that for any x > 0, $e^{-x} + x - 1 > 0$. Thus,

$$0 < F(1, \alpha_{\infty}, \delta, C) < F(1, \alpha_0, \delta, C) < F(1, \alpha, \delta, C)$$
(M6)

for any 0 < C < T, $\delta > 0$, and $\alpha > \alpha_0$, which yields $U'_{A}(1) > 0$. Additionally,

$$\frac{\partial^2 F}{\partial s_A^2}(s_A, \alpha, \delta, C) = 2 + e^{\frac{\delta C}{s_A}} \left[\frac{\delta C(2s_A - \delta C)}{s_A^2} - 2 \right] \equiv f(s_A, C, \delta).$$

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Because

$$\frac{\partial f}{\partial \delta}(s_A, C, \delta) = -\frac{\delta^2 C^3 e^{\frac{\delta C}{s_A}}}{s_A^3} < 0,$$

we get that $f(s_A, C, \delta) < \lim_{\delta \to 0} f(s_A, C, \delta) = 0$ and, therefore, $\frac{\partial^2 F}{\partial s_A^2}(s_A, \alpha, \delta, C) < 0$.

Consequently, because with respect to $s_A \in [C/T, 1]$ the function $F(\cdot)$ is concave and for $s_A = 1$ its value is positive (consult (M6)), this function can cross zero at most once for $s_A \in [C/T, 1]$. We denote by $s_0(\alpha, \delta, C)$ a point on [C/T, 1] at which $F(s_0, \alpha, \delta, C) = 0$ for given α, δ , and C. Because the sign of $F(\cdot)$ is the same as the sign of $U'_A(\cdot)$, there are two possibilities.

- (i) Such $s_0(\alpha, \delta, C)$ does not exist in the interval [C/T, 1] and, thus, $F(s_A, \alpha, \delta, C) > 0$ in the entire interval [C/T, 1], which is equivalent to $U_A(s_A)$ constituting an increasing function up to $s_A = 1$.
- (ii) Such $s_0(\alpha, \delta, C) \in [C/T, 1]$ exists, in which case the function $U_A(s_A)$ is decreasing in the interval $[C/T, s_0(\alpha, \delta, C))$, with $s_A = C/T$ being a local (border) maximum, and increasing on $(s_0(\alpha, \delta, C), 1]$, with $s_A = 1$ being another border maximum. However,

$$U_A(C/T) = \int_0^T e^{-\delta t} u(2 - C/T) dt < \int_0^T e^{-\delta t} u(2) dt = U_H, \text{ and from the proof of}$$

Lemma M1 and condition (M2) it follows that $U_A(1) > U_H$. Consequently, $U_A(C/T) < U_A(1)$.

Pooling together (i) and (ii), we conclude that $s_A^* = 1$ is a global maximum. Q.E.D.

In sum, the optimal duration of saving of a single individual is $T_A \equiv T_A(s_A^*) = C/s_A^* = C$. The corresponding maximum lifetime utility is

$$U_{A} \equiv U_{A}(1) = \int_{0}^{T_{A}} e^{-\delta t} u(0) dt + \int_{T_{A}}^{T} e^{-\delta t} u(\alpha) dt,$$
 (M7)

which is the same as expression (1) presented in the main text of the paper.

Case 2: Saving jointly

We first analyze a joint saving agreement under the assumption of complete compliance, that is, an agreement where the risk of reneging is not taken into consideration, which is equivalent to setting the affinity level p_i^j at 1. Once we derive the optimal saving rates in a complete-compliance agreement, we will show that allowing $p_i^j < 1$ will not affect the solutions of the corresponding utility maximization problems.

The individuals who save jointly choose three optimal saving rates in order to maximize their lifetime utility. Initially, when the individuals are in their home country, then until the draw determines who will be the first to migrate, they are indistinguishable. We therefore assume that each of them saves at the common rate $0 \le s_{H1} \le 1$, accumulating together the amount of $2s_{H1}$ per month. Following the draw and the migration of one of the individuals, the individuals can potentially save at different rates. We therefore denote by $s_D \in [0, \alpha]$ the saving rate of the individual who is already in the destination country, and by $s_{H2} \in [0, 1]$ the saving rate of the individual who is still in the home country.

Under a complete-compliance agreement, the only random event is the one in which it is determined who will be the winner of the draw. The winner will be able to take the migration journey after $T_W(s_{H1}) \equiv \frac{C}{2s_{H1}}$ months, whereas the loser of the draw will be able to migrate after an additional period of $\frac{C}{s_D + s_{H2}}$ months; that is, at the point in time $T_L(s_{H1}, s_{H2}, s_D) \equiv \frac{C}{2s_{H1}} + \frac{C}{s_D + s_{H2}}$. Thus, the expected utility of an individual entering a complete-compliance, two-person joint saving scheme is

$$EU_{CC}(s_{H1}, s_{H2}, s_D) \equiv \frac{1}{2}U_W(s_{H1}, s_{H2}, s_D) + \frac{1}{2}U_L(s_{H1}, s_{H2}, s_D),$$
(M8)

where subscript CC stands for complete-compliance, and where

$$U_{W}(s_{H1}, s_{H2}, s_{D}) \equiv \int_{0}^{T_{W}(s_{H1})} e^{-\delta t} u (1 - s_{H1}) dt + \int_{T_{W}(s_{H1})}^{T_{L}(s_{H1}, s_{H2}, s_{D})} e^{-\delta t} u (\alpha - s_{D}) dt + \int_{T_{L}(s_{H1}, s_{H2}, s_{D})}^{T} e^{-\delta t} u (\alpha) dt$$
(M9)

and

$$U_{L}(s_{H1}, s_{H2}, s_{D}) \equiv \int_{0}^{T_{W}(s_{H1})} e^{-\delta t} u \left(1 - s_{H1}\right) dt + \int_{T_{W}(s_{H1})}^{T_{L}(s_{H1}, s_{H2}, s_{D})} e^{-\delta t} u \left(1 - s_{H2}\right) dt + \int_{T_{L}(s_{H1}, s_{H2}, s_{D})}^{T} e^{-\delta t} u (\alpha) dt$$
(M10)

are, respectively, the lifetime utility of the winner of the draw, and the lifetime utility of the loser of the draw.

In the following lemma we specify the saving rates that maximize $EU_{CC}(s_{H1}, s_{H2}, s_D)$, which we correspondingly denote by s_{H1}^* , s_{H2}^* , and s_D^* .

Lemma M3. Assuming that condition (M2) is met, the two individuals who are optimizing $EU_{CC}(s_{H1}, s_{H2}, s_D)$ will choose rates of saving that amount to their entire spare incomes, that is, $s_{H1}^* = s_{H2}^* = 1$, $s_D^* = \alpha$.

Proof. We consider the following maximization problem:

$$\max_{s_{H_1}, s_{H_2}, s_D} \left\{ EU_{CC}(s_{H_1}, s_{H_2}, s_D) \right\}$$

s.t. $0 \le s_{H_1}, s_{H_2} \le 1, 0 \le s_D \le \alpha, \frac{C}{2s_{H_1}} + \frac{C}{s_{H_2} + s_D} \le T.$ (M11)

The first two constraints in (M11) express the assumption that savings cannot be higher than the corresponding spare incomes. The third constraint in (M11) means that the time needed to accumulate the funds required to cover the costs of migration by the two individuals cannot be longer than the working life of each of them.¹⁶

In order to show that the global maximum of problem (M11) is obtained for $s_{H1}^* = s_{H2}^* = 1$ and $s_D^* = \alpha$, we will proceed as follows. First, without loss of generality, we will rewrite problem (M11) so that the objective function $EU_{CC}(s_{H1}, s_{H2}, s_D)$ will be replaced by a simpler function of two variables, $EU(s_{HH}, s_{HD})$. Then, in Supportive Lemmas M1 and M2 we will derive properties of the derivatives of $EU(s_{HH}, s_{HD})$ with respect to s_{HH} and s_{HD} . These properties will allow us to identify the maximum of $EU(s_{HH}, s_{HD})$ and, consequently, the maximum of $EU_{CC}(s_{H1}, s_{H2}, s_D)$.

By combining (M8), (M9), and (M10) we obtain

$$EU_{CC}(s_{H1}, s_{H2}, s_D) = \frac{1}{2} \int_{0}^{\frac{C}{2S_{H1}}} e^{-\delta t} (4 - 2s_{H1}) dt$$

+ $\frac{1}{2} \int_{\frac{C}{2S_{H1}}}^{\frac{C}{2S_{H1}} + \frac{C}{S_{H2} + S_D}} e^{-\delta t} (3 + \alpha - s_{H2} - s_D) dt$
+ $\frac{1}{2} \int_{\frac{C}{2S_{H1}} + \frac{C}{S_{H2} + S_D}}^{T} e^{-\delta t} 2(1 + \alpha) dt.$

We simplify the maximization problem in (M11) by merging the two saving amounts s_{H2} and s_D into one variable, $s_{HD} \equiv s_{H2} + s_D$, and we denote $s_{HH} \equiv 2s_{H1}$. Because we will show that the solution of the problem defined in (M12) below is to set s_{HD} as high as possible, it follows that this solution translates to a well-defined solution of (M11) with respect to s_{H2} and s_D ; both amounts will have to be set at their upper limits. Namely

$$\max_{s_{HH}, s_{HD}} \left\{ EU(s_{HH}, s_{HD}) \right\}$$
s.t. $0 < s_{HH} \le 2, 0 < s_{HD} \le 1 + \alpha$, (M12)

¹⁶We do not contemplate a possibility that, for example, the migrant who turned out to be the second to go decides not to migrate but stay in the home country and boost his consumption by spending there the funds sent by the first migrant; the saving scheme is aimed at expediting the migration of both individuals.

where

$$EU(s_{HH}, s_{HD}) \equiv \frac{1}{2} \left[\int_{0}^{C/s_{HH}} e^{-\delta t} (4 - s_{HH}) dt + \int_{C/s_{HH}}^{C/s_{HH} + C/s_{HD}} e^{-\delta t} (3 + \alpha - s_{HD}) dt + \int_{C/s_{HH} + C/s_{HD}}^{T} e^{-\delta t} 2(1 + \alpha) dt \right],$$

and, thus, $EU_{CC}(s_{H1}, s_{H2}, s_D) = EU(2s_{H1}, s_{H2} + s_D)$.

Compared to (M11), in (M12) we changed the permitted ranges of the variables to strict inequalities with respect to zero, and we have done so because this is necessary for the third condition in (M11) to hold. However, for the sake of simplifying the analysis that follows, in (M12) we omit this condition; at the end of the proof of Supportive Lemma M3 below we show that, nonetheless, this condition is satisfied in the solution of problem (M12). In what follows, we assume that α satisfies condition (M2).

We next draw on a property of the derivative of $EU(\cdot)$ with respect to s_{HH} that is similar to the property found for $U'_A(\cdot)$ in the proof of Lemma M2. That is, we first show that $\frac{\partial EU}{\partial s_{HH}}(s_{HH}, s_{HD})$ is positive for the border value of $s_{HH} = 2$, and that it can cross zero at most once in the range $s_{HH} \in (0, 2)$. Therefore, in the search for maxima, we can concentrate on the borders where $s_{HH} = 2$ and where $s_{HH} \rightarrow 0$. With regard to the border $s_{HH} = 2$, $\frac{\partial EU}{\partial s_{HD}}(2, s_{HD})$ is positive for $s_{HD} = 1 + \alpha$, and it can cross zero at most once for $s_{HD} \in (0, 1 + \alpha)$ and, thus, either a maximum is obtained for $(s_{HH}, s_{HD}) = (2, 1 + \alpha)$, or for $s_{HD} \rightarrow 0$ the function $EU(\cdot)$ grows beyond $EU(2, 1 + \alpha)$. With regard to the border $s_{HD} \rightarrow 0$, we get that the function $\lim_{s_{HH} \to 0} EU(s_{HH}, s_{HD})$ is constant with respect to s_{HD} and, therefore, all that remains in order to resolve existence and pinpoint the global maximum of $EU(\cdot)$ on the set $(s_{HH}, s_{HD}) = (0, 2] \times (0, 1 + \alpha]$ is to compare the values of $EU(2, 1 + \alpha)$, $\lim_{s_{HH} \to 0} EU(2, s_{HD})$, and $\lim_{s_{HH} \to 0} EU(s_{HH}, s_{HD})$. We note that

$$\frac{\partial EU}{\partial s_{HH}}(s_{HH}, s_{HD}) = \frac{e^{-\frac{\delta C(s_{HH}+s_{HD})}{s_{HH}s_{HD}}} \left\{ \delta C \left[\alpha + s_{HD} - 1 + e^{\frac{\delta C}{s_{HD}}} \left(\alpha + s_{HH} - s_{HD} - 1 \right) \right] + e^{\frac{\delta C}{s_{HD}}} \left(1 - e^{\frac{\delta C}{s_{HH}}} \right) s_{HH}^2 \right\}}{2\delta s_{HH}^2}.$$
(M13)

We further note that the sign of $\frac{\partial EU}{\partial s_{HH}}(s_{HH}, s_{HD})$ for $s_{HH} \in (0, 2]$ and for $s_{HD} \in (0, 1+\alpha]$ is the same as that of the expression inside the curly brackets in (M13), which we

denote by

$$G(s_{HH}, s_{HD}, \alpha, \delta, C) \equiv \delta C \left[\alpha + s_{HD} - 1 + e^{\frac{\delta C}{3\mu D}} \left(\alpha + s_{HH} - s_{HD} - 1 \right) \right]$$
$$+ e^{\frac{\delta C}{3\mu D}} \left(1 - e^{\frac{\delta C}{3\mu H}} \right) s_{HH}^2.$$

In the following supportive lemma we present two properties of the $G(\cdot)$ function.

Supportive Lemma M1. $G(s_{HH}, s_{HD}, \alpha, \delta, C)$ is a concave function with respect to $s_{HH} \in (0, 2]$ for every $s_{HD} \in (0, 1 + \alpha]$, and $G(2, s_{HD}, \alpha, \delta, C) > 0$ for every $s_{HD} \in (0, 1 + \alpha]$.

Proof. We first deal with the concavity property. We note that

$$\frac{\partial^2 G}{\partial s_{HH}^2}(s_{HH}, s_{HD}, \alpha, \delta, C) = \frac{e^{\frac{\delta C}{\delta_{HD}}} \left[2s_{HH}^2 - e^{\frac{\delta C}{\delta_{HH}}} \left(\delta^2 C^2 - 2\delta C s_{HH} + 2s_{HH}^2\right)\right]}{s_{HH}^2}.$$
 (M14)

Denoting the expression in square brackets in (M14), which determines the sign of $\frac{\partial^2 G}{\partial s_{HH}^2}(s_{HH}, s_{HD}, \alpha, \delta, C)$, by

$$g(s_{HH},\alpha,\delta,C) \equiv 2s_{HH}^2 - e^{\frac{\delta C}{S_{HH}}} \left(\delta^2 C^2 - 2\delta C s_{HH} + 2s_{HH}^2\right),$$

we obtain

$$\frac{\partial g}{\partial \delta}(s_{HH}, \alpha, \delta, C) = -\frac{\delta^2 C^3 e^{\frac{\delta C}{\delta_{HH}}}}{s_{HH}} < 0.$$

Thus, for $\delta > 0$, we get that

$$g(s_{HH}, \alpha, \delta, C) < \lim_{\delta \to 0} g(s_{HH}, \alpha, \delta, C) = 0,$$

from which we get that $\frac{\partial^2 G}{\partial s_{HH}^2}(s_{HH}, s_{HD}, \alpha, \delta, C) < 0.$

To determine the sign of $G(2, s_{HD}, \alpha, \delta, C)$, we note that

$$G(2, s_{HD}, \alpha, \delta, C) = e^{\frac{\delta C}{\delta HD}} \left[4 + \delta C \left(1 + \alpha - s_{HD} \right) \right] + \delta C \left(\alpha + s_{HD} - 1 \right) - 4e^{\frac{\delta C}{\delta HD} + \frac{1}{2}},$$

from which we obtain

$$\frac{\partial G}{\partial \alpha}(2, s_{HD}, \alpha, \delta, C) = \delta C \left(1 + e^{\frac{\delta C}{\delta_{HD}}}\right) > 0,$$

so that, consequently, for $\alpha > \alpha_0$ (as defined in (M2)) it follows that

$$G(2, s_{HD}, \alpha, \delta, C) > G(2, s_{HD}, \alpha_0, \delta, C).$$
(M15)

Treating α_0 in (M2) as a function of T, we get that

$$\alpha_0'(T) = -\frac{e^{\delta(T+C)} \left(e^{\delta C} - 1\right)\delta}{\left(e^{\delta T} - e^{\delta C}\right)^2} < 0, \tag{M16}$$

that

$$\lim_{T \to \infty} \alpha_0(T) = e^{\delta C} \equiv \alpha_\infty, \tag{M17}$$

and that

$$\frac{dG(2, s_{HD}, \alpha_0(T), \delta, C)}{dT} = \delta C \left(1 + e^{\frac{\delta C}{\delta_{HD}}} \right) \alpha'_0(T) < 0.$$
(M18)

Thus, from (M18) we get that

$$G(2, s_{HD}, \alpha_0, \delta, C) > \lim_{T \to \infty} G(2, s_{HD}, \alpha_0(T), \delta, C) = G(2, s_{HD}, \alpha_\infty, \delta, C)$$

= $\delta C \left(e^{\delta C} + e^{\frac{\delta C}{\delta_{HD}} + \delta C} + s_{HD} - 1 \right) + e^{\frac{\delta C}{\delta_{HD}}} \left[4 + \delta C \left(1 - s_{HD} \right) \right] - 4e^{\frac{\delta C}{\delta_{HD}} + \frac{\delta C}{2}}.$ (M19)

Without loss of generality, we replace the term $\delta C > 0$ in (M19) by a variable r > 0, denoting

$$g(s_{HD}, r) \equiv r\left(e^{r} + e^{\frac{r}{s_{HD}} + r} + s_{HD} - 1\right) + e^{\frac{r}{s_{HD}}}\left[4 + r\left(1 - s_{HD}\right)\right] - 4e^{\frac{r}{s_{HD}} + \frac{r}{2}}.$$

Then,

$$\frac{\partial g}{\partial s_{HD}}(s_{HD},r) = \frac{re^{\frac{r}{3HD} + \frac{r}{2}} \left\{ 4 - re^{\frac{r}{2}} + s_{HD}^2 e^{-\frac{r}{3HD} - \frac{r}{2}} - e^{-\frac{r}{2}} \left[4 + r\left(1 - s_{HD}\right) + s_{HD}^2 \right] \right\}}{s_{HD}^2}.$$
(M20)

We denote the expression in curly brackets in (M20) by

$$\gamma(s_{HD}, r) \equiv 4 - re^{\frac{r}{2}} + s_{HD}^2 e^{-\frac{r}{S_{HD}} - \frac{r}{2}} - e^{-\frac{r}{2}} \left[4 + r \left(1 - s_{HD} \right) + s_{HD}^2 \right].$$

Then

$$\frac{\partial \gamma}{\partial s_{HD}}\left(s_{HD}, r\right) = e^{-\frac{r}{s_{HD}} - \frac{r}{2}} \left[e^{\frac{r}{s_{HD}}} \left(r - 2s_{HD}\right) + r + 2s_{HD} \right].$$
(M21)

We denote the expression inside the square brackets in (M21) by

$$\theta(r, s_{HD}) \equiv e^{\frac{r}{s_{HD}}}(r - 2s_{HD}) + r + 2s_{HD},$$

and we get that

$$\lim_{r \to 0} \theta(r, s_{HD}) = 0, \tag{M22}$$

that

$$\frac{\partial^2 \theta}{\partial r^2}(r, s_{HD}) = \frac{r e^{\frac{r}{S_{HD}}}}{s_{HD}^2} > 0, \tag{M23}$$

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and that

$$\frac{\partial \theta}{\partial r}(r, s_{HD}) = \frac{e^{\frac{r}{s_{HD}}}(r - s_{HD}) + s_{HD}}{s_{HD}^2} \xrightarrow[r \to 0]{} 0. \tag{M24}$$

Combining (M24) and (M23) yields $\frac{\partial \theta}{\partial r}(r, s_{HD}) > 0$ which, together with (M22), yields $\theta(r, s_{HD}) > 0$ and, finally, we get that

$$\frac{\partial \gamma}{\partial s_{HD}} (s_{HD}, r) > 0. \tag{M25}$$

From (M25) we get that for every $s_{HD} > 0$ and r > 0,

$$\gamma(s_{HD}, r) < \lim_{s_{HD}\to\infty} \gamma(s_{HD}, r) = \frac{1}{2} \Big[8e^{\frac{r}{2}} - 8 - 2r - 2re^{r} + r^{2} \Big].$$

Let

$$\varphi(r) \equiv 8e^{\frac{r}{2}} - 8 - 2r - 2re^{r} + r^{2}.$$

It follows that

$$\lim_{r \to 0} \varphi(r) = 0, \tag{M26}$$

that

$$\varphi''(r) = -\left(2e^r - 2\right) - \left(2e^r - 2e^{\frac{r}{2}}\right) - 2re^r < 0, \tag{M27}$$

and that

$$\varphi'(r) = 2\left[2e^{\frac{r}{2}} + r - e^{r}(1+r) - 1\right] \xrightarrow[r \to 0]{} 0.$$
 (M28)

Combining (M27) and (M28) yields $\varphi'(r) < 0$ which, together with (M26), implies that for every r > 0, $\varphi(r) < 0$ and, consequently, $\gamma(s_{HD}, r) < 0$, so that also

$$\frac{\partial g}{\partial s_{HD}}(s_{HD}, r) < 0. \tag{M29}$$

Now (M29) implies that for every $s_{HD} > 0$ and r > 0

$$g(s_{HD}, r) > \lim_{s_{HD} \to \infty} g(s_{HD}, r) = 4 - 4e^{\frac{r}{2}} + 2re^{r} - r^{2} \equiv \rho(r).$$

Because $\lim_{r\to 0} \rho(r) = 0$ and $\rho'(r) = 2(e^r - e^{\frac{r}{2}}) + 2r(e^r - 1) > 0$, then $0 < \rho(r) < g(s_{HD}, r)$ for every $s_{HD} > 0$ and r > 0 which, upon incorporating (M15) and (M19), gives us that

 $0 < G(2, s_{HD}, \alpha_{\infty}, \delta, C) < G(2, s_{HD}, \alpha_0, \delta, C) < G(2, s_{HD}, \alpha, \delta, C)$. This completes the proof of Supportive Lemma M1. Q.E.D.

Returning to the main line of the proof of Lemma M3, from the properties of the function $G(\cdot)$ in Supportive Lemma M1 we infer that the derivative of $EU(\cdot)$ with respect to s_{HH} in (M13) is positive on the right-hand border of the permitted range $(s_{HH} = 2)$, and that it can cross zero only once in the range $s_{HH} \in (0, 2)$.

Consequently,

$$\forall_{s_{HD}\in(0,1+\alpha]}\forall_{s_{HH}\in(0,2)}EU(s_{HH},s_{HD})<\max\left\{EU(2,s_{HD}),\lim_{s_{HH}\to0}EU(s_{HH},s_{HD})\right\},$$

namely for every $s_{HD} \in (0, 1 + \alpha]$ either the maximum with respect to s_{HH} is obtained at the right border ($s_{HH} = 2$), or it cannot be obtained if the function increases as $s_{HH} \rightarrow 0$ beyond the level $EU(2, s_{HD})$ and, thus, we can constrain our search for maxima to the borders $s_{HH} = 2$ and $s_{HH} \rightarrow 0$.

Investigating first the border $s_{HH} = 2$,

$$\frac{\partial EU}{\partial s_{HD}}(2, s_{HD}) = \frac{e^{-\frac{\delta C}{2} - \frac{\delta C}{\delta_{HD}}} \left[\left(1 - e^{\frac{\delta C}{\delta_{HD}}} \right) s_{HD}^2 + \delta C \left(\alpha + s_{HD} - 1 \right) \right]}{2\delta s_{HD}^2}.$$
 (M30)

We note that the sign of $\frac{\partial EU}{\partial s_{HD}}(2, s_{HD})$ for $s_{HD} \in (0, 1 + \alpha]$ is the same as that of the expression inside the square brackets in (M30), which we denote by

$$H(s_{HD}, \alpha, \delta, C) \equiv \left(1 - e^{\frac{\delta C}{S_{HD}}}\right) s_{HD}^2 + \delta C \left(\alpha + s_{HD} - 1\right).$$
(M31)

We next will show that $H(s_{HD}, \alpha, \delta, C)$ is concave with respect to $s_{HD} \in (0, 1 + \alpha]$ and, using Supportive Lemmas M2 and M3 below, we will also show that $H(1 + \alpha, \alpha, \delta, C) > 0$.

Supportive Lemma M2. $H(s_{HD}, \alpha, \delta, C)$ is a concave function with respect to $s_{HD} \in (0, 1 + \alpha]$, and $H(1 + \alpha, \alpha, \delta, C) > 0$.

Proof. Attending first to the concavity property, from (M31) we get that

$$\frac{\partial^2 H}{\partial s_{HD}^2}(s_{HD}, \alpha, \delta, C) = 2 + e^{\frac{\delta C}{s_{HD}}} \left[\frac{\delta C(2s_{HD} - \delta C)}{s_{HD}^2} - 2 \right] \equiv h(s_{HD}, \alpha, \delta, C).$$

We next get that

$$\frac{\partial h}{\partial \delta}(s_{HD}, \alpha, \delta, C) = -\frac{\delta^2 C^3 e^{\frac{\delta C}{S_{HD}}}}{s_{HD}^3} < 0,$$

and, therefore,

$$h(s_{HD}, \alpha, \delta, C) < \lim_{\delta \to 0} h(s_{HD}, \alpha, \delta, C) = 0,$$

which yields $\frac{\partial^2 H}{\partial s_{HD}^2}(s_{HD}, \alpha, \delta, C) < 0.$

To determine the sign of $H(1 + \alpha, \alpha, \delta, C)$, we denote

$$H(1+\alpha,\alpha,\delta,C) = 2\alpha\delta C - (1+\alpha)^2 \left(e^{\frac{\delta C}{1+\alpha}} - 1\right) \equiv \mu(\alpha,\delta,C).$$

We get that

$$\frac{\partial \mu}{\partial \alpha}(\alpha, \delta, C) = e^{\frac{\delta C}{1+\alpha}} \left[\delta C - 2(1+\alpha)\right] + 2(1+\alpha+\delta C).$$

that

$$\frac{\partial^2 \mu}{\partial \alpha^2}(\alpha, \delta, C) = 2 - \frac{e^{\frac{\delta C}{1+\alpha}} \left[\delta^2 C^2 + 2(1+\alpha)^2 - 2\delta C(1+\alpha)\right]}{(1+\alpha)^4},$$

and that

$$\frac{\partial^3 \mu}{\partial \alpha^3}(\alpha, \delta, C) = \frac{\delta^3 C^3 e^{\frac{\delta C}{1+\alpha}}}{(1+\alpha)^4} > 0.$$
(M32)

From (M32) we get that

$$\frac{\partial^2 \mu}{\partial \alpha^2}(\alpha, \delta, C) < \lim_{\alpha \to \infty} \frac{\partial^2 \mu}{\partial \alpha^2}(\alpha, \delta, C) = 0,$$

which, in turn, implies that

$$\frac{\partial \mu}{\partial \alpha}(\alpha, \delta, C) > \lim_{\alpha \to \infty} \frac{\partial \mu}{\partial \alpha}(\alpha, \delta, C) = \delta C > 0.$$
 (M33)

We note that (M33) implies that for $\alpha > \alpha_0$ (recalling (M2)),

$$\mu(\alpha, \delta, C) > \mu(\alpha_0, \delta, C)$$

Treating α_0 as a function of *T*, from (M16) and (M33) it follows that

$$\frac{d\mu(\alpha_0(T),\delta,C)}{dT} = \alpha'_0(T)\frac{\partial\mu}{\partial\alpha}(\alpha,\delta,C) < 0.$$

Recalling (M17), we get that

$$\mu(\alpha_0, \delta, C) > \lim_{T \to \infty} \mu(\alpha_0(T), \delta, C) = \mu(\alpha_\infty, \delta, C)$$
$$= 2e^{\delta C} \delta C - \left(1 + e^{\delta C}\right)^2 \left(e^{\frac{\delta C}{1 + e^{\delta C}}} - 1\right).$$

Without loss of generality, we replace $\delta C > 0$ in the preceding expression with a variable r > 0, and we rewrite this expression as

$$f(r) = 2e^{r}r - (1+e^{r})^{2} \left(e^{\frac{r}{1+e^{r}}} - 1\right).$$
(M34)

We next show that f(r) > 0 for r > 0. Using the following representation of the exponential function

$$e^{y} = 1 + \frac{y}{1!} + \frac{y^{2}}{2!} + \frac{y^{3}}{3!} + \frac{y^{4}}{4!} + \dots + \frac{y^{n}}{n!} + \dots,$$
 (M35)

we can likewise write

$$e^{\frac{r}{1+e^r}} - 1 = \frac{r}{1+e^r} + \frac{r^2}{2!(1+e^r)^2} + \frac{r^3}{3!(1+e^r)^3} + \frac{r^4}{4!(1+e^r)^4} + \dots + \frac{r^n}{n!(1+e^r)^n} + \dots,$$

and, therefore,

$$(1+e^{r})^{2}\left(e^{\frac{r}{1+e^{r}}}-1\right) = r(1+e^{r}) + \frac{r^{2}}{2!} + \frac{r^{3}}{3!(1+e^{r})} + \frac{r^{4}}{4!(1+e^{r})^{2}} + \dots + \frac{r^{n}}{n!(1+e^{r})^{n-2}} + \dots$$
(M36)

Using (M35) again,

$$e^{r}r = r + \frac{r^{2}}{1!} + \frac{r^{3}}{2!} + \frac{r^{4}}{3!} + \dots + \frac{r^{n}}{(n-1)!} + \dots,$$

and, thus,

$$2e^{r}r = e^{r}r + e^{r}r = e^{r}r + r + \frac{r^{2}}{1!} + \frac{r^{3}}{2!} + \frac{r^{4}}{3!} + \dots + \frac{r^{n}}{(n-1)!} + \dots$$

$$= r(e^{r}+1) + \frac{r^{2}}{1!} + \frac{r^{3}}{2!} + \frac{r^{4}}{3!} + \dots + \frac{r^{n}}{(n-1)!} + \dots$$
(M37)

Combining (M36) and (M37),

$$\begin{split} &2e^{r}r - \left(1 + e^{r}\right)^{2}\left(e^{\frac{r}{1+e^{r}}} - 1\right) \\ &= \left[\frac{r^{2}}{1!} - \frac{r^{2}}{2!}\right] + \left[\frac{r^{3}}{2!} - \frac{r^{3}}{3!(1+e^{r})}\right] + \left[\frac{r^{4}}{3!} - \frac{r^{4}}{4!(1+e^{r})^{2}}\right] + \dots \\ &+ \left[\frac{r^{n}}{(n-1)!} - \frac{r^{n}}{n!(1+e^{r})^{n-2}}\right] + \dots \\ &= r^{2}\left[\frac{1}{1!} - \frac{1}{2!}\right] + r^{3}\left[\frac{1}{2!} - \frac{1}{3!(1+e^{r})}\right] + r^{4}\left[\frac{1}{3!} - \frac{1}{4!(1+e^{r})^{2}}\right] + \dots \\ &+ r^{n}\left[\frac{1}{(n-1)!} - \frac{1}{n!(1+e^{r})^{n-2}}\right] + \dots \\ &= \sum_{n=2}^{\infty} r^{n}\left[\frac{1}{(n-1)!} - \frac{1}{n!(1+e^{r})^{n-2}}\right] > 0, \end{split}$$

where the inequality is due to r > 0, and because $\frac{1}{(n-1)!} - \frac{1}{n!(1+e^r)^{n-2}} > 0$ for any $n \ge 2$. Consequently, f(r) > 0 for r > 0.

In sum: $0 < \mu(\alpha_{\infty}, \delta, C) < \mu(\alpha_0, \delta, C) < \mu(\alpha, \delta, C)$, which yields $H(1 + \alpha, \alpha, \delta, C) > 0$. This completes the proof of Supportive Lemma M2. Q.E.D.

We now return to the main line of the proof of Lemma M3, noting that from the properties of the $H(\cdot)$ function displayed in Supportive Lemma M2 we infer that the derivative with respect to s_{HD} in (M30) is positive on the right-hand border of the permitted range ($s_{HD} = 1 + \alpha$), and that it can cross zero only once in the range $s_{HD} \in (0, 1 + \alpha)$. Therefore, either the maximum at the border $s_{HH} = 2$ is obtained for $s_{HD} = 1 + \alpha$, or it cannot be obtained if the function increases for $s_{HD} \rightarrow 0$ beyond the level $EU(2, 1 + \alpha)$. Namely

$$\forall_{s_{HD}\in(0,1+\alpha)} EU(2, s_{HD}) < \max\left\{EU(2, 1+\alpha), \lim_{s_{HD}\to 0} EU(2, s_{HD})\right\}.$$

For the point $(s_{HH}, s_{HD}) = (2, 1 + \alpha)$,

$$EU(2, 1+\alpha) = \frac{1}{\delta} \left[1 + \alpha e^{-\frac{(3+\alpha)\delta C}{2(1+\alpha)}} - (1+\alpha)e^{-\delta T} \right].$$
 (M38)

Comparing the expression for $EU(2, 1 + \alpha)$ in (M38) with U_A (recalling (M7)), and drawing on the assumption that under condition (M2) saving alone for migration is rational, we see that

$$EU(2, 1+\alpha) > U_A > U_H, \tag{M39}$$

where the first inequality in (M39) is due to $\frac{(3 + \alpha)}{2(1 + \alpha)} < 1$ for any $\alpha > 1$.

For the limit $s_{HD} \rightarrow 0$ at the border $s_{HH} = 2$,

$$\lim_{s_{HD}\to 0} EU(2, s_{HD}) = \frac{1}{2\delta} \left[2 + (1 + \alpha) \left(e^{-\delta C/2} - 2e^{-\delta T} \right) \right]$$

and from a comparison of this last expression with (M38), we get that

$$EU(2, 1 + \alpha) - \lim_{s_{HD} \to 0} EU(2, s_{HD}) = \frac{1}{2\delta} \left[2\alpha e^{-\frac{(3+\alpha)\delta C}{2(1+\alpha)}} - (1+\alpha)e^{-\frac{\delta C}{2}} \right].$$

Now, we denote

$$J(\alpha, C, \delta) \equiv 2\alpha e^{-\frac{(3+\alpha)\delta C}{2(1+\alpha)}} - (1+\alpha)e^{-\frac{\delta C}{2}}$$

and we replace the variable α with $b = \frac{2}{\alpha + 1}$. Because $\alpha > \alpha_0 > 1$, then $b \in (0, b_0)$, where $b_0 = \frac{2}{\alpha_0 + 1} < 1$. Thus,

$$J\left(\frac{2}{b}-1, C, \delta\right) = \frac{2}{b}e^{-\frac{\delta C(1+b)}{2}} \left[2-b-e^{\frac{\delta Cb}{2}}\right].$$
 (M40)

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Obviously, $\frac{2}{b}e^{-\frac{\delta C(1+b)}{2}} > 0$ and, therefore, the sign of $J\left(\frac{2}{b}-1, C, \delta\right)$ is the same as that of the expression inside the square brackets in (M40). We denote

$$K(b, C, \delta) \equiv 2 - b - e^{\frac{\delta Cb}{2}}$$

Then
$$\frac{\partial K}{\partial b}(b, C, \delta) = -\left(1 + \frac{1}{2}e^{\frac{\delta C b}{2}}\delta C\right) < 0$$
 and, therefore, $K(b, C, \delta) > K(b_0, C, \delta)$.
Because $b_0 = \frac{2}{\alpha_0 + 1} < 1$, it follows from (M2) that

$$b_0(T) = \frac{2\left[e^{\delta(T-3C/2)} - 1\right]}{e^{\delta T} + e^{\delta(T-3C/2)} - 2}$$

where we have expressed b_0 as a function of T. Then,

$$b_0'(T) = \frac{2\left[e^{\delta T} - e^{\delta(T - 3C/2)}\right]\delta}{\left[e^{\delta T} + e^{\delta(T - 3C/2)} - 2\right]^2} > 0$$

and

$$\lim_{T\to\infty}b_0(T)=\frac{2}{1+e^{\delta 3C/2}}\equiv b_\infty.$$

Furthermore,

$$\frac{dK(b_0(T), C, \delta)}{dT} = -\frac{1}{2}b'_0(T)\left(2 + \delta C e^{\frac{\delta C b_0(T)}{2}}\right) < 0,$$

and, thus,

$$K(b_0(T), C, \delta) > \lim_{T \to \infty} K(b_0(T), C, \delta) = K(b_\infty, C, \delta) = 2 - e^{\frac{\delta C}{1 + e^{\delta C}}} - \frac{2}{1 + e^{\delta C}}.$$

Without loss of generality, we once again replace $\delta C > 0$ in the preceding expression by r > 0, and we denote $\kappa(r) \equiv 2 - e^{\frac{r}{1+e^r}} - \frac{2}{1+e^r}$. We note that $\kappa(0) = 0$, and that

$$\kappa'(r) = \frac{2e^r - e^{\frac{r}{1+e^r}} + e^{\left(1 + \frac{1}{1+e^r}\right)r}(r-1)}{\left(1 + e^r\right)^2}$$

Let

$$I(r) \equiv 2e^r - e^{\frac{r}{1+e^r}} + e^{\left(1+\frac{1}{1+e^r}\right)r} \left(r-1\right).$$

For $r \in (0, 1)$, $I(r) > 2e^r - e^{\frac{r}{2}} + e^{\left(1 + \frac{1}{2}\right)r} \left(r - 1\right)$. We denote $M(r) \equiv 2e^r - e^{\frac{r}{2}} + e^{\frac{3}{2}r} \left(r - 1\right)$

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for $r \in [0, 1)$. We then get that

$$M(0) = 0$$

that

$$M'(r) = \frac{1}{2}e^{\frac{r}{2}} \left[4e^{\frac{r}{2}} - e^{r} \left(1 - 3r \right) - 1 \right],$$

M'(0) = 1,

that

and that

$$M''(r) = \frac{1}{4}e^{\frac{r}{2}} \left[8e^{\frac{r}{2}} + e^{r} (9r+3) - 1 \right] > 0$$

for $r \in (0, 1)$ and, thus, M'(r) > 0, and M(r) > 0 for $r \in (0, 1)$. For $r \geq 1$,

$$I(r) \ge 2e^r - e^{\frac{r}{2}} > 0,$$

and, therefore, I(r) > 0 for all r > 0. It then follows that $\kappa'(r) > 0$ for r > 0 which, combined with $\kappa(0) = 0$, implies that $\kappa(r) > 0$ for r > 0.

Thus, we conclude that $0 < K(b_{\infty}, C, \delta) < K(b, C, \delta) = J\left(\frac{2}{b} - 1, C, \delta\right)$, and that $EU(2, 1 + \alpha) - \lim_{s_{HD} \to 0} EU(2, s_{HD}) > 0$. Therefore, the maximum at the border $s_{HH} = 2$ is obtained for $s_{HD} = 1 + \alpha$.

Now, at the border $s_{HH} \rightarrow 0$, the limit function is

$$\lim_{s_{HH}\to 0} EU(s_{HH}, s_{HD}) = \frac{1}{\delta} \left[2 - (1+\alpha)e^{-\delta T} \right],$$

this limit is constant with respect to s_{HD} , and

$$U_{H}-\lim_{s_{HH}\to 0}EU(s_{HH},s_{HD})=\frac{e^{-\delta T}(\alpha-1)}{\delta}>0,$$

so that adding in (M39) yields $EU(2, 1 + \alpha) > \lim_{s_{HH} \to 0} EU(s_{HH}, s_{HD})$. Summing up the discussion pertaining to the maximization problem (M12), the global maximum is obtained at $(s_{HH}^*, s_{HD}^*) = (2, 1 + \alpha)$. Returning to the original problem (M11), we obviously get that $s_{H1}^* = 1$, $s_{H2}^* = 1$, $s_D^* = \alpha$, and because

$$\frac{C}{2s_{H1}^*} + \frac{C}{s_{H2}^* + s_D^*} = \frac{C}{2} + \frac{C}{1+\alpha} < \frac{C}{2} + \frac{C}{2} = C < T,$$

the last condition in (M11) is also satisfied. This finally concludes the proof of Lemma M3. Q.E.D.

We see that when they save at the optimal rates, the time of departure of the winner of the draw and the time of departure of the loser of the draw are, respectively,

$$T_{W} \equiv T_{W}(s_{H1}^{*}) = \frac{1}{2}C\frac{C}{2s_{H1}^{*}} = \frac{1}{2}C, \text{ and}$$

$$T_{L} \equiv T_{L}(s_{H1}^{*}, s_{H2}^{*}, s_{D}^{*}) = \frac{C}{2s_{H1}^{*}} + \frac{C}{s_{D}^{*} + s_{H2}^{*}} = \frac{1}{2}C + \frac{1}{1+\alpha}C = \frac{3+\alpha}{2(1+\alpha)}C. \text{ Thus, the}$$

maximum level of utility attainable under a complete-compliance agreement is

$$EU_{CC}(1,1,\alpha) = \frac{1}{2}U_W(1,1,\alpha) + \frac{1}{2}U_L(1,1,\alpha) = \int_0^{T_L} e^{-\delta t}u(0)dt + \int_{T_L}^T e^{-\delta t}u(\alpha)dt.$$

By comparison of the right-hand side of the preceding expression with (3), we see that

$$EU_{CC}(1,1,\alpha) = U_W^H = U_L, \tag{M41}$$

where U_W^H and U_L are, respectively, the lifetime utility of an honest winner ((2) in the main text of the paper), and the lifetime utility of a loser of the draw ((3) in the main text of the paper).

We next dispose of the assumption that joint saving is of the complete-compliance type, so as to incorporate the possibility that the winner of the draw will renege, neglecting to contribute funds to enable the loser of the draw to migrate. We show that under assumption (M2), the cheated individual will find it desirable to start saving alone from scratch at the rate s_{Ch} (subscript *Ch* stands for cheated).¹⁷ We denote the utility of the loser of the draw when the winner of the draw reneges by

$$U_{Ch}(s_{H1}, s_{Ch}) \equiv \int_{0}^{T_{W}(s_{H1})} e^{-\delta t} u(1 - s_{H1}) dt + \int_{T_{W}(s_{H1})}^{T_{Ch}(s_{H1}, s_{Ch})} e^{-\delta t} u(1 - s_{Ch}) dt + \int_{T_{Ch}(s_{H1}, s_{Ch})}^{T} e^{-\delta t} u(\alpha) dt,$$

where $T_{Ch}(s_{H1}, s_{Ch}) \equiv \frac{C}{2s_{H1}} + \frac{C}{s_{Ch}}$ is the point in time at which the cheated individual

can take the migration journey after saving alone at the rate s_{Ch} . In the following lemma we show, given that condition (M2) regarding α holds, the cheated individual indeed finds it rational to start saving from scratch, and that the saving rates that maximize $U_{Ch}(s_{H1}, s_{Ch})$ are $s_{H1}^* = s_{Ch}^* = 1$.

Lemma M4. If C < 2T/3 and the condition (M2) is met, then saving alone in order to migrate after being cheated is a desirable choice for an individual, that is, there exists $0 \le s_{H1}, s_{Ch} \le 1$ such that $U_{Ch}(s_{H1}, s_{Ch}) > U_H$. For such α , the function $U_{Ch}(s_{H1}, s_{Ch})$ obtains a maximum at $s_{H1}^* = s_{Ch}^* = 1$.

¹⁷In the main text of the paper we provide a rationale why striking a joint saving agreement with yet another individual is not an appealing option for a cheated individual.

Proof. The proof of this lemma is structured as follows. First, we will show that conditions C < 2T/3 and (M2) are sufficient for saving from scratch in order to migrate to be a desirable act for an individual after being cheated in a joint saving scheme. Next, we will define an optimization problem for the utility function $U_{Ch}(s_{H1}, s_{Ch})$, and we will exhibit Supportive Lemma M3, which serves to reveal a property of the derivative of $U_{Ch}(s_{H1}, s_{Ch})$ with respect to s_{H1} . This property will allow us to determine that the maximum value of $U_{Ch}(s_{H1}, s_{Ch})$ is obtained at $s_{H1}^* = s_{Ch}^* = 1$.

To show that if conditions C < 2T/3 and (M2) hold, then an individual who was cheated in a joint saving scheme will be inclined to save for migration from scratch, we let $s_{H1} = s_{Ch} = 1$. Then, the lifetime utility of a cheated individual is

$$U_{Ch}(1,1) = \frac{1}{\delta} \left[1 + \alpha e^{-3\delta C/2} - (1+\alpha) e^{-\delta T} \right].$$

The requirement that the migration of a cheated individual after saving $s_{H1} = s_{Ch} = 1$ per month is a gainful option is tantamount to the condition $U_{Ch}(1, 1) - U_H > 0$, where U_H defined in (M1) is the lifetime utility of an individual who spends his entire working life in the home country. We note that

$$U_{Ch}(1,1) - U_{H} = \frac{1}{\delta} \left[\alpha e^{-3\delta C/2} - (\alpha - 1)e^{-\delta T} - 1 \right],$$

which translates into the condition

$$\alpha > \frac{e^{\delta T} - 1}{e^{\delta (T - 3C/2)} - 1}$$

for $U_{Ch}(1, 1) - U_H > 0$, which is (M2). When condition (M2) holds, not migrating after a period of saving any amount s > 0 in an unsuccessful joint saving, which is surmised by the utility function

$$U_{Ch2}(s) = \int_{0}^{C/s} e^{-\delta t} u(1-s) dt + \int_{C/s}^{T} e^{-\delta t} u(1) dt,$$

is not preferred to saving the amount of 1 per month while in a futile joint saving, and saving alone from scratch the amount of 1 per month. To see this consideration clearly, we note that, obviously, $U_{Ch2}(s) < U_H$, and then, by incorporating (M2), we get that $U_{Ch2}(s) < U_H < U_{Ch}(1, 1)$.

We formulate the maximization problem of a cheated individual as

$$\max_{S_{H_1}, s_{Ch}} \left\{ U_{Ch}(s_{H_1}, s_{Ch}) \right\}
= \max_{S_{H_1}, s_{Ch}} \left\{ \frac{2 - s_{H_1}}{\delta} \left(1 - e^{-\frac{\delta C}{2s_{H_1}}} \right) + \frac{2 - s_{Ch}}{\delta} \left(e^{-\frac{\delta C}{2s_{H_1}}} - e^{-\frac{\delta C}{2s_{H_1}}} - \frac{\delta C}{s_{Ch}} \right)
+ \frac{1 + \alpha}{\delta} \left(e^{-\frac{\delta C}{2s_{H_1}}} - \frac{\delta C}{s_{Ch}} - e^{-\delta T} \right) \right\}$$
(M42)

s.t. $0 < s_{H_1}, s_{Ch} \le 1, \frac{C}{2s_{H_1}} + \frac{C}{s_{Ch}} \le T.$

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Replicating steps that are similar to the ones taken in the course of the proof of Lemma M3, assuming that α satisfies condition (M2), and ignoring temporarily the second constraint in (M42) which, as shown at the end of the proof of Lemma M4, is satisfied at the optimum anyway, we get that

$$\frac{\partial U_{Ch}}{\partial s_{Ch}}(s_{H1}, s_{Ch}) = \frac{e^{-\frac{\delta C}{2S_{H1}} - \frac{\delta C}{s_{Ch}}} \left[s_{Ch}^2 \left(1 - e^{\frac{\delta C}{S_{Ch}}}\right) + \delta C \left(\alpha + s_{Ch} - 1\right)\right]}{2\delta s_{Ch}^2}.$$
 (M43)

The expression inside the square brackets in (M43) is the same as that inside the square brackets in (M4) for $s_A = s_{Ch}$. Therefore, drawing on the proof of Lemma M2, we note that $\frac{\partial U_{Ch}}{\partial s_{Ch}}(s_{H1}, 1) > 0$, and $\frac{\partial U_{Ch}}{\partial s_{Ch}}(s_{H1}, s_{Ch})$ can cross zero only once for $s_{Ch} \in (0, 1)$. Consequently,

$$\forall_{s_{H1}\in(0,1)}\forall_{s_{Ch}\in(0,1)}U_{Ch}(s_{H1},s_{Ch}) < \max\left\{U_{Ch}(s_{H1},1),\lim_{s_{Ch}\to 0}U_{Ch}(s_{H1},s_{Ch})\right\}$$

Thus, just as in the case of the function $EU(\cdot)$ in the proof of Lemma M3, either the maximum is reached at the border $s_{Ch} = 1$, or it is not obtained if the function increases as $s_{Ch} \rightarrow 0$.

Next, we deal with the properties of $U_{Ch}(s_{H1}, 1)$. We get that

$$\frac{\partial U_{Ch}}{\partial s_{H1}}(s_{H1},1) = \frac{e^{-\frac{\delta C}{2s_{H1}} - \delta C} \left\{ \delta C \left[\alpha + e^{\delta C} \left(s_{H1} - 1 \right) \right] + 2e^{\delta C} \left(1 - e^{\frac{\delta C}{2s_{H1}}} \right) s_{H1}^2 \right\}}{2\delta s_{H1}^2}.$$
 (M44)

Denoting the expression inside the curly brackets in (M44), which determines the sign of $\frac{\partial U_{Ch}}{\partial s_{H1}}(s_{H1}, 1)$, by $L(s_{H1}, \alpha, \delta, C)$, we show that $L(\cdot)$ is a concave function of $s_{H1} \in (0, 1]$, and that $L(1, \alpha, \delta, C) > 0$. This we do by using the following supportive lemma.

Supportive Lemma M3. For the function

$$L(s_{H1},\alpha,\delta,C) \equiv \delta C \left[\alpha + e^{\delta C} \left(s_{H1} - 1 \right) \right] + 2e^{\delta C} \left(1 - e^{\frac{\delta C}{2S_{H1}}} \right) s_{H1}^2$$

it holds: (i) that the function is concave with respect to $s_{H1} \in (0, 1]$, and (ii) that $L(1, \alpha, \delta, C) > 0$.

Proof. We begin by proving part (i). From

$$\frac{\partial L}{\partial s_{H1}}(s_{H1},\alpha,\delta,C) = e^{\delta C} \left[\delta C + e^{\frac{\delta C}{2s_{H1}}} \left(\delta C - 4s_{H1} \right) + 4s_{H1} \right]$$

we get that

$$\frac{\partial^2 L}{\partial s_{H1}^2}(s_{H1},\alpha,\delta,C) = e^{\delta C} \left\{ 4 + e^{\frac{\delta C}{2s_{H1}}} \left[\frac{\delta C \left(4s_{H1} - \delta C \right)}{2s_{H1}^2} - 4 \right] \right\} \equiv l(s_{H1},\alpha,\delta,C).$$

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Because

$$\frac{\partial l}{\partial \delta}(s_{H1}, \alpha, \delta, C) = -\frac{\delta^2 C^3 e^{\frac{\delta C}{2s_{H1}}}}{2s_{H1}^3} < 0,$$

then $l(s_{H_1}, \alpha, \delta, C) < \lim_{\delta \to 0} l(s_{H_1}, \alpha, \delta, C) = 0$, which yields $\frac{\partial^2 L}{\partial s_{H_1}^2}(s_{H_1}, \alpha, \delta, C) < 0$.

Proceeding to prove part (ii), from the definition of the function $L(\cdot)$ we get that

$$L(1, \alpha, \delta, C) = 2e^{\delta C} - 2e^{\frac{3\delta C}{2}} + \alpha \delta C.$$

We note that

$$\frac{\partial L}{\partial \alpha}(1, \alpha, \delta, C) = \delta C > 0,$$

and, consequently, upon taking into account (M2), $L(1, \alpha, \delta, C) > L(1, \alpha_0, \delta, C)$. Treating α_0 as a function of *T*, we then get from (M2) that

$$\alpha_{0}'(T) = -\frac{e^{\delta T + 3\delta C/2} \left(e^{3\delta C/2} - 1\right)\delta}{\left(e^{\delta T} - e^{3\delta C/2}\right)^{2}} < 0$$

and that

$$\lim_{T\to\infty}\alpha_0(T)=e^{3\delta C/2}\equiv\alpha^\infty.$$

Then,

$$\frac{dL(1,\alpha_0(T),\delta,C)}{dT} = \alpha_0'(T)\frac{\partial L}{\partial \alpha}(1,\alpha,\delta,C) < 0,$$

and, thus,

$$L(1, \alpha_0, \delta, C) > \lim_{T \to \infty} L(1, \alpha_0(T), \delta, C) = L(1, \alpha^{\infty}, \delta, C)$$
$$= 2e^{\delta C} - 2e^{\frac{3\delta C}{2}} + e^{\frac{3\delta C}{2}} \delta C = e^{\delta C} \left[2 + e^{\frac{\delta C}{2}} \left(\delta C - 2 \right) \right].$$

Without loss of generality, we once again replace the product $\delta C > 0$ in the preceding expression by a variable r > 0, denoting the term in square brackets by $v(r) = 2 + e^{\frac{r}{2}}(r-2)$. We note that

$$\nu'(r) = \frac{1}{2}re^{\frac{r}{2}} > 0$$

so that $v(r) > \lim_{r \to 0} v(r) = 0$. Consequently, $0 < L(1, \alpha^{\infty}, \delta, C) < L(1, \alpha_0, \delta, C) < L(1, \alpha, \delta, C)$, which concludes the proof of Supportive Lemma M3. Q.E.D.

Returning to the proof of Lemma M4, from the properties of the function $L(\cdot)$ displayed in Supportive Lemma M3, we infer that either the maximum of $U_{Ch}(s_{H1}, 1)$ is obtained for $s_{H1} = 1$, or that it cannot be obtained if this function increases for $s_{H1} \rightarrow 0$ beyond the level $U_{Ch}(1, 1)$.

Recalling (M2), in $(s_{H1}, s_{Ch}) = (1, 1)$,

$$U_{Ch}(1,1) > U_H,$$
 (M45)

whereas for $s_{Ch} = 1$ and $s_{H1} \rightarrow 0$,

$$\lim_{s_{H1}\to 0} U_{Ch}(s_{H1}, 1) = \frac{1}{\delta} \left[2 - (1+\alpha)e^{-\delta T} \right].$$
(M46)

Comparing $\lim_{s_{H_1}\to 0} U_{Ch}(s_{H_1}, 1)$ with U_H (as defined in (M1)) yields

$$U_{H} - \lim_{s_{H_{1}} \to 0} U_{Ch}(s_{H_{1}}, 1) = \frac{1}{\delta} (\alpha - 1)e^{-\delta T} > 0,$$
(M47)

and, consequently, upon making use of (M45), we get that $U_{Ch}(1,1) > U_H > \lim_{s_{H_1} \to 0} U_{Ch}(s_{H_1},1)$ and, therefore, for $s_{Ch} = 1$ the maximum is obtained at $s_{H_1} = 1$.

We now investigate the case of $s_{Ch} \rightarrow 0$. We get that

$$\lim_{S_{Ch}\to 0} U_{Ch}(s_{H1}, s_{Ch}) = \frac{1}{\delta} \left[2 + s_{H1} \left(e^{-\frac{\delta C}{2S_{H1}}} - 1 \right) - (1+\alpha) e^{-\delta T} \right]$$

and then

$$\frac{\partial^2}{\partial s_{H1}^2} \left[\lim_{s_{Ch} \to 0} U_{Ch}(s_{H1}, s_{Ch}) \right] = \frac{\delta C^2 e^{-\frac{\delta C}{2S_{H1}}}}{4s_{H1}^3} > 0.$$

Consequently, the function $\lim_{s_{Ch}\to 0} U_{Ch}(s_{H1}, s_{Ch})$ is convex with respect to $s_{H1} \in (0, 1]$, which implies that either it obtains a maximum for $s_{H1} = 1$, or a maximum cannot be obtained if the function increases for $s_{H1} \to 0$ beyond the level $\lim_{s_{Ch}\to 0} U_{Ch}(1, s_{Ch})$. We note that

$$\lim_{s_{H_1}\to 0}\left[\lim_{s_{Ch}\to 0}U_{Ch}(s_{H_1},s_{Ch})\right]=\frac{1}{\delta}\left[2-(1+\alpha)e^{-\delta T}\right].$$

Comparing this last expression with (M46), and using (M47), we see that

$$\lim_{s_{H_1}\to 0} \left[\lim_{s_{C_h}\to 0} U_{C_h}(s_{H_1}, s_{C_h}) \right] = \lim_{s_{H_1}\to 0} U_{C_h}(s_{H_1}, 1)$$

and, therefore, $U_{Ch}(1, 1) > U_H > \lim_{s_{H1} \to 0} \left[\lim_{s_{Ch} \to 0} U_{Ch}(s_{H1}, s_{Ch}) \right]$. Also

$$\lim_{s_{ch}\to 0} U_{Ch}(1, s_{Ch}) = \frac{1}{\delta} \left[1 + e^{-\delta C/2} - (1+\alpha)e^{-\delta T} \right],$$

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and

$$U_{H} - \lim_{s_{Ch} \to 0} U_{Ch}(1, s_{Ch}) = \frac{1}{\delta} \left[\left(1 - e^{-\delta C/2} \right) + (\alpha - 1)e^{-\delta T} \right] > 0$$

because $(1 - e^{-\delta C/2}) > 0$ for $\delta C > 0$ and $\alpha > 1$. Therefore,

$$U_{Ch}(1, 1) > U_H > \lim_{s_{Ch} \to 0} U_{Ch}(1, s_{Ch})$$

Summing up the analysis pertaining to the maximization problem (M42), $U_{Ch}(1,1)$ is the global maximum over $(s_{H1}, s_{Ch}) = (0,1] \times (0,1]$, obtained for $(s_{H1}^*, s_{Ch}^*) = (1,1)$. Because

$$\frac{C}{2s_{H1}^*} + \frac{C}{s_{Ch}^*} = \frac{C}{2} + \frac{C}{1} = \frac{3C}{2}$$

and because we have assumed that C < 2T/3, then also the constraint $\frac{C}{2s_{H1}} + \frac{C}{s_{Ch}} \le T$ in (M42) is satisfied for $(s_{H1}^*, s_{Ch}^*) = (1, 1)$. This concludes the proof of Lemma M4. Q.E.D.

It follows then that after $T_W = C/2$ months of a futile joint saving, the cheated individual will spend an additional *C* months saving, this time doing so alone. We denote by $T_{Ch} \equiv T_{Ch}(s_{H1}^*, s_{Ch}^*) = C/(2s_{H1}^*) + C/s_{Ch}^* = 3C/2$ the point in time at which he will at last be able to migrate. Thus, the utility of the loser of the draw when the winner of the draw reneges, namely U_{Ch} , as introduced in (4) in the main text of the paper, is

$$U_{Ch} = U_{Ch}(1, 1) = \int_{0}^{T_{Ch}} e^{-\delta t} u(0) dt + \int_{T_{Ch}}^{T} e^{-\delta t} u(\alpha) dt.$$

Then, the expected utility of an honest would-be migrant j (an individual who is inclined to keep his part of the agreement if he were the winner of the draw) derived from striking a joint saving agreement with individual i is

$$EU^{H}(s_{H1}, s_{H2}, s_{D}, s_{Ch})$$

$$= \frac{1}{2}EU_{CC}(s_{H1}, s_{H2}, s_{D}) + \frac{1}{2}\left[pEU_{CC}(s_{H1}, s_{H2}, s_{D}) + (1-p)U_{Ch}(s_{H1}, s_{Ch})\right]$$
(M48)
$$= \frac{1+p_{i}^{j}}{2}EU_{CC}(s_{H1}, s_{H2}, s_{D}) + \frac{1-p_{i}^{j}}{2}U_{Ch}(s_{H1}, s_{Ch}).$$

Lemma M5. If T > 3C/2 and condition (M2) is met, then the saving rates $s_{H1}, s_{H2}, s_D, s_{Ch}$ that maximize $EU^H(s_{H1}, s_{H2}, s_D, s_{Ch})$ are $s_{H1}^* = s_{H2}^* = s_{Ch}^* = 1$, and $s_D^* = \alpha$ for any level of p.

Proof. We can write the maximization problem that pertain to the expected utility defined in (M48) as

$$\max_{s_{H1}, s_{H2}, s_{D}, s_{Ch}} \left\{ EU^{H}(s_{H1}, s_{H2}, s_{D}, s_{Ch}) \right\}$$

=
$$\max_{s_{H1}, s_{H2}, s_{D}, s_{Ch}} \left\{ \frac{1 + p_{i}^{j}}{2} EU_{CC}(s_{H1}, s_{H2}, s_{D}) + \frac{1 - p_{i}^{j}}{2} U_{Ch}(s_{H1}, s_{Ch}) \right\}$$

First, we note that because $(1 + p_i^j) \ge 0$ and $(1 - p_i^j) \ge 0$, there is no "trade-off" between the maximized functions $EU_{CC}(\cdot)$ and $U_{Ch}(\cdot)$ (both functions enter the objective function with non-negative coefficients). Second, the maximization of $EU_{CC}(\cdot)$ and the maximization of $U_{Ch}(\cdot)$ each yields the same result with respect to the common variable s_{H_1} ; that is, in both cases $s_{H_1}^* = 1$.

Summing up: the values of s_{H1} , s_{H2} , s_D , s_{Ch} that maximize $EU^H(\cdot)$ are the same as those that maximize $EU_{CC}(\cdot)$ (s_{H1}^* , s_{H2}^* , s_D^* from Lemma M3), and $U_{Ch}(\cdot)$ (s_{H1}^* , s_{Ch}^* from Lemma M4) separately. Q.E.D.

Thus, the optimal level of the expected utility of a honest migrant, namely EU^{H} , as introduced in (5) in the main text of the paper, is

$$\begin{split} EU^{H} &= EU^{H}(s_{H1}^{*}, s_{H2}^{*}, s_{D}^{*}, s_{Ch}^{*}) = \frac{1 + p_{i}^{j}}{2} EU_{CC}(1, 1, \alpha) + \frac{1 - p_{i}^{j}}{2} U_{Ch}(1, 1) \\ &= \frac{1 + p_{i}^{j}}{2} EU_{CC} + \frac{1 - p_{i}^{j}}{2} U_{Ch} \\ &= \frac{1 + p_{i}^{j}}{2} U_{W}^{H} + \frac{1 - p_{i}^{j}}{2} U_{Ch}, \end{split}$$

where the last equality above follows from (M41).

The expected utility of a dishonest would-be migrant j from striking a joint saving agreement with individual i is

$$EU^{D}(s_{H1}, s_{H2}, s_{D}, s_{Ch})$$

= $\frac{1}{2}U^{D}_{W}(s_{H1}) + \frac{1}{2}[p^{j}_{i}EU_{CC}(s_{H1}, s_{H2}, s_{D}) + (1 - p^{j}_{i})U_{Ch}(s_{H1}, s_{Ch})],$

where superscript D stands for dishonest, and

$$U_W^D(s_{H1}) = \int_0^{T_D(s_{H1})} e^{-\delta t} u(1 - s_{H1}) dt + \int_{T_D(s_{H1})}^T e^{-\delta t} u(\alpha) dt$$

is the utility of a dishonest individual who ends up as the winner of the draw, and who uses the savings of his co-saver to embark on migration after $T_D(s_{H1}) \equiv C/(2s_{H1})$ months. By proceeding in a manner that is similar to the ones undertaken in proving Lemmas M4 and M5, it can be shown that for the dishonest individual, the saving rates that maximize $EU^D(s_{H1}, s_{H2}, s_D, s_{Ch})$ are $s_{H1}^* = s_{H2}^* = s_{Ch}^* = 1$, and $s_D^* = \alpha$. Then, $T_D \equiv T_D(s_{H1}^*) = C/(2s_{H1}^*) = C/2 = T_W$,

$$U_{W}^{D} = U_{W}^{D}(s_{H1}^{*}) = \int_{0}^{T_{W}} e^{-\delta t} u(0) dt + \int_{T_{W}}^{T} e^{-\delta t} u(\alpha) dt,$$

and the expected utility of a dishonest would-be migrant, namely EU^{D} , as introduced in (6) in the main text of the paper, is

$$\begin{split} EU^{D} &= EU^{D}(s_{H1}^{*}, s_{H2}^{*}, s_{D}^{*}, s_{Ch}^{*}) = \frac{1}{2}U_{W}^{D} + \frac{1}{2} \left[p_{i}^{j}EU_{CC} + \left(1 - p_{i}^{j} \right) U_{Ch} \right] \\ &= \frac{1}{2}U_{W}^{D} + \frac{1}{2} \left[p_{i}^{j}U_{L} + \left(1 - p_{i}^{j} \right) U_{Ch} \right], \end{split}$$

where the last equality above follows from (M41).