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Sense and Sensitivity: An Argument Against Reporting Multiple Net Present Values

Abstract

We critically assess an almost universal Benefit-Cost Analysis (BCA) practice. In addition to the central Net Present Value (NPV), analysts frequently also report multiple additional values in what is commonly referred to as 'NPV sensitivity analysis'. This practice is generally justified with reference to the future net benefits to the asset being risky, or because the correct discounting model is difficult to identify. We explain why, despite the fact that this is recommended as best practice across multiple prestigious and influential sources, the reporting of more than one NPV either lacks sufficient theoretical support or reflects decisions taken at an inappropriate level within the organizational hierarchy. As a consequence, this practice may confuse decision-makers more than help them. We illustrate this point in relation to a number of current guidelines across the public and private sectors and with particular focus on the US Environmental Protection Agency's latest estimates of the Social Cost of Carbon.

JEL-Codes: H430, G310, L510, Q510, Q540.

Keywords: benefit-cost analysis, net present value, sensitivity analysis, social cost of carbon, regulatory analysis.

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1 Introduction

We focus on one particular practice in benefit-cost analysis (BCA) that is prevalent, to the point of being almost universal, across public sector discounting guidance and long-established norms in the private sector: 'Net Present Value (NPV) sensitivity analysis'. In addition to a central NPV (or NPSV in some social contexts), it is generally recommended that additional NPVs should also be reported to assist in the economic appraisal of potential new 'assets' that might be projects, policies, regulations, or secondary market financial investments. Our central purpose is to ask the question: "What decision-relevant information is provided to the BCA process by this reporting of more than one Net Present Value?".

Consider a single monetized benefit, d_{it} , that an organization will receive from some asset, i, at some future time, t. This benefit's 'present value', p_{it} , is the value of d_{it} as of today. Consistent with general guidance, we define p_{it} to be the value that, if spent today in exchange for d_{it} at time t, would leave either expected social welfare (for social BCA) or expected long-term shareholder utility (for private sector BCA) unchanged. Under exponential discounting, $p_{it} = E[d_{it}] \exp(-tr_{it})$ for some asset-specific discount rate r_{it} . If, in practice, the organization knows the value, I_{it} , that it would need to spend today in order to acquire this single future benefit, then its Net Present Value is $p_{it} - I_{it}$. The NPV therefore quantifies the extent to which acquiring the asset is expected to change social or shareholder well-being. The more positive the NPV, the stronger the economic appraisal case for proceeding.

In Section 2 we describe the 'pricing kernel' theory of discounting that underlies such analysis, and we explain how many of the major discounting models—the Ramsey Rule, the opportunity cost of capital approach of discounting future net benefits using a Treasury Bond yield, the Consumption Capital Asset Pricing Model (CCAPM), and the market-based Capital Asset Pricing Model (CAPM)—are all nested within it. This section introduces a key distinction between 'sensitivity analysis' and 'model uncertainty'. The future net benefit, d_{it} , is, in almost all commercial and social settings, very difficult to estimate at the BCA appraisal stage. For this reason, in practice the analyst undertaking the BCA evaluation is very likely to be encouraged to consider a finite set, $\{d_{it}^s\}$, of possible future outcomes, reflecting varying levels of success of the asset. 'NPV sensitivity analysis' values (denoted by the superscript s) can then be calculated; $p_{it}^s - I_{it} = d_{it}^s \exp(-tr_{it}) - I_{it}$, where the discount rate r_{it} is generally assumed to be fixed across different asset outcomes. By contrast, 'model uncertainty' (denoted by the superscript u), is a term we use to describe situations where there may be uncertainty about r_{it} for more fundamental reasons that cannot be quantified. This may be because there is disagreement about the conceptual approach that should be taken to discounting, the specific values of individual parameters within any given approach, or whether investment or consumption will be displaced. Now the analyst makes different judgments about r_{it}^{u} , calculates $p_{it}^{u} = E[d_{it}] \exp(-tr_{it}^{u})$ and uses a range of these values in a similar way to p_{it}^{s} . This is conceptually distinct from sensitivity analysis as we have defined it above, but both are widespread in the practices that we describe.

In Section 3, we discuss the application of NPV models in three real-world contexts. The first

two relate to general public sector BCA. Until recently in the United States, this was laid out in *Circular A-4* (OMB, 2003) for regulatory BCA, and *Circular A-94* (OMB, 1992) for the BCA of public spending from Federal grant money. Following President Biden's directive to modernize the regulatory review process, draft updates to both of these Circulars were published in April 2023 (OIRA, 2023a; OMB, 2023a) and a public consultation process was put in place to elicit input from outside government. In November 2023, final versions became operational (OIRA, 2023c). The new versions differ from previous guidance in a range of ways and we consider both in this paper. Public sector BCA practice in the United Kingdom is laid out in the *Green Book* (HM Treasury, 2022a), and this is our second example. Our third example considers BCA best practice in the private sector when appraising the impact of a potential project on long-term shareholder well-being. This section also highlights the widespread reporting of multiple NPVs in these three examples, both in terms of sensitivity analysis and model uncertainty.

In Section 4, we turn in more detail to one recent, and highly influential, specific example; the US Environmental Protection Agency's (USEPA, 2023) recent updates to its estimates of the Social Cost of Greenhouse Gas (SC-GHG) emissions. We treat this example separately because of the subtle ways in which it combines NPV sensitivity analysis and model uncertainty by allowing r_{it} to vary alongside d_{it} .

Section 5 presents the main arguments of this paper, illustrated by a simple numerical example. We argue that, as $p_{it} = E[d_{it}] \exp(-tr_{it})$, the present value is the product of an expectation and a fixed discount factor. This gives a single, easily quantified, real number around which it is not possible to report a sensitivity. Put bluntly, one cannot report a sensitivity around the number 38. Thus, that there exists a finite set of identifiable real outcomes, $\{d_{it}^s\}$, does not imply that there is an equivalent set of NPV sensitivity analysis values, $\{p_{it}^s - I_{it}\}$, with economic meaning at the time when the economic appraisal is being undertaken. This lack of theoretical support leads to practical problems with NPV sensitivity values: the appropriate discount rate to apply to each scenario is not clear, each sensitivity value does not have a clear interpretation in terms of welfare outcomes, the values are non-additive, and it is not obvious how an analyst should use such sensitivity analysis to compare investments within the organization with others elsewhere in the economy.

With respect to model uncertainty, resolving conceptual disputes about how to undertake discounting is an extremely complex issue, but ultimately a choice must be taken on whether or not to press ahead with the asset. We argue that dealing with such uncertainty should not be passed on to individual project analysts whose knowledge relates to the characteristics of the project itself and not the economy at large. Instead, the bodies that are setting the overall BCA guidelines should decide on the discounting framework themselves because they are best placed to do so and they can then also be confident that there is consistent treatment across different economic appraisals. Fundamentally, there is an appropriate level within the organizational hierarchy at which the decisions about such model uncertainty should be taken based on expertise. In Section 6 we try to understand why reporting multiple NPVs remains such common practice. This is not a straightforward task because the sources that recommend it do not link to references which provide formal justification, and we are not aware of detailed academic literature on NPV sensitivity analysis that may guide us. What we can evidence is that this practice was commonplace in both the public and private sectors long before more formal risk-adjusted discounting models were deployed. We therefore argue that it was an early heuristic to deal with risk and uncertainty in investment appraisals, which then became accepted practice that has not subsequently been dropped once more formal discounting techniques were introduced. We therefore conjecture that the reporting of multiple NPVs has remained widespread for a mixture of four reasons: (i) there has been a failure for organizations to "unlearn" previous practice when this has been replaced by more formal methods; (ii) that NPV sensitivity analysis is useful during the construction of the central NPV itself, and therefore reporting multiple NPV values at the conclusion of a BCA process is an easy task to complete; (iii) that managers often have difficultly assigning probabilities to future scenarios, and (iv) that managers may find the additional quantitative information reassuring as a means of justifying their decisions.

To our knowledge, the practical usefulness of non-central NPVs in the capital allocation process is unproven and we present arguments that would suggest that reporting multiple NPVs may actually be detrimental to good decision making under risk and uncertainty. In light of our finding, we believe that international guidance across the public and private sectors on NPV sensitivity analysis requires revision. Specifically, we ask organizations to be clearer about the economic interpretation of the individual values and ranges of p_{it}^s, p_{it}^u , as well as to provide more specific guidance to analysts about how such values should be used to inform BCA decisions.

2 Discounting theory

Consider a specific asset, denoted by i, that will have a future profile of monetized net benefits denoted by $d_{i1}, d_{i2}, \ldots, d_{iT}$. Its present value is denoted by P_i . This is the value that an organization would give up today in exchange for the future stream of benefits while keeping either expected social welfare or expected long-term shareholder utility unchanged. The NPV is given by $P_i - I_i$, where I_i is the known total current cost of acquiring the asset. Assume, in line with most guidance and best practice, that the present value of any portfolio of assets is equal to the sum of the present value of its component parts. It is then possible to deconstruct $P_i = \sum_{t=1}^{T} p_{it}$, where p_{it} is the present value of the single net benefit d_{it} and no other.¹ We add three further assumptions. First, there is a single probability space $(\Omega, \mathcal{F}, \mathcal{P})$. We also assume that the values d_{it} do not depend on future organizational decisions.² Finally we assume that, under $(\Omega, \mathcal{F}, \mathcal{P})$, d_{it} has finite mean and variance. Under these conditions, there exists at least one random variable π_t , known as the pricing

¹For a recent discussion of the order of aggregation, see Li *et al.* (2024). Nesje & Piacquadio (2024) derive pricing kernels for non-additive settings, which can be interpreted as generalizations of equation 1 below.

²This assumption rules out Real Option methods for undertaking BCA appraisals. Our main results can be extended to Real Options frameworks because such valuation methods can be expressed in terms of pricing kernels (e.g., Edge, 2011). We make this assumption to focus our attention on NPVs.

kernel (or stochastic discount factor), such that $p_{it} = E[d_{it}\pi_t]$: see Appendix. This pricing kernel is a function of the maturity of the benefit, t, but is the same for all assets: π_t is not dependent on i. From this it follows that for continuous discounting:

$$p_{it} = E[d_{it}] \exp(-tr_{it}), \quad \text{where} \quad r_{it} = \underbrace{-\frac{1}{t} \ln\left(E[\pi_t]\right)}_{\text{Risk-free rate}} \underbrace{-\frac{1}{t} \ln\left(1 + \operatorname{Cov}\left[\frac{d_{it}}{E[d_{it}]}, \frac{\pi_t}{E[\pi_t]}\right]\right)}_{\text{Project-specific risk premium}}$$
(1)

One particular asset of note is the zero-coupon Treasury bond with maturity t and redemption value of \$1, which we assume to be risk-free.³ If we denote this bond through the subscripts ft, then $d_{ft} = 1$, $p_{ft} = \exp(-tr_{ft})$, and $r_{ft} = -(1/t)\ln(E[\pi_t])$. This equals the first term on the right hand side of the expression for r_{it} . The final term is then the project-specific risk premium that arises from the correlation between d_{it} and π_t . This is a highly generic formula for both social and corporate NPV analysis.

While the Hilbert space mathematics that is outlined in the Appendix assures the existence of at least one pricing kernel, it does not help identify it and therefore economic assumptions must be introduced in order to give interpretation to π_t . We focus on two very common approaches to NPV analysis which underpin all the examples that we will consider in this paper. First, the standard normative model for governmental policy analysis assumes that social welfare, W, is derived from a time-separable utility of consumption, $U(c_t)$, in the presence of a representative agent: $W = \sum_{t=0}^{T} \exp(-\rho t)U(c_t)$, where ρ is the utility discount rate. In this case, $\pi_t = \exp(-\rho t)U'(c_t)/U'(c_0)$; see, for example, Cochrane (2001, Section 1.2). With isoelastic marginal utility, $U'(c_t) = c_t^{-\eta}$ where η represents the elasticity of the marginal rate of consumption, then $\pi_t = \exp(-t(\rho + \eta g_t))$, where $g_t = (1/t) \ln(c_t/c_0)$ is the per-period real logarithmic per-capita aggregate consumption growth rate. This framework leads to the Ramsey Rule, $r_{ft} = \rho + \eta g_t$, when g_t is non-stochastic, its extensions when g_t is stochastic, and other well-known normative models of social discounting such as the Consumption Capital Asset Pricing Model (CCAPM) — see, for example, Gollier (2012).

Contrasting with this, by far the most commonly applied model by large corporations assumes that shareholders are mean-variance decision makers (Markowitz 1952). They evaluate opportunities over a single time period and, within this investment window, they make decisions to maximize their expected returns for the given level of risk (as measured by the standard deviation of returns) that they are prepared to tolerate. In this case, $\pi_t = A + Br_{mt}$ where A, B are constants and r_{mt} is the return to the 'market portfolio' that is diversified across all risky assets with weights proportional to their market capitalization; see, e.g., Cochrane (2001, Section 9.1). This leads directly to the Capital Asset Pricing Model (CAPM). The social opportunity cost of capital (SOC) approach can also be interpreted as $\pi_t = A + Br_{mt}$ with B = 0 when Treasury bond yields are used to estimate the SOC.

A key difference between these two approaches is that, in the former, analysts make explicit normative judgments about how discounting *should* be undertaken. They do not calibrate to any

³In practice, even Treasury bonds entail a small element of risk. If the bond payoff is uncorrelated with the pricing kernel, then this result still holds even in the presence of risk.

rate of return that may be observed in financial markets or elsewhere. Calibration is instead 'normative'. By contrast, in the CAPM framework, the risk-free rate and the expected return to the market portfolio are both exogenous (determining the values of the constants A, B) and other expected asset returns are benchmarked against these two values based on their systematic risk, or 'betas'. This approach is therefore often referred to as 'positive' in that rates must be consistent with those that prevail in the economy. However, by basing our analysis on equation 1, which spans the normative/positive divide across guidelines, we have an integrated framework that allows us to provide a general resolution to our core question.

This framework also allows us to formalize our distinction between the terms 'sensitivity analysis' and 'model uncertainty', which we lay out in Table 1. There are two dimensions to consider. Uncertainty can be quantifiable in the sense that it can be described by a probability density function, in alignment with the assumption that there is a well-defined probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Aternatively, such uncertainty may be unquantifiable. In another dimension, uncertainty may relate either to the asset-specific future benefits (d_{it}) , features of the macroeconomy (π_t) , or both.

We use the term 'NPV sensitivity analysis' in the situation where there is quantifiable assetspecific uncertainty about the future net benefits, d_{it} , as captured by the set $\{d_{it}^s\}$ — this is the left-hand box in the first row of Table 1. By contrast, when there is unquantifiable economy-wide uncertainty around π_t , then we refer to this as 'model uncertainty' — this is the right-hand box in the second row of Table 1. This may be because there are conceptual uncertainties around the appropriate choice of a discounting framework resulting from different parties disagreeing over which welfare function is most relevant. There can also be irreconcilable differences (Drupp et al., 2018) or differences in estimates that cannot be easily integrated out under the expectations operator (Dahlquist and Ibert, 2024) over the choice of the core parameters ρ , η , A, B within any given discounting framework. Another reason for model uncertainty under some BCA frameworks arises when it is unclear whether investing in the asset will displace investment or consumption within the economy. These are the two key cases, out of the six possible outlined in Table 1, that are the focus of this paper because these are the most common justifications for reporting multiple NPVs.

Three other cases are also of interest. When there is unquantifiable uncertainty over d_{it} then this is a situation of Knightian uncertainty over the asset outcome: we return to this case in Subsection 6.3. Another case arises when there is quantifiable uncertainty over the stochastic process that describes aggregate consumption growth (g_t) which impacts on the risk-free rate in a normative setting through a precautionary savings motive. Such uncertainty can be integrated out under the expectations operator (Gollier, 2016) but we would argue that this should be done at a higher level in the organizational hierarchy than any specific BCA appraisal team because the risk-free rate is fixed across the economy. Third, there may be quantifiable uncertainty about the nature of the correlation between g_t and d_{it} for social discounting, and between market returns (r_{mt}) and d_{it} for private-sector discounting. This is particularly relevant for understanding discounting in USEPA (2023) and mixes macroeconomic and asset-specific outcomes, and we return to this in

Section 4. Unquantifiable joint uncertainty over d_{it} , π_t , which is the final possible case, has not featured heavily in either the academic literature or BCA practice and therefore we do not consider it further.

	Quantifiable	Unquantifiable
Project-specific (d_{it})	A finite set of possible quanti- fied asset outcomes, $\{d_{it}^s\}$, each of which is separately discounted to give a range of present values. This is what we refer to as 'NPV sensitivity analysis'.	Possible asset outcomes are un- quantifiable (e.g., in the sense of Knightian uncertainty). The theo- retical case for using NPV analysis as the appropriate basis for BCA is then unclear.
Economy-wide (π_t)	Uncertainty in macroeconomic variables, such as g_t , can be integrated out under the expectations operator. Should be undertaken at a relatively highly level in the organizational hierarchy as it does not require asset knowledge.	Fundamental disagreements over the approach to discounting (e.g., normative vs. positive), irrecon- cilable differences over parameter values, and/or whether consump- tion or investment will be dis- placed. We refer to this as 'model uncertainty'.
Both (d_{it}, π_t)	Quantifying the correlation be- tween the asset and macroeco- nomic outcomes to determine the asset CCAPM or CAPM beta. Mixes specific knowledge of the as- set with that of the wider econ- omy. Particularly relevant for the USEPA interquantile ranges.	Not commonly incorporated into BCA analysis at present.

Table 1: Differentiating between 'sensitivity analysis' and 'model uncertainty'.

3 Public and for-profit discounting

We now show that this pricing kernel framework, and hence our arguments against reporting multiple NPVs which follow in Section 5, can be applied to all the social and corporate NPV practices that we described in the Introduction. We then describe the ways in which both sensitivity analysis and model uncertainty are recommended practices in all these settings. We defer a discussion of the USEPA's (2023) recommendations for the Social Cost of Greenhouse Gas emissions to Section 4 because of the subtle way in which it intertwines the issues that are of relevance to this paper.

3.1 BCA practice

The urgency with which the *Stern Review* (Stern, 2006) recommended strong governmental climate change action refocused many public sector economists' attentions on the importance of NPV analysis. As the *Review*'s conclusions were subsequently shown to be highly dependent on some

of its discounting assumptions — particularly its choice of a low normative utility discount rate (see, for example, Nordhaus, 2007; Stern, 2008) — this led to a vigorous academic debate on appropriate discounting choices. This reflected an earlier observation by Weitzman (2001), who had stressed that "the choice of an appropriate discount rate is one of the most critical problems in all of economics" (*ibid.*, p.260). The rapid expansion of the academic discounting literature that followed from the *Stern Review* has more recently found its way into international policy guidelines on social discounting (Groom *et al.*, 2022). For example, a 2023 White House briefing announcing proposed changes to Federal BCA guidance in the United States explicitly noted that "…parts of the federal regulatory review process haven't been updated since the 1990s, and since then we've seen substantial advances in scientific and economic knowledge".

In the UK, HM Treasury's (2022a) Green Book takes an explicitly normative approach through the application of a social rate of time preference (SRTP, or sometimes just STP). The risk-free component of the discount rate is set through the calibration of the Ramsey Rule parameters $\rho = 0.5\%$ and $\eta = 1$, and a forecast of expected annual real per-capita consumption growth of 2%. The Green Book also recommends an explicitly declining discount rate structure (see, for example, Cropper et al., 2014) to account for current uncertainty over future economic growth, resulting in lower longer-horizon discount rates. In addition, there is a premium to ρ denoted by L = 1% that is added to the discount rate for all projects in order to account for 'catastrophic' and 'systematic' risk. As the Green Book explains, "The risks contained in L could, for example, be disruptions due to unforeseeable and rapid technological advances that lead to obsolescence, or natural disasters that are not directly connected to the appraisal. L also includes a small premium for 'systemic risk' because costs and benefits are usually positively correlated to real income per capita" (HM Treasury 2022a, Section 14.2). This speaks to a normative CCAPM framework for estimating risk premiums, with the Treasury not attempting to estimate differences in systematic risk across different public projects, programmes and policies.

In the US, by contrast, the new version of *Circular A-4* (OIRA, 2023b) is explicitly positive in nature, taking a social opportunity cost approach where "no project should be accepted if its return is less than the return available on alternative projects" (Burgess & Zerbe, 2013: p.391). The proposed discount rate of 2% reflects the average real yields on 10-year Treasury securities over the previous 30 years (1.7%) together with an adjustment for the difference in price growth between personal consumer expenditure compared to the normal Consumer Price Index (0.3%). Appendix D of *Circular A-94* follows this guidance by also taking 2% as the risk-free component of the discount rate. This is a substantial change in value from the previous guidance where "As a default position, OMB Circular A-94 states that a real discount rate of 7 percent should be used as a base-case for regulatory analysis. The 7 percent rate is an estimate of the average before-tax rate of return to private capital in the U.S. economy" (OMB 2003, p.33). This is also a positive social discount rate, based on observed average returns to capital rather than explicit normative considerations of social welfare. Both the previous and current versions of *Circular A-4* note the specific issues that arise in an intergenerational context and point to the use of declining discount rates. Many other countries take one or other of the SOC and SRTP approaches or some balance of the two (Groom *et al.*, 2022).

US Federal BCA practices now adjust for risk predominantly through the use of certainty equivalents. These are mathematically equivalent to risk-adjusted discount rates, but now uncertainty is accounted for in the net benefit, d_{it}^* , of the NPV equation through the rearrangement of equation 1:

$$p_{it} = E[d_{it}^*]E[\pi_t] \quad \text{where} \quad d_{it}^* = d_{it} + \operatorname{Cov}\left(d_{it}, \frac{\pi_t}{E[\pi_t]}\right)$$
(2)

Circular A-4 states that "... you should endeavor to estimate certainty equivalents when risk is material to your analysis. This approach is favored over the use of higher discount rates as a means of accounting for risk due to its potential for greater accuracy" (OIRA, 2023b, p.81). It justifies this by saying that "However, as a general matter, using discount rates to account for risk requires rigid assumptions about the form that risks take over time, and therefore creates the potential for increased inaccuracy relative to the certainty equivalents approach" (*ibid.*, p.81).⁴ Circular A-4 does, though, allow for an adjustment to the discount rate instead, although cautioning that "Any agency that wishes to account for risk using alternative discount rates in primary or sensitivity analyses should provide specific justification for their approach, and should confer with OMB before proceeding" (*ibid.*, p.82). Circular A-94 also expresses a clear preference for the use of certainty-equivalents. However, it also notes that "In the absence of calculating certainty equivalents, systematic risk should be captured through the discount rate for a project by applying a risk premium, which reflects the extent to which a project is less (or more) valuable on net by virtue of the positive (or negative) correlation of its benefits with times when aggregate consumption is high" (OMB, 2023b, p.14). Appendix D of Circular A-94 calibrates this risk premium at 1.1%, which it justifies on the basis of "the standard consumption capital asset pricing model" but, in fact, is instead using the market-based CAPM as it comprises of an average public sector CAPM 'beta' of 0.45 multiplied by 2.5%, which is a leverage-adjusted equity market risk premium. Circular A-94 also allows, with specific OMB approval, for analysts to change this if they think the benefits they are appraising have a different risk profile to the average project.

In contrast to the formal procedures issued by governments for BCA in the public sector, in the commercial world there are no standardized guidelines that regulate for-profit discounting. Instead there are norms and best practices that individual firms may choose to implement, and these are often summarized in leading textbooks and similar sources. While, in all the social BCA examples we consider, discounting is undertaken in real, inflation-adjusted, terms, corporate discounting is almost invariably nominal. Despite the extensive academic debates over, and policy updates to, social NPV best practice over the last two decades, corporate BCA norms remain largely unchanged. In his Presidential Address to the American Finance Association, John Graham (2022) compared current behaviour, as surveyed across Chief Financial Officers, to the results from a similar survey

 $^{^{4}}$ Given the mathematical equivalence between equations 1 and 2, we are unsure of the justification for this statement.

reported in a seminal study twenty year earlier (Graham & Harvey, 2001). His results show how little the practice of private sector discounting has changed over this period, both in terms of the methods used and the costs of capital applied.⁵ A number of companies, particularly smaller ones (annual revenue < \$1bn), use simple rules, such as payback periods, often in preference to NPV analysis because of "liquidity needs, a lack of sophistication, and a preference for simple decision rules" (*ibid.*, p.1988). Amongst large firms, by contrast, NPV or its closely related counterpart, the Internal Rate of Return (IRR), are very heavily applied in practice.⁶ Discount rates are commonly based on estimates of the firm's Weighted Average Costs of Capital. For over 80% of large firms that estimate a cost of equity, this is undertaken always or almost always by applying the CAPM.⁷

3.2 Sensitivity analysis

Within these different discounting frameworks, the practice of sensitivity analysis and model uncertainty is widespread to the point of being almost universal, with these two issues sometimes being combined into a single issue. For example, the current version of *Circular A-94* states:

"Discounted net benefits and other outcomes should be recomputed to determine how sensitive outcomes are to changes in major assumptions. The assumptions that deserve the most attention will depend on the dominant benefit and cost elements and the areas of greatest uncertainty of the project being analyzed. For example, in analyzing a retirement program, one could consider analyzing alternative assumptions about the number of beneficiaries, future wage growth, inflation, and the discount rate. In general, it is worthwhile to consider sensitivity analyses for estimates of: (i) important inputs to benefits and costs; (ii) the discount rate; (iii) the general inflation rate; and (iv) the distribution of key variables." (OMB, 2023b, pp.13–14)

Point (i) in this quotation refers, in our nomenclature, to uncertainty over d_{it} and is therefore what we have referred to as 'sensitivity analysis'. Point (ii), by contrast, refers to uncertainty over π_t for the risk-free component of the discount rate and is therefore what we describe as 'model uncertainty'. This distinction will be central to the arguments that we make in Section 5 because the

 $^{^{5}}$ A potential reason why social BCA practice has advanced in a way that corporate BCA practice has not relates to the nature of the controversies that were stirred by the *Stern Review*. By focusing on climate change, the maturities of the costs and benefits that were being considered by Stern are far longer than those that arise in corporate contexts, where projected cash flows almost never extend beyond a few decades outside of a few sectors, such as utilities. Because of the compounding effects of interest, the choice of discount rate is more critical for intergenerational projects and policies than those that span a few years or decades. Second, Stern took a SRTP position on discounting, while Nordhaus (2007) and others argued that the social discount rate should, instead, be based on social opportunity costs. While this normative/positive divide is now seen as being overly polarising (e.g., Drupp *et al.*, 2018), it nevertheless fueled many of the developments in the field of social discounting. For corporations, such issue are irrelevant as it is accepted that positive approaches must prevail in a for-profit context.

⁶The Internal Rate of Return is the discount rate or rates that sets the NPV equal to zero. The number of IRRs is only certain to be unique if the net cash flow time series changes sign only once.

⁷In small companies, the use of the CAPM has declined very significantly over the last twenty years, with little apparent use of economic models to estimate the cost of equity.

former is microeconomic and asset-specific, while the latter depends on π_t which is macroeconomic and economy-wide.

In terms of sensitivity analysis, *Circular A-4* states: "Sensitivity analysis can be used to find 'switch points,' critical parameter values at which estimated net benefits change sign or the alternative with the most net benefits switches" OIRA (2023b, p.71). The *Green Book* also makes a large number of statements about the requirement for sensitivity analysis in social BCA, including that it "...can demonstrate, for example, the changes in key assumptions required to change the preferred option on an NPSV" (HM Treasury, 2022a, Section 5.8).

Similar practices are also commonplace in corporate finance, as evidenced by statements from leading textbooks in the field. For example, Ross, Westerfield & Jordon (2017, p.294) say that "The basic idea with a sensitivity analysis is to freeze all the variables except one and then see how sensitive our estimate of the NPV is to changes in that one variable". Brearley *et al.* (2023, p.278), another highly influential textbook, also notes in relation to one of their numerical examples that "The fourth and fifth columns of the table show what happens to the project's net present value if the variables are set one at a time to their optimistic and pessimistic values". Similar statements can be found in many other corporate finance sources. A 2014 Harvard Business Review refresher article that focuses on the use of NPV notes that what "... managers need to keep in mind is that the calculation is based on several assumptions and estimates, which means there's lots of room for error. You can mitigate the risks by double-checking your estimates and doing sensitivity analysis after you've done your initial calculation".

3.3 Model uncertainty

In addition to sensitivity analysis, model uncertainty is also captured in both UK and US governmental guidance. When referring to intergenerational effects, the Green Book (Section 14.5) considers two different utility discount rates: $\rho = 0$ as well as the usual value of $\rho = 0.5\%$. Within a positive setting, the previous version of Circular A-4 says that "For regulatory analysis, you should provide estimates of net benefits using both 3 percent and 7 percent" (OMB, 2003, p.34) because new regulation might replace consumption or capital investment, with the lower 3% rate representing historic average real Treasury bond yields that preceded the time that the previous guidance was issued. The treatment of risk in these discount rates is somewhat unclear, though. Circular A-4 explains that the gap between the 3% and 7% rates is driven by "economic distortions, including taxes" (*ibid.*, p.33). However, as observed by Li & Pizer (2021), such frictions are unlikely to be sufficiently severe to cause such a large difference in rates and that risk is also likely to play a role. In the current version of Circular A-4, the 7% capital displacement rate has been removed, with the new draft guidance recommending that a shadow price of capital approach be employed instead. However, model uncertainty is re-introduced through a separate mechanism. It now recognizes alternative approaches to discounting, noting that "...you may also analyze the welfare effects of your regulation in an economic model in which the evolution of the discount rates is endogenous. If you take such an approach to discounting, a number of assumptions need to be made in order to inform the selection of parameter values." (OIRA, 2023b, p.77), with similar advice being given for determining the declining discount rate schedule in an intergenerational context (*ibid.*, p.81). The final version of this circular does, though, give less weight to the normative Ramsey rule than the draft version circulated in April 2023 (OIRA, 2023a). This was discussed in the public consultation response: "OMB has decided to reduce Circular A-4's discussion of the Ramsey approach at this time, while noting the potential utility of model-based approaches that endogenize the discount rate. OMB believes that this balances the potential virtues of using such model-based approaches in appropriate circumstances with the virtues of setting a default approach to discounting that is descriptive and reflects real interest rate data in the Treasury and TIPS markets" (OMB, 2023c, p.89).

Model uncertainty in the private sector can be broadly categorized into two distinct forms. First, there are a range of models that corporations commonly apply alongside the CAPM to estimate their cost of equity capital. For example, Graham (2022) reports that over the last 20 years, there has been an increased move by large companies to also estimate the cost of equity using multi-factor models, such as the Arbitrage Pricing Theory, which have yet another representation for π_t without deviating from the general framework of equation 1. Real Option frameworks may also be used, which go beyond standard NPV analysis. In addition, within the confines of the CAPM, there is parameter uncertainty over the appropriate values to use for the equity premium and nominal risk-free rate, which in turn determine the variables A, B within the pricing kernel $\pi_t = A + Br_{mt}$. In particular, following Mehra & Prescott (1985), there has been a very extensive literature that aims to estimate the value of the equity premiums with no consensus yet reached on this point. For example, in a survey with over 1,500 responses from academics, analysts and corporate managers. Fernandez et al. (2022) report that estimates of the market risk premium in the United States ranged from 3% to 15% with a mean of 5.6%; see also Dahlquist & Ibert (2024). Corporations may therefore perturb their chosen value of the equity premium when calculating NPVs beyond the central one.

4 Discounting in USEPA (2023)

We now turn in more detail to the 'dynamic discounting' approach introduced by Newell, Pizer & Prest (2022).⁸ This helped provide the economic foundations for both an impactful interdisciplinary study on the Social Cost of Carbon (SCC) published in *Nature* by Rennert *et al.* (2022) and, crucially, for the US Environmental Protection Agency's recently updated estimates of the social costs of carbon and other greenhouse gas emissions for policy purposes (USEPA, 2023). These recommendations are not just of relevance in the United States but are also likely to be influential in the setting of future policies in other jurisdictions. The mathematical foundation for this approach is given in equation 2.5.3 in USEPA (2023):

⁸The term 'dynamic discounting' was first used to describe this approach in the draft version of the current updated guidance: USEPA (2022).

$$SCC = E\left[\int_0^T e^{-t(\rho + \eta g_t)} d_t dt\right],\tag{3}$$

where d_t is the monetized flow of damages today and in the future that result from emitting one additional ton of CO₂ into the atmosphere today. This has clear interpretation within the context of our general pricing framework, with the pricing kernel $\pi_t = \exp(-t(\rho + \eta g_t))$ as is standard for a consumption-based asset pricing model. Therefore, by applying equation 1 with this pricing kernel, a value for SCC can be derived that is identical to those reported from equation 3. This framework allows for both risk premiums (through the correlation between π_t and d_t) and declining discount rates. However, rather than applying equation 1, Newell, Pizer & Prest (2022) instead discretize time and then take the mean across a large number of simulations, S:

$$SCC = \frac{1}{S} \sum_{s=1}^{S} \sum_{t=1}^{T} e^{-t(\rho + \eta g_{st})} d_{st}.$$
(4)

The dynamic discounting approach implicitly rephrases this as:

$$SCC = \frac{1}{S} \sum_{s=1}^{S} SCC_s, \quad \text{where} \quad SCC_s = \sum_{t=1}^{T} e^{-r_{st}t} d_{st} \quad \text{and} \quad r_{st} = \rho + \eta g_{st}.$$
(5)

The specific sub-case when $\eta = 0$ is called 'constant discounting' by Newell, Pizer & Prest (2022) and is used as a contrast with their more general dynamic discount rate approach. While equation 5 gives an identical estimate of the SCC to equation 1, it also produces S interim values of r_{st} and SCC_s in a way that equation 1 does not. This interim information is used in Newell, Pizer & Prest (2022), where Table 2 reports median as well as mean values of SCC_s. Figure 5 plots all values of SCC_s against g_{st} and the associated text states "95% of all SCC draws fall between \$20 and \$220 per ton when using our discounting rule" (p.1042). Rennert *et al.* (2022) refer to interquantile ranges of SCC_s in a number of places; for example, "Our mean SC-CO₂ estimate using the preferred discounting scheme is \$185 per tCO₂ (\$44 - \$413 per tCO₂: 5% - 95% range, in 2020 US dollars)" (p.689). Figure 3.1.1 in USEPA (2023) also gives interquantile ranges for the Social Cost of Carbon for 2030 based on different climate damage models.

In the nomenclature of this paper, but not Newell, Pizer & Prest (2022), the interquantile ranges of SCC_s and the associated r_{st} for any given (ρ, η) pair reflect what we have called sensitivity analysis because they depend on quantifiable uncertainty concerning d_{st} and also here g_{st} . As we foreshadowed in the Introduction and Section 2, this extends our definition of 'NPV sensitivity analysis' because it jointly allows for perturbations in future net benefits (through d_t) and the discount rate (through g_t) in ways that are internally consistent. As Newell, Pizer & Prest (2022), observe (s subscripts added here for consistency of notation), "... future economic growth is uncertain, meaning g_{st} is a random variable. This in turn implies that the discount rate, r_{st} , is also a random variable so long as $\eta \neq 0$ " (p.1019).⁹ Such stochasticity is clear from equation 5,

⁹A distinction is that Newell, Pizer & Prest (2022) is referring to single-period growth and discount rates, while

where r_{st} varies with g_{st} which is the *ex-post* realization of growth along simulation run s.

In terms of model uncertainty, in order to derive empirical values for SCC_s it is necessary to calibrate equation 5. Following from recommendations made by the National Academy of Sciences, Engineering & Medicine (NASEM, 2017), Newell, Pizer & Prest (2022) apply a 'hybrid approach' to discounting that calibrates the normative Ramsey equation through the positive use of market interest rates. Parameter values for (ρ, η) are estimated through the following equation:

$$(\hat{\rho}, \hat{\eta}) = \arg\min_{\rho \ge 0, \eta} \sum_{t=2021}^{2300} \left(r_t^{ce} - \left(\rho - \frac{1}{t} \ln \left(E \left[e^{-\sum_{\tau=1}^t \eta G_\tau} \right] \right) \right) \right)^2.$$
(6)

This requires three key inputs. The single-period growth process, $G_{\tau} = \ln(c_{\tau}/c_{\tau-1})$, is estimated in a way that is consistent with the stochastic process in Müller, Stock & Watson (2022). The longterm process for r_t^{ce} is then set in a way that is consistent with the declining term structure of Bauer & Rudebusch (2023), which is estimated based on econometric models of historic Treasury bond yields. Trial values of η are applied, with the associated value of ρ in each case chosen so that the modelled near-term rate is close to a specified value, which is the third input. The final pair, (η, ρ) , is the one that minimises the loss function in equation 6.

What is of particular relevance to this paper is that, rather than deciding on a single short-term rate on which to estimate (ρ, η) , the studies give a range of possible values instead. Newell, Pizer & Prest (2022, p.1030) use 1.5%, 2%, 3% and 5%. Rennert et al. (2022, Figure 2) use 1.5%, 2%, 2.5% and 3%. USEPA (2023, Table ES.1.) use 1.5%, 2% and 2.5%. These give four possible pairs of (ρ, η) for both Newell, Pizer & Prest (2022) and Rennert *et al.* (2022), and three pairs for USEPA (2023). Figure 2 in Rennert et al. (2022) is particularly clear in this regard as it shows the distribution of SCC_s for each of the four pair values for (ρ, η) based on the four different short-term rates to which these are calibrated, with Figure 3.1.1 in USEPA (2023) sharing many of these features. While Newell, Pizer & Prest (2022, p.1028) refer to their highest (5%) and lowest (1.5%) rates as 'sensitivity cases', within the nomenclature of this paper, they instead reflect model uncertainty because they impact the pricing kernel through (ρ, η) , which is independent of the specific pricing case of the social cost of greenhouse gas emissions. Further academic support for this practice of reporting non-central values in USEPA (2023) is given by Fraas et al. (2023), who state that "Uncertainties around input parameters should be traced through to SC-GHG estimates" (*ibid.*, p.201) for sensitivity analysis and "In the near term, adopt constant rates of 3% and 5% for computing SC-GHGs" (*ibid.*, p.195) for model uncertainty.

5 The case against reporting multiple NPVs

Having laid out the theoretical framework of NPV analysis and shown how this is applied within public and private sector BCA practices, we next critique the reporting of more than one NPV.

In relation to 'NPV sensitivity analysis', the present value of an asset is a single real number; a

this discussion concerns the t-period discount rate. This does not alter the substance of the argument.

scalar. This is because it is calculated as the expectation of the product of two random variables: $p_{it} = E[d_{it}\pi_t]$ where $\pi_t = \exp(-t(\rho + \eta g_t))$ or $\pi_t = A + Br_{mt}$ in the core models that we consider. An expectations operator converts stochastic variables into a single real number. This point can also be seen through the more general discounting formula, $p_{it} = E[d_{it}] \exp(-tr_{it})$. The term $E[d_{it}]$ is a scalar because it is an expected value, and so is the discount factor because the discount rate itself is a single real number. In the presence of uncertainty, $r_{ft} = -(1/t) \ln(E[\pi_t])$, which includes the *ex-ante* expectation, rather than *ex post* realization, of π_t . If the single-period growth rate $G_{\tau} = \ln(c_{\tau}/c_{\tau-1}) \sim N(\mu, \sigma^2)$ for all τ and G_{τ} is not autocorrelated, then it is easily shown that the constant risk-free discount rate is given by $r_{ft} = \rho + \eta \mu - 0.5\eta^2 \sigma^2$; see, for example, Gollier (2011).¹⁰ The risk-free discount rate is a function of an *ex-ante* expectation and variance of a random variable. Therefore, r_{ft} in itself is not stochastic, a characteristic which also holds true for r_{it} which equals r_{ft} plus a non-stochastic risk premium as given in equation 1. Similarly, the value of the SCC, from equation 3, is the expected value of a function of random variables; it is an *expected* present value. As with r_{ft} and r_{it} , such an expected value is not, in and of itself, stochastic. It is just one number.

There is no concept of 'sensitivity' or interquantile ranges in relation to scalars. One cannot ask someone to undertake sensitivity analysis around the number 38, for example, or enquire what its interquantile ranges are. Because of this we make our core observation in relation to NPV sensitivity analysis: 'NPV sensitivity analysis' values and interquantile ranges cannot be economically interpreted as sensitivities and interquantile ranges around the central NPV.

Instead, practitioners using such an approach must believe that these values have a different, and investment decision relevant, economic interpretation. For example, it is tempting to conclude that, if the range of values $\{d_{it}^s\}$ give a distribution of possible asset outcomes, then their associated NPVs give a probability distribution of the potential *ex-post* welfare effects from investing in the asset. Under such an interpretation, NPV sensitivity analysis would allow for a broader range of questions to be answered than just "to what extent will this asset change expected welfare?", including: "will this asset increase welfare with more than 50% probability?", or the related question "which of these two assets has the higher probability of giving greater future welfare?". There may also be interest in analyzing the bottom quartile (say) of the welfare probability distribution so that assets that may present the greatest threats to welfare are avoided.

Yet a range of issues arise when asking these questions. These can be divided into two broad categories which we consider in the next two subsections: (i) how useful is information about *ex-post* welfare quantiles in the *ex-ante* BCA decision-making process?, and (ii) do NPV sensitivity analysis values actually reveal information about welfare quantiles? We then more tangibly illustrate these difficulties in Subsection 5.3 through a numerical example.

¹⁰Autocorrelated consumption growth leads to a non-flat, and generally declining, term structure of discount rates. See, for example, Cropper *et al.* (2014). Note that $g_t = (1/t) \sum_{\tau=1}^t G_{\tau}$.

5.1 The usefulness of welfare quantile information

There are three broad potential pitfalls that may arise when incorporating additional information about the probability distribution of *ex-post* welfare outcomes, beyond the mean of the distribution, into an *ex-ante* BCA capital investment process.

5.1.1 The potential double-counting of risk

If the correct pricing kernel, as given by equation 1, has been applied, then the central present value already accounts for the uncertainty over the *ex-post* welfare outcomes and society's/shareholdes' aversion to such uncertainty. To then additionally consider other properties of the welfare distribution, such as its quantile values, may easily result in a simple double-counting of risk. This issue is particularly manifest in the USEPA (2023) approach. Newell, Pizer & Prest (2022) note that allowing for the correlation between damages and economic growth, compared to constant discounting when $\eta = 0$, "...is not only important for determining the expected value of the SCC, but it also reduces the overall uncertainty around it" (p.1044). Yet they do not simultaneously explicitly recognize that risk has already been fully accounted for in their central estimate of the SCC because equation 4 allows for the correlation between d_t and g_t .

5.1.2 Aggregation within the organization

One of the key properties of central NPVs is that they are additive. As a consequence, the present value of a project is the sum of the present values of the component parts of the project; the present value of an organization is the sum of the present values of the assets in the organization; the present value of a portfolio with holdings in a range of organizations is the sum of the present value of each holding. This follows immediately from the mathematical result that, for any stochastic variables x, y, E[x + y] = E[x] + E[y]. But this additivity property, which is trivial for expectations, does not in general, hold for many other statistical properties of the distribution. And, specifically, it does not generally hold for quantiles, including the median. The quantile risk to the organization cannot be deduced from aggregating individual quantiles that arise from different NPV sensitivity analyses. This introduces a range of additional complexities for the organization, including the need to decide at what scale the sensitivity analysis should be undertaken to ensure that the welfare of society or shareholders are being protected through BCA analysis.

5.1.3 Comparison with external projects

If an analyst is undertaking NPV sensitivity analysis, they will need to consider which assets to include in their workings. If they are considering a binary 'invest or not' decision in one asset, they are likely to only consider sensitivity analysis with respect to that particular asset. Similarly, if they are choosing between two different assets, they may run sensitivity analysis on both. But one of the key purposes of BCA is to compare opportunities in the organization with those elsewhere in the economy. By benchmarking against the risk-free asset and market portfolio in the CAPM, the capital allocation process in the private sector explicitly considers whether shareholders would be better served by passively investing their capital in a well-diversified tracker fund instead of *any* asset within the firm. Similar reasoning lies behind the SOC approach to social discounting. Therefore the analyst must consider whether this induces a requirement to also include external assets in the sensitivity analysis and, if so, which ones.

This presents a further theoretical problem. The two-fund portfolio separation theorem of Sharpe (1964) establishes that, for mean-variance decision-makers, the market portfolio and the risk-free asset are the *only* external assets that need to be considered in a BCA. But this argument applies to the central NPV only and not to NPV sensitivities. Given this, even if the market portfolio and risk-free asset were included in the sensitivity analysis, we know of no theory that would reassure us that this was sufficient. Similarly, within the SOC approach, it is not clear which external assets to include within the sensitivity analysis.

5.2 The welfare implications of NPV sensitivity analysis

Even if an organization continues to believe that welfare quantile information is useful, the question arises as to whether NPV sensitivity values and interquantile ranges can be sensibly interpreted in the context of welfare. This issue revolves around the choice of the discount rate. While the pricing kernel framework makes clear, through equation 1, what the appropriate discount rate should be for the central NPV, it does not establish that the same rate should be used for NPV sensitivity analysis. Given this theoretical gap, there are three choices that may seem intuitively suitable to apply to the multiple different values d_{it}^s . First, since each is a single outcome, one possibility would be to use the risk-free rate r_{ft} . A second choice might be to apply the standard discount rate, given by equation 1, that is used for the calculation of the central NPV. A third option for the discount rate is to mimic the process used in USEPA (2023).

Yet all of these choices of discount rates for NPV sensitivity analysis raise important questions. If a risk-free discount rate is applied, then this is theoretically the correct choice for determining the present value of the asset assuming that there is only one possible asset outcome. This, though, immediately contradicts the reason for why sensitivity analysis is required in the first place. If r_{it} is a risk-adjusted discount rate, then, from equation 1, the risk-adjustment accounts for the covariance between d_{it} and π_t . But such a covariance must equal zero between a single realization, d_{it}^s , and π_t . Therefore applying a risk-adjusted discount rate to individual outcomes in NPV sensitivity analysis is theoretically inconsistent. Finally, if the discount rate is based on USEPA (2023), then using *ex-post* values of g_{st} rather than *ex-ante* means, variances and covariances, r_{st} and SCC_s can be interpreted as the perfect hindsight values that the policy maker wishes, at time t = T, she had enacted at time t = 0 contingent on state s being the actual path that the economy takes. This is reminiscent of the scenario-dependent discount rates in Weitzman (1998). However, as Weitzman makes clear (*ibid.*, p.204), it is the expected present value that is of relevance to the policy maker before the relevant scenario has been revealed.¹¹ Therefore, while all three of these discount rate

¹¹In addition, Weitzman models the immediate resolution of uncertainty, which is not consistent with the way that

choices seem intuitively reasonable, each is problematic and lack the sound theoretical foundations that would robustly tie them to welfare outcomes.

5.3 A simple numerical example

We illustrate the issues raised in the previous two subsections through a simple numerical example. Consider an organization that is choosing between four different possible investment opportunities, each of which will cost an initial amount I = \$99.1 to acquire. In all cases, the organization will divest of its investment after one time period (t = 1). The first asset is risk-free and will pay off \$100 with certainty next period. Alternatively, the organization can invest in any one of three risky assets. There is a general, financial market tradable, well diversified market portfolio available to all investors, and two primary assets that are available only to this organization.

The analyst who is responsible for the investment appraisal decides to undertake NPV sensitivity analysis based on ten potential future states of the world and these are described in Table 2. This includes some low probability scenarios with relatively significant changes in consumption (both positive and negative) together with states that are more likely but involve a smoother consumption transition. The analyst's estimate of how much the initial \$99.1 will be worth in one period if placed in the market portfolio and the two organization-specific assets, contingent on the future state of the world, are also given in this Table 2.

		Macr	oeconomic	Asset-	related
Scenario	Probability	g_t	Market	Asset 1	Asset 2
1	2%	20.00%	155	134	82
2	3%	17.00%	132	118	89
3	10%	12.00%	125	120	111
4	15%	8.00%	105	92	90
5	20%	3.00%	113	100	75
6	20%	0.00%	87	71	135
7	15%	-5.00%	85	127	90
8	10%	-9.00%	83	116	98
9	3%	-12.00%	75	66	113
10	2%	-20.00%	69	57	120

Table 2: An example of NPV sensitivity analysis. Ten possible future scenarios are presented with different macro-economic and asset-specific outcomes and different probabilities associated with each state. g_t refers to real per-capita consumption growth.

Table 3 provides summary statistics on consumption growth, g_t , and the values of the market portfolio and two assets one time period from now. Consumption is expected to grow at 1.5%, with

stochasticity evolves under the consumption growth process of Müller, Stock & Watson (2022) that is employed by Newell, Pizer & Prest (2022).

standard deviation of 8.0%, and all three assets have the same expected future value as the risk-free asset next period (to within a small rounding error): \$100.0. The correlation matrix shows that the market portfolio and Asset 1 are pro-cyclical, while Asset 2 is counter-cyclical.

	Expected	Standard		Correla	tion matri	x
	Value	Deviation	g_t	Market	Asset 1	Asset 2
g_t	1.5%	8.0%	100%			
Market	100.0	18.0	90%	100%		
Asset 1	100.0	21.4	19%	36%	100%	
Asset 2	100.0	20.9	-18%	-43%	-59%	100%

Table 3: Summary statistics for the macroeconomic and asset-specific variables.

The consumption growth information provided in Table 3 is sufficient to calculate the pricing kernel for each scenario, s, $\pi_{st} = \exp(-(\rho + \eta g_{st}))$, under the Consumption CAPM (CCAPM) approach. We set $\rho = 1.5\%$ and $\eta = 5.^{12}$ Because the probabilities of each state, θ_s , are known, we can calculate the risk-free rate, $r_{ft} = -\ln(E[\pi_t]) = -\ln(\sum_{s=1}^{10} \theta_s \pi_{st})$, and the central present value of the four investment opportunities $E[d_{it}\pi_t] = \sum_{s=1}^{10} \theta_s d_{it}^s \pi_{st}$ under the CCAPM. These are given in the second column of Table 4. The present value of the risk-free asset is \$99.1 = I, making this a zero NPV opportunity. By contrast, the present value of the market portfolio and Asset 1 are below the initial amount invested, making these negative NPV because of their pro-cyclical nature and the fact they have the same expected value next period as the risk-free asset. Asset 2 is positive NPV, by contrast, because it is counter-cyclical.

The third column of Table 4 shows that, when the risk-free asset and market porfolio are priced through the CCAPM, the endogenized risk-free rate and equity premium lie within the range of what is generally considered reasonable. Based on these values, we can also calculate the central present values of the two organization-specific assets using the market-based CAPM. These are given in the fourth column of Table 4. This again shows that Asset 1 is negative NPV, while Asset 2 is positive NPV. The ranking of the projects, based on the extent to which the investment opportunities are expected to increase either expected shareholder utility or social welfare are clear: Asset $2 \succ \text{Risk-free asset} \succ \text{Asset } 1 \succ \text{Market Portfolio.}$

5.3.1 NPV sensitivity analysis on this example

Table 5 reports the discounted present values for each of the ten scenarios under risk-free, riskadjusted (CCAPM), and USEPA (2023) discounting methods.¹³ For the last of these cases, we

¹²The standard deviation of consumption growth, at 8.0%, and the value of $\eta = 5$, are both higher than standard estimates to overcome the equity premium and risk-free rate puzzles.

¹³For Assets 1 and 2, the results from risk-adjusted discounting are largely unaltered if we use the CAPM rather than CCAPM. The CAPM cannot, though, be used to discount the market portfolio because the equity premium is exogenous within this model.

	Cons	umption CAPM	I	Market-based CA	PM
	PV	Discount rate	\mathbf{PV}	Discount rate	Beta
Market portfolio Risk-free asset	$92.9 \\ 99.1$	$7.31\%\ 0.88\%$			
Asset 1 Asset 2	$97.6 \\ 100.7$	2.38%- $0.68%$	96.5 102.2	$3.63\% \\ -2.16\%$	0.409 -0.454

Table 4: The present value (PV) and discount rate for the four investment opportunities. The Consumption CAPM (CCAPM) has pricing kernel given by $\pi_t = \exp(-(\rho + \eta g_t))$ with $\rho = 1.5\%$ and $\eta = 5$. The price and discount rate of the two assets are given based both on the CCAPM and markets-based CAPM where the equity premium and risk-free rate are derived from the top part of the table. Note that the CAPM is based on simple returns, while the CCAPM is based on logarithmic returns.

calculate the risk-free rate as given by equation 5, $r_{st} = \rho + \eta g_{st}$, depending on the consumption growth rate in that scenario, s. For example, in Scenario 10, $r_{10t} = 0.015 + 5 \times -0.2 = -0.985$ and the present value of the risk-free asset is therefore $267.8 = 100 \times \exp(-(-0.985))$ in this scenario. The difference between the present values calculated using this technique, and the volatility of the present value of Asset 2 in particular when compared against the undiscounted values, are of note.¹⁴

If the analyst has been asked to compare Assets 1 and 2 only, under risk-free discounting, Asset 1 has the higher present value in seven of the ten scenarios, with combined probability of 75%. This is the same under USEPA (2023) discounting because, in both cases, the same discount rate is applied to both assets for any given scenario. Therefore their ranking can be deduced directly from the undiscounted benefits given in Table 2. Under risk-adjusted discounting, Asset 2 now has a higher present value than Asset 1 in Scenario 4, which occurs with 15% probability.

If the analyst also includes the risk-free asset and the market portfolio within the sensitivity analysis, under risk-free discounting Asset 1 now only has the highest discounted value under two scenarios with combined probability of 25%. In Scenarios 1–5 (combined probability of 50%) where Asset 1 has a higher present value than Asset 2, the market portfolio's present value is greater than both. Again, these results hold for discounting under the USEPA (2023) procedure. With risk-adjusted discounting, all three risky assets have the highest present values in three scenarios (combined probability 35% for Asset 1, 25% for Asset 2 and the market portfolio). In Scenario 4, the present values of all three risky assets are below \$99.1, making the risk-free asset the highest present value in this case (15% probability).

What is clear from this is that the ranking of assets by which scenarios occur with greatest probability points to Asset 1 and the market portfolio being the best investment opportunities even though it has already been established in Table 4 that these assets will decrease expected

¹⁴If, for each asset, these values are multiplied by their associated probabilities, θ_s , and then summed across s, then this gives the CCAPM present value in Table 4 as implied by equation 5.

		Risk-1	free discou	uting	Risk-adj	justed disc	counting	ns	EPA disco	unting	
Scenario	Probability	Asset 1	Asset 2	Market	Asset 1	Asset 2	Market	Risk-free rate	Asset 1	Asset 2	Market
1	2%	132.83	81.28	153.64	130.84	82.56	144.07	101.50%	48.56	29.72	56.17
2	3%	116.97	88.22	130.84	115.22	89.60	122.69	86.50%	49.68	37.47	55.58
3	10%	118.95	110.03	123.90	117.17	111.75	116.18	61.50%	64.88	60.01	67.58
4	15%	91.19	89.21	104.08	89.83	90.61	97.59	41.50%	60.75	59.43	69.34
5 C	20%	99.12	74.34	112.01	97.64	75.51	105.03	16.50%	84.79	63.59	95.81
6	20%	70.38	133.82	86.24	69.33	135.92	80.86	1.50%	69.94	132.99	85.70
2	15%	125.89	89.21	84.25	124.01	90.61	79.00	-23.50%	160.64	113.84	107.52
×	10%	114.98	97.14	82.27	113.27	98.67	77.15	-43.50%	179.22	151.41	128.23
6	3%	65.42	112.01	74.34	64.44	113.77	69.71	-58.50%	118.47	202.83	134.62
10	2%	56.50	118.95	68.40	55.66	120.82	64.13	-98.50%	152.64	321.34	184.77
	÷	-	-	-	-	-	c -	L • F 35	•	-	

Table 5:	The	discounted	value	of (each	scenario-dependent	asset	net	oenefit,	$d^s_{it},$	under	risk-free,	risk-adjusted	(CCAPM),	and
USEPA(2	2023)-:	style discoun	ting.												

welfare. We also note that the inferences that are likely to be drawn from this type of analysis will differ depending on whether or not the project analyst includes external assets into the sensitivity analysis.

If, instead, the organization wanted to use NPV sensitivity analysis to consider the lowest 25% quantile (quartile) of potential outcomes, for Asset 1 that is a risk-free discounted present value of \$70.38 in Scenario 6 and \$88.22 for Asset 2 in Scenario 2. While this does point to Asset 2 being preferable to Asset 1, the risk-free asset guarantees a discounted payoff of \$99.1, making this the appropriate choice if the tail of the distribution is the greatest organizational concern. In addition, if the firm is able to find the resources to invest in both of Assets 1 and 2, then the bottom quartile outcome is not equal to the sum of these two values, \$158.60, but is instead higher at \$177.43 in Scenario 9. This illustrates the non-additivity of quantiles and that the level of aggregation therefore matters when sensitivity analysis is undertaken.

5.4 Model uncertainty

The situation is rather different when it comes to model uncertainty. For any given pricing kernel, $E[d_{it}\pi_t]$ can be calculated to give a unique real number for p_{it} . However, while the existence of π_t is a mathematical truth under the assumptions given in the Appendix, it may take different forms. To take an example, within a normative setting, agents may differ in their views about normative parameters (ρ, η) within the same model or about the general approach to intergenerational distribution; see, for example, Drupp et al. (2018) and Nesje et al. (2023). As emphasised by Freeman & Groom (2015) and Heal & Millner (2014), such differences are ethical in nature and cannot be integrated out under the expectations operator. While any thoughtful organization will recognise the difficulties involved in identifying the pricing kernel, this variable is does not depend on the individual asset being appraised. For this reason, it is not clear what benefit is gained by sharing these difficulties with individual project analysts whose expertise relates to the asset being assessed, not the broader economic principles that underlie the identification of π_t . Therefore, both because of the expertise of the people involved, and to ensure consistency across appraisals, model uncertainty concerning π_t should be resolved at the institutional level before guidance is given to individual analysts. This leads to our core observation in relation to model uncertainty: There is a correct level within the organizational hierarchy where the pricing kernel should be specified. This level lies above the individual project analyst. The same pricing kernel should be applied consistently across all BCA exercises within the same organization.

There is some evidence that incorporating model uncertainty has actively hindered policymaking. Under the previous version of *Circular A-4*, both 3% and 7% rates needed to be applied. As Sunstein (2014) noted, "...as a general rule, the choice between 7% and 3% has not mattered to the ultimate decision about whether and how to proceed" (p.200). This rather made the 3% rate redundant, if not necessarily unhelpful to appraisals conducted at that time. Yet, in the case of estimates of the Social Cost of Carbon, which are highly sensitive to the discount rate because of the many centuries that carbon dioxide stays in the atmosphere, having two rates was demonstrably problematic. Under the Obama administration, and re-introduced under the Biden administration, the lower 3% rate was applied. The first Trump administration, by contrast, applied the higher 7% rate, which was one important reason why that administration estimated a much lower estimate of the Social Cost of Carbon: $3-5t/CO_2$ under Trump compared to $43t/CO_2$ under Biden according to Stanford News. That the OMB was unable to decide on a single rate did, in this instance, prove unhelpful in providing policy guidance that transitioned smoothly across administrations. The current version of *Circular A-4* now focuses on a single discount rate of 2%.

If an organization continues to give project analysts a range of discount rates that should be used, it should clarify what conclusions should be drawn if the sign and/or ordering of NPV calculations differ depending on the different rates. For example, HM Treasury's (2022a) Green Book says for intergenerational discounting which applies two different values of ρ (referred to here as 'STPR'), "When applying this approach the Net Present Social Value (NPSV) using the standard STPR and the reduced rate STPR should both be included in the results of the appraisal and explained clearly. The difference between these two estimates of NPSV provides an estimate of the intergenerational wealth transfer attributable to pure social time preference which should be part of the explanation of the approach" (Section 14.5). However, it does not make clear the practical use of understanding this difference for making policy choices and nor does it ensure that there is consistency across different appraisals.

6 Why is NPV sensitivity analysis still recommended?

Given the arguments that we have laid out above, the question arises as to why the reporting of multiple NPVs remains so widely recommended. While all the examples that we have considered have recommend this practice to assist in decision making in ways that capture both sensitivity analysis and model uncertainty, they tend not to provide a theoretical argument, or references, to support it. Indeed as Liu (2022) notes "...sensitivity analysis on the NPV method is seldom studied in the existing literature" (*ibid.*, p.1861) with some studies being limited to showing how such methods can be implemented (e.g., Mangiero, & Kraten, 2017). To the best of our knowledge, the most rigorous work in this field is around strong NPV-consistent metrics (see, for example, Magni & Marchioni, 2018, 2020), but this literature does not provide a case for why NPV sensitivity analysis should be undertaken in the first place. The lack of formal support for reporting multiple NPVs contrasts with the otherwise rigorous policy and practice recommendations that are contained more generally in governmental guidance on BCA. We present a range of potential reasons that may provide something of an explanation.

6.1 The history of NPV sensitivity analysis

We first conjecture that the way in which risk and uncertainty were historically incorporated into BCA continues to play a key role in the continued widespread practice of NPV sensitivity analysis.¹⁵

¹⁵We greatly thank Diderik Lund for pointing us in this direction.

The first US consolidated Federal guidance on discounting was contained in the initial publication of *Circular A-94* (Congressional Record, 1969).¹⁶ Although this was a very short document, NPV sensitivity analysis was already a central component of what was expected, and it was explicit that risk-adjustment should not be in the discount rate:

"6. Uncertainty and risk. Since future events cannot be predicted with certainty, yearly costs and benefits actually realized in the future are liable to differ from the values expected for them in the present. In some cases, the range of variation can be anticipated and the sensitivity of proposed programs or projects to future contingencies can be evaluated.

a. The 'most likely' estimates of expected yearly costs and benefits should be supplemented with minimum and maximum estamates. Present value total cost and benefit should be calculated for each of these estimates. The likelihood that each of the possible benefit and cost estimates will occur also should be discussed.

b. Uncertainty and risk should be treated explicitly in alternative calculations of expected yearly benefits and costs. Generally, uncertainty should not be reflected in the discount rate." (*ibid.*)

Similarly, for UK social BCA, the earliest version of the document that ultimately transitioned into the *Green Book* also stressed the importance of sensitivity analysis by noting that "... where there is any doubt about what assumptions ought to be built into the calculations (eg the timing of particular inputs or outputs, technological standards, or even the correct figures for some of the basic data) one must see how 'sensitive' the result is to changes in any of the assumptions" (HM Treasury, 1973, p.6).¹⁷ This also reflects contemporaneous best practice in the corporate sector, where the widespread take-up of discounted cash flow methods occurred between the late 1950s and mid 1980s (e.g., Klammer 1972; Pike 1996; Haka, 2007). Mao (1970) gives a fascinating qualitative description of best practice at a time when payback period and accounting rates of return were still preferred to discounting cashflow techniques in major firms: "In some companies, three sets of figures are forecast: optimistic, pessimistic, and most likely. The optimistic and pessimistic figures denote the range of possibilities. The 'most likely' figure does not mean the mode: it is a conservative estimate which the executives consider as having a probability of about .75 of being attained or exceeded" (*ibid*, p. 356).

All of these uses of multiple values significantly preceded the application of formal risk-adjusted discount rates or their mathematical counterpart, certainty equivalents. Indeed, these only explicitly came into use in US Federal guidance in the most recent versions of *Circular A-4*. In the UK,

¹⁶This followed from the recommendations of the Joint Economics Committee (1968) of Congress, which had reported that "... there exists no consensus among agencies on the appropriate interest rate concept for discounting the future streams of benefits and costs of public investments" (p.3) and that "... the interest rates applied by agencies in discounting their proposed investment alternatives range from a minimum of 0 percent to a maximum of 20 percent" (p.3).

¹⁷See Spackman (2013) for an expert review of the historical development of social discounting practices in the UK.

the use of the CAPM was considered and rejected for use within nationalised industries in 1979, while the recognition of systematic risk was recognised for the first time in the 1984 version of the *Green Book* (Spackman, 2013). For corporate discounting, by the time of a 1980 survey of Fortune 1000 firms, Gitman & Mercurio (1982, p.26) report that "...it is interesting to note that, except for sensitivity analysis, the majority of respondent firms do *not* employ the techniques listed" (their emphasis). While 70% report using sensitivity analysis, under 30% report using the CAPM or beta. By the turn of the millennium, Graham & Harvey (2001) found that 73% of firms who calculate the cost of equity capital did so by using the CAPM, while about half reported using sensitivity analysis. In the UK, Pike (1996) found that, in the period between 1975 and 1992, the use of sensitivity analysis rose from 18% to 88% of firms. In the former year, no firm reported the use of (CAPM) beta analysis, while in the latter year only 20% of firms used beta, and most of those respondents said that they used it 'rarely' (*ibid.* p. 86).¹⁸

Given this evidence, we conjecture that the reporting of NPV sensitivity analysis became an organizational heuristic for the treatment of risk and uncertainty in public and private sector BCA before detailed economic models became sufficiently well established to be implemented in formal guidelines and best practice. Given the arguments that we have already presented, the continued use and development of such techniques can, we believe, be broadly understood as a failure of "organizational unlearning".¹⁹

6.2 Sensitivity analysis during the BCA process

It has been suggested to us that the reason why NPV sensitivity analysis is so prevalent is because it is useful during the process of constructing the NPV itself. All organizations struggle from a lack of resources, including those needed to undertake an absolutely thorough BCA exercise. For this reason, it is accepted that the time and effort put into such appraisals should not be excessive. As an example, the Financial Conduct Authority in the UK is required to undertake BCAs of its proposed policies by the Financial Services & Markets Act (2000), but in a way that is explicitly "...proportionate to the size, cost, strategic priority and risk of the intervention" (FCA, 2024, p.3). Such considerations of proportionality makes managers assess where their limited analytical resources can best be deployed. This requires them to identify the factors that are most likely to affect the final success of the asset, with NPV sensitivity analysis being a highly useful tool for this task. Time and effort can then be concentrated on ensuring that the estimates of the most critical variables are as accurate as possible. Having done the NPV sensitivity analysis as part of the process, it is then easy to include in the final report.

We can trace this line of reasoning back to the original HM Treasury (1973) document in the UK: "If a change does not greatly affect the result, the uncertainty can be safely ignored. If it does, then one needs to think hard about the reliability of the doubtful figures, and if necessary to re-

 $^{^{18}}$ More informal use of higher rates for riskier projects was also prevalent during this period, rising from around 40% of firms in 1975 to 65% of firms by 1992.

¹⁹For recent reviews of the literatue on organizational unlearning, see, for example, Klammer & Gueldenberg (2019), Sharma & Lenke (2021), and Kluge (2023).

check them or to get fresh advice from a qualified source." (*ibid.*, p.6), as well as much more recent guidance in the United States: "If benefit or cost estimates depend heavily on particular inputs, approaches, or assumptions, it is often informative to make those details explicit and carry out sensitivity analyses using plausible alternatives. If the value of net benefits changes from positive to negative (or vice versa) or if the relative ranking of regulatory options changes with alternative plausible inputs, approaches, or assumptions, you should generally consider conducting further analysis to inform the determination of which of the alternatives is more appropriate" (OIRA, 2023b, pp.74–75; see also OMB, 2003, p.42).

There is a key distinction, though, between using NPV sensitivity analysis as part of the BCA process in order to report the most accurate central NPV possible given the organizational resource constraints, compared to reporting the sensitivity analysis itself at the end of the process. The former is distinct from the practices that we have critiqued earlier, while our arguments apply to the latter. Because it is useful as part of the process does not automatically imply that capital allocation decisions should be based on any reported NPV other than the central one.

6.3 Knightian uncertainty

Another reason that practitioners have suggested to us for the continued use of NPV sensitivity analysis is that they believe it is too difficult in practice to explicitly assign probabilities to different scenarios. This is captured in the current version of *Circular A-4*, which says that "Sensitivity analysis is especially valuable when the information is lacking to carry out a formal probabilistic simulation" (OIRA, 2023b, p.71). This corresponds to the right-hand box in the first row of Table 1. There are two possibilities to consider here. The first is that there are genuine Knightian uncertainties, where the probabilities are genuinely 'unknowable'. Yet, in such cases, without the identification of a single formal probability space, the underlying theoretical justification that underlies the use of NPVs, and which we have reviewed above, is no longer applicable. Instead, it is necessary to deploy some other decision theory to handle this uncertainty, such as minimax regret discounting (Iverson, 2013) in the case that only the support of probabilities is known.²⁰ Alternatively, it may be that practitioners believe that it is just too difficult and time-consuming to estimate the probabilities. This may be the case, but we stress that, unless the probability space is identified, it is not clear on what theoretical basis even the central NPV is being calculated.

6.4 The reassurance of quantification

Given the key role played by the central NPV, it is both relatively straightforward and enticing to report other values as well. Because what is certain is that the central NPV alone is insufficient to make good investment, policy, and regulatory choices. There are many reasons why an organization may decide to deviate from the NPV decision rule. In a for-profit context, a poorly performing asset may impact on a firm's liquidity and hence its ultimate survival while, in the public sector, a failed

²⁰This can also lead to an identified single economic value for the asset without requiring sensitivity analysis, but such techniques lie well beyond the types of appraisal that are commonly applied in practice.

project or policy might have adverse political consequences, or a project may have many benefits that are difficult to quantify.²¹ This need to go beyond the central NPV is widely recognised in practice. As *Circular A-4* makes clear: "You will find that you cannot conduct a good regulatory analysis according to a formula. Conducting high-quality analysis requires competent professional judgment" (OIRA, 2023b, p.4). As a consequence, as the OMB (2023b) *Circular A-94* guidance stresses, the NPV rule should only 'generally' be followed. The *Green Book* (HM Treasury, 2022a) also makes clear that the NPV process is only one part of a good BCA.

Yet, the binary nature of the investment process puts a number of pressures on the management of an organization. For example, in a US Federal context, a proposed regulation can proceed "only if the benefits justify the costs and only if the chosen approach maximizes net benefits (unless the law requires otherwise)" (Sunstein, 2014, p.170) and this, at least in principle, makes BCA decisions subject to regulatory review.²² Under such scrutiny, managers may seek reassurance in having their decisions backed by a battery of quantitative information, including NPV sensitivity analysis.

In reality, though, the choice of whether to invest in an asset or not is, ultimately, taken by people who are very likely to be driven by organizational pressures and qualitative judgments as well as empirical values. This has been particularly clearly described by Graham (2022), who notes that "NPV is often used to support decisions that are driven by other strategic or qualitative objectives". Graham, Harvey & Puri (2015, pp. 463–466) also make clear the 'messy' nature of real world capital allocation processes:

"NPV ranking is the most important factor that affects capital allocation across divisions within U.S. firms ... The second most important factor affecting capital allocation in the United States is the reputation of the divisional manager ... CEOs focus more on the person (i.e., the manager's reputation), rather than on the division the manager represents ... Divisional manager confidence (in whether the project will succeed) has great influence on corporate capital allocation decisions ... Approximately half of CEOs list their "gut feel" as an important factor affecting how to allocate capital across divisions ... Overall, the popularity of "gut feel" emphasizes that decisions are made by people, not by corporate entities."

That investment decisions will ultimately be made based on professional experience and organizational politics may come uneasily to many who take comfort in the sense of scientific validity that is conferred by NPV sensitivity analysis and interquantile ranges. Ultimately, though, there is a simple danger of over-quantification.

 $^{^{21}}$ Sunstein (2014) gives a detailed example of this in the context concerning the dignity benefits of providing wheelchair access to public buildings.

²²There are restrictions on how this may be done, with Sunstein noting that "It is noteworthy, and revealing, that the Precautionary Principle does not appear in the governing Executive Orders; cost-benefit balancing is endorsed instead." (p.181). Considering the potential role for ranges of NPVs to influence policymaking in settings that are consistent with existing Executive Orders and yet distinct from the discounting framework of equation 1 is beyond the scope of this paper. Non-expected decision rules are also not common in existing guidance (Groom et al., 2022).

We stress that we believe that, in many situations, quantitative scenario analysis is likely to be helpful. For example, a corporation may wish to consider many potential outcomes for cash balance forecasting to ensure that there is not an unacceptable risk of insolvency. Central and non-central NPV calculations do not allow such questions to be answered. Our arguments apply only to sensitivity around the sum of the discounted values of future net benefits.

7 Conclusion

In this paper we have questioned whether the reporting of non-central NPVs helps in real world investment, policy, and regulatory decision making. Reporting such values is widely recommended in both the public and private sectors, although this practice conflates at least two separate concepts. First, future net cash flows are uncertain in ways that can be quantified, and perturbing these to derive non-central NPVs is what we have described as 'NPV sensitivity analysis'. We have shown that these results do not have clear theoretical interpretation in commonly used models. The NPV is an expected value and hence just a single real number. There is no concept of sensitivity when it comes to a single value. Alternatively, there may be more conceptual uncertainty about the form of the discount rate which cannot be integrated out under the expectations operator. We have referred to this as 'model uncertainty'. In this case, we have argued that such issues should be resolved into a single choice at an appropriate 'higher level' in the organizational hierarchy by those setting the overall guidance. If not, because the pricing kernel is economy-wide, it lies outside the expertise of individual project analysts. What is more, if different analysts make different choices on the pricing kernel that lie within the bounds of what the organization allows, then there may be inconsistent decision making across different appraisals and a 'gaming' of the appraisals process (Nesje and Lund, 2018).

These points, though, are theoretical. A case could be made for reporting non-central NPVs on the more practical grounds that they are useful. While we cannot establish that non-central NPVs are *never* helpful, to our knowledge there is no empirical evidence showing that such values generally improve real-world decision making. This contrasts with the need for scientific support that is generally imposed by organizations on their BCA methods. We have also argued that the US Federal choice of having 3% and 7% discount rates under the previous version of *Circular A-4* has proved a barrier to a smoothly-managed evolution of climate change mitigation regulations across administrations. On the basis of the evidence we have presented, the assumption that reporting non-central NPVs can only help the decision-maker rather than providing harmful noise may not be as innocuous as it at first appears.

The case that we have made still clearly allows for the use of additional information beyond the central NPV when making project, policy, and regulatory decisions. As almost all guidance makes clear — perhaps most explicitly in the UK Treasury's (2022a) *Green Book* — the NPV calculation is just one part of a good appraisals process. For example, it is almost certainly helpful for private sector companies with limited liquidity to undertake cash balance sensitivity analysis before investing in a major project. But such quantitative sensitivity analysis should not be confused with the information provided by non-central NPVs. In addition, we believe that qualitative management decision-making judgement should be more explicitly recognised and this may not always be aided by additional quantitative information. Questions of strategic fit, the ethical trade-offs between different choices, and the ability of a managerial team to deliver on implementation are amongst the many factors that will influence whether an organization's board will support an investment or regulatory decision. Such issues cannot be incorporated in any meaningful way into non-central NPVs.

Benefit-cost analysis and decision-making is a complex organizational process that requires quantitative inputs and qualitative management judgements. We believe it is important that, in all areas, there is precision around why additional information is provided and how that should be used in the decision-making process. This paper has shown that this is currently lacking in relation to the reporting of non-central NPVs. Further theoretical and empirical work is required if such reporting is to remain central to scientifically informed public and private sector benefit-cost analysis.

Appendix: Pricing kernels

The generality of the pricing equation $p_{it} = E[d_{it}\pi_t]$, where π_t is the pricing kernel for t that is the same for all i, can be traced back to the initial application of Hilbert spaces mathematics to financial economics by Harrison & Kreps (1979), Chamberlain & Rothschild (1983) and Chamberlain, (1983).²³ This framework is based around the fact that, in a given probability space, $(\Omega, \mathcal{F}, \mathcal{P})$, the set of random variables, \mathcal{L}^2 , with finite second moment $E[x^2] < \infty$ is known to be a Hilbert space with inner product $\langle x, y \rangle = E[xy]$ for $x, y \in \mathcal{L}^2$ and associated norm $||x|| = (E[x^2])^{1/2}$. Denote by $\mathcal{M} \subset \mathcal{L}^2$ the linear span of the payoffs from all assets with finite variance (the 'asset span'). Then \mathcal{M} is also a Hilbert space in its own right under the same inner product and norm. Within this Hilbert space, there exists a linear functional $p: \mathcal{M} \to \Re$ such that, for any scalars, a, b, p(ax+by) = ap(x) + bp(y) where $x, y \in \mathcal{M}$. The economic interpretation of this is that p(ax+by)is the value of a portfolio constructed from the two assets, with p(x), p(y) being the individual asset values and a, b representing the number of each asset held in the portfolio. By appyling the Riesz Representation Theorem to \mathcal{M} , it follows immediately that there exists at least one $\pi \in \mathcal{M}$, such that, for all $x \in \mathcal{M}$, $p(x) = \langle x, \pi \rangle = E[x\pi]$, with π being unique if and only if the market is complete. Therefore this result depends on only three assumptions: (i) that there is an identified probability space, (ii) that the payoff to assets have finite variance, and (iii) that portfolio values are linear in the prices of their underlying assets. Almost all asset pricing models can therefore be represented through this equation, with discussions over theoretical asset pricing amounting to disagreements over the functional form of π_t .

 $^{^{23}}$ LeRoy & Werner (2014, Chapter 17) and other theoretical asset pricing textbooks give a more complete account of this approach for readers without a formal background in functional analysis. More rigorous treatment is given in, for example, Hansen & Richard (1987).

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