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A Note on Barriers to Capital Accumulation and Income Levels

John S. Landon-Lane* and Peter E. Robertson

October, 2005

Abstract

In this paper we clarify the impact that barriers to capital accumulation can have on a two-sector neoclassical growth model’s ability to explain the observed differences in incomes across countries. We show that the effect of barriers to technology adoption in a two sector model is necessarily identical to a one-sector model when there are no factor market imperfections and each sector has identical technologies. We also show that this result generalizes to the case when the technologies are different across the sectors.

J.E.L Classifications: F0, O0, O4

Keywords: Economic Growth, Economic Development, Barriers, Capital Accumulation.

Running Head: Barriers to Accumulation.

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1 Introduction

What accounts for the large differences in GDP per capita across countries? Several empirical studies have suggested that that barriers to capital accumulation may be responsible for much of the observed differences.\(^1\) Nevertheless Prescott (1998) shows that in a standard one sector growth model, barriers to capital accumulation on income levels cannot account for the disparities in income levels.\(^2\)

In a recent contribution to this debate, Restuccia (2004) claims that the effects of barriers to capital accumulation are substantially amplified in a two-sector economy with technology choice. The purpose of this article is to show that this result is misleading. Specifically we show that the effects of barriers to accumulation in the model considered by Restuccia (2004) are necessarily the same as the effects of barriers in a similarly calibrated standard one sector model. We also briefly discuss how two-sector models might amplify the effects of barriers to capital accumulation in the presence of imperfect factor markets.

2 The Model

Following Restuccia (2004) we consider an economy with a modern sector and a traditional sector. The outputs of each sector are perfect substitutes and hence there exists a choice of production techniques for aggregate output. Denoting output per worker in the modern sector as \(Y_m\) and output per worker in the traditional sector as \(Y_a\), the production functions for each sector are

\[
\begin{align*}
Y_{m,t} &= K_{m,t}^\alpha (\tilde{B}_t N_{m,t})^{1-\alpha} \\
Y_{a,t} &= K_{a,t}^\psi \lambda (\tilde{A}_t N_{a,t})^{1-\lambda} L^{(1-\psi) \lambda},
\end{align*}
\]

where \(L\) is a fixed factor, \(K_{m,t}\) and \(K_{a,t}\) are the capital stocks in each sector, \(N_t\) is the total number of workers, and \(N_{m,t}\) and \(N_{a,t}\) are the stocks of labor employed in each sector. Capital’s income share in the modern sector is \(\alpha\). In the traditional sector capital services and a fixed factor have income shares equal to \(\psi \lambda\), and \((1-\psi)\lambda\) respectively.

Labor grows exogenously at a rate \(N_{t+1}/N_t = \phi > 1\), modern sector labor productivity, \(\tilde{B}_t\), grows exogenously at rate \(\tilde{B}_{t+1}/\tilde{B}_t = \gamma\), where \(\gamma > 1\), and traditional labor productivity, \(\tilde{A}_t\), grows at a rate \(\tilde{A}_{t+1}/\tilde{A}_t = \eta\), where \(\eta = \gamma^{(1-\psi \alpha)/(1-\alpha)} \phi^{\alpha (1-\psi)}/(1-\alpha)\). It

\(^1\)See for example, Easterly (1993), Jones (1994), and Restuccia and Urrutia (2001).

\(^2\)An important caveat is given by Parente, Rogerson and Wright (2000), who consider a model with unmeasured home production.
is also convenient to define $A_t \equiv \tilde{A}_t^{1-\lambda} L^{(1-\psi) \lambda}$ and $B_t \equiv \tilde{B}_t^{1-\alpha}$.

Nominal GDP per worker is $Y_t = Y_{m,t} + Y_{a,t}$, and aggregate income must equal aggregate expenditure, so that $Y_t = C_t + X_t$ where $X_t$ is nominal investment spending measured in terms of the price of consumption, and $C_t$ is consumption spending.

The aggregate capital stock is $K_t = K_{m,t} + K_{a,t}$, and capital accumulation is given by

\begin{equation}
K_{t+1} - K_t = X_t/\pi - \delta K_t
\end{equation}

where $\pi$ is a measure of barriers to accumulation.\(^3\)

The agents in the economy are households and firms. Firms choose factor demands to maximize profits, taking factor prices for labor and capital, $w_t$, $r_t$, and land rents, $\theta_t$, as given. Households own factors of production and finance their consumption by renting factor services to firms. An infinitely lived representative household chooses a sequence of consumption values, $C_t$ to maximize the utility function,

\begin{equation}
U_t = \sum_{t=0}^{\infty} \beta^t N_t u(C_t)
\end{equation}

where $u(C) = c^{1-(1/\sigma)}/(1 - (1/\sigma))$, $\sigma \neq 1$, and $u(C) = \ln(c)$, $\sigma = 1$.

### 2.1 Equilibrium

Given initial conditions $K_0$, $N_0$, and the level of barriers $\pi$, a competitive equilibrium consists of sequences of factor allocations $\{K_{m,t}, K_{a,t}, N_{m,t}, N_{a,t}\}_{t=0}^{\infty}$, factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, land rents, $\{\theta_t\}_{t=0}^{\infty}$, and consumption decisions $\{C_t\}_{t=0}^{\infty}$, such that: (i) household’s maximize utility given factor allocations and factor rental rates; (ii) firms maximize profits given factor rental rates; (iii) firms earn zero profits, and; (iv) for each period, $t$, markets clear such that $Y_t = C_t + X_t$, $K_t = K_{m,t} + K_{a,t}$ and $N = N_{m,t} + N_{a,t}$. A balanced growth equilibrium is a competitive equilibrium such that $K_t$, $K_{m,t}$, $K_{a,t}$, $Y_{a,t}$, $Y_{m,t}$, $Y_t$ and $C_t$ are all growing at the rate $\phi \gamma$, and $n_a = N_{a,t}/N_t$ and $n_m = N_{m,t}/N_t$ are constant.

Profit maximization by firms implies that the market clearing conditions for capital and labor are\(^4\)

\begin{equation}
\alpha Y_{m,t}/K_{m,t} = (\psi \lambda) Y_{a,t}/K_{a,t} = r_t + \delta
\end{equation}

\(^3\)This model of barriers to accumulation was introduced by Parente and Prescott (1994).

\(^4\)For brevity we have suppressed the first order condition for Land.
and

\[(5) \quad (1 - \alpha) Y_{m,t}/N_{m,t} = (1 - \lambda) Y_{a,t}/N_{a,t} = w_t.\]

Utility maximization gives rise to an Euler equation that describes the path of consumption over time. On the balanced growth path the marginal product of capital is constant and equal to \(r^* + \delta\), and the Euler equation can be written as

\[(6) \quad (r^* + \delta)/\pi = \gamma/\beta^\sigma - (1 - \delta)\]

Hence changes in the level of barriers to accumulation will result in a new steady state where \(r^* + \delta\) has increased or decreased in strict proportion to \(\pi\).

### 2.2 The Effect of Barriers on Income

The preceding model is equivalent to the model studied by Restuccia (2004) if we impose the restriction that \(\lambda = \alpha\), and \(\sigma = 1\). In this case the effects of barriers on GDP per capita, relative to the standard one sector model, are particularly transparent. To see this first note that the labor market clearing condition, (5) implies that GDP per worker and output per worker in both sectors are equal. Dropping time subscripts we have

\[(7) \quad Y/N = Y_m/N_m = Y_a/N_a.\]

Consequently GDP per worker, \(y \equiv Y/N\), can be expressed as a function of the modern sector factor employment ratio alone,

\[(8) \quad y = B (K_m/N_m)^\alpha.\]

However, since the return to capital is constant in a steady state, we have \(K_m/N_m = ((r^* + \delta)/(B \alpha))^{1/\alpha - 1}\). Hence steady state GDP per worker is equal to

\[(9) \quad y = B^{1/(1-\alpha)} ((r^* + \delta)/\alpha)^\alpha/(\alpha-1).\]

This expression, however, is identical to the equivalent expression in the standard aggregate model. Now consider the ratio of of GDP per worker, \(y'/y\) where \(y'\) and \(y\) are the respective values of GDP per worker in two economies with barriers \(\pi'\) and \(\pi\). It follows
from (6) and (9) that

$$y'/y = (\pi'/\pi)^{-\alpha/(1-\alpha)}.$$  \hspace{1cm} (10)

The preceding discussion is summarized in the following result.

**Result 1** *In the steady state equilibrium of the two-sector model described by (1) to (3):*

(i) *the ratio $(r^* + \delta)/\pi$ is constant, for given parameters, $\gamma$, $\beta$, and $\sigma$, and $\delta$.*

(ii) *there is a constant exponential mapping between the level of barriers to capital accumulation and the steady state level of nominal GDP per capita, which is identical to the standard one sector Ramsey model.*

Thus the relationship between barriers to accumulation and differences in GDP per worker in this two-sector model, is the same as the standard Ramsey model. Following Prescott (1998), we conclude that this two-sector model does not provide a useful a theory of international income differences.

### 2.3 Numerical results

The model considered by Restuccia (2004) is a special case with the restriction $\lambda = \alpha$. In this section we use numerical methods to show that our preceding results also apply to the more general two-sector model when $\lambda \neq \alpha$. For ease of comparison we use the parameters chosen by Restuccia (2004), which are reported in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ramsey</th>
<th>Restuccia</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.020</td>
<td>1.020</td>
<td>growth of aggregate productivity of 1.02</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.019</td>
<td>1.019</td>
<td>growth rate of labor of 1.019</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.350</td>
<td>0.350</td>
<td>capital income share in modern sector of 0.35</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>–</td>
<td>0.350</td>
<td>non-labor share in traditional sector of 0.35</td>
</tr>
<tr>
<td>$\psi$</td>
<td>–</td>
<td>0.714</td>
<td>income elasticity of land of 0.1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.041</td>
<td>0.041</td>
<td>investment to output ratio of 0.2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.928</td>
<td>0.928</td>
<td>capital to output ratio of 2.5</td>
</tr>
<tr>
<td>$B$</td>
<td>0.726</td>
<td>0.724</td>
<td>normalization of aggregate output to 1</td>
</tr>
<tr>
<td>$A$</td>
<td>–</td>
<td>0.584</td>
<td>employment in agriculture share 0.02</td>
</tr>
</tbody>
</table>

Table 2 reports the ratio nominal GDP (NGDP) in the counterfactual ($\pi > 1$) to the benchmark ($\pi = 1$), for different values of $\pi$. As expected, these values correspond
with the values predicted by (10) for the case where $\lambda = \alpha$. We also report the value of real GDP (RGDP) and it can be seen that both models produce very similar values.\footnote{Real GDP is defined as $Z_t = C_t + X_t / \pi$. It can be seen that in these terms, barriers have a slightly smaller effect on income levels in the two sector model compared to the Ramsey model.}

Table 2: Cross-Country Income Differences: One-sector vs Two-Sector

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>NGDP</th>
<th>RGDP</th>
<th>$K/Y$</th>
<th>$r^* + \delta$</th>
<th>$n_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>2.500</td>
<td>0.140</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.689</td>
<td>0.620</td>
<td>1.250</td>
<td>0.280</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>0.474</td>
<td>0.403</td>
<td>0.625</td>
<td>0.560</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.381</td>
<td>0.318</td>
<td>0.417</td>
<td>0.840</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.326</td>
<td>0.269</td>
<td>0.313</td>
<td>1.120</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>0.289</td>
<td>0.237</td>
<td>0.250</td>
<td>1.400</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>0.262</td>
<td>0.214</td>
<td>0.208</td>
<td>1.680</td>
<td>-</td>
</tr>
</tbody>
</table>

Restuccia Two-Sector Model ($\lambda = \alpha$)

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>NGDP</th>
<th>RGDP</th>
<th>$K/Y$</th>
<th>$r^* + \delta$</th>
<th>$n_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>2.500</td>
<td>0.139</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>0.689</td>
<td>0.620</td>
<td>1.236</td>
<td>0.278</td>
<td>0.058</td>
</tr>
<tr>
<td>4</td>
<td>0.474</td>
<td>0.406</td>
<td>0.598</td>
<td>0.557</td>
<td>0.169</td>
</tr>
<tr>
<td>6</td>
<td>0.381</td>
<td>0.323</td>
<td>0.381</td>
<td>0.835</td>
<td>0.315</td>
</tr>
<tr>
<td>8</td>
<td>0.326</td>
<td>0.277</td>
<td>0.270</td>
<td>1.114</td>
<td>0.490</td>
</tr>
<tr>
<td>10</td>
<td>0.289</td>
<td>0.247</td>
<td>0.202</td>
<td>1.392</td>
<td>0.691</td>
</tr>
<tr>
<td>12</td>
<td>0.262</td>
<td>0.227</td>
<td>0.155</td>
<td>1.670</td>
<td>0.915</td>
</tr>
</tbody>
</table>

Next we consider the effects of barriers in more general version of the model where $\lambda \neq \alpha$. Table 3 shows the effects of barriers for the case $\lambda = 0.30$ and $\lambda = 0.25$. It can be seen that there is now a much larger labor reallocation effect in response to changes in $\pi$. With $\lambda = 0.25$, a barrier of $\pi = 4$ is sufficient to shift all labor to the traditional sector. Nevertheless, the effects of barriers on income levels are still of negligible difference from the standard model. The numerical results show that our critique of Restuccia (2004) does not depend on his assumption that $\lambda = \alpha$. Even when we relax this constraint, the effects of the barriers only differ slightly from the standard Ramsey model. Thus the two-sector extension of the Ramsey model described above does not provide a compelling explanation for differences in international income levels, even when we allow for $\lambda$ to be different from $\alpha$. The results show, moreover, that barriers to accumulation induce very large changes.
Table 3: Cross-Country Income Differences: $\lambda < \alpha$

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>NGDP</th>
<th>RGDP</th>
<th>K/Y</th>
<th>$r^* + \delta$</th>
<th>$n_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>2.500</td>
<td>0.139</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>0.675</td>
<td>0.612</td>
<td>1.155</td>
<td>0.277</td>
<td>0.169</td>
</tr>
<tr>
<td>4</td>
<td>0.429</td>
<td>0.402</td>
<td>0.271</td>
<td>0.554</td>
<td>1.000</td>
</tr>
<tr>
<td>8</td>
<td>0.380</td>
<td>0.351</td>
<td>0.135</td>
<td>1.109</td>
<td>1.000</td>
</tr>
</tbody>
</table>

$\lambda = 0.30$

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>NGDP</th>
<th>RGDP</th>
<th>K/Y</th>
<th>$r^* + \delta$</th>
<th>$n_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>2.500</td>
<td>0.139</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>0.685</td>
<td>0.618</td>
<td>1.210</td>
<td>0.278</td>
<td>0.099</td>
</tr>
<tr>
<td>4</td>
<td>0.458</td>
<td>0.403</td>
<td>0.503</td>
<td>0.556</td>
<td>0.490</td>
</tr>
<tr>
<td>8</td>
<td>0.339</td>
<td>0.305</td>
<td>0.180</td>
<td>1.111</td>
<td>1.000</td>
</tr>
</tbody>
</table>

in the allocation of labor and capital between sectors. The fact that these large factor movements do not change output levels relative to the standard Ramsey model is a consequence of efficient factor markets. If follows, that two-sector models may have amplification effects, if factor markets are distorted. For example Landon-Lane and Robertson (2005) show that incorporating barriers to labor mobility into two sector models can substantially amplify the effects of barriers to accumulation.\(^6\)

3 Conclusion

Substantial debate exists as to whether differences in factor accumulation or productivity growth are the principal cause of differences in income levels across countries. Our aim has been to clear the record on the effects of barriers to accumulation in a particular class of growth models. Specifically the effect of these barriers on income levels is not amplified by introducing technology choice, as proposed by Restuccia (2004). A promising avenue for further research, however, is to consider the effects of barriers to accumulation in two-sector models with imperfect factor markets.

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\(^6\)Also see Robertson (1999) and Chanda and Dalgaard (2003) who both find that distortions in labor markets can provide important interactions between accumulation and aggregate productivity, in otherwise relatively standard disaggregated growth models. As noted, barriers can have a large effect on measured incomes in models with home production, Parente et al. (2000).
References


