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Flexible or Uniform? Optimal Pricing Strategies of Chains

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Flexible or Uniform? Optimal Pricing Strategies of Chains

Abstract

We develop a model of spatial competition with two heterogeneous in their market access chains, choosing between third-degree price discrimination in their local markets (flexible pricing) and a unified chain-level price (uniform pricing). The markets are interconnected with each other via consumers who commute between them and can make purchases in locations where they do not reside. Our model supports an asymmetric equilibrium, in which the two pricing strategies co-exist: the larger chain uses uniform pricing, while the smaller chain employs flexible pricing. We also find that the chains never choose the pricing strategies that maximize the total consumer surplus.

JEL-Codes: D210, L110, L200, R320.

Keywords: spatial competition, price discrimination, uniform pricing, commuters.

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1 Introduction

In the retail sector, large players form chains of stores located in markets with different demand characteristics and face a choice of which prices to set in these markets. Regarding the latter, the traditional theory predicts that retail chains find it profitable to adjust their prices to local demand conditions (third-degree price discrimination, or flexible pricing).¹ At the same time, a number of empirical studies document that, instead of price discriminating consumers in local markets, some retail chains commit to a unified chain-level price (uniform pricing).² In fact, different pricing strategies can co-exist: some chains adjust their prices to local markets using a flexible pricing strategy; while other chains (selling the same products) commit to uniform pricing across their local markets.³ In this paper, we develop a novel framework of spatial competition that sheds light on the trade-offs retail chains face when choosing their pricing strategies, and explains the co-existence of flexible and uniform pricing strategies across chains and its effects on social welfare and consumer surplus.

In our model, we consider an economy with three local markets: two markets are peripheral and one market is central, – and each market is viewed as a Hotelling linear city. There are two competing chains that sell a homogeneous good and have their stores located in these markets. Each chain can select between two pricing strategies. It can commit to setting the same price of its good in all markets (uniform pricing) or, alternatively, it can use flexible pricing, choosing the price for each market separately. The decisions of the chains, which strategy to employ, are made simultaneously and independently. We then add two key ingredients in the above framework. First, we assume that the markets are interconnected with each other: there are consumers who commute from their peripheral markets to the central one and make some of their purchases there.⁴ In the case of uniform pricing used by a certain chain, the commuters are informed about the price set by this chain in the central market, while in the case of flexible pricing, the commuters do not observe the price and form some beliefs about its level. Second,

¹For example, see [Beckmann \(1976\)](#), [Holahan \(1975\)](#), and [Thisse and Vives \(1988\)](#).

²The uniform pricing strategy of some chains is discussed and confirmed by many studies among others analyzing pricing patterns in movie-theater industry ([Orbach and Einav, 2007](#)), car rental market ([Cho and Rust, 2010](#)), iTunes Music store songs ([Shiller and Waldfogel, 2011](#)), retail markets in the US ([Nakamura, 2008](#); [Hitsch, Hortacsu, and Lin, 2019](#); [DellaVigna and Gentzkow, 2019](#)) and France ([Allain, Chambolle, Turolla, and Villas-Boas, 2017](#)).

³[Li, Gordon, and Netzer \(2018\)](#) report the co-existence of flexible and uniform pricing policies used by major U.S. retailers for selling digital cameras; [Adams and Williams \(2019\)](#) show similar results regarding zone pricing in the retail home-improvement industry. In our paper, we document in Section 3 that around 41% of pharmacy chains in Moscow, Russia, stick to uniform pricing, while their competitors choose flexible pricing.

⁴For instance, this can be work commute, a social or recreational trip.

the chains are asymmetric in their access to the peripheral markets. Specifically, one of the chains is not present in one of the peripheral markets: that is, the other chain is a local monopolist there and, therefore, is bigger in terms of the number of stores. We show that these two assumptions result in the possibility of asymmetric equilibrium, when the bigger chain finds it profitable to use uniform pricing, while the smaller chain sticks to the flexible pricing strategy. This is in contrast to what existing studies on optimal pricing policies for chains find, where the equilibrium either features only flexible pricing or allows for uniform pricing but needs to be symmetric.⁵

The key assumptions of our model and the equilibrium outcome are consistent with some empirical regularities we observe in the data. In particular, we consider pharmacy chains in Moscow, Russia, where both flexible and uniform pricing are widespread, and document the following three facts: (1) the uniform pricing chains are bigger and more spread around the city; (2) the uniform pricing chains have more markets where they face no competition from other chains; (3) the uniform pricing chains tend to have their affiliates/stores in markets where more commuters arrive to. Thus, our theoretical framework, despite being rather stylized, turns out to be rich enough to capture some empirical patterns.

The intuition behind our main finding is based on the following trade-off. On the one hand, uniform pricing reduces the ability of a chain using this strategy to extract consumer surplus. On the other hand, given competition between the chains, uniform pricing allows the chain to attract commuters from the peripheral markets who are informed about the price level the chain sets, and thereby to increase its profits. This second effect, associated with uniform pricing, never emerges if there are no commuters or if they have perfect information about prices in the central market. In the paper, we show that the asymmetric equilibrium takes place when the relative size of the peripheral market, where only one retail chain is present, is neither too large nor too small and the consumer valuation of the homogeneous product sold by the chains is not high. In all other cases, both chains find it profitable to use flexible pricing. This result emphasizes the importance of the firm asymmetry assumption for generating the co-existence of flexible and uniform pricing. In particular, when the chains are symmetric in their access to the peripheral markets (the size of the market with only one chain present there is equal to zero), both chains always use the flexible pricing strategy. Finally, it is worth noting that the smaller chain always sticks to flexible pricing in the equilibrium. Specifically, we find that uniform pricing turns out to be a dominant strategy

⁵For example, see [Thisse and Vives \(1988\)](#) and [Dobson and Waterson \(2005, 2008\)](#).

for the smaller chain, only when the larger chain also uses uniform pricing and the relative size of the peripheral market with only one chain is sufficiently large, which, as we show, cannot be the case in the equilibrium.

In the paper, we also explore the relationship between pricing strategies chosen by the chains and social welfare and consumer surplus. In particular, we show that the social welfare is maximized when the chains equally share the markets where both of them present. This corresponds to the equilibrium in which both chains use flexible pricing, implying that the asymmetric equilibrium, where the larger chain employs uniform pricing, is inefficient. Regarding the consumer surplus, we find that, depending on the parameters in the model, it is maximized either when both chains use uniform pricing or when the larger chain uses flexible pricing, while the smaller one sticks to uniform pricing. This finding, in turn, implies that the pricing strategies maximizing the consumer surplus are never chosen by the chains in the market equilibrium.

In what follows, we discuss the related literature in Section 2 and summarize the empirical firm-level findings in Section 3. In Section 4 and Section 5, we present our theoretical framework and results. Section 6 concludes.

2 Related Literature

Our paper focuses on the theoretical analysis of the choice between uniform pricing and third-degree price discrimination, or flexible pricing, and primarily contributes to the literature that tries to explain the uniform pricing phenomenon. [Thisse and Vives \(1988\)](#) examine how two firms with exogenously given locations set prices in geographically dispersed markets that are not connected to each other. They find that flexible pricing is the unique equilibrium even if the firms can commit to their pricing strategies (namely, flexible or uniform) before setting exact prices. In contrast to [Thisse and Vives \(1988\)](#), we show that, in the model of spatial competition, the presence of interconnected local markets with uninformed commuters can support an equilibrium where at least one of the firms adopts uniform pricing. [Holmes \(1989\)](#) considers the case of two firms that produce differentiated goods, which are imperfect substitutes, and operate in two markets where they compete in prices. As [Holmes \(1989\)](#) shows, flexible pricing results in less total profit than uniform pricing does if a market where the

equilibrium price under discrimination is smaller (or a “weak” market) displays “higher cross-price elasticity but lower industry elasticity of demand” than the other market.⁶ [Corts \(1998\)](#) extends the model of [Holmes \(1989\)](#) by making firms asymmetric and shows that a unilateral commitment to uniform pricing can actually soften competition and increase the firms’ profits. Similar to our setting, [Corts \(1998\)](#) requires sufficient heterogeneity between the firms to make uniform pricing preferable. However, [Corts \(1998\)](#) does not explain the co-existence of uniform and flexible pricing strategies.

[Armstrong and Vickers \(2001\)](#) relate the choice of uniform pricing to the degree of competition in different markets and conclude that firms can commit to this strategy only if the markets are sufficiently uncompetitive. [Dobson and Waterson \(2005\)](#) extend this idea and assume that firms, selling differentiated goods, differ in their access to markets. Specifically, they consider a framework with one chain firm present in two isolated markets. In one of these markets, the chain does not face any competition, while in the other market it competes with a local (non-chain) producer. [Dobson and Waterson \(2005\)](#) show that uniform pricing relaxes competition in the contested market and can be profitable for the chain firm if (1) the price elasticity in this market is not less than the price elasticity in the monopolistic market and (2) the goods sold by the firms are sufficiently different. In their follow-up paper, [Dobson and Waterson \(2008\)](#) introduce two chains to the baseline model and look at their incentives to engage in uniform pricing in this new setting. However, [Dobson and Waterson \(2008\)](#) do not allow asymmetric equilibria to be played – both chains must stick to the same pricing strategy. On the one hand, similar to [Dobson and Waterson \(2005, 2008\)](#), we assume that chains differ in their access to local markets. On the other hand, we consider a framework where chains sell a homogeneous good and the markets are interconnected via uninformed commuters, which has non-trivial implications for the optimal pricing strategies. Moreover, in our model, chains can choose freely between flexible pricing and uniform pricing, and this allows us to characterize conditions under which the two pricing strategies co-exist. [Gabrielsen, Johansen, and Straume \(2023\)](#) consider a model of spatial competition of symmetric retail chains that compete in the price and the local quality of their products. As in [Thisse and Vives \(1988\)](#), they show that in the absence of quality competition, the chains always choose flexible pricing. However, if the level of quality competition is sufficiently high, there exist equilibria where at least one of the chains employs the uniform pricing

⁶[Holmes \(1989\)](#) focuses on symmetric equilibrium where both firms set identical market-specific prices under discrimination and select the same price under uniform pricing.

strategy. In our paper, the chains compete in the price of a homogeneous good, and the co-existence of the two pricing strategies in the equilibrium becomes possible because (1) the chains differ in their access to local markets and (2) the markets are interconnected with each other via commuters who observe prices only in locations where they reside.

The present paper also contributes to the literature on the effects of uniform pricing on social welfare and consumer surplus. Overall, the evidence on the relationship between the uniform pricing strategy and social welfare and consumer surplus is mixed. [Miravete, Seim, and Thurk \(2020\)](#) consider the liquor market in Pennsylvania where markups for spirits are fixed by law. With the prohibited price discrimination, the winners are low-income households and small specialized firms. [Fabra and Reguant \(2020\)](#) show theoretically that uniform pricing benefits medium-size buyers at the expense of their small- and large-size peers and illustrate numerically that this strategy may drive the total surplus up, which is different from our analytical findings. [Vélez-Velásquez \(2024\)](#) investigates the consequences of a ban on price discrimination in broadband carriers in Colombia. The paper shows that this policy resulted in large welfare transfers from low-income households towards high-income consumers and was accompanied by the price increase for the Internet provision. Our paper contributes to this literature by showing that in the environments similar to ours – where chain firms differ in their access to local markets and these markets are interconnected via uninformed commuters, – uniform pricing is always detrimental for social welfare, while consumer surplus, however, increases when one or both chains engage in uniform pricing.

Finally, our paper is related to a broader literature that provides explanations for uniform pricing and, overall, low price dispersion. [Levy, Bergen, Dutta, and Venable \(1997\)](#) justify the uniform pricing phenomenon with managerial inertia. A number of studies argue that uniform pricing can serve as a signal for loyal customers and results from their perception of a fair price ([Kahneman, Knetsch, and Thaler, 1986a,b](#); [Chen and Cui, 2013](#)). Another explanation is related to the presence of pricing zones, where prices are uniform within a certain geographic area ([Hoch, Kim, Montgomery, and Rossi, 1995](#); [Adams and Williams, 2019](#)). Uniform pricing is quite common in online retailing ([Brynjolfsson and Smith, 1999](#); [Brown and Goolsbee, 2002](#); [Cavallo, 2018](#); [Clay, Krishnan, and Wolff, 2001](#)). For instance, [Ater and Rigbi \(2018\)](#) explain this with an increased transparency that limits third-degree price discrimination online. Finally, the uniform pricing strategy can be optimal when consumers have low search costs

(Chandra and Tappata, 2011) or as a barrier to entry into a market (Feng, Li, and Shuai, 2023).

3 Empirical Evidence on Uniform Pricing

In this section, we present several stylized facts that illustrate how chains that use uniform pricing differ from their rivals engaged in flexible pricing. Specifically, we focus on pharmacy chains in Moscow, Russia, where both flexible and uniform pricing are widespread. We consider pharmacy chains with *at least four affiliates*. A chain is classified as using uniform pricing if its online prices do not differ across affiliates; otherwise, we say that this chain use flexible pricing.⁷ In total, our sample includes 26 uniform pricing chains with 1,895 affiliates and 37 flexible pricing chains with 1,317 affiliates, respectively. On average, prices differ by 8% across affiliates of a given flexible-pricing chain with the highest dispersion of 14% for care products and the lowest dispersion of about 1% for optics (see Appendix B for more details). The data we use contain information on the location and pricing strategy for each pharmacy chain’s affiliate.⁸

We collect information about the locations of pharmacies as well as some other market-specific characteristics from [2gis.ru](#) maps designed for local business promotion. Also, we use open data from the [official portal](#) of Moscow government. To identify a market, we propose two alternative approaches. First, we define a market by pharmacies’ geographical closeness. Specifically, we find the clusters of pharmacies such that the distance between two pharmacies within each cluster does not exceed one kilometer. This approach gives us 896 markets in total. Second, we create a one-kilometer grid that fully covers the map of Moscow and obtain 731 markets.

With these data in hand, we document three main stylized facts and highlight the difference between the chains that use uniform pricing and their rivals with flexible pricing. These patterns are quite indicative and seem to be in line with the key assumptions of the theoretical model we introduce in Section 4.

⁷Among smaller chains, only a few of them can be clearly identified as uniform pricing chains, whereas for the rest we cannot find reliable information on prices at all.

⁸We limit our analysis to the “old” Moscow borders that mostly coincide with the Moscow Automobile Ring Road (MKAD). Appendix A depicts the area of Moscow that we analyze.

Stylized fact 1: Uniform pricing chains are bigger and more spread around the city. On average, the chains that use uniform pricing have a higher number of affiliates and are present in more markets (see Table 1). To measure how spread a chain is across the markets, we compute the Herfindahl-Hirschman index. We find that, on average, the uniform pricing chains have affiliates that are more evenly distributed around the city, and this is mainly driven by the markets located outside of the Garden Ring and the Third Transport Ring.

TABLE 1: AVERAGE CHAIN SIZE

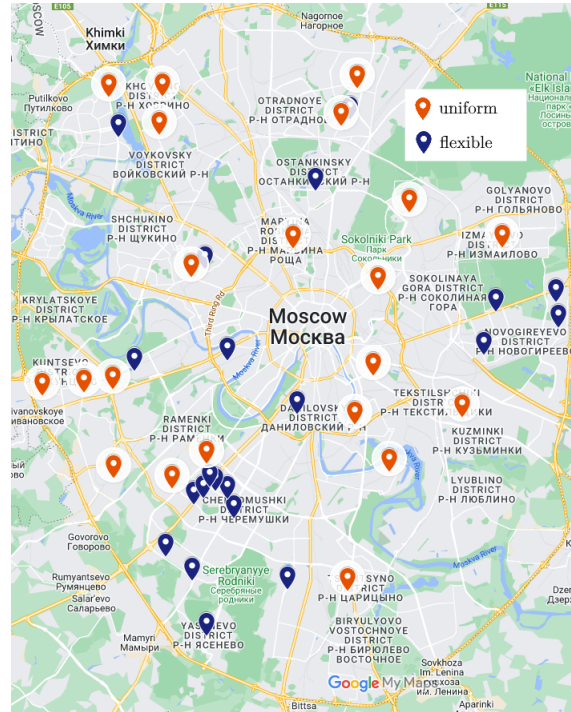
	Pricing			Pricing		
	Flexible	Uniform	p -value	Flexible	Uniform	p -value
Number of affiliates	34.514	72.885	0.138			
inside 1 st ring	2.688	7.417	0.042			
inside 2 nd ring	5.742	13.273	0.066			
inside 1 st ring (share)	0.057	0.047	0.645			
inside 2 nd ring (share)	0.198	0.192	0.899			
	Market defined by clustering			Market defined by grid		
Number of markets	31.216	58.962	0.137	30.811	58.077	0.133
Average number of affiliates in a market	1.036	1.085	0.087	1.039	1.088	0.101
HHI						
Total	0.110	0.064	0.017	0.109	0.064	0.016
Periphery – outside 1 st ring	0.120	0.067	0.012	0.119	0.066	0.011
Periphery – outside 2 nd ring	0.140	0.085	0.049	0.138	0.086	0.055

Note: 26 uniform pricing chains and 46 flexible pricing chains. 1st ring is the Garden Ring; 2nd ring is the Third Transport Ring.

To make our comparison more vivid, we take two medium-size chains that have the same number of affiliates – Zdravcity (a uniform pricing chain) and Dezhurnaya Apteka (a flexible pricing chain). In Figure 1, we show the respective locations of these two chains' affiliates on the city map. As one can see, for the uniform pricing chain (Zdravcity), the affiliates are located relatively far from each other, while for the flexible pricing chain (Dezhurnaya Apteka), there are zones of high concentration (e.g., in the South-West).

Stylized fact 2: Uniform pricing chains have more markets where they face no competition from other chains. As Table 2 indicates, the affiliates of the uniform pricing chains are more likely to be the only ones in the respective markets. One could expect that such a monopolistic position in a number of markets should increase the benefits from adjusting prices to local demand (Varian, 1980). However, the empirical evidence suggests quite the opposite – even though these chains can exploit their monopoly

FIGURE 1: COMPARISON OF TWO EQUAL SIZE CHAINS – UNIFORM VS. FLEXIBLE PRICING



power in more locations, they tend to commit to uniform pricing more often than their rivals with a smaller number of monopolistic markets do. Moreover, the uniform pricing chains face less competition from “big players” – top-5 and top-10 chains, based on the number of affiliates. Interestingly, the uniform pricing chains are less likely to compete with each other in a particular market – as Table 3 reveals, the affiliates of the uniform pricing chains have a larger share of markets where all their rivals use flexible pricing.

TABLE 2: A NUMBER OF MONOPOLY MARKETS FOR CHAINS WITH UNIFORM AND FLEXIBLE PRICING STRATEGIES

	Pricing			Pricing		
	Flexible	Uniform	<i>p</i> –value	Flexible	Uniform	<i>p</i> –value
	Market defined by clustering			Market defined by grid		
Number of monopoly markets	2.378	6.077	0.083	1.189	3.077	0.087
Share of monopoly markets	0.086	0.101	0.509	0.036	0.045	0.467
Number of ‘big 5’ competitors	2.784	1.654	0.016	3.081	1.885	0.022
Number of ‘big 10’ competitors	3.703	2.154	0.015	4.243	2.962	0.059
No ‘big 5’ competitors	0.189	0.346	0.180	0.189	0.154	0.718
No ‘big 10’ competitors	0.162	0.308	0.196	0.081	0.115	0.664
Share of markets with no ‘big 5’ competitors	0.162	0.196	0.355	0.125	0.152	0.254
Share of markets with no ‘big 10’ competitors	0.126	0.162	0.290	0.070	0.100	0.130

Note: 26 uniform pricing chains and 37 flexible pricing chains.

TABLE 3: THE JOINT DISTRIBUTION OF LOCATIONS FOR UNIFORM AND FLEXIBLE PRICING CHAINS

	Pricing			Pricing		
	Flexible	Uniform	p -value	Flexible	Uniform	p -value
	Market defined by clustering			Market defined by grid		
N uniform pricing chains	3.498	2.931	0.000	3.508	2.995	0.000
No uniform pricing chains	0.131	0.178	0.000	0.097	0.113	0.137

Note: 1,895 affiliates of uniform pricing chains and 1,317 affiliates of flexible pricing chains.

Stylized fact 3: Uniform pricing chains tend to have their affiliates in markets where more commuters arrive to. In the data, we find that markets covered by the uniform pricing chains seem to attract more commuters: in other words, there is a positive relationship between the presence of the uniform pricing chains in a market and the “attractiveness” of this market to commuters. In our analysis, in the absence of detailed data on internal migration in Moscow, we rely on information about the inflow and outflow of passengers for each metro station.⁹ A net passenger outflow serves as a good proxy for location centrality, which can be in turn associated with the attractiveness of a location and, thereby, with a higher number of commuters there.¹⁰ For instance, metro stations located next to large business areas are characterized by large positive net passenger outflows – the number of outflow travelers to these stations significantly exceeds the number of inflow travelers.

As Table 4 indicates, on average, there are more metro stations and crosswalks in markets where the uniform pricing chains have their affiliates; these markets also have a significantly higher outflow/inflow ratio. We then focus only on metro stations with a positive net passenger outflow and, for each market, compute the total number of such stations as well as their total net passenger outflows. We find that markets having affiliates of the uniform pricing chains tend to have a higher number of metro stations with a positive net passenger outflow and a higher total passenger outflow. We report similar findings for the number of business centers and free parking zones,¹¹ where the former is greater and the latter is smaller for markets covered by the uniform pricing chains. Finally, we show that it is more likely that

⁹Moscow is fully covered by the metro network, which accounts for 65% of all travels around the city.

¹⁰This measure is used to determine location centrality by [Han and Liu \(2018\)](#); [Pan and Lai \(2019\)](#); [Moylan and Somwrita \(2019\)](#) among others.

¹¹Typically, there are fewer free parking zones in business areas.

markets with the uniform pricing chains have main railway stations and other stations, by which regional trains to the Moscow area pass by.

TABLE 4: MARKETS AND COMMUTERS

	Pricing			Pricing		
	Flexible	Uniform	p -value	Flexible	Uniform	p -value
Market defined by clustering				Market defined by grid		
N metro stations	0.606	0.713	0.000	0.569	0.683	0.000
Passenger outflow/inflow	0.611	0.722	0.000	0.573	0.692	0.000
Positive net passenger outflow						
Absolute sum	23,941.630	33,744.960	0.000	22,608.800	33,905.100	0.000
N of stations	0.339	0.442	0.000	0.312	0.419	0.000
N business centers	1.754	2.403	0.000	1.583	2.248	0.000
N crosswalks	2.273	3.010	0.000	2.093	2.524	0.001
N free parking zones	8.647	7.754	0.009	6.988	6.231	0.000

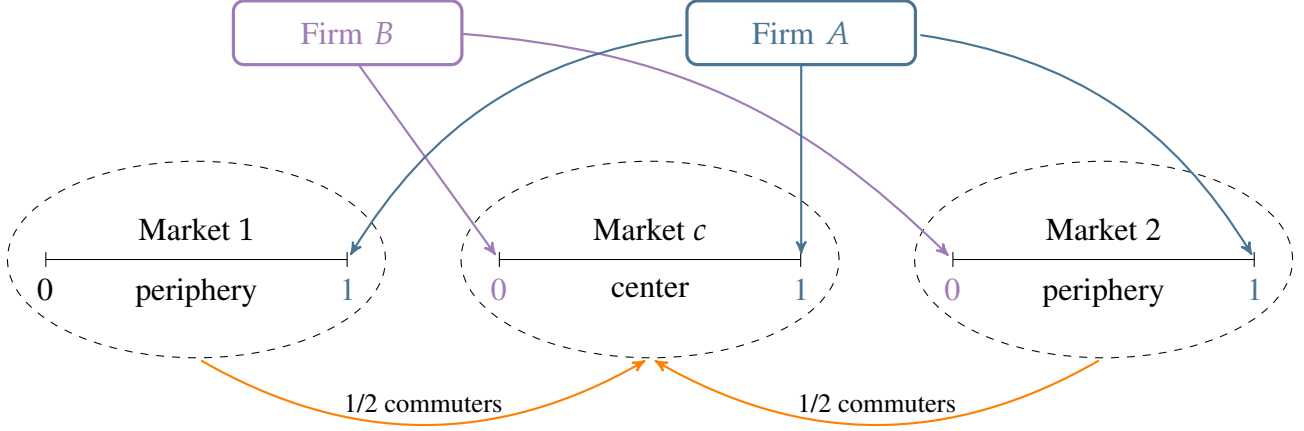
Note: 1,895 affiliates of uniform pricing chains and 1,317 affiliates of flexible pricing chains. Net passenger outflow is the difference between passenger outflow and inflow.

4 The Baseline Model

Firms and markets. We consider an economy that consists of three markets: two periphery markets (denoted by 1 and 2) and one central market (denoted by c); and two chain firms labeled with A and B . Each market can be viewed as a Hotelling linear city on the $[0, 1]$ interval. We assume that firm A operates in all three markets, while firm B is active only in the central market and market 2. That is, firm A is the only seller in market 1 and, therefore, acts there as a monopolist. In the other markets, firm A competes with firm B . Notice that the above assumption implies heterogeneity across the firms in terms of their presence in a certain market, which is the only source of firm heterogeneity in the model. It is also worth noting that we do not consider in the present paper the mechanism of how firms choose which market to serve, the distribution of firms across the markets is exogenously given in the model.

We also assume that each firm has exactly one store in market 2 and in market c and firm A has one store in market 1. We denote their respective stores by A_j , where $j \in \{1, 2, c\}$, and B_k with $k \in \{2, c\}$. Without the loss of generality, we assume that the stores of firm B (respectively A) are located at 0 (respectively 1) in each market. The firms sell homogeneous good produced according to a constant returns to scale technology with zero marginal and fixed costs of production. The assumption about zero

FIGURE 2: BASELINE MODEL: MARKETS AND FIRMS



marginal cost of production simplifies the analysis of the equilibrium in the model and at the same time does not affect the qualitative implications of the model and the intuition behind them. The firms do not face any capacity constraints and, hence, can serve any demand.

Consumers. In each market $i \in \{1, 2, c\}$, there is a mass of consumers that are uniformly distributed over the $[0, 1]$ interval with density $n_i > 0$: that is, the total number of consumers in market i is n_i . We assume that consumers leave for two periods, $t = 1, 2$. Each consumer needs one unit of a good per period, and this unit gives her the value of $v > 0$. In the first period (that can be viewed as a weekend, for instance), consumers from the peripheral markets can shop only in their “home” markets. In the second period (working days), half of consumers *from each periphery market* travels to the central market and makes purchases there. This assumption can be interpreted as that in each peripheral market there is a share of consumers who need to commute to the center during working days and cannot postpone their purchases until returning to the home market. In the model, we normalize this share to $1/2$. On the one hand, this allows us avoiding unnecessary complications in the model. On the other hand, this assumption does not seem too restrictive, as the effects of a change in this share on the equilibrium outcome can be at least partly (if not completely) represented by the effects of corresponding changes in the market densities n_i . For instance, a rise in the share of “commuters” makes the presence of “peripheral” consumers in the central market more pronounced, which in turn can be achieved by reducing the relative size of the central market. Consumers from the central market always make their purchases in their home market and, thereby, never travel. Figure 2 depicts the market structure.

In each market, each consumer is characterized by location $x \in [0, 1]$. As in a standard Hotelling model, to reach a certain store in her home market located at $l \in \{0, 1\}$, she needs to pay the cost of $\tau \cdot |x - l|$, where $\tau > 0$ holds. In the central market, each commuter arrives at location $y \in [0, 1]$, which is uncorrelated with her home location x , and all commuters are uniformly distributed over the $[0, 1]$ interval. Similarly, to reach a certain store in the central market, a commuter at y needs to pay the cost of $\tau \cdot |y - l|$.

Information and pricing. We define by p_{Aj} and p_{Bk} prices set by firms A and B in the markets with $j \in \{1, 2, c\}$ and $k \in \{2, c\}$. We assume that consumers *know prices only in their home market*. Namely, they observe (1) p_{A1} in market 1, (2) p_{A2} and p_{B2} in market 2, and (3) p_{Ac} and p_{Bc} in market c . When commuters arrive to market c , they generally do not have information about p_{Ac} and p_{Bc} , but form expectations about them. We denote these expected prices by \tilde{p}_{Ac} and \tilde{p}_{Bc} , respectively, and assume that they do not depend on consumers' identities.

Firms set prices to maximize their profits over two periods. We consider two pricing strategies. First, a firm can set different prices in different markets. We denote this strategy as *flexible pricing*. In this case, the prices set by the firm in peripheral markets are not informative about the prices it sets in the central market. Alternatively, a firm can choose to set identical prices in all the markets (for instance, $p_{A1} = p_{A2} = p_{Ac} \equiv p_A$). We refer to this strategy as *uniform pricing*.¹² Under such a strategy, the price in the peripheral markets set by the firm, using this strategy, is perfectly informative about the price set by that firm in the central market. We assume that both firm's pricing strategy and prices are set for two time periods, i.e., the firms commit to their choices. The game proceeds in two stages.

Stage 1: Firm A and firm B simultaneously choose their pricing strategies (flexible or uniform) and commit to them.

Stage 2:

1. The firms set prices in all markets simultaneously and non-cooperatively.
2. Consumers observe prices set by the firms in their home markets:

- $t = 1$: Given these prices, consumers decide which store to visit in their home market.

¹²In the paper, we assume away firm strategies, when uniform pricing is "partial" with identical prices set on the subset of the markets.

- $t = 2$: Half of consumers, or commuters, travel from the periphery to the central market. Given her beliefs, each commuter chooses which store to visit in market c . Consumers who stay at their home locations, shop exactly like at $t = 1$.

We solve the game by backward induction. At Stage 2, we consider a *fulfilled expectations equilibrium* (for example, see [Katz and Shapiro, 1985](#)) that can be defined as follows.

Definition. *The equilibrium at Stage 2 is a collection of prices $\mathbf{p} = \{p_i\}_{i \in \{A1, A2, Ac, B2, Bc\}}$ and beliefs $\tilde{\mathbf{p}} = \{\tilde{p}_i\}_{i \in \{Ac, Bc\}}$ such that:*

- *each consumer maximizes her utility given \mathbf{p} and $\tilde{\mathbf{p}}$,*
- *\mathbf{p} maximizes firms' profits given $\tilde{\mathbf{p}}$, and*
- *beliefs are consistent, i.e., $\mathbf{p} = \tilde{\mathbf{p}}$.*

At Stage 1, we consider a Nash equilibrium. In the next subsection, we characterize the aggregate consumer demand in the model.

4.1 Demand Characterization

As the first step in deriving the equilibrium at Stage 2, we represent the aggregate demand for the product sold by each store in each market as a function of observed firms' prices and consumers' beliefs about the unobserved prices.

4.1.1 The Peripheral Markets

We start with the peripheral markets. In market 1, there is only one store of firm A located at 1. Thus, a consumer located at x buys from it if and only if (we normalize the utility in the case of zero consumption of the good to zero):

$$v - p_{A1} - \tau(1 - x) \geq 0 \Leftrightarrow \frac{p_{A1} - v}{\tau} + 1 \leq x.$$

As a result, the aggregate demand in market 1 (here, we take into account consumer demand in both periods) is given by

$$D^{A,1}(p_{A1}) = \begin{cases} 0 & , p_{A1} > v, \\ \frac{3n_1}{2} \left(\frac{v-p_{A1}}{\tau} \right) & , p_{A1} \in [v-\tau, v], \\ \frac{3n_1}{2} & , p_{A1} < v-\tau, \end{cases} \quad (1)$$

where the coefficient $3/2$ captures the fact that at $t = 2$ one-half of the consumers leaves market 1 and travels to market c . Later in the paper, we determine the condition on the parameters such that $v \geq p_{A1} + \tau$ in the equilibrium, implying that all consumers in market 1 purchase the good.

In market 2, the firms compete with each other. A consumer located at x chooses to purchase the product in the store of firm B if and only if:

$$v - p_{B2} - \tau x \geq v - p_{A2} - \tau(1-x) \Leftrightarrow x \leq \frac{p_{A2} - p_{B2} + \tau}{2\tau}$$

and

$$v - p_{B2} - \tau x \geq 0.$$

The latter inequality holds if

$$v - p_{B2} \geq \frac{p_{A2} - p_{B2} + \tau}{2} \Leftrightarrow v \geq \frac{p_{A2} + p_{B2} + \tau}{2}. \quad (2)$$

If the inequality in (2) takes place, then, given the prices, all consumers purchase the good either from firm A or firm B . In the paper, we consider the equilibrium where this is the case and formulate later the corresponding constraint on the parameters. Checking for corner solutions, we obtain that the aggregate demand for the good sold by firm i in market 2 is given by:

$$D^{i,2}(p_{i2}, p_{j2}) = \begin{cases} 0 & , p_{i2} > p_{j2} + \tau, \\ \frac{3n_2}{2} \left(\frac{p_{j2} - p_{i2} + \tau}{2\tau} \right) & , p_{i2} \in [p_{j2} - \tau, p_{j2} + \tau], \\ \frac{3n_2}{2} & , p_{i2} < p_{j2} - \tau, \end{cases} \quad (3)$$

where $i, j \in \{A, B\}$ and $i \neq j$.

4.1.2 The Central Market

The aggregate demand functions for the good sold in the central market have two ingredients. First, there are local consumers who are aware of the prices set by the firms in this market. As for market 2, the aggregate demand over two time periods of these consumers can be written as follows (recall that these consumers shop only in their home market):

$$D^{i,c}(p_{ic}, p_{jc}) = \begin{cases} 0 & , p_{ic} > p_{jc} + \tau, \\ 2n_c \left(\frac{p_{jc} - p_{ic} + \tau}{2\tau} \right) & , p_{ic} \in [p_{jc} - \tau, p_{jc} + \tau], \\ 2n_c & , p_{ic} < p_{jc} - \tau, \end{cases}$$

where $i, j \in \{A, B\}$ and $i \neq j$. Similarly to the assumption about the prices set by the firms at market 2, we assume that

$$v \geq \frac{p_{Ac} + p_{Bc} + \tau}{2}. \quad (4)$$

Second, there are commuters from the peripheral markets. At the moment, we express their demand for the good sold by the firms in terms of their beliefs about the prices. In particular, we have that the aggregate demand of commuters is

$$D_{com}^{i,c}(\tilde{p}_{ic}, \tilde{p}_{jc}) = \begin{cases} 0 & , \tilde{p}_{ic} > \tilde{p}_{jc} + \tau, \\ \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{jc} - \tilde{p}_{ic} + \tau}{2\tau} \right) & , \tilde{p}_{ic} \in [\tilde{p}_{jc} - \tau, \tilde{p}_{jc} + \tau], \\ \frac{n_1 + n_2}{2} & , \tilde{p}_{ic} < \tilde{p}_{jc} - \tau, \end{cases}$$

where $i, j \in \{A, B\}$ and $i \neq j$. As before, we assume that

$$v \geq \frac{\tilde{p}_{Ac} + \tilde{p}_{Bc} + \tau}{2}. \quad (5)$$

In the next section, we derive the equilibrium at Stage 2. Specifically, we find firms' optimal pricing and profits given the pricing strategies they choose at Stage 1.

4.2 The Equilibrium at Stage 2

As mentioned before, to characterize the optimal pricing strategies of the firms, we apply backward induction. First, we solve for the equilibrium at Stage 2 (the profit maximization problem) for each of the four possible combinations of pricing strategies. Second, we focus on Stage 1 and find the Nash equilibrium of the game where the firms choose their pricing strategies independently and simultaneously. We denote the four possible scenarios of firms' pricing strategies as $\{FF, UF, FU, UU\}$ where F and U stand for flexible and uniform pricing strategies, respectively, and the first and second letters represent firm A and B . For instance, FU means that firm A chooses flexible pricing, while firm B chooses uniform pricing.

As discussed before, in order to make the problem non-trivial, we impose the following constraints on prices in the equilibrium:

$$\left\{ \begin{array}{l} p_{A1} \leq v - \tau, \\ p_{i2} \in (p_{j2} - \tau, p_{j2} + \tau), \quad \forall i \neq j \text{ where } i, j \in \{A, B\} \\ p_{ic} \in (p_{jc} - \tau, p_{jc} + \tau), \quad \forall i \neq j \text{ where } i, j \in \{A, B\} \\ v \geq \frac{p_{A2} + p_{B2} + \tau}{2}, \\ v \geq \frac{p_{Ac} + p_{Bc} + \tau}{2}. \end{array} \right. \quad (6)$$

The first inequality indicates that for given p_{A1} , all consumers in market 1 purchase the good from firm A . The next two inequalities ensure that *in equilibrium* no firm faces zero demand in market 2 and market c , respectively. The last two inequalities guarantee that *all* consumers want to purchase the good in market 2 and market c , correspondingly. Notice that for the commuters we have the inequality (5), which includes their beliefs about the prices. However, since in the equilibrium beliefs are consistent, inequality (5) is in fact equivalent to the last inequality in (6).

4.2.1 The FF Equilibrium

Under flexible pricing of both firms, firm A solves (condition (6) is verified later):

$$\max_{p_{A1}, p_{A2}, p_{Ac} \geq 0} \left\{ \pi_A^{FF} = \frac{3n_1}{2} \left(\frac{v - p_{A1}}{\tau} \right) p_{A1} + \frac{3n_2}{2} \left(\frac{p_{B2} - p_{A2} + \tau}{2\tau} \right) p_{A2} \right. \\ \left. + \left(2n_c \left(\frac{p_{Bc} - p_{Ac} + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Bc} - \tilde{p}_{Ac} + \tau}{2\tau} \right) \right) p_{Ac} \right\}, \quad (7)$$

where the first (the second) entry refers to consumers who shop in market 1 (in market 2) in both periods and the third entry corresponds to consumers who shop in market c , including the commuters arriving at $t = 2$. Firm B , which operates only in market 2 and market c , solves:

$$\max_{p_{B2}, p_{Bc} \geq 0} \left\{ \pi_B^{FF} = \frac{3n_2}{2} \left(\frac{p_{A2} - p_{B2} + \tau}{2\tau} \right) p_{B2} \right. \\ \left. + \left(2n_c \left(\frac{p_{Ac} - p_{Bc} + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Ac} - \tilde{p}_{Bc} + \tau}{2\tau} \right) \right) p_{Bc} \right\}. \quad (8)$$

Under flexible pricing of firm A , the price set in market 1, where only firm A operates, does not affect the other markets. As a result, solving for p_{A1} , when the solution of the above maximization problem with respect to p_{A1} is interior, implies that

$$\frac{\partial \pi_A^{FF}}{\partial p_{A1}} = \frac{3n_1}{2\tau} (v - 2p_{A1}) = 0,$$

resulting in

$$p_{A1} = \frac{v}{2}.$$

It is straightforward to see from (1) that $p_{A1} = v/2$ is interior, when $v < 2\tau$. If $v \geq 2\tau$ holds, then the optimal price is $v - \tau$ and, as a result, all consumers in market 1 purchase the good from firm A . Hereafter, we assume that v is sufficiently large (the first inequality in condition (6) is satisfied):

$$v \geq 2\tau \quad (9)$$

that leads to $p_{A1} = v - \tau$ in case when firm A sticks to flexible pricing. The demand in market 1 is then $3n_1/2$ and the profit of firm A in this market is $3n_1(v - \tau)/2$.

Next, we formulate a proposition that characterizes the equilibrium at Stage 2 when both firms use flexible pricing.

Proposition 1. *If both firms use flexible pricing and*

$$v \geq \max \left(2\tau, \frac{3}{2}\tau + \frac{\tau(n_1 + n_2)}{4n_c} \right), \quad (10)$$

then the equilibrium price vector is:

$$\begin{aligned} p_{A1}^{FF} &= v - \tau, \\ p_{A2}^{FF} &= p_{B2}^{FF} = \tau, \\ p_{Ac}^{FF} &= p_{Bc}^{FF} = \tau + \frac{\tau(n_1 + n_2)}{4n_c}, \end{aligned}$$

and the firms earn:

$$\begin{aligned} (\pi_A^{FF})^* &= \frac{3n_1}{2}(v - \tau) + \frac{3n_2\tau}{4} + n_c\tau \left(1 + \frac{n_1 + n_2}{4n_c} \right)^2, \\ (\pi_B^{FF})^* &= \frac{3n_2\tau}{4} + n_c\tau \left(1 + \frac{n_1 + n_2}{4n_c} \right)^2. \end{aligned}$$

Proof. See Appendix C. □

When both firms use flexible pricing, the only source of heterogeneity between them is market 1 that is unavailable to firm B . This explains why firms' equilibrium profits differ. One can notice that the equilibrium prices in market 2 are similar to those in a standard Hotelling model with two firms. In market c , commuters' expectations about p_{Ac} and p_{Bc} turn out to be important, increasing the equilibrium price set by each firm by $\tau(n_1 + n_2)/4n_c$ compared to the price observed in market 2. Interestingly, p_{Ac}^{FF} and p_{Bc}^{FF} increase in $(n_1 + n_2)/n_c$, that is, a rise in the share of commuters with respect to the number of local consumers in the central market leads to higher prices in market c .

4.2.2 The UU Equilibrium

When both firms adopt uniform pricing (recall that in this case, the firms set the same price for all markets where they are active), commuters from market 2 do not face any uncertainty about p_{Ac} and p_{Bc} : that is, their beliefs about the prices are exactly the prices the firms set. At the same time, commuters from market 1 know only the price set by firm A (as firm B does not present in market 1) and form some expectations \tilde{p}_{Bc} about p_{Bc} . Taking the above into account, we have the following firm profit maximization problems (we omit in the below the market indexes in the prices and beliefs):

$$\begin{aligned} \max_{p_A \geq 0} \left\{ \pi_A^{UU} = p_A \left(\frac{3n_1}{2} + \left(\frac{p_B - p_A + \tau}{2\tau} \right) 2(n_2 + n_c) + \frac{n_1}{2} \left(\frac{\tilde{p}_B - p_A + \tau}{2\tau} \right) \right) \right\}, \\ \max_{p_B \geq 0} \left\{ \pi_B^{UU} = p_B \left(\left(\frac{p_A - p_B + \tau}{2\tau} \right) 2(n_2 + n_c) + \frac{n_1}{2} \left(\frac{p_A - \tilde{p}_B + \tau}{2\tau} \right) \right) \right\}. \end{aligned}$$

The constraints in (6) can be then rewritten as follows

$$\begin{cases} p_A \leq v - \tau, \\ p_i \in (p_j - \tau, p_j + \tau) \quad , \forall i \neq j \text{ where } i, j \in \{A, B\}, \\ v \geq \frac{p_A + p_B + \tau}{2}. \end{cases} \quad (11)$$

The next proposition characterizes the equilibrium at Stage 2 when both firms choose uniform pricing.

Proposition 2. *If*

$$v \geq 2\tau + \tau \frac{52n_1(n_2 + n_c) + 7n_1^2}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)}, \quad (12)$$

and both firms use uniform pricing, then the equilibrium price vector is:

$$\begin{aligned} p_A^{UU} &= \tau + \tau \frac{52n_1(n_2 + n_c) + 7n_1^2}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)}, \\ p_B^{UU} &= \tau + \frac{8\tau n_1}{12(n_2 + n_c) + n_1}, \end{aligned}$$

and the firms earn:

$$\begin{aligned}\left(\pi_A^{UU}\right)^* &= \frac{4\tau \left(2n_1^2 + 17n_1(n_2 + n_c) + 12(n_2 + n_c)^2\right)^2}{(n_1 + 4n_2 + 4n_c)(n_1 + 12n_2 + 12n_c)^2}, \\ \left(\pi_B^{UU}\right)^* &= \frac{9\tau(n_2 + n_c)(3n_1 + 4n_2 + 4n_c)^2}{(n_1 + 12n_2 + 12n_c)^2}.\end{aligned}$$

Proof. See Appendix C. □

As can be seen from the above proposition, in the equilibrium condition (11) is equivalent to (12). One can also see that the prices charged by the firms are higher than τ , the price set in a standard Hotelling model with two firms, and depend on the relative market size $n_1/(n_c + n_2)$. This is due to imperfect information of commuters from market 1 about the price set by firm B in the central market. In particular, if $n_1 = 0$, in this case all commuters are informed, $p_A^{UU} = p_B^{UU} = \tau$.

4.2.3 The FU Equilibrium

In this subsection, we assume that firm A uses flexible pricing, while firm B uniform. In this case, the firm profit maximization problems are given by

$$\begin{aligned}\max_{p_{A1}, p_{A2}, p_{Ac} \geq 0} \left\{ \pi_A^{FU} &= \frac{3n_1}{2} \left(\frac{v - p_{A1}}{\tau} \right) p_{A1} + \frac{3n_2}{2} \left(\frac{p_B - p_{A2} + \tau}{2\tau} \right) p_{A2} \right. \\ &+ \left(2n_c \left(\frac{p_B - p_{Ac} + \tau}{2\tau} \right) + \frac{n_1}{2} \left(\frac{\tilde{p}_B - \tilde{p}_{Ac} + \tau}{2\tau} \right) + \frac{n_2}{2} \left(\frac{p_B - \tilde{p}_{Ac} + \tau}{2\tau} \right) \right) p_{Ac} \Big\}, \\ \max_{p_B \geq 0} \left\{ \pi_B^{FU} &= \left(\frac{3n_2}{2} \left(\frac{p_{A2} - p_B + \tau}{2\tau} \right) + 2n_c \left(\frac{p_{Ac} - p_B + \tau}{2\tau} \right) \right) p_B + \right. \\ &\left. \left(\frac{n_2}{2} \left(\frac{\tilde{p}_{Ac} - p_B + \tau}{2\tau} \right) + \frac{n_1}{2} \left(\frac{\tilde{p}_{Ac} - \tilde{p}_B + \tau}{2\tau} \right) \right) p_B \right\}.\end{aligned}\tag{13}$$

Condition (6) can be rewritten in the following way:

$$\begin{cases} p_{A1} \leq v - \tau, \\ p_{A2}, p_{Ac} \in (p_B - \tau, p_B + \tau), \\ v \geq \frac{p_{A2} + p_B + \tau}{2}, \\ v \geq \frac{p_{Ac} + p_B + \tau}{2}. \end{cases} \quad (14)$$

As before, we formulate a proposition that characterizes the equilibrium firm prices and profits in the *FU* equilibrium.

Proposition 3. *If*

$$v \geq \max \left\{ 2\tau, \frac{4(n_1 + 6n_2 + 6n_c)(6n_c + n_1 + n_2)}{(8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2))} \tau \right\}, \quad (15)$$

$$n_1 \in \left[0, 2n_2 + n_c + \sqrt{9n_2^2 + 54n_2n_c + 49n_c^2} \right), \quad (16)$$

and only firm B uses uniform pricing, then the equilibrium price vector is:

$$\begin{aligned} p_{A1}^{FU} &= v - \tau, \\ p_{A2}^{FU} &= \frac{2(n_1 + 6n_2 + 6n_c)(8n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau, \\ p_{Ac}^{FU} &= \frac{4(4n_c + n_1 + n_2)(n_1 + 6n_2 + 6n_c)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau, \\ p_B^{FU} &= \frac{-8n_c(4n_c + n_1 + n_2) + (4n_1 + 13n_2 + 16n_c)(8n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau, \end{aligned}$$

and the firms earn:

$$\begin{aligned}
(\pi_A^{FU})^* &= \frac{3n_1}{2} (v - \tau) + \frac{3n_2}{2} \left(\frac{p_B^{FU} - p_{A2}^{FU} + \tau}{2\tau} \right) p_{A2}^{FU} + \frac{4n_c + n_1 + n_2}{2} \left(\frac{p_B^{FU} - p_{Ac}^{FU} + \tau}{2\tau} \right) p_{Ac}^{FU}, \\
(\pi_B^{FU})^* &= \left(\frac{3n_2}{2} \left(\frac{p_{A2}^{FU} - p_B^{FU} + \tau}{2\tau} \right) + \frac{4n_c + n_1 + n_2}{2} \left(\frac{p_{Ac}^{FU} - p_B^{FU} + \tau}{2\tau} \right) \right) p_B^{FU}.
\end{aligned}$$

Proof. See Appendix C. □

Note that in the statement of the above proposition, we do not substitute for the equilibrium prices in the expressions for the equilibrium profits, as in this case they look too complex. Another thing to notice is the condition on the market sizes in (16). The idea behind it is that, if market 1 is sufficiently large relative to market 2 and the central market, the prices chosen by firms are such that $p_{A2}^{FU} + \tau \leq p_B^{FU}$, which contradicts (14). The intuition is the following. If market 1 is large, commuters from this market to the central one play an important role for firms, when they set their prices at the central market. Moreover, these commuters are not informed about the prices both firms choose in the central market (this is because firm A uses flexible pricing, while firm B is not present in market 1). As a result, for sufficiently high n_1 , firm B , that uses uniform pricing, decides to choose such a high price p_B^{FU} to increase the profit in the central market that it loses its consumers in market 2. The latter does not happen, if market 2 is sufficiently large relative to market 1. Interestingly, p_B^{FU} is also lower than $p_{A2}^{FU} + \tau$, if the central market is large enough. This is because consumers in the central market are informed about the prices set by the firms there, which implies a downward pressure on the prices the firms choose competing with each other.

4.2.4 The UF Equilibrium

Finally, in this subsection, we assume that firm A uses uniform pricing, while firm B flexible. In this case, condition (6) can be rewritten in the following way:

$$\begin{cases} p_A \leq v - \tau, \\ p_{B2}, p_{Bc} \in (p_A - \tau, p_A + \tau), \\ v \geq \frac{p_A + p_{B2} + \tau}{2}, \\ v \geq \frac{p_A + p_{Bc} + \tau}{2}. \end{cases} \quad (17)$$

The firm profit maximization problems are in turn given by

$$\begin{aligned} \max_{p_A \geq 0} \left\{ \pi_A^{UF} = \frac{3n_1}{2} p_A + \frac{3n_2}{2} \left(\frac{p_{B2} - p_A + \tau}{2\tau} \right) p_A \right. \\ \left. + \left(2n_c \left(\frac{p_{Bc} - p_A + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Bc} - p_A + \tau}{2\tau} \right) \right) p_A \right\}, \\ \max_{p_{B2}, p_{Bc} \geq 0} \left\{ \pi_B^{UF} = \frac{3n_2}{2} \left(\frac{p_A - p_{B2} + \tau}{2\tau} \right) p_{B2} + \right. \\ \left. \left(2n_c \left(\frac{p_A - p_{Bc} + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{p_A - \tilde{p}_{Bc} + \tau}{2\tau} \right) \right) p_{Bc} \right\}. \end{aligned} \quad (18)$$

As before, we formulate a proposition that characterizes the equilibrium firm prices and profits in the UF equilibrium.

Proposition 4. *If*

$$v \geq \frac{6(4n_c + 3n_1 + 4n_2)(8n_c + n_1 + n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)} \tau \quad (19)$$

$$n_1 \in \left[0, \frac{5n_2 - 20n_c + \sqrt{5(45n_2^2 + 360n_2n_c + 464n_c^2)}}{10} \right), \quad (20)$$

and only firm A uses uniform pricing, then the equilibrium price vector is:

$$\begin{aligned}
p_A^{UF} &= \frac{(12n_1 + 9n_2)(8n_c + n_1 + n_2) + 4(4n_c + n_1 + n_2)(6n_c + n_1 + n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)}\tau, \\
p_{B2}^{UF} &= \frac{3(4n_c + 3n_1 + 4n_2)(8n_c + n_1 + n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)}\tau, \\
p_{Bc}^{UF} &= \frac{6(4n_c + n_1 + n_2)(4n_c + 3n_1 + 4n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)}\tau,
\end{aligned}$$

and the firms earn:

$$\begin{aligned}
(\pi_A^{UF})^* &= \left(\frac{3n_1}{2} + \frac{3n_2}{2} \left(\frac{p_{B2}^{UF} - p_A^{UF} + \tau}{2\tau} \right) + \left(2n_c + \frac{n_1 + n_2}{2} \right) \left(\frac{p_{Bc}^{UF} - p_A^{UF} + \tau}{2\tau} \right) \right) p_A^{UF}, \\
(\pi_B^{UF})^* &= \frac{3n_2}{2} \left(\frac{p_A^{UF} - p_{B2}^{UF} + \tau}{2\tau} \right) p_{B2}^{UF} + \left(2n_c + \frac{n_1 + n_2}{2} \right) \left(\frac{p_A^{UF} - p_{Bc}^{UF} + \tau}{2\tau} \right) p_{Bc}^{UF}.
\end{aligned}$$

Proof. See Appendix C. □

Similar to the *FU* equilibrium, there is an additional constraint on the market sizes. If market 1 is sufficiently large, the prices chosen by firms are such that $p_{B2}^{UF} + \tau \leq p_A^{UF}$, which contradicts (17). As in the previous section, for sufficiently high n_1 , the firm that uses uniform pricing decides to choose such a high price to increase the profit in the central market that it loses its consumers in market 2.

In the next section, we consider the equilibrium at the first stage.

5 Optimal Pricing Strategies

In this section, we find the equilibrium at Stage 1 and analyze its properties. As one can see, the expressions for the profits derived in the previous section are quite complex to be compared in the general case with arbitrary market sizes. Therefore, in the below analysis we consider the special case with $n_2 = n_c \equiv n$ and $n_1 = a \cdot n$, where $a \geq 0$ captures how large market 1 is relative to the other markets. In particular, when $a = 0$, we have the case with completely symmetric firms in terms their access to the markets; when, for instance, $a = 1$, all the markets have the same density of local consumers at each point of the unit interval, but only firm *A* serves market 1.

5.1 The Equilibrium at Stage 1

First, we impose the constraints on v and a to ensure that both firms face non-zero demand in all the markets where they are present. Specifically, we need that the constraints on the parameters formulated in Propositions 1–4 are satisfied. It is relatively straightforward to show that the latter constraints can be rewritten in the following way:

$$\begin{cases} v \geq \tilde{v}(a) \\ a \in \left[0, \frac{\sqrt{4345}-15}{10}\right) \end{cases}, \quad (21)$$

where

$$\tilde{v}(a) = \max \left\{ 2\tau, \frac{\tau(7+a)}{4}, \frac{3\tau(3a^2+56a+128)}{(a^2+32a+192)}, \frac{4\tau(a^2+19a+84)}{(27a+211)}, \frac{6\tau(3a^2+35a+72)}{(2a^2+45a+211)} \right\}. \quad (22)$$

The next lemma simplifies the expression for \tilde{v} .

Lemma 1. *For $a \in \left[0, \frac{\sqrt{4345}-15}{10}\right)$, the expression for $\tilde{v}(a)$ in (22) is simplified to*

$$\tilde{v}(a) = \frac{6\tau(3a^2+35a+72)}{2a^2+45a+211}.$$

Proof. See Appendix C. □

Thus, we assume that

$$\begin{cases} v \geq \tilde{v}(a) = \frac{6\tau(3a^2+35a+72)}{2a^2+45a+211} \\ a \in \left[0, \frac{\sqrt{4345}-15}{10}\right) \end{cases}. \quad (23)$$

Next, we formulate the main theorem in the paper.

Theorem 1. *Suppose conditions in (23) hold. Then, there exist \bar{v} , where*

$$\bar{v}(a) = \tau \left(\frac{(a+8)(16a^2+165a+221)^2}{6a(2a^2+45a+211)^2} - \frac{2}{3a} \left(\frac{a+5}{4} \right)^2 + \frac{2a-1}{2a} \right) > \tilde{v}(a),$$

$\hat{a} \approx 0.34$, and $\tilde{a} \approx 2.84$ such that:

- For $a \in [0, \hat{a})$, the unique Nash equilibrium of Stage 1 is (F, F) for any feasible v ,

- For $a \in [\hat{a}, \tilde{a})$, the Nash equilibrium of Stage 1 depends on v :
 - for $v \in [\tilde{v}(a), \bar{v}(a))$, the unique Nash equilibrium of Stage 1 is (U, F) , and
 - for $v \geq \bar{v}(a)$, the unique Nash equilibrium of Stage 1 is (F, F) ,
- For $a \in [\tilde{a}, (\sqrt{4345} - 15) / 10)$, the unique Nash equilibrium of Stage 1 is (F, F) for any feasible v .

Proof. See Appendix C. □

In the proof, we show that if firm A uses flexible pricing, firm B always responds with the same strategy. When firm A sticks to uniform pricing, the response of firm B depends on the value of a but is not affected by parameter v . If a is sufficiently small, firm B finds strategy F more attractive, and flexible pricing turns out to be its dominant strategy. Otherwise, firm B responds to the competitor's action U with uniform pricing as well, which can change the equilibrium of the game. Firm A 's choice depends on both a and v , and the effect of a on its strategy is non-monotone.¹³ For each set of parameters specified in Theorem 1, either firm A or firm B has a dominant strategy, which makes the Nash equilibrium of the Stage 1 game unique.¹⁴

Theorem 1 has the following interpretation. When a is small enough, market 1 does not seem to be decisive for firm A in terms of its pricing strategy. In fact, the situation is close to that when $a = 0$ and firms are completely symmetric, which results in sticking to the same pricing strategy F of both firms irrespective of the value of v . With a being sufficiently high, market 1 is so large relative to the other markets that firm A finds it optimal to set a separate price for this market to extract more surplus, which leads again to the equilibrium with flexible pricing. When, however, a is neither too big nor too small, the situation is different and the value of v , which defines the surplus firm A can extract from market 1, becomes important. If v is relatively low, this surplus turns out to be rather small. Then, setting the same price in all the markets where firm A is present helps it to attract commuters, who do not observe the price of firm B in market c , and this positive effect outweighs losses from no full surplus extraction

¹³Parameter τ has no direct effect on firm A 's strategy choice.

¹⁴Although there can be equilibria where one of the firms randomizes between strategy F and strategy U , all of them correspond to boundary cases (for example, $v = \bar{v}$). Since such equilibria are extremely specific and very unrealistic, we do not look at them separately.

in market 1. As a result, we observe (U, F) as the Nash equilibrium (firm B 's dominant strategy is still flexible pricing). With v high enough, the surplus firm A extracts from market 1 under flexible pricing is rather large compared to the competitive benefits uniform pricing brings. Thus, flexible pricing is more attractive for firm A in this case.

5.2 Uniform Pricing and Social Welfare

As Theorem 1 states, there are only two strategy profiles – (F, F) and (U, F), – that can be a Nash equilibrium of the Stage 1 game. In other words, in some cases firm A benefits from setting the same price in all the markets where it is present. In this section, we examine how this affects the social welfare evaluated *in equilibrium*. As in Section 5.1, we assume that conditions in (23) hold and both firms face non-zero demand in the markets where they operate. We focus on utilitarian welfare measured as

$$W^j = \sum_{i \in \{1, 2, c\}} W_i^j,$$

where $j \in \{FF, UU, FU, UF\}$ corresponds to pricing strategies chosen by the firms and W_i^j is the social welfare in market $i \in \{1, 2, c\}$ given by

$$W_i^j = CS_i^j + \left(\pi_{iA}^j\right)^* + \left(\pi_{iB}^j\right)^*$$

with CS_i^j being the corresponding consumer surplus.

The first thing to show is that the social welfare in market 1, or W_1^j , does not depend on the price set by firm A in this market. If the conditions in (23) hold, all consumers in market 1 buy from firm A . Then, the consumer surplus (taken over two periods) in this market can be found as

$$CS_1^j = \left(n_1 + \frac{n_1}{2}\right) \int_0^1 (v - p_{A1} - \tau(1 - x)) dx = \frac{3n_1}{2} \left(v - p_{A1} - \frac{\tau}{2}\right),$$

and firm A receives

$$\left(\pi_{1A}^j\right)^* = \frac{3n_1}{2} p_{A1}.$$

As a result, we have that

$$W_1^j = CS_1^j + \left(\pi_{1A}^j\right)^* = \frac{3n_1}{2} \left(v - \frac{\tau}{2}\right).$$

Note that the above holds for any pricing strategies $j \in \{FF, UU, FU, UF\}$. This finding allows us to claim that social welfare only depends on the prices the firms set in markets 2 and c , where they compete with each other. The next proposition states when social welfare turns out to be the highest.

Proposition 5. *Suppose the conditions in (23) hold. Then, social welfare achieves its maximum, when both firms set equal prices in markets 2 and C : $p_{A2} = p_{B2}$ and $p_{Ac} = p_{Bc}$: that is, under the (F, F) pricing strategy.*

Proof. See Appendix C. □

In the proof of Proposition 5, we first define the social welfare as a function of consumers' threshold types in markets 2 and c , respectively.¹⁵ Then, we show that the social welfare is maximized when the firms equally share these two markets, which is possible if and only if they set identical prices. This corresponds to the (F, F) Nash equilibrium of the Stage 1 game, and this equilibrium is feasible given conditions in (23). Hence, the Nash equilibrium of the Stage 1 game where firm A uses uniform pricing is inefficient compared to the (F, F) equilibrium.

To quantify efficiency losses associated with uniform pricing, we define

$$\Delta_W = W^{FF} - W^{UF} > 0,$$

where $\Delta_W > 0$ follows from Propositions 1, 4, and 5. Then, we investigate how Δ_W depends on parameter a that captures the size of market 1 relative to the other markets. As Theorem 1 states, firm A chooses uniform pricing if a is neither too big nor too small and v is rather low. This allows us to restrict attention to $a \in [\hat{a}, \tilde{a})$ and $v \in [\tilde{v}(a), \bar{v}(a))$, where the threshold values for a and v are defined in Theorem 1.¹⁶ The following lemma states how Δ_W responds to changes in parameter a .

Lemma 2. *Suppose $a \in [\hat{a}, \tilde{a})$ and $v \in [\tilde{v}(a), \bar{v}(a))$ hold. Then, the efficiency losses associated with uniform pricing by firm A are increasing in parameter a , or $\partial \Delta_W / \partial a > 0$.*

¹⁵Actually, the proof of Proposition 5 does not rely on the assumption of $n_2 = n_c \equiv n$, $n_1 = a \cdot n$.

¹⁶Notice that $a \in [\hat{a}, \tilde{a})$ and $v \in [\tilde{v}, \bar{v})$ imply conditions in (23).

Proof. See Appendix C. □

This result is quite intuitive. If a increases, market 1, where firm A acts as a monopolist, becomes larger relative to the other two markets where it faces competition from firm B . As a result, the advantage of firm A over firm B and its ability to extract consumer surplus improve, too. This power imbalance, however, leads to deadweight losses (compared to the case when both firms set identical market-specific prices), and social welfare declines.

The above analysis implies that the allocation maximizing the social welfare determined as the sum of the consumer surplus and firms' profits involves equal prices and market shares of the firms in all markets. We then focus solely on the consumer surplus and investigate how it depends on the pricing strategies adopted by firms. In other words, we explore which pair of the pricing strategies maximizes the total consumer surplus defined as

$$CS^j = \sum_{i \in \{1, 2, c\}} CS_i^j,$$

where $j \in \{FF, UU, FU, UF\}$. The following proposition summarizes our findings.

Proposition 6. *Assume that conditions in (23) hold. Then, there exists $\bar{a} \approx 0.2583 \in \left[0, \left(\sqrt{4345} - 15\right) / 10\right)$ such that:*

1. *For a being sufficiently small: $a \in [0, \bar{a}]$; (U, U) maximizes the total consumer surplus for any feasible v ,*
2. *For a being large enough: $a \in \left(\bar{a}, \left(\sqrt{4345} - 15\right) / 10\right)$; there exists $\hat{v}(a) > \tilde{v}(a)$ such that:*
 - *(F, U) maximizes the total consumer surplus for $v \in [\tilde{v}(a), \hat{v}(a)]$, and*
 - *(U, U) maximizes the total consumer surplus for $v > \hat{v}(a)$.*

Proof. See Appendix C. □

As Proposition 6 implicitly states, the strategy profiles that maximize the total consumer surplus are never chosen by the firms in the equilibrium (see Theorem 1). This result is rather intuitive, since profit maximization is generally achieved at the cost of consumer surplus, and the following corollary formalizes our observation.

Corollary 1. *Assume that conditions in (23) hold. Then, the Nash equilibrium of the Stage 1 game never maximizes the total consumer surplus.*

To highlight the intuition behind Proposition 6, it is important to understand what happens to the consumer surplus at the market level. In the Appendix we show that, if the relative size of market 1 is sufficiently small, which corresponds to a low enough, strategy profile (U, U) is the best for consumers in the central market. The intuition behind this result is the following. In the case of flexible pricing, the presence of uninformed commuters in the central market creates upward pressure on the firm prices, making them higher than they would be in the case of uniform pricing, when the presence of market 2 with perfectly informed consumers “smooths” the price level. In other words, when market 1 is relatively small, flexible pricing leads to relatively high prices in the central market and low prices in market 2, while uniform pricing results in some “average” pricing in the markets, which benefits consumers in the central market.

When the relative size of market 1 rises, it starts playing a more important role. In particular, the advantage of firm A over its rival becomes more pronounced, as in the case of uniform pricing there is a substantial amount of commuters from market 1, who are informed about the price of firm A , but not informed about the price of firm B . As a result, the uniform price set by firm A increases, and consumers in market c turn out to be worse off if firm A does not switch to flexible pricing. This implies that, when a is sufficiently high, strategy profile (F, U) appears to be the best for consumers in the central market (the formal proof can be found in the Appendix).

In market 2, there are no uninformed consumers and the firms compete with each other. As we show in the Appendix, in this market, the (F, F) strategy profile is always the best for the consumers, as uniform pricing “brings” higher prices from the central market. In contrast, consumer surplus in market 1, where only firm A is active, is maximized when both firms stick to uniform pricing, and this finding is not novel in the literature (for example, see [Holmes, 1989](#); [Dobson and Waterson, 2005, 2008](#)). The idea behind is that using flexible pricing, firm A can extract as much surplus from market 1 as possible (up to the point where some consumers stop buying its good). With uniform pricing, its ability to price discriminate becomes limited – essentially, firm A commits not to use its monopolistic power in market 1, – and consumer surplus in this market increases.

Hence, the overall effect of different pricing strategies on the total consumer surplus formulated in Proposition 6 is determined by the weighted effects on the consumer surplus in each market discussed above and depends on the relative size of market 1 (parameter a) and the value of the good (parameter v). In particular, when a is sufficiently small, the total consumer surplus is maximized under (U, U), and this is mainly driven by consumers who shop in market c . If a increases and more commuters from market 1 arrive to market c , the value of the good, v , becomes important. For v being sufficiently small, the positive effect of uniform pricing adopted by firm A on CS_1 , which is closely related to the value of v , is smaller than the negative effect of this pricing strategy on CS_c , and the total consumer surplus is higher under (F, U) than in case of (U, U). When v increases, consumers in market 1 gain much more than their peers in market c lose under uniform pricing by firm A , and (U, U) maximizes the total consumer surplus. Note that under the assumption of $n_2 = n_c = n$, $n_1 = an$, market 2 appears to be not that decisive, because it is always smaller than market c where commuters arrive to.¹⁷

The findings in this section provide novel insights on the relationship between firms' pricing strategies and social welfare. If a policymaker is concerned about the total consumer surplus maximization, under certain conditions it can be optimal to force both chain firms to use uniform pricing, as uniform pricing seems to be a good way to limit the ability of chain firms to price discriminate. At the same time, the total social welfare in this case appears to be lower than when both firms use flexible pricing. These results may call for policies aimed at restricting or facilitating firms' incentives to engage in uniform pricing (for example, by bringing more competition to markets where one of the chains acts as a monopolist or has excessive market power for some other reasons), depending on the goals of a policymaker.

6 Discussion and Conclusions

This paper enlarges the support for the uniform pricing phenomenon by introducing interconnection of heterogeneous markets via commuters. We show that if two chains are alike and have access to almost the same set of markets, both firms engage in flexible pricing. When heterogeneity between the chains increases, a larger firm, which acts as a monopolist in some of the markets, can benefit from uniform

¹⁷If we relax this assumption and let the size of market 2 change, (F, F) can also maximize the total consumer surplus under certain conditions.

pricing, while its rival still sticks to flexible pricing. The main trade-off for the former firm is driven by the interplay between the surplus it can extract from consumers in monopolized markets under flexible pricing and the opportunity of exploiting uninformed commuters to the central market, where the other chain is also presented, that uniform pricing gives.

As for welfare implications, uniform pricing, adopted by at least one of the firms, unambiguously reduces the total surplus. At the same time, consumer surplus increases when both chain firms or only a smaller one set uniform prices across different markets because this limits their ability to price discriminate. This sheds light on the policies a regulator should implement depending on the goals that need to be achieved.

Although our baseline model is stylized, it allows us to explain the co-existence of uniform and flexible pricing and to capture some empirical regularities. The two forces that make uniform pricing possible in equilibrium are (1) the presence of commuters who generally have no information about prices in a market where they do not reside, and (2) the fact that one of the firms has no access to a subset of markets. If at least one of these assumptions is relaxed, we get back to a model with isolated markets / symmetric firms, where the only possible equilibrium of the Stage 1 game is flexible pricing for both firms. Indeed, in the absence of market 1 and uninformed commuters, firms clearly have no incentive to use the uniform pricing strategy. If the firms are identical, which corresponds to the case when the size of market 1 is zero (or $n_1 = 0$), it is possible to show that both firms choose flexible pricing irrespective of the sizes of market 2 and the central market. This result, in fact, follows from Propositions 1 and 2. Specifically, if $n_1 = 0$, then according to the propositions, in the case of uniform pricing both firms set their prices equal to τ , while flexible pricing allows them to discriminate consumers shopping in the central market by setting prices strictly higher than τ , thanks to uninformed commuters from market 2. This in turn implies that flexible pricing is more preferable for both firms than uniform pricing. Other assumptions made in Section 4 are not pivotal for our results, and we discuss some of them below.

In the baseline model, we assume that in the second period, half of consumers from each periphery market travel to the central market and make purchases there. The main reason for this simplification is to keep the analytical tractability of the model. In particular, our focus in the paper is the role of the firm asymmetry represented by the size of market 1. If we introduce one more parameter capturing the share

of commuters from each market, the qualitative analysis becomes much more complicated. Moreover, as stated previously, if we choose another reasonable value for the share of commuters, this will not affect our result qualitatively, but only quantitatively. At the same time, it may be interesting and promising to explore the role of the interplay between the size of market 1 and the share of commuters in determining the equilibrium pricing strategies.

Another interesting extension can be to let some commuters select a market where to shop in the second period, which they are not allowed to do in the present model. This would help us to endogenize the share of commuters and may provide some additional insights, but at the cost of having an even less analytically tractable model, compared to the case with the exogenous share of commuters represented by a parameter. Intuitively, our main results should remain the same if the share of commuters who must shop in the central market in the second period (for example, due to preference shocks or other reasons) is not too small. However, we do not provide a detailed analysis here and leave this question for future research.

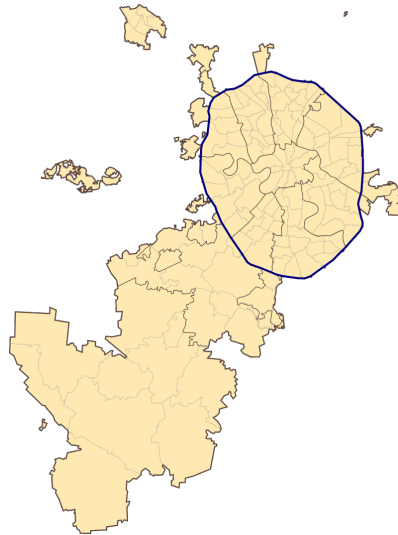
One more potentially interesting extension is to further explore the role of consumer heterogeneity. In the paper, we allow consumers to differ in their location (both within and between markets) and commuting status. However, there can be other sources of heterogeneity (e.g., different preferences, income level). For instance, it may be interesting to disentangle the role of market size in terms of the number of consumers residing there and in terms of their income on the firms' pricing strategies. This research direction is also left for future analysis.

A Map of Moscow

By the end of the 1960s, Moscow was limited by the Moscow Automobile Ring Road (MKAD). In 1969, the city of Zelenograd officially became the first “enclave” part of Moscow outside the MKAD. Finally, the borders of Moscow substantially changed in 2011–2012 with the inclusion of new territories (so-called “new” Moscow) to the South-West of the city.

To make sure that our sample is rather homogenous in geographical and population characteristics, we limit our attention to “old” Moscow borders that mostly coincide with the MKAD. Therefore, we exclude two big cities – Zelenograd and Troitsk, – that are officially located in Moscow but are geographically remote and potentially different in demand. The same concerns hold for “new” Moscow, which is still poorly connected to the city center via public transportation and is distinct not just geographically but also in population characteristics and urban organization.

FIGURE 3: THE MOSCOW AUTOMOBILE RING ROAD (MKAD) AND MOSCOW BORDERS



B Price Dispersion

In this Appendix, we discuss price dispersion across different affiliates of a given flexible-pricing pharmacy chain. The information on assortment and prices is scrapped from a well-known online aggregator apteka.ru that allows consumers to find out where and at what price a drug they need is available. Since this online platform substantially simplifies consumer search, the majority of chain

TABLE 5: PRICE DISPERSION IN FLEXIBLE PRICING CHAINS

	Price dispersion		$\frac{\max(\text{price}) - \min(\text{price})}{\text{mean}(\text{price})}$		N
	Mean	SD	Mean	SD	
Total	0.0797	0.1056	0.1224	0.2032	35,239
Need a recepe	0.0676	0.0608	0.1200	0.1132	6,538
Has limit price	0.0572	0.0576	0.1002	0.1014	3,094
By category					
Drugs	0.0730	0.0743	0.1173	0.1319	15,654
Medical cosmetics	0.0738	0.1107	0.1147	0.2438	6,428
Medical devices	0.1028	0.1592	0.1432	0.2979	4,178
BAD	0.0745	0.0938	0.1252	0.1976	4,129
Care products	0.1406	0.1949	0.1560	0.3019	1,481
Hygiene products	0.0808	0.0920	0.1296	0.1770	947
Food products	0.0795	0.1024	0.1136	0.1904	615
Homeopathy	0.0785	0.1315	0.1190	0.2478	264
Therapeutic nutrition	0.0512	0.0943	0.0814	0.1240	124
Optics	0.0082	0.0341	0.0148	0.0548	90
Disinfectants	0.1342	0.1282	0.2134	0.3011	67
Other	0.0917	0.1182	0.1188	0.2096	1211

Note: A drug is defined at the level of its trade name, dosage, producer, and country.

pharmacies, or 58% of drug stores in Moscow, post information on their assortment and prices there.¹⁸ Still, the information provision is voluntary, and this limits our sample of pharmacies for which data on assortment and prices are available.¹⁹

On average, prices for a given product differ by 8% across different chain locations (see Table 5). Differences in prices are slightly lower for drugs which require a prescription or those for which price is regulated. We note that according to the Russian law, as long as a price is not more than 15% higher than a price limit, pharmacy is free to choose a price for a drug for which a certain price limit applies. As for the categories of products sold by flexible-pricing pharmacies, the highest dispersion results for care products, disinfection, and medical cosmetics.

¹⁸We acknowledge that online prices can differ from offline ones (Cavallo, 2017) and therefore consider only a market of online pre-orders, either via an aggregator or a pharmacy's own website.

¹⁹Notably, two large pharmacy chains – 36,6 and Apteki Stolichki, – entered the platform only recently, and we do not have information about assortment and prices for some of their affiliates.

C Proofs

The proof of Proposition 1 Suppose conditions (6) hold. Then, taking into account that we solve for optimal p_{A1} in the main text, the system of the first order conditions for optimal p_{A2} , p_{Ac} , p_{B2} , p_{Bc} is as follows:

$$\begin{aligned}\frac{\partial \pi_A^{FF}}{\partial p_{A2}} &= \frac{3n_2}{4\tau} (p_{B2} - 2p_{A2} + \tau) = 0, \\ \frac{\partial \pi_A^{FF}}{\partial p_{Ac}} &= 2n_c \left(\frac{p_{Bc} - p_{Ac} + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Bc} - \tilde{p}_{Ac} + \tau}{2\tau} \right) - \frac{n_c}{\tau} p_{Ac} = 0, \\ \frac{\partial \pi_B^{FF}}{\partial p_{B2}} &= \frac{3n_2}{4\tau} (p_{A2} - 2p_{B2} + \tau) = 0, \\ \frac{\partial \pi_B^{FF}}{\partial p_{Bc}} &= 2n_c \left(\frac{p_{Ac} - p_{Bc} + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Ac} - \tilde{p}_{Bc} + \tau}{2\tau} \right) - \frac{n_c}{\tau} p_{Bc} = 0.\end{aligned}$$

As can be seen, for p_{A2} and p_{B2} , the solution is the same as in a standard Hotelling model:

$$p_{B2}^{FF} = p_{A2}^{FF} = \tau.$$

Constraint (6) requires

$$v \geq \frac{p_{A2} + p_{B2} + \tau}{2} = \frac{3\tau}{2},$$

which holds as $v \geq 2\tau$.

For p_{Ac} and p_{Bc} , we first solve for prices as the functions of \tilde{p}_{Ac} and \tilde{p}_{Bc} and then find consistent beliefs by setting $\tilde{p}_{Ac} = p_{Ac}$ and $\tilde{p}_{Bc} = p_{Bc}$. Specifically, from the above first order conditions, we have

$$\frac{n_c}{\tau} p_{Bc} + \frac{n_c}{\tau} p_{Ac} = 2n_c + \frac{n_1 + n_2}{2} \iff p_{Ac} = 2\tau + \tau \frac{n_1 + n_2}{2n_c} - p_{Bc}.$$

This in turn implies that

$$\begin{aligned}p_{Bc} &= \tau + \tau \frac{n_1 + n_2}{4n_c} + \frac{n_1 + n_2}{12n_c} (\tilde{p}_{Ac} - \tilde{p}_{Bc}), \\ p_{Ac} &= \tau + \tau \frac{n_1 + n_2}{4n_c} - \frac{n_1 + n_2}{12n_c} (\tilde{p}_{Ac} - \tilde{p}_{Bc}).\end{aligned}$$

Setting $\tilde{p}_{Ac} = p_{Ac}$ and $\tilde{p}_{Bc} = p_{Bc}$, we have

$$p_{Bc}^{FF} = p_{Ac}^{FF} = \tau + \tau \frac{n_1 + n_2}{4n_c}.$$

These prices are compatible with constraint (6) if and only if:

$$v \geq \frac{p_{Ac} + p_{Bc} + \tau}{2} = \frac{3\tau + \tau \frac{n_1 + n_2}{2n_c}}{2}.$$

Finally, we compute the equilibrium profits:

$$\left(\pi_A^{FF}\right)^* = \frac{3n_1}{2} (v - \tau) + \frac{3n_2}{4} \tau + n_c \tau \left(1 + \frac{n_1 + n_2}{4n_c}\right)^2,$$

$$\left(\pi_B^{FF}\right)^* = \frac{3n_2}{4} \tau + n_c \tau \left(1 + \frac{n_1 + n_2}{4n_c}\right)^2.$$

The Proof of Proposition 2 The first order conditions with respect to p_A and p_B are given by

$$\frac{\partial \pi_A^{UU}}{\partial p_A} = 0 \iff$$

$$\left(\frac{n_2 + n_c}{\tau} + \frac{n_1}{4\tau}\right) p_A = \frac{3n_1}{2} + \left(\frac{p_B - p_A + \tau}{2\tau}\right) 2(n_2 + n_c) + \frac{n_1}{2} \left(\frac{\tilde{p}_B - p_A + \tau}{2\tau}\right),$$

$$\frac{\partial \pi_B^{UU}}{\partial p_B} = 0 \iff$$

$$\frac{n_2 + n_c}{\tau} p_B = \left(\frac{p_A - p_B + \tau}{2\tau}\right) 2(n_2 + n_c) + \frac{n_1}{2} \left(\frac{p_A - \tilde{p}_B + \tau}{2\tau}\right).$$

Adding the conditions implies that

$$\left(\frac{n_2 + n_c}{\tau} + \frac{n_1}{4\tau}\right) p_A + \frac{n_2 + n_c}{\tau} p_B = 2(n_2 + n_c + n_1) \iff$$

$$p_A = \frac{2\tau(n_2 + n_c + n_1) - (n_2 + n_c)p_B}{n_2 + n_c + \frac{n_1}{4}}. \quad (24)$$

Substituting into the first order condition for p_B , we derive

$$2(n_2 + n_c) p_B = \left(\frac{2\tau(n_2 + n_c + n_1) - (n_2 + n_c) p_B}{n_2 + n_c + \frac{n_1}{4}} + \tau \right) \left(n_2 + n_c + \frac{n_1}{4} \right) - \frac{n_1}{4} \tilde{p}_B \Rightarrow$$

$$3(n_2 + n_c) p_B = 2\tau(n_2 + n_c + n_1) + \tau \left(n_2 + n_c + \frac{n_1}{4} \right) - \frac{n_1}{4} \tilde{p}_B.$$

Taking into account that in the equilibrium, $\tilde{p}_B = p_B$, we have

$$p_B = \frac{2\tau(n_2 + n_c + n_1) + \tau \left(n_2 + n_c + \frac{n_1}{4} \right)}{3(n_2 + n_c) + \frac{n_1}{4}} = \tau + \frac{2\tau n_1}{3(n_2 + n_c) + \frac{n_1}{4}}.$$

Then, substituting the derived expression for p_B into (24), we have

$$p_A = \tau + \tau n_1 \frac{52(n_2 + n_c) + 7n_1}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)}.$$

Let us then check whether $p_i \in (p_j - \tau, p_j + \tau)$, $\forall i \neq j$ where $i, j \in \{A, B\}$. Specifically, we need to check whether $p_A < p_B + \tau$ and $p_A > p_B - \tau$. For the former, we need to check

$$\tau + \tau n_1 \frac{52(n_2 + n_c) + 7n_1}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)} < 2\tau + \frac{2\tau n_1}{3(n_2 + n_c) + \frac{n_1}{4}} \iff$$

$$n_1 \frac{52(n_2 + n_c) + 7n_1}{4(n_2 + n_c) + n_1} < 12(n_2 + n_c) + 9n_1 \iff$$

$$23(n_2 + n_c)^2 + (n_2 + n_c - n_1)^2 > 0.$$

For the latter,

$$\tau + \tau n_1 \frac{52(n_2 + n_c) + 7n_1}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)} > \frac{8\tau n_1}{12(n_2 + n_c) + n_1} \iff$$

$$12(n_2 + n_c) + \frac{24n_1(n_2 + n_c)}{4(n_2 + n_c) + n_1} > 0.$$

As can be seen, the inequalities hold. We also find

$$p_A - p_B = \tau n_1 \frac{52(n_2 + n_c) + 7n_1}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)} - \frac{8\tau n_1}{12(n_2 + n_c) + n_1}$$

$$= \frac{\tau n_1}{12(n_2 + n_c) + n_1} \left(\frac{20(n_2 + n_c) - n_1}{4(n_2 + n_c) + n_1} \right).$$

This in fact implies that

$$p_A > p_B \iff 20(n_2 + n_c) > n_1.$$

Moreover,

$$p_A - p_B + \tau = \frac{12\tau(n_2 + n_c)(3n_1 + 4(n_2 + n_c))}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)} > 0,$$

$$p_B - p_A + \tau = 2\tau \left(\frac{n_1^2 - 2n_1(n_2 + n_c) + 24(n_2 + n_c)^2}{(12(n_2 + n_c) + n_1)(4(n_2 + n_c) + n_1)} \right) > 0.$$

Note that

$$p_A + \tau > \frac{p_A + p_B + \tau}{2} \iff p_A - p_B + \tau > 0.$$

This immediately implies that condition (11) is satisfied when $v \geq p_A + \tau$.

Finally, the profits are then given by

$$\begin{aligned} (\pi_B^{UU})^* &= p_B \left(\left(\frac{p_A - p_B + \tau}{2\tau} \right) \left(2(n_2 + n_c) + \frac{n_1}{2} \right) \right) \\ &= 9\tau(n_2 + n_c) \left(\frac{4(n_2 + n_c) + 3n_1}{12(n_2 + n_c) + n_1} \right)^2, \\ (\pi_A^{UU})^* &= p_A \left(\frac{3n_1}{2} + \left(\frac{p_B - p_A + \tau}{2\tau} \right) \left(2(n_2 + n_c) + \frac{n_1}{2} \right) \right) \\ &= \frac{p_A}{2} \left(3n_1 + \frac{n_1^2 - 2n_1(n_2 + n_c) + 24(n_2 + n_c)^2}{12(n_2 + n_c) + n_1} \right) \\ &= p_A \left(\frac{17n_1(n_2 + n_c) + 2n_1^2 + 12(n_2 + n_c)^2}{12(n_2 + n_c) + n_1} \right) \\ &= 4\tau \frac{\left(12(n_2 + n_c)^2 + 17n_1(n_2 + n_c) + 2n_1^2 \right)^2}{(4(n_2 + n_c) + n_1)(12(n_2 + n_c) + n_1)^2}. \end{aligned}$$

The Proof of Proposition 3 The first order conditions with respect to p_{A1} , p_{A2} , p_{Ac} , and p_B are given by

$$\frac{\partial \pi_A^{FU}}{\partial p_{A1}} = 0 \iff \frac{3n_1}{2} \left(\frac{v - 2p_{A1}}{\tau} \right) = 0,$$

$$\frac{\partial \pi_A^{FU}}{\partial p_{A2}} = 0 \iff \frac{3n_2}{2} \left(\frac{p_B - 2p_{A2} + \tau}{2\tau} \right) = 0,$$

$$\frac{\partial \pi_A^{FU}}{\partial p_{Ac}} = 0 \iff$$

$$2n_c \left(\frac{p_B - 2p_{Ac} + \tau}{2\tau} \right) + \frac{n_1}{2} \left(\frac{\tilde{p}_B - \tilde{p}_{Ac} + \tau}{2\tau} \right) + \frac{n_2}{2} \left(\frac{p_B - \tilde{p}_{Ac} + \tau}{2\tau} \right) = 0,$$

$$\frac{\partial \pi_B^{FU}}{\partial p_B} = 0 \iff$$

$$\frac{3n_2}{2} \left(\frac{p_{A2} - 2p_B + \tau}{2\tau} \right) + 2n_c \left(\frac{p_{Ac} - 2p_B + \tau}{2\tau} \right) = - \left(\frac{n_2}{2} \left(\frac{\tilde{p}_{Ac} - 2p_B + \tau}{2\tau} \right) + \frac{n_1}{2} \left(\frac{\tilde{p}_{Ac} - \tilde{p}_B + \tau}{2\tau} \right) \right).$$

Solving for p_{A1} is straightforward:

$$\frac{3n_1}{2} \left(\frac{v - 2p_{A1}}{\tau} \right) = 0 \iff p_{A1} = \frac{v}{2},$$

and this does not contradict condition (14) if and only if $v \geq 2\tau$ holds. When firm A chooses $p_{A1} = \frac{v}{2}$ for $v \geq 2\tau$, demand in market 1 is equal to

$$\frac{3n_1}{2} \cdot \min \left\{ 1, \frac{v - p_{A1}}{\tau} \right\} = \frac{3n_1}{2} \cdot \min \left\{ 1, \frac{v}{2\tau} \right\} = \frac{3n_1}{2}.$$

Notice that $\min \left\{ 1, \frac{v - p_{A1}}{\tau} \right\} = 1$ if and only if $p_{A1} \leq v - \tau$ holds, that is, any p_{A1} that satisfies this condition gives the highest possible demand in market 1. Then, the profit-maximizing price p_{A1} for $v \geq 2\tau$ essentially solves

$$\max_{p_{A1} \in [0, v - \tau]} \left\{ \frac{3n_1}{2} \cdot p_{A1} \right\}.$$

Since the objective function is strictly increasing in $p_{A1} \in [0, v - \tau]$, it is optimal to choose the largest feasible p_{A1} . Thus, firm A must set $p_{A1} = v - \tau$ for $v \geq 2\tau$ in order to maximize its profit.

Next, we solve for p_{A2} , p_{Ac} , and p_B . Adding the first order conditions for p_{Ac} and p_B implies that

$$2n_c \left(\frac{-p_{Ac} + 2\tau - p_B}{2\tau} \right) + \frac{n_1}{2} + \frac{n_2}{2} \left(\frac{5\tau - 7p_B + 3p_{A2}}{2\tau} \right) = 0. \quad (25)$$

Then, we use the first order condition for p_{A2} to express p_{A2} as a function of p_B and substitute this into (25), which results in

$$p_B = \frac{-8n_c p_{Ac} + 4n_1 \tau + 13n_2 \tau + 16n_c \tau}{(8n_c + 11n_2)}. \quad (26)$$

Imposing $p_{Ac} = \tilde{p}_{Ac}$, $p_B = \tilde{p}_B$ (that must hold in any fulfilled expectations equilibrium) in the first order condition for p_{Ac} implies

$$(p_B + \tau) \left(1 + \frac{n_1 + n_2}{4n_c} \right) = p_{Ac} \left(2 + \frac{n_1 + n_2}{4n_c} \right).$$

Substituting (26) into the latter, we derive

$$p_{Ac} = \frac{4(4n_c + n_1 + n_2)(n_1 + 6n_2 + 6n_c)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau.$$

Using this expression, we recover p_B and p_{A2} from (26) and the first order condition for p_{A2} , respectively:

$$p_B = \frac{-8n_c(4n_c + n_1 + n_2) + (4n_1 + 13n_2 + 16n_c)(8n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau > 0,$$

$$p_{A2} = \frac{2(n_1 + 6n_2 + 6n_c)(8n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau.$$

Let us then check when condition (14) holds for the given price vector. First, we focus on $p_{A2} \in (p_B - \tau, p_B + \tau)$, which implies $p_B \in (p_{A2} - \tau, p_{A2} + \tau)$:

$$\begin{aligned} p_{A2} > p_B - \tau &\iff \\ \frac{2(50n_2n_c + 4n_2n_1 + 5n_2^2 + 48n_c^2 + 2n_cn_1 - n_1^2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)} \tau > 0 &\iff \\ n_1 \in \left[0, 2n_2 + n_c + \sqrt{9n_2^2 + 54n_2n_c + 49n_c^2} \right), \end{aligned}$$

$$p_{A2} < p_B + \tau \iff$$

$$\frac{-2(n_1 + 6n_2 + 6n_c)(8n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)}\tau < 0.$$

Thus, both firms face non-zero demand in market 2 if and only if n_1 is sufficiently small.

Second, we look at $p_{Ac} \in (p_B - \tau, p_B + \tau)$, which implies $p_B \in (p_{Ac} - \tau, p_{Ac} + \tau)$:

$$p_{Ac} > p_B - \tau \iff$$

$$\frac{2(8n_cn_1 + 56n_cn_2 + 48n_c^2 + 11n_1n_2 + 11n_2^2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)}\tau > 0,$$

$$p_{Ac} < p_B + \tau \iff$$

$$\frac{-16n_c(n_1 + 6n_2 + 6n_c)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)}\tau < 0,$$

and these two conditions hold for any $n_1, n_2, n_c, \tau > 0$.

Third, we check the constraints on v :

$$v \geq \frac{p_{A2} + p_B + \tau}{2} \iff$$

$$v \geq \frac{3(n_1 + 6n_2 + 6n_c)(8n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)}\tau,$$

$$v \geq \frac{p_{Ac} + p_B + \tau}{2} \iff$$

$$v \geq \frac{4(n_1 + 6n_2 + 6n_c)(6n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)}\tau.$$

It is straightforward to see that the latter inequality implies the former for any $n_1, n_2, n_c, \tau > 0$. Then, we need

$$v \geq \max \left\{ 2\tau, \frac{4(n_1 + 6n_2 + 6n_c)(6n_c + n_1 + n_2)}{8n_c(4n_c + n_1 + n_2) + (8n_c + n_1 + n_2)(8n_c + 11n_2)}\tau \right\}$$

to make sure that condition (14) holds.

The Proof of Proposition 4 The first order conditions with respect to p_A , p_{B2} , and p_{Bc} are given by:

$$\begin{aligned}\frac{\partial \pi_A^{UF}}{\partial p_A} &= 0 \iff \\ \frac{3n_1}{2} + \frac{3n_2}{2} \left(\frac{p_{B2} - 2p_A + \tau}{2\tau} \right) + 2n_c \left(\frac{p_{Bc} - 2p_A + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Bc} - 2p_A + \tau}{2\tau} \right) &= 0, \\ \frac{\partial \pi_B^{UF}}{\partial p_{B2}} &= 0 \iff \frac{3n_2}{2} \left(\frac{p_A - 2p_{B2} + \tau}{2\tau} \right) = 0, \\ \frac{\partial \pi_B^{UF}}{\partial p_{Bc}} &= 0 \iff 2n_c \left(\frac{p_A - 2p_{Bc} + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{p_A - \tilde{p}_{Bc} + \tau}{2\tau} \right) = 0.\end{aligned}$$

Since both profit functions are strictly concave in respective prices, a price vector that solves the system of first order conditions characterizes a maximum. We use the first order condition for p_{B2} to express p_{B2} as a function of p_A :

$$\frac{\partial \pi_B^{UF}}{\partial p_{B2}} = 0 \iff p_{B2} = \frac{p_A + \tau}{2}.$$

Then, we express p_{Bc} from the first order condition for p_{Bc} and impose $\tilde{p}_{Bc} = p_{Bc}$ that needs to hold in any fulfilled expectations equilibrium:

$$\begin{aligned}4n_c \left(\frac{p_A - 2p_{Bc} + \tau}{4\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{p_A - p_{Bc} + \tau}{2\tau} \right) &= 0 \iff \\ p_{Bc} &= \frac{(p_A + \tau)(4n_c + n_1 + n_2)}{(8n_c + n_1 + n_2)}.\end{aligned}$$

Substituting these two expressions into the first order condition for p_A and taking $\tilde{p}_{Bc} = p_{Bc}$, we solve for p_A :

$$\frac{3n_1}{2} + \frac{3n_2}{2} \left(\frac{p_{B2} - 2p_A + \tau}{2\tau} \right) + 2n_c \left(\frac{p_{Bc} - 2p_A + \tau}{2\tau} \right) + \frac{n_1 + n_2}{2} \left(\frac{\tilde{p}_{Bc} - 2p_A + \tau}{2\tau} \right) = 0 \iff$$

$$\frac{3n_1}{2} + \frac{9n_2}{2} \left(\frac{\tau - p_A}{4\tau} \right) + \frac{(4n_c + n_1 + n_2)}{2} \left(\frac{p_A(-12n_c - n_1 - n_2) + 2\tau(6n_c + n_1 + n_2)}{2\tau(8n_c + n_1 + n_2)} \right) = 0 \iff$$

$$p_A = \frac{(12n_1 + 9n_2)(8n_c + n_1 + n_2) + 4(4n_c + n_1 + n_2)(6n_c + n_1 + n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)} \tau,$$

Now, we can recover p_{B2} and p_{Bc} :

$$\begin{aligned} p_{Bc} &= \frac{(p_A + \tau)(4n_c + n_1 + n_2)}{(8n_c + n_1 + n_2)} = \\ &= \frac{6(4n_c + n_1 + n_2)(4n_c + 3n_1 + 4n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)} \tau, \\ p_{B2} &= \frac{p_A + \tau}{2} = \\ &= \frac{3(4n_c + 3n_1 + 4n_2)(8n_c + n_1 + n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)} \tau. \end{aligned}$$

Let us then check when condition (17) holds for the given price vector. First, we look at $p_{B2} \in (p_A - \tau, p_A + \tau)$, which implies $p_A \in (p_{B2} - \tau, p_{B2} + \tau)$:

$$\begin{aligned} p_{B2} &> p_A - \tau \iff \\ 96n_c^2 + 100n_2n_c + 5n_1n_2 + 10n_2^2 &> 20n_1n_c + 5n_1^2 \iff \\ n_1 &\in \left[0, \frac{5n_2 - 20n_c + \sqrt{5(45n_2^2 + 360n_2n_c + 464n_c^2)}}{10} \right), \\ p_{B2} &< p_A + \tau \iff \end{aligned}$$

$$(4n_c + 3n_1 + 4n_2)(8n_c + n_1 + n_2) < 2(4n_c + 3n_1 + 4n_2)(8n_c + n_1 + n_2),$$

and the latter condition holds for any $n_1, n_2, n_c, \tau > 0$. Second, we focus on $p_{Bc} \in (p_A - \tau, p_A + \tau)$, which implies $p_A \in (p_{Bc} - \tau, p_{Bc} + \tau)$:

$$p_{Bc} > p_A - \tau \iff$$

$$48n_c^2 + 56n_2n_c - 4n_1n_c + 2n_1^2 + 13n_1n_2 + 11n_2^2 > 0,$$

$$p_{Bc} < p_A + \tau \iff (12n_1 + 18n_2)4n_c + (4n_c + n_1 + n_2)24n_c > 0,$$

and these two conditions also hold for any $n_1, n_2, n_c, \tau > 0$.

Third, we check the constraints on v . It is straightforward to show that $p_{Bc} \geq p_{B2}$. In addition, we have that $p_{Bc} < p_A + \tau$ implying

$$p_A + \tau > \frac{p_A + p_{Bc} + \tau}{2}.$$

As a result, it is sufficient to take

$$v \geq p_A + \tau = \frac{6(4n_c + 3n_1 + 4n_2)(8n_c + n_1 + n_2)}{9n_2(8n_c + n_1 + n_2) + 2(4n_c + n_1 + n_2)(12n_c + n_1 + n_2)}\tau$$

to make sure that condition (17) holds.

The Proof of Lemma 1 First, it is straightforward to see that

$$\frac{3\tau(3a^2 + 56a + 128)}{a^2 + 32a + 192} \geq 2\tau$$

holds for any $a \geq 0$. One can also show that

$$\frac{3(3a^2 + 56a + 128)}{a^2 + 32a + 192} > \frac{7+a}{4} \iff \frac{7a+104}{a^2 + 32a + 192} > \frac{1-1/a}{4}.$$

The latter holds for any $a \in [0, (\sqrt{4345} - 15)/10]$. Similarly,

$$\frac{3(3a^2 + 56a + 128)}{a^2 + 32a + 192} > \frac{4(a^2 + 19a + 84)}{27a + 211} \iff \frac{7a+104}{a^2 + 32a + 192} > \frac{4a+22-86/a}{27a+211},$$

which also holds for any $a \in [0, (\sqrt{4345} - 15)/10]$. Finally, it can be shown that for any $a \geq 0$,

$$\frac{6\tau(3a^2 + 35a + 72)}{2a^2 + 45a + 211} > \frac{3\tau(3a^2 + 56a + 128)}{a^2 + 32a + 192}.$$

This finishes the proof.

The proof of Theorem 1 We find a Nash equilibrium of the game between the firms. The profits under different pricing strategies look as follows:

$$\begin{aligned}
(\pi_A^{FF})^* &= n \left(\frac{3a(v-\tau)}{2} + \frac{3\tau}{4} + \tau \left(\frac{a+5}{4} \right)^2 \right) = \frac{n(37\tau - 14a\tau + 24av + a^2\tau)}{16}, \\
(\pi_A^{UU})^* &= \frac{16n\tau(a^2 + 17a + 24)^2}{(a+8)(a+24)^2}, \quad (\pi_A^{UF})^* = \frac{n\tau(a+8)(16a^2 + 165a + 221)^2}{4(2a^2 + 45a + 211)^2}, \\
(\pi_A^{FU})^* &= n \left(\frac{3a}{2}(v-\tau) + \tau \frac{(a+12)^2(3(a+9)^2 + 16(a+5)^2)}{(27a+211)^2} \right) \Longleftrightarrow \\
(\pi_A^{FU})^* &= \frac{n(185184\tau - 41067a\tau - 17152a^2\tau - 847a^3\tau + 38a^4\tau + av(133563 + 34182a + 2187a^2))}{2(27a+211)^2}, \\
(\pi_B^{FF})^* &= \frac{n\tau(a^2 + 10a + 37)}{16}, \quad (\pi_B^{UU})^* = \frac{18n\tau(3a+8)^2}{(a+24)^2}, \\
(\pi_B^{UF})^* &= \frac{9n\tau(3a+8)^2(19a^2 + 214a + 643)}{4(2a^2 + 45a + 211)^2}, \quad (\pi_B^{FU})^* = \frac{2n\tau(4a^2 + 57a + 221)^2}{(27a+211)^2}.
\end{aligned}$$

Next, we check when F is a dominant strategy for firm A . We have

$$\begin{aligned}
(\pi_A^{FF})^* > (\pi_A^{UF})^* &\Longleftrightarrow v > \tau \left(\frac{(a+8)(16a^2 + 165a + 221)^2}{6a(2a^2 + 45a + 211)^2} - \frac{2}{3a} \left(\frac{a+5}{4} \right)^2 + \frac{2a-1}{2a} \right) \equiv v_1, \\
(\pi_A^{FU})^* > (\pi_A^{UU})^* &\Longleftrightarrow v > \tau \left(\frac{32(a^2 + 17a + 24)^2}{3a(a+8)(a+24)^2} - \frac{2(a+12)^2(3(a+9)^2 + 16(a+5)^2)}{3a(27a+211)^2} + 1 \right) \equiv v_2.
\end{aligned}$$

Using the Matlab software, we find that there exists such a_1 (≈ 0.44) that $v_1 > v_2$ for $a \in (a_1, (\sqrt{4345} - 15)/10)$ and for $a \in [0, a_1]$, $v_1 \leq v_2$. We also find that $v_1 \geq \tilde{v} \Leftrightarrow a \in [a_2, a_3]$, where $a_2 \approx 0.34$ and $a_3 \approx 2.84$. Finally, $v_2 \geq \tilde{v} \Leftrightarrow a \in [a_4, a_5]$, where $a_4 \approx 0.29$ and $a_5 \approx 1.65$.

Then, we check when F is a dominant strategy for firm B . We have

$$\left(\pi_B^{FF}\right)^* > \left(\pi_B^{FU}\right)^* \iff \frac{a^2 + 10a + 37}{16} > \frac{2(4a^2 + 57a + 221)^2}{(27a + 211)^2},$$

which holds for any a ; and

$$\left(\pi_B^{UF}\right)^* > \left(\pi_B^{UU}\right)^* \iff \frac{19a^2 + 214a + 643}{(2a^2 + 45a + 211)^2} > \frac{8}{(a + 24)^2},$$

which in turn holds for $a < a_6$, where $a_6 \approx 3.09$. Hence, if firm A plays strategy F , it is always better for firm B to respond with strategy F . At the same time, firm B may find strategy U profitable, when firm A sticks to uniform pricing and $a \geq a_6$. Overall, we have $0 < a_4 < a_2 < a_1 < a_5 < a_3 < a_6 < (\sqrt{4345} - 15) / 10$.

Finally, we look at different intervals for a and find the Nash equilibrium of the game between the firms:

1. $a \in [0, a_4) \Rightarrow$ it is $\tilde{v} > \max\{v_1, v_2\}$, and F is a dominant strategy for both firms. As a result, (F, F) is a unique Nash equilibrium:

		Firm B	
		F	U
Firm A	F	$(\pi_A^{FF})^*, (\pi_B^{FF})^*$	$(\pi_A^{FU})^*, (\pi_B^{FU})^*$
	U	$(\pi_A^{UF})^*, (\pi_B^{UF})^*$	$(\pi_A^{UU})^*, (\pi_B^{UU})^*$

2. $a \in [a_4, a_2) \Rightarrow$ it is $v_2 \geq \tilde{v} > v_1$, and there are two cases to consider:

- (a) $v \in [\tilde{v}, v_2) \Rightarrow$ firm A chooses F when firm B plays F and selects U when the rival sticks to U . At the same time, F is still a dominant strategy for firm B under $a \in [a_4, a_2)$, and (F, F) is a unique Nash equilibrium:

		Firm B	
		F	U
Firm A	F	$(\pi_A^{FF})^*, (\pi_B^{FF})^*$	$(\pi_A^{FU})^*, (\pi_B^{FU})^*$
	U	$(\pi_A^{UF})^*, (\pi_B^{UF})^*$	$(\pi_A^{UU})^*, (\pi_B^{UU})^*$

(b) $v \geq v_2 \Rightarrow F$ is a dominant strategy for firm A , and this is equivalent to Case 1 where (F, F) is the unique Nash equilibrium.

3. $a \in [a_2, a_1) \Rightarrow$ it is $v_2 > v_1 \geq \tilde{v}$, and we must look at three cases:

(a) $v \in [\tilde{v}, v_1) \Rightarrow U$ is a dominant strategy for firm A . Since F is a dominant strategy for firm B under $a \in [a_2, a_1)$, the unique Nash equilibrium turns out to be (U, F):

		Firm B	
		F	U
Firm A	F	$(\pi_A^{FF})^*, (\pi_B^{FF})^*$	$(\pi_A^{FU})^*, (\pi_B^{FU})^*$
	U	$(\pi_A^{UF})^*, (\pi_B^{UF})^*$	$(\pi_A^{UU})^*, (\pi_B^{UU})^*$

(b) $v \in [v_1, v_2) \Rightarrow$ firm A chooses F when firm B plays F and selects U when the rival sticks to U , which is equivalent to Case 2.(a) considered above.

(c) $v \geq v_2 \Rightarrow F$ is a dominant strategy for firm A , and this is equivalent to Case 1 where (F, F) is the unique Nash equilibrium.

4. $a \in [a_1, a_5) \Rightarrow$ it is $v_1 \geq v_2 > \tilde{v}$, and there are three cases to consider:

(a) $v \in [\tilde{v}, v_2) \Rightarrow U$ is a dominant strategy for firm A , while F is still a dominant strategy for firm B . This is equivalent to Case 3.(a) where (U, F) is the Nash equilibrium.

(b) $v \in [v_2, v_1) \Rightarrow$ firm A chooses U when firm B plays F and selects F when the rival sticks to U . Since F is a dominant strategy for firm B , the unique Nash equilibrium is (U, F):

		Firm B	
		F	U
Firm A	F	$(\pi_A^{FF})^*, (\pi_B^{FF})^*$	$(\pi_A^{FU})^*, (\pi_B^{FU})^*$
	U	$(\pi_A^{UF})^*, (\pi_B^{UF})^*$	$(\pi_A^{UU})^*, (\pi_B^{UU})^*$

(c) $v \geq v_1 \Rightarrow F$ is a dominant strategy for firm A , and this is equivalent to Case 1 where (F, F) is the Nash equilibrium.

5. $a \in [a_5, a_3) \Rightarrow$ it is $v_1 > \tilde{v} \geq v_2$, and we must look at two cases:

- (a) $v \in [\bar{v}, v_1) \Rightarrow$ firm A chooses U when firm B plays F and selects F when the rival sticks to U , which is equivalent to Case 4.(b) where (U, F) is the unique Nash equilibrium.
- (b) $v \geq v_1 \Rightarrow F$ is a dominant strategy of firm A , and this is equivalent to Case 1 where (F, F) is the unique Nash equilibrium.
6. $a \in [a_3, a_6) \Rightarrow$ it is $\bar{v} \geq v_1 > v_2$, and F is a dominant strategy for firm A for any feasible v . This is equivalent to Case 1 where (F, F) is the unique Nash equilibrium.
7. $a \in [a_6, (\sqrt{4345} - 15) / 10) \Rightarrow$ it is $\bar{v} \geq v_1 > v_2$, and F is a dominant strategy for firm A . At the same time, firm B chooses F when firm A plays F and selects U when the rival sticks to U . Then, (F, F) is the unique Nash equilibrium:

		Firm B	
		F	U
Firm A	F	$(\pi_A^{FF})^*, (\pi_B^{FF})^*$	$(\pi_A^{FU})^*, (\pi_B^{FU})^*$
	U	$(\pi_A^{UF})^*, (\pi_B^{UF})^*$	$(\pi_A^{UU})^*, (\pi_B^{UU})^*$

Taking $\bar{v} = v_1$, $\hat{a} = a_2$, and $\tilde{a} = a_3$ we obtain the statement of the theorem.

The proof of Proposition 5 First, we derive an expression for W^j , $j \in \{FF, UU, FU, UF\}$, analytically and show how it depends on the prices set by the firms. Consider market 2, where all consumers have the same information about local prices and define

$$\tilde{x}_2^j = \frac{p_{A2}^j - p_{B2}^j + \tau}{2\tau}.$$

Then, the consumer surplus (taken over two periods) in this market can be found as

$$\begin{aligned} CS_2^j &= \left(n_2 + \frac{n_2}{2}\right) \left(\int_0^{\tilde{x}_2^j} (v - p_{B2}^j - \tau x) dx + \int_{\tilde{x}_2^j}^1 (v - p_{A2}^j - \tau(1-x)) dx \right) \\ &= \frac{3n_2}{2} \left((v - p_{B2}^j) \tilde{x}_2^j - \frac{\tau}{2} (\tilde{x}_2^j)^2 + (v - p_{A2}^j - \tau) (1 - \tilde{x}_2^j) + \frac{\tau}{2} (1 - (\tilde{x}_2^j)^2) \right) \end{aligned}$$

$$= \frac{3n_2}{2} \left(\left(-p_{B2}^j + p_{A2}^j + \tau \right) \tilde{x}_2^j - \tau \left(\tilde{x}_2^j \right)^2 + v - p_{A2}^j - \frac{\tau}{2} \right).$$

The firms in turn obtain

$$\pi_{A2}^j = \frac{3n_2}{2} \left(1 - \tilde{x}_2^j \right) p_{A2}^j, \quad \pi_{B2}^j = \frac{3n_2}{2} \tilde{x}_2^j p_{B2}^j,$$

and we get the following expression for W_2^j :

$$\begin{aligned} W_2^j &= CS_2^j + \pi_{A2}^j + \pi_{B2}^j \\ &= \frac{3n_2}{2} \left(\left(-p_{B2}^j + p_{A2}^j + \tau \right) \tilde{x}_2^j - \tau \left(\tilde{x}_2^j \right)^2 + v - p_{A2}^j - \frac{\tau}{2} + \left(1 - \tilde{x}_2^j \right) p_{A2}^j + \tilde{x}_2^j p_{B2}^j \right) \\ &= \frac{3n_2}{2} \left(v - \frac{\tau}{2} + \tau \tilde{x}_2^j \left(1 - \tilde{x}_2^j \right) \right). \end{aligned}$$

Next, we focus on market c . Here, consumers can have different information about the prices set by the firms in market c , which affects their shopping decisions. However, since we evaluate the social welfare *in equilibrium*, commuters' expectations must coincide with the actual prices observed in market c . As a result, commuters behave exactly as local consumers in market c , and it is enough to define only one threshold type:

$$\tilde{x}_c^j = \frac{p_{Ac}^j - p_{Bc}^j + \tau}{2\tau}.$$

We use \tilde{x}_c^j to characterize the consumer surplus and social welfare (taken over two periods) in market c :

$$\begin{aligned} CS_c^j &= \left(2n_c + \frac{n_1}{2} + \frac{n_2}{2} \right) \left(\int_0^{\tilde{x}_c^j} \left(v - p_{Bc}^j - \tau x \right) dx + \int_{\tilde{x}_c^j}^1 \left(v - p_{Ac}^j - \tau (1 - x) \right) dx \right) \\ &= \left(2n_c + \frac{n_1}{2} + \frac{n_2}{2} \right) \left(\left(-p_{Bc}^j + p_{Ac}^j + \tau \right) \tilde{x}_c^j - \tau \left(\tilde{x}_c^j \right)^2 + v - p_{Ac}^j - \frac{\tau}{2} \right), \\ \pi_{Ac}^j &= \left(2n_c + \frac{n_1}{2} + \frac{n_2}{2} \right) \left(1 - \tilde{x}_c^j \right) p_{Ac}^j, \quad \pi_{Bc}^j = \left(2n_c + \frac{n_1}{2} + \frac{n_2}{2} \right) \tilde{x}_c^j p_{Bc}^j \implies \\ W_c^j &= CS_c^j + \pi_{Ac}^j + \pi_{Bc}^j = \left(2n_c + \frac{n_1}{2} + \frac{n_2}{2} \right) \left(v - \frac{\tau}{2} + \tau \tilde{x}_c^j \left(1 - \tilde{x}_c^j \right) \right). \end{aligned}$$

Thus, we obtain the expression for W^j :

$$W^j = \sum_{i \in \{1, 2, c\}} W_i^j = \frac{3n_1}{2} \left(v - \frac{\tau}{2} \right) + \frac{3n_2}{2} \left(v - \frac{\tau}{2} + \tau \tilde{x}_2^j \left(1 - \tilde{x}_2^j \right) \right) +$$

$$\left(2n_c + \frac{n_1}{2} + \frac{n_2}{2}\right) \left(v - \frac{\tau}{2} + \tau \tilde{x}_c^j (1 - \tilde{x}_c^j)\right).$$

As one can notice, p_{A2}^j , p_{B2}^j , p_{Ac}^j , and p_{Bc}^j affect W^j only through \tilde{x}_2^j and \tilde{x}_c^j , respectively; moreover, W^j is additively separable in \tilde{x}_2^j and \tilde{x}_c^j . Hence, it is straightforward to see that W^j is maximized when $\tilde{x}_2^j = 1/2$ and $\tilde{x}_c^j = 1/2$, which corresponds to²⁰

$$p_{A2}^j = p_{B2}^j \text{ and } p_{Ac}^j = p_{Bc}^j.$$

Finally, we must show that the maximum of W^j defined by $p_{A2}^j = p_{B2}^j$ and $p_{Ac}^j = p_{Bc}^j$ is attainable. This follows immediately from (1) Proposition 1 that supports $p_{A2} = p_{B2}$ and $p_{Ac} = p_{Bc}$ in case when both firms use flexible pricing and (2) the conditions in (23) that make the (F, F) equilibrium well-defined (namely, both firms face non-zero demand in all the markets where they are present). This finishes the proof.

The proof of Lemma 2 We have that, if both firms use flexible pricing, $\tilde{x}_2^{FF} = 1/2$ and $\tilde{x}_c^{FF} = 1/2$, which results in

$$W^{FF} = \frac{3an}{2} \left(v - \frac{\tau}{2}\right) + \frac{3n}{2} \left(v - \frac{\tau}{2} + \frac{\tau}{4}\right) + \frac{n}{2} (a+5) \left(v - \frac{\tau}{2} + \frac{\tau}{4}\right).$$

When firm A uses uniform pricing, while firm B still sticks to flexible pricing, we derive that

$$W^{UF} = \frac{3an}{2} \left(v - \frac{\tau}{2}\right) + \frac{3n}{2} \left(v - \frac{\tau}{2} + \tau \tilde{x}_2^{UF} (1 - \tilde{x}_2^{UF})\right) + \frac{n}{2} (a+5) \left(v - \frac{\tau}{2} + \tau \tilde{x}_c^{UF} (1 - \tilde{x}_c^{UF})\right),$$

where

$$\begin{aligned} \tilde{x}_2^{UF} &= \frac{3(3a+8)(9+a)}{2(9(9+a) + 2(5+a)(13+a))} = \frac{9a^2 + 105a + 216}{2(2a^2 + 45a + 211)}, \\ \tilde{x}_c^{UF} &= \frac{3((2a+3)(9+a) - (5+a)(2a-1))}{9(9+a) + 2(5+a)(13+a)} = \frac{36a + 96}{2a^2 + 45a + 211}. \end{aligned}$$

²⁰The Hessian matrix computed for W_j with respect to \tilde{x}_2 and \tilde{x}_c is negative semi-definite.

Then, after some simplifications we can express the efficiency losses from uniform pricing, or $\Delta_W = W^{FF} - W^{UF}$, as

$$\begin{aligned}\Delta_W &= \frac{\tau n}{8} \left(3 \left(1 - 4\tilde{x}_2^{UF} \left(1 - \tilde{x}_2^{UF} \right) \right) + (5 + a) \left(1 - 4\tilde{x}_c^{UF} \left(1 - \tilde{x}_c^{UF} \right) \right) \right) \\ &= \frac{\tau n}{8} \frac{4a^5 + 59a^4 + 2785a^3 + 14009a^2 - 2969a + 1880}{(2a^2 + 45a + 211)^2}.\end{aligned}$$

It can be easily verified that Δ_W is strictly increasing with respect to a on $[\hat{a}, \tilde{a})$ interval, which proves the lemma.

The proof of Proposition 6 Using the proof of Proposition 5, it is straightforward to show that for $j \in \{FF, UU, FU, UF\}$:

$$\begin{aligned}CS^j &= \sum_{i \in \{1, 2, c\}} CS_i^j = \\ &= \frac{3an}{2} \left(v - p_{A1}^j - \frac{\tau}{2} \right) + \frac{3n}{2} \left(\left(-p_{B2}^j + p_{A2}^j + \tau \right) \tilde{x}_2^j - \tau \left(\tilde{x}_2^j \right)^2 + v - p_{A2}^j - \frac{\tau}{2} \right) + \\ &\quad n \left(\frac{5+a}{2} \right) \left(\left(-p_{Bc}^j + p_{Ac}^j + \tau \right) \tilde{x}_c^j - \tau \left(\tilde{x}_c^j \right)^2 + v - p_{Ac}^j - \frac{\tau}{2} \right),\end{aligned}$$

where

$$\tilde{x}_2^j = \frac{p_{A2}^j - p_{B2}^j + \tau}{2\tau}, \quad \tilde{x}_c^j = \frac{p_{Ac}^j - p_{Bc}^j + \tau}{2\tau}.$$

First, we show that

$$\begin{aligned}CS^{UU} &> CS^{UF} \iff \\ \frac{3n\tau (399a^7 + 26109a^6 + 601455a^5 + 6469672a^4 + k_3a^3 + k_2a^2 + k_1a + k_0)}{8(a+8)(2a^3 + 93a^2 + 1291a + 5064)^2} &> 0,\end{aligned}$$

where

$$k_0 = 152678400, k_1 = 216685568, k_2 = 124013760, k_3 = 37535392.$$

The latter obviously holds for any n, τ, a , implying that strategy profile (U, U) results in higher total consumer surplus than (U, F) does. Also, we can show that $CS^{FU} > CS^{FF}$ holds for any set of

parameters that satisfy conditions in (23):

$$CS^{FU} > CS^{FF} \iff \frac{n\tau (309a^4 + 5727a^3 + 35284a^2 + 90799a + 123205)}{8(27a + 211)^2} > 0.$$

Then, it is left to check when $CS^{UU} > CS^{FU}$ holds. In particular, we have that

$$CS^{UU} > CS^{FU} \iff$$

$$v > \hat{v} = \frac{-35a^7 + 5526a^6 + 406381a^5 + 8952108a^4 + 85861128a^3 + m_2a^2 + m_1a - m_0}{a(a+8)(27a^2 + 859a + 5064)^2}\tau,$$

where

$$m_0 = 38169600, m_1 = 552112384, m_2 = 367864656.$$

One can also check that

$$\hat{v} > \tilde{v} \iff a > \bar{a} \approx 0.2583,$$

which has a non-empty intersection with conditions in (23). This leads us to the conclusion that for $a \in [0, \bar{a}]$, (U, U) maximizes the total consumer surplus for any feasible v . For $a \in (\bar{a}, (\sqrt{4345} - 15)/10)$, (F, U) maximizes the total consumer surplus for $v \in (\tilde{v}, \hat{v}]$ and (U, U) maximizes the total consumer surplus for $v > \hat{v}$. This proves the proposition.

Consumer surplus in different markets under various pricing strategies Assume that conditions in (23) hold. First, we consider market 1. When firms play (F, F) or (F, U), there is no difference in the surplus consumers obtain in market 1 – the pricing strategy of firm B has no effect on the price firm A sets in market 1. Also, we can show that $CS_1^{UU} > CS_1^{UF}$ holds for any feasible set of parameters:

$$CS_1^{UU} > CS_1^{UF} \iff \frac{9an(15a^3 + 127a^2 + 472a + 640)}{2(a+8)(2a^2 + 45a + 211)}\tau > 0.$$

Finally, we investigate when (U, U) results in higher consumer surplus in market 1 than (F, F) does:

$$CS_1^{UU} > CS_1^{FF} = CS_1^{FU} \iff v > \frac{9a^2 + 168a + 384}{a^2 + 32a + 192} \tau,$$

and we can show that

$$\begin{aligned} \frac{9a^2 + 168a + 384}{a^2 + 32a + 192} \tau < \tilde{v} &\iff \\ -\frac{3(15a^3 + 127a^2 + 472a + 640)}{2a^4 + 109a^3 + 2035a^2 + 15392a + 40512} \tau &< 0 \end{aligned}$$

holds for any feasible a . This allows us to conclude that (U, U) maximizes consumer surplus in market 1 under conditions in (23).

Next, we focus on market 2. It is straightforward to show that (F, F) maximizes consumer surplus in this market for any parameters that satisfy conditions in (23):

$$CS_2^{FF} > CS_2^{UU} \iff \frac{3an(29a^3 + 1376a^2 + 14912a + 64512)}{8(a^2 + 32a + 192)} \tau > 0,$$

$$CS_2^{FF} > CS_2^{FU} \iff \frac{3n(-4a^4 + 264a^3 + 4717a^2 + 19650a + 6305)}{8(27a + 211)^2} \tau > 0,$$

where

$$\begin{aligned} -4a^4 + 264a^3 + 4717a^2 + 19650a + 6305 &> 0 \forall a \in \left[0, \frac{\sqrt{4345} - 15}{10}\right), \\ CS_2^{FF} > CS_2^{UF} &\iff \frac{3n(35a^4 + 1770a^3 + 21452a^2 + 76710a + 6305)}{8(2a^2 + 45a + 211)^2} \tau > 0. \end{aligned}$$

In the end, we check what strategy profile – (F, F), (U, U), (U, F), or (F, U), – maximizes consumer surplus in market c . We can show that consumer surplus in market c is always higher under (U, U) than under (U, F):

$$CS_c^{UU} > CS_c^{UF} \iff \frac{3n(30a^8 + 3335a^7 + 118261a^6 + t_5a^5 + t_4a^4 + t_3a^3 + t_2a^2 + t_1a + t_0)}{2(2a^4 + 109a^3 + 2035a^2 + 15392a + 40512)^2} \tau > 0,$$

where

$$t_0 = 247249920, t_1 = 385464064, t_2 = 251952176, t_3 = 88002392, t_4 = 17458929, t_5 = 1961365.$$

Moreover, (F, U) outperforms (F, F) for any parameters that satisfy conditions in (23):

$$CS_c^{FU} > CS_c^{FF} \iff \frac{n(297a^4 + 6519a^3 + 49435a^2 + 149749a + 142120)}{8(27a + 211)^2} \tau > 0,$$

and we need to focus on (U, U) and (F, U) only. Imposing conditions in (23), the former strategy profile results in higher consumer surplus in market c if and only if:

$$CS_c^{UU} > CS_c^{FU} \iff a \in [0, \check{a}),$$

where $\check{a} \approx 0.3359$. Then, we conclude that (U, U) maximizes consumer surplus in market c for a sufficiently small, or $a \in [0, \check{a})$, and (F, U) maximizes consumer surplus in market c when a is large enough, or $a \in [\check{a}, (\sqrt{4345} - 15) / 10)$.

References

- ADAMS, B., AND K. R. WILLIAMS (2019): “Zone Pricing in Retail Oligopoly,” *American Economic Journal: Microeconomics*, 11(1), 124–156.
- ALLAIN, M.-L., C. CHAMBOLLE, S. TUROLLA, AND S. B. VILLAS-BOAS (2017): “Retail Mergers and Food Prices: Evidence from France,” *The Journal of Industrial Economics*, 65(3), 469–509.
- ARMSTRONG, M., AND J. VICKERS (2001): “Competitive Price Discrimination,” *RAND Journal of Economics*, 32(4), 579–605.
- ATER, I., AND O. RIGBI (2018): “The Effects of Mandatory Disclosure of Supermarket Prices,” CESifo Working Paper Series 6942, CESifo.
- BECKMANN, M. J. (1976): “Spatial Price Policies Revisited,” *The Bell Journal of Economics*, 7(2), 619–630.
- BROWN, J. R., AND A. GOOLSBEE (2002): “Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry,” *Journal of Political Economy*, 110(3), 481–507.
- BRYNJOLFSSON, E., AND M. SMITH (1999): “Frictionless Commerce? A Comparison of Internet and Conventional Retailers,” *Management Science*, 46, 563–585.
- CAVALLO, A. (2017): “Are Online and Offline Prices Similar? Evidence from Large Multi-channel Retailers,” *American Economic Review*, 107(1), 283–303.
- (2018): “More Amazon Effects: Online Competition and Pricing Behaviors,” NBER Working Papers 25138, National Bureau of Economic Research, Inc.
- CHANDRA, A., AND M. TAPPATA (2011): “Consumer Search and Dynamic Price Dispersion: An Application to Gasoline Markets,” *RAND Journal of Economics*, 42(4), 681–704.
- CHEN, Y., AND T. H. CUI (2013): “The Benefit of Uniform Price for Branded Variants,” *Marketing Science*, 32(1), 36–50.
- CHO, S., AND J. RUST (2010): “The Flat Rental Puzzle,” *Review of Economic Studies*, 77(2), 560–594.

- CLAY, K., R. KRISHNAN, AND E. WOLFF (2001): “Prices and Price Dispersion on the Web: Evidence from the Online Book Industry,” *Journal of Industrial Economics*, 49(4), 521–539.
- CORTS, K. S. (1998): “Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment,” *RAND Journal of Economics*, 29(2), 306–323.
- DELLAVIGNA, S., AND M. GENTZKOW (2019): “Uniform Pricing in U.S. Retail Chains*,” *The Quarterly Journal of Economics*, 134(4), 2011–2084.
- DOBSON, P. W., AND M. WATERSON (2005): “Chain-Store Pricing Across Local Markets,” *Journal of Economics & Management Strategy*, 14(1), 93–119.
- (2008): “Chain-Store Competition: Customized vs. Uniform Pricing,” Economic Research Papers 269789, University of Warwick - Department of Economics.
- FABRA, N., AND M. REGUANT (2020): “A Model of Search with Price Discrimination,” *European Economic Review*, 129, 103571.
- FENG, H., Y. LI, AND J. SHUAI (2023): “Uniform Pricing as a Barrier to Entry,” *Journal of Industrial Economics*, 71(1), 176–191.
- GABRIELSEN, T. S., B. O. JOHANSEN, AND O. R. STRAUME (2023): “National Pricing with Local Quality Competition,” *Journal of Economics Management Strategy*, 32(1), 48–74.
- HAN, J., AND J. LIU (2018): “Urban Spatial Interaction Analysis Using Inter-City Transport Big Data: A Case Study of the Yangtze River Delta Urban Agglomeration of China,” *Sustainability*, 10(12), 1–16.
- HITSCH, G. J., A. HORTACSU, AND X. LIN (2019): “Prices and Promotions in U.S. Retail Markets: Evidence from Big Data,” NBER Working Papers 26306, National Bureau of Economic Research, Inc.
- HOCH, S. J., B.-D. KIM, A. L. MONTGOMERY, AND P. E. ROSSI (1995): “Determinants of Store-Level Price Elasticity,” *Journal of Marketing Research*, 32(1), 17–29.
- HOLAHAN, W. L. (1975): “The Welfare Effects of Spatial Price Discrimination,” *The American Economic Review*, 65(3), 498–503.

- HOLMES, T. J. (1989): “The Effects of Third-Degree Price Discrimination in Oligopoly,” *American Economic Review*, 79(1), 244–250.
- KAHNEMAN, D., J. L. KNETSCH, AND R. THALER (1986a): “Fairness as a Constraint on Profit Seeking: Entitlements in the Market,” *American Economic Review*, 76(4), 728–741.
- KAHNEMAN, D., J. L. KNETSCH, AND R. H. THALER (1986b): “Fairness and the Assumptions of Economics,” *The Journal of Business*, 59(4), 285–300.
- KATZ, M. L., AND C. SHAPIRO (1985): “Network Externalities, Competition, and Compatibility,” *The American Economic Review*, 75(3), 424–440.
- LEVY, D., M. BERGEN, S. DUTTA, AND R. VENABLE (1997): “The Magnitude of Menu Costs: Direct Evidence from Large U. S. Supermarket Chains,” *The Quarterly Journal of Economics*, 112(3), 791–824.
- LI, Y., B. R. GORDON, AND O. NETZER (2018): “An Empirical Study of National vs. Local Pricing by Chain Stores Under Competition,” *Marketing Science*, 37(5), 812–837.
- MIRAVETE, E. J., K. SEIM, AND J. THURK (2020): “One Markup to Rule Them All: Taxation by Liquor Pricing Regulation,” *American Economic Journal: Microeconomics*, 12(1), 1–41.
- MOYLAN, E., AND S. SOMWRITA (2019): “Defining Urban Centres Using Alternative Data Sets,” *Findings*.
- NAKAMURA, E. (2008): “Pass-Through in Retail and Wholesale,” *American Economic Review*, 98(2), 430–437.
- ORBACH, B. Y., AND L. EINAV (2007): “Uniform Prices for Differentiated Goods: The Case of the Movie-theater Industry,” *International Review of Law and Economics*, 27(2), 129–153.
- PAN, J., AND J. LAI (2019): “Spatial pattern of population mobility among cities in China: Case study of the National Day plus Mid-Autumn Festival based on Tencent migration data,” *Cities*, 94, 55–69.
- SHILLER, B., AND J. WALDFOGEL (2011): “Music for a Song: An Empirical Look at Uniform Pricing and Its Alternatives,” *Journal of Industrial Economics*, 59(4), 630–660.

- THISSE, J., AND X. VIVES (1988): “On the Strategic Choice of Spatial Price Policy,” *American Economic Review*, 78(1), 122–37.
- VARIAN, H. R. (1980): “A Model of Sales,” *The American Economic Review*, 70(4), 651–659.
- VÉLEZ-VELÁSQUEZ, J. S. (2024): “Quantifying the Effects of Price Discrimination Under Imperfect Competition,” *The Journal of Industrial Economics*, 72(1), 429–458.