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Efficient Use of Information and Social Value of Information*

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Abstract

This paper analyzes equilibrium and welfare for a tractable class of economies (games) with externalities, strategic complementarity or substitutability, and heterogeneous information. First, we characterize the *equilibrium use of information*; complementarity heightens the sensitivity of equilibrium actions to public information, raising aggregate volatility, whereas substitutability heightens the sensitivity to private information, raising cross-sectional dispersion. Next, we define and characterize an efficiency benchmark designed to address whether the equilibrium use of information is optimal from a social perspective; the *efficient use of information* reflects the social value of aligning choices across agents. Finally, we examine the comparative statics of equilibrium welfare with respect to the information structure; the *social value of information* is best understood by classifying economies according to the inefficiency, if any, in the equilibrium use of information. We conclude with a few applications, including production externalities, beauty contests, business cycles, and large Cournot and Bertrand games.

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1 Introduction

Many environments—including economies with network externalities, incomplete financial markets, or monopolistic competition—feature a coordination motive: an agent’s optimal action depends not only on his expectation of exogenous “fundamentals” but also on his expectation of other agents’ actions. Furthermore, different agents may have different information about the fundamentals and hence different beliefs about other agents’ actions. Although the equilibrium properties of such environments have been extensively studied, their welfare properties are far less understood. Filling this gap is the goal of this paper.

To fix ideas, consider the following example. A large number of investors are choosing how much to invest in a new sector. The profitability of this sector depends on an uncertain exogenous productivity parameter (the fundamentals) as well as on aggregate investment. The investors thus have an incentive to align their choices. This coordination motive makes investment highly sensitive to public news about the fundamentals. Furthermore, more precise public information, by reducing investors’ reliance on private information, may dampen the sensitivity of aggregate investment to the true fundamentals and instead amplify its sensitivity to the noise in public information.

It is tempting to give a normative connotation to these positive properties, but this would not be wise. Is the heightened sensitivity of investment to public information, and its consequent heightened volatility, undesirable from a social perspective? Furthermore, does this mean that public information disseminated, for example, by policy makers or the media can reduce welfare? To answer the first question, one needs to understand the *efficient use of information*; to answer the second, one needs to understand the *social value of information*. In this paper we undertake these two tasks in an abstract framework that is tractable yet flexible enough to capture a number of applications.

Because we allow for various strategic and external effects, there is no simple answer to the questions raised above. For example, there are economies where welfare would be higher if agents were to raise their reliance on public information, and economies where the converse is true. Similarly, there are economies where any information is socially valuable, and economies where welfare decreases with both private and public information. This is consistent with the folk theorem that “anything goes” in a second-best world.

Our contribution is to identify a clear structure for “*what goes when*”. The instrument that permits this is an appropriate efficiency benchmark: the best society may attain maintaining information decentralized.

The environment. A large number of ex-ante identical small agents takes a continuous action. Payoffs depend, not only on one’s own action, but also on the mean and the dispersion of actions in the population—this is the source of external and strategic effects. Agents observe noisy private and public signals about the underlying fundamentals—this is the source of dispersed heterogeneous

information. We allow for either strategic complementarity or strategic substitutability, but restrict attention to economies in which the equilibrium is unique. Finally, we assume that payoffs are quadratic and that information is Gaussian, which makes the analysis tractable.

Equilibrium use of information. The equilibrium use of information depends crucially on the private value that agents assign to aligning their choices with those of others. The latter can be measured by the slope of best responses with respect to aggregate activity. This slope, which we call the *equilibrium degree of coordination*, conveniently summarizes how strategic complementarity or substitutability impacts equilibrium behavior: the higher this slope, the higher the sensitivity of equilibrium actions to public information relative to private.

This result is intuitive. When actions are strategic complements, agents wish to coordinate their actions; and because public information is a relatively better predictor of others' actions, agents find it optimal to rely more on public information relative to a situation in which actions are strategically independent. When instead actions are strategic substitutes, agents wish to differentiate from one another, and thus find it optimal to rely more on private information.

This result also has interesting observable implications. Noise in public information generates non-fundamental aggregate volatility (that is, common variation in actions due to noise); noise in private information generates non-fundamental cross-sectional dispersion (that is, idiosyncratic variation in actions due to noise). It follows that complementarity contributes to higher volatility, substitutability to higher dispersion.

Efficient use of information. To address whether the heightened volatility or dispersion featured in equilibrium is socially undesirable, one needs to compare the equilibrium to an appropriate efficiency benchmark. The one that best serves this goal is the strategy—the mapping from primitive information to actions—that maximizes ex-ante utility. This strategy identifies the best society could do under the sole constraint that information cannot be centralized or otherwise communicated among the agents. Comparing equilibrium to this benchmark isolates the discrepancy, if any, between private and social incentives in the use of available information.

The efficient use of information depends crucially on the social value of aligning choices across agents. The latter can be measured as follows. Consider a fictitious game in which agents' payoffs are manipulated so that the equilibrium coincides with the efficient strategy of the actual economy. The slope of the best responses with respect to the mean activity in this fictitious game identifies the degree of complementarity (or substitutability) that society would like the agents to perceive for the efficient outcome to obtain as an equilibrium. This slope, which we call the *socially optimal degree of coordination*, is unique and summarizes how much society values alignment.

Just as the relative sensitivity of the equilibrium allocation to public information is pinned down by the equilibrium degree of coordination, the relative sensitivity of the efficient allocation is pinned down by the socially optimal degree of coordination. One can thus understand the inefficiency, if

any, in the equilibrium use of information simply by comparing the equilibrium and the optimal degree of coordination. The question is then what determines the latter.

We first show that the optimal degree of coordination increases with social aversion to dispersion, and decreases with social aversion to volatility. This is intuitive: a higher degree of coordination perceived by the agents implies lower sensitivity to private noise (lower dispersion) at the expense of higher sensitivity to public noise (higher volatility).

We next relate the optimal degree of coordination to the primitives of the economy. When payoffs are independent across agents, all that matters for welfare is the level of noise, not its composition; as a result, the welfare costs of dispersion and volatility are completely symmetric, implying that the optimal degree of coordination is zero. Complementarity reduces social aversion to volatility by alleviating concavity (or “diminishing returns”) at the aggregate level; as a result, complementarity contributes to a positive optimal degree of coordination and, symmetrically, substitutability to a negative. The impact of strategic effects on the efficient use of information thus parallels their impact on the equilibrium use of information. However, the optimal degree of coordination—and the efficient use of information—also depends on other external effects that affect social preferences over volatility and dispersion without affecting private incentives.

Social value of information. Our efficiency benchmark is a useful instrument for assessing the social value of information in equilibrium. In particular, we show how the comparative statics of equilibrium welfare with respect to the information structure can be understood by classifying economies according to the type of inefficiency, if any, exhibited by the equilibrium.

First, consider economies in which the equilibrium is efficient under both complete and incomplete information. In this case, equilibrium welfare necessarily increases with both private and public information. This is because, in these economies, the equilibrium coincides with the solution to the planner’s problem, in which case an argument analogous to Blackwell’s theorem ensures that any source of information is welfare-improving.

Next, consider economies in which the equilibrium is inefficient only under incomplete information. Public information can now reduce equilibrium welfare, when the equilibrium degree of coordination is higher than the socially optimal one. Intuitively, more precise public information reduces the noise in the agents’ forecasts about the fundamentals, but also facilitates closer alignment of their choices. The first effect necessarily improves welfare in economies in which the inefficiency vanishes under complete information; but the latter effect can reduce welfare if the equilibrium degree of coordination is excessively high. Symmetrically, welfare can decrease with private information when the equilibrium degree of coordination is lower than the optimal one.

Finally, consider economies in which inefficiency pertains even under complete information; this is the case when distortions other than incomplete information create a gap between the complete-information equilibrium and the first best. In this case, welfare can decrease with *both* private

and public information—a possibility not present in the previous two classes of economies. This is because less noise necessarily brings the equilibrium activity closer to its complete-information counterpart, but now this may mean taking it further away from the first-best level.

Applications. We conclude the paper by illustrating how our results help understand the inefficiency of equilibrium, and the social value of information, in specific applications.

We start with an incomplete-market competitive economy in which production decisions take place under incomplete information about future demand. In this economy, actions are strategic substitutes, leading in equilibrium to high sensitivity to private information and high dispersion; however, the equilibrium use of information is efficient, implying that the equilibrium dispersion is just right and that any type of information is welfare-increasing.

Next we consider a typical model of production spillovers, like the one outlined at the beginning of the introduction. Complementarities in investment choices amplify the volatility of aggregate investment; however, the equilibrium degree of coordination is actually lower than the optimal one, so that the amplified volatility is anything but excessive. Moreover, because coordination is socially valuable, welfare necessarily increases with the precision of public information, despite the adverse effect the latter can have on volatility.

In contrast, the equilibrium degree of coordination is inefficiently high in economies that resemble Keynes’ beauty-contest metaphor for financial markets and that are stylized in the example of Morris and Shin (2002). As a result, more precise public information can reduce welfare in these economies—but this is only because coordination is socially undesirable.

Keynesian frictions such as monopolistic competition or incomplete markets are often the source of macroeconomic complementarities. It is tempting to draw a relation between such models and beauty contests: if the coordination motive originates in a market friction, isn’t it safe to presume that it is socially unwarranted? The answer is no. Consider, for example, new-Keynesian models of the business cycle. These models typically feature complementarity in pricing decisions originating in monopolistic competition, but also a disutility from cross-sectional price dispersion (Woodford, 2001; Hellwig, 2005; Roca, 2005). The latter effect heightens social aversion to dispersion, thereby contributing to a higher optimal degree of coordination than the equilibrium one—the opposite of what holds in beauty contests.

This observation helps explain why Hellwig (2005) and Roca (2005) find public information to be welfare improving in their models—a result they use to make a case for transparency in central bank communication. However, this result is highly sensitive to the nature of the underlying business-cycle shocks. We highlight this point by constructing an example that features two types of shocks: one that affects the equilibrium and the first-best allocation symmetrically, and another that drives fluctuations in the gap between the two. Whereas information about the former shock increases welfare, information about the latter decreases it. A case for “constructive ambiguity”

can thus be made if the business cycle is driven by shocks to “mark-ups”, “wedges”, or other distortions.

The above examples have a macro flavor, but our results are also relevant for micro applications. Our last example analyzes how information affects expected industry profits in oligopolistic industries with many small firms. We find that information-sharing among firms, or other improvements in commonly available information, necessarily increases profits in Bertrand games (where firms compete in prices), but not in Cournot games (where firms compete in quantities).

Related literature. To the best of our knowledge, this paper is the first one to conduct a complete welfare analysis for the class of economies considered here. The closest ascendants are Cooper and John (1988), who examine economies with complementarities but complete information, and Vives (1984, 1990), who examines a class of limit-competitive economies that is a special case of the more general class considered in this paper (see Section 6.1). Also related are Vives (1990) and Raith (1996), who examine the value of information sharing in oligopolies (see Section 6.5).

The social value of information, on the other hand, has been the subject of a vast literature, going back at least to Hirshleifer (1971). More recently, Morris and Shin (2002) drew attention to models with complementarities. In their model, public information can reduce welfare. In contrast, public information is necessarily welfare improving in the investment game of Angeletos and Pavan (2004) and the monetary economy of Hellwig (2005). These models are isomorphic from a positive perspective, but deliver completely different normative results, leaving a mystery around the question of why this is so. We resolve the mystery here by showing how the social value of information depends, not only on the form of strategic interaction, but also on other external effects that determine the gap between equilibrium and efficient use of information.

The literature on rational expectations has emphasized how the aggregation of dispersed private information in markets can improve allocative efficiency (e.g., Grossman, 1981). Laffont (1985) and Messner and Vives (2001), on the other hand, highlight how *informational* externalities can generate inefficiency in the private collection and use of information. Although the information structure here is exogenous, the paper provides an input into this line of research by studying how the welfare effects of information depend on *payoff* externalities.

The paper also contributes to the recent debate on central-bank transparency. While earlier work focused on incentive issues (e.g., Canzoneri, 1985; Atkeson and Kehoe, 2001; Stokey, 2002), recent work emphasizes the role of coordination. Morris and Shin (2002, 2005) and Heinemann and Cornand (2004) argue that central-bank disclosures can reduce welfare if financial markets behave like beauty contests; Svensson (2005) and Woodford (2005) question the practical relevance of this result; Hellwig (2005) and Roca (2005) argue that disclosures improve welfare by reducing price dispersion. While all these papers focus exclusively on whether coordination is inefficiently high or not, we argue that perhaps a more important dimension is the source of the business cycle.

The rest of the paper is organized as follows. We introduce the model in Section 2. We examine the equilibrium use of information in Section 3, the efficient use of information in Section 4, and the social value of information in Section 5. We turn to applications in Section 6 and conclude in Section 7. The Appendix contains proofs omitted in the main text.

2 The model

Actions and payoffs. We will analyze an economy with a continuum of agents. However, to clarify the assumptions we make about payoffs, it is useful to start with the finite-player version of the game, in which the number of agents is $J \in \mathbb{N}$.

Each agent i chooses an action $k_i \in \mathbb{R}$. His payoff is given by $u_i = \tilde{U}(k_i, k_{-i}, \theta)$, where \tilde{U} is a twice-differentiable function, $k_{-i} \equiv (k_j)_{j \neq i}$ is the vector of other agents' actions, and $\theta \in \mathbb{R}$ is an exogenous random payoff-relevant variable (the fundamentals).¹ We assume that $\tilde{U}(k_i, k_{-i}, \theta)$ is symmetric in k_{-i} in the sense that $\tilde{U}(k_i, k_{-i}, \theta) = \tilde{U}(k_i, k'_{-i}, \theta)$ for any k_{-i} and k'_{-i} such that k'_{-i} is a permutation of k_{-i} . We further impose that \tilde{U} is quadratic, which ensures linearity of best responses as well as linearity in the structure of the efficient allocations; this assumption is essential for keeping the analysis tractable under incomplete information, but might also be viewed as a second-order approximation of a broader class of concave economies. Let $K_{-i} \equiv \frac{1}{J-1} \sum_{j \neq i} k_j$ denote the mean and $\sigma_{-i} \equiv [\frac{1}{J-1} \sum_{j \neq i} (k_j - K_{-i})^2]^{1/2}$ the dispersion of the actions of agent i 's opponents.² Under the aforementioned two assumptions, payoffs can be rewritten as

$$u_i = U(k_i, K_{-i}, \sigma_{-i}, \theta), \quad (1)$$

where U is quadratic and its partial derivatives satisfy $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = 0$ and $U_{\sigma}(k, K, 0, \theta) = 0$ for all (k, K, θ) .³ That is, dispersion has only a second-order, non-strategic external effect.

Consider now the continuum-player version of this economy and let Ψ denote the cumulative distribution function for action k in the cross-section of the population. The continuum-player analogue of (1) is

$$u = U(k, K, \sigma_k, \theta), \quad (2)$$

where $K \equiv \int k d\Psi(k)$ is the mean, and $\sigma_k^2 \equiv [\int (k - K)^2 d\Psi(k)]^{1/2}$ the dispersion, of individual actions in the population. From here on, we restrict attention to the continuum-player case.

To ensure that equilibrium is unique and bounded, we assume $U_{kk} < 0$ and $-U_{kK}/U_{kk} < 1$. The first condition imposes concavity at the individual level, ensuring that best responses are well

¹The analysis easily extends to multi-dimensional fundamentals ($\theta \in \mathbb{R}^N$ for $N \geq 2$). See Section 6.4 for an example and the working-paper version of this article (Angeletos and Pavan, 2006a) for further details.

²Usually dispersion is defined as the variance rather than the standard deviation; since this distinction is immaterial for qualitative purposes, here we use the two notions interchangeably.

³Equivalently, $U(k_i, K_{-i}, \sigma_{-i}, \theta) = (k_i, K_{-i}, \theta)' \mathbf{M} (k_i, K_{-i}, \theta) + U_{\sigma\sigma} \sigma^2 / 2$, where \mathbf{M} is a 3×3 matrix.

defined; the second requires that the slope of best responses with respect to aggregate activity is less than one, which is essentially the same as imposing uniqueness of equilibrium.⁴ Similarly, to ensure that the first-best allocation is unique and bounded, we assume $U_{kk} + 2U_{kK} + U_{KK} < 0$ and $U_{kk} + U_{\sigma\sigma} < 0$. As we will explain later, these conditions impose concavity at the aggregate level; if either one were violated, infinite ex ante utility could be obtained by introducing random noise in the actions of the agents. Finally, to make the analysis interesting, we assume $U_{k\theta} \neq 0$; this rules out the trivial case where the fundamental θ is irrelevant for equilibrium behavior.

Other than these restrictions, the payoff structure is quite flexible: it allows for either strategic complementarity ($U_{kK} > 0$) or strategic substitutability ($U_{kK} < 0$), as well as for positive or negative externality with respect to the mean ($U_K \neq 0$) or the dispersion ($U_\sigma \neq 0$) of activity.

Information. Following the pertinent literature, we introduce incomplete information by assuming that agents observe noisy private and public signals about the underlying fundamentals. Before agents move, nature draws θ from a Normal distribution with mean μ and variance σ_θ^2 . The realization of θ is not observed by the agents. Instead, agents observe private signals $x_i = \theta + \xi_i$ and a public signal $y = \theta + \varepsilon$, where ξ_i and ε are, respectively, idiosyncratic and common noises, independent of one another as well as of θ , with variances σ_x^2 and σ_y^2 .

For future reference, note that the common posterior for θ given public information alone is Normal with mean $z \equiv \mathbb{E}[\theta|y] = \lambda y + (1 - \lambda)\mu$ and variance σ_z^2 , where $\lambda \equiv \sigma_y^{-2}/\sigma_z^{-2}$ and $\sigma_z \equiv (\sigma_y^{-2} + \sigma_\theta^{-2})^{-1/2}$. In what follows we will often identify public information with z rather than y . Private posteriors, on the other hand, are Normal with mean $\mathbb{E}[\theta|x_i, y] = (1 - \delta)x_i + \delta z$ and variance $\text{Var}[\theta|x_i, y] = \sigma^2$, where

$$\sigma^{-2} \equiv \sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2} > 0 \quad \text{and} \quad \delta \equiv \frac{\sigma_y^{-2} + \sigma_\theta^{-2}}{\sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2}} \in (0, 1). \quad (3)$$

3 Equilibrium use of information

Each agent chooses k so as to maximize $\mathbb{E}[U(k, K, \sigma_k^2, \theta)|x, y]$. The solution to this optimization problem gives the best response for the individual. The fixed point is the equilibrium.

The information set of agent i is given by the realizations of x_i and y , whereas the state of the world is given by the realizations of θ , y , and $(x_i)_{i \in [0,1]}$. Since the private errors ξ_i are i.i.d. across agents, K and σ_k , as well as any other aggregate variable, are functions of (θ, y) alone. Letting $P(x|\theta, y)$ denote the conditional c.d.f. of x given (θ, y) , an equilibrium is defined as follows.

⁴To be precise, our model admits a unique equilibrium under complete information whenever $-U_{kK}/U_{kk} \neq 1$; but for $-U_{kK}/U_{kk} > 1$ this uniqueness is an artifact of the simplifying assumption that the action space is unbounded. See the Supplementary Material for a detailed discussion.

Definition 1 An equilibrium is a strategy $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that, for all (x, y) ,

$$k(x, y) = \arg \max_{k'} \mathbb{E}[U(k', K(\theta, y), \sigma_k(\theta, y), \theta) \mid x, y], \quad (4)$$

where $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$ and $\sigma_k(\theta, y) = [\int_x [k(x, y) - K(\theta, y)]^2 dP(x|\theta, y)]^{1/2}$ for all (θ, y) .

Definition 2 A linear equilibrium is any strategy satisfying (4) that is linear in x and y .

It is useful to consider first the complete-information benchmark. When θ is known, the unique equilibrium is $k_i = \kappa(\theta)$ for all i , where $\kappa(\theta)$ is the unique solution to $U_k(\kappa, \kappa, 0, \theta) = 0$. Because U is quadratic, κ is linear: $\kappa(\theta) = \kappa_0 + \kappa_1\theta$, where $\kappa_0 \equiv -U_k(0, 0, 0, 0)/(U_{kk} + U_{kK})$ and $\kappa_1 \equiv -U_{k\theta}/(U_{kk} + U_{kK})$. The incomplete-information equilibrium is then characterized as follows.

Proposition 1 Let $\kappa(\theta) = \kappa_0 + \kappa_1\theta$ denote the complete-information equilibrium allocation and

$$\alpha \equiv \frac{U_{kK}}{|U_{kk}|}. \quad (5)$$

(i) A strategy $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ is an equilibrium if and only if, for all (x, y) ,

$$k(x, y) = \mathbb{E}[(1 - \alpha) \cdot \kappa(\theta) + \alpha \cdot K(\theta, y) \mid x, y] \quad (6)$$

where $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$ for all (θ, y) .

(ii) A linear equilibrium exists, is unique, and is given by

$$k(x, y) = \kappa_0 + \kappa_1[(1 - \gamma)x + \gamma z], \quad (7)$$

where

$$\gamma = \delta + \frac{\alpha\delta(1 - \delta)}{1 - \alpha(1 - \delta)}. \quad (8)$$

Part (i) states that any equilibrium—linear or not—must solve (6). This condition has a simple interpretation. An agent's best response is an affine combination of his expectation of some given “target” and his expectation of aggregate activity. The target is simply the complete-information equilibrium, $\kappa(\theta)$. The slope of the best response with respect to aggregate activity, α , is what we call the *equilibrium degree of coordination*; it captures the private value agents assign to aligning their choices.

Part (ii) establishes that there exists a unique linear solution to (6). Because the best response of an agent is linear in his expectations of θ and K , and because his expectation of θ is linear in x and y (or, equivalently, in x and z), it is natural to conjecture that there do not exist solutions to

(6) other than the linear one. This conjecture can be verified at least for $\alpha \in (-1, 1)$, following the same argument as in Morris and Shin (2002).⁵

As evident from condition (8), the sensitivity of the equilibrium to private and public information depends, not only on the relative precision of the two (captured by δ), but also on the private value of coordination (captured by α). When $\alpha = 0$, the incomplete-information equilibrium strategy is simply the best predictor of the complete-information equilibrium allocation: condition (7) reduces to $k(x, y) = \mathbb{E}[\kappa(\theta) | x, y]$. Accordingly, the weights on x and z are simply the Bayesian weights: $\gamma = \delta$ if $\alpha = 0$. When, instead, $\alpha \neq 0$, equilibrium behavior is tilted towards public or private information depending on whether agents' actions are strategic complements or substitutes. In particular, complementarity raises the relative sensitivity to public information ($\gamma > \delta$ when $\alpha > 0$), while substitutability raises the relative sensitivity to private information ($\gamma < \delta$ when $\alpha < 0$).

To understand better this result, consider the best response of an agent to a given strategy by the other agents. To simplify, let $\kappa(\theta) = \theta$. When the other agents' strategy is $k(x, y) = (1 - \gamma)x + \gamma z$ for some arbitrary γ , the mean action is $K(\theta, y) = (1 - \gamma)\theta + \gamma z$ and an agent's best response is

$$\begin{aligned} k'(x, y) &= \mathbb{E}[(1 - \alpha)\theta + \alpha K(\theta, y) | x, y] \\ &= (1 - \alpha\gamma)\mathbb{E}[\theta | x, y] + \alpha\gamma z \\ &= (1 - \gamma')x + \gamma'z \end{aligned}$$

where $\gamma' = \delta + \alpha\gamma(1 - \delta)$. Thus, as long as other agents put a positive weight on public information ($\gamma > 0$) and actions are strategic complements ($\alpha > 0$), the best response is to put a weight on z higher than the Bayesian one ($\gamma' > \delta$), and the more so the higher the other agents' weight or the stronger the complementarity. Symmetrically, the converse is true in the case of strategic substitutability ($\alpha < 0$). The reason is that public information is a relatively better predictor of other agents' activity than private information. In equilibrium, this leads an agent to adjust upwards his reliance on public information when he wishes to align his choice with other agents' choices (i.e., $\gamma > \delta$ when $\alpha > 0$), and downwards when he wishes to differentiate his choice from those of others (i.e., $\gamma < \delta$ when $\alpha < 0$).

Another way to appreciate this result is to consider its observable implications. If information were complete (i.e., $\sigma = 0$), then all agents would choose $k = \kappa(\theta)$. Incomplete information affects equilibrium behavior in two ways. First, common noise generates *non-fundamental volatility*, that is, variation in aggregate activity around the complete-information level. Second, idiosyncratic

⁵To be precise, the argument in Morris and Shin (2002) is incomplete in that it presumes that $\alpha^t \bar{E}^t K \rightarrow 0$ as $t \rightarrow \infty$ where \bar{E}^t denotes the t -th order iteration of the average-expectation operator. With $\alpha \in (-1, 1)$, $\alpha^t \rightarrow 0$ as $t \rightarrow \infty$, but one also needs to ensure that $\bar{E}^t K$ remains bounded. Since K is unbounded, this is not obvious. However, this problem is easily bypassed by imposing bounds on the action space.

noise generates *dispersion*, that is, variation in the cross-section of the population. The following is then a direct implication of the result that γ increases with α .

Corollary 1 *Stronger complementarity decreases the dispersion and increases the non-fundamental volatility of equilibrium activity: $d\text{Var}(k - K)/d\alpha < 0 < d\text{Var}(K - \kappa)/d\alpha$.*

4 Efficient use of information

We now introduce an efficiency benchmark that addresses whether higher welfare could be obtained if agents were to use their available information in a different way than what they do in equilibrium. This efficiency benchmark is interesting on its own right, because it helps us understand whether the heightened volatility or dispersion originating in strategic effects is socially undesirable. But it will also serve as an instrument for understanding the welfare effects of information in equilibrium.

Letting $P(\theta, y)$ denote the c.d.f. of joint distribution of (θ, y) , we define our efficiency benchmark as follows.

Definition 3 *An efficient allocation is a strategy $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ that maximizes*

$$\mathbb{E}u = \int_{(\theta, y)} \int_x U(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) dP(x|\theta, y) dP(\theta, y),$$

where $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$ and $\sigma_k(\theta, y) = [\int_x [k(x, y) - K(\theta, y)]^2 dP(x|\theta, y)]^{1/2}$ for all (θ, y) .

The strategy defined above is the one that maximizes ex-ante utility subject to the sole constraint that information cannot be transferred from one agent to another. It can be understood as the solution to a “team problem,” where agents get together before they receive information, choose cooperatively a strategy for how to use the information they will receive, and then adhere to this strategy. It is also the solution to a “planner’s problem,” where the planner can perfectly control how an agent’s action depends on his own information, but can not make an agent’s action depend on other agents’ private information. This efficiency benchmark thus identifies what is the best a society could do if its agents were to internalize their payoff interdependencies and appropriately adjust their use of available information, but without communicating with one another.⁶ Comparing equilibrium to this allocation thus permits to isolate the inefficiency that originates in the way equilibrium processes available information.

⁶Our efficiency concept is thus different from standard constrained-efficiency concepts that assume costless communication and instead focus on incentive constraints (e.g., Mirrlees, 1971; Holmstrom and Myerson, 1983). Instead, it shares with Hayek (1945) and Radner (1962) the idea that information is dispersed and can not be communicated to a “center”.

We now turn to the characterization of the efficient allocation. Let

$$W(K, \sigma_k, \theta) \equiv U(K, K, \sigma_k, \theta) + \frac{1}{2} U_{kk} \sigma_k^2 = \int U(k, K, \sigma_k^2, \theta) d\Psi(k)$$

denote welfare under a utilitarian aggregator. We are interested in allocations that maximize ex-ante utility; this is just a convenient instrument for computing ex-ante utility. Next, let $\kappa^*(\theta)$ be the unique solution to $W_K(\kappa^*, 0, \theta) = 0$; that is, $\kappa^*(\theta) = \kappa_0^* + \kappa_1^* \theta$, where $\kappa_0^* = -W_K(0, 0, 0) / W_{KK}$ and $\kappa_1^* = -W_{K\theta} / W_{KK}$. Ex-ante utility for any arbitrary strategy $k(x, y)$ is given by

$$\mathbb{E}u = \mathbb{E}W(\kappa^*, 0, \theta) + \frac{W_{KK}}{2} \mathbb{E}(K - \kappa^*)^2 + \frac{W_{\sigma\sigma}}{2} \mathbb{E}(k - K)^2, \quad (9)$$

where $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK}$ and $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma}$ (see the Appendix for the proof). Clearly, $W_{KK} < 0$ and $W_{\sigma\sigma} < 0$ imply that $\mathbb{E}u \leq \mathbb{E}W(\kappa^*, 0, \theta)$, which proves that $\kappa^*(\theta)$ is the first-best allocation. If, instead, W_{KK} and/or $W_{\sigma\sigma}$ were positive, infinite ex-ante utility could be obtained by inducing arbitrarily random variation in activity—which explains why, to start with, we imposed $U_{kk} + 2U_{kK} + U_{KK} < 0$ and $U_{kk} + U_{\sigma\sigma} < 0$.

Proposition 2 *Let $\kappa^*(\theta) = \kappa_0^* + \kappa_1^* \theta$ denote the first-best allocation and let*

$$\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}} = 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{kk} + U_{\sigma\sigma}}. \quad (10)$$

(i) *An allocation $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ is efficient under incomplete information if and only if, for almost all (x, y) ,*

$$k(x, y) = \mathbb{E}[(1 - \alpha^*)\kappa^*(\theta) + \alpha^*K(\theta, y) \mid x, y] \quad (11)$$

where $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$ for all (θ, y) .

(ii) *The efficient allocation exists, is unique for almost all (x, y) , and is given by*

$$k(x, y) = \kappa_0^* + \kappa_1^* [(1 - \gamma^*)x + \gamma^*z], \quad (12)$$

where

$$\gamma^* = \delta + \frac{\alpha^* \delta (1 - \delta)}{1 - \alpha^* (1 - \delta)}. \quad (13)$$

This result characterizes the efficient allocation among all possible strategies, not only the linear ones; that the efficient strategy turns out to be linear is because of the combination of quadratic payoffs and Gaussian information.

In equilibrium, each agent's action was an affine combination of his expectation of κ , the complete-information equilibrium, and of his expectation of aggregate activity, K . The same is true for the efficient allocation if we replace κ with κ^* and α with α^* . In this sense, condition (11) is the analogue for efficiency of what the best response is for equilibrium. This idea is formalized by the following result.

Proposition 3 *Given an economy $\mathbf{e} = (U; \sigma, \delta, \mu, \sigma_\theta)$, let $\mathcal{U}(\mathbf{e})$ be the set of payoffs U' such that the economy $\mathbf{e}' = (U'; \sigma, \delta, \mu, \sigma_\theta)$ admits an equilibrium that coincides with the efficient allocation for \mathbf{e} .*

- (i) *For every \mathbf{e} , $\mathcal{U}(\mathbf{e})$ is non-empty.*
- (ii) *For every \mathbf{e} , $U' \in \mathcal{U}(\mathbf{e})$ only if $\alpha' \equiv -U'_{kK}/U'_{kk} = \alpha^*$.*

Part (i) says that the efficient allocation of any given economy \mathbf{e} can be understood as the unique linear equilibrium of a fictitious game \mathbf{e}' in which the information structure is the same as in \mathbf{e} but where private incentives are adjusted so as to coincide with the social incentives of the actual economy. Indeed, because our efficiency concept allows “a planner” to perfectly control the incentives of the agents, it is as if the planner (whose objective is the true U) can design the payoffs U' perceived by the agents. Part (ii) then explains why we identify α^* with the *optimal degree of coordination*: α^* describes the level of complementarity (if $\alpha^* > 0$) or substitutability (if $\alpha^* < 0$) that the planner would like the agents to perceive for the equilibrium of the fictitious game to coincide with the efficient allocation of the true economy.⁷

To understand better the forces behind the determination of the optimal degree of coordination, consider the set of strategies that, for some arbitrary $\alpha' < 1$, solve $k(x, y) = \mathbb{E}[(1 - \alpha')\kappa^*(\theta) + \alpha'K(\theta, y) \mid x, y]$ for almost all (x, y) , where $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$ for all (θ, y) . For any such strategy, condition (9) can be restated as $\mathbb{E}u = \mathbb{E}W(\kappa^*, 0, \theta) - \mathcal{L}^*$, where

$$\mathcal{L}^* \equiv \frac{-W_{KK}}{2} \text{Var}(K - \kappa^*) + \frac{-W_{\sigma\sigma}}{2} \text{Var}(k - K) \quad (14)$$

measures the welfare losses due to volatility and dispersion.⁸ Different α' then lead to different \mathcal{L}^* ; the efficient allocation thus corresponds to the α' that minimizes \mathcal{L}^* . In words, when the planner controls how agents use information, it is as if he controls the degree of coordination perceived by the agents (i.e., α'). Because a higher degree of coordination means a higher sensitivity to public information and a lower sensitivity to private information, a higher degree of coordination trades off higher volatility for lower dispersion. It is then not surprising that the optimal degree of coordination reflects social preferences over volatility and dispersion.

Corollary 2 *The optimal degree of coordination (α^*) decreases with social aversion to volatility ($-W_{KK}$) and increases with social aversion to dispersion ($-W_{\sigma\sigma}$).*

Recall that $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK}$ and that $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma}$. As with equilibrium, the optimal degree of coordination is increasing in U_{kK} , the level of complementarity. But unlike equilibrium, the optimal degree of coordination depends also on U_{KK} and $U_{\sigma\sigma}$, two second-order

⁷Here we use this result only to give a precise *meaning* to our notion of the socially optimal degree of coordination. However, this also suggests an *implementation* for certain environments (Angeletos and Pavan, 2006a).

⁸This follows from (9) using the fact that any such strategy satisfies $\mathbb{E}[k(\theta, y)] = \mathbb{E}[K(\theta, y)] = \mathbb{E}[\kappa^*(\theta)]$.

external effects that do not affect private incentives. A more negative $U_{\sigma\sigma}$, by increasing social aversion to dispersion, contributes to a higher α^* , while a more negative U_{KK} , by increasing social aversion to volatility, contributes to a lower α^* . In the absence of these effects, the optimal degree of coordination is twice as large as the equilibrium one ($\alpha^* = 2\alpha$), reflecting the internalization of the externality associated with the complementarity. More generally, from conditions (5) and (10), we have that $\alpha \geq \alpha^*$ if and only if $U_{kK} \leq -U_{KK} + U_{\sigma\sigma}[U_{kK}/U_{kk} - 1]$

Finally, just as α pinned down the relative sensitivity of the equilibrium allocation to public and private information, α^* pins down the corresponding sensitivity of the efficient allocation. Comparing the two gives the following result.

Corollary 3 *The relative sensitivity of the equilibrium allocation to public noise—and the consequent volatility of the equilibrium allocation—is inefficiently high if and only if the equilibrium degree of coordination is higher than the optimal one (i.e., $\gamma \geq \gamma^* \Leftrightarrow \alpha \geq \alpha^*$).*

5 Social value of information

We now turn to the comparative statics of equilibrium welfare with respect to the information structure.⁹ For this purpose, we find it useful to decompose the information structure into its *accuracy* and its *commonality*, where by accuracy we mean the precision of the agents' forecasts about θ and by commonality we mean the correlation of forecast errors across agents. We also find it useful to classify economies according to the type of inefficiency, if any, exhibited in equilibrium.

A useful decomposition of information. Let $v_i \equiv \theta - \mathbb{E}[\theta|x_i, y]$ denote agent i 's forecast error about θ . One can show that $\text{Var}(v_i) = \sigma^2$ and, for $i \neq j$, $\text{Corr}(v_i, v_j) = \delta$. We accordingly identify the *accuracy* of information with σ^{-2} and its *commonality* with δ .

Clearly, there is a one-to-one mapping between (σ_x, σ_z) and (δ, σ^{-2}) ; any change in the information structure can thus be decomposed into an accuracy and a commonality effect. For many applied questions, one is interested in the comparative statics of equilibrium welfare with respect to the precisions of public and private information—and this is also what we do when we turn to applications in Section 6. However, from a theoretical perspective, this decomposition is more insightful. When there are no payoff interdependencies across agents, the distinction between private and public information is irrelevant—all that matters for welfare is the level of noise, not its composition. With strategic interactions, instead, the commonality of information becomes crucial, for it affects the agents' ability to forecast one another's actions—and it is only in this sense that public information is different than private.

A useful classification of economies. The inefficiency, if any, of the equilibrium can be understood by comparing κ with κ^* and α with α^* .

⁹Throughout this section, when we refer to equilibrium, we mean the unique linear equilibrium of Proposition 1.

Proposition 4 *The economy $\mathbf{e} = (U; \sigma, \delta, \mu, \sigma_\theta)$ is efficient if and only if U is such that*

$$\kappa(\theta) = \kappa^*(\theta) \quad \forall \theta \quad \text{and} \quad \alpha = \alpha^*.$$

The condition $\kappa = \kappa^*$ means that the equilibrium is efficient under *complete* information. However, efficiency under complete information alone does not guarantee efficiency under *incomplete* information. What is also necessary is $\alpha = \alpha^*$, that is, efficiency in the equilibrium degree of coordination.

In what follows, we classify economies according to the type of inefficiency, if any, featured in equilibrium. In particular, we start with economies that are efficient under both complete and incomplete information ($\kappa = \kappa^*$ and $\alpha = \alpha^*$), continue with economies that are inefficient only when information is incomplete ($\kappa = \kappa^*$ but $\alpha \neq \alpha^*$), and conclude with the case of economies that are inefficient even under complete information ($\kappa \neq \kappa^*$).

Note that this taxonomy uses only properties of the payoff function U . This is because, within the class of quadratic economies examined in this paper, whether the aforementioned two conditions are satisfied for any given economy depends on the payoff structure of this economy but not on its information structure.¹⁰

Efficient economies ($\kappa = \kappa^*$ and $\alpha = \alpha^*$). Efficient economies exhibit a clear relation between the form of strategic interaction and the social value of information.

Proposition 5 *Consider economies in which $\kappa = \kappa^*$ and $\alpha = \alpha^*$.*

- (i) *Welfare necessarily increases with σ^{-2} .*
- (ii) *Welfare increases with δ if $\alpha > 0$, decreases if $\alpha < 0$, and is independent if $\alpha = 0$.*

As highlighted in the previous section, the impact of information on welfare at the efficient allocation is summarized by the impact of noise on volatility and dispersion; see condition (14). An increase in accuracy (for given commonality) reduces both volatility and dispersion and therefore necessarily increases welfare. On the other hand, an increase in commonality (for given accuracy) is equivalent to a reduction in dispersion at the expense of volatility.¹¹ Such a substitution is

¹⁰Indeed, it is easy to verify that $\alpha = \alpha^*$ if and only if $U_{kK} + U_{KK} - U_{\sigma\sigma}[U_{kK}/U_{kk} - 1] = 0$ and that $\kappa_0 = \kappa_0^*$ and $\kappa_1 = \kappa_1^*$ if and only if

$$U_K(0, 0, 0, 0) = -U_k(0, 0, 0, 0) \left[\frac{U_{kK} + U_{KK}}{U_{kK} + U_{kk}} \right] \quad \text{and} \quad U_{K\theta} = \left[\frac{U_{kK} + U_{KK}}{U_{kK} + U_{kk}} \right] U_{k\theta}.$$

¹¹This informal discussion presumes that higher δ reduces dispersion and increases volatility which, as can be seen from the proof of Corollary 1, is true if and only if $\alpha \in (-\frac{1}{1-\delta}, \frac{1}{1+\delta})$. The result in Proposition 5, however, does not rely on this restriction. Both volatility and dispersion increase with δ when $\alpha < -\frac{1}{1-\delta}$, whereas they both decrease when $\alpha > \frac{1}{1+\delta}$, implying that welfare necessarily decreases with δ in the former case and increases in the latter.

welfare-improving if and only if the social cost of dispersion is higher than that of volatility, which is the case in efficient economies if and only if $\alpha (= \alpha^*)$ is positive.

We now turn to the welfare effects of private and public information.

Proposition 6 *Consider economies in which $\kappa = \kappa^*$ and $\alpha = \alpha^*$.*

(i) *Welfare increases with the precision of either private or public information, no matter the degree of complementarity or substitutability.*

(ii) *The social value of public information relative to private increases as the degree of complementarity increases:*

$$\frac{\partial \mathbb{E}u / \partial \sigma_z^{-2}}{\partial \mathbb{E}u / \partial \sigma_x^{-2}} = \frac{\sigma_x^{-2}}{(1 - \alpha) \sigma_z^{-2}}.$$

Private and public information have symmetric effects on the accuracy of information, but opposite effects on commonality. While accuracy necessarily increases welfare, the impact of commonality depends on α . Nevertheless, the accuracy effect always dominates. This is because, when the equilibrium is efficient, it coincides with the solution to a planner's problem. The planner can never be worse off with a reduction in either σ_z or σ_x , for he can always replicate the initial distributions of z and x by adding noise to the new distributions.¹² It follows that *any* source of information is welfare-improving, no matter the form of strategic interaction—which explains part (i) of the proposition. At the same time, the form of strategic interaction matters for the *relative* value of different sources of information. Complementarity, by generating a positive value for commonality, raises the value of public information relative to private, while the converse is true for substitutability—which explains part (ii).

Economies that are inefficient only under incomplete information ($\kappa = \kappa^*$ but $\alpha \neq \alpha^*$). This case is of special interest, for it identifies economies where the equilibrium coincides with the first-best allocation on average (in the sense that $\mathbb{E}k = \mathbb{E}k^*$) but it fails to be efficient in its response to noise (in the sense that $\gamma \neq \gamma^*$). This type of inefficiency crucially affects the social value of commonality, but not that of accuracy.

Proposition 7 *Consider economies in which $\kappa = \kappa^*$ but $\alpha \neq \alpha^*$.*

(i) *Welfare necessarily increases with σ^{-2} .*

(ii) *Welfare increases with δ if $\alpha^* \geq \alpha > 0$ and decreases with it if $\alpha^* \leq \alpha < 0$.*

In these economies, the welfare losses associated with incomplete information continue to be the weighted sum of volatility and dispersion, as in condition (14). Because higher accuracy reduces both volatility and dispersion, part (i) is immediate. To understand part (ii), note that, for given α

¹²The planner's problem we defined in the previous section did not give the planner the option to add such noise. However, if we were to give the planner such an option, he would never use it, because $W_{KK} < 0$ and $W_{\sigma\sigma} < 0$.

(and hence given equilibrium strategies and given volatility and dispersion), a higher α^* means only a lower social cost to volatility relative to dispersion. It follows that, relatively to the case where $\alpha^* = \alpha$, inefficiently low coordination ($\alpha^* > \alpha$) increases the social value of commonality, whereas inefficiently high coordination ($\alpha^* < \alpha$) reduces it. Combining this with the result in Proposition 5 that, when $\alpha^* = \alpha$, welfare increases with δ if and only if $\alpha > 0$, gives the result in part (ii).

Consider now the social value of private and public information. Once the equilibrium degree of coordination is inefficient, it is possible that welfare decreases with an increase in the precision of a specific source of information; but, because accuracy is still welfare-improving, this can happen only through an adverse commonality effect.

Corollary 4 *Consider economies in which $\kappa = \kappa^*$ but $\alpha \neq \alpha^*$.*

(i) *Welfare can decrease with the precision of public [private] information only if it decreases [increases] with the commonality of information.*

(ii) *$\alpha^* \geq \alpha \geq 0$ suffices for welfare to increase with the precision of public information, whereas $\alpha^* \leq \alpha \leq 0$ suffices for it to increase with the precision of private information.*

Economies that are inefficient even under complete information ($\kappa \neq \kappa^*$). In this class of economies, incomplete information contributes to welfare losses not only through volatility and dispersion but also through a novel first-order effect. Indeed, equilibrium welfare can now be expressed as $\mathbb{E}u = \mathbb{E}W(\kappa, 0, \theta) - \mathcal{L}$, where $\mathbb{E}W(\kappa, 0, \theta)$ is expected welfare under the complete-information allocation and

$$\mathcal{L} = -Cov(K - \kappa, W_K(\kappa, 0, \theta)) + \frac{|W_{KK}|}{2} \cdot Var(K - \kappa) + \frac{|W_{\sigma\sigma}|}{2} \cdot Var(k - K) \quad (15)$$

are the welfare losses due to incomplete information (see the Appendix for a derivation). The last two terms in \mathcal{L} are the familiar second-order effects: volatility and dispersion. The covariance term is the novel first-order effect: a positive correlation between $K - \kappa$, the “aggregate error” due to incomplete information, and W_K , the social return to aggregate activity, contributes to higher welfare, whereas a negative correlation between the two contributes to lower welfare.

As shown in the Appendix, $Cov(K - \kappa, W_K) = |W_{KK}|\phi v$, where

$$v \equiv Cov(K - \kappa, \kappa) = -\frac{1}{1 - \alpha + \alpha\delta} \kappa_1^2 \sigma^2 \quad \text{and} \quad \phi \equiv \frac{Cov(\kappa, \kappa^* - \kappa)}{Var(\kappa)} = \frac{\kappa_1^* - \kappa_1}{\kappa_1}. \quad (16)$$

Note that v captures the covariance between the “aggregate error” due to incomplete information ($K - \kappa$) and the complete-information equilibrium (κ), whereas ϕ captures the covariance between the latter and the complete-information “efficiency gap” ($\kappa^* - \kappa$). Below we explain how the welfare effects of information depend on ϕ .

First consider the social value of accuracy. A higher σ^{-2} implies v closer to zero, because less noise brings K closer to κ for any given θ . How this affects welfare depends on whether bringing

K closer to κ also means bringing it closer to the first-best allocation. This in turn depends on the correlation between κ and κ^* . Intuitively, less noise brings K closer to κ^* when $\phi > 0$, but further away when $\phi < 0$. Combining this with the unambiguous effect of accuracy on volatility and dispersion, we conclude that higher accuracy necessarily increases welfare when $\phi > 0$, but can reduce welfare when ϕ is sufficiently negative.

Proposition 8 *There exist functions $\underline{\phi}', \bar{\phi}' : (-\infty, 1)^2 \rightarrow \mathbb{R}$, with $\underline{\phi}' \leq \bar{\phi}' < 0$, such that welfare increases with σ^{-2} for all (σ, δ) if $\phi > \bar{\phi}'(\alpha, \alpha^*)$, and decreases with it for all (σ, δ) if $\phi < \underline{\phi}'(\alpha, \alpha^*)$.*

Next, consider the social value of commonality. The impact of δ on second-order welfare losses (i.e., volatility and dispersion) remains the same as in Proposition 7. But now this must be combined with the impact of δ on first-order losses, which is captured by the product ϕv . The impact of δ on v depends on the sign of α : higher commonality increases the covariance between $K - \kappa$ and κ when $\alpha > 0$, but decreases it when $\alpha < 0$. How this in turn affects welfare depends on the sign of ϕ , the covariance between κ and the efficiency gap $\kappa^* - \kappa$. It follows that the sign of the first-order effect of δ is given by the sign of the product of α and ϕ . Combining these observations, and noting that the first-order effect dominates when ϕ is sufficiently away from zero, we conclude that ϕ sufficiently high [low] suffices for the welfare effect of commonality to have the same [opposite] sign as α .

Proposition 9 *There exist functions $\underline{\phi}, \bar{\phi} : (-\infty, 1)^2 \rightarrow \mathbb{R}$, with $\underline{\phi} \leq \bar{\phi}$, such that the following are true:*

- (i) *When $\alpha = 0$, welfare increases with δ if $\alpha^* > 0$, and decreases with it if $\alpha^* < 0$.*
- (ii) *When $\alpha > 0$, welfare increases with δ for all (σ, δ) if and only if $\phi > \bar{\phi}(\alpha, \alpha^*)$, and decreases with it for all (σ, δ) if and only if $\phi < \underline{\phi}(\alpha, \alpha^*)$.*
- (iii) *When $\alpha < 0$, welfare increases with δ for all (σ, δ) if and only if $\phi < \underline{\phi}(\alpha, \alpha^*)$, and decreases with it for all (σ, δ) if and only if $\phi > \bar{\phi}(\alpha, \alpha^*)$.*

Finally, consider the social value of private and public information. Because a sufficiently extreme ϕ suffices for the first-order effect of accuracy to dominate all other effects, we have the following result.

Corollary 5 *For any α and α^* , ϕ sufficiently high ensures that welfare increases with both the precision of private and public information, whereas ϕ sufficiently low ensures the converse.*

Another direct implication of Propositions 8 and 9 is that Proposition 7 and Corollary 4, which applied to economies where $\kappa = \kappa^*$, extend to economies where the efficiency gap $\kappa^* - \kappa$ is either

constant or positively correlated with κ .¹³ In particular, when $\phi \geq 0$, $\alpha^* \geq \alpha \geq 0$ suffices for public information to be welfare-improving, while $\alpha^* \leq \alpha \leq 0$ suffices for private information to be welfare-improving. In contrast, Corollary 5 ensures that welfare decrease with *both* types of information when the efficiency gap $\kappa^* - \kappa$ is sufficiently negatively correlated with κ (i.e. when ϕ is sufficiently low). These observations will prove useful for certain applications.

Summary. Three principles emerge through the analysis in this section. First, even if one is ultimately interested in the comparative statics of equilibrium welfare with respect to the precisions of private and public information, it is insightful to decompose these comparative statics into their effects through the accuracy and the commonality of information. Second, the social value of accuracy relies crucially on the inefficiency (if any) of the complete-information equilibrium: accuracy can *not* reduce welfare if the complete-information equilibrium is efficient, or more generally if $\phi \geq 0$, no matter the equilibrium and optimal degrees of coordination. Third, the impact of commonality relies crucially on the relation between the equilibrium and the socially optimal degree of coordination: when the equilibrium degree of coordination is inefficiently high, commonality can reduce welfare even if the complete-information equilibrium is efficient.

6 Applications

In the previous section we have showed how understanding the inefficiency, if any, in the equilibrium use of information sheds light on the social value of information within a flexible abstract framework. We now show how our results can guide welfare analysis within specific applications.

6.1 Efficient competitive economies

We start with an incomplete-market competitive economy in which production choices are made under incomplete information about future demand. There is a continuum of households, each consisting of a consumer and a producer, and two goods. Let q_{1i} and q_{2i} denote the respective quantities purchased by consumer i (the consumer living in household i). His preferences are given by

$$u_i = v(q_{1i}, \theta) + q_{2i}, \quad (17)$$

where $v(q, \theta) = \theta q - bq^2/2$, $\theta \in \mathbb{R}$, and $b > 0$. His budget is

$$pq_{1i} + q_{2i} = e + \pi_i, \quad (18)$$

¹³To see this, note that, since $\bar{\phi}' < 0$, $\phi \geq 0$ suffices for welfare to increase with accuracy. Furthermore, since, as shown in the Appendix, $\bar{\phi} < 0$ when either $\alpha^* \geq \alpha > 0$ or $\alpha^* \leq \alpha < 0$, we have that $\phi \geq 0$ also suffices for welfare to increase with the commonality of information in the first case and to decrease with it in the latter.

where p is the price of good 1 relative to good 2, e is an exogenous endowment of good 2, and π_i are the profits of producer i (the producer living in household i), which are also denominated in terms of good 2. Profits in turn are given by

$$\pi_i = pk_i - C(k_i) \quad (19)$$

where k_i denotes the quantity of good 1 produced by household i and $C(k)$ the cost in terms of good 2, with $C(k) = k^2/2$.¹⁴

The random variable θ represents a shock in the relative demand for the two goods. Exchange and consumption take place once θ has become common knowledge, while production takes place when information is still incomplete. Consumer i chooses (q_{1i}, q_{2i}) so as to maximize (17) subject to (18). The implied (inverse) demand function for good 1 is $p = \theta - bq_{1i}$. Clearly, all households consume the same quantity of good 1, which together with market clearing gives $q_{1i} = K$ for all i and $p = \theta - bK$, where $K = \int k d\Psi(k)$. It follows that $u_i = v(K, \theta) - pK + e + \pi_i = bK^2/2 + e + \pi_i$, with $\pi_i = pk_i - C(k_i) = (\theta - bK)k_i - k_i^2/2$. Hence this example is nested in our model with

$$U(k, K, \sigma_k, \theta) = (\theta - bK)k - k^2/2 + bK^2/2 + e.$$

It is then easy to check that $\kappa^*(\theta) = \kappa(\theta) = \theta/(1+b)$ and $\alpha^* = \alpha = -b < 0$.

That the complete-information equilibrium is efficient ($\kappa = \kappa^*$) is just a consequence of the first welfare theorem. What is interesting is that the equilibrium is efficient also under incomplete information. This is because the strategic substitutability perceived by the agents coincides with the one that the planner would like them to perceive ($\alpha^* = \alpha$). The following result is then immediate.

Corollary 6 *In the competitive economy described above, the heightened cross-sectional dispersion featured in equilibrium due to strategic substitutability in production choices is efficient. Moreover, welfare increases with both private and public information.*

The aforementioned competitive economy is an example of an efficient economy with strategic substitutability. For examples of efficient economies with strategic complementarity, we refer the reader to the common-interest games in Angeletos and Pavan (2006b) and Morris and Shin (2006); in those games, the equilibrium features heightened volatility instead of heightened dispersion, but again there is nothing inefficient about it. Also, the example considered here is closely related to the one in Vives (1988). He considers an incomplete-information quadratic Cournot game and shows that the maximal expected social surplus is obtained by the equilibrium allocation in the limit as the number of firms goes to infinity. Because this limit essentially coincides with the competitive economy considered here, the efficiency of this economy also follows from Vives' analysis.

¹⁴Implicit behind this cost function is a quadratic production frontier. The resource constraints are therefore given by $\int q_{1i} di = \int k_i di$ and $\int q_{2i} di = e - \frac{1}{2} \int k_i^2 di$ for good 1 and 2, respectively.

6.2 Investment complementarities

The canonical model of production externalities can be nested in our framework by interpreting k as investment and defining individual payoffs as follows:

$$U(k, K, \sigma_k, \theta) = A(K, \theta)k - C(k), \quad (20)$$

where $A(K, \theta) = (1 - a)\theta + aK$ represents the private return to investment, with $a \in (0, 1/2)$, $\theta \in \mathbb{R}$ represents exogenous productivity, and $C(k) = k^2/2$ represents the cost of investment. The important ingredient is that the private return to investment increases with the aggregate level of investment—the source of both complementarity and externality in this class of models.¹⁵

It is easy to verify that $\kappa(\theta) = \theta$ and $\kappa^*(\theta) = \frac{1-a}{1-2a}\theta$, and hence that $\phi > 0$; because of the spillover, the social return to investment increases with θ more than the private return. Furthermore, apart from the complementarity ($U_{kK} = a > 0$), there are no other second-order external effects ($U_{KK} = U_{\sigma\sigma} = 0$) and hence $\alpha = a > 0$ and $\alpha^* = 2\alpha > \alpha$; that is, the agents' private incentives to coordinate are anything but excessive from a social perspective. Because Proposition 7 and Corollary 4 extend to economies in which $\phi > 0$, we have the following result.

Corollary 7 *In the investment economy described above, the heightened volatility featured in equilibrium is not excessive. Moreover, welfare increases with both the accuracy and the commonality of information, and hence with the precision of public information.*

Economies with frictions in financial markets—in which complementarities emerge through collateral constraints, missing assets, or other types of market incompleteness—are often related to economies with investment complementarities such as the one considered here. Although this analogy is appropriate for many positive questions, it need not be so for normative purposes. As the examples in the next two subsections illustrate, the result in Corollary 7 depends on the absence of certain second-order external effects (i.e., $U_{KK} = U_{\sigma\sigma} = 0$) and on positive correlation between equilibrium and first-best activity (i.e., $\phi \geq 0$). Whether these properties are shared by mainstream models of financial frictions is an open question.

6.3 “Beauty contests” versus other Keynesian frictions

Keynes contended that financial markets often behave like “beauty contests” in the sense that traders try to forecast and outbid one another’s forecasts instead of simply bidding for the fundamental value of the asset—the presumption being that, for some unspecified reason, this is socially undesirable. Making sense of this idea with proper micro-foundations is an open question, but one

¹⁵ Variants of this example are common in the macroeconomics literature, as well as in models of network externalities and technology adoption. This is also the example we examined in Angeletos and Pavan (2004, Section 2), although there we computed welfare conditional on θ , thus omitting the effect of $\phi \neq 0$ on welfare losses.

possible shortcut, followed by Morris and Shin (2002), is to consider a game in which payoffs are given by

$$u_i = -(1-r)(k_i - \theta)^2 - r(L_i - \bar{L}),$$

where $r \in (0,1)$. Here, $L_i = \int (k_j - k_i)^2 dj = (k_i - K)^2 + \sigma_k^2$ is the mean square-distance of other agents' actions from agent i 's action, $\bar{L} = \int L_j dj = 2\sigma_k^2$ is the cross-sectional mean of L_i , and $r \in (0,1)$. The first term in u_i is meant to capture the value of taking an action close to a fundamental "target" θ . The L_i term introduces a private value for taking an action close to other agents' actions. Finally, the \bar{L} term is an ad hoc externality that ensures that there is no social value in doing so.¹⁶

This example is nested in our framework with

$$U(k, K, \sigma_k, \theta) = -(1-r)(k - \theta)^2 - r(k - K)^2 + r\sigma_k^2.$$

It follows that $\kappa^*(\theta) = \kappa(\theta) = \theta$, $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{\sigma\sigma} = 2r$, and hence $\alpha = r > 0 = \alpha^*$. The key here is that private motives to coordinate are not warranted from a social perspective ($\alpha > 0 = \alpha^*$), and that the inefficiency of equilibrium vanishes as information becomes complete ($\kappa = \kappa^*$). The following is then an immediate implication of Corollary 4.

Corollary 8 *In beauty-contest economies (defined as economies in which $\kappa = \kappa^*$ and $\alpha > 0 = \alpha^*$), welfare can decrease with the precision of public information, but only when it decreases with the commonality of information—and this is possible only because coordination is excessively high.*

It is tempting to extend the lesson from this example to other environments in which the complementarity appears to be socially unwarranted because it originates from a market friction. To see why this need not be appropriate, consider the incomplete-information Keynesian business-cycle models recently examined by Woodford (2002), Hellwig (2005), Lorenzoni, (2005), and Roca (2005). In these models, complementarity in pricing choices originates from monopolistic competition—a market friction. However, imperfect substitutability across goods implies that noise-driven cross-sectional dispersion in relative prices creates a negative externality ($U_{\sigma\sigma} < 0$), contributing towards a *higher* optimal degree of coordination—exactly the opposite of what happens in the aforementioned beauty-contest economy. This helps explain why Hellwig (2005) and Roca (2005), in contrast to Morris and Shin (2002), find that welfare necessarily increases with public information.¹⁷

¹⁶Indeed, aggregating across agents gives $W(K, \sigma_k, \theta) = -(1-r) \int (k_i - \theta)^2 di$, so that, from a social perspective, it is as if utility were simply $u_i = -(k_i - \theta)^2$, in which case there is of course no social value to coordination.

¹⁷In these models, the business cycle is efficient under complete information (i.e. $\kappa = \kappa^*$). Combining this property with the fact that $\alpha^* > \alpha > 0$, the result in Hellwig (2005) can be read as a special case of Corollary 4.

6.4 Inefficient fluctuations

The examples examined so far illustrated how strategic and second-order external effects may tilt the social trade-off between volatility and dispersion, thus affecting the relation between α and α^* ; but they all featured $\phi \geq 0$, thus ensuring that accuracy is welfare improving. We now consider an economy in which the efficiency gap $\kappa^* - \kappa$ can co-vary negatively with κ (i.e., $\phi < 0$), as in the case of recessions that are inefficiently deep. We also highlight the role of different shocks by allowing for two types of fundamentals.

Agents engage in an investment activity for which private and social returns differ:

$$U(k, K, \sigma_k, \theta) = (\theta_1 + \theta_2)k - k^2/2 - \lambda\theta_2 K,$$

for some $\lambda \in (0, 1)$. One can interpret the last term as the impact of a “wedge” or “mark-up” that introduces a gap between private and social returns: the private return to investment is $\theta_1 + \theta_2$, the wedge is $\lambda\theta_2$, and the social return is $\theta_1 + (1 - \lambda)\theta_2$.

Although our analysis has been limited to a single-dimensional fundamental, it easily extends to the multi-dimensional case.¹⁸ First, note that the complete-information equilibrium is $\kappa(\theta_1, \theta_2) = \theta_1 + \theta_2$, whereas the first-best allocation is $\kappa^*(\theta_1, \theta_2) = \theta_1 + (1 - \lambda)\theta_2$. Next, let $\phi_1 \equiv \frac{Cov(\kappa, \kappa^* - \kappa | \theta_2)}{Var(\kappa | \theta_2)}$ and $\phi_2 \equiv \frac{Cov(\kappa, \kappa^* - \kappa | \theta_1)}{Var(\kappa | \theta_1)}$, and note that $\phi_1 = 0$ but $\phi_2 = -\lambda < 0$. Finally, note that $U_{kK} = U_{KK} = U_{\sigma\sigma} = 0$ and hence $\alpha^* = \alpha = 0$. If there were only one fundamental, then welfare would increase with both private and public information if $\phi > -1/2$ and would decrease with both types of information if $\phi < -1/2$. A similar result holds here in that $\phi_1 = 0$ ensures that any information about θ_1 is welfare improving, while $\phi_2 < -1/2$ suffices for welfare to decrease with any information about θ_2 .

Corollary 9 *Consider the economy described above and suppose $\lambda > 1/2$. Welfare necessarily increases with private or public information about the efficient source of the business cycle (θ_1), and decreases with private or public information about the inefficient source (θ_2).*

The recent debate on the merits of transparency in central bank communication has focused on the role of complementarities in new-Keynesian models (e.g., Morris and Shin, 2002; Svensson, 2005; Woodford, 2005; Hellwig, 2005; Roca, 2005). The example here suggests that this debate might be missing a critical element—the potential inefficiency of equilibrium fluctuations under complete information. For example, we conjecture that the result in Hellwig (2005) and Roca (2005) that public information has a positive effect on welfare relies on the property that, in their model, the business cycle is efficient under complete information. This is because, in these models,

¹⁸See the working-paper version (Angeletos and Pavan, 2006a). Here θ_1 and θ_2 are two independent normal random variables. Agents receive independent private and public signals for each of the two fundamentals, $x_n^i = \theta_n + \xi_n^i$ and $y_n = \theta_n + \varepsilon_n$, $n = 1, 2$, where ξ_n^i and ε_n are independent of one another as well as of θ_1 and θ_2 .

the monopolistic mark-up and the consequent efficiency gap are constant over the business cycle. But if business cycles are driven primarily by shocks in mark-ups or other distortions that induce a countercyclical efficiency gap, it is possible that providing markets with information that helps predict these shocks may reduce welfare.

6.5 Cournot versus Bertrand

We conclude with two IO applications: a Cournot-like game in which firms compete in quantities and actions are strategic substitutes and a Bertrand-like game in which firms compete in prices and actions are strategic complements. Efficiency and value of information are now evaluated from the perspective of firms; “welfare” is identified with expected total profits.

First, consider Cournot. The demand faced by a firm is given by $p = a_0 + a_1\theta - a_2q - a_3Q$ (with $a_0, a_1, a_2, a_3 > 0$), where p denotes the price at which the firm sells each unit of its product, q the quantity it produces, Q the average quantity in the market, and θ an exogenous demand shifter. Individual profits are given by $u = pq - C(q)$, where $C(q) = c_1q + c_2q^2$ is the cost function (with $c_1, c_2 > 0$).

This model is nested in our framework with $k \equiv q$, $K \equiv Q$ and

$$U(k, K, \sigma_k, \theta) = (a_0 - c_1 + a_1\theta - a_3K)k - (a_2 + c_2)k^2,$$

It is easy to check that $\phi = \frac{\alpha}{2(1-\alpha)} < 0$; under complete information, both the monopoly and the Cournot quantity increase with the demand intercept, but the monopoly one less so than the Cournot one. Moreover, $\alpha^* = 2\alpha < \alpha < 0$; firms would be better off if they were to perceive a stronger degree of strategic substitutability in their quantity choices and thereby increase their reliance on private information. Using these results together with the formulas for the bounds $\bar{\phi}'$ and $\bar{\phi}$ (see the Appendix), we can show that $\phi > \bar{\phi}'$ and $\phi > \bar{\phi}$. By Proposition 8 and 9, then, total profits increase with accuracy and decrease with commonality. This ensures that expected profits necessarily increase with the precision of private information, but opens the door to the possibility that they decrease with that of public. In the Appendix we verify that this is possible if $\alpha < -1$.

Corollary 10 *In the Cournot game described above, firms’ actions are strategic substitutes, but less so than what is collectively optimal (i.e., $\alpha^* < \alpha < 0$). Expected total profits necessarily increase with the precision of private information, but can decrease with that of public.*

Next, consider Bertrand. Demand is now given by $q = b_0 + b_1\theta' - b_2p + b_3P$, where q denotes the quantity sold by the firm, p the price the firm sets, P the average price in the market, and θ' an exogenous demand shifter ($b_0, b_1, b_2, b_3 > 0$); we naturally impose $b_3 < b_2$, so that an equal increase in p and P reduces q . Individual profits are $u = pq - C(q)$, where $C(q) = c_1q + c_2q^2$ (with $c_1, c_2 > 0$).

This model is nested in our framework with $k \equiv p - c_1$, $K \equiv P - c_1$ (actions are now prices), and

$$U(k, K, \sigma_k, \theta) = b_2[(\theta - k + bK)k - c(\theta - k + bK)^2],$$

where $\theta \equiv b_0/b_2 + b_1/b_2\theta' - c_1(1 - b)$, $b \equiv b_3/b_2 \in (0, 1)$, and $c \equiv c_2b_2 > 0$; without loss of generality, we let $b_2 = 1$. It is easy to check that $\phi > 0$, meaning that the Bertrand price reacts too little to θ as compared to the monopoly price, and that $\alpha^* > \alpha > 0$, meaning that firms would be better off if they were to perceive a stronger complementarity in their pricing decisions. It follows that expected profits increase with both the accuracy and the commonality of information. This immediately implies that more precise public information necessarily increases expected profits; that ϕ is sufficiently high turns out to ensure that the same is true also for private information.

Corollary 11 *In the Bertrand game described above, firms' actions are strategic complements, but less so than what is collectively optimal (i.e., $\alpha^* > \alpha > 0$). Expected total profits increase with the precision of both public and private information.*

If we interpret information sharing among firms as an increase in the precision of public information, then the aforementioned results imply that information-sharing is profit-enhancing under Bertrand competition, but not necessarily under Cournot competition. This result is closely related to Vives (1984, 1990) and Raith (1996) who examine the impact of information-sharing in Cournot and Bertrand oligopolies with a finite number of firms.¹⁹

7 Concluding remarks

This paper examined equilibrium and welfare in a rich class of economies with externalities, strategic complementarity or substitutability, and dispersed information.

Certain modeling choices—namely the quadratic specification for the payoff structure and the Gaussian specification for the information structure—were dictated by the need for tractability, but do not appear to be essential for the main insights. We expect our analysis to be a good benchmark also for more general environments with a unique equilibrium and concave payoffs.²⁰

On the other hand, the restrictions to unique equilibrium and concave payoffs are essential for our results. First, when complementarities are strong enough that multiple equilibria emerge under common knowledge, then the information structure matters not only for the local properties of any given equilibrium but also for the determinacy of equilibria (e.g., Morris and Shin, 2003);

¹⁹For example, it is easy to check that Raith's payoff specification is nested in our framework with $\alpha^* = 2\alpha$ and $\phi > \max\{\bar{\phi}, \bar{\phi}'\}$. If it were not for the difference in the number of players and the information structure, his Proposition 4.2 would be a special case of our Propositions 8 and 9.

²⁰Indeed, an interesting extension is to check whether our results are second-order approximations of this more general class of economies.

the social value of information may then critically depend on equilibrium selection (e.g., Angeletos and Pavan 2004, Sec. 3). And second, when aggregate welfare exhibits convexity over some region, society may prefer a lottery to the complete-information equilibrium;²¹ when this is the case, more noise in public information may improve welfare to the extent that aggregate volatility mimics such a lottery. Therefore, multiple equilibria and payoff convexities introduce effects that our model has ruled out. Extending the analysis in these directions is an interesting, but also challenging, next step for future research.

Another promising direction is extending the analysis to environments with endogenous information structures. This is interesting, not only because the endogeneity of information is important per se, but also because inefficiencies in the *use* of information are likely to interact with inefficiencies in the *collection* or *aggregation* of information. For example, in economies with a high social value for coordination, the private collection of information can reduce welfare by decreasing the correlation of expectations across agents and thereby hampering coordination. Symmetrically, in environments where substitutability is important, the aggregation of information through prices or other channels could reduce welfare by increasing correlation in beliefs.

The aforementioned extensions are important for developing a more complete picture of the welfare properties of large economies with heterogeneous information. The use of the efficiency benchmark identified in this paper as an instrument to assess these welfare properties is the core methodological contribution of this paper.

Appendix

Proof of Proposition 1. *Part (i).* Take any strategy $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ (not necessarily a linear one) and let $K(\theta, y) = \mathbb{E}[k(x, y) | \theta, y]$. A best-response is a strategy $k'(x, y)$ that solves, for all (x, y) , the first-order condition

$$\mathbb{E}[U_k(k', K, \sigma_k, \theta) | x, y] = 0. \quad (21)$$

Using $U_k(k', K, \sigma_k, \theta) = U_k(\kappa, \kappa, 0, \theta) + U_{kk} \cdot (k' - \kappa) + U_{kK} \cdot (K - \kappa)$, where κ stands for the complete-information equilibrium allocation and the fact that κ solves $U_k(\kappa, \kappa, 0, \theta) = 0$ for all θ , (21) reduces to

$$\mathbb{E}[U_{kk} \cdot (k' - \kappa) + U_{kK} \cdot (K - \kappa) | x, y] = 0,$$

or equivalently $k'(x, y) = \mathbb{E}[(1 - \alpha)\kappa + \alpha K | x, y]$. In equilibrium, $k'(x, y) = k(x, y)$ for all x, y , which gives (6).

²¹Indeed, this is necessarily the case when welfare is locally convex around the complete-information equilibrium and the lottery has small variance and expected value equal to the complete-information equilibrium.

Part (ii). Because $\mathbb{E}[\theta|x, y]$, and hence $\mathbb{E}[\kappa|x, y]$, is linear in (x, z) , it is natural to look for a solution to (6) that is linear in x and z , where $z = \lambda y + (1 - \lambda)\mu$. Thus suppose

$$k(x, y) = a + bx + cz \quad (22)$$

for some $a, b, c \in \mathbb{R}$. Then $K(\theta, y) = a + b\theta + cz$ and (6) reduces to

$$k(x, y) = (1 - \alpha)\kappa_0 + \alpha a + [(1 - \alpha)\kappa_1 + \alpha b] \mathbb{E}[\theta|x, y] + \alpha cz$$

Substituting $\mathbb{E}[\theta|x, y] = (1 - \delta)x + \delta z$, we conclude that (22) is a linear equilibrium if and only if a, b and c solve $a = (1 - \alpha)\kappa_0 + \alpha a$, $b = (1 - \delta)[(1 - \alpha)\kappa_1 + \alpha b]$, and $c = \delta[(1 - \alpha)\kappa_1 + \alpha b] + \alpha c$. Equivalently $a = \kappa_0$, $b = \kappa_1(1 - \alpha)(1 - \delta)/[1 - \alpha(1 - \delta)]$, and $c = \kappa_1\delta/[1 - \alpha(1 - \delta)]$. Note that $b + c = \kappa_1$ always; $b = c = 0$ whenever $\kappa_1 = 0$; and $b_1 \in (0, \kappa_1)$ and $c \in (0, \kappa_1)$ whenever $\kappa_1 \neq 0$. Letting $\gamma \equiv c/\kappa_1 \in (0, 1)$ gives (7)-(8). ■

Proof of Corollary 1. From condition (7), $k - K = \kappa_1[(1 - \gamma)(x - \theta)]$ and $K - \kappa = \kappa_1\gamma(z - \theta)$. Using $Var(x - \theta) = \sigma_x^2$, $Var(z - \theta) = \sigma_z^2 = (\sigma_y^{-2} + \sigma_\theta^{-2})^{-1}$ and $\delta = \sigma_z^{-2}/\sigma^{-2}$, together with (8), we have

$$Var(k - K) = \kappa_1^2 [(1 - \gamma)^2 \sigma_x^2] = \kappa_1^2 \left[\frac{(1 - \alpha)^2 (1 - \delta)}{(1 - \alpha + \alpha\delta)^2} \sigma^2 \right], \quad (23)$$

$$Var(K - \kappa) = \kappa_1^2 \gamma^2 \sigma_z^2 = \kappa_1^2 \left[\frac{\delta}{(1 - \alpha + \alpha\delta)^2} \sigma^2 \right]. \quad (24)$$

It is then easy to check that $\partial Var(k - K)/\partial \alpha < 0 < \partial Var(K - \kappa)/\partial \alpha$, which proves the result. For future reference, also note that both volatility and dispersion increase with σ , whereas dispersion decreases with δ if and only if $\alpha > -\frac{1}{1 - \delta}$ and volatility increases with δ if and only if $\alpha < \frac{1}{1 + \delta}$. ■

Proof of Condition (9). Given any strategy $k : \mathbb{R}^2 \rightarrow \mathbb{R}$, ex-ante utility is given by

$$\mathbb{E}u = \int_{(\theta, y)} \int_x U(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) dP(x|\theta, y) dP(\theta, y)$$

where $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$ and $\sigma_k(\theta, y) = [\int_x [k(x, y) - K(\theta, y)]^2 dP(x|\theta, y)]^{1/2}$. (To economize on notation, we henceforth suppress the dependence of k, K, σ_k on x, θ, y .) A second-order Taylor expansion around $k = K$ gives

$$U(k, K, \sigma_k, \theta) = U(K, K, \sigma_k, \theta) + U_k(K, K, \sigma_k, \theta) \cdot (k - K) + \frac{U_{kk}}{2} \cdot (k - K)^2.$$

It follows that ex-ante utility can be rewritten as

$$\mathbb{E}u = \int_{(\theta, y)} W(K, \sigma_k, \theta) dP(\theta, y) \quad (25)$$

A second-order Taylor expansion of $W(K, \sigma_k, \theta)$ around $K = \kappa^*$ and $\sigma_k = 0$ gives

$$W(K, \sigma_k, \theta) = W(\kappa^*, 0, \theta) + W_K(\kappa^*, 0, \theta) \cdot (K - \kappa^*) + W_\sigma(\kappa^*, 0, \theta) \sigma_k + \frac{W_{KK}}{2} (K - \kappa^*)^2 + \frac{W_{\sigma\sigma}}{2} \sigma_k^2.$$

By definition of κ^* , $W_K(\kappa^*, \theta) = 0$. Along with the fact that $W_\sigma(\kappa^*, 0, \theta) = 0$ and the fact that $\int_{(\theta, y)} \sigma_k^2(\theta, y) dP(\theta, y) = \mathbb{E}[k(x, y) - K(\theta, y)]^2$ gives the result. ■

Proof of Proposition 2. The Lagrangian for the program in Definition 3 can be written as

$$\begin{aligned} \Lambda = & \int_{(\theta, y)} \int_x U(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) dP(x|\theta, y) dP(\theta, y) \\ & + \int_{(\theta, y)} \lambda(\theta, y) [K(\theta, y) - \int_x k(x, y) dP(x|\theta, y)] dP(\theta, y) \\ & + \int_{(\theta, y)} \eta(\theta, y) [\sigma_k^2(\theta, y) - \int_x (k(x, y) - K(\theta, y))^2 dP(x|\theta, y)] dP(\theta, y) \end{aligned}$$

Because the program is concave, the solution is given by the first order conditions for $K(\theta, y)$, $\sigma_k(\theta, y)$, and $k(x, y)$:

$$\int_x [U_K(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) + \lambda(\theta, y) + 2\eta(\theta, y)(k(x, y) - K(\theta, y))] dP(x|\theta, y) = 0 \quad (26)$$

for almost all (θ, y)

$$\int_x U_\sigma(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) dP(x|\theta, y) + 2\eta(\theta, y)\sigma_k(\theta, y) = 0 \quad (27)$$

for almost all (θ, y)

$$\int_\theta [U_k(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) - \lambda(\theta, y) - 2\eta(\theta, y)(k(x, y) - K(\theta, y))] dP(\theta|x, y) = 0 \quad (28)$$

for almost all (x, y)

where $P(\theta|x, y)$ denotes the cumulative distribution function of an agent's posterior about θ given (x, y) . Noting that $U_K(k, K, \sigma_k, \theta)$ is linear in its arguments and using $K(\theta, y) = \int_x k(x, y) dP(x|\theta, y)$, condition (26) can be rewritten as $-\lambda(\theta, y) = U_K(K(\theta, y), K(\theta, y), \sigma_k(\theta, y), \theta)$. Next, noting that $U_\sigma(k, K, \sigma_k, \theta) = U_{\sigma\sigma}\sigma_k$, condition (27) can be rewritten as $-2\eta(\theta, y) = U_{\sigma\sigma}$. Replacing $\lambda(\theta, y)$ and $\eta(\theta, y)$ into (28), we conclude that an allocation $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ is efficient if and only if, for almost all (x, y) , it satisfies

$$\begin{aligned} \mathbb{E}[& U_k(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta) + \\ & + U_K(K(\theta, y), K(\theta, y), \sigma_k(\theta, y), \theta) + U_{\sigma\sigma}[k(x, y) - K(\theta, y)] \mid x, y] = 0. \end{aligned} \quad (29)$$

Consider now part (i) in the proposition. Because U is quadratic in (k, K, θ) and linear in σ_k^2 , condition (29) can be rewritten as

$$\begin{aligned} \mathbb{E}[& U_k(\kappa^*, \kappa^*, 0, \theta) + U_{kk} \cdot (k(x, y) - \kappa^*) + U_{kK} \cdot (K - \kappa^*) + \\ & + U_K(\kappa^*, \kappa^*, 0, \theta) + (U_{kK} + U_{KK}) \cdot (K - \kappa^*) + U_{\sigma\sigma}(k(x, y) - K(\theta, y)) \mid x, y] = 0. \end{aligned}$$

Using $U_k(\kappa^*, \kappa^*, 0, \theta) + U_K(\kappa^*, \kappa^*, 0, \theta) = 0$, by definition of the first-best allocation, the above reduces to

$$\mathbb{E}[U_{kk}(k(x, y) - \kappa^*) + (2U_{kK} + U_{KK})(K - \kappa^*) + U_{\sigma\sigma}(k(x, y) - K(\theta, y)) \mid x, y] = 0,$$

which gives (11).

Next, consider part (ii). Uniqueness follows from the fact that the planner's problem in Definition 3 is strictly concave. The characterization follows from the same steps as in the proof of Proposition (1) replacing α with α^* and $\kappa(\cdot)$ with $\kappa^*(\cdot)$. ■

Proof of Proposition 3. Consider first part (ii). Because the (unique) efficient allocation of \mathbf{e} is linear, *only* the linear equilibrium of the economy \mathbf{e}' can coincide with the efficient allocation of the true economy \mathbf{e} . Now, take any U' satisfying $\alpha' \equiv -U'_{kK}/U_{kk} < 1$. By Proposition 1, any equilibrium of $\mathbf{e}' = (U'; \sigma, \delta, \mu, \sigma_\theta)$ is a function $k(x, y)$ that solves

$$k(x, y) = \mathbb{E}[(1 - \alpha')\kappa' + \alpha'K(\theta, y) \mid x, y] \quad \forall (x, y), \quad (30)$$

where $\kappa'(\theta) = \kappa'_0 + \kappa'_1\theta_1$ is the unique solution to $U'_k(\kappa', \kappa', 0, \theta) = 0$ and $K(\theta, y) = \mathbb{E}[k(x, y) \mid \theta, y]$. The unique linear solution to (30) is the function

$$k(x, y) = \kappa'_0 + \kappa'_1 [(1 - \gamma')x + \gamma'z],$$

where $\gamma' = \delta + \frac{\alpha'\delta(1-\delta)}{1-\alpha'(1-\delta)}$. For this function to coincide with the efficient allocation of \mathbf{e} for all (x, y) , it is necessary and sufficient that $\kappa'(\cdot) = \kappa^*(\cdot)$ and $\alpha' = \alpha^*$, which proves part (ii).

For part (i), it suffices to let

$$U'(k, K, \sigma_k, \theta) = U(k, K, \sigma_k, \theta) + U_K(K, K, \sigma_k, \theta)k,$$

in which case it is immediate that $\kappa'(\cdot) = \kappa^*(\cdot)$ and $\alpha' = \alpha^*$. ■

Proof of Proposition 4. The result follows directly from the proof of Proposition 3 together with the definitions of $\kappa(\cdot)$, $\kappa^*(\cdot)$, α and α^* . ■

Proof of Proposition 5. Consider the set \mathcal{K} of linear strategies that satisfy

$$k(x, y) = \mathbb{E}[(1 - \alpha')\kappa + \alpha'K \mid x, y]$$

for some $\alpha' < 1$, where $K(\theta, y) = \int_x k(x, y) dP(x \mid \theta, y)$ for all (θ, y) . Such strategies have the structure $k(x, y) = \kappa_0 + \kappa_1 [(1 - \gamma')x + \gamma'z]$, where $\gamma' = \delta + \frac{\alpha'\delta(1-\delta)}{1-\alpha'(1-\delta)}$. Clearly, the equilibrium (and hence also the efficient) allocation is nested with $\alpha' = \alpha (= \alpha^*)$. For any strategy in \mathcal{K} , $\mathbb{E}u = \mathbb{E}W(\kappa, 0, \theta) - \frac{|W_{\sigma\sigma}|}{2}\Omega$ where

$$\Omega \equiv \frac{|W_{KK}|}{|W_{\sigma\sigma}|} \text{Var}(K - \kappa) + \text{Var}(k - K).$$

Using $|W_{KK}|/|W_{\sigma\sigma}| = 1 - \alpha^*$ together with the formulas (23)-(24) for dispersion and volatility (replacing γ with γ'), we have that

$$\Omega = \kappa_1^2 \left\{ (1 - \alpha^*) \frac{\gamma'^2}{\delta} + \frac{(1 - \gamma')^2}{1 - \delta} \right\} \sigma^2.$$

Note that $\mathbb{E}u$ depends on α' and (δ, σ) only through Ω . Because the efficient allocation is nested with $\alpha' = \alpha^*$, it must be that $\alpha' = \alpha^*$ maximizes $\mathbb{E}u$, or equivalently that $\gamma' = \gamma^*$ solves $\partial\Omega/\partial\gamma' = 0$; that is,

$$(1 - \alpha^*) \frac{\gamma^*}{\delta} = \frac{1 - \gamma^*}{1 - \delta}. \quad (31)$$

Next note that Ω increases with σ , and hence $\mathbb{E}u$ decreases with σ (equivalently, increases with the accuracy σ^{-2}). Finally, consider the effect of δ . By the envelope theorem,

$$\frac{d\Omega}{d\delta} = \frac{\partial\Omega}{\partial\delta} \Big|_{\gamma'=\gamma^*} = \kappa_1^2 \left\{ -\frac{(1 - \alpha^*)\gamma^{*2}}{\delta^2} + \frac{(1 - \gamma^*)^2}{(1 - \delta)^2} \right\} \sigma^2$$

Using (31), we thus have that $d\mathbb{E}u/d\delta > [<]0$ if and only if $\gamma^*/(1 - \gamma^*) > [<] \delta/(1 - \delta)$, which is the case if and only if $\alpha^* > [<]0$. Using $\alpha = \alpha^*$ (by efficiency) then gives the result. ■

Proof of Proposition 6. Part (i) follows from the Blackwell-like argument in the main text; it can also be obtained by noting that

$$\mathcal{L}^* = \omega \kappa_1^2 \left\{ \frac{(1 - \alpha) \sigma_x^2 \sigma_z^2}{\sigma_x^2 + (1 - \alpha) \sigma_z^2} \right\},$$

where $\omega \equiv |W_{\sigma\sigma}|/2$, and hence

$$\begin{aligned} \frac{\partial \mathbb{E}u}{\partial \sigma_z^{-2}} &= -\frac{\partial \mathcal{L}^*}{\partial \sigma_z^2} \left(-\frac{1}{[\sigma_z^{-2}]^2} \right) = \omega \kappa_1^2 \frac{(1 - \alpha) \sigma_x^2 \sigma_z^4}{[\sigma_x^2 + (1 - \alpha) \sigma_z^2]^2} > 0 \\ \frac{\partial \mathbb{E}u}{\partial \sigma_x^{-2}} &= -\frac{\partial \mathcal{L}^*}{\partial \sigma_x^2} \left(-\frac{1}{[\sigma_x^{-2}]^2} \right) = \omega \kappa_1^2 \frac{(1 - \alpha)^2 \sigma_z^2 \sigma_x^4}{[\sigma_x^2 + (1 - \alpha) \sigma_z^2]^2} > 0 \end{aligned}$$

Part (ii) is then immediate. ■

Proof of Proposition 7. Equilibrium welfare is $\mathbb{E}u = \mathbb{E}W(\kappa, 0, \theta) - \mathcal{L}^*$, where

$$\mathcal{L}^* = \frac{|W_{\sigma\sigma}|}{2} \{ (1 - \alpha^*) \text{Var}(K - \kappa) + \text{Var}(k - K) \} \quad (32)$$

$$= -\frac{|W_{\sigma\sigma}|}{2} (\alpha^* - \alpha) \text{Var}(K - \kappa) + \frac{|W_{\sigma\sigma}|}{2} \{ (1 - \alpha) \text{Var}(K - \kappa) + \text{Var}(k - K) \}. \quad (33)$$

(i) Because $\text{Var}(K - \kappa)$ and $\text{Var}(k - K)$ are both increasing in σ , welfare necessarily decreases with σ (equivalently, increases in accuracy σ^{-2}).

(ii) Consider the "canonical case" in which $\text{Var}(k - K)$ is decreasing and $\text{Var}(K - \kappa)$ increasing in δ . By Proposition 5, the second term in (33) decreases with δ if $\alpha > 0$ and increases if $\alpha < 0$. It follows that $\alpha^* \geq \alpha > 0$ suffices for \mathcal{L}^* to decrease (and hence welfare to increase) with δ , whereas $\alpha^* \leq \alpha < 0$ suffices for \mathcal{L}^* to increase (and hence welfare decrease) with δ . ■

Proof of Condition (15). By (25), we have $\mathbb{E}u = \mathbb{E}W(K, \sigma_k, \theta)$. A Taylor expansion of $W(K, \sigma_k, \theta)$ around $K = \kappa$ and $\sigma_k = 0$ gives

$$W(K, \sigma_k, \theta) = W(\kappa, 0, \theta) + W_K(\kappa, 0, \theta)(K - \kappa) + \frac{W_{KK}}{2}(K - \kappa)^2 + W_\sigma(\kappa, 0, \theta)\sigma_k + \frac{W_{\sigma\sigma}}{2}\sigma_k^2.$$

Using the fact that $W_\sigma(\kappa, 0, \theta) = 0$ and the fact that $\mathbb{E}[\sigma_k^2] = \mathbb{E}[(k - K)^2]$ we thus have that

$$\mathbb{E}u = \mathbb{E}W(\kappa, 0, \theta) + \mathbb{E}[W_K(\kappa, 0, \theta) \cdot (K - \kappa)] + \frac{W_{KK}}{2} \cdot \mathbb{E}[(K - \kappa)^2] + \frac{W_{\sigma\sigma}}{2} \cdot \mathbb{E}[(k - K)^2].$$

In equilibrium, $\mathbb{E}k = \mathbb{E}K = \mathbb{E}\kappa$ and therefore, $\mathbb{E}[W_K(\kappa, 0, \theta) \cdot (K - \kappa)] = \text{Cov}[W_K(\kappa, 0, \theta), (K - \kappa)]$, $\mathbb{E}[(K - \kappa)^2] = \text{Var}(K - \kappa)$ and $\mathbb{E}[(k - K)^2] = \text{Var}(k - K)$, which gives the result. ■

Proof of Propositions 8 and 9. We prove the two results together, in three steps. Step 1 computes the welfare losses due to incomplete information. Step 2 derives the comparative statics. Step 3 characterizes the bounds $\underline{\phi}$, $\bar{\phi}$, $\underline{\phi}'$, $\bar{\phi}'$.

Step 1. The property that W is quadratic, along with the fact that $W_K(\kappa^*, 0, \theta) = 0$ (by definition of the first best), and $W_{KK} < 0$, imply that

$$W_K(\kappa, 0, \theta) = W_K(\kappa^*, 0, \theta) + W_{KK} \cdot (\kappa - \kappa^*) = |W_{KK}| \cdot (\kappa^* - \kappa).$$

It follows that

$$\text{Cov}(K - \kappa, W_K(\kappa, 0, \theta)) = |W_{KK}| \cdot \text{Cov}(K - \kappa, \kappa^* - \kappa). \quad (34)$$

Because $K - \kappa = \kappa_1 \gamma(z - \theta)$, $z - \theta = [\lambda(\varepsilon) + (1 - \lambda)(\mu_\theta - \theta)]$, and ε and θ are mutually orthogonal, we have that

$$\text{Cov}(K - \kappa, \kappa^* - \kappa) = \text{Cov}(\kappa_1 \gamma(z - \theta), (\kappa_1^* - \kappa_1) \theta) = (\kappa_1^* - \kappa_1) \kappa_1 \gamma \text{Cov}(\theta, z - \theta).$$

Using $\phi \equiv (\kappa_1^* - \kappa_1)/\kappa_1$, $\gamma = \delta/(1 - \alpha + \alpha\delta)$, and $\text{Cov}(\theta, z - \theta) = -(1 - \lambda) \text{Var}(\theta) = -(\sigma_\theta^{-2}/\sigma_z^{-2})\sigma_\theta^2 = -\sigma_z^2 = \sigma^2/\delta$, we have that

$$\text{Cov}(K - \kappa, \kappa^* - \kappa) = \phi \left\{ -\frac{1}{1 - \alpha + \alpha\delta} \kappa_1^2 \sigma^2 \right\} \quad (35)$$

while

$$\text{Cov}(K - \kappa, \kappa) = \kappa_1^2 \gamma \text{Cov}(z - \theta, \theta) = -\frac{1}{1 - \alpha + \alpha\delta} \kappa_1^2 \sigma^2.$$

Substituting (34), (35), (23) and (24) into (15), using $v = (1 - \alpha^*)|W_{\sigma\sigma}|$, and rearranging, one obtains that

$$\mathcal{L} = \omega \Lambda(\alpha, \alpha^*, \phi, \delta) \kappa_1^2 \sigma^2 \quad (36)$$

where $\omega \equiv |W_{\sigma\sigma}|/2$ and

$$\Lambda(\alpha, \alpha^*, \phi, \delta) \equiv \frac{(1 - \alpha^*) [2\phi(1 - \alpha + \alpha\delta) + \delta] + (1 - \alpha)^2(1 - \delta)}{(1 - \alpha + \alpha\delta)^2}. \quad (37)$$

Step 2. $\mathbb{E}W(\kappa, 0, \theta)$ is independent of (δ, σ) and hence the comparative statics of welfare with respect to (δ, σ) coincide with the opposite of those of \mathcal{L} . Also note that

$$\frac{\partial \mathcal{L}}{\partial \sigma^2} = \omega \kappa_1^2 \Lambda(\alpha, \alpha^*, \phi, \delta) \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \delta} = \omega \kappa_1^2 \sigma^2 \frac{\partial \Lambda(\alpha, \alpha^*, \phi, \delta)}{\partial \delta}.$$

We thus only need to understand the sign of Λ and that of $\partial\Lambda/\partial\delta$.

Note that $\Lambda > [<] 0$ if and only if $\phi > [<] g(\alpha, \alpha^*, \delta)$, where

$$g(\alpha, \alpha^*, \delta) = -\frac{(1-\alpha)^2(1-\delta) + \delta(1-\alpha^*)}{2(1-\alpha + \alpha\delta)(1-\alpha^*)} < 0.$$

Letting

$$\underline{\phi}'(\alpha, \alpha^*) \equiv \min_{\delta \in [0,1]} g(\alpha, \alpha^*, \delta) \quad \text{and} \quad \bar{\phi}'(\alpha, \alpha^*) \equiv \max_{\delta \in [0,1]} g(\alpha, \alpha^*, \delta),$$

we then have that $\partial\mathcal{L}/\partial\sigma^2 > 0$ [< 0] for all $\delta \in [0, 1]$ if $\phi > \bar{\phi}'$ [$< \underline{\phi}'$], whereas $\partial\mathcal{L}/\partial\sigma^2$ alternates sign as δ varies if $\phi \in (\underline{\phi}', \bar{\phi}')$.

Next, consider the effect of commonality. By condition (37),

$$\frac{\partial\Lambda}{\partial\delta} = \frac{\alpha^2[(1-\delta)(1-\alpha) - \delta] - \alpha^*(1-\alpha - \alpha\delta) - 2\alpha\phi(1-\alpha^*)(1-\alpha + \alpha\delta)}{(1-\alpha + \alpha\delta)^3}.$$

When $\alpha = 0$, this reduces to $\partial\Lambda/\partial\delta = -\alpha^*$ and hence $\partial\mathcal{L}/\partial\delta > [<] 0$ if and only if $\alpha^* < [>] 0$.

When instead $\alpha \neq 0$,

$$\frac{\partial\Lambda}{\partial\delta} = \frac{2(1-\alpha^*)}{[1-\alpha + \alpha\delta]^2} \alpha[f(\alpha, \alpha^*, \delta) - \phi],$$

where

$$f(\alpha, \alpha^*, \delta) \equiv \frac{\alpha^2[(1-\delta)(1-\alpha) - \delta] - \alpha^*(1-\alpha - \alpha\delta)}{2\alpha(1-\alpha + \alpha\delta)(1-\alpha^*)}.$$

Because $\alpha^* < 1$, $\text{sign}[\partial\mathcal{L}/\partial\delta] = \text{sign}[\alpha] \cdot \text{sign}[f(\alpha, \alpha^*, \delta) - \phi]$. Let

$$\underline{\phi}(\alpha, \alpha^*) \equiv \min_{\delta \in [0,1]} f(\alpha, \alpha^*, \delta) \quad \text{and} \quad \bar{\phi}(\alpha, \alpha^*) \equiv \max_{\delta \in [0,1]} f(\alpha, \alpha^*, \delta).$$

If $\phi \in (\underline{\phi}, \bar{\phi})$, then $\partial\mathcal{L}/\partial\delta$ alternates sign as δ varies between 0 and 1, no matter whether $\alpha > 0$ or $\alpha < 0$. Hence, $\phi < \underline{\phi}$ is necessary and sufficient for $\partial\mathcal{L}/\partial\delta > 0 \forall \delta$ when $\alpha > 0$ and for $\partial\mathcal{L}/\partial\delta < 0 \forall \delta$ when $\alpha < 0$, whereas $\phi > \bar{\phi}$ is necessary and sufficient for $\partial\mathcal{L}/\partial\delta < 0 \forall \delta$ when $\alpha > 0$ and for $\partial\mathcal{L}/\partial\delta > 0 \forall \delta$ when $\alpha < 0$.

Step 3. Note that both f and g are monotonic in δ , with

$$\frac{\partial f}{\partial\delta} = 2\frac{\partial g}{\partial\delta} = \frac{(1-\alpha)}{(1-\alpha^*)(1-\alpha + \alpha\delta)^2}(\alpha^* - \alpha)$$

When $\alpha^* = \alpha$, both f and g are independent of δ , and

$$\underline{\phi}'(\alpha, \alpha^*) = \underline{\phi}(\alpha, \alpha^*) = \bar{\phi}(\alpha, \alpha^*) = \bar{\phi}'(\alpha, \alpha^*) = -\frac{1}{2} < 0.$$

When instead $\alpha^* > \alpha$, both f and g are strictly increasing in δ , so that

$$\begin{aligned} \underline{\phi}(\alpha, \alpha^*) &= f(\alpha, \alpha^*, 0) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1), \\ \underline{\phi}'(\alpha, \alpha^*) &= g(\alpha, \alpha^*, 0) < \bar{\phi}'(\alpha, \alpha^*) = g(\alpha, \alpha^*, 1), \end{aligned}$$

and when $\alpha^* < \alpha$, both f and g are strictly decreasing in δ , so that

$$\begin{aligned}\underline{\phi}(\alpha, \alpha^*) &= f(\alpha, \alpha^*, 1) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) \\ \underline{\phi}'(\alpha, \alpha^*) &= g(\alpha, \alpha^*, 1) < \bar{\phi}'(\alpha, \alpha^*) = g(\alpha, \alpha^*, 0).\end{aligned}$$

Consider first the case $\alpha \in (0, 1)$. If $\alpha^* > \alpha$, then $\alpha^2 + (1 - 2\alpha)\alpha^* > 0$ (using the fact that $\alpha^* < 1$) and therefore

$$\underline{\phi}(\alpha, \alpha^*) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1) = -\frac{\alpha^2 + (1 - 2\alpha)\alpha^*}{2\alpha(1 - \alpha^*)} < 0.$$

If instead $\alpha^* < \alpha$, then

$$\underline{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1) = -\frac{\alpha^2 + (1 - 2\alpha)\alpha^*}{2\alpha(1 - \alpha^*)} < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) = -\frac{\alpha^* - \alpha^2}{2\alpha(1 - \alpha^*)}$$

and therefore $\underline{\phi} < 0$ if and only if $\alpha > 1/2$ or $\alpha^* > -\alpha^2/(1 - 2\alpha)$, while $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$. Because $-\alpha^2/(1 - 2\alpha) < 0$ whenever $\alpha < 1/2$, we conclude that, for $\alpha \in (0, 1)$, $\underline{\phi} < 0$ if and only if $\alpha > 1/2$ or $\alpha^* > -\alpha^2/(1 - 2\alpha)$, and $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$.

Next, consider the case $\alpha \in (-\infty, 0)$. If $\alpha^* > \alpha$, then

$$\underline{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) = \frac{\alpha^* - \alpha^2}{(-2\alpha)(1 - \alpha^*)} < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1) = \frac{\alpha^2 + (1 - 2\alpha)\alpha^*}{(-2\alpha)(1 - \alpha^*)}$$

and hence $\underline{\phi} < 0$ if and only if $\alpha^* < \alpha^2$, while $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1 - 2\alpha)$. If instead $\alpha^* < \alpha$, then $\alpha^* < 0 < \alpha^2$ and hence

$$\underline{\phi}(\alpha, \alpha^*) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) = \frac{\alpha^* - \alpha^2}{(-2\alpha)(1 - \alpha^*)} < 0.$$

We conclude that, for $\alpha \in (-\infty, 0)$, $\underline{\phi} < 0$ if and only if $\alpha^* < \alpha^2$, and $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1 - 2\alpha)$.

Finally, note that

$$g(\alpha, \alpha^*, 0) = -\frac{(1 - \alpha)}{2(1 - \alpha^*)} < 0 \quad \text{and} \quad g(\alpha, \alpha^*, 1) = -\frac{1}{2} < 0.$$

Hence, $\underline{\phi}' = -\frac{(1 - \alpha)}{2(1 - \alpha^*)} < -1/2 = \bar{\phi}'$ for $\alpha^* > \alpha$, $\underline{\phi}' = \bar{\phi}' = -1/2$ for $\alpha = \alpha^*$, and $\underline{\phi}' = -1/2 < \bar{\phi}' = -\frac{(1 - \alpha)}{2(1 - \alpha^*)} < 0$ for $\alpha^* < \alpha$. ■

Proof of Corollary 5. Using the formula for the \mathcal{L} function given in the proof of Propositions 8 and 9, we have that, after some tedious algebra,

$$\frac{\partial \mathcal{L}}{\partial \sigma_z^2} = \omega \kappa_1^2 \sigma_x^4 \left\{ \frac{(1 - \alpha^*) \sigma_x^2 + (1 - \alpha)(1 - 2\alpha + \alpha^*) \sigma_z^2}{[\sigma_x^2 + (1 - \alpha) \sigma_z^2]^3} + 2\phi \frac{(1 - \alpha^*)}{[\sigma_x^2 + (1 - \alpha) \sigma_z^2]^2} \right\} \quad (38)$$

$$\frac{\partial \mathcal{L}}{\partial \sigma_x^2} = \omega \kappa_1^2 \sigma_z^4 (1 - \alpha) \left\{ \frac{(1 - \alpha - 2\alpha^*) \sigma_x^2 + (1 - \alpha)^2 \sigma_z^2}{[\sigma_x^2 + (1 - \alpha) \sigma_z^2]^3} + 2\phi \frac{(1 - \alpha^*)}{[\sigma_x^2 + (1 - \alpha) \sigma_z^2]^2} \right\} \quad (39)$$

where $\omega \equiv |W_{\sigma\sigma}|/2$. The result is then immediate. ■

Proof of Corollary 10. That welfare increases with private information follows from the property that $\alpha < 0$ and $\phi > \bar{\phi}$ (which ensures that welfare decreases with commonality) and the property that $\phi > \bar{\phi}'$ (which ensures that welfare increases with accuracy). As for the effect of public information, substituting $\alpha^* = 2\alpha$ and $\phi = \frac{\alpha}{2(1-\alpha)}$ in (36)-(37), we have that

$$\mathcal{L} = \frac{\sigma_x^2 \sigma_z^2 [(1-2\alpha) \sigma_x^2 + (1-2\alpha + \alpha^3) \sigma_z^2]}{(1-\alpha) (\sigma_x^2 + \sigma_z^2) (\sigma_x^2 + (1-\alpha) \sigma_z^2)}$$

and hence

$$\frac{\partial \mathcal{L}}{\partial \sigma_z^2} = \frac{\sigma_x^2 [(1-\alpha^2) (1-\alpha)^2 \sigma_z^4 + (1-2\alpha) \sigma_x^4 + 2(1-2\alpha + \alpha^3) \sigma_x^2 \sigma_z^2]}{(1-\alpha) (\sigma_x^2 + \sigma_z^2)^2 [\sigma_x^2 + (1-\alpha) \sigma_z^2]^2}$$

Note that the denominator is always positive. When $\alpha \in [-1, 0)$, the numerator is also positive for all σ_x and σ_z . When instead $\alpha < -1$, we can find values for σ_x and σ_z such that the numerator is negative. (Indeed, it suffices to take σ_z high enough, for then the term $(1-\alpha^2) (1-\alpha)^2 \sigma_z^4$, which is negative when $\alpha < -1$, necessarily dominates the other two terms in the numerator.) It follows that the social value of public information is necessarily positive when $\alpha \in [-1, 0)$, but can be negative when $\alpha < -1$. ■

Proof of Corollary 11. That welfare necessarily increases with public information follows directly from the fact that $\alpha^* > \alpha > 0$ and $\phi > 0$. For the social value of private information, after some tedious algebra, it is possible to show that

$$\frac{\partial \mathcal{L}}{\partial \sigma_x^2} = \frac{\sigma_z^4 [\lambda_1 \sigma_x^4 + \lambda_2 \sigma_x^2 \sigma_z^2 + \lambda_3 \sigma_z^4]}{2(1+c)(1+2c)(\sigma_x^2 + \sigma_z^2)^2 [(b+2bc) \sigma_z^2 - 2(1+c)(\sigma_x^2 + \sigma_z^2)]^2}$$

where λ_1, λ_2 , and λ_3 are positive functions of b and c . (This result has been obtained with Mathematica; the code and the formulas for the λ 's are available upon request.) It follows that welfare also increases with the precision of private information. ■

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