Armstrong, Christopher S.; Larcker, David F.; Su, Che-Lin

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Stock Options and Chief Executive Officer Compensation†

Christopher S. Armstrong  
Armstrong.Christopher@gsb.stanford.edu

David F. Larcker  
Larcker.David@gsb.stanford.edu

Stanford University  
Graduate School of Business  
518 Memorial Way  
Stanford, CA 94305-5015

Che-Lin Su  
c-su@kellogg.northwestern.edu

CMS-EMS  
Kellogg School of Management  
Northwestern University  
580 Leverone Hall  
2001 Sheridan Road  
Evanston, IL 60208-2014

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Stock Options and Chief Executive Officer Compensation

Abstract

Although stock options are commonly observed in chief executive officer (CEO) compensation contracts, there is theoretical controversy about whether stock options are part of the optimal contract. Using a sample of Fortune 500 companies, we solve an agency model calibrated to the company-specific data and we find that stock options are almost always part of the optimal contract. This result is robust to alternative assumptions about the level of CEO risk-aversion and the disutility associated with their effort. In a supplementary analysis, we solve for the optimal contract when there are no restrictions on the contract space. We find that the optimal contract (which is characterized as a state-contingent payoff to the CEO) typically has option-like features over the most probable range of outcomes.
Stock Options and Chief Executive Officer Compensation

Over the past decade there has been an explosion in the use of equity-based compensation (especially stock options) for top executives (e.g., Murphy (1999) and Ittner, Lambert, and Larcker (2003)). Despite the growing popularity of stock options, there is considerable academic and professional debate regarding the relative costs and benefits of equity-based compensation. Some observers view these plans as providing high-powered incentives that align the interests of employees with those of shareholders. They also contend that stock options help attract and retain scarce managerial and technical talent. However, critics claim that options give away too much value by diluting the interests of shareholders. Perhaps based on this claim, some companies are dropping their stock option in favor of restricted stock (e.g., Carter, Lynch, and Tuna (2007) and Frederic W. Cook & Co. (2006)).

One especially pointed academic critique is that stock options are an inefficient mechanism for compensating executives relative to restricted stock (e.g., Meulbroek (2001) and Hall and Murphy (2002)). Similarly, Dittmann and Maug (2007) conclude that stock options should almost never be part of the compensation contract for chief executive officers (CEOs). In contrast, Kadan and Swinkels (2006) develop and test an agency model where stock options dominate restricted stock when non-viability (or bankruptcy) risk is zero. Aseff and Santos (2005) also suggest that option grants are a powerful instrument for providing incentives to the agent. Thus, there is considerable debate in the prior literature about the optimality of stock option compensation for senior-level executives.

The purpose of this paper is to further investigate the use of stock options in compensation contracts for CEOs. We first develop an agency model that mimics the real world contracting problem between the board (acting on behalf of shareholders) and the CEO.
Some of the important features of our model are that the CEO’s compensation contract is limited to fixed salary, at-the-money stock options, and restricted stock, the fixed salary is assumed to be nonnegative (i.e., there is limited liability for the agent), the agent is assumed to have power utility where wealth plays an important role, outside wealth consists of a fixed component and a portfolio of pre-existing stock and stock options with a stochastic payoff, and the CEO exerts effort that affects all the moments of the lognormal distribution of stock price. We then employ numerical methods to solve this bi-level optimization problem for the optimal CEO compensation contract for a subsample of firms from the Fortune 500 during the 2000 to 2004 time period.

Our analysis produces three important results. First, in marked contrast to the conclusions by Meulbroek (2001), Hall and Murphy (2002) and Dittmann and Maug (2007) that do not solve the complete bi-level optimization problem between the principal and agent, we find that stock options are almost always an important part of the optimal CEO compensation contract. Second, consistent with Aseff and Santos (2005), restricting the compensation contract to fixed salary, at-the-money stock options, and restricted stock produces roughly the same expected payoff to owners as the optimal unrestricted second-best compensation contract. This result suggests that simple observed compensation contracts are robust to restrictions on the contract space. Finally, similar to the observations made by Core, Guay, and Verrecchia (2003), the incentive effects of fixed salary, at-the-money stock options, and restricted stock for some CEOs are dominated by the level and composition of the executive’s pre-existing wealth. For these CEOs, the principal’s choice of compensation contract is essentially the amount of fixed salary that is necessary to satisfy the outside reservation wage (i.e., the individual rationality constraint).

The remainder of the paper consists of six sections. The relevant prior research on ob-
served executive compensation contracts is reviewed in Section I. We specify our agency model and develop our numerical optimization approach in Section II. Section III discusses our sample and measurement choices. The contracting results for our sample are presented in Section IV. Section V provides sensitivity and validation analyses. Conclusions and limitations are discussed in Section VI.

I. Prior Research

The analysis of compensation contract choice, especially the use of stock options and restricted stock, has been a popular topic for analytical and numerical research. For example, Meulbroek (2001) argues that risk averse and undiversified executives do not place enough value on the risky payout they will receive from an option to justify the cost given up by shareholders (and implicitly the incentives provided by the options). However, Meulbroek (2001) does not model the incentive effects of the stock options and this makes it problematic for her to assess the net benefit to shareholders from using stock options. Similarly, using the certainty equivalent approach of Lambert, Larcker, and Verrecchia (1991), Hall and Murphy (2002) conclude that restricted stock (which is essentially an option with an exercise price of zero) dominates options with a non-zero exercise price. However, their numerical results are also based on a “partial equilibrium” analysis that does not formally incorporate the cost of the option, the value to the employee, or the incentives provided by the options into an optimization program. Since the incentives provided by stock options are a key reason for their use in compensation contracts, it is impossible to make substantive conclusions about the relative desirability (and optimality) of stock options or restricted stock unless incentives are actually modeled in the analysis.

In contrast to Meulbroek (2001) and Hall and Murphy (2002), Kadan and Swinkels (2006)
analyze and provide some empirical tests of a fully specified optimization model where the agent’s compensation contract consists of salary and either stock options or restricted stock (i.e., a stock option with an exercise price of zero), but not both.\textsuperscript{1} Their formulation departs from the traditional agency model by incorporating a minimum payment constraint or limited liability (e.g., Innes (1990)) and a positive probability that stock price is equal to zero, which they term “non-viability risk”. Using the first order approach (FOA) to represent the agent’s optimization problem, Kadan and Swinkels (2006) find that stock options dominate restricted stock when non-viability risk is zero.\textsuperscript{2} Using a sample of firms from ExecuComp, they also find that the probability of bankruptcy (as a measure of non-viability) risk is positively related to the use of restricted stock. Since the probability of non-viability risk is likely to be low for most firms, the results in Kadan and Swinkels (2006) imply that stock options should be part of the optimal CEO compensation contract.\textsuperscript{3}

Aseff and Santos (2005) examine a standard agency model where the agent takes either a high or low action which results in a continuous stock price outcome. They also assume that the FOA can be used to represent the agent’s problem. The agent’s salary is bounded from below (but can be negative), the compensation contract consists of only fixed salary

\textsuperscript{1}Feltham and Wu (2001) also develop a fully specified optimization model that includes either stock options or restricted stock. They find that restricted stock dominates (does not necessarily dominate) option-based contracts that when the agent affects only the mean (both the mean and the variance) of the outcome, However, their model structure and solution technique exhibit several problematic features such as a mean-variance approximation to the agent’s expected utility which is unlikely to be accurate when the agent’s payoff is skewed with stock option contracts, reliance on the first-order approach which is inappropriate for this setting, and unconstrained salary for the agent.

\textsuperscript{2}In order to justify the FOA, Kadan and Swinkels (2006) assume that the distribution of $F(x|e)$, or the cumulative distribution of stock price given the agent’s choice of effort, satisfies the convexity of the distribution function (CDFC). It is interesting to think about what type distribution satisfies this assumption. In their numerical examples, $F(x|e)$ is set to either $(1 - e + cx)$ or $(x + (1 - 2x)(1 - 2e)/2$. It is difficult to image how these distributions translate into the real world distributions or how they are useful for motivating empirical tests of hypotheses generated by a model making these distributional assumptions. In Section II.C, we show that it is generally problematic to use the FOA for analyzing compensation contracts that involve stock options.

\textsuperscript{3}This hypothesis is somewhat at odds with the general observation that young technology firms (with a high probability of bankruptcy) aggressively use stock options, as opposed to restricted stock, in the executive compensation programs (e.g., Ittner, Lambert, and Larcker, 2003).
and stock options, agent wealth is explicitly considered in the model, and the power function is used to represent the agent’s utility function. The primitive model inputs are developed by selecting parameters to mimic observed compensation payments and stock prices for a typical firm. Their numerical results suggest that the cost of moral hazard (where the agent selects the low action) to the principal is large, but that the use of a simple stock option contract can motivate the agent to select the high action with a very small additional cost. Thus, Aseff and Santos (2005) show that stock options are an important component of the observed executive compensation contracts.

Finally, Dittmann and Maug (2007) consider an agency model with a number of realistic features and use the FOA to assess whether observed CEO compensation contracts are optimal. In particular, their analysis assumes that observed compensation contracts implement the optimal action and asks whether the principal can write a contract that induces the same action with less cost. Thus, their analysis abstracts from the typical agency model by focusing only on the contract design (which is only one level (i.e., the “upper-level” or the principal’s) of the bi-level optimization). They find the very surprising result that stock options should almost never be part of the optimal compensation contract for CEOs.

Although this is a provocative conclusion, there are two questionable aspects in their analysis. First, they appear to assume that the beginning stock price anticipates the optimal effort that will be selected by the agent for a given compensation contract. If stock options are issued at-the-money and the strike price already reflects the expected optimal level of effort, then stock options have little incentive effect because the payoff to the agent (i.e., the intrinsic value) will be very small in expectation. Thus, it is not surprising that stock options do not enter the “optimal” contract in the analysis by Dittmann and Maug (2007). Second, their analysis relies on the ability of the FOA to construct a measure for the incentives
imposed on the agent. As we demonstrate below, the combination of lognormal stock price and power utility for the agent renders the FOA invalid, and consequently, their use of the utility-adjusted pay for performance sensitivity is problematic.

This brief literature review illustrates that there is controversy regarding the use of stock options in executive compensation plans. In order to provide some insight into the optimal use of stock options, we develop an agency model that mimics many of the features of the “real world” contracting problem between shareholders and the CEO. We also incorporate a number of the structural features from Aseff and Santos (2005), Kadan and Swinkels (2006), and Dittmann and Maug (2007) into our model.

II. Model
A. Basic Model Structure

We assume that the traditional moral hazard model is an appropriate representation of the contracting problem involving shareholders and the CEO. Our model is based on a traditional single period agency setting with a risk neutral principal and a risk and effort averse agent. Rather than selecting a set of assumptions to produce mathematical tractability, we develop the structure of our model based on features of the contracting environment that are observed in the real world. The cost associated with this choice is that the resulting model will be mathematically intractable and numerical methods are necessary to generate

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4 We could have also used the adverse selection (or hidden information) rather than the moral hazard (or hidden action) framework for modeling executive compensation. Although there is some debate about which model best approximates contracting with a CEO, Milgrom (1987) and Hagerty and Segal (1988) show that the adverse selection and moral hazard models are fundamentally similar and this modeling choice is largely arbitrary. Although this is essentially true in our modeling, the use of limited liability complicates the transformation from a moral hazard to an adverse selection model.

5 Our model only focuses on incentive issues. We do not consider other potentially important determinants of contract choice such as taxes, executive selection, and differential accounting treatments (e.g., salary versus stock options). This is a limitation of our analysis, as well as the prior research reviewed in Section I.
solutions. However, we believe that the insights produced by such a model outweigh the absence of a closed form solution for the contract.

In our model, the risk and effort averse agent has an additively separable utility function defined over terminal wealth (which consists of pre-existing wealth and the current period’s compensation) and effort. The agent’s disutility of effort is a convex and increasing function of effort. The agent selects an effort level to maximize the expected utility of flow compensation provided by the principal and existing wealth less the disutility of effort. We assume that the agent’s effort choice is made to satisfy the incentive compatibility (IC) constraints. Finally, we assume that the effort choice affects both the mean and variance of the stock price distribution.6

The risk neutral principal selects a compensation contract to maximize the expected value of the firm net of the expected compensation payment to the agent. In our primary analysis, the contract space is constrained to include fixed salary, stock options that are granted at-the-money (similar to most actual option grants), and restricted stock. Thus, the principal selects the level of salary, number of stock options, and number of restricted shares in the flow pay for the agent. Although this is a simplified characterization of actual executive compensation contracts, base salary, stock options, and restricted stock capture the majority of the value of compensation paid to executives. Similar to observed compensation arrangements, we also require the salary to be non-negative (i.e., the agent has limited liability).

The principal also observes the dollar level and individual components of the agent’s wealth at the beginning of the period. This is a reasonable assumption for the stock options

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6As discussed below, the agent’s action affects one of the parameters of the lognormal stock price distribution, which affects all of the moments of the price distribution, including the mean and variance.
and shares owned by the agent since these amounts are disclosed in proxy statements, but it is perhaps more questionable for the other cash component of agent wealth. We assume that the compensation payment satisfies the traditional (ex ante) individual rationality (IR) constraint that the expected utility of compensation less the cost of effort is greater than or equal to the utility of the outside reservation wage that the agent can earn in the labor market. This reservation wage is assumed to be constant and known to both the agent and the principal.

The structure of our basic agency model (exclusive of the agent’s pre-existing holdings and fixed wealth) is given by the following program (#1):

$$\max_{\alpha, \beta_1, \beta_2, a} \mathbb{E}[NP - (\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a]\$$

subject to

$$a \in \arg\max_{\tilde{a}} \{\mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|\tilde{a}] - D(\tilde{a})\} \quad (IC)$$

$$\mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a] - D(a) \geq U \quad (IR)$$

$$\alpha \geq 0 \quad (LL)$$

$$\beta_1, \beta_2 \geq 0 \quad (SS)$$

$$\beta_1 + \beta_2 \leq N \quad (TS)$$

where $N$ is the number of shares outstanding, $^7 P$ is the terminal per share price of the firm’s stock, $\alpha$ is the fixed salary payment, $\beta_1$ is the number of shares of restricted stock granted to the agent, $\beta_2$ is the number of options granted to the agent with a strike price of $K$, $D(a)$ is the agent’s disutility of effort, and $U$ is the agent’s reservation utility. (IC) and (IR) denote the agent’s incentive compatibility and individual rationality constraints, respectively, (LL)

$^7$Note that number of shares granted to the agent (i.e., $\beta_1$) is a reduction to the principal’s ownership of the firm, $N$. However, rather than modeling the options granted to the agent (i.e., $\beta_2$) as a reduction of the principal’s equity in only certain states (i.e., when $P > K$), we model stock options as if a cash payment is made to satisfy this claim upon the realization of the stock price.
is the limited liability constraint, (SS) is the short selling constraint that precludes the agent from short sales or writing call options, and (TS) is the total shares constraint which prevents the agent’s equity-based compensation from exceeding the firm’s total shares outstanding.

One important feature missing in program (1) is the role of the agent’s pre-existing fixed wealth and equity portfolio holdings of stock and options on the firm’s stock. Although the principal’s choice variables are the same as the case without pre-existing wealth, the flow compensation parameters only alter the agent’s incentives incremental to those produced by the pre-existing wealth. When we incorporate pre-existing wealth into the optimization, the principal’s problem is characterized by the following maximization program (2).

\[
\begin{align*}
\text{maximize} & \quad \mathbb{E}[(N - S)P - \text{Compensation} - \text{Options}|a] \\
\text{subject to} & \quad a \in \arg\max \{\mathbb{E}[U(\text{Wealth} + \text{Compensation})|\tilde{a}] - D(\tilde{a})\} \quad (IC') \\
& \quad \mathbb{E}[U(\text{Wealth} + \text{Compensation})|a] - D(a) \geq U' \quad (IR') \\
& \quad \alpha \geq 0 \quad (LL) \\
& \quad \beta_1, \beta_2 \geq 0 \quad (SS) \\
& \quad \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 \leq N \quad (TS')
\end{align*}
\]

where \( S \) is the agent’s pre-existing shares and \( \text{Compensation} \) is the agent’s compensation in the current period with the following payoff:

\[
\text{Compensation} = \alpha + \beta_1 P + \beta_2 \max\{P - K, 0\}.
\]

\( \text{Options} \) represents the payoff from the agent’s pre-existing options which, as discussed further below, fall into three different categories, and \( \beta_3, \beta_4, \) and \( \beta_5 \) \((K1, K2, \text{and } K3)\) are the
number (exercise price) of options in each category. The payoff for the pre-existing option is defined as follows:

\[
Options = \beta_3 \max\{P - K_1, 0\} + \beta_4 \max\{P - K_2, 0\} + \beta_3 \max\{P - K_3, 0\}.
\]

Wealth is sum of the agent’s pre-existing fixed wealth, shares, and stock options. \(U'\) is the agent’s reservation utility for both wealth and compensation and is defined as follows:

\[
U' = \mathbb{E}[U(\text{Wealth} + \text{External Wage})]
\]

The new total shares constraint (TS') precludes the agent from owning more shares and options (both pre-existing and from the current period’s compensation) than there are shares of the firm outstanding. The remaining constraints are similar to those discussed above for program (#1).

B. Concerns Regarding to the First Order Approach

The analytical and numerical analyses reviewed in Section II rely on the validity of the FOA in their solution technique. This approach replaces the continuum of the agent’s (IC) constraints with the first-order condition for an optimum. This “relaxed” version of the problem is more amenable to solution by standard nonlinear optimization techniques. While there are sufficient conditions where the FOA is known to be appropriate (e.g., Rogerson (1985), Jewitt (1988), and Araujo and Moreira (2001)), there are no known necessary conditions for its application. Moreover, the sufficient conditions found in the literature are highly special-

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8The three categories are options granted last period, exercisable options, and unexercisable options. This choice is related to the data that are available for developing model parameters (e.g., Core and Guay, 2002).

9We model the payoff from the pre-existing options as a contingent cash payment from the principal to the agent for the realized intrinsic value of the options.
ized (e.g., the convexity of the distribution function condition (CDFC) and the monotone likelihood ratio property (MLRP)) and can easily fail in the economic setting in the papers discussed in Section II as well as our model. Thus, it is important to verify the validity of the FOA before proposing a solution strategy for program (#2).

It is straightforward to demonstrate the failure of the FOA for our problem using actual company examples (we describe the sample and measurement choices below). The agent’s expected utility versus effort choices under the optimal compensation contract (consisting of salary, at-the-money stock options, and restricted stock) is plotted in Figure 1 for Archer Daniels Midland Co. and Paccar Inc. For both companies, this function has a “double hump” and expected utility is not a concave function of effort. Since this type of agent response raises concerns about the validity of the FOA, we do not use the “relaxed” version for generating our numerical solutions.10

C. Solution Strategy

We represent our model using discrete actions by the agent and continuous compensation contract parameters. The use of discrete actions allows us to employ the solution techniques of Grossman and Hart (1983) and avoid the reliance on the validity of the FOA. The Grossman and Hart (1983) approach also facilitates the search for a globally optimal solution for the compensation contract.11

Since there are only a finite number of actions, the Grossman and Hart (1983) approach

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10 Note that it is not necessary to show that the agent’s expected utility is not a concave function of the action, but it is sufficient. The agent’s expected utility could be concave in action for any (or all) given contract(s) and the FOA could still fail.

11 Our action space is much larger than the typical binary action space (i.e., high or low action) that is common in most prior research. We use 101 discrete actions by the agent and 501 discrete stock prices for each action in our numerical analysis.
first replaces the agent’s incentive compatibility constraint (IC) with the following set of inequalities:

\[ \mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a] - D(a) \geq \mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a_i] - D(a_i) \]

for each of the agent’s \( i = 1, \ldots, M \) possible actions. We introduce a binary variable \( y_i \in \{0, 1\} \) associated with each action \( a_i \in A \) so that \( y = (y_1, \ldots, y_M) \in R^M \). Finally, let \( e^M \) denote the vector of all ones in \( R^M \). The program for the optimal contract in program (#1) can then be reformulated as the following mixed-integer non-linear program (MINLP), which we refer to as program (#3)\(^{12}\):

\[
\begin{align*}
\text{maximize} & \quad \mathbb{E} \left[ NP - (\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\}) \left( \sum_{i=1}^{M} y_i a_i \right) \right] \\
\text{subject to} & \quad y \in \arg\max_{\tilde{y} \in \{0, 1\}, \sum \tilde{y}_i = 1} \left\{ \mathbb{E} \left[ U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\}) \left( \sum_{i=1}^{M} \tilde{y}_i a_i \right) \right] - D(\sum_{i=1}^{M} \tilde{y}_i a_i) \right\} \\
& \quad \mathbb{E} \left[ U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\}) \left( \sum_{i=1}^{M} y_i a_i \right) \right] - D(\sum_{i=1}^{M} y_i a_i) \geq U \\
& \quad \alpha \geq 0, \quad \beta_1 + \beta_2 \leq N, \quad \beta_1, \beta_2 \geq 0 \\
& \quad e_M^T y = 1, \quad y_i \in \{0, 1\} \quad \text{for all} \ i = 1, \ldots, M.
\end{align*}
\]

Program (#3) has \( Q \) nonlinear variables (where \( Q \) is the number of stock price outcomes for each action), \( M \) binary variables, one linear constraint, and \((M+1)\) nonlinear constraints. Since the agent will choose one, and only one action, the number of possible combinations on the binary vector \( y \) is only \( M \). Thus, we can solve \( M \) nonlinear (nonconvex) programs, where \( y_i = 1 \) (for \( i = 1, \ldots, M \)) and the other \( y_{i-1} = 0 \). Among those \( M \) solutions, we then select

\(^{12}\)In order to ease the notation in the text, these programs do not include agent wealth. The inclusion of wealth is a simple extension to the programs.
the feasible solution with the largest value of the objective function. Rather than solving
the program \#3 using a mixed-integer nonlinear program solver such as MINLP (Fletcher
and Leyffer, 1999) or BARON (Sahinidis and Tawarmalani, 2004), we follow Su and Judd
(2006) and transform our problem into the following MPEC formulation, which we refer to
as program (\#4):

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{M} \delta_i \mathbb{E} \left[ NP - (\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a_i \right] \\
\text{subject to} & \quad 0 \leq \delta_j \perp \sum_{i=1}^{M} \delta_i \cdot \left( \mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a_i] - D(a_i) \right) \\
& \quad - \left( \mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a_j] - D(a_j) \right) \geq 0 \\
& \quad \sum_{i=1}^{M} \delta_i \mathbb{E}[U(\alpha + \beta_1 P + \beta_2 \max\{P - K, 0\})|a_i] - D(a_i) \geq U \\
& \quad \alpha \geq 0, \quad \beta_1 + \beta_2 \leq N, \quad \beta_1, \beta_2 \geq 0 \\
& \quad \sum_{i=1}^{M} \delta_i = 1
\end{align*}
\]

In general, this program has only \((M + Q)\) variables and \(M\) complementarity constraints
with one linear constraint and one nonlinear constraint. The complementary constraints
require that if an (IC) constraint is not active (binding), then its multiplier must be zero. If
the particular (IC) constraint is active, then \(\delta_i = 1\) and \(\delta_{-i} = 0\), for that particular action,
and we solve the corresponding nonlinear program. One advantage of this formulation is
that it enables more flexibility in the choice of nonlinear programming solvers. This enables
us to check the robustness of our solutions (by comparing solutions from different solvers
such as KNITRO and SNOPT).
III. Sample and Measurement Choices

A. Sample

Our sample consists of 46 firms from the Fortune 500 where there was no CEO turnover during the time period from 2000 to 2004. A list of these companies and their ticker symbols are presented in Table I. These selection criteria obviously reduce our ability to generalize our results. We impose these criteria because we use the four-year period from 2001 to 2004 to compare the model results to actual CEO compensation and assess the validity of our model in Section V.B.\(^\text{13}\) Despite our modest sample size and the requirement for a constant CEO, we believe that our sample is sufficient for providing insight into the use of stock options in CEO compensation contracts for individual companies.

The descriptive statistics for our sample are presented in Table II (Panel A). Since we are selecting firms from the Fortune 500, it is not surprising that the mean (median) firm has very large with a market capitalization of $53,794 million ($12,871 million). In addition, this sample spans a variety of industrial and service sectors of the economy.

B. Measurement of Model Parameters

We assume that agent’s utility function can be characterized as a member of power class of functions, or 

\[
U(W + s) = \frac{1}{1 - \delta}(W + s)^{1+\delta}
\]

for \(\delta \geq 0\), where \(\delta\) is the coefficient of relative risk aversion, \(W\) is the agent’s pre-existing wealth, and \(s\) is the current period (or flow) compensation. This utility function exhibits decreasing absolute and constant relative risk aversion (CRRA). This choice is supported by the prior empirical work by Friend and Blume (1975) and Litzenberger and Ronn (1986). We adopt the power utility rather than the more

\(^{13}\)It is also necessary to use a sample much smaller than studies such as Dittmann and Maug (2007) because the computational time required to solve our bi-level optimization is on the order of several hours for each program for each company.
common (at least in analytical work) negative exponential utility (CARA) because we believe that managerial wealth is an important factor for understanding executive incentives. Friend and Blume (1975) estimate the risk aversion parameter for the power utility to be between two and three. Kocherlakota (1990) argues that this parameter is probably higher (perhaps in excess of ten), although Lucas (1994) suggests that the parameter should be around 2.5. Consistent with prior research, we set the coefficient of relative risk aversion to two in our primary analyses.\footnote{As discussed further below, we conduct a sensitivity analysis on a subset of our sample companies. For these companies we solve for the optimal restricted second-best contract under different combinations of the risk-aversion coefficient (i.e., 0.5, 2, and 4) and the disutility parameter.}

Since a complete measure of CEO wealth is not available from public data, we develop a proxy for this parameter. We assume that CEO wealth is composed of a fixed (nonstochastic) portion that is uncorrelated with stock price and a stochastic portion composed of existing stock options and shares owned by the CEO. We estimate the fixed dollar amount of CEO wealth as five times cash compensation (salary plus bonus) plus an estimate of the value for the supplemental executive retirement plan (measured as the present value, discounted at the risk-free rate, of a 15 year annuity equal to 60% of the CEO’s salary and bonus in the most recent year that starts paying out five years after the current year).

The stochastic wealth consists of shares of stock, restricted stock, and stock options owned by the CEO. Since complete information about the executive holdings are not available, we use the Core and Guay (2002) one-year approximation method with the information reported in the first proxy statement of our sample period (i.e., for the 2000 fiscal year end). This proxy statement reports the agent’s stock and restricted stock holdings from prior periods (which we group together and refer to as “pre-existing stock”), the number of exercisable options and their inferred average strike price (“pre-existing exercisable options”), the num-
ber of unexercisable options and the inferred average strike price (“pre-existing unexercisable options”), and the number and actual strike price of any option grants from the year prior to the proxy (“pre-existing new options”). The one-year approximation method assumes that the unexercisable (exercisable) options have a remaining life of one year (four years) less than the life of the newly granted options. This distinction, however, is lost in our single period setup, because we implicitly assume that all of the pre-existing option grants, as well as any new grants in the optimal compensation package have the same life and, accordingly, the same potential time value. The mean (median) fixed wealth for CEO sample is about $27.36 ($25.57) billion (Table II, Panel B). Moreover, CEOs also have substantial wealth invested in their company’s equity though both stock and option holdings.

Consistent with a large body of finance research and the basic distributional assumption for the Black-Scholes model, we assume that the firm’s stock price is characterized by a two parameter ($\mu$ and $\sigma$) lognormal distribution. We assume that the agent’s action impacts only the $\mu$ parameter (i.e., we assume that $\sigma^2$, the variance of the returns process, is exogenous) which shifts the mean of the underlying normal returns distribution and affects all moments of the lognormal price distribution. Specifically, a shift in $\mu$ will affect the mean ($exp[\mu + \sigma^2/2]$) and variance ($[exp(\sigma^2) - 1] \cdot exp[2\mu + \sigma^2]$) of the lognormal distribution. This enables us to capture the natural risk-return tradeoff associated with agent effort because increases in effort increase both the mean and variance of the lognormal price distribution.

The parameter $\sigma$ is measured using the standard deviation of daily returns over the prior

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15If there was more than a single option grant in the prior year, we aggregate the options together as if there were a single grant of the total number of options with a strike price that preserves the sum of the total Black-Scholes value of the individual grants. Thus, we fix the number of options in the aggregate grant equal to the total number of options in the individual grants, and the Black-Scholes value of the aggregate grant equal to the sum of the Black-Scholes value of the individual grants and solve for the unique strike price and use the resulting number as the strike price for the “pre-existing new options”.

16For the typical option grant with a ten year life, the one-year approximation method implies an estimated life of nine years and six years, respectively, for the unexercisable and exercisable options.

17This assumption implies that returns are normally distributed, with mean $\mu$ and variance $\sigma^2$. 
year. The mean (median) annual $\sigma$ for our sample is 0.489 (0.456)

One especially crucial modeling choice is the “production technology” that translates the agent’s effort (e.g., choice of strategy, operational investments, long-term investments, and other similar managerial tasks) into $\mu$. We arbitrarily restrict (and implicitly scale) agent effort to take discrete integer values between zero and 100. We also assume that $\mu$ is a piecewise linear function of the agent’s effort (see the illustrative examples for Archer Daniels Midland Co. and Paccar Inc. in Panels A and B of Figure 2, respectively). At an effort equal to zero, we assume the firm earns the risk-free rate of return. Since this return is less than the firm’s estimated cost of capital, $\mu$ of the lognormal price distribution will be negative, which implies a negative expected abnormal return. At an effort level of 29 (or the 30th action), we assume that $\mu$ is equal to zero, which implies the firm’s expected return will equal its cost of capital. At an effort level of 100, we assume that the firm earns an annual rate of return equal to the annualized return implied by the high four-year target price reported by Value Line. $\mu$ implied by intermediate effort choices are (piecewise-) linearly interpolated between these three points. We report distributional statistics for the slope of each piece in Table II (Panel A) and we find that the production technology is concave (convex) for 24 (22) of the firms in our sample (untabulated).

Finally, in order to calculate each agent’s reservation utility for the (IR) constraint, we assume that the agent’s compensation in the external labor market over the next four years would equal four times the median (three-digit SIC) industry compensation for the most re-

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18We estimate the cost-of-capital for each company using the Capital Asset Pricing Model with a risk-free rate and market-risk premium equal to 5.24% and 6.00%, respectively (which are approximately the prevailing rates at the beginning of our sample period). Each company’s Beta was estimated using monthly returns over the prior 60 months. These values are reported in Table II (Panel A).

19Note that the use of analysts’ price forecasts assumes that analysts do not incorporate the expected effect on the firm’s stock price of the CEO’s actions that are induced by the observed contract. Using the highest analyst forecasts rather than the average analyst forecast mitigates this problem because the highest analyst forecast is more likely to have an idiosyncratic measurement error that overstates the maximum outcome from agent effort.
cent year for all CEO’s in the Fortune 500. We use four years in this computation because this captures the approximate term for a CEO and we are using the four-year Value Line forecast for returns. The agent’s expected utility from the pre-existing (fixed and stochastic) wealth plus the industry median compensation is evaluated over the firm’s price distribution induced by an action equal to zero (i.e., the firm’s expected return is equal to the risk-free rate less the cost-of-capital) and the agent experiences no associated disutility of effort.

C. Scaling Constants

One common issue in numerical analysis concerns the choice of scaling for the objective functions and constraints. Since the agent’s utility is defined over consumption of both flow compensation and wealth, it is necessary to scale these figures in order to produce a utility number that is “reasonable” for numerical analysis. For example, if the risk aversion parameter is equal to two, the utility of $100 million dollars of non-stochastic flow compensation and wealth is \( \frac{1}{1-2}(-100,000,000)^{-2} = -10^{-8} \), which is very close to zero from a computational perspective. Further, the agent’s marginal utility is \((100,000,000)^{-2} = 10^{-16} \), which is numerically indistinguishable from zero for conventional levels of precision.

In order to mitigate these types of numerical issues, we deflate the agent’s monetary consumption (both pre-existing wealth and flow compensation) by 129,000,000, which is approximately the median value of total wealth for the CEO’s in the Fortune 500. This

20Because the median industry compensation for all industries represented in our sample includes stock options, we used the industry median annual Black-Scholes value of the options granted. We then calculate the company-specific number of at-the-money options that would yield the industry median Black-Scholes value and use this number for the industry median compensation.

21The estimated total wealth for the executives in our sample is the sum of fixed wealth, value of their stock holdings, and the Black-Scholes value of their various option holdings. Although many papers show that a risk averse executive values an employee stock option at less than its Black-Scholes value (e.g., Lambert, Larcker, and Verrecchia, 1991), this should provide a reasonable approximation for computing a scaling multiplier.
scaling serves to “shift” the agent back on the utility function where both the (1) overall expected utility from consumption is a smaller value (but larger in absolute value) and (2) marginal utility of consumption is a larger value (e.g., the agent’s marginal utility in the example above would be $(\frac{100,000,000}{129,000,000})^{-2} = 1.6641$). Since utility is a scale-free construct, this approach is empirically valid.

A more critical scaling parameter is the multiplier for the agent’s disutility of effort. We assume that the disutility function, $D(a)$, is equal to a scaling parameter ($\lambda$) multiplied by the square of effort, or $\lambda \cdot a^2$. Since the agent’s utility is additively separable in monetary consumption and disutility of effort, this multiplier scales the agent’s disutility of effort to ensure that it is of the same “order of magnitude” as the utility from consumption. We estimate this multiplier by determining the value of $\lambda$ that will result in the observed compensation contract for the median firm in our sample. Specifically, we assume that the agent takes an action of 29 (i.e., the action that yields expected returns equal to the hypothetical firm’s cost-of-capital) and then solve for the multiplier for which the principal would select a contract that is most similar to the median contract values observed in the data.22 A crucial point to emphasize is that the arbitrary consumption multiplier (i.e., 129,000,000) also affects the calculation of the disutility multiplier because the disutility multiplier is calculated using the scaled median values for our sample. However, this preserves the relative unscaled values of the marginal utility from consumption and the marginal disutility of effort.

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22This approach for solving for the disutility multiplier assumes that our model represents the actual contracting process between the principal and agent.
IV. Results

A. Unconstrained Second-Best Solution

The results for the second-best solution with an unconstrained compensation contract are computed using the basic structure of program #4 with two key changes. First, the unconstrained solution consists of a cash payment for each stock price outcome (as opposed to a salary, stock option, and restricted stock contract) and second, agent wealth is included in the problem. We solved for the optimal unrestricted contract for each of the 46 companies in our sample and we present results related to the distribution of optimal contracts across the 46 companies in Table III. In addition, the typical shape of the optimal unconstrained contract is illustrated in Figure 2 for Archer Daniels Midland Co. (Panel A) and Paccar Inc. (Panel B). The first plot in both panels of Figure 2 shows that the unconstrained compensation function is convex for low stock prices and becomes concave at higher stock prices. Both contracts provide zero payment to the agent until the realized stock price is fairly close to the expected stock price given the optimal action. In this region, the contract is highly convex (e.g., for Archer Daniels Midland Co., a change in the stock price from $12 to $13 produces an increase in the fixed payment from $0 to $178 million). This part of the contract is very similar to an option. However, unlike an option, the payment is also zero for very high observed stock prices. This occurs because the principal is likely to infer that these high outcomes are due to a high random outcome (i.e., “good luck”) as opposed to the agent providing a high level of effort. Stated differently, for very high outcomes, the principal is almost certain that agent deviated from taking the desired (i.e., optimal) action so the principal pays the agent nothing in these cases.

The fixed payments to the agent are substantially larger than the typical flow compensation for CEOs (even after considering that the payments in Figure 2 are for a four year
period). Although these payments to the agent are large (about $200 to $500 million), the expected payoff to the principal is also extremely large for high levels of agent effort (approximately $11 billion for both companies). In these cases, the principal is only paying 6.27% and 2.75% (for Archer Daniels Midland Co. and Paccar Inc., respectively) of the change in expected value of the firm to the agent. The magnitude of the sharing rule given these levels of wealth creation is consistent with the Haubrich (1994) critique of the Jensen and Murphy (1990) challenge to the agency model.\(^{23}\)

**B. Constrained Second-Best Solution**

The constrained second-best contract (consisting of salary, at-the-money stock options, and restricted stock over four years) results are computed using the approach in program #4 and the results are presented in Table IV. As expected, the optimal agent effort is less than or equal to the effort level observed in the unconstrained contract case. Similar to Aseff and Santos (2005), we find that, on average, there is only a modest loss in expected payoff to the principal when the constrained contract is used rather than the more complicated unconstrained second-best contract. For our sample, the mean (median) loss in expected stock price caused by using a constrained contract is only $273 ($52) million.

Before discussing our results, it is important to demonstrate that the model and parameter choices produce compensation contracts that are reasonably similar to the range of actual CEO compensation contracts. In Table V, we compare the second-best constrained optimal contracts to the actual CEO contracts for our sample companies. Although there

\(^{23}\)These results provide some insights into the recent movement of executives from public companies to private equity firms (e.g., Thornton, 2006; Guerrera, 2006). It may be possible for private equity firms to implement something like the unconstrained second-best contract because there are no external constituencies to satisfy or they have a more analytical economic approach to contract design. If this is the case, our model provides a rational economic explanation for compensation payments to private equity partners on the order of several hundred million dollars.
are some differences between the optimal contract and the observed contracts in terms of composition, the level of the total compensation is very similar. This illustrates that the optimal contract contracts are plausible relative to contracts that are observed in practice.

The distributional statistics in Table IV show that options are almost always part of the optimal contract and are almost always the primary source of new equity incentives (rather than restricted stock).\textsuperscript{24} Similar to the optimal unrestricted contract presented in Table III and Figure 2, options allow for the agent to receive payment only if a certain stock price is achieved (namely, the beginning stock price in the case of options issued at-the-money, which is the case in our analysis and in almost every observed compensation contract). In the case of very low outcomes (assuming that a relatively high outcome is desired), the principal can infer that the agent was unlikely to have taken the desired action and therefore does not pay the agent. In the case of a very high outcome, the principal can likewise infer that the agent was unlikely to have taken the desired action and does not want to pay the agent. This can be achieved in the case of the unrestricted contract from the previous section, but not with typical stock options where the payoff is not constrained. Although this is a limitation of stock options relative to the unrestricted second-best contract, this structure dominates restricted stock because stock pays the agent for both very low and very high outcomes where the agent probably did not take the optimal action.

Although stock options are the dominant source of new equity incentives in most of the optimal restricted contracts, they are not the only source (i.e., at least 1,000 shares of restricted stock appear in the contract for 24 firms, but not for the remaining 22 firms). We conjecture that the optimal restricted contract for certain companies relies, at least in

\textsuperscript{24}We make the distinction between new equity incentives which come from any restricted stock and stock options in the optimal contract and pre-existing equity incentives which come from the agent’s previously held stock and stock options. In addition, since the agent’s utility function exhibits wealth effects, any salary payment can also have an indirect on both the agent’s new and pre-existing equity incentives.
part, on restricted stock rather than solely stock options for primarily two reasons. First, since we assume the agent’s utility function exhibits wealth effects, restricted stock provides a way for the principal to “insure” the agent in very low outcome states. In particular, for very low stock price realizations, the agent’s marginal utility is extremely high (since the agent’s wealth is concentrated in firm-specific holdings). Therefore, shares provide some consumption to the agent in these states, unlike stock options. Second, when the principal wants to induce a relatively low action (e.g., because it is either too costly or impossible to induce the agent to take a high action), shares of restricted stock become relatively less expensive (compared to stock options) less expensive, so they become part of the optimal contract.

Although it is impractical to present the elements of the optimal restricted second-best contract for each company, a few examples are instructive. First, consider the case of Lyondell Chemical Co. (LYO). This company is relatively small (market capitalization of $1.881 million) with a high volatility (sigma of 0.57, which is above the 75th percentile of 0.54), average risk (Beta of 0.73 which is roughly equal to the median value of 0.72) and low growth (high VL forecast of 13.43% which is close to the value at the 25th percentile of 13.25%). The CEO’s pre-existing holdings consist of a large amount of fixed wealth (about $31 million), relatively low share holdings (about 679,000 shares), and a relatively high number of options (about 535,000 new, 309,000, unexercisable and 187,000 exercisable stock options). The optimal restricted contract consists of salary of about $4,664,000, about 657,000 shares of restricted stock and about 9,000 stock options and this induces an optimal effort of 26, which results in expected returns of slightly below the firm’s cost of capital. In this case, the firm’s low growth prospects make a high action less desirable to the principal. This coupled with the fact that the agent has relatively high-powered incentives from his pre-existing wealth imply that the optimal contract will consist of primarily salary (which
is the most cost-effective instrument for satisfying the agent’s IR constraint).

A second (and more common) pattern is illustrated by the case of Proctor & Gamble Co. (PG). The company is very large (market capitalization of $150 billion), has slightly below average growth prospects (high VL forecast of 18.49% compared with the median value of 21.15%), is relatively volatile (sigma of 0.538, which is slightly below the 75th percentile) and has average risk (Beta of 0.46 which is slightly below the median value of 0.47). The CEO’s pre-existing holdings are also fairly typical (with fixed wealth of about $11.4 million, about 641,000 shares of stock and about 787,000 total stock options). The optimal contract calls for (almost) no salary or shares but about 3,500,000 stock options. In this case, the principal wants to induce a relatively high level of effort (optimal effort of 89), but using restricted stock to induce this effort would be too costly because the agent would receive a payoff in every state up to (and beyond) the expected stock price given the optimal action – even those states of a very low outcome which are almost improbable conditional on the agent taking the optimal action. Since there is a relatively high expected payoff, at-the-money stock options satisfy the agent’s IR constraint in expectation.25

The components of the second-best constrained contract also vary considerably across firms. There are three cases where the salary, number of at-the-money stock options, and restricted share are trivial in magnitude (Hewlett Packard, United Technologies, and Harley Davidson). For these companies, flow pay has minimal incentives effects and serves primarily to satisfy the agent’s IR constraint and agent incentives are primarily produced by the pre-existing exogenous wealth.26 For four companies (Rohm & Haas, Smithfield Foods, General Mills, and Deere) the optimal constrained compensation contract is essentially all fixed

\footnote{Recall that the agent’s IR constraint is an ex ante constraint that must only be satisfied in expectation rather than an ex post constraint which would have to be satisfied in every outcome.}

\footnote{We confirmed this point by computing the agent’s effort choice after constraining flow pay to zero.}
salary. In these companies, additional equity incentives are too costly for principal and salary is used either to satisfy the agent’s (IR) constraint and/or mitigate the agent’s risk aversion. Another feature of companies with a very large salary component in flow pay is that they tend to exhibit small values of systematic risk (Beta). The absence of stock options and restricted stock in the flow pay is a result of the low expected benefit in the production function from using equity incentives to increase agent effort (up to action 30). Although the production technology for these companies is likely to be convex after moving beyond action 30, the expected benefit to the principal needs to be very high in order to compensate the agent for the substantial disutility incurred at high levels of effort.

In 34 out of the 46 companies in our sample, the optimal second-best restricted contract calls for a material amount (which we define as more than 10,000 units) of new equity incentives (i.e., either restricted stock or stock options). The optimal contract for the remaining 12 companies consists almost entirely of salary. In 31 of the 34 cases where the contract calls for new equity incentives, the contract consists of more options than shares of stock. Further, in 27 of the 34 cases, the optimal contract calls for at least twice as many options as shares of stock, and in 24 of the cases, the optimal contract calls for more than five times the number of options as shares of stock. Although it is difficult to make a comparison of stock and option in terms of their relative importance in producing incentives and the relative valuation to the CEO, the number of options vastly exceeds the number of shares in the most optimal contracts. This result contrasts with the conclusions of Meulbroek (2001), Hall and Murphy (2002), and Dittmann and Maug (2007). Thus, stock options dominate restricted stock for most companies after the incentive effects of stock options are explicitly considered in the analysis (which is absent from the Hall and Murphy (2002) analysis) and incentives are correctly modeled (the limitation in Dittmann and Maug (2007)).
V. Extensions
A. Sensitivity Analysis

Although we calibrated our model in the previous section to observed data, it is still necessary to make assumptions about certain functional forms and parameters. In order to assess the sensitivity of the result that stock options are usually a component of the optimal contract, we solved for the optimal restricted second-best contract using a variety of alternative parameter estimates. In particular, two important parameters for our model are the risk aversion parameter and the disutility parameter. Therefore, we selected five companies (ADM, BC, DOV, SLE, and TXT) where options are a large component of the optimal restricted second-best contract and solved for the optimal contract varying both the risk-aversion parameter and the disutility multiplier.

In the previous section, we used a coefficient of relative risk-aversion of 2.0 and a disutility multiplier of 0.000075. We estimated the model for the five companies using a coefficient of absolute risk-aversion of 0.5 and 4.0. Since the disutility multiplier and the CEOs’ reservation utilities are a function of the assumed level of risk-aversion, we estimated a new disutility multiplier within each level of risk-aversion. We assume that our original disutility multiplier was “low”, so we multiply this figure by five to yield a “high” disutility multiplier. Therefore, we estimate the optimal restricted second-best contract for five additional sets of parameters.

The results of the sensitivity analysis for the five selected companies are presented in Table VI. There are a few noteworthy general observations. First, for the combination of low risk-aversion and a low disutility multiplier (i.e., the first row for each company), the use of restricted stock is virtually nonexistent, but stock options are an important part of the optimal contract. This combination implies that the principal wants to induce a relatively high level of effort from the agent (i.e., the maximum level of effort in four of the five cases)
which makes using restricted stock relatively costly. This again occurs because the expected terminal stock price is high and using restricted stock provides the agent with a payoff over the entire range of stock prices. Options, however, do not result in a payoff for the agent below the exercise price, so the principal does not compensate the agent over this part of the stock price distribution.

Focusing on the case where the agent’s coefficient of relative risk aversion is 2.0, as the disutility parameter increases (from 0.000075 to 0.000375), the optimal contract (1) induces a lower action and (2) supplants stock options with restricted stock in all five cases. With a higher level of disutility, it is more costly to provide incentives to the agent, so the principal accepts a lower level of effort at the optimum. And opposite of the first result discussed, when the principal wants to induce a lower level of effort, restricted stock becomes relatively less expensive for the principal because the restricted stock will result in less compensation to the agent over the stock price distribution.

Finally, as the degree of risk-aversion increases, the use of stock options decreases. Although it is difficult to compare results across different classes of risk-aversion (because of differences in the agents’ reservation utilities and differences in the disutility multiplier), this result indicates that as the agent’s risk aversion increases, the cost of using stock options more than outweighs the increase in incentive benefits (if any) from using stock options. Since the payoff associated with stock options is riskier than fixed salary (which is risk-free by definition) and restricted stock, the principal is required to pay the agent a higher risk-premium to use this form of compensation, which more than offsets any potential increased incentive benefit from their use.
B. Validation

Although the optimal second-best constrained contract is presented in Table IV, it is also instructive to estimate the agent effort and expected payoff to the principal using the actual compensation paid to the CEO during the subsequent four years (2002-2005). In particular, we use the actual compensation contract (which, like the optimal restricted contract, consists primarily of a fixed salary, restricted stock, and at-the-money stock options) as an input for our model and then compute the induced agent effort and expected payoff to the principal. Although the contracts are different than the constrained second-best contract, the agent’s effort choice is often the same with the actual observed flow compensation (although the expected payoff to the principal is lower in these cases). Another interesting output from these computations is that we can also compute the expected value of the \( \theta \) parameter of the lognormal price distribution induced by the observed compensation contract. If our model captures the important features of the contracting environment and if compensation contracts have an important impact on firm performance, then we should observe a positive association between expected firm performance (conditional on the observed contract) and actual firm performance.\(^{27}\) In Figure 4, we plot the average monthly excess returns (controlling for the three Fama-French (1993) factors and Carhart’s (1997) fourth momentum factor) over the four year time period from 2002 to 2005 versus the predicted performance induced by the actual contract. An ordinary least squares analysis reveals that the slope coefficient is 0.050 \((p < 0.05, \text{two-tail})\), intercept is 0.010 \((p < 0.01, \text{two-tail})\), and the \( R^2 \) (adjusted \( R^2 \)) is equal to 8.41% (6.33%). These results are consistent with our expectations and provide some validation of our agency model (and the associated functional forms and

\(^{27}\)This analysis assumes that the market does not fully impound the expected optimal action in the beginning price. If the model is correct and the market perfectly anticipates the optimal action then this should be reflected in beginning price and there should be no relation between predicted and future realized abnormal returns.
parameter estimates).

VI. Summary and Conclusions

In this paper, we develop and analyze a moral hazard agency model based on observed characteristics of executives, typical compensation plans, and stylized features of the contracting environment. Some of these features are (i) a compensation contract that consists of fixed salary, restricted stock, and stock options granted at-the-money, (ii) fixed salary that is greater than or equal to zero (i.e., limited liability), (iii) power utility for the agent, (iv) pre-existing wealth (both fixed and stochastic) which we show plays an important role in designing the optimal contract, (v) lognormally distributed stock prices, and (vi) a production function where agent effort affects all the moments of the distribution of stock price. We believe that this model structure captures many of the important observed features of contracting arrangements between owners (i.e., principals) and CEOs (i.e., agents).

Given the constraints and distributional assumptions of our model, it is not possible to develop a closed form mathematical solution to the principal’s problem. Therefore, we solve the principal-agent model using numerical optimization methods. We represent our model using discrete actions by the agent, discrete stock prices, and continuous compensation contract parameters. The use of discrete actions allows us to employ the solution techniques of Grossman and Hart (1983).

For our sample of firms, we find that the optimal unrestricted contract (which is characterized by state-contingent cash payments) exhibits option-like characteristics (i.e., a convex payoff) over the lower range of outcomes. The optimal contract typically becomes concave for high outcomes and then results in no payoff to the agent for very high outcomes (relative
to the expected outcome when the optimal action is taken). This result occurs because from an informativeness perspective, the relatively high outcome was unlikely to be the result of an optimal action choice by the agent.

When we restrict the contract space to consist of only fixed salary, restricted stock, and stock options granted at-the-money, we find that the optimal compensation contract frequently includes large quantities of stock options, which mimic the payoff from the optimal unrestricted contract over the most likely range of outcomes (given the action is taken by the agent). These numerical results are at odds with the numerical results and conclusions by Meulbroek (2001), Hall and Murphy (2002), and Dittmann and Maug (2007). The primary reasons for these differences are that we solve the complete bi-level optimization problem for both the principal and agent (rather than simply focusing on the agent’s problem in isolation) and we do not rely on the first-order approach in our solution technique (which we demonstrate is invalid in this contracting setting).

Constraining the compensation contract to fixed salary, at-the-money stock options, and restricted stock (as opposed to an unrestricted compensation contracts) also produces roughly the same expected payoff to owners in most cases. This result is consistent with Aseff and Santos (2005) and suggests that simple observed compensation contracts can be fairly close to the optimal unrestricted contract. Similar to Core, Guay, and Verrecchia (2003), we find that the incentive effects of fixed salary, at-the-money stock options, and restricted stock are dominated by the level and composition of CEO wealth for some companies.

Finally, for a subsample of the firms in our analysis, we demonstrate that the above results are reasonably robust to our choice of parameters for agent risk aversion and disutility of effort. We also find that the firm performance predicted using our model and the observed compensation payments is able to explain some of the actual excess stock price performance.
of our firms. These results suggest that our model has at least some degree of validity.

Our analytical and empirical results are subject to a variety of limitations related to the specific assumptions used in our model. First, we rely on a lognormal distribution of stock prices and our model captures only the risk-return tradeoff inherent in this specific distribution. Although this is a somewhat standard assumption in the finance literature, there are other reasonable ways to describe the impact on the agent’s effort choice on the distribution of stock price outcomes. Second, our choice of the production function assumes that the agent’s productivity is a specific piecewise linear function of both the firm’s cost of capital and the analyst long-term price forecasts. Third, our model includes only a single action that leads to a change in the distribution of stock price. The role of accounting information and accounting-based compensation contracts (e.g., annual bonus or performance plans) is ignored. Fourth, we assume that the power utility function describes the executives’ preferences for monetary consumption and that a quadratic cost function describes the agent’s disutility for effort. Finally, our analysis is conducted in a single period setting. This requires us to abstract away from undoubtedly important features of real world contracting settings, such as the early exercise of stock options (and thus their time value), inter-temporal effort allocation, and consumption smoothing.
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Figure 1
Agent Effort and Expected Utility for the Optimal Restricted Compensation Contract for Archer Daniels Midland Co. and Paccar Inc.

This figure plots the agent’s expected utility as a function of his action under the optimal restricted contract (which consists of a fixed salary, restricted stock, and at-the-money stock options). This optimization represents the lower-level (i.e., the agent’s) optimization of the complete moral hazard problem. The figure illustrates that there are multiple local optima which is a sufficient condition for demonstrating the failure of the first-order approach as a solution technique.
Figure 2
Production Technology of the Relation Between Agent Effort and $\mu$ of the Returns/Price Distribution

This figure plots the agent’s production function for Archer Daniels Midland Co. (ADM) and Paccar Inc. (PCAR). The production function translates the agent’s action into the $\mu$ parameter of the lognormal distribution of stock price (which corresponds to the mean of a normally distributed returns distribution). The production function is a piecewise linear function between (1) the risk-free rate of return and the firm’s cost of capital (calculated using the Capital Asset Pricing Model) and (2) the firm’s cost of capital and the high Value Line price forecast. All returns are expressed relative to the firm’s cost of capital, so for an action of zero, the return is the risk-free rate less the firm’s cost of capital.
Figure 3

Optimal Unrestricted Second-best Contracts for the Optimal Action for Archer Daniels Midland (ADM) and Paccar (PCAR)

The first figure shows the optimal unrestricted second-best contract (which consists of a state-contingent cash payment to the agent) and the probability density function of stock price, conditional on the optimal action of 75, for Archer Daniels Midland Co. (ADM). The fixed payment to the agent (in millions of dollars) is on the left vertical axis and the probability of the realized stock price is on the right vertical axis and both are plotted against realized stock price which is on the horizontal axis. The second figure shows the agent’s utility (on the left vertical axis) and the probability density function of stock price, conditional on the optimal action of 75, for ADM. In both figures, a dashed vertical line indicates the expected stock price (i.e., the mean of the lognormal distribution) of $75.60 given the optimal action of 75.
Figure 3 (continued)

The first figure shows the optimal unrestricted second-best contract (which consists of a state-contingent cash payment to the agent) and the probability density function of stock price, conditional on the optimal action of 88, for Paccar Inc. (PCAR). The fixed payment to the agent (in millions of dollars) is on the left vertical axis and the probability of the realized stock price is on the right vertical axis and both are plotted against realized stock price which is on the horizontal axis. The second figure shows the agent’s utility (on the left vertical axis) and the probability density function of stock price, conditional on the optimal action of 88, for PCAR. In both figures, a dashed vertical line indicates the expected stock price (i.e., the mean of the lognormal distribution) of $18.08 given the optimal action of 88.
Figure 4

Plot of Actual Excess Return versus Predicted Excess Return

This figure shows the actual excess returns plotted against the predicted excess over the period 2001 to 2004. The sample is the 46 firms in the Fortune 500 in 2000 that remained in the Fortune 500 every year from 2001 to 2004 and had the same CEO from 2001 to 2004. Actual excess returns are calculated using the three Fama-French (1993) factors and Carhart’s (1997) momentum factor. Predicted excess returns are calculated as $\mu$ using the agent’s optimal response to the observed compensation contract (of fixed salary, restricted stock, and stock options) along with pre-existing wealth and the assumption about firm-specific the production function. The estimated intercept and slope are 0.010 and 0.050, respectively.
Table I
Sample Companies

This table lists the sample of 46 firms companies used in this study in alphabetical order. The sample was constructed using the companies in the Fortune 500 in 2000 remained in the Fortune 500 every year from 2001 to 2004 and had the same CEO from 2001 to 2004. The company name and ticker symbol are listed for each company.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Symbol</th>
<th>Name</th>
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<td>AAPL</td>
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<td>HORMEL FOODS CORP /DE/</td>
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<tr>
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<td>ARCHER DANIELS MIDLAND CO</td>
<td>INTC</td>
<td>INTEL CORP</td>
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<td>AMGN</td>
<td>AMGEN INC</td>
<td>ITW</td>
<td>ILLINOIS TOOL WORKS INC</td>
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<td>ASD</td>
<td>AMERICAN STANDARD COMPANIES INC</td>
<td>K</td>
<td>KELLOGG CO</td>
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<td>AVON PRODUCTS INC</td>
<td>LLY</td>
<td>LILLY ELI &amp; CO</td>
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<td>LXK</td>
<td>LEXMARK INTERNATIONAL INC /KY/</td>
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<td>LYO</td>
<td>LYONDELL CHEMICAL CO</td>
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<td>CONAGRA FOODS INC /DE/</td>
<td>MRK</td>
<td>MERCK &amp; CO INC</td>
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<tr>
<td>CL</td>
<td>COLGATE PALMOLIVE CO</td>
<td>PBI</td>
<td>PITNEY BOWES INC /DE/</td>
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<td>CUMMINS INC</td>
<td>PCAR</td>
<td>PACCAR INC</td>
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<td>CISCO SYSTEMS INC</td>
<td>PFE</td>
<td>PFIZER INC</td>
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<tr>
<td>DD</td>
<td>DUPONT E I DE NEMOURS &amp; CO</td>
<td>PG</td>
<td>PROCTER &amp; GAMBLE CO</td>
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<tr>
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<td>DEERE &amp; CO</td>
<td>PX</td>
<td>PRAXAIR INC</td>
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<td>DF</td>
<td>DEAN FOODS CO/</td>
<td>QCOM</td>
<td>QUALCOMM INC /DE</td>
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<td>DOVER CORP</td>
<td>ROH</td>
<td>ROHM &amp; HAAS CO</td>
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<td>SFD</td>
<td>SMITHFIELD FOODS INC</td>
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<td>EATON CORP</td>
<td>SII</td>
<td>SMITH INTERNATIONAL INC</td>
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<td>GD</td>
<td>GENERAL DYNAMICS CORP</td>
<td>SLE</td>
<td>LEE SARA CORP</td>
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<tr>
<td>GIS</td>
<td>GENERAL MILLS INC</td>
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<td>HDI</td>
<td>HARLEY DAVIDSON INC</td>
<td>TXT</td>
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<tr>
<td>HNZ</td>
<td>HEINZ H J CO</td>
<td>UTX</td>
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<tr>
<td>HPQ</td>
<td>HEWLETT PACKARD CO</td>
<td>WWY</td>
<td>WRIGLEY WM JR CO</td>
</tr>
</tbody>
</table>
Table II

Panel A: Descriptive Statistics for Sample Companies

This table provides distributional statistics of variables related to the 46 companies (listed in Table I) used in this study. Shares outstanding is the number of shares outstanding (in millions) at the end of the 2001 fiscal year. Price per share is the market price per share of common stock (in dollars) at the end of the 2001 fiscal year. Market capitalization is the number of shares outstanding multiplied by the price per share (in millions of dollars) at the end of the 2001 fiscal year. Sigma is the annualized standard deviation of daily returns over the 2001 fiscal year. Beta is calculated from the Capital Asset Pricing Model (CAPM) using the monthly return series over the 60 months prior to the end of the 2001 fiscal year end. Cost-of-capital is the company-specific cost of equity capital calculated using the Capital Asset Pricing Model (CAPM) with a risk-free rate of 5.24% and a market-risk premium of 6%. High VL Forecast is annualized return implied by the Value Line high long-term target price. Slope 1 and Slope 2 are the slopes of the production function that translates the agent’s action (in the set \{0, 1, ..., 100\}) to the \(\mu\) parameter of the lognormal price distribution (which corresponds to the mean of the normal returns distribution). Slope 1 is the slope between actions 0 and 30 where \(\mu\) ranges between the risk-free rate and the firm’s cost-of-capital. Slope 2 is the slope between actions 30 and 100 where \(\mu\) ranges between the firm’s cost-of-capital and the annual return implied by the high Value Line price forecast.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
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<th>25th Percentile</th>
<th>50th Percentile</th>
<th>75th Percentile</th>
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<tr>
<td>Shares Outstanding</td>
<td>988</td>
<td>1,676</td>
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<tr>
<td>Price per Share</td>
<td>47.20</td>
<td>22.25</td>
<td>34.60</td>
<td>46.32</td>
<td>61.64</td>
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<tr>
<td>Market Capitalization</td>
<td>53,794</td>
<td>95,395</td>
<td>7,844</td>
<td>12,871</td>
<td>46,344</td>
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<tr>
<td>Sigma</td>
<td>0.4891</td>
<td>0.1532</td>
<td>0.3888</td>
<td>0.4560</td>
<td>0.5435</td>
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<tr>
<td>Beta</td>
<td>0.71</td>
<td>0.33</td>
<td>0.47</td>
<td>0.73</td>
<td>0.97</td>
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<tr>
<td>Cost-of-capital</td>
<td>0.095</td>
<td>0.020</td>
<td>0.081</td>
<td>0.096</td>
<td>0.111</td>
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<tr>
<td>High VL Forecast</td>
<td>20.05%</td>
<td>8.76%</td>
<td>13.25%</td>
<td>21.15%</td>
<td>24.68%</td>
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<tr>
<td>Slope 1</td>
<td>0.00142</td>
<td>0.00067</td>
<td>0.00094</td>
<td>0.00147</td>
<td>0.00194</td>
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<tr>
<td>Slope 2</td>
<td>0.00148</td>
<td>0.00124</td>
<td>0.00056</td>
<td>0.00160</td>
<td>0.00218</td>
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</tbody>
</table>
Panel B: Descriptive Statistics for Sample CEOs

This table provides distributional statistics of variables related to the portfolios of the CEOs of the 46 companies (listed in Table I) used in this study. *Fixed wealth* is the executive’s total non-stochastic wealth (in dollars) which is estimated as five times the sum of the 2000 salary and bonus payment, plus an estimated SERP payment which is calculated as the present value of 60% of the 2000 salary and bonus paid out over 15 years starting five years into the future. *Shares of stock* is the total number of shares of stock and restricted stock held by the executive as of the end of the 2001 fiscal year. *New options* is the number of options granted in the prior year (i.e., fiscal year 2000). *New strike* is the exercise price (in dollars) of the options granted in the prior year (i.e., fiscal year 2000). If there was more than one grant in the prior year, *new strike* is a blended strike price calculated as the strike price of the total number of options that would produce an equivalent value to the total Black-Scholes value of all grants. *Unexercisable options* is the number of unexercisable options reported on the proxy statement for the end of the 2001 fiscal year. *Unexercisable strike* is the estimated average exercise price (in dollars) of the unexercisable options using the Core and Guay (2002) one-year approximation approach. *Exercisable options* is the number of exercisable options reported on the proxy statement for the end of the 2001 fiscal year. *Exercisable strike* is the estimated average exercise price (in dollars) of the exercisable options using the Core and Guay (2002) one-year approximation approach. *Reservation utility* is the agent’s expected utility over the pre-existing wealth (consisting of fixed wealth, shares of stock, new options, unexercisable options, and exercisable options) and four times the most recent (i.e., fiscal year 2000) median industry compensation assuming the executive exerts an action of zero and incurs no disutility of effort.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
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<th>25th Percentile</th>
<th>50th Percentile</th>
<th>75th Percentile</th>
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<tr>
<td>Fixed Wealth</td>
<td>27,607,313</td>
<td>16,314,577</td>
<td>16,117,084</td>
<td>25,567,048</td>
<td>33,083,224</td>
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<td>Shares of Stock</td>
<td>8,971,551</td>
<td>21,286,121</td>
<td>692,873</td>
<td>1,037,915</td>
<td>3,722,520</td>
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<tr>
<td>New Options</td>
<td>1,375,999</td>
<td>5,858,256</td>
<td>173,750</td>
<td>325,000</td>
<td>637,500</td>
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<td>New Strike</td>
<td>36.01</td>
<td>23.05</td>
<td>19.28</td>
<td>37.18</td>
<td>44.87</td>
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<td>Unexercisable Options</td>
<td>890,238</td>
<td>1,481,403</td>
<td>63,957</td>
<td>293,959</td>
<td>1,104,567</td>
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<td>Unexercisable Strike</td>
<td>34.04</td>
<td>22.87</td>
<td>16.34</td>
<td>35.88</td>
<td>48.56</td>
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<td>Exercisable Options</td>
<td>1,720,076</td>
<td>3,501,367</td>
<td>236,234</td>
<td>556,965</td>
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<td>Exercisable Strike</td>
<td>29.67</td>
<td>19.61</td>
<td>15.86</td>
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<td>38.91</td>
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<td>Reservation Utility</td>
<td>-2.0086</td>
<td>1.9755</td>
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<tr>
<td>Wealth Only Reservation Utility</td>
<td>-94.8032</td>
<td>622.9967</td>
<td>-3.5516</td>
<td>-2.4658</td>
<td>-1.5215</td>
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Table III

Unrestricted Second-Best Solutions

This table presents distributional statistics of parameters and values associated with the optimal second-best unconstrained (i.e., unrestricted) contract. The optimal second-best unconstrained contract is a state-contingent cash payment to the agent for each stock price. Optimal action is the non-negative integral action value taken by the agent. $\mu$ is the value of the $\mu$ parameter of the lognormal price distribution under the optimal action. Scaled objective is the value of the principal’s objective function at the optimal contract. Participation is the value of the agent’s participation (i.e., (IR)) constraint at the optimal contract, which is the agent’s expected utility at the optimal contract. Objective is the value of the principal’s objective function (in millions of dollars) for the optimal contract scaled by the scaling multiplier (i.e., 129,000,000). This can be interpreted as the payoff to the principal (i.e., total firm value), net of the compensation paid to the agent and the value of the agent’s pre-existing equity holdings. Mean Price Dist. is the expected value of the price distribution following the agent’s action induced by the optimal contract. Median Price Dist. is the median value of the price distribution following the agent’s action induced by the optimal contract. Variance Price Dist. is the variance of the price distribution following the agent’s action induced by the optimal contract. Skewness Price Dist. is the normalized third central moment of the price distribution following the agent’s action induced by the optimal contract. This measures the degree of asymmetry in the distribution and values greater (less) than zero indicates positive (negative) skewness. Kurtosis Price Dist. is the normalized fourth central moment of the price distribution (minus three) following the agent’s action induced by the optimal contract.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>25th Percentile</th>
<th>50th Percentile</th>
<th>75th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Action</td>
<td>45</td>
<td>31</td>
<td>25</td>
<td>33</td>
<td>74</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0378</td>
<td>0.0597</td>
<td>-0.0035</td>
<td>0.0033</td>
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<td>Scaled Objective</td>
<td>53,064</td>
<td>99,029</td>
<td>9,317</td>
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<td>Participation</td>
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<td>1.3302</td>
<td>-2.5320</td>
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<td>Objective</td>
<td>411.35</td>
<td>767.67</td>
<td>72.23</td>
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<td>Mean Price Dist.</td>
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<td>21.66</td>
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<td>Median Price Dist.</td>
<td>33.80</td>
<td>16.78</td>
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<td>Variance Price Dist.</td>
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<td>16,660</td>
<td>1,884</td>
<td>3,630</td>
<td>7,700</td>
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<td>Skewness Price Dist.</td>
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<td>3.27</td>
<td>0.07</td>
<td>0.09</td>
<td>0.19</td>
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<tr>
<td>Kurtosis Price Dist.</td>
<td>360,026</td>
<td>2,296,037</td>
<td>28</td>
<td>62</td>
<td>208</td>
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</tbody>
</table>
Table IV

Restricted Second-Best Solutions

This table presents distributional statistics of parameters and values associated with the optimal second-best constrained (i.e., restricted) contract which is the solution to program #4. The contract space is restricted to include only fixed salary, restricted stock, and at-the-money stock options. Optimal action is the non-negative integral action value taken by the agent (in the set \{0, 1, … , 100\}). Mu is the value of the \( \mu \) parameter of the lognormal price distribution under the optimal action. Salary is the amount of the fixed payment (in thousands of dollars) to the agent in the optimal contract. Shares is the number of shares granted to the agent in the optimal contract. Options is the number of at-the-money call options (in thousands) on the firm’s stock granted to the agent in the optimal contract. Strike is the exercise price (in dollars) of the at-the-money call options granted to the agent in the optimal contract. Scaled objective is the value of the principal’s objective function at the optimal contract. Participation is the value of the agent’s participation (i.e., (IR)) constraint at the optimal contract, which is the agent’s expected utility at the optimal contract. Objective is the value of the principal’s objective function (in millions of dollars) for the optimal contract scaled by the scaling multiplier (i.e., 129,000,000). This can be interpreted as the payoff to the principal (i.e., total firm value), net of the compensation paid to the agent and the value of the agent’s pre-existing equity holdings. Mean Price Dist. is the expected value of the price distribution following the agent’s action induced by the optimal contract. Median Price Dist. is the median value of the price distribution following the agent’s action induced by the optimal contract. Variance Price Dist. is the variance of the price distribution following the agent’s action induced by the optimal contract. Skewness Price Dist. is the normalized third central moment of the price distribution following the agent’s action induced by the optimal contract. This measures the degree of asymmetry in the distribution and values greater (less) than zero indicates positive (negative) skewness. Kurtosis Price Dist. is the normalized fourth central moment of the price distribution (minus three) following the agent’s action induced by the optimal contract.

<table>
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<tr>
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<th>Mean</th>
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<th>50(^{th}) Percentile</th>
<th>75(^{th}) Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Action</td>
<td>44</td>
<td>31</td>
<td>23</td>
<td>31</td>
<td>73</td>
</tr>
<tr>
<td>Mu</td>
<td>0.0361</td>
<td>0.0591</td>
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<td>0.0861</td>
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<tr>
<td>Salary</td>
<td>8,280,176</td>
<td>36,213,657</td>
<td>192</td>
<td>26,478</td>
<td>2,401,238</td>
</tr>
<tr>
<td>Shares</td>
<td>90,429</td>
<td>217,063</td>
<td>63</td>
<td>1,445</td>
<td>59,537</td>
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<tr>
<td>Options</td>
<td>4,488,695</td>
<td>6,587,179</td>
<td>4,082</td>
<td>1,167,597</td>
<td>7,596,392</td>
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<tr>
<td>Strike</td>
<td>47.20</td>
<td>22.25</td>
<td>34.60</td>
<td>46.32</td>
<td>61.64</td>
</tr>
<tr>
<td>Scaled Objective</td>
<td>52,792</td>
<td>98,653</td>
<td>9,257</td>
<td>13,678</td>
<td>44,963</td>
</tr>
<tr>
<td>Participation</td>
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<td>Objective</td>
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<td>764.75</td>
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<td>Mean Price Dist.</td>
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<td>70.56</td>
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<td>Median Price Dist.</td>
<td>33.61</td>
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<td>45.42</td>
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<td>7,700</td>
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<tr>
<td>Skewness Price Dist.</td>
<td>0.73</td>
<td>3.27</td>
<td>0.07</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>Kurtosis Price Dist.</td>
<td>360,026</td>
<td>2,296,037</td>
<td>28</td>
<td>62</td>
<td>208</td>
</tr>
</tbody>
</table>
Table V
Comparison of Actual (Four-Year Aggregate) to the Restricted Second-Best Optimal Compensation

This table presents distributional statistics for the observed (i.e., actual) contracts for the sample companies and the optimal restricted second-best contract from program #4. Actual Salary is the annualized average amount of the expected fixed payment (in dollars) to the agent and is computed as the sum of salary, bonus, other compensation, and the target long-term incentive from performance share plans. Actual Shares is the annualized average number of shares granted to the agent in the optimal contract. Actual Options is the annualized average number of at-the-money call options on the firm’s stock granted to the agent in the optimal contract. Optimal Salary is the annualized average amount of the fixed payment (in dollars) to the agent in the optimal contract. Optimal Shares is the annualized average number of shares granted to the agent in the optimal contract. Optimal Options is the annualized average number of at-the-money call options on the firm’s stock granted to the agent in the optimal contract. Optimal Salary, Optimal Shares, and Optimal Options are drawn from Table IV. ∆ Salary is the difference between Actual Salary and Optimal Salary. ∆ Shares is the difference between Actual Shares and Optimal Shares. ∆ Options is the difference between Actual Options and Optimal Options.

<table>
<thead>
<tr>
<th></th>
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<th>Standard Deviation</th>
<th>25th Percentile</th>
<th>50th Percentile</th>
<th>75th Percentile</th>
</tr>
</thead>
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<tr>
<td>Actual Salary</td>
<td>3,911,474</td>
<td>3,098,408</td>
<td>2,438,839</td>
<td>3,159,117</td>
<td>4,400,750</td>
</tr>
<tr>
<td>Actual Shares</td>
<td>1,503,151</td>
<td>3,017,305</td>
<td>-</td>
<td>450,563</td>
<td>1,744,228</td>
</tr>
<tr>
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Table VI

Sensitivity Analysis for Risk-Aversion and Disutility Multiplier

This table presents the results of the sensitivity analysis for the optimal restricted second-best contract (from program #4) for five companies (ADM, BC, DOV, SLE, and TXT). For each company, six optimal contracts are present for three different levels of risk aversion and two different levels of the disutility multiplier. Company is the ticker symbol of the company. Optimal Action is the non-negative integral action value taken by the agent (in the set \(\{0, 1, \ldots, 100\}\)). Reservation Utility is the agent’s expected utility over the pre-existing wealth (consisting of fixed wealth, shares of stock, new options, unexercisable options, and exercisable options) and four times the most recent (i.e., fiscal year 2000) median industry compensation assuming the executive exerts an action of zero and incurs no disutility of effort. Disutility Multiplier is a parameter from program #4. Risk Aversion is the agent’s coefficient of relative risk aversion from the power utility function. Salary is the amount of the fixed payment (in dollars) to the agent in the optimal contract. Shares is the number of shares granted to the agent in the optimal contract. Options is the number of at-the-money call options on the firm’s stock granted to the agent in the optimal contract. Objective is the value of the principal’s objective function (in millions of dollars) for the optimal contract scaled by the scaling multiplier (i.e., 129,000,000). This can be interpreted as the payoff to the principal (i.e., total firm value), net of the compensation paid to the agent and the value of the agent’s pre-existing equity holdings. Participation is the value of the agent’s participation (i.e., (IR)) constraint at the optimal contract, which is the agent’s expected utility at the optimal contract.

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