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Innovation and Startup Acquisition

Abstract

In this paper, we consider two platforms that compete for the development of a new product to integrate into their ecosystems. The new product can be developed either inhouse by the platforms or by an independent startup active only in the technology market. The presence of the startup affects the platforms' R&D efforts through an *insurance effect*, which reduces the cost of failure in innovation, and a *competition effect*, which diminishes the returns to innovation. The magnitude of these effects depends on the attitude of the competition authorities towards the acquisition of the startup by one of the platforms. We show that allowing acquisitions stimulates platform innovation, but at the cost of a more concentrated market structure. We also compare the funding of the startup by independent venture capitalists or by the platforms themselves, and investigate how the merger regime influences the direction of the startup's innovation.

JEL-Codes: D430, G340, K210, L400, L860.

Keywords: innovation, startup acquisitions, mergers, digital, big tech, competition policy.

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1 Introduction

Large digital platforms, such as Amazon, Apple, Google, Microsoft, and others, offer a wide range of services. A key aspect of competition among them is the continuous development of new products, features, functionalities, and services. For example, over time, they have introduced innovations such as geo-localization, navigation systems, search engines, payment solutions, AI, virtual reality, and cloud solutions into their ecosystems.

New products can be developed *in-house* by incumbent platforms, or acquired *externally* from startups. This can be done either by acquiring the startups or by buying a license for their products. Startup acquisition and product licensing are thus two alternatives for integrating new products into an ecosystem. The objective of this paper is to compare these two strategies and their impact on platform innovation.

Product or technology transfer through acquisition can be efficient, especially in digital industries (Cabral, 2021). However, it raises two important concerns. First, startup acquisitions are not only driven by product integration; they may also aim at eliminating competitive threats. Killer acquisitions have been documented in the pharmaceutical industry (Cunningham *et al.*, 2021) and raise concerns in the digital economy as well (Motta and Peitz, 2021). Second, platforms may buy innovations from startups instead of developing them themselves. Acquisitions of startups may thus crowd out in-house innovation by platforms, a phenomenon that is referred to as *'reverse' killer acquisitions* (Crawford et al. 2020).¹

The magnitude of these anti-competitive effects is potentially large. It is well documented that in the digital economy, the acquisition of young startups is a pervasive phenomenon (Gautier and Lamesch, 2021; Gautier and Maitry, 2024) and that acquisitions have become the main exit route for startups (Ederer and Pellegrino, 2023). Furthermore, digital ecosystems expand primarily through acquisitions (Heidhues *et al.*, 2024).

In this paper, we study the interplay between the possibility of startup acquisition and the competition through innovation between digital ecosystems. We consider a model where two platforms compete to develop a new product that complements a core product in their ecosystems. A startup conducts R&D to create an alternative version of this new product. However, it does not offer the core product, so it is active only in the technology market, not in the final market. If the startup's R&D is successful, its product is both a substitute and a complement to the platforms' products. It is a substitute because the platforms can acquire the startup's product if they fail to develop their own. It is a complement because the startup's product has no value on its own; it can only be used in combination with the platforms' core products.

In our model, the startup can monetize its technology in two ways: by licensing its product to the platform(s), or by being acquired by one of them. There are two key differences between

¹The perspective of being acquired can also change the direction and the intensity of innovation by startups (Bryan and Hovenkamp, 2020; Dijk *et al.*, 2024).

these two forms of technology transfer. First, licensing is non-exclusive, so both platforms can acquire a license. In contrast, acquisition makes the product technology exclusive to the buyer. Second, the platforms compete to acquire the startup, whereas, by definition, they do not have to compete for a non-exclusive license.

Platforms consider acquiring the startup under two circumstances. First, if the startup is successful but both platforms have failed, the platforms can obtain the product technology by acquiring the startup. We refer to this case as *technology acquisition*. Second, if the startup and only one of the platforms are successful, the unsuccessful platform may seek to acquire the startup to access its technology and compete with the other platform. However, the successful platform may have an incentive to acquire the startup to prevent its rival from entering the market and secure its monopolistic position. In such a case, the acquirer has two versions of the new product, its own and the one developed by the startup. Since the startup's project is redundant with the platform's, the platform will discontinue the startup's innovation, an example of *killer acquisition*. We show that when platforms compete for acquisition, the successful platform will buy the startup and killer acquisitions occur.

In this environment, we consider a competition authority that regulates mergers. Specifically, it can allow all types of mergers, only mergers motivated by technology acquisition, or prohibit them all. The merger regime chosen by the competition authority affects the *ex-post* market structure, and thus the platforms' incentives to develop the product *ex-ante*.

The presence of the startup in the technology market affects the platforms' incentives to invest in R&D in two ways. First, the option of relying on the startup's technology *ex-post* reduces the cost of failure in innovation (*insurance effect*). This creates an opportunity cost that reduces the incentives of the platforms to invest in R&D. Second, the startup reduces the returns to innovation for a platform when it succeeds in developing the new product, because the rival platform can access the startup's technology if its R&D project fails (*competition effect*). The magnitude of these effects depends on the merger regime in place.

We show that a more lenient merger policy results in higher R&D efforts by the platforms. Therefore, allowing startup acquisitions induces platforms to perform more R&D. However, under all merger regimes, the startup's R&D crowds out the platforms' own R&D efforts, and this effect is stronger when mergers are prohibited.

While a more lenient merger regime stimulates platform innovation, it does not mean that consumers or even platforms are better off. Indeed, even if the insurance provided by the startup reduces the platforms' R&D efforts, it also benefits them. For this reason, platforms may prefer a regime that prohibits mergers, particularly when the startup offers a relatively cheap license for its product. We also show that platforms prefer a complete ban on all types of acquisitions to a ban that only targets killer acquisitions.

For consumers, there is a classic trade-off between a more competitive market structure and higher R&D efforts. Allowing startup acquisitions boosts the platforms' R&D efforts, but it makes

the market less competitive. Conversely, prohibiting startup acquisitions promotes the diffusion of the startup's innovation, but it comes at the cost of reduced platform innovation.

To finance its product development and its growth, the startup needs funding, which can come from either by a venture capitalist or one of the platforms. We show that venture capitalists are more likely to fund a startup when (at least some) mergers are allowed, i.e., when startup acquisitions facilitate entry. Platforms can be a substitute for venture capitalists, and they are willing to fund the startup's product development if it gives them a first-mover advantage in acquiring the startup. In this case, the competition for acquisition is reduced and the funding platform can buy the startup at a lower price. Finally, we show that the merger regime can also influence the direction of the startup's innovation. In particular, we find that allowing startup acquisitions increases the likelihood that the startup follows a different innovation path than the platforms.

Literature review

Innovation is an important dimension of competition among firms and several papers have examined the impact of mergers on the incentives to innovate of both merger insiders and outsiders (see, among others, Federico *el al.*, 2018; Motta and Tarantino, 2021; Bourreau *el al.*, 2024; see also Haucap *el al.*, 2019 for empirical evidence).

While these papers focus primarily on horizontal mergers in a given industry, recently the focus has shifted to the acquisition of a small, actual or potential, competitor by a dominant incumbent. Given the empirical evidence provided by Cunningham *el al.* (2020), there is a growing concern that some of these acquisitions are not driven by technology acquisition (Cabral, 2021; Varian 2021), but are aimed at eliminating a competitive threat. Like our paper, several papers have focused on startup acquisitions, often in the context of digital industries where these acquisitions are massive. All of these papers analyze the impact of startup acquisitions on the incentives to innovate of the startup and/or of the incumbent(s), considering both the intensity and the direction of innovation. We contribute to this literature by focusing on the impact of startup entry and acquisition on the incentives to innovate of competing platforms, considering that the startup's product is a complement and not a substitute to those of the platforms.

Phillips and Zhdanov (2013) study an R&D race between small and large firms. They show that large firms may find it disadvantageous to engage in an R&D race with smaller competitors as they can gain access to innovation through acquisition. Letina *et al.* (2024) consider a model where an incumbent and an entrant choose a set of innovation projects. Innovation projects are considered duplicates if both firms invest in the same project, and new projects if only one firm invests. The incumbent has the option to acquire the entrant, and its motivation could be to eliminate a competitive threat. The authors show that the investment incentives are lower when acquisitions are prohibited. Specifically, the entrant has less incentive to invest in duplicate projects when acquisitions are prohibited because the possibility of selling its innovation to the incumbent is eliminated. Also, a ban on acquisitions can lead the incumbent to invest in more projects, especially duplicates, because it can no longer rely on acquisitions to eliminate potential competition. Similarly, Henkel *et al.* (2015) show that entrants choose more radical innovations to maximize their return from acquisition. Dijk *et al.* (2024) consider a startup that invests in a portfolio of projects and show that the chosen portfolio is biased to maximize the acquisition rents, a classic 'entry for buyout' strategy (Rasmussen, 1988). In the case of acquisitions, portfolio reallocation can improve welfare and should be balanced against the anti-competitive effect of the merger. Fons-Rosen *et al.* (2021) show in a general equilibrium model that allowing acquisitions generates more startup entry but less innovation by incumbents. Furthermore, the startups' ideas are less often implemented by acquiring incumbents. They collect data to estimate the net effect and find it to be negative.

With few dominant platforms acquiring startups on a large scale, the digital sector has received a lot of attention. Several papers consider a situation where a startup innovates and develops a product that directly competes with products of incumbent platforms. Motta and Peitz (2021) consider a startup that is financially constrained, and show that acquisition by a platform can alleviate these constraints. However, the acquiring platform inefficiently stops too many projects, thereby reducing competition. Motta and Shelegia (2024) show that the platform can influence the direction of the startup's innovation. Without the acquisition, the platform can threaten the startup to copy its project, thereby reducing its value. To avoid this, the startup redirects its R&D to projects that do not directly compete with those of the incumbent, i.e., it develops a complement rather than a substitute. Katz (2021) considers a dynamic model of competition for the market, which is the case in markets with strong network effects. He shows that incumbents can buy an innovative entrant and later shut it down, but this strategy is not necessarily inefficient because it preserves the network effects on the incumbent's platform.

Bryan and Hovenkamp (2020) consider, as in our paper, a startup that develops a component that does not directly compete with the platforms, but that is used by them to improve their production process, focusing on the startup's incentives to innovate. They consider two competing platforms, a leader and a laggard, and show that in a 'laissez-faire' regime, the startup is acquired by the leader and there is no licensing to the laggard, resulting in too little diffusion of innovation. The startup prefers innovations that benefit the leader, resulting in an inefficient direction of innovation. Finally, they show that there is an inefficient rate of innovation by the startup. Prohibiting acquisitions by the leader or imposing licensing obligations can reduce some of these inefficiencies.

While most of the literature has focused on the incentives to innovate of one or more entrants facing an incumbent, looking at both the intensity and the direction of innovation, our model analyzes the incentives to innovate of two competing platforms facing a startup. We show that while startup entry can crowd out intrinsic innovation by the platforms, a well-established result, this effect can be even stronger when mergers are not allowed, as startups have an alternative strategy to monetize their products (licensing). We also show that competition between platforms to acquire the startup eliminates the safeguard provided by the startup, and the insurance effect is no longer present. All that remains is the cost of protecting a monopolistic position, which requires buying the startup and later killing it.

The rest of the paper is organized as follows. In Section 2, we set up the model and solve the benchmark case with a monopoly platform. In Section 3, we solve the equilibrium with competing platforms. In Section 4, we compare the different merger regimes in terms of R&D effort, platform profits, and consumer surplus. In Section 5, we consider three extensions of the base-line setting. We consider the startup's entry decision, possible funding by one of the platforms, and the direction of the startup's innovation project. Finally, we conclude in Section 6. The proofs of our main results are relegated to the Appendix.

2 Model

2.1 Platforms

We consider two competing platforms, 1 and 2. The platforms have a core product, product *A*, and invest in R&D to develop a complementary product, product *B*. Product *B* has no value on its own and is only valuable when combined with product *A*.

If both platforms successfully develop product *B* and combine it with *A*, they form a duopoly in the market for the combined product A + B, and each of them has a gross profit of π^D . If only one platform develops product *B*, it operates as a monopolist in market A + B and earns a profit of π^M . The other platform operates only in market *A* and has a profit of $\bar{\pi}$, which we normalize to zero without loss of generality. Finally, if neither platform develops product *B*, they both operate only in market *A* and make a profit of $\bar{\pi} = 0$. We assume that $\pi^M \ge 2\pi^D > 0$, which is a standard assumption.

R&D is probabilistic. If platform *i* invests $C(p_i) = \phi_i \frac{p_i^2}{2}$ in R&D, it successfully develops product *B* with probability p_i , where $p_i \in [0, 1]$. The firms' R&D projects are probabilistically independent. For simplicity, we assume that $\phi_1 = \phi_2 = \phi > 0$, so the platforms are fully symmetric *ex ante*.

To ensure that the equilibrium is stable,² we assume that:

Assumption 1. $\phi > \pi^M - \pi^D$.

This assumption also ensures that R&D efforts are always interior. Note that as $\pi^M \ge 2\pi^D$, Assumption 1 implies that $\phi > \pi^D$, which will be useful in the analysis.

²The equilibrium is stable if the slope of the best response functions is less than 1 in absolute terms.

2.2 Startup

In addition to the platforms, a startup *S* develops its own version of product *B*. The startup successfully develops the product with probability $p_S \in (0, 1)$, which we consider exogenous. For example, the startup could be financially constrained and unable to respond if the platforms increase their R&D efforts. Therefore, we take the startup's R&D effort as given.

The startup is only active in the technology market and does not compete with the platforms in the downstream market, for example, because it lacks product *A*. Thus, if the platforms have developed their own version of *B*, the startup's product has no added value, and its profit is normalized to zero. On the other hand, if a platform's R&D project was not successful, it can combine its product *A* with the startup's version of *B*.

When a platform uses the startup's product, it pays a license fee equal to a share β of its profits, where $\beta \in (0, 1)$ reflects the bargaining power of the startup. We assume that the startup offers a non-exclusive license, meaning that *S* will license its product to the two platforms if they both request it.³

2.3 Startup acquisition and competition policy

As an alternative to licensing, the platforms have the option of acquiring the startup. The acquisition takes place after the startup has completed its R&D project. So, the platforms know whether the startup's *B* product is available. Thus, if the startup fails to innovate, it will not be acquired. Once acquired, the startup's product becomes exclusive to the acquiring platform. We assume that the platforms compete for the acquisition, with the startup accepting the highest bid, if it exceeds its outside option (i.e., licensing its technology).

Competition policy can prohibit all mergers or only some of them. Specifically, we assume that there is a competition authority that decides when a merger between a platform and the startup is allowed. The competition authority can allow all mergers, prohibit all mergers, or prohibit only "killer acquisitions," when a platform acquires the startup only to deprive its rival of product *B*. The competition authority chooses the merger regime that maximizes consumer surplus. The rules set by the competition authority are known to the firms before they decide on their level of R&D effort.

2.4 Timing of the game

The timing of the game is as follows:

- 1. The competition authority decides on its merger policy.
- 2. The platforms decide on their level of effort in R&D.

³In our model, exclusive licensing by the startup would be equivalent to an acquisition.

- 3. The platforms observe the success of the R&D projects and decide whether to compete for the acquisition of the startup (if the acquisition of the startup is allowed by the competition authority). The startup is acquired by the highest bidder, if any. If no platform acquires the startup, the platforms can license the startup's product if it has been successfully developed.
- 4. Profits are realized.

We look for the subgame perfect equilibrium of this game.

Note that depending on the R&D outcome and the merger policy in place, the market structure in Stage 4 can be a duopoly, a monopoly, or no active platform. The consumer surplus associated with these different market configurations is CS^D , CS^M , and CS^{\emptyset} , respectively, and we make the natural assumption that $CS^D \ge CS^M > CS^{\emptyset} = 0$.

2.5 Monopoly platform benchmark

To understand how the presence of the startup affects the competition between the platforms, it is useful to consider as a benchmark the situation where there is only one (monopoly) platform and the startup. As in the baseline model, if its project fails, the monopoly platform can acquire the startup's technology if it is successful, either through a license or an acquisition. In this case, the platform pays $\beta \pi^M$ to the startup. Denoting by *p* the platform's R&D effort, the expected profit of the monopoly platform is

$$\Pi = p\pi^{M} + (1-p)p_{S}(1-\beta)\pi^{M} - \phi \frac{p^{2}}{2}$$

The profit-maximizing effort for the monopoly platform is $p^M = \min\left\{1, \frac{(1-p_S(1-\beta))\pi^M}{\phi}\right\}$ and the corresponding profit when p^M is interior is $\Pi^M = (1-\beta)p_S\pi^M + \phi(p^M)^2/2$. We can easily show that:

Lemma 1. Suppose that there is a monopoly platform and a startup. The platform's equilibrium innovation effort decreases in p_S and increases in β . Its equilibrium profit increases in p_S and decreases in β .

Proof. The first part of the lemma is immediate from the expression of p^M . Moreover, when p^M is interior, we have $\frac{\partial \Pi^M}{\partial p^s} = (1 - \beta)\pi^M(1 - p^M) > 0$ and $\frac{\partial \Pi^M}{\partial \beta} = -p_S\pi^M(1 - p^M) < 0$.

In this monopoly benchmark, we observe a *reverse killer (acquisition) effect* (Crawford *et al.*, 2020): the monopoly platform invests *less* in R&D when a startup invests in a similar project. The startup provides the monopolist with insurance against failure in R&D, which reduces the platform's innovation effort. In other words, the research efforts of the monopolist and the startup are *substitutes*, with the latter crowding out the former.

This reduction in research effort is larger when the startup has a higher probability of success (p_S is higher) and when the price paid for the startup's technology is lower (β is lower). Despite a lower intrinsic research effort, the presence of the startup increases the monopoly platform's profit because the insurance more than compensates for the lower research effort.

3 Equilibrium with competing platforms

In this section, we solve for the equilibrium when platforms compete. First, we define the different possible research outcomes.

The two platforms and the startup perform R&D to develop product *B*. Thus, at the end of the research phase, we can have 0, 1, 2, or 3 firms that have successfully developed product *B*. This leads to the following possible cases:

Case 0: With probability $(1 - p_1)(1 - p_2)(1 - p_S)$, all firms fail and there is no product *B*. All firms have zero profit.

Case 1: With probability p_1p_2 , both platforms have their version of product *B*. The market is a duopoly and the startup has no value, even if it has successfully developed product *B*.

Case 2: With probability $p_i(1 - p_j)(1 - p_S)$, only platform *i* is successful and it operates as a monopoly.

Case 3: With probability $(1 - p_1)(1 - p_2)p_S$, only the startup is successful.

Case 4: With probability $p_i(1-p_j)p_s$, platform *i* and the startup are successful, platform *j* fails.

In the following, we first consider the case where the competition authority prohibits the acquisition of the startup, so that licensing is the only way to obtain the startup's technology. Then, we consider the case where platforms can acquire the startup. Table 1 shows the profits of the platforms in the different cases for all possible merger regimes.

	Merger regimes			
	No acquisition	All acquisitions allowed	Killer acquisitions prohibited	
Case 1	Duopoly	Duopoly	Duopoly	
	$\pi_1 = \pi_2 = \pi^D$	$\pi_1 = \pi_2 = \pi^D$	$\pi_1 = \pi_2 = \pi^D$	
Case 2	Monopoly platform <i>i</i>	Monopoly platform <i>i</i>	Monopoly platform <i>i</i>	
	$\pi_i=\pi^M$, $\pi_j=0$	$\pi_i = \pi^M, \pi_j = 0$	$\pi_i=\pi^M$, $\pi_j=0$	
Case 3	Duopoly	Monopoly & acquisition	Monopoly & acquisition	
	$\pi_1=\pi_2=(1-\beta)\pi^D$	$\pi_i = \pi_j = 0$	$\pi_i = \pi_j = 0$	
Case 4	Duopoly	Monopoly <i>i</i> & acquisition	Duopoly	
	$\pi_i=\pi^D, \pi_j=(1-\beta)\pi^D$	$\pi_i=\pi^M-\pi^D, \pi_j=0$	$\pi_i=\pi^D, \pi_j=(1-\beta)\pi^D$	

Table 1: Firms' profits in the different market configurations and merger regimes.

3.1 No acquisition

If the platforms cannot acquire the startup, the only way to access its technology is through licensing. The market structure in Stage 4 of the game then depends on the research outcomes defined above (see also Table 1):

- In Case 1, both platforms innovate and the market is a duopoly: $\pi_1 = \pi_2 = \pi^D$, $\pi^S = 0$.
- In Case 2, only platform *i* successfully innovates and it monopolizes the market: $\pi_i = \pi^M$, $\pi_i = 0$, $\pi^S = 0$.
- In Case 3, both platforms fail but the startup succeeds. The platforms buy a license from *S* and the market is a duopoly: $\pi_1 = \pi_2 = (1 \beta)\pi^D$, $\pi^S = 2\beta\pi^D$.
- In Case 4, platform *i* innovates but platform *j* fails. Platform *j* buys a license from *S* and the market is a duopoly: $\pi_i = \pi^D$, $\pi_j = (1 \beta)\pi^D$, $\pi^S = \beta \pi^D$.

Platform *i*'s expected profit in Stage 2 is then:

$$\pi_i = p_i(1-p_j)\pi^M + p_i p_j \pi^D + (1-p_i) p_S(1-\beta)\pi^D - p_i(1-p_j) p_S(\pi^M - \pi^D) - \phi \frac{p_i^2}{2}.$$
 (1)

The first-order condition (FOC) with respect to p_i is:⁴

$$\frac{d\pi_i}{dp_i} = (1 - p_j)\pi^M + p_j\pi^D \underbrace{-p_S(1 - \beta)\pi^D}_{\text{Insurance effect}} \underbrace{-p_S(1 - p_j)(\pi^M - \pi^D)}_{\text{Competition effect}} - \phi p_i = 0.$$
(2)

The FOC (2) shows that the presence of the startup in the technology market affects the platforms' R&D incentives through an *insurance effect* and a *competition effect*.

First, the startup provides an insurance against the failure of an R&D project; the platform can make positive profits even if it fails to develop product *B*, as shown by the third term in (1). Thus, the possibility of acquiring the startup's technology represents an opportunity cost for inhouse R&D, as it can be seen in the corresponding term in the FOC (2). This opportunity cost reduces the incentives to invest in R&D, which we call the *insurance effect*.

Second, the startup reduces the returns to innovation for the platform if it succeeds in developing product *B*. Without the startup, if platform *i* succeeds while platform *j* fails, platform *i* can safely enjoy a monopoly position. With the startup, the benefits of innovation are reduced, because the competing platform can obtain a license to the startup's technology, as shown by the fourth and last term in (1). This *competition effect*, identified in the FOC (2), also reduces the platform's incentives to innovate.

Since we look for a symmetric equilibrium, we can set $p_j = p_i$ in the FOC (2). Solving for p_i , we then obtain the symmetric equilibrium level of R&D:

$$p^{N} = \frac{(1 - p_{S})\pi^{M} + p_{S}\beta\pi^{D}}{\phi + (1 - p_{S})(\pi^{M} - \pi^{D})}.$$
(3)

Note that $p^N < 1$ because $\phi > \pi^D$ under Assumption 1.

⁴The second-order condition is satisfied as $\phi > 0$.

3.2 Startup acquisition

Now, we consider the case where the platforms can acquire the startup in Stage 3. The platforms consider acquiring the startup only if they both fail and the startup succeeds (case 3), or if one of them fails and the startup succeeds (case 4). In all other cases, there is no reason to acquire the startup, either because it fails and has no value (cases 0 and 2), or because both platforms succeed (case 1).

We model the acquisition as a two-stage game: in stage 1, the platforms simultaneously decide whether to make an offer to acquire *S*; then, in stage 2, there is an auction to acquire *S* among the platforms that expressed interest in the previous stage. The startup is acquired by the platform making the highest offer, provided that the startup accepts the offer. We assume that in the case of indifference, a platform prefers to make an offer to acquire the startup than to make no offer.

Let us first consider the acquisition game when both platforms have failed but *S* has developed product *B* (case 3). If no platform expresses interest in stage 1, *S* remains independent and licenses its product to both platforms. If only one platform decides to make an offer, it can acquire the startup in stage 2, provided that the offer is large enough to be accepted by *S*. In this case, the startup's outside option is to license its technology to both platforms, in which case it earns a profit of $2\beta\pi^D$. Thus, the platform has to pay $2\beta\pi^D + \epsilon$, where ϵ is very small, to acquire *S*. Such a deal is profitable for the platforms decide to make an offer, they compete à la Bertrand to acquire *S*, and the acquisition price is equal to π^M . So, the startup can capture all the monopoly profits and the platforms make no profit. Table 2 shows the payoffs in stage 1 of the acquisition game, in anticipation of the equilibrium outcome in stage 2.

		Platform 2		
		Acquire	Not acquire	
Platform 1	Acquire	(0,0)	$(\pi^M - 2\beta\pi^D, 0)$	
riau01111 1	Not acquire	$(0,\pi^M-2\beta\pi^D)$	$((1-\beta)\pi^D,(1-\beta)\pi^D)$	

Table 2: Acquisition game, case 3 - technology acquisition.

In this game, it is not an equilibrium for both firms not to acquire *S*. Thus, acquisition always occurs. As we have assumed that in the case of indifference, platform *i* prefers to make an offer than no offer, there is a unique equilibrium where platforms compete to acquire the startup.

Lemma 2. If both platforms fail to develop product B while S successfully develops product B, one platform acquires the startup and the startup extracts the entire monopoly profit.

In the following, we will refer to this case as *technology acquisition*.

⁵After acquiring the startup, platform *i* has no incentive to offer a license to its competitor, as $\pi^M > 2\pi^D$.

Now, consider that platform *i* and the startup are successful, but platform *j* is not (case 4). If platform *i* does not bid to acquire *S*, *S* can be acquired by platform *j* at a price of $\beta \pi^D$. If platform *i* decides to compete for the acquisition, it wins the auction and monopolizes the market. In this case, the acquisition price is $\pi^D + \epsilon$. Table 3 shows the payoffs in stage 1 of the acquisition game, in anticipation of the equilibrium outcome in stage 2.

		Platform <i>j</i>		
		Acquire	Not acquire	
Platform <i>i</i>	Acquire	$(\pi^M-\pi^D,0)$	$(\pi^M - \beta \pi^D, 0)$	
	Not acquire	$(\pi^D,(1-\beta)\pi^D)$	$(\pi^D, (1-\beta)\pi^D)$	

Table 3: Acquisition game, case 4 – killer acquisition.

Note that acquisition is a dominant strategy for platform *i*. Therefore, in equilibrium, platform *i* acquires *S* at a price of π^{D} .

Lemma 3. If platform *i* and the startup successfully develop product *B*, but platform *j* does not, platform *i* acquires the startup at a price of π^D .

In equilibrium, the acquirer buys a startup that poses a competitive threat. Since the startup is not active in the downstream market, it does not represent a threat in itself, but because it can be acquired by the rival platform. Moreover, as the startup's technology is redundant with the platform's own technology, the acquiring platform can discontinue the startup's version of product *B*. Thus, we call this case a *killer acquisition*.

3.3 Innovation efforts under acquisition

In this section, we study the platforms' innovation efforts when the acquisition of the startup can take place. First, we consider the case where the competition authority allows both technology and killer acquisitions. Then, we consider the case where killer acquisitions are prohibited.

3.3.1 All types of acquisitions are allowed

When all types of acquisition are allowed, cases 0, 1 and 2 are the same as under no acquisition (licensing). However, cases 3 and 4 are affected by the possibility of acquisition.

In case 3, both platforms fail, but the startup succeeds. The platforms compete to acquire *S* and the market is a monopoly (see Lemma 2). The firms' profits are $\pi_i = \pi_j = 0$, $\pi^S = \pi^M$. In case 4, platform *i* innovates, but platform *j* fails. Platform *i* acquires *S*, and the market is a monopoly (see Lemma 3). We have $\pi_i = \pi^M - \pi^D$, $\pi_j = 0$, and $\pi^S = \pi^D$.

The expected profit of platform *i* is then:

$$\pi_i = p_i (1 - p_j) \pi^M + p_i p_j \pi^D - p_i (1 - p_j) p_S \pi^D - \phi \frac{p_i^2}{2}, \tag{4}$$

and the first-order condition with respect to p_i is:

$$\frac{d\pi_i}{dp_i} = (1 - p_j)\pi^M + p_j\pi^D \underbrace{-p_S(1 - p_j)\pi^D}_{\text{competition effect}} - \phi p_i = 0.$$
(5)

Note that only the competition effect is at play; platform *i* purchases the startup to secure its monopoly position if platform *j* has not developed product *B* and tries to buy the startup's technology. Consequently, it became costly to maintain the monopolistic position, and the benefits of innovation are reduced. There is no insurance effect, because if platform *i* fails to develop product *B*, the startup either captures all downstream profits (case 3) or is acquired by the competing platform (case 4).

Replacing p_j for p_i and solving the FOC (5) for p_i , we obtain the symmetric equilibrium level of R&D,

$$p^{A} = \frac{\pi^{M} - p_{S}\pi^{D}}{\phi + \pi^{M} - (1 + p_{S})\pi^{D}},$$
(6)

where $p^A \in (0, 1)$, as $\pi^M > p_S \pi^D$ and $\phi > \pi^D$ under Assumption 1.

3.3.2 Killer acquisitions are prohibited

Finally, consider the case where killer acquisitions are prohibited, which means that a platform is not allowed to acquire the startup if it owns a similar technology (here, product *B*). Thus, in case 4, where platform *i* innovates but platform *j* fails, platform *i* cannot acquire *S*. In this case, platform *j* can obtain the startup's technology at a price of $\beta \pi^D$, and the market is a duopoly.⁶ Thus, we have $\pi_i = \pi^D$, $\pi_j = (1 - \beta)\pi^D$, and $\pi^S = \beta \pi^D$.

Platform *i*'s expected profit is then:

$$\pi_i = p_i(1-p_j)\pi^M + p_i p_j \pi^D + (1-p_i)p_j p_s(1-\beta)\pi^D - p_i(1-p_j)p_s(\pi^M - \pi^D) - \phi \frac{p_i^2}{2}.$$
 (7)

The first-order condition with respect to p_i is:

$$\frac{d\pi_i}{dp_i} = (1 - p_j)\pi^M + p_j\pi^D \underbrace{-p_S p_j(1 - \beta)\pi^D}_{\text{insurance effect}} \underbrace{-p_S(1 - p_j)(\pi^M - \pi^D)}_{\text{competition effect}} - \phi p_i = 0.$$
(8)

The insurance effect kicks in because killer acquisitions are prohibited. The competition effect here comes from the fact that if platform *j* fails, it can license the startup's technology, which dissipates part of the return to innovation $(\pi^M - \pi^D)$.

Replacing p_j for p_i into the FOC (8) and solving for p_i , we obtain the symmetric equilibrium level of R&D effort,

$$p^{K} = \frac{(1 - p_{S})\pi^{M} + p_{S}\pi^{D}}{\phi + (\pi^{M} - \pi^{D}) - p_{S}(\pi^{M} - (2 - \beta)\pi^{D})},$$
(9)

with $p^K < 1$, because $\phi > \pi^D$ under Assumption 1.

⁶Note that the price paid by platform j for *S*'s technology is the same whether it acquires the startup or buys a license.

4 Comparison of merger regimes

In this section, we compare R&D effort (Section 4.1), profits (Section 4.2), and consumer surplus (Section 4.3) under the three merger regimes.

4.1 R&D effort

First, we show that R&D is more intense when startup acquisitions are allowed.

Lemma 4. For $p_S > 0$, R&D efforts in the three merger regimes compare as follows: $p^N < p^K < p^A$.

As discussed above, the presence of the startup in the technology market affects the platforms' R&D through an *insurance effect* and a *competition effect*, both of which reduce the platforms' R&D incentives. The magnitude of the two effects differs across the three merger regimes. Comparing the FOC (2), (5) and (8), we can see that the magnitude of the insurance effect is highest when no acquisitions are allowed (regime N) and lowest when they are all allowed (regime A). Moreover, for a given p_j , the competition effect is the same in regimes N and K, with a higher magnitude than in regime A. Therefore, the overall impact of the insurance and competition effects is highest in regime N and lowest in regime A. From a policy point of view, a more 'lenient' merger policy would thus stimulate R&D.

Next, we analyze how the R&D effort is affected by the startup's ability to develop the innovation, i.e., its probability of success p_s .

Lemma 5. In the three merger regimes, the R&D effort decreases with p_S . Moreover, we have:

$$\left|\frac{\partial p^A}{\partial p_S}\right| < \left|\frac{\partial p^K}{\partial p_S}\right| < \left|\frac{\partial p^N}{\partial p_S}\right|.$$

In all regimes, the magnitude of the insurance and competition effects increases with the startup's ability to develop the innovation, p_S . Therefore, in-house R&D decreases with p_S . Moreover, since the magnitude of the two effects is highest in regime N and lowest in regime A, in-house R&D is more (resp., less) sensitive to the startup's ability to innovate in regime N (resp., A).

Finally, we characterize the variation of the R&D effort with respect to the licensing rate β . We can show that:

Lemma 6. p^N and p^K are both increasing in β , while p^A does not depend on β .

Proof. Immediate from the expressions of p^N , p^K , and p^A .

The magnitude of the insurance effect is lower when the license rate β is higher. Indeed, a higher value of β means that the insurance is more expensive. Therefore, it increases the incentives to invest in R&D. Finally, when all types of acquisition are allowed, licensing does not occur in equilibrium. Therefore, p^A does not depend on β .

4.2 Platforms' profits

We now compare the equilibrium profits of the platforms in the three merger regimes, which can be written as follows:

$$\pi^{N} = \frac{\phi}{2} \left(p^{N} \right)^{2} + (1 - \beta) p_{S} \pi^{D}, \qquad (10)$$

$$\pi^A = \frac{\phi}{2} \left(p^A \right)^2, \tag{11}$$

$$\pi^{K} = \frac{\phi}{2} \left(p^{K} \right)^{2} + (1 - \beta) p_{S} p^{K} \pi^{D}.$$
(12)

The first term in the profit equations is the net benefit of the innovation effort, that is, the expected profit from a successful innovation, net of R&D costs. The second term in (10) and (12) represents the value of the insurance from the startup's presence, which is nonzero when all acquisitions are banned (regime N) or only killer acquisitions are banned (regime K).

When all acquisitions are allowed (regime *A*), the value of the insurance is zero, because the platforms dissipate this value in their competition to acquire the startup. Therefore, a higher probability of success for the startup always hurts the platforms.

Lemma 7. When all acquisitions are allowed, the platforms' profit π^A decreases in p_S .

Proof. Immediate from the fact that p^A decreases in p_S (Lemma 4).

This result is a direct consequence of the competition effect discussed above. A higher probability of a startup's success p_s decreases the benefit of a successful innovation for the platform, thus reducing its profit. At the limit, when all acquisitions are allowed, the platforms are better off if there is no startup ($p_s = 0$).

When some or all the acquisitions are prohibited, the effect of the startup's probability of success on platform profits is ambiguous. A higher p_S increases the risk of profit dissipation if the startup succeeds and some platforms fail. On the other hand, a higher p_S also increases the profits of the platforms in case of failure (insurance effect). For this reason, π^K and π^L are not always monotone in p_s .

We now turn to a comparison of platform profits in the different regimes. We can show that the regime preferred by the platforms depends on the startup's license fee β and probability of success p_S through a simple threshold.

Proposition 1. There is a threshold $\tilde{\beta}(p_S) \in [0,1)$ such that the platforms prefer the acquisition regime if $\beta \ge \tilde{\beta}(p_S)$, and the licensing (no acquisition) regime otherwise. Furthermore, the platforms always prefer a ban on all types of acquisitions to a ban on killer acquisitions only.

The fact that the platforms prefer a ban on all types of acquisitions to a ban on killer acquisitions only can be easily understood by looking at Table 1: there is only one case where the market outcome differs between the two regimes, when only one platform failed (case 3). In this case, the outcome for the platforms is worse when technology acquisitions are possible, because the startup extracts all their profits. As a consequence, their profit is (weakly) lower in regime *K* compared to regime *N* (with equality when $\beta = 1$).

When compared to the acquisition regime A, the licensing regime N provides an insurance value to the platforms, but at the cost of a more important competition effect. When β is high, the value of the insurance is low and platforms prefer regime A where the competition effect is smaller. Conversely, when β is low, the value of the insurance is high and platforms may prefer regime N.

Figure 1 illustrates these results. It shows which merger regime yields the highest profits for the platforms as a function of p_S on the horizontal axis and β on the vertical axis. The white area corresponds to the case where the licensing regime is preferred, and the gray area corresponds to the case where it is the acquisition regime that is preferred (we use the same color code for the other figures). Platforms prefer the acquisition regime when the license fee β is high, and a higher probability of success p_S tends to induce platforms to prefer this regime for a larger set of values of β .⁷

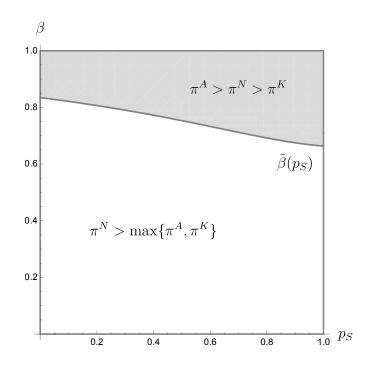


Figure 1: Merger regime maximizing platform profits for $\pi^M = 1, \pi^D = 0.33, \phi = 0.7$.

4.3 Consumer surplus and competition policy

In the first stage, the competition authority chooses the merger regime that maximizes consumer surplus. Consumer surplus depends only on the (ex-post) market structure, specifically

⁷Note however that $\tilde{\beta}(p_S)$ is not always monotone decreasing.

on the number of active platforms – it does not matter to consumers whether a platform develops the innovation in-house or obtains it through licensing or acquisition.

In equilibrium, the market for the combined product A + B can be a monopoly, a duopoly, or non-existant if there is no innovation, with a corresponding consumer surplus of CS^M , CS^D and $CS^{\emptyset} = 0$, and $CS^D \ge CS^M > CS^{\emptyset} = 0$.

For each merger regime $r \in \{N, A, K\}$, we can calculate the probability θ_M^r of having a monopoly, the probability θ_D^r of having a duopoly, and the probability θ_{ϕ}^r that the market does not emerge, with $\theta_M^r + \theta_D^r + \theta_{\phi}^r = 1$. The expected consumer surplus is then:

$$CS^r = \theta^r_M CS^M + \theta^r_D CS^D.$$

For consumers, there is a classic trade-off between, on the one hand, the diffusion of innovation in the market, for which it is better to prohibit mergers because they lead to monopolization, and, on the other hand, the incentives of platforms to innovate, which are highest when mergers are allowed (Lemma 4). On this basis, we can isolate two special cases:

Proposition 2. Consumer surplus is highest under:

- 1. The no-acquisition regime when the cost of R&D is very high $(\phi \rightarrow \infty)$;
- 2. The acquisition regime when market prices are regulated.

Proof. 1. When $\phi \to \infty$, the platforms' R&D efforts go to 0 for all merger regimes. We find that $CS^A = CS^K \approx p_S CS^M < p_S CS^D \approx CS^N$.

2. If prices are regulated, then $\pi^D = \pi^M/2$ and $CS^M = CS^D$. What matters for consumer surplus is the probability of innovation. In any regime *r*, it is given by $1 - (1 - p_S)(1 - p^r)^2$. Since $p^A = p^K > p^N$ (we have $p^A = p^K$ because $\pi^D = \pi^M/2$), it follows that $CS^A = CS^K > CS^N$.

When the cost of R&D ϕ is very high, platforms don't invest in R&D. Innovation comes only from the startup. Therefore, acquisitions must be prohibited to avoid the monopolization of the innovation and to maximize its diffusion.

When prices are regulated (which can be the case, for example, for some pharmaceutical products), consumer surplus is the same under a monopoly as under a duopoly ($CS^M = CS^D$). So, what matters for consumers is the probability of innovation. Therefore, they are better off if startup acquisitions are allowed, as this stimulates innovation.

In the general case, which regime is preferable for consumers depends on the relative importance of CS^D and CS^M , the license fee β , and the probability of success p_S . Figure 2 shows the areas where licensing (no acquisition) is preferred and where allowing startup acquisitions is preferred, as a function of p_S and β . For this example, we have used the demand model of Singh and Vives (1984) to set values for consumer surplus consistent with the values for profits used in Figure 1. Specifically, we calibrate the parameters of the demand model so that $\pi^M = 1$

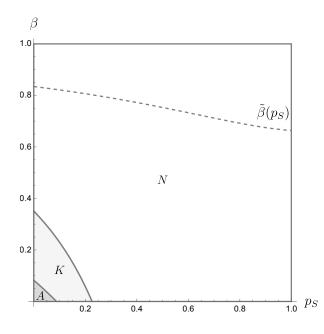


Figure 2: Merger regime maximizing consumer surplus with $CS^M = 1/2$ and $CS^D = 1.51$.

and $\pi^D = 0.33$. We then calculate consumer surplus and obtain $CS^M = 1/2$ and $CS^D = 1.51$ (see the Appendix for details).

When $\beta \ge \beta(p_S)$, the platforms prefer the acquisition regime (see Proposition 1), but it is optimal for the competition authority to prohibit all mergers. When $\beta < \tilde{\beta}(p_S)$, the platforms prefer the licensing (no acquisition) regime. This is also the regime chosen by the competition authority when the startup's probability of success, p_S , is not too low. For low values of the startup's probability of success, p_S , and of the license fee, β , the optimal merger policy can be to allow (some) acquisitions, while platforms would still prefer a ban and the licensing regime.

5 Extensions

In this section, we extend the baseline model in three directions. First, we study the startup's entry decision. Second, we analyze the case where a platform can fund the startup, giving the funding platform a first-mover advantage in the acquisition game. Third, we study how the merger regime affects the startup's innovation direction.

5.1 Venture financing and startup entry

In our model, the startup makes a profit if its R&D project is successful and at least one platform fails to develop its own version of product B. However, to develop its product, the startup must invest in R&D. Suppose that the startup must incur a fixed cost F for its R&D project. In this section, we assume that a venture capitalist (VC) can provide such funding. Assuming that the VC is risk neutral, he will do so if the startup's expected profit exceeds the R&D cost F. As the startup's expected profit depends on the merger regime, the merger regime affects the startup's

entry. In the following, we thus compare the startup's profits in the three regimes to see their impact on the startup's entry and on the VC's decision to fund the startup's R&D project.

The startup's profits in the three merger regimes are as follows:

$$\pi_{S}^{N} = p_{S} \left[(1 - p^{N})^{2} \cdot 2\beta \pi^{D} + 2p^{N} (1 - p^{N}) \cdot \beta \pi^{D} \right] = 2p_{S} (1 - p^{N}) \beta \pi^{D},$$
(13)

$$\pi_{S}^{A} = p_{S} \left[(1 - p^{A})^{2} \pi^{M} + 2p^{A} (1 - p^{A}) \pi^{D} \right],$$
(14)

$$\pi_{S}^{K} = p_{S} \left[(1 - p^{K})^{2} \pi^{M} + 2p^{K} (1 - p^{K}) \beta \pi^{D} \right].$$
(15)

Inspection of the profit functions shows that for a given level of R&D effort by the platforms, the startup always prefers to be acquired and also prefers no ban on killer acquisitions. This is due to the fact that $\pi^M > 2\pi^D$ and competition for acquisition always pays more than licensing, even if the startup manages to capture all the benefits of its innovation ($\beta = 1$). However, this may not be true from an ex-ante perspective, because the possibility of acquiring the startup increases the innovation effort of the platforms, with a lower probability of being acquired as a consequence. For this reason, the startup may be better off if acquisitions are prohibited.

First, we show that if the cost of R&D is high enough,⁸ the startup always prefers a regime in which at least some acquisitions are allowed:

Lemma 8. If $\phi > \pi^M$, the startup prefers a ban on killer acquisitions to a ban on all acquisitions $(\pi_S^K > \pi_S^N)$.

The lemma shows that the startup always prefers a regime that allows at least some acquisitions to a regime that prohibits all of them. Thus, to determine which merger regime the startup prefers, it suffices to compare its profit under the two acquisition regimes. This comparison is complicated, but when the cost of R&D is high enough, we can show the following:

Proposition 3. If $\phi > \pi^M + \pi^D$, there is a threshold $\tilde{p}_S(\beta)$ such that the startup prefers a ban on killer acquisitions if $p_S \ge \tilde{p}_S(\beta)$, and no ban (all acquisitions allowed) otherwise.

Figure 3 below shows which merger regime yields the highest profit for the startup for the same parameter values as Figures 1 and 2. In all cases, remember that the startup prefers that acquisitions are at least partially allowed. If its probability of success is low, the startup prefers that all types of acquisitions be allowed. Conversely, if its probability of success is high, it prefers a ban on killer acquisitions.

A consequence of Proposition 3 is that the possibility for a startup to find venture funding depends on the merger regime in place. When startup acquisitions are prohibited, the startup can expect a lower profit, and therefore, it may not find funding to develop its project.

⁸For lower values of the R&D cost, the opposite can be true.

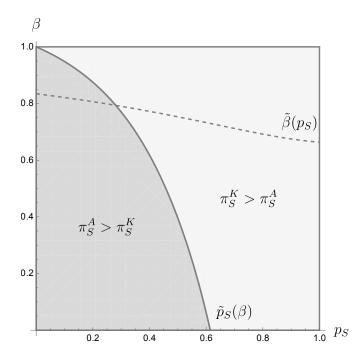


Figure 3: Merger regime maximizing startup's profits for $\pi^M = 1, \pi^D = 0.33, \phi = 0.7$.

5.2 Venture funding by a platform

The startup needs funds to develop its product and this funding can come from venture capitalists, as discussed in the previous section. But it can also come from the platforms themselves. In fact, many digital companies, such as Google or Microsoft, support the development of young startups through their own venture funds. In this section, we thus discuss the case where the platforms can provide venture funding to the startup.

We consider that when a platform provides venture funding to the startup, it changes the acquisition game. In particular, if platform *i* funded the startup at an early stage, it has the opportunity to make a first offer to acquire S. If this offer is accepted, *i* acquires *S*; otherwise, if *S* rejects the offer, the other platform *j* can make an acquisition offer. In other words, the acquisition game becomes sequential and we give the funding platform a first-mover advantage. To simplify the analysis, we focus on the case where all acquisitions are allowed by the competition authority.

Acquisition game with venture financing. As in the baseline case, acquisition can occur in two circumstances: when *S* succeeds and both platforms fail (case 3), and when *S* succeeds and only one platform fails (case 4).

First, suppose that both platforms fail. In this case, if one platform acquires *S*, it will monopolize the market for product A + B. In the acquisition game, the funding platform *i* makes the first offer, and if rejected, platform *j* has the opportunity to make an offer. As usual, we solve the game backwards. In the second stage, if the startup rejects the offer from *j*, it remains independent and sells a license to both platforms, making a profit of $2\beta\pi^D$. Therefore, platform *j* offers

this amount to acquire *S* and monopolize the market, as it makes a higher profit with acquisition $(\pi^M - 2\beta\pi^D)$ than with licensing $((1 - \beta)\pi^D)$. In the first stage, platform *i* anticipates that if its offer is rejected, the startup will be acquired by *j* and that it will not be able to compete in the market. Therefore, platform *i* offers $2\beta\pi^D$ and *S* accepts the deal. Hence:

Lemma 9. If both platforms fail to develop product B while S successfully develops product B, the platform that provided venture funding acquires the startup at a price of $2\beta\pi^D$.

Compared to the baseline case without venture funding, the competition for acquisition is less intense, resulting in a lower acquisition price, $2\beta\pi^D$ instead of π^M . A startup that received venture funding from a platform is no longer able to capture all of the monopoly profits when it is the only firm with a successful innovation.

Second, consider the case where the startup and one platform succeed, while the other platform fails. If the funding platform *i* is successful, it acquires *S* at a price equal to $\beta \pi^D$, which is the amount that firm *j* is willing to pay in the second stage. The successful platform acquires *S* and secures its monopolistic position; we have a killer acquisition as in the baseline case, but at a lower price.

If the unsuccessful platform is the funding platform *i*, it anticipates that in the second stage, platform *j* will make an offer to *S* to secure its monopolistic position. This offer will be equal to $\beta \pi^D$, the profit of the startup if it remains independent. Therefore, in the first round, platform *i* offers the same amount and acquires *S*. In this case, the two platforms share the market. Summing up, we have the following result:

Lemma 10. If only one platform and the startup successfully develop product B, the startup is acquired by the platform that provided venture funding for a price of $\beta \pi^D$.

There are two differences compared to the baseline case. First, the acquisition price decreases from π^D to $\beta\pi^D$. Second, the startup is not acquired by the successful platform, but by the venture capitalist. If the venture capitalist was successful, we have a killer acquisition. If it failed, it acquires *S* and competes with the other platform. In this case, it prevents a killer acquisition by its competitor.

Innovation effort by the platforms. Given the equilibrium of the acquisition game described above, the profits of the funding platform *i* and non-funding platform *j* are as follows:

$$\begin{aligned} \pi_i^V &= p_i(1-p_j)\pi^M + p_i p_j \pi^D - p_S p_i(1-p_j)\beta\pi^D \\ &+ p_S(1-p_i)\left(p_j(1-\beta)\pi^D + (1-p_j)(\pi^M-2\beta\pi^D)\right) - \phi \frac{p_i^2}{2}, \\ \pi_j^V &= p_j(1-p_i)\pi^M + p_j p_i \pi^D - p_S p_j(1-p_i)(\pi^M-\pi^D) - \phi \frac{p_j^2}{2}. \end{aligned}$$

The first-order conditions with respect to the R&D efforts are:

$$\frac{d\pi_i^V}{dp_i} = (1-p_j)\pi^M + p_j\pi^D \underbrace{-p_S(1-p_j)\beta\pi^D}_{\text{competition effect}} \underbrace{-p_S\left(p_j(1-\beta)\pi^D + (1-p_j)(\pi^M - 2\beta\pi^D)\right)}_{\text{insurance effect}} - \phi p_i = 0,$$

$$\frac{d\pi_j^V}{dp_j} = (1-p_i)\pi^M + p_i\pi^D \underbrace{-p_S(1-p_i)(\pi^M - \pi^D)}_{\text{competition effect}} - \phi p_j = 0.$$

For the funding platform *i*, there is both a competition effect and an insurance effect. The competition effect is reduced because the competition for acquisition is less intense and the platform acquires the startup more often and at a lower price. There is also an insurance effect, as the venture capitalist has the option to buy the startup if it fails and make some profit. This insurance effect is stronger than in the baseline case, regardless of the merger regime.

For the non-funding platform j, there is only a competition effect that reduces the return to innovation, and this effect is stronger than in the baseline case. Indeed, if both platform j and the startup are successful, platform j can no longer buy and kill S to monopolize the market, but must share the market, which reduces the payoff to innovation.

Solving the first-order conditions, we obtain the equilibrium R&D efforts:

$$p_i^V = \frac{\phi((1-p_S)\pi^M + \beta p_S \pi^D) - (1-p_S)(\pi^M - \pi^D)((1-p_S)\pi^M + p_S \pi^D)}{\phi^2 - (1-p_S)^2(\pi^M - \pi^D)^2},$$

$$p_j^V = \frac{\phi((1-p_S)\pi^M + p_S \pi^D) - (1-p_S)(\pi^M - \pi^D)((1-p_S)\pi^M + \beta p_S \pi^D)}{\phi^2 - (1-p_S)^2(\pi^M - \pi^D)^2}.$$

Assumption 1 ensures that the equilibrium is stable and that the R&D efforts are positive.

We can show that venture funding by one of the platforms reduces R&D:

Lemma 11. The funding platform *i* makes less R&D effort than its competitor, i.e., $p_i^V < p_j^V$. The total innovation effort is also lower compared to the baseline case: $p_i^V + p_j^V < 2p^A$.

Proof. We have:

$$p_i^V - p_j^V = \frac{-p_S(1-\beta)\pi^D}{\phi - (1-p_S)(\pi^M - \pi^D)} < 0,$$

and

$$p_i^V + p_j^V = \frac{2(1 - p_S)\pi^M + p_S(1 + \beta)\pi^D}{\phi + (1 - p_S)(\pi^M - \pi^D)} < 2p^A.$$

We also find that the platform that provides funding to the startup has a higher profit than the other platform, i.e., $\pi_i^V > \pi_j^V$.

We can now turn to the initial stage of the game where the platforms decide whether to fund the startup. We assume that if both platforms propose funding, then each platform has an equal chance of funding the startup. If no platform proposes funding, we assume that the startup can find an outside investor, provided that $\pi_S^A \ge F$. The game played by the platforms can then be represented as follows:

		Platform 2		
		Fund	No fund	
Platform 1	Fund	$\left(\frac{1}{2}(\pi_i^V - F + \pi_j^V), \frac{1}{2}(\pi_i^V - F + \pi_j^V)\right)$	$\left(\pi_i^V - F, \pi_j^V\right)$	
	No fund	$\left(\pi_{j}^{V},\pi_{i}^{V}-F\right)$	$\left(\pi^{A},\pi^{A} ight)$	

Table 4: Funding game.

Proposition 4. When the platforms can provide venture funding to the startup:

- If $F \le \min\{\pi_i^V \pi_j^V, \pi_i^V \pi^A\}$, there is a unique equilibrium where both platforms offer funding to the startup;
- If $\pi_i^V \pi^A < F \le \pi_i^V \pi_j^V$, there are two equilibria where either both platforms offer funding or neither does;
- If $\pi_i^V \pi_j^V < F \le \pi_i^V \pi^A$, there are two equilibria where only one platform offers funding.

So, if $F \leq \max\left\{\pi_i^V - \pi_j^V, \pi_i^V - \pi^A\right\} \equiv \Delta \pi$, there is an equilibrium with platform funding. It is interesting to compare this threshold with the threshold for funding by an independent VC, which occurs when $F \leq \pi_S^A$. Figure 4 shows the areas where $\Delta \pi > \pi_S^A$ and $\Delta \pi < \pi_S^A$. In the former case, venture funding is more likely to happen from the platforms than from an independent VC; in the latter case, the opposite is true. The figure shows that funding is more likely to come from independent VCs when the probability of innovation is low, and from platforms when it is high.

5.3 Direction of innovation

In the baseline model, we have shown that the merger policy affects the intensity of innovation by the platforms and, in Section 5.1, we have shown that it also affects the startup's entry. In this subsection, we argue that merger policy can also influence the direction of the startup's innovation (see also Brian and Hovenkamp and Moraga).

Suppose that there are two possible innovation paths to develop product *B*. The two platforms follow the same path, which we call 'conventional'. The startup can decide to follow the same conventional path or an alternative path, which we call 'radical'.

Only one research path is successful. Specifically, we assume that the radical path is successful with probability θ^r . If it fails, the conventional path succeeds. So, this path is successful with probability $\theta^c = 1 - \theta^r$. There is no other difference between the two paths, e.g. regarding the quality of the innovation. The timing of the game is then modified as follows:

- 1. The competition authority decides on its merger policy.
- 2. The startup chooses a research path, conventional or radical.

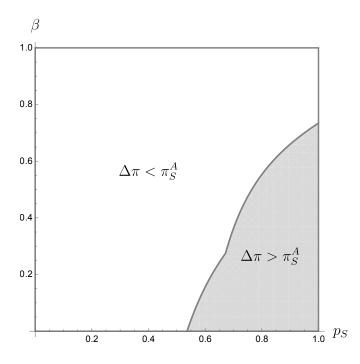


Figure 4: Comparison of thresholds for funding from platforms ($\Delta \pi$) and from an independent VC (π_S^A) for $\pi^M = 1, \pi^D = 0.33, \phi = 0.7$.

- 3. The platforms decide on their level of effort in R&D.
- 4. Nature decides on which innovation path is successful.
- 5. The platforms observe the success of the R&D projects and they play the continuation game as in the baseline case.

As usual, we solve the game backwards.

If the startup chooses the radical research path, its R&D project is successful with probability $\theta^r p_s$. In this case, as the only successful innovator, the startup earns a profit equal to π^M if mergers are allowed and to $2\beta\pi^D$ if they are not.⁹

If instead the startup chooses the conventional path, the game is the same as in the baseline case. Thus, if the conventional path is successful, the startup earns the expected profit π_S^N if acquisitions are prohibited, π_S^A if they are allowed, and π_S^K if killer acquisitions are prohibited.

Therefore, the startup chooses the radical innovation path if:

$$p_S \theta^r 2\beta \pi^D \ge \theta^c \pi_S^N$$
 in regime N, (16)

$$p_S \theta^r \pi^M \ge \theta^c \pi_S^A$$
 in regime A, (17)

$$p_S \theta^r \pi^M \ge \theta^c \pi_S^K$$
 in regime K. (18)

Comparing these expressions, we can establish that:

⁹It does not matter whether killer acquisitions are allowed or not, because the case where one platform succeeds while the other fails cannot arise when the conventional path failed.

Proposition 5. *The startup is less likely to choose the radical innovation path when mergers are not allowed.*

Therefore, allowing startup acquisitions pushes startups toward more radical (differentiated) research paths. Note also that if the startup chooses the radical path, killer acquisitions are eliminated.

6 Conclusion

In our setting, startup innovation complements platform innovation. The startup's product has no intrinsic value and must be combined with the platforms' core products, which are oneway essential complements (Chen and Nalebuff, 2006). However, the startup's innovation is also a substitute, as it can displace organic innovation by the platforms. Indeed, in the digital economy, many startups develop new functionalities for users of established platforms. In doing so, they build up on the existing networks of the incumbent platforms rather than developing their own. However, this strategy implies that the startup also competes with the platforms, which have their own research programs to develop (sometimes comparable) functionalities.

In this context, we examine the incentives of platforms to engage in R&D. A well-established result in the literature (Phillips and Zhdanov, 2013) is that the possibility to acquire the startup crowds out platform innovation, as the startup's innovation is a substitute for their own research efforts. In this paper, we show that this effect is even more pronounced when platforms are prohibited from acquiring the startup. In fact, the startup has other options to monetize its product, and we focus on non-exclusive licensing to the platforms. With licensing, the platforms have access to the startup's product if it is successful, which reduces their R&D efforts. This effect is exacerbated by the fact that a platform that successfully innovates is more likely to face competition, as the other platform can adopt the startup's innovation if it fails to innovate.

When startup acquisitions are allowed, the platforms can buy the startup if they fail to innovate. However, the competition among them to acquire the startup dissipates their profits, which are captured by the startup. For this reason, the startup's innovation is no longer a substitute for the platforms' own R&D efforts.

Acquisitions can also be used to deprive a competitor of access to the startup's innovation, which is a typical example of a killer acquisition, where a platform acquires a startup to eliminate a competitive threat. Killer acquisitions occur when one platform and the startup have successfully innovated. In this case, the acquisition of the startup allows to restrict the competitor's access to the startup's innovation.

For competition authorities, the decision of whether or not to allow startup acquisitions by incumbent platforms involves a classic trade-off between innovation incentives on the one hand and the diffusion of innovation on the other hand. Prohibiting mergers promotes platform innovation but limits the diffusion of innovation in the market. When platform innovation is lim-

ited, for instance when platforms develop functionalities outside their core business for which they have high R&D costs, the diffusion of innovation is the primary objective. In this case, prohibiting startup acquisitions clearly benefits consumers. Conversely, when prices are fixed or regulated, the *ex-post* market structure is no longer a concern, and consumers are better off if the competition authority focuses on innovation incentives. In such a case, allowing startup acquisitions is efficient.

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Appendix

Singh and Vives illustrative model

The utility of the representative consumer is $U(q_1, q_2) = a(q_1 + q_2) - (q_1^2 + 2\gamma q_1 q_2 + q_2^2)/2$, where q_1 and q_2 are the quantities of the firms, *a* is the intercept of demand, and γ measures the substitutability between the products. Maximizing the net utility $U(q_1, q_2) - p_1 q_1 - p_2 q_2$ with respect to quantities, we find the demand function of firm = 1,2:

$$D_i(p_i, p_j) = \frac{a(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2}$$

Assuming zero marginal costs, firms maximize their profit $p_i D_i(p_i, p_j)$ with respect to their price p_i . We find the equilibrium duopoly profit:

$$\pi^{D} = \frac{a^{2}(1-\gamma)}{(1+\gamma)(2-\gamma)^{2}}.$$

The monopoly price, which maximizes p(a - p), is $p^M = a/2$ and therefore the monopoly profit is $\pi^M = a^2/4$. We then calibrate this model using the values in Figure 1. We have $\pi^M = a^2/4 = 1$, so a = 2. Then, we look for the value of γ such that $\pi^D = 0.33$ and find $\gamma \approx 0.78$.

Consumer surplus is given by the net utility of the representative consumer. We find that:

$$CS^{D} = \frac{a^{2}}{(1+\gamma)(2-\gamma)^{2}} \approx 1.51$$
 and $CS^{M} = \frac{a^{2}}{8} = \frac{1}{2}$.

Finally, the consumer surplus in the three regimes is:

$$CS^{N} = 2(1 - p_{S})(1 - p_{N})p_{N}CS^{M} + [p_{S} + (1 - p_{S})p_{N}^{2}]CS^{D}$$

$$CS^{A} = (1 - p_{A})[p_{S} + (2 - p_{S})p_{A}]CS^{M} + p_{A}^{2}CS^{D}$$

$$CS^{K} = (1 - p_{K})(p_{S} + p_{K}(2 - 3p_{S}))CS^{M} + p_{K}(p_{K} + 2p_{S}(1 - p_{K}))CS^{D}$$

Proofs

Proof of Lemma 4 To show that $p^K > p^N$, we calculate the derivative of platform *i*'s profit when killer acquisitions are banned (regime *K*) at $p_i = p_j = p^N$. Using the FOC (8), we have:

$$\left. \frac{d\pi_i}{dp_i} \right|_{p_i = p^N, p_j = p^N} = p_S(1 - p^N)(1 - \beta)\pi^D > 0,$$

and therefore $p^{K} > p^{N}$ since the SOC holds. Similarly, using the FOC (5), when all acquisitions are allowed (regime *A*), we have:

$$\left. \frac{d\pi_i}{dp_i} \right|_{p_i = p^K, p_j = p^K} = p_S p^K (1 - \beta) \pi^D + p_S (1 - p^K) (\pi^M - 2\pi^D) > 0,$$

and therefore $p^A > p^K$ since the SOC holds.

Proof of Lemma 5 We have:

$$\frac{dp^N}{dp_S} = \frac{\beta \pi^D \left(\pi^M - \pi^D\right) - \phi \left(\pi^M - \beta \pi^D\right)}{\left[\phi + (1 - p_S)(\pi^M - \pi^D)\right]^2} < 0,$$

because $\phi > \beta \pi^D$ under Assumption 1 and $\pi^M - \beta \pi^D > \pi^M - \pi^D$,

$$\frac{dp^{A}}{dp_{S}} = \frac{\pi^{D}(\pi^{D} - \phi)}{\left[\phi + \pi^{M} - (1 + p_{S})\pi^{D}\right]^{2}} < 0$$

as $\pi^D - \phi < 0$ under Assumption 1, and

$$\frac{dp^{K}}{dp_{S}} = -\frac{\phi(\pi^{M} - \pi^{D}) - \pi^{D}(\beta\pi^{M} - \pi^{D})}{\left[\phi + (\pi^{M} - \pi^{D}) - p_{S}(\pi^{M} - (2 - \beta)\pi^{D})\right]^{2}} < 0,$$

using the fact that $\phi > \pi^D$ and $\beta < 1$.

Moreover, we have:

$$p^{A} - p^{K} = p_{S} \frac{\phi(\pi^{M} - 2\pi^{D}) - \pi^{D}(\beta\pi^{M} - 2\pi^{D} + p_{S}\pi^{D}(1 - \beta))}{\left[\phi + \pi^{M} - (1 + p_{S})\pi^{D}\right] \left[\phi + (\pi^{M} - \pi^{D}) - p_{S}(\pi^{M} - (2 - \beta)\pi^{D})\right]},$$

and

$$p^{K} - p^{N} = \frac{p_{S}(1 - \beta)\pi^{D} \left[\phi - \pi^{D} + p_{S}(1 - \beta)\pi^{D}\right]}{\left[\phi + (\pi^{M} - \pi^{D}) - p_{S}(\pi^{M} - (2 - \beta)\pi^{D})\right] \left[\phi + (1 - p_{S})(\pi^{M} - \pi^{D})\right]}$$

Note that the denominator of $p^A - p^K$ is decreasing in p_S , while we can show that the numerator is increasing in p_S , using the fact that $\phi > \pi^D$. Similarly, we can see that the numerator of $p^K - p^L$ is increasing in p_S , while the denominator is decreasing in p_S . This proves the second statement of the lemma.

Proof of Proposition 1 First, we compare the profits in the no acquisition and killer acquisition regimes, π^N and π^K . We have $\pi^N > \pi^K$ if $\beta < 1$ and $\pi^N = \pi^K$ if $\beta = 1$. Indeed, comparing the profit functions in regimes *N* and *K*, given by equations (1) and (7), respectively, we can see that $\pi_i^N(p_i, p_j) = \pi_i^K(p_i, p_j) + (1 - p_i)(1 - p_j)p_s(1 - \beta)\pi^D$. So, $\pi_i^N(p^K, p^K) > \pi_i^K(p^K, p^K) = \pi^K$ if $\beta < 1$. Since $\pi_i^N(p^N, p^N) = \pi^N \ge \pi_i^N(p^K, p^K)$, it follows that $\pi^N > \pi^K$ if $\beta < 1$. If $\beta = 1$, $\pi_i^N(p_i, p_j) = \pi_i^K(p_i, p_j)$, and therefore, $\pi^N = \pi^K$.

As $\pi^N \ge \pi^K$, which regimes is preferred by the platforms depends on the comparison of π^N and π^A :

- (i) We have $\pi^N < \pi^A$ when $\beta = 1$. Indeed, if $\beta = 1$, $\pi^A = \phi (p^A)^2 / 2$ and $\pi^N = \phi (p^N)^2 / 2$ from equations (10) and (11). Then, $\pi^A > \pi^N$ follows from the fact that $p^A > p^N$ (Lemma 4).
- (ii) π^N decreases with β while π^A is constant. Indeed, we have:

$$\frac{\partial \pi^N}{\partial \beta} = -p_S \pi^D \frac{(1-p_S)^2 (\pi^M - \pi^D)^2 + \phi \left[\phi + (1-p_S)\pi^M - \pi^D \left(2(1-p_S) + \beta p_S\right)\right]}{(\phi + (1-p_S)(\pi^M - \pi^D))^2} < 0$$

using $\pi^M > 2\pi^D$. Moreover, as $\pi^A = \phi (p^A)^2 / 2$ and p^A does not depend on β , then π^A does not depend on β .

(iii) Now, assume that $\pi^N > \pi^A$ when $\beta = 0$. Since π^N decreases with β , while π^A does not depend on β , and $\pi^A > \pi^N$ when $\beta = 1$, there exists a threshold $\tilde{\beta}(p_S) < 1$ such that $\pi^N > \pi^A$ for $\beta < \tilde{\beta}(p_S)$ and $\pi^N \le \pi^A$ for $\beta \ge \tilde{\beta}(p_S)$. If, instead, we have $\pi^A > \pi^N$ when $\beta = 0$, then the acquisition regime is always preferred by the platforms, i.e., $\tilde{\beta}(p_S) = 0$.

Summing up, if $\beta < \tilde{\beta}(p_s)$, then $\pi^N > \max\{\pi^A, \pi^K\}$. If $\beta \ge \tilde{\beta}(p_s), \pi^A > \pi^N \ge \pi^K$.

Proof of Lemma 8 Let $k \equiv \pi^M / (2\pi^D)$ and $\alpha \equiv \phi / \pi^M$. As $\pi^M > 2\pi^D$, we have k > 1. Moreover, if $\phi > \pi^M$, we have $\alpha > 1$. We can then write:

$$\pi_{S}^{N} - \pi_{S}^{K} = 2p_{S}\pi^{D} \underbrace{\left[1 - 2k\alpha - (1 - \beta)p_{S}\right]}_{(-)} Y,$$

with

$$Y = \underbrace{\frac{-\beta}{(2k-1)(1-p_{S})+2k\alpha}}_{(-)} + \underbrace{\frac{k(k\alpha-1+p_{S})+\beta p_{S}+2\beta k(1-p_{S})+k(k\alpha-\beta p_{S})}{\left[(2k-1)(1-p_{S})+2k\alpha+p_{S}(1-\beta)\right]^{2}}_{(+)}$$

We have $\pi_S^N - \pi_S^K < 0$ if and only if Y > 0, that is, if

$$\begin{split} z &= \left[k(k\alpha - 1 + p_S) + \beta p_S + 2\beta k(1 - p_S) + k(k\alpha - \beta p_S)\right] \left[(2k - 1)(1 - p_S) + 2k\alpha\right] \\ &\quad - \beta \left[(2k - 1)(1 - p_S) + 2k\alpha + p_S(1 - \beta)\right]^2 > 0. \end{split}$$

The rest of the proof consists in proving that z > 0 for all $\alpha \in [1, +\infty]$. We find that $z''(\alpha) > 0$, so $z'(\alpha)$ is increasing. Since z'(1) > 0, then $z(\alpha)$ is increasing. Finally, we find that z(1) > 0, which proves that $z(\alpha) > 0$ for all $\alpha \in [1, +\infty]$. Therefore, we have $\pi_S^N < \pi_S^K$.

Proof of Proposition 3 Let $\bar{\pi}_S^A \equiv \pi_S^A / p_S$ and $\bar{\pi}_S^K \equiv \pi_S^K / p_S$. First, if $p_S = 0$, we have

$$\bar{\pi}_{S}^{A} - \bar{\pi}_{S}^{K} = \frac{2\pi^{D}\pi^{M}(1-\beta)(\phi - \pi^{D})}{\left(\phi + \pi^{M} - \pi^{D}\right)^{2}} > 0.$$

Second, using (14) and (15), we find that:

$$\frac{1}{2}\frac{\partial(\bar{\pi}_{S}^{A}-\bar{\pi}_{S}^{K})}{\partial p_{S}} = \underbrace{\left[-\pi^{M}+\pi^{D}+(\pi^{M}-2\pi^{D})p^{A}\right]}_{(-)}\underbrace{\frac{\partial p^{A}}{\partial p_{S}}}_{(-)} - \underbrace{\left[-\pi^{M}+\beta\pi^{D}+(\pi^{M}-2\beta\pi^{D})p^{K}\right]}_{(-)}\underbrace{\frac{\partial p^{K}}{\partial p_{S}}}_{(-)}$$

From Lemma 5, we have

$$\left|\frac{\partial p^A}{\partial p_S}\right| < \left|\frac{\partial p^K}{\partial p_S}\right|.$$

Besides, we have

$$\begin{split} \left[\pi^{M} - \beta \pi^{D} - (\pi^{M} - 2\beta \pi^{D})p^{K}\right] - \left[\pi^{M} - \pi^{D} - (\pi^{M} - 2\pi^{D})p^{A}\right] = \\ (1 - \beta)\pi^{D} + (\pi^{M} - 2\pi^{D})p^{A} - (\pi^{M} - 2\beta \pi^{D})p^{K} = \\ (\pi^{M} - 2\beta \pi^{D})(p^{A} - p^{K}) + (1 - \beta)\pi^{D}(1 - 2p^{A}) > 0, \end{split}$$

since $p^A > p^K$ from Lemma 4 and $p^A \le 1/2$, which holds because $\phi > \pi^M + \pi^D$. So, we conclude that

$$\frac{1}{2} \frac{\partial \left(\bar{\pi}_{S}^{A} - \bar{\pi}_{S}^{K} \right)}{\partial p_{S}} \leq 0$$

If follows that there exists a threshold $\tilde{p}_{S}(\beta)$ such $\pi_{S}^{A} \ge \pi_{S}^{K}$ if $p_{S} \ge \tilde{p}_{S}(\beta)$, and $\pi_{S}^{A} < \pi_{S}^{K}$ otherwise. We have $\tilde{p}_{S}(\beta) < 1$ if $\pi_{S}^{A} - \pi_{S}^{K}|_{p_{S}=1} < 0$ and we set $\tilde{p}_{S}(\beta) = 1$ if $\pi_{S}^{A} - \pi_{S}^{K}|_{p_{S}=1} \ge 0$.

Proof of Proposition 5 Using equation (13), condition (16) can be written as:

$$\frac{\theta^r}{\theta^c} \ge \frac{\pi_S^N}{2p_S\beta\pi^D} = 1 - p^N.$$

Similarly, using equation (14), condition (17) can be written as:

$$\frac{\theta^r}{\theta^c} \geq \frac{\pi_S^A}{p_S \pi^M} = \left(1 - p^A\right)^2 + p^A \left(1 - p^A\right) \frac{2\pi^D}{\pi^M}.$$

Therefore, the startup is more likely to be radical in regime A than in regime N if

$$1 - p^{N} \ge (1 - p^{A})^{2} + p^{A} (1 - p^{A}) \frac{2\pi^{D}}{\pi^{M}}.$$

Since $2\pi^D/\pi^M < 1$, we have

$$(1-p^{A})^{2} + p^{A}(1-p^{A})\frac{2\pi^{D}}{\pi^{M}} < (1-p^{A})^{2} + p^{A}(1-p^{A}) = 1-p^{A}.$$

Using Lemma 4, we can then write:

$$1 - p^{N} > 1 - p^{A} > (1 - p^{A})^{2} + p^{A} (1 - p^{A}) \frac{2\pi^{D}}{\pi^{M}}.$$

Finally, using a similar argument, we can show that the startup is less likely to choose the radical path in regime *N* compared to regime *K*.