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Existence and Computation of Pure-Strategy
Equilibria in Models of Legislative Bargaining
with Reconsideration¹

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Abstract

Diermeier and Fong (2008a) recently proposed a legislative bargaining model with reconsideration in the context of a distributive policy environment. In this paper we prove general existence and necessary conditions for pure-strategy stationary equilibria for *any* finite policy space and preference profile. We also provide an algorithm to compute these equilibria. In any such equilibrium proposal power is endogenously limited compared to the case of no reconsideration. The general model is then applied to various policy environments, including a model with spatial preferences. In the context of a public goods environment we can show that lack of commitment increases policy efficiency.

JEL Classification Numbers: C72, D72, D78.

Keywords: Legislative bargaining, reconsideration, evolving default, proposal power, lack of commitment, distributive politics, public goods, spatial model.

1 Introduction

Since the seminal paper by Baron and Ferejohn (1989), legislative institutions have been modeled as a sequence of proposal making and voting by members of a legislature. The legislative bargaining game has been extensively applied in public finance (Battaglini and Coate, 2007, 2008), comparative constitutions (Persson, Roland and Tabellini, 2000, 2007), and government formation (Diermeier and Merlo, 2000; Baron and Diermeier, 2001), to name just a few. In all variants of these models, it is commonly assumed that a policy is adopted once a proposal is accepted by some majoritarian voting rule.

A different approach was recently proposed by Bernheim, Rangel and Rayo (2006) and Diermeier and Fong (2008a) to model legislative bargaining. The idea is that while the legislature is in session it can choose to modify any approved bill at will. In other words, passage of a bill does not prevent the legislature from reconsidering the same policy issue, and a passed bill simply serves as a default policy during the process of possible reconsideration.

Bernheim, Rangel and Rayo (2006) study such a legislative process under the assumption of a fixed, commonly known number of proposal rounds and predetermined sequence of proposers. They show that if the proposal protocol is sufficiently inclusive, the last proposer has nearly dictatorial power. This suggests that allowing for reconsideration would only increase the proposal power to a maximal extent.

In contrast, Diermeier and Fong (2008a) develops an analytical framework in which policy-making proceeds until nobody is able to change the previously approved policy, so the final round of legislative bargaining is endogenously determined rather than exogenously given. In a distributive model with a single, persistent proposer, Diermeier and Fong show that a new phenomenon emerges. That is, voters without

proposal power have an incentive to protect each other from exploitation by the proposer. Intuitively, voter J will not approve of a proposal that gives another voter K a low allocation because any proposer would then have an incentive to reconsider the bill, choose K as a cheaper coalition partner, and exploit J . By voting to protect K , voter J effectively safeguards his bargaining position in case the policy will be reconsidered. In equilibrium, this creates endogenous limits on proposal power.

In this paper, we generalize the distributive model of Diermeier and Fong (2008a) to a model with an arbitrary finite policy space and preference profile. Following Diermeier and Fong, we focus on the case with a single, persistent proposer throughout the legislative session. That is, the same legislator is granted the power to make policy proposals and to initiate reconsideration of an approved policy. This setup is different from most legislative bargaining models, e.g. Baron and Ferejohn (1989) and Battaglini and Coate (2007, 2008), which typically assume that all legislators are randomly selected to make proposals in turn before a policy is made. Focusing on a single proposer allows us to isolate our main result, how mutual protection of voters leads to limits on proposer power. We do note, however, that a single-proposer model may be a better representation of certain legislative institutions. For example, comparative scholars have long observed that constitutional features of a parliamentary system lead to high levels of agenda control by the executive, i.e. the cabinet. Moreover, in many cases that power is concentrated within the prime minister (Döring 1995). In the U.S. Congress, committees may effectively control proposal power over certain policy issues and be able to block amendments (Knight, 2005). Unless the composition of a committee is changed, proposal power is persistently concentrated in the same group of legislators.¹ Finally, in a general

¹Diermeier and Fong (2008a) also consider the case of a committee as a (collective) persistent proposer.

context the politicians who can successfully submit policy proposals are typically political leaders, who control power persistently unless contested (Myerson, 2008).

The core of our analysis is an algorithm we propose to construct a set of policy alternatives (henceforth a stable set) which persist as default in equilibrium. Since the policy converges in the long run, any policy alternative not in a stable set cannot appear as a final policy outcome. With this algorithm we prove the existence and necessary conditions of a pure-strategy stationary Markov perfect equilibrium.

In general, equilibrium existence, especially of pure-strategy equilibria, is not assured in dynamic legislative bargaining models.² Indeed, a pure-strategy stationary equilibrium rarely exists if the default policy evolves endogenously, unless one makes assumptions about random shocks on preferences and the transition of the default (Duggan and Kalandrakis, 2007).³ The existence of pure-strategy equilibria makes our model tractable for applications in dynamic policy environments or specific contexts, e.g. in public finance, and the proposed algorithm provides a "toolkit" for solving such models (e.g. Diermeier and Fong, 2008b, 2008c).

We show that, in any such pure-strategy equilibrium, the proposer faces endogenous limits to his proposal power compared to the case where reconsideration is not allowed. So, he is unambiguously worse off with the possibility of reconsideration. Paradoxically, more power *de jure* reduces *de facto* power, as measured by favorable outcomes. The findings of Diermeier and Fong (2008a) thus generalize to any policy environment that can be modeled as a finite choice space. Later in this paper we illustrate three possible policy environments.

²See Banks and Duggan (2000, 2006) for positive analysis of a general legislative bargaining game with a fixed, exogenous status quo, and Duggan and Kalandrakis (2007) for equilibrium existence and positive properties of a general dynamic legislative bargaining game in which the status quo in one period depends on the policy chosen in the previous period.

³See Kalandrakis (2004; 2007), Fong (2006), Battaglini and Palfrey (2007), and Bowen and Zahran (2007) for examples of mixed-strategy stationary equilibria for various legislative bargaining games in which the default policy endogenously evolves.

The possibility of reconsideration can be interpreted as lack of commitment by the proposer. Whereas it has been commonly understood that lack of commitment by policymakers could be a source of policy inefficiency, the model considered here may yield the opposite conclusion.⁴ As the proposer has an incentive to fully exploit the legislators with low bargaining positions in the future, a majority of legislators implicitly coordinates to vote against any proposal that expropriates each other too much. Therefore, lack of commitment by the proposer induces the legislature as a whole to commit to choosing the policy from an effectively smaller set of policy alternatives. In some policy environments, some less efficient policies, e.g. more unequal allocations, are excluded from possible policy outcomes, so the possibility of reconsideration (i.e. lack of commitment) leads to more equal distributions and enhances policy efficiency.

In the Appendix we also present a variant of the model with a continuous-time framework where a policy, once enacted, is in effect until it is reformed through the political process. With a single, permanent proposer that model shares the same mathematical structure as the one presented here if legislative actions can be taken with sufficient frequency. The alternative model provides an additional interpretation of our analytical framework.⁵

The remainder of this paper is organized as follows. Section 2 presents the model with a general, finite policy space. Section 3 defines a stationary Markov perfect

⁴The commitment problem was first formally addressed by Kydland and Prescott (1977). More recent political economy studies of government policies include Persson and Svensson (1989), Tabellini and Alesina (1990), Besley and Coate (1998), Acemoglu and Robinson (2001), and Baron, Diermeier and Fong (2008), to name only a few. See Acemoglu (2003) for a comprehensive survey of the commitment literature in political economy.

⁵Our paper thus is also linked to the growing literature on dynamic legislative bargaining in which the status quo in one period is the policy that has been enacted. This literature was initiated by Baron (1996) and further developed by Baron and Herron (2003), Fong (2006), Baron, Diermeier and Fong (2008) in spatial setups, by Kalandrakis (2004, 2007), Battaglini and Palfrey (2007), Bowen and Zahran (2007) in distributive setups, and by Duggan and Kalandrakis (2007) in a general model.

equilibrium. Section 4 proposes an algorithm to construct an equilibrium stable set, and proves the existence and necessary conditions of an equilibrium. Section 5 discusses some general implications of the model. Section 6 illustrates these implications through three specific models. Finally, the Appendix provides an alternative interpretation of the model in continuous time.

2 The Model

Let $N = \{1, 2, \dots, n\}$ be a set of n players in the legislature, where $n = 2m + 1$ and $m \in \mathbb{N}$. The legislature must collectively choose a policy x from a *finite* policy space denoted by X .⁶ Preferences of any player ℓ are represented by a von Neumann-Morgenstern utility function $u_\ell(x)$. We refer to $u = (u_1, u_2, \dots, u_n)$ as a preference profile.

There is one *proposer* in the legislature. Assume this position is occupied by player 1. The proposer is conferred the sole power to make policy proposals from the policy space during the legislative session. All other players, $i \in N \setminus \{1\}$, are referred to as *voters*.

The legislature selects a policy over the course of potentially infinite rounds of proposal making and voting, where the number of rounds depends on exogenous factors and the decision made by the proposer.

As the legislative session commences, an initial default $x^0 \in X$ is exogenously given. The initial default is interpreted as the policy that has been enacted prior to the legislative session. Since then, activities prior to round t establish a default $x^{t-1} \in X$. In round t , the proposer either chooses to make a proposal $y^t \in X$ or pass

⁶A discrete policy space limits the extent to which utilities are transferable among the players, and is critical to our main results. From the perspective of modeling real-life policy issues, however, this assumption seems innocuous. For example, entitlement programs usually involve a minimal spending unit, even if it is very small, say a dollar.

the proposal round. A "pass" means inaction by the proposer and, for mathematical convenience, is modeled as a proposal $y^t = x^{t-1}$. The proposal y^t is then put to an immediate vote against x^{t-1} . If it is approved by majority rule, it replaces x^{t-1} as the default policy and $x^t = y^t$. If it is not approved, the default policy remains the same and $x^t = x^{t-1}$. The policy that survives as default till the end of the legislative session is implemented.

In contrast to Bernheim et al. (2006) who assume a fixed number of proposal rounds, in the model considered here the last proposal round is not predetermined. We say the legislative session *ends endogenously* after proposal round t , if the default x^t established by the first t rounds of proposal making and voting is such that the proposer will choose to pass any possible proposal round $t' > t$. In addition, after any proposal round the legislative session may be terminated exogenously with probability $1 - \delta$, where $\delta \in [0, 1)$ is the probability that the proposer will have an opportunity to reconsider the policy that emerges from the current round.

We interpret δ as a parameter of the legislative institution, since various legislative rules, unmodeled here, may affect the likelihood of chances for reconsideration. For example, the case of $\delta = 0$ is associated with the agenda-setting model of Romer and Rosenthal (1978). In this paper we intend to focus on institutions where legislative actions is very likely to continue until the session ends endogenously, i.e. we study the case in which $\delta < 1$ with δ sufficiently close to 1.⁷ This assumption is maintained throughout this paper, although not repeated unless it is necessary.

⁷The case of $\delta = 1$ admits a plethora of equilibria, as the bargaining position of a voter is solely determined by what she believes to happen eventually.

3 Equilibrium Definition

As is customary in the legislative bargaining literature we focus the analysis on stationary Markov perfect equilibria, in which the players condition their strategies only on the prevailing default policy. We thus drop the superscript t for the proposal round from the notations.

Let $f : X \rightarrow X$ be the (pure) proposal strategy of the proposer. In particular, $f(x)$ denotes the policy that the proposer proposes when the prevailing default is x .⁸

Let $U_\ell(x)$ be the expected utility of player ℓ if policy x is approved. With probability $1 - \delta$ the legislative session is exogenously terminated after the current proposal round and this player receives a utility of $u_\ell(x)$. With probability δ the proposer has a chance to reconsider the approved policy and make a new proposal according to f . In this case, player ℓ receives a continuation utility of $U_\ell(f(x))$. Thus,

$$U_\ell(x) = (1 - \delta) u_\ell(x) + \delta U_\ell(f(x)). \quad (1)$$

We refer to $U_\ell : X \rightarrow \mathbb{R}$ as the value function of player ℓ .

We make two technical assumptions regarding how the players break indifference. First, any player votes against a policy proposal if and only if passage of the proposal makes him strictly worse off. Second, the proposer never proposes any shift in policy that is destined to be vetoed by a majority of voters. None of our qualitative results depend on the first assumption. The second one simplifies the notation, but is otherwise innocuous, since making a losing proposal is equivalent to remaining at the prevailing default.

⁸Diermeier and Fong (2008a) also construct a mixed-strategy stationary equilibria for the game with a distributive policy. In those equilibria the sole proposer strategically designs a sequence of proposals with randomization to achieve his ideal point eventually. Diermeier and Fong argue that if the legislature needs to make a collective decision on whether to discuss a policy, i.e. to put it on the agenda, these mixed-strategy equilibria disappear and only the pure-strategy equilibria survive.

In each proposal round, the proposer selects a policy proposal to maximize his expected utility. A policy is politically feasible if, as a proposal it would be approved by a majority of players. By the second assumption above, the maximization problem of the proposer is subject to the constraint of political feasibility. By the first assumption above, the feasibility constraint is equivalent to an incentive compatibility constraint that requires that a majority of players are weakly better off with the proposal policy than with the prevailing default. To sum up, given any default $x \in X$, $f(x)$ must solve

$$\begin{aligned} \max_{y \in X} \quad & U_1(y) \\ \text{s.t.} \quad & |\{\ell \in N : U_\ell(y) \geq U_\ell(x)\}| \geq m + 1, \end{aligned} \tag{2}$$

where, for any finite set A , $|A|$ denotes the number of its elements. We are now ready to define an equilibrium.

Definition 1 A stationary Markov perfect equilibrium is a proposal strategy f and a set of value functions $\{U_\ell\}_{\ell=1}^n$ such that:

1. Given f , $\{U_\ell(x)\}_{\ell=1}^n$ solve the equation system defined by (1).
2. Given $\{U_\ell\}_{\ell=1}^n$, $f(x)$ solves problem (2) of the proposer for any default $x \in X$.

For any proposal strategy f and any $x \in X$, let $f^0(x) \equiv x$ and $f^t(x) \equiv f(f^{t-1}(x))$ for all $t \in \mathbb{N}$. A policy path, $\{f^t(x)\}_{t=0}^\infty$, thus traces the evolution of default along an equilibrium path that starts with an initial default $x \in X$. We further restrict attention to equilibria where any policy path converges. Given that we assume a single persistent proposer, it is natural to rule out the equilibria where the policy oscillates until the legislative session is exogenously terminated.⁹

⁹See Diermeier and Fong (2008a) for an example of such equilibrium.

Definition 2 A stationary Markov perfect equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$ is *well-behaved* if and only if, for all $x \in X$, there exists $T \in \mathbb{N}$ such that $f(f^T(x)) = f^T(x)$.

In this paper we characterize any well-behaved, pure-strategy stationary Markov perfect equilibrium and simply call it an *equilibrium*.

4 Analysis

4.1 An Algorithm

For any equilibrium $(f, \{U_\ell\}_{\ell=1}^\infty)$, let

$$S_f = \{x \in X : f(x) = x\}$$

be the equilibrium stable set and refer to any $x \in S_f$ as a stable policy. In words, a stable policy persists as default. In principle, the boundary of a stable set depends on the equilibrium proposal strategy, so two different equilibria may imply two distinct stable sets.

Let $z_1 \in X$ be an ideal point of the proposer. As the proposer would never make a new proposal to replace his ideal point as default, any equilibrium stable set is nonempty since $z_1 \in S_f$.

Our characterization of an equilibrium rests on an algorithm that could be applied to construct an equilibrium stable set. For a more concise presentation, we write $y \succeq x$ if and only if $u_1(y) \geq u_1(x)$ and there exists $M \subset N \setminus \{1\}$ such that $|M| = m$ and $u_i(y) \geq u_i(x)$ for all $i \in M$.

Algorithm 1 Construct a policy set $\widehat{S} \subset X$ through the following steps:

1. Let $Y_0 \equiv X$ and $K = 0$.

2. Let $k = K$.

3. Let C_k be any nonempty subset of

$$C_k^* \equiv \arg \max_{x \in Y_k} u_1(x), \quad (3)$$

$$D_k \equiv \{x \in Y_k \setminus C_k : \exists y \in C_k \text{ s.t. } y \succeq x\}, \quad (4)$$

and

$$Y_{k+1} \equiv Y_k \setminus (C_k \cup D_k). \quad (5)$$

4. If $Y_{k+1} = \emptyset$, then let

$$\widehat{S} \equiv \bigcup_{k'=0}^k C_{k'}. \quad (6)$$

Otherwise let $K = k + 1$ and repeat Steps 2-4.

Let \mathcal{S} be the collection of all policy sets that can be constructed by the algorithm.

This algorithm has three technical features. First, $\widehat{S} \neq \emptyset$ for all $\widehat{S} \in \mathcal{S}$. Any policy set constructed by the algorithm must contain at least an ideal policy of the proposer. Second, the iteration must end in finite rounds given that the policy space is finite. Third, multiple policy sets may be constructed by the algorithm, due to the degree of freedom in constructing C_k when C_k^* is not a singleton. If the maximization problem defined by (3) has a unique solution in every round of the iteration, a unique policy set is constructed by the algorithm.

4.2 Equilibrium Existence

Theorem 1 applies the algorithm to characterize a class of equilibria.

Theorem 1 For any $\widehat{S} \in \mathcal{S}$, there exists an equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$ such that:

1. for all $x \in X$ and all $\ell \in N$,

$$U_\ell(x) = (1 - \delta)u_\ell(x) + \delta u_\ell(f(x)); \quad (7)$$

2. for all $x \in \widehat{S}$, $f(x) = x$;

3. for all $x \notin \widehat{S}$, $f(x)$ is an element of

$$F(x; \widehat{S}) \equiv \{y \in \widehat{S} : y \succeq x\} \neq \emptyset. \quad (8)$$

Proof of Theorem 1. Consider a proposal strategy f and a set of value functions $\{U_\ell\}_{\ell=1}^n$ that satisfy conditions 1-3 for some \widehat{S} constructed by the algorithm along with $\{C_k, C_k^*, D_k, Y_k\}$. Through a series of claims we prove that $(f, \{U_\ell\}_{\ell=1}^n)$ constitute an equilibrium. Claim 1 shows that $F(x; \widehat{S}) \neq \emptyset$ for all $x \notin \widehat{S}$ and therefore $f(x)$ is well-defined. Claims 2 and 5 provide instrumental results useful for the rest of the proof. Claim 3 shows that $\{U_\ell(x)\}_{\ell=1}^n$ solve the equation system defined by (1), so Condition 1 of Definition 1 is satisfied. Claims 4 and 6 jointly show that $f(x)$ solves problem (2) of the proposer for any default $x \in X$, so Condition 2 of Definition 1 is satisfied. Respectively, Claims 4 and 6 prove that $f(x)$ is politically feasible and that no other politically feasible policy can do strictly better than $f(x)$ for the proposer.

CLAIM 1. For all $x \notin \widehat{S}$, $F(x; \widehat{S}) \neq \emptyset$.

PROOF. Take any $x \notin \widehat{S}$. Without loss of generality, assume that $x \in D_k$ for some $k \in \mathbb{Z}_+$. Note that $C_k \neq \emptyset$ since $D_k \neq \emptyset$. Then take any $y \in C_k$. By (4) and (6), $y \succeq x$ and $y \in \widehat{S}$. Therefore $y \in F(x; \widehat{S})$.

CLAM 2. For all $x \in X$ and $\ell \in N$, (a) $U_\ell(f(x)) = u_\ell(f(x))$; and (b) $u_\ell(x) > u_\ell(f(x))$ if and only if $U_\ell(x) > U_\ell(f(x))$.

PROOF. These directly follow (7) and the fact that $f(f(x)) = f(x)$ for all x .

CLAIM 3. For all $\ell \in N$, U_ℓ satisfies equation (1).

PROOF. This directly follows (7) and Claim 2.

CLAIM 4. For all $x \in X$, (a) $U_1(f(x)) \geq U_1(x)$; and (b) there exists $M \subset N \setminus \{1\}$ such that $|M| = m$ and $U_i(f(x)) \geq U_i(x)$ for all $i \in M$.

PROOF. The claim is obviously true for all $x \in \widehat{S}$, so take any $x \notin \widehat{S}$. By (8), $u_1(f(x)) \geq u_1(x)$ and there exists $M \subset N \setminus \{1\}$ such that $|M| = m$ and $u_i(f(x)) \geq u_i(x)$ for all $i \in M$. Then by Claim 2, for all $j \in M \cup \{1\}$, $U_j(f(x)) \geq U_j(x)$.

CLAIM 5. For all $x, y \in X$ and $\ell \in N$, if $u_1(f(x)) > u_1(f(y))$ then $U_\ell(x) > U_\ell(y)$, $U_\ell(x) > U_\ell(f(y))$ and $U_\ell(f(x)) > U_\ell(y)$.

PROOF. By (7) and given that $\delta \in (0, 1)$ is sufficiently large, $U_1(y)$ and $U_1(x)$ are sufficiently close to $u_1(f(y))$ and $u_1(f(x))$, respectively. The rest directly follows.

CLAIM 6. For all $x, y \in X$, either $U_1(f(x)) \geq U_1(y)$, or there exists $M_+ \subset N$ such that $|M_+| \geq m + 1$ and $U_i(x) > U_i(y)$ for all $i \in M_+$.

PROOF. Let $k(x), k(y) \in \mathbb{Z}_+$ be such that $f(x) \in C_{k(x)}$ and $f(y) \in C_{k(y)}$. We discuss the three cases below. Case 1. Suppose that $u_1(f(x)) > u_1(f(y))$. Then by Claim 5, $U_1(f(x)) > U_1(y)$. Case 2. Suppose that $u_1(f(x)) < u_1(f(y))$. Then $k(x) > k(y)$.¹⁰ This implies that $f(x) \in Y_{k(y)} \setminus (C_{k(y)} \cup D_{k(y)})$. By definition of $D_{k(y)}$, there exists $M_+ \subset N$ such that $|M_+| = m + 1$ and $u_i(f(x)) > u_i(f(y))$ for all $i \in M_+$. Then by Claims 5, $U_i(x) > U_i(y)$ for all $i \in M_+$. Case 3. Suppose that $u_1(f(x)) = u_1(f(y))$. If $u_1(y) > u_1(f(y))$, then by Claim 5, $U_1(y) > U_1(f(y))$. This contradicts the optimality of $f(y)$ for the proposer. Therefore, it must be the

¹⁰Suppose to the contrary that $k(x) \leq k(y)$, then $f(y) \in Y_{k(x)}$. Since $f(x) \in C_{k(x)}$, $u_1(f(x)) \geq u_1(f(y))$. This contradicts the condition that $u_1(f(x)) < u_1(f(y))$.

case that $u_1(y) \leq u_1(f(y)) = u_1(f(x))$. Then by Claim 2, $U_1(f(x)) = U_1(f(y)) \geq U_1(y)$. ■

Theorem 1 has several notable implications. First, in dynamic legislative bargaining games existence of a pure-strategy stationary equilibria is usually not assured. Duggan and Kalandrakis (2007) prove the existence of a pure-strategy stationary equilibrium for a general class of dynamic legislative bargaining games in which the default policy endogenously evolves, but only with some suitably assumed randomness on preferences and the dynamic process of the default. While our model does not satisfy the sufficient conditions of Duggan and Kalandrakis, Theorem 1 shows that a class of pure-strategy equilibria still exists given that the policy space is finite and the probability of a chance to reconsider is sufficiently high.

Second, for any $\widehat{S} \in \mathcal{S}$, there exists at least one equilibrium $(f, \{U_\ell\}_{\ell=1}^\infty)$ in which the equilibrium stable set is exactly the constructed policy set, i.e.

$$S_f = \widehat{S}.$$

Third, reconsideration of a passed bill does not actually occur in any equilibrium characterized by Theorem 1. However, the possibility of reconsideration changes the nature of the policy-making game, as the proposer is endogenously constrained to select a policy proposal from some $\widehat{S} \in \mathcal{S}$ instead of the whole policy space X .

Fourth, there may exist multiple equilibria. Technically, multiplicity arises due to the degrees of freedom to construct C_k by (3), when C_k^* is more than a singleton, and to construct $f(x)$ by (8) for all $x \notin \widehat{S}$, when $F(x; \widehat{S})$ is more than a singleton.

Intuitively, different equilibria result from self-fulfilling beliefs of the players. In every proposal round all players anticipate the equilibrium strategies to be played in all subsequent rounds, and based on this common belief they calculate their reserva-

tion values that determine the strategies for the current proposal round. Stationarity requires that all players' beliefs on future strategies be consistent with their strategies in the current round. Therefore, players' expectations determine their current play. As we focus on institutions in which $\delta \in (0, 1)$ is sufficiently large, the players' future becomes disproportionately important as the players are mainly concerned about the stable policies that they would eventually reach. Multiple equilibria thus result from the existence of multiple pairs of belief and strategy that are consistent with each other.

Some additional assumptions could be made to reduce the set of equilibria. In these cases the proposed algorithm could be modified accordingly. For example, Diermeier and Fong (2008a) characterize equilibria in which the stable set is symmetric with respect to positions of the voters and pin down a unique equilibrium stable set. Diermeier and Fong (2008b,c) assume that in the legislature there is not only a proposer, who is granted proposal power by the legislative procedures, but also a coordinating legislator who ensures common beliefs among the players. Whereas the proposer controls *de jure* power to submit proposals, the coordinator controls *de facto* power to direct players' beliefs and actions. In principle, the coordinator could be the same as or different from the proposer, and this may depend on various features in a legislature including norms or conventions. A general discussion of equilibrium selection or equilibrium refinement, however, is beyond the scope of this paper.

Given the possibility of multiple equilibria, it is important to characterize a set of necessary conditions so as to capture common properties that prevail in all equilibria. We do this in the next section.

4.3 Necessary Conditions

Given the requirement of well-behavedness, any policy path induced by an equilibrium proposal strategy f must converge. For any $x \in X$, let $f^\infty(x) \equiv \lim_{t \rightarrow \infty} f^t(x)$. Given that the policy space is finite, $f^\infty(x) = f^t(x)$ for all t sufficiently large. Therefore $f(f^\infty(x)) = f^\infty(x)$ and $f^\infty(x) \in S_f$ for all $x \in X$. For any initial default x , we will call $f^\infty(x)$ the *final policy outcome* if the legislative session lasts for sufficiently many proposal rounds so that in realization the stable set is reached. This section focuses on general properties of an equilibrium stable set and the bounds on final policy outcomes.

Lemma 1 first provides conditional inequalities useful for development of the main results.

Lemma 1 Consider any equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$. For all $x \in X$ and $\ell \in N$:

1. $U_\ell(f(x)) \geq U_\ell(x) \Leftrightarrow U_\ell(f(x)) \geq u_\ell(x)$.
2. $u_\ell(f^\infty(x)) < u_\ell(x) \Rightarrow U_\ell(f(x)) < u_\ell(x)$.

Proof. Part 1 directly follows (1). To prove Part 2, take any $x \in X$ and $\ell \in N$ such that $u_\ell(f^\infty(x)) < u_\ell(x)$. Let $T \in \mathbb{N}$ be such that (a) $f^t(x) = f^\infty(x)$ for all $t \geq T$, and (b) either $T = 1$ or $f^{t+1}(x) \neq f^t(x)$ for all $t \leq T - 1$. Then

$$U_\ell(f(x)) = (1 - \delta) \left(\sum_{t=1}^{T-1} \delta^{t-1} u_\ell(f^t(x)) \right) + \delta^{T-1} u_\ell(f^\infty(x)).$$

Given that $\delta \in (0, 1)$ is sufficiently large, $U_\ell(f(x))$ is sufficiently close to $u_\ell(f^\infty(x))$ so that $U_\ell(f(x)) < u_\ell(x)$. ■

Theorem 2 shows that, in any equilibrium, the final policy outcome must be weakly preferred to the initial default by a majority of players including the proposer.

Theorem 2 For any equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$ and for any $x \in X$,

$$f^\infty(x) \succeq x;$$

i.e. (A) $u_1(f^\infty(x)) \geq u_1(x)$, and (B) there exists $M \subset N \setminus \{1\}$ such that $|M| = m$ and $u_i(f^\infty(x)) \geq u_i(x)$ for all $i \in M$.

Proof of Theorem 2. Part A. Suppose that $u_1(x) > u_1(f^\infty(x))$. Then by Lemma 1, $u_1(x) > U_1(f(x))$ and $U_1(x) > U_1(f(x))$. This contradicts the optimality of $f(x)$ for the proposer. Part B. Suppose to the contrary that there exists $M_+ \subset N \setminus \{1\}$ such that $|M_+| = m + 1$ and $u_i(x) > u_i(f^\infty(x))$ for all $i \in M_+$. Then by Lemma 1, for all $i \in M_+$, $u_i(x) > U_i(f(x))$ and $U_i(x) > U_i(f(x))$. This contradicts political feasibility of $f(x)$. ■

Intuitively, as we assume that the proposer is sufficiently likely to have a chance to reconsider any passed bill, players are mainly concerned about how their proposal making and voting will lead to the final *stable* policy outcome. In other words, if it takes more than one proposal round to reach the final policy outcome, any policy approved in a transitional proposal round only contributes insignificantly to the calculation of expected utilities by the players. What the players care about a transitional policy is which stable policy it would eventually lead to. The proposer, for example, would avoid proposing any policy that would eventually transition to a stable policy that makes him strictly worse off. Similarly, no policy is politically feasible if it would lead to some stable policy where a majority of voters would be strictly worse off.

Regardless of the initial default, the final policy outcome must be a stable policy. Therefore, we need to characterize the equilibrium stable set. This is done by Theorem

3.

Theorem 3 For any equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$,

$$S_f \in \mathcal{S}.$$

In other words, there exists $\widehat{S} \in \mathcal{S}$ such that $f^\infty(x) \in \widehat{S}$ for all $x \in X$, i.e.,

$$f(x) = x \Leftrightarrow x \in \widehat{S}.$$

Proof of Theorem 3. Take any equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$. The proof proceeds by math induction through Claims 1-5.

CLAIM 1. For any $\{C_k, C_k^*, D_k, Y_k\}$ constructed by the algorithm, $C_0^* \cap S_f \neq \emptyset$.

PROOF. Suppose that $C_0^* \cap S_f = \emptyset$. Note that $C_0^* \neq \emptyset$ so take any $x \in C_0^*$. Since $f^\infty(x) \in S_f$, $f^\infty(x) \notin C_0^*$ by supposition. Then $u_1(x) > u_1(f^\infty(x))$ by (3) for $k = 0$. By Lemma 1, $u_1(x) > U_1(f(x))$ and $U_1(x) > U_1(f(x))$. This contradicts the optimality of $f(x)$ for the proposer.

CLAIM 2. Take any $K \in \mathbb{Z}_+$ and let $\{C_k, C_k^*, D_k, Y_k\}$ be constructed by the algorithm such that $Y_K \neq \emptyset$ and $C_K \subseteq S_f$. Then $(D_K \setminus C_K^*) \cap S_f = \emptyset$.

PROOF. Suppose that $(D_K \setminus C_K^*) \cap S_f \neq \emptyset$ and take any $x \in (D_K \setminus C_K^*) \cap S_f$. Also take any $y \in C_K$. Since $C_K \subseteq S_f$, $y \in S_f$. By (3) and (4) for $k = K$, (a) $u_1(y) > u_1(x)$ and (b) there exists $M \subset N \setminus \{1\}$ such that $|M| = m$ and $u_i(y) \geq u_i(x)$ for all $i \in M$. Since $x, y \in S_f$, $U_\ell(x) = u_\ell(x)$ and $U_\ell(y) = u_\ell(y)$ for all $\ell \in N$. Therefore, $U_1(y) > U_1(x)$ and $U_i(y) \geq U_i(x)$ for all $i \in M$. This implies that $f(x) \neq x$ and $x \notin S_f$, which is a contradiction.

CLAIM 3. There exists $\{C'_k, C_k^*, D'_k, Y'_k\}$ constructible by the algorithm such that

$C'_1 \subseteq S_f$ and $D'_1 \cap S_f = \emptyset$.

PROOF. Let $\{C'_k, C_k, D'_k, X'_k\}$ be constructed by the algorithm such that $C'_1 = C_1 \cap S_f$. By Claim 1, $C'_1 \neq \emptyset$. By construction, $C'_1 \subseteq S_f$ and $(C_1 \setminus C'_1) \cap S_f = \emptyset$. By Claim 2, $(D'_1 \setminus C_1) \cap S_f = \emptyset$. Note that $D'_1 = (D'_1 \setminus C_1) \cup (C_1 \setminus C'_1)$. Therefore $D'_1 \cap S_f = \emptyset$.

CLAIM 4. Take any $K \in \mathbb{N}$ and let $\{C_k, C_k^*, D_k, Y_k\}$ be constructed by the algorithm such that, for all $k \leq K$, $Y_k \neq \emptyset$, $C_k \subseteq S_f$ and $D_k \cap S_f = \emptyset$. If $Y_{K+1} \neq \emptyset$, then (A) $x \in Y_{K+1} \Rightarrow f^\infty(x) \in Y_{K+1}$, and (B) $C_{K+1}^* \cap S_f \neq \emptyset$.

PROOF. Part A. Take any $x \in Y_{K+1}$ and suppose that $f^\infty(x) \notin Y_{K+1}$. Since $f^\infty(x) \in S_f$, $f^\infty(x) \notin \bigcup_{k=0}^K D_k$. Then $f^\infty(x) \in \bigcup_{k=0}^K C_k$ by (5) and (6). Without loss of generality assume that $f^\infty(x) \in C_k$ for some $k \leq K$. Since $x \in Y_{K+1} \subset Y_k$ and $x \notin (C_k \cup D_k)$, $u_1(f^\infty(x)) \geq u_1(x)$ and $f^\infty(x) \not\prec^* x$. This implies that there exists $M_+ \subset N \setminus \{1\}$ such that $|M_+| = m + 1$ and $u_i(x) > u_i(f^\infty(x))$ for all $i \in M_+$. By Lemma 1, for all $i \in M_+$, $u_i(x) > U_i(f(x))$ and $U_i(x) > U_i(f(x))$. This contradicts political feasibility of $f(x)$.

Part B. The argument is in parallel to that for Claim 1. Suppose that $C_{K+1}^* \cap S_f = \emptyset$. Note that $C_{K+1}^* \neq \emptyset$ since $Y_{K+1} \neq \emptyset$. So take any $x \in C_{K+1}^*$. Note that $x \in Y_{K+1}$ and therefore $f^\infty(x) \in Y_{K+1}$ by Part A of the claim. Since $f^\infty(x) \in S_f$, $f^\infty(x) \notin C_{K+1}^*$ by supposition. Since $x \in C_{K+1}^*$ and $f^\infty(x) \in Y_{K+1} \setminus C_{K+1}^*$, $u_1(x) > u_1(f^\infty(x))$ by (3) for $k = K + 1$. By Lemma 1, $u_1(x) > U_1(f(x))$ and $U_1(x) > U_1(f(x))$. This contradicts the optimality of $f(x)$ for the proposer.

CLAIM 5. Suppose that, for some $K \in \mathbb{N}$, $\{C_k, C_k^*, D_k, Y_k\}$ is constructed by the algorithm such that, for all $k \leq K$, $C_k \subseteq S_f$ and $D_k \cap S_f = \emptyset$. If $Y_{K+1} \neq \emptyset$, then there exists $\{C'_k, C_k, D'_k, Y'_k\}$ constructible by the algorithm such that, for all $k \leq K + 1$, $C'_k \subseteq S_f$ and $D'_k \cap S_f = \emptyset$.

PROOF. The argument is in parallel to that for Claim 3. Let $\{C'_k, C_k^{*'}, D'_k, Y'_k\}$ be constructed by the algorithm such that $C'_k = C_k$ for all $k \leq K$ and $C'_{K+1} = C_{K+1}^{*'} \cap S_f$. By Claim 4, $C'_{K+1} \neq \emptyset$. By construction, $C'_{K+1} \subseteq S_f$ and $(C_{K+1}^{*'} \setminus C'_{K+1}) \cap S_f = \emptyset$. Since $C'_{K+1} \subseteq S_f$, $(D'_{K+1} \setminus C_{K+1}^{*'}) \cap S_f = \emptyset$ by Claim 2. Note that $D'_{K+1} = (D'_{K+1} \setminus C_{K+1}^{*'}) \cup (C_{K+1}^{*'} \setminus C'_{K+1})$. Therefore $D'_{K+1} \cap S_f = \emptyset$. ■

Whereas Theorem 1 shows that any policy set constructed by the algorithm is the stable set in some equilibrium, Theorem 3 shows that the stable set in any equilibrium is constructible by the algorithm. These theorems thus jointly show that the collection of all possible equilibrium stable sets is identical to the collection of all policy sets constructible by the algorithm.

Finally, Theorems 2 and 3 jointly imply that, given any initial default $x \in X$, the final policy outcome could be any policy in $F(x; \widehat{S})$ for some $\widehat{S} \in \mathcal{S}$, where F is defined by (8). Recall Part 3 of Theorem 1 and we can see that restricting the focus to equilibria in which no reconsideration occurs does not reduce the set of possible policy outcomes, given that the legislative session lasts for sufficiently many rounds so that a stable policy is reached. Therefore, if we only focus on policy outcomes we miss nothing by ignoring equilibria in which reconsideration actually does occur.¹¹

5 Implications

Our analysis has hinted the additional endogenous constraints the sole proposer faces when he is granted power to reconsider an approved policy. In this section we will make this point very clear.

Assume any initial default $x \in X$. We first consider a standard agenda-setting

¹¹See Diermeier and Fong (2008a) for an example of an equilibrium in which a passed bill may be reconsidered.

institution, in which there is only one round of proposal making and voting, i.e. $\delta = 0$ (Romer and Rosenthal, 1978; Baron and Ferejohn, 1989). Let $g(x)$ denote any policy outcome from this institution. Then $g(x)$ must be politically feasible, i.e.

$$g(x) \in G(x) \equiv \{y \in X : y \succeq x\}, \quad (9)$$

and maximizing the proposer's utility, i.e.

$$u_1(g(x)) \geq u_1(y) \text{ for all } y \in G(x). \quad (10)$$

We then consider the institution that allows reconsideration with $\delta \in (0, 1)$ sufficiently large. By Theorems 2 and 3, the final policy outcome x^* in any equilibrium must satisfy

$$x^* \in F(x; \widehat{S}) = G(x) \cap \widehat{S} \quad (11)$$

for some $\widehat{S} \in \mathcal{S}$.

A comparison of (9) and (11) shows that the proposer effectively faces a more stringent constraint when he is granted power to reconsider passed bills than when he is not allowed to do so. With $\delta \in (0, 1)$ sufficiently large, the proposer must make a policy choice such that eventually the policy converges to some $\widehat{S} \in \mathcal{S}$. With this additional constraint, in this case the value of proposal power is in general more limited than if reconsideration was not allowed. The next theorem formalizes this insight.

Theorem 4 Assume, as elsewhere in this paper, that $\delta \in (0, 1)$ is sufficiently large.

For any equilibrium $(f, \{U_\ell\}_{\ell=1}^\infty)$ and any default $x \in X$,

$$U_1(f(x)) \leq U_1(f^\infty(x)) \leq u_1(g(x)),$$

where $u_1(g(x))$ would be the equilibrium utility of the proposer if reconsider was not allowed, i.e. $\delta = 0$.

Proof of Theorem 4. Take any $x \in X$. First note that, for any $t \in \mathbb{Z}_+$, the proposer cannot be strictly worse off by choosing $f^{t+1}(x)$ when the default is $f^t(x)$. Therefore,

$$U_1(f^t(x)) \leq U_1(f^{t+1}(x))$$

and as a consequence $U_1(f(x)) \leq U_1(f^\infty(x))$. Also note that $f^\infty(x) \in G(x)$ by (11). Then by (10), $U_1(f^\infty(x)) = u_1(f^\infty(x)) \leq u_1(g(x))$. ■

Theorem 4 implies that, granting the sole proposer to initiate reconsideration only limits the value of his proposal power. In other words, if the sole proposer could choose, he would have committed to the institution in which he was restricted to making a proposal once and for all with no possibility for reconsideration. Counter-intuitively, more power granted by the legislative procedure in this case leads to less valuable power in practice.

A critical prerequisite for this somewhat counterintuitive statement to be true is the institution of majoritarian voting. Knowing that the proposer may use his power to exploit the voters in the future, a majority of voters implicitly coordinates to constrain the proposer, which provides an endogenous commitment device that benefits the voters yet harms the proposer. Whereas Diermeier and Fong (2008a) clearly illustrates this mechanism in a distributive model, in this paper we show that their results robustly hold with any finite but otherwise general policy environment,

e.g. with public goods provision. Our theory thus provides a novel explanation for why empirically estimated values of proposal power may be smaller than predicted by the models of Baron and Ferejohn (1989).¹²

The possibility of reconsideration can be interpreted as lack of commitment by the proposer. It has been commonly understood that lack of commitment by policy-makers could be a source of policy inefficiency (Kyddland and Prescott, 1977; Persson and Svensson, 1989; Aghion and Bolton, 1990; Tabellini and Alesina, 1990; Besley and Coate, 1998; Acemoglu and Robinson, 2001; Acemoglu, 2003). However, our model illustrates a mechanism that works in the opposite direction in majoritarian environments. In fact, lack of commitment by a proposer with persistent power may lead to a less unequal allocation of public resources and more efficient policy outcome. This is because the possibility of reconsideration induces the legislature to "commit" to choosing a final policy outcome that falls in the stable set, which is typically smaller than the whole policy space. Our study thus suggests the importance of understanding the interaction of collective decision rules with commitment technologies.

6 Policy Environments

In this section, we discuss a few commonly studied policy environments to explain how the algorithm can be applied to solve specific models and to illustrate the key insights obtained in the previous general analysis. In order to reduce notational complexity, we only consider a legislature with three players, indexed by $\ell \in \{1, 2, 3\}$, where player 1 is assumed to be the sole proposer.

¹²See Knight (2005), for example, for a recent empirical study of the U.S. Congress.

6.1 A Distributive Model

This example is a simplified version of Diermeier and Fong (2008a). The policy is to divide $\pi \in \mathbb{N}$ units of fixed benefits among the three players. The policy space is therefore $X = \{x \in \mathbb{Z}_+^3 : \sum_{\ell=1}^3 x_\ell = \pi\}$. Given any policy $x = (x_1, x_2, x_3) \in X$, player ℓ receives x_ℓ units of benefits and derives a utility of $u_\ell(x) = x_\ell$.¹³

By the algorithm, for all $k \in [0, \frac{1}{2}\pi]$,

$$\begin{aligned} Y_k &= \{x \in X : \min\{x_2, x_3\} \geq k\}, \\ C_k &= C_k^* = \{x \in X : x_2 = x_3 = k\}, \\ D_k &= \{x \in X : \max\{x_2, x_3\} > \min\{x_2, x_3\} = k\}, \end{aligned}$$

and a unique policy set

$$\widehat{S} = \{x \in X : x_2 = x_3\}$$

is constructed. By Theorem 3, this is the unique policy set in any equilibrium. By Theorems 2 and 3, for any initial default $x \in X$, the final policy outcome $f^\infty(x)$ is such that $f_2^\infty(x) = f_3^\infty(x) \geq \min\{x_2, x_3\}$ and $f_1^\infty(x) = \pi - 2 \min\{x_2, x_3\}$. Intuitively, reconsideration leads to more egalitarian allocations as the possibility of reconsideration induces the two voters to "defend" the benefits for each other. In particular, a voter may decline a policy proposal if the other voter is substantially expropriated, as this prevents the proposer from playing off the voters against each other in the future. In equilibrium the proposer must allocate the same amount of benefits to both voters. As a consequence, the proposer receives strictly less than what he would do if reconsideration was not allowed, i.e. $x_1 + \max\{x_2, x_3\}$.

¹³The same analysis applies if $u_\ell(x) = v(x_\ell)$, where $v(\cdot)$ is a strictly increasing function.

6.2 Public Goods Production

This example is an extension of the previous one. Assume that the three players must jointly produce benefits that they can divide and consume. In this case a policy $x = (x_1, x_2, x_3)$ specifies not only allocation but also size of the total benefits. The policy space is therefore $X = \mathbb{Z}_+^3$. Public production is costly. The cost function is assumed to be quadratic and given by

$$\kappa(x) = \frac{1}{2}\phi \left(\sum_{\ell=1}^3 x_\ell\right)^2,$$

where ϕ is a constant marginal cost of production. Each player ℓ is assumed to share equally the production cost, and for any policy $x \in X$, derive a utility of

$$u_\ell(x) = \mu x_\ell - \frac{1}{3}\kappa(x),$$

where μ is a common marginal utility of benefits consumption.¹⁴

This example can be interpreted as a model with distortionary taxation and the provision of local public goods (Diermeier and Fong, 2008b, 2008d). In particular, x_ℓ could be the local public good for the geographical district or the socioeconomic group that legislator ℓ represents, and the production cost $\kappa(x)$ of public goods include the forgone private consumption of the individuals and the deadweight loss that any distortionary tax, e.g. a proportional labor income tax, may incur.

The initial default is assumed to be $x^0 = (0, 0, 0)$. That is, if no agreement is made in the legislature, there will be no production and no consumption of the benefits. If the policy was chosen by a benevolent dictator, the size of total benefits would be $\pi^* \equiv \frac{\mu}{\phi}$, at which level marginal social cost of production is equal to marginal

¹⁴For technical convenience, assume that the values of μ and ϕ are such that $\frac{\mu}{\phi}$ is an integer.

utility of benefits consumption. Here, however, a policy is made through the political process of legislative bargaining.

In one-round closed-rule legislative bargaining, the proposer needs to satisfy one voter, for example j , at his reservation value $U_j(x^0) = 0$ and can fully expropriate the other voter. By proposing any policy x associated with $\pi_x \equiv \sum_{\ell=1}^3 x_\ell$ units of total benefits, the proposer then must offer j at least $\frac{1}{3\mu}\kappa(\pi)$ units of the benefits to compensate her for the production cost, and can take at most $\left[\pi_x - \frac{1}{3\mu}\kappa(\pi_x)\right]$ units for himself. The proposer thus selects a policy to maximize effectively

$$\mu \left[\pi_x - \left(\frac{1}{3\mu} \kappa(\pi_x) \right) \right] - \frac{1}{3} \kappa(\pi_x) = \mu \pi_x - \frac{2}{3} \kappa(\pi_x).$$

Since the proposer only internalizes the costs paid by himself and voter j , in equilibrium there is generally overproduction of the benefits.

With reconsideration and for $\delta \in (0, 1)$ sufficiently large, an application of the algorithm shows that for any policy in the unique equilibrium stable set, the two voters must receive an equal amount of benefits for each level of total benefits production. This follows because each voter is induced to protect the benefits of the other voter in order to secure her own long-term bargaining position in the legislature. Therefore, by proposing any policy x associated with π_x units of total benefits, the proposer must offer both voters at least $\frac{1}{3\mu}\kappa(\pi)$ units of the benefits to compensate their production costs and therefore can take no more than $\left[\pi_x - 2 \left(\frac{1}{3\mu} \kappa(\pi_x) \right)\right]$ units. Otherwise neither voter would accept the policy proposal. In this case, the proposer selects a policy to maximize effectively

$$\mu \left[\pi_x - 2 \left(\frac{1}{3\mu} \kappa(\pi_x) \right) \right] - \frac{1}{3} \kappa(\pi_x) = \mu \pi_x - \kappa(\pi_x).$$

Note that any politically feasible policy x thus requires the proposer to internalize fully all costs and gains of benefits production. As a consequence, in equilibrium the size of benefits production is socially efficient. With the possibility of reconsideration, social welfare defined by aggregate utility is unambiguously improved.

6.3 A Spatial Model

Spatial models are usually set in a continuous policy space, but their main insights can also be captured in a discrete policy as in the following setup based on Fong (2005).¹⁵ Here the three players must collectively choose a policy from a multi-dimensional policy space that includes 7 policy alternatives in three categories:

1. **Ideal Point.** For all $\ell \in \{1, 2, 3\}$, let z_ℓ be the ideal point of player ℓ . In other words, $u_\ell(z_\ell) = a > u_\ell(x)$ for all $x \neq z_\ell$. Without loss of generality, we normalize the utility functions so that $a > 0$ and $u_\ell(z_i) = 0$ for all $i \neq \ell$.
2. **Bilateral Compromise.** For any distinct $i, j \in \{1, 2, 3\}$, let z_{ij} be a policy that equally benefits players i and j but disadvantages the third player ℓ . Assume that $0 < u_i(z_{ij}) = u_j(z_{ij}) = b < a$, $u_\ell(z_{ij}) = d$, and $2b > a$. We call z_{ij} a bilateral compromise for players i and j because it attains a higher joint utility for the two players than any of their ideal points. For expositional purpose, in this section we only consider the case in which $d \leq 0$. That is, whenever players i and j reach their bilateral compromise, the third party ℓ is worse off than if z_i or z_j is chosen.
3. **Centrist Policy.** Let z_{123} be the centrist policy in the policy space. Assume that $0 < u_\ell(z_{123}) = c < b$ for all ℓ , and $3c > \max\{a, 2b + d\}$. That is, the

¹⁵See also Diermeier and Merlo (2000) for a similar model in the context of coalition bargaining.

centrist policy attains the maximal joint utility of all three players and delivers the same utility to each player. However, for any player ℓ , the centrist policy is worse than his ideal point and any bilateral compromise between himself and some of the other players.

To sum up, $X = \{z_1, z_2, z_3, z_{12}, z_{13}, z_{23}, z_{123}\}$. For any player ℓ and any other distinct j and k , we have $u_\ell(z_\ell) > u_\ell(z_{\ell j}) = u_\ell(x_{\ell k}) > u_\ell(z_{123}) > u_\ell(x_j) = u_\ell(x_k) \geq u_\ell(x_{jk})$.

By the algorithm,

$$\begin{aligned} C_1 &= C_1^* = \{z_1\}, \\ D_1 &= \{z_2, z_3, z_{12}, z_{13}\}, \\ C_2 &= C_2^* = \{z_{123}\}, \\ C_3 &= C_3^* = \{z_{23}\}, \end{aligned}$$

and the unique policy set

$$\widehat{S} = \{z_1, z_{123}, z_{23}\}$$

is constructed. By Theorem 3, \widehat{S} is the unique equilibrium stable set. Below we highlight three noteworthy features of this model.

First, although z_{23} is the policy the proposer dislikes the most, in any equilibrium he is not able to change it if z_{23} is the initial default. If $\delta = 0$, the proposer would be able to pass z_{12} , for example, to increase his utility, as player 2 would be indifferent. However, given that $\delta \in (0, 1)$ is sufficiently large, player 2 would not accept such proposal because she anticipates that with z_{12} as the new default in the next proposal round the proposer would ally with the cheaper player 3 and pass his ideal point, z_1 , which delivers less utility to player 2 than z_{23} . By this logic, player 2 will not approve

any policy change that could benefit the proposer. Similarly player 3 will do the same. As a consequence, the policy cannot be moved and the proposer is not able to use his power to exploit the voters. Moreover, the proposer ends up with the *least* utility among all players. This happens when the proposer is sufficiently disadvantaged by the initial default.

Second, by a similar argument the socially efficient policy, z_{123} , is sustainable. If $\delta = 0$, the proposer would be able to move the policy to either z_{12} or z_{13} and policy efficiency measured as the aggregate utility would decrease. Here with $\delta \in (0, 1)$ sufficiently large, mutual protection by the voters constrain the proposer and force him to retain the efficient default. This case, again, illustrates our earlier point that lack of commitment by the proposer serves as a commitment device for the legislature as a whole to sustain more efficient policies.

Third, with z_{12} (or z_{13}) as default, two different final policy outcomes are possible in equilibrium. In one equilibrium, the proposer seeks voting support from player 3 and moves the policy from z_{12} to his ideal point z_1 , i.e. $f(z_{12}) = z_1$. In this case player 3 is better off and the proposer's utility is maximized. This is also the equilibrium outcome in the case of no reconsideration. In the other equilibrium, the proposer still seeks voting support from player 3 but he is only able to move the policy from z_{12} to the centrist policy z_{123} , given the constraint imposed by 3. This equilibrium emerges due to self-fulfilling beliefs in the case with $\delta \in (0, 1)$ sufficiently large. If players anticipate that the policy will transition to and stabilize at the centrist policy, the voters' reservation values would be so high that they would not approve the proposal of z_1 . The possibility of reconsideration thus supports a more efficient equilibrium outcome than what would have been chosen in the case of no reconsideration.

7 Conclusion

In this paper we prove general existence for pure-strategy stationary equilibria in a model of legislative bargaining with reconsideration. We also define an algorithm to compute the equilibrium stable set. We then characterize the equilibrium stable set and show that in any pure-strategy stationary equilibrium the possibility of reconsideration endogenously limits the power of the single, persistent proposer. The results of the model hold for any finite policy space and can therefore be applied in various policy domains. We discuss three such domains: a distributive model, a model of public goods production, and a finite variant of a spatial model. Our analysis shows that the proposer would be better off by being able to commit not to reconsider a policy proposal in the future, yet such lack of commitment may lead to efficiency gains, in some examples even recovering the first-best solution.

Appendix

A Continuous-Time Model

We present a continuous-time model in which a policy, once enacted, is in effect until it is replaced by a new policy. This model provides an alternative interpretation of the one presented in the main text, because the two models have an identical mathematical structure. An equilibrium defined and characterized for the model in the main text is also an equilibrium for this model with continuous time.

In the legislature there are a group of $N = \{1, 2, \dots, n\}$ players, where $n \geq 3$ is odd. Time is continuous and infinite. A policy $x \in X$ continuously delivers a flow of utility $u_\ell(x)$ to player ℓ as long as it is implemented. The policy could be understood as a continuing government program so that, once implemented, it is in effect until replaced by a new policy chosen through the political process.¹⁶ As we assume in the main text, the policy space X is finite.

Let $x(t)$ denote the policy that is implemented in instant t . Then the expected utility of player ℓ is given by

$$E \left[\int_0^\infty \frac{1}{\rho} e^{-\rho t} u_\ell(x(t)) dt \right],$$

where $\rho > 0$ measures impatience of the players, and expectation is taken over any exogenous randomness. The utility function is scaled by $\frac{1}{\rho}$ in order to simplify the exposition of the algebra.

The policy is made in the legislature through the political process. There is a sole, permanent proposer, say player 1.¹⁷ The proposer is granted power to make a policy

¹⁶See Baron (1996) and Baron and Herron (2003) for examples of such policies.

¹⁷This assumption of a permanent proposer could be relaxed. Diermeier and Fong (2008c) consider a related model in which a proposer may be contested and lose his power following a Poisson process.

proposal at any time, with the only requirement that any two consecutive proposals must be separated by $\varepsilon > 0$ units of time.

A status quo is defined as the policy that has been implemented. Once a new proposal is made, it is immediately subject to majority voting against the status quo. Without loss of generality we assume that proposal making and voting are done instantly so they do not take any time. If the proposal is approved by a majority of players, it is enacted right away; otherwise the status quo remains in place. In this context, reconsideration refers to a change of the policy that has been enacted.

We could imagine that the continuous time consists of a sequence of time periods. Each period lasts for ε units of time, and legislative actions are required to take place in the beginning of every period. Suppose that a policy $x \in X$ is chosen and enacted in the beginning of period t . Then during this period every player ℓ receives a utility of $(1 - \exp(-\rho\varepsilon))u_\ell(x)$ and discounts his utility derived in the next period by $\exp(-\rho\varepsilon)$. The per-period discount factor, $\delta \equiv \exp(-\rho\varepsilon) \in (0, 1)$, is a function of period length ε , and impatience ρ of the players. A stationary Markov perfect equilibrium can then be defined in the same way as it is done for the model in the main text.

The case in which the per-period discount factor $\delta \in (0, 1)$ is sufficiently large is equivalent to the case in which the time interval of a period is sufficiently small. This approximates the situation in which the proposer can propose to change a policy at any time he wants and as frequent as possible. Formally, $\lim_{\varepsilon \rightarrow 0} \delta = 1$ for any $\rho > 0$.

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