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## **ORIGINAL RESEARCH PAPER**



# What to offer if consumers do not want what they need? A simultaneous evaluation approach with an application to retirement savings products

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# Abstract

Standard economic models of rational decision making provide information on how people should decide. In practice, human decisions are influenced by numerous behavioral patterns that lead to systematic deviations from rationally optimal behavior. In the context of retirement savings, this can result in substantial pension gaps, and hence in a reduction of the standard of living in the retirement phase. The aim of this work is to introduce a general framework to (simultaneously) assess and evaluate the objectively rational utility and the subjectively perceived attractiveness. We illustrate the approach by means of an application to retirement savings products. Such a combined approach can help to identify or design retirement savings products that create a high (albeit not the maximum possible) objective utility while at the same time being subjectively of high (albeit not maximum possible) attractiveness. We argue that a focus on such products might lead to improved consumer decisions compared to observed decisions that are often driven by subjective attractiveness (resulting in rather low objective utility).

**Keywords** Retirement savings · Old-age provision · Utility maximization · Behavioral insurance · Prospect theory · Simultaneous evaluation

JEL Classification  $D14 \cdot G11 \cdot G22 \cdot G41 \cdot J26 \cdot J32$ 

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# 1 Introduction

Decision-making processes depend on many factors, e.g., available choices, personal characteristics and motives, the importance and complexity of the decision, the decision situation, etc., and often involve a trade-off between being accurate and spending as little time and effort as possible, cf., Tversky and Kahneman [62], Payne et al. [48], and Kahneman and Tversky [36]. Heuristics (consciously and subconsciously applied) often help people to make acceptable decisions with reasonable effort, cf., Kahneman et al. [37] and Gigerenzer [25]. However, especially in the case of complex decisions under uncertainty, like in the context of retirement planning, heuristics may not work very well. As a consequence, intuitively attractive choices may not be very suitable to achieve the objective goal of the decision. In the literature, this is known as systematic error or bias, cf., Tversky and Kahneman [60]. Accordingly, for such decisions there is a considerable risk that the subjectively most attractive choice at the time of decision making may be objectively suboptimal, cf., Kahneman and Klein [34].

In reality, we observe that people tend to spend more time on important decisions (which can also include seeking for expert advice). For example, many people consult advisors to support their decision-making on retirement planning. This can help to incorporate information in a more accurate way, and, ideally, lead to objectively better decisions, cf., Harvey and Fischer [30]. However, since many heuristics affect the decision process subconsciously, certain objectively disadvantageous attributes of a choice can still have a (strong) subjective attractiveness, cf., Chaiken and Maheswaran [15]. This is particularly relevant for decisions that have to be made only once in a lifetime (or very rarely) for which it is not possible to learn from own experience. For example, myopic loss aversion makes retirement savings products which reduce (or eliminate) short-term losses, e.g., by annual guarantees, which are very popular in many countries, although they can significantly reduce long term returns and are (at least for most people) objectively not optimal, cf., Benartzi and Thaler [4] and Ruß and Schelling [50]. Further, the results of Ruß and Schelling [50] indicate that in the case of retirement savings, the objectively optimal choice is perceived as significantly less attractive than the subjectively most attractive choice. In such situations, the subjective attractiveness of specific attributes may dominate the decision to such an extent that even opposing information and expert advice can have only limited impact on the decision. This is demonstrated by the high market share of retirement savings products<sup>1</sup> with annual guarantees in many markets, e.g., Germany, although most of this products are bought after a financial advisor had been consulted.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup> Note that by retirement savings products, we mean both, products that pay a lump sum at maturity, as well as the deferment period of deferred annuities.

<sup>&</sup>lt;sup>2</sup> Most retirement savings products in Germany are sold with the involvement of some type of financial advisor - the market share of direct sales in 2020 was only 3.4% (in terms of premiums of new contracts), cf., GDV [23]. When it comes to product types, unit-linked or index-linked products with guarantees have a significant market share. If, for example, we exclude products that are not for retirement savings (like term life insurance, etc.) from the tables on pages 10/11 respectively 8/9 in GDV [24], their market share is 46% for products with single premium respectively of 56% for products with regular premium

In these cases, minimizing the possible negative emotions associated with the choice seems to play an important role (such emotions cannot only arise when making a choice against the subjectively attractive choice but also in the opposite scenario), cf., Larrick [40], Luce et al. [44], and Luce et al. [45], Loewenstein et al. [41]. A "suitable compromise between" the subjectively attractive and the objective choice might be preferable in such a case, since negative emotions will probably be smaller than with the theoretically optimal decision and the decision is objectively better than the subjectively preferred one.

Vast literature shows that people often consider various attributes of choices and tend to make compromises. Most of the literature, however, considers the willingness for compromise in terms of attributes like price vs. quality (rather than in terms of subjective attractiveness vs. objective utility), cf., Simonson [54] and Simonson and Tversky [56]. Overall, the results of this stream of literature indicate that many people prefer choices which are compromising or middle option in the set of considered options. Verbeke [64] analyzes in the willingness for compromise on taste for health in the case of functional foods, which is closer related to the question analyzed in our paper. Bell et al. [3] describe that people "may be viewed as having divided minds with different aspirations," and "that decision making, even for the individual, is an act of compromise among the different selves". In this spirit, a targeted offer of "compromise products" and tailored advice of these products could help people to make objectively better decision (compared to their subjectively most attractive choice) also in retirement planning. However, it is essential that a suitable compromise exists and that it is included in the set of considered options, otherwise the decision could also be to postpone or to opt for the status quo, cf., Simonson [55] and Luce [43].

In this paper, we introduce and discuss an approach which simultaneously considers two different preference formulations; one to asses the preference from an objective, rational point of view, and one to asses the subjective attractiveness at the time of decision making. Our proposed approach is generally applicable (for example with respect to the underlying preference formulations) and is aimed to provide practical aid to identify and design suitable compromises. To the best of our knowledge, there are no similar (quantitative) approaches (e.g., optimization approaches, specific measures or classification systems) in the existing literature which evaluate both aspects simultaneously.

We illustrate our approach by analyzing common unit-linked retirement savings products (with and without terminal guarantees as well as with annual lock-in guarantee features) using the same setting as Ruß and Schelling [50].<sup>3</sup> We apply

Footnote 2 (continued)

payment. Note that the products sold in Germany in this segment mostly fall in the category of static and dynamic hybrid products (cf., Kochanski and Karnarski [39]) or certain equity-index-linked products (cf., Alexandrova et al. [1]), whereas in this paper we use stylized guaranteed unit-linked products to make our results applicable in a more general framework.

<sup>&</sup>lt;sup>3</sup> Note that in this paper, we do not consider the application to the annuity payout phase. The specification used for the savings phase cannot be immediately transferred to products paying lifelong or other regular benefits, since, for example, it would require the use of a different certainty equivalent value as the concept of a certainty equivalent return used for the savings phase cannot directly be applied in

Multi Cumulative Prospect Theory (MCPT) which is based on the popular Cumulative Prospect Theory (CPT) introduced by Kahneman and Tversky [35] and Tversky and Kahneman [63], and which can explain observed decisions of long investment horizon-investors more accurately, cf., Ruß and Schelling [50].<sup>4</sup> To assess the objective preference from a rational point of view we consider Expected Utility Theory (EUT).

The application illustrates how products that constitute a suitable compromise between objective utility and subjective attractiveness can be identified and designed. We perform a detailed analysis for various parametrizations of the underlying preference formulations, e.g., with respect to the individual's risk aversion and loss aversion. The results of this application show that loss aversion has a significant influence on the suggested ("favorable") compromise. For moderate values of loss aversion the favorable product combines attributes of the optimal EUT and the preferable MCPT product, which are high stock ratios and low annual guarantees. Hence, in these cases the approach identifies suitable compromises which create a high (albeit not the maximum possible) objective utility while being subjectively appealing. For a high loss aversion, the favorable compromise is very similar to the preferable MCPT product and has an objective utility which is far below the maximum. In this case, our approach indicates limits for the willingness to compromise, i.e., settings in which the subjective attractiveness of certain attributes dominate to such an extent that it is very unlikely that the individual will choose an objectively better product. Especially in these cases, it seems essential to apply further measures to objectively improve the decision, e.g., reducing the impact of loss aversion by suitable information, framing, etc. Overall, the application illustrates how the approach can contribute to the design as well as consulting of retirement savings products to improve decisions.

The remainder of this paper is organized as follows. In Sect. 2, we motivate the approach, discuss limitations, and introduce the conceptual framework of the simultaneous evaluation. In Sect. 3, we consider the application to retirement savings. To this end, we specify the simultaneous evaluation approach, the underlying preference formulations, as well as the capital market and the considered products. Further, we present and discuss the main findings as well as various sensitivity analyses. Sect. 4 summarizes and gives an outlook for future research.

Footnote 3 (continued)

these circumstances. Moreover, the measurement of subjective (and possibly also objective) attractiveness would need to be adapted. For an alternative possibility to measure objective attractiveness, cf., Alonso-García and Sherris [2]; subjective attractiveness of regular payouts may heavily depend on the way the respective products are framed, cf., Schelling [53] for potential impact of framing on subjective attractiveness. Nevertheless, the topic becomes increasingly important since innovative (particularly unitlinked guaranteed) products in the retirement phase become more and more popular in many countries, cf., ifa [33] for an overview over respective products in Germany.

<sup>&</sup>lt;sup>4</sup> In contrast to CPT, MCPT can explain the demand for products with cliquet or ratch-up guarantees ( Ruß and Schelling [50]), the demand for so-called life-cycle funds which decrease their risk exposure over time Graf et al. [26], and the demand for traditional participating life insurance products which make use of collective return smoothing elements Ruß and Schelling [51].

# 2 Motivation and discussion of the method

In this section, we introduce and discuss the terms of subjective attractiveness and objective utility. Also, we discuss various settings, where our approach is applicable as well as limitations. Further, we identify desirable properties of a "combined preference function" that simultaneously evaluates objective utility and subjective attractiveness. Based on these properties, we develop in the next chapter an exemplary preference function that will be used in the numerical analyses in this paper.

## 2.1 On the preference formulations

Previous research has focused, on the one hand, on how individuals actually make decisions, and on the other hand on how individuals should ideally make decisions. This has given rise to two essential model approaches where the distinction in practice is not always sharp, cf. Slovic et al. [57] and Slovic et al. [58].

*Normative models* describe how individuals should make decisions. They measure the goodness (often stated in terms of *objective utility*) of a choice using objective and normative criteria under the most complete and correct information set possible. For example, typically normative models fulfill basic principles of rational decision making such as transitivity (if A is better than B, and B is better than C, then A is also better than C). Further, a useful normative model, which is able to describe how an individual should make decisions, can be tuned to the specific characteristics and needs of the individual under consideration (e.g., by the individuals risk aversion or specific constraints). In practice (e.g., for financial decisions), the objective utility of a choice under uncertainty is often measured based on the Expected Utility Theory (EUT).

Descriptive models aim to describe and predict actual decision making based on the subjective attractiveness. Subjective attractiveness refers to the individuals' own perception and evaluation of a choice which depends on cognitive capacities and limitations to process information. Mostly subconsciously applied heuristics help individuals to get an (initial) intuitive feeling about choices and to speed up the decision-making process. Common examples are availability, anchoring and the default heuristic, cf., Kahneman et al. [37] and Benartzi and Thaler [5]. While heuristics are often helpful to quickly derive reasonable decisions, they can in some cases lead to severe and systematic biases (i.e., systematic deviations compared to the objective choice of a normative model), cf., Tversky and Kahneman [60]. In particular, for complex choices under uncertainty, heuristics can lead to cognitive biases like (myopic) loss aversion or probability distortion, which can adversely affect the decision. For example, low probability events can have a disproportionate impact on the decision due to probability distortion. Hence, descriptive models like the popular Cumulative Prospect Theory (CPT) usually incorporate such insights from the field of psychology to better describe actual decision making, cf. Kahneman and Tversky [35] and Tversky and Kahneman [63].

It is worth noting that both approaches use models which can only approximate the target variables (objective utility respectively subjective attractiveness). Further, also primal normative models can be modified and applied in such a way that they recognize cognitive limitations and have a better descriptive power. For example, Subjective Expected Utility Theory considers subjective beliefs instead of true probabilities, cf., Savage [52]. Consequently, all applications discussed below depend on a careful and accurate model selection (tailored to the specific application).

#### 2.2 Why and when do we need a compromise?

Normative and descriptive models describe how people should make decisions and how they actually face decisions, respectively. A third stream of literature deals with the prescriptive side of decision making i.e., the question, how to help people to make objectively better decisions recognizing their cognitive biases, cf., Bell et al. [3]. One possibility is to present and describe the objective of the decision, the set of choices and related information in such a way that objectively better choices become more appealing. Ideally, such efforts result in objectively optimal decisions. Unfortunately, this is not always the case. Actual decisions can be influenced by subconsciously applied heuristics and negative emotions associated with the choice. Hence, objectively disadvantageous attributes of a choice can have a high subjective attractiveness adversely impacting on the decision. In such cases, compromises which address cognitive concerns but which are objectively better than the subjectively most attractive choice can improve the decision.

Compromises (in the above mentioned sense) are important whenever the objectively best choice is perceived as subjectively very unattractive and at the same time the subjectively attractive choice comes with a significantly lower objective utility. This includes complex decisions under uncertainty where it is difficult for the decision-maker to process relevant information accurately, where cognitive biases have a strong and crucial impact on the decision, and where it is difficult to help people to overcome their cognitive biases. In addition, it is important that the decision-making process allows for information gathering and an act of compromise, i.e., the decision should not be made quickly and solely intuitively. Otherwise, the intuitive choice with the highest subjective attractiveness will be chosen. This is usually not the case when it comes to important decisions, e.g., when a lot of money is at stake, and where people have a desire to make objectively good decisions which they do not regret later. Also, product providers have an interest to avoid unsatisfied customers and poor reputation in the long run. Hence, they should have an interest to steer their customers away from the subjectively most attractive (but objectively not optimal) product.

In what follows, we will present a model that can help identify suitable compromises in the above sense. This can be helpful for decisions that fulfill the following properties: Firstly, it should be a decision problem in which the objective utility and the subjective attractiveness of the choices can be determined quantitatively. Secondly, our approach is particularly useful when the design of a suitable compromise is not obvious. This is often the case when the choices have a variety of different attributes and specifications. For example, a compromise is rather obvious for an investor who intends to invest his money in bonds and stocks only. In this case, an equal split between bonds and stocks seems to be a reasonable compromise between a pure bond investment (which addresses the need for security) and a pure stock investment (which addresses the desire for gains). This compromise can also be observed to be appealing to many people.<sup>5</sup> However, the identification of a favorable compromise is significantly more difficult, if the set of the considered options also includes, for example, investments with guarantees of different types (e.g., annually or at maturity only) and different levels. Without quantitative methods, it is difficult to assess the combined impacts of the various different attributes on the objective utility and the subjective attractiveness. We therefore believe that particularly in such cases it is important to develop approaches that can identify a suitable compromise, which can then be marketed by product providers and recommended by advisors whenever biases prevent the optimal choice.

#### 2.3 The simultaneous evaluation approach

For the sake of simplicity, we assume a set C of  $n \in \mathbb{N}$  different choices (e.g., products)  $c_i$  for i = 1, ..., n.<sup>6</sup> Let  $P_1$  and  $P_2$  denote two preference formulations. In the remainder of this paper we consider the case that  $P_1$  specifies a normative model and  $P_2$  a descriptive model.<sup>7</sup> Further, we assume that under both preference formulations we can derive unique finite real valued certainty equivalent values (e.g., a fixed amount, a fixed annual return, etc.) for all  $c_i \in C$  and that higher certainty equivalent values are preferred. The use of certainty equivalent values makes it possible to derive quantities under different models that have comparable ranges. We denote the certainty equivalent values of  $c_i$  by  $r_i^{P_1}$  and  $r_i^{P_2}$ . We say that choice  $c_i$  dominates  $c_j$  (j = 1, ..., n) if  $r_i^{P_1} \ge r_j^{P_1}$  and  $r_i^{P_2} \ge r_j^{P_2}$  with at least one of the relations being strict.

Further, we denote the *combined preference function* for the simultaneous evaluation by  $K : D_{P_1} \times D_{P_2} \to I$ , where  $D_{P_1}, D_{P_2}$  are intervals in  $\mathbb{R}$  and I is an interval in  $\mathbb{R} \cup \{-\infty, \infty\}$ . We then use the notation  $K(c_i) := K(r_i^{P_1}, r_i^{P_2})$  for the *K*-value of choice  $c_i$ . We say that  $c_i$  is strictly preferred over  $c_j$  (denoted as  $c_i > c_j$ ) if  $K(c_i) > K(c_j)$  and call the relation as *indifferent* if  $K(c_i) = K(c_j)$  (denoted as  $c_i > c_j$ ). If there is a  $c_i$  such that  $K(c_i) > K(c_j)$  for all  $c_j \in C$  then we call  $c_i$  the favorable compromise choice. Next, we introduce some desirable properties of the function K:

- 1. Completeness, i.e., for all  $c_i, c_i \in C$  we have either  $c_i > c_i, c_i \sim c_i$ , or  $c_i > c_i$ .
- 2. *Transitivity*, i.e., for all  $c_i, c_j, c_k \in C$  it holds that if  $K(c_i) \ge K(c_j)$  and  $K(c_j) \ge K(c_k)$  then  $K(c_i) \ge K(c_k)$ .

<sup>&</sup>lt;sup>5</sup> Balanced funds with a with an approximately equal allocation (50/50 or 60/40) are among the most popular fund investments, cf., Morningstar [46].

<sup>&</sup>lt;sup>6</sup> Note that under some additional assumptions the approach can also be extended to a setting with infinitely many choices.

<sup>&</sup>lt;sup>7</sup> Note that our approach is not restricted to this consideration. For example, one could specify both  $P_1$  and  $P_2$ , under EUT, but with different specifications or subjective beliefs.

The properties 1) and 2) ensure that the relation > defines a *preference order* and are fulfilled since *K* maps to  $\mathbb{R} \cup \{-\infty, \infty\}$ .

- 3. The preference function *K* suggests only choices which lay on the "efficient frontier", i.e., a choice that is dominated by some other choice can never be the favorable compromise choice, since there exists a better choice. Consequently, *K* should prevail *preference dominance*, i.e., for all  $c_i, c_j \in C$  it holds that if  $c_i$  dominates  $c_j$  then  $K(c_i) > K(c_j)$ , and if  $r_i^{P_1} = r_j^{P_1}$  and  $r_i^{P_2} = r_j^{P_2}$  then  $K(c_i) = K(c_j)$ .
- 4. The combined preference function K is aimed to identify suitable compromises. Hence, it should prefer choices with medium certainty equivalent values under both,  $P_1$  and  $P_2$  over choices with a very high value in one component and a very low value in the other. This can be ensured if the resulting indifference curves of K are (strictly) convex. The property is in line with findings that show that many people prefer choices which are compromising or middle option in the set of considered options, cf. Simonson and Tversky [56].
- 5. Lastly, the relation between two choices  $(c_i \text{ and } c_j)$  should not change if other choices are added to or removed from the set of choices C.<sup>8</sup> Note that, while the order of two choices under standard models of rational choice cannot be influenced by the set of choices, this may be different for specific descriptive models, e.g., Prospect Theory (if the reference point or the probability distortion depends on the set of choices), cf., e.g., Guevara and Fukushi [27] for an overview and other examples. Under such a descriptive model, property 5 will be violated (independent of the choice of *K*).

# 3 Application for retirement savings products

In this section, we apply our approach to retirement savings products. We closely follow the framework of Ruß and Schelling [50] and consider several types of common guaranteed products as well as a product without guarantee (Sect. 3.2). Further, we use the underlying preference formulations discussed therein to describe the objective and the subjective preferences (Sect. 3.3). Additionally, we specify a concrete choice of the function K (Sect. 3.4). Finally, we perform extensive numerical analyses (3.5), identify suitable compromises, and discuss the impact of various parameters.

<sup>&</sup>lt;sup>8</sup> Formally, this means for all sets of choices  $\mathcal{A}$  and  $\mathcal{B}$  with  $c_i, c_j \in \mathcal{A} \cap \mathcal{B}$  it holds that if  $K^{\mathcal{A}}(c_i) \Box K^{\mathcal{A}}(c_j)$ then  $K^{\mathcal{B}}(c_i) \Box K^{\mathcal{B}}(c_j)$  with  $\Box \in \{>, =\}$ . We say that the preference function K specifies preferences which are independent of other choices if this is true for all possible pairs  $c_i$  and  $c_j$ . The effect that the inclusion of a dominated choice affects the preference order of the remaining choices is documented in the literature as "attraction effect", "decoy effect" or "menu effect", cf., e.g., Huber et al. [32].

### 3.1 Motivation for the application

There is a large amount of literature on the optimal (rational utility-maximizing) design of retirement savings products (e.g., Branger et al. [10], Chen et al. [16]). However, findings from behavioral economics show that numerous human behavioral patterns can cause a deviation between actual and optimal decisions (e.g., Benartzi and Thaler [5], Beshears et al. [7], Brown et al. [13] or Richter et al. [49] and references therein). Due to its complexity and its long-term nature, decisions in the context of retirement savings seem particularly prone to such deviations. Consequently, the literature shows that there are significant deviations between actual and objectively optimal decisions in retirement savings (cf., e.g., Chen et al. [17], Fuino et al. [22], or Luca et al. [42]). In addition, non-optimal decisions in this context can result in considerable negative consequences on the standard of living in the retirement phase. Therefore, in recent years, an increasing number of studies have examined the question of how to support individuals to make objectively better decisions. For example, framing, i.e., the way products are presented and explained can possibly be used to make products with high objective utility more appealing (cf., Brown et al. [12], Beshears et al. [8] or Brown et al. [14]).

Nevertheless, existing literature in this field typically either analyzes optimal product design from a rational perspective (e.g., Nielsen and Steffensen [47]) or the question of the perceived subjective attractiveness of retirement savings products at the time of decision making (e.g., Dierkes et al. [20] or Ebert et al. [21]).<sup>9</sup> We argue that ideally, products should be designed in a way that constitutes a suitable compromise between creating a high (albeit not the maximum possible) objective utility and being subjectively appealing. This could help to reduce the deviation between the objective utility of the optimal decision and the decision actually made. Beyond product design, the approach can also help financial advisors by providing information about suitable compromise products. This can help to ensure that a compromise product is in the set of considered options at all.

#### 3.2 Financial market and considered products

We choose the financial market model and corresponding products identical to Ruß and Schelling [50]. Hence, we assume a Black and Scholes [9] financial market model with a risky asset *S* following a geometric Brownian motion with drift  $\mu > r \ge 0$  and volatility  $\sigma > 0$ , where *r* denotes the constant interest rate. The portfolio value process *V* invests the fraction  $\theta \in [0, 1]$  in the risky asset *S*, and the fraction  $1 - \theta$  in the risk free asset *B* with fixed annual return *r*. We assume continuous rebalancing.<sup>10</sup> The portfolio value process is the basis for all considered products.

<sup>&</sup>lt;sup>9</sup> This also applies to other fields of research, for example, in decision making in financial markets Kliger and Levy [38], energy markets Häckel et al. [28] or tax evasion Dhami & Al-Nowaihi (2007) [19]. Similar, for the broadly related stream of literature which focuses on comparing different risk measures, cf., Benoit et al. [6] or Emmer et al. [59].

<sup>&</sup>lt;sup>10</sup> For details on the financial market model we refer to Ruß and Schelling [50].

For all products, we assume a fixed maturity date *T* and a single premium of 1 paid at the beginning of the contract. The product that invests in the underlying *V* without guarantee is referred to as *constant mix (cm)* product. For the guaranteed contracts, the investment premium  $\alpha$  describes the fraction of the premium that is allocated to the investment *V*, while the remaining part  $1 - \alpha$  is used to finance the guarantee, where the guarantee rate is denoted by *g*. We only consider fair contracts with an identical initial arbitrage free price of 1.<sup>11</sup>

We consider three types of guarantees: a terminal guarantee and two different annual guarantee features. The payoff at maturity T of a product with a (terminal) roll-up guarantee feature is given by

$$A_T^{rol} := \max\left(e^{gT}, \alpha V_T\right) = \alpha V_T + \left[e^{gT} - \alpha V_T\right]^+.$$

Further, we consider products with annual guarantee features which aim to protect interim gains. The so-called *ratch-up* guarantee feature is defined by the following payoff:

$$A_T^{rat} := \max\left(e^{gT}, \alpha V_1, \dots, \alpha V_T\right)$$

This product essentially pays the highest portfolio value at any annual lock-in date or a roll-up with rate g, whichever is higher.

Moreover, we define the *cliquet* guarantee feature by the payoff:

$$A_T^{cli} := \alpha \prod_{i=1}^I \max\left(e^g, \frac{V_i}{V_{i-1}}\right).$$

In each period, this product earns the greater of the guaranteed rate and the performance of the underlying portfolio.

### 3.3 Choice of the preference formulations

In this section, we briefly introduce and discuss the considered normative and descriptive models which have been chosen to be in line with Ruß and Schelling [50].

We use EUT with power utility function  $u(x) = x^{\gamma}$  as a normative model which implies constant relative risk aversion (CRRA) for a risk aversion parameter  $\gamma \in (0, 1)$ . As descriptive model we use MCPT which can better explain observed behavior of long-term investors than CPT.<sup>12</sup> In particular, there are many popular long-term investment products which neither an EUT-investor nor a CPT-investor

<sup>&</sup>lt;sup>11</sup> To be in line with Ebert et al. [21] and Ruß and Schelling [50] we only consider products with  $\alpha \ge 0.6$ and use closed form solutions to calculate the fair annual guaranteed rate g for a given  $\alpha$ . For more details and closed form solutions for the arbitrage free prices of the different products we refer to Ruß and Schelling [50].

<sup>&</sup>lt;sup>12</sup> A more detailed description is given in Appendix A.

would buy (cf., Ebert et al. [21], Ruß and Schelling [50] or Graf et al. [26]).<sup>13</sup> Ruß and Schelling [50] show that the demand for these products can be explained by taking into account potential interim value changes which is possible under MCPT. MCPT is based on three main components: Firstly, an S-shaped value function, which allows to distinguish between gains and losses with respect to a certain reference point (which in our case is set to the initial premium paid). The risk attitude is controlled by the parameter *a* and the loss aversion by the parameter  $\lambda$ . Secondly, a probability distortion function with distortion parameter  $\zeta$  which particularly takes into account that small probabilities are overweighted in decision making. Thirdly, multiple reference points and evaluation periods which assumes that potential future value fluctuations affect a product's subjective attractiveness already at the time of decision making.

In both models we can derive certainty equivalent returns (CE returns) which describe the fixed annual return that an investor would regard equally desirable as the considered contract, cf., Appendix A. More precisely, under EUT, the CE return can be computed by  $r^{EUT} = \frac{1}{T} \ln \left( \left( E[X^{\gamma}] \right)^{\frac{1}{\gamma}} \right)$ , where X represents the terminal payoff of the considered choice. Under MCPT it holds that the CE return  $(r^{MCPT})$  solves the following equation

$$0 = \sum_{t=1}^{T} \rho^{t} \left( e^{r^{MCPT}t} - e^{r^{MCPT}(t-1)} \right)^{a} - MCPT(c) \quad \text{if } MCPT(c) \ge 0$$
$$0 = -\lambda \sum_{t=1}^{T} \rho^{t} \left| e^{r^{MCPT}t} - e^{r^{MCPT}(t-1)} \right|^{a} - MCPT(c) \quad \text{if } MCPT(c) < 0,$$

where MCPT(c) denotes the MCPT value of choice *c*. Note that for preference formulations which only consider the terminal payoff, like EUT, the CE return corresponds to the internal rate of return of the certainty equivalent payoff. However, MCPT-investors derive value from interim value changes (in our case annually), i.e., from the path which results in the terminal payoff. Consequently, the concept of a certainty equivalent payoff does not make sense here since a representative certainty equivalent value must consider interim value changes.<sup>14</sup> As we need to condense the information into one single number, this can either be done in terms of a constant absolute or relative interim value change. In line with Ruß and Schelling [50] we use the latter which is easy to apply and interpret in the context of investment decisions.<sup>15</sup> However, we also considered instead a constant absolute annual value change and found that the results are qualitatively very similar.

<sup>&</sup>lt;sup>13</sup> Note that while CPT often cannot satisfactorily explain the demand for retirement savings products for the savings period, there is vast literature which shows that it provides an explanation why annuitization rates are very low, cf., Hu and Scott [31], Brown et al. [11], and Chen et al. [17].

<sup>&</sup>lt;sup>14</sup> This is very similar to deriving constant consumption streams for preference models which consider actual consumption over multiple periods, cf., e.g., Cocco et al. [18].

<sup>&</sup>lt;sup>15</sup> Note that the calculation of the CE return is also possible in a setting with regular premiums, cf., Graf et al. [26].

### 3.4 The combined preference function

In this subsection, we propose an explicit combined preference function which will be used in our analysis. A natural starting point for the choice of the combined preference function K could be the weighted average or a simple transformation of it. However, it can easily be shown, that this - as well as other "simple" functions would violate some of the previously stated desired properties. In what follows we specify a combined preference function which fulfills all five properties defined above on the set of considered contracts. While we consider the suggested formulation of K appropriate for our applications, we do not claim that it is the only suitable or the best (in whatever sense) formulation.

Firstly, we restrict the set of considered choices C to products that yield at least a fixed minimal acceptable certainty equivalent return  $b \in [-1, \infty)$  under both preference formulations. If not stated otherwise, we set b = 0, i.e., we only consider products which have a non-negative CE return under both preference formulation.<sup>16</sup> This constraint is motivated by the fact that we do not consider products as suitable compromises that are particularly unattractive or objectively disadvantageous. Further, it simplifies the notation of the *K*-function.<sup>17</sup> Secondly, we define  $K : [b, \infty) \times [b, \infty) \rightarrow [0, \infty)$  as

$$K(c_i) := (1 - \omega) \left( r_i^{P_1} - b \right)^{\beta} + \omega \left( r_i^{P_2} - b \right)^{\beta}$$
(1)

with preference weight  $\omega \in [0, 1]$ . In the application, we focus on the case  $\omega = 0.5$ , which represents the situation that both preference formulations are equally weighted. Further,  $\beta \in (0, 1)$  determines the degree of convexity of the indifference curves<sup>18</sup> and we focus on the case  $\beta = 0.5$ . It is straightforward to show that the properties 1 - 5 hold for all  $c_i \in C$ , cf., Appendix B.

### 3.5 Numerical analysis

### 3.5.1 Base case

For the sake of comparability with Ebert et al. [21] and Ruß and Schelling [50], we consider a five-year investment horizon (T = 5) and the following financial parameters unless stated otherwise:  $\mu = 0.06$ ,  $\sigma = 0.3$ , r = 0.03.<sup>19</sup> In all subsequent figures

<sup>&</sup>lt;sup>16</sup> Note that if *b* depends on the set of choices (e.g., maximum or minimum) property 5 of the *K*-function could be violated. This can be seen with similar arguments as used to show that, e.g., descriptive models with a choice-depended reference point (to which gains and losses are evaluated) can explain the attraction effect, cf., e.g., Simonson and Tversky [56].

<sup>&</sup>lt;sup>17</sup> This constraint could alternatively also be taken into account by a suitable choice of the *K*-function for negative values. This constraint could alternatively be taken into account by a suitable choice of the *K*-function for negative values.

<sup>&</sup>lt;sup>18</sup> The lower  $\beta$ , the stronger the emphasis on a well balanced compromise.

<sup>&</sup>lt;sup>19</sup> Detailed descriptions of the product characteristics (guaranteed rates, distribution of terminal values, distribution of annual value changes etc.) can be found in Ebert et al. [21] and Ruß and Schelling [50].

and tables, a product without guarantee is denoted by  $\circ$ , a roll-up product is denoted by +, a ratch-up product is denoted by  $\Diamond$ , and a cliquet product is denoted by \*. We investigate 41 stock ratios  $\theta$  between 0% and 100% in steps of 2.5%. For products with a guarantee, we consider for each  $\theta > 0$  eight levels of  $\alpha$  between 0.6 and 0.95 in steps of 0.05. Thus, we investigate 41 different products without guarantee and  $40 \cdot 8 = 320$  different fairly priced products for each guarantee type.<sup>20</sup> If not stated otherwise, we assume a probability distortion of  $\zeta = 0.65$ , which is in line Tversky and Kahneman with [63], and consider different combinations of the other preference parameters ( $\lambda$ , a,  $\gamma$ ). All results are based on Monte-Carlo simulations with sample size of 20, 000. The favorable product is then determined by computing the CE return under both preference formulations for all products. Subsequently, for each product in the considered set (i.e., both CE returns are at least 0), the *K*-value is computed with Formula (1) and the favorable product is the one with the highest *K*-value.

When applying MCPT standalone, more complex guaranteed products, in particular certain cliquet products, are preferred in all considered cases, which is in line with the findings in Ruß and Schelling [50].

Based on these results we now investigate favorable compromises under the simultaneous evaluation approach. First, we assume that the risk aversion under EUT coincides with the risk attitude under MCPT, i.e.,  $\gamma = a$ , and we fix it at 0.88, as suggested by Tversky and Kahneman [63]. Note that this is a simplifying assumption as the risk aversion (under EUT) and the risk attitude (under MCPT) are influenced by different aspects. However, we will independently vary  $\gamma$ , a, as well as  $\lambda$  to analyze the impact of the different parameters. Further, we will discuss for which combinations it is possible to find a favorable compromises.

Figure 1 shows the MCPT and EUT CE returns of the products for different levels of loss aversion  $\lambda$ . In this figure, we observe that CE returns of the products follow certain lines, where one line of symbols represents one level of  $\alpha$  and all lines start at the same point (black triangle), where  $\theta = 0$  and  $r^{EUT} = r^{MCPT} = 3\%$ . Symbols at the outer end of each line have a higher stock ratio. Moreover, the vertical (horizontal) dotted line shows the highest possible CE under EUT (MCPT). The favorable compromise based on the combined preference function *K* is indicated by a red symbol. Further, the solid black lines are indifference curves under the K-measure. We observe that for increasing loss aversion the subjective attractiveness of products which allow for interim losses decreases while the subjective attractiveness of products with a positive annual guarantee remains unchanged. Hence, the favorable compromise also changes with increasing loss aversion.

Without loss aversion ( $\lambda = 1$ ), a cliquet product is favorable (stock ratio of 100%, annual guaranteed rate of -29.15%, i.e., only protecting against very severe annual

Footnote 19 (continued)

Note that the applications are presented in a rather simple model framework and therefore their aim is not to give actual advise for specific products but rather to illustrate the approach.

<sup>&</sup>lt;sup>20</sup> Note that fairly priced ratch-up features are not possible for all combinations of  $\theta$  and  $\alpha$ , cf., Ruß and Schelling [50] for more details.

losses). The stock ratio is the same as for the EUT optimal and MCPT preferable product. However, the guarantee of the favorable compromise is lower than for the MCPT preferable product (-19.58%, i.e., also only protecting against severe annual losses). Compared to the EUT optimal product the EUT CE is slightly lower (5.17% instead of 5.46%) whereas the MCPT CE is higher by about twice the difference (7.66% instead of 7.05%). On the other hand, the EUT CE is 0.25% higher than for the preferable MCPT product while the MCPT CE is only 0.1% lower.

If loss aversion is higher (1.25 or 1.75), the favorable compromise remains a cliquet product with  $\theta = 100$ , but the guaranteed rate increases to -16.4% or -9.4%, respectively. The MCPT CE returns of the favorable products exceed the MCPT CE returns of the EUT optimal product by a large amount (1.78% respectively 4.66%) while the EUT CE is reduced by a significantly smaller amount (0.66% respectively 1.03%). On the other hand, when the favorable compromise products are compared to the MCPT preferred products, MCPT CE's are only reduced by 0.23% respectively 0.47% whereas EUT CE increases by 0.37% respectively 0.97%. In a nutshell, the favorable compromise has a significantly higher EUT CE than the subjectively most attractive product, but remains attractive for MCPT investors.

For a high loss aversion of 2.25, the favorable compromise coincides with the MCPT preferable product and hence is not a compromise in a strict sense. It is a cliquet product with a positive guarantee (0.14%) and a stock ratio of 50% and comes with an EUT CE of 3.46% and an MCPT CE of 4.79%. This result suggests that a promotion of objectively better products alone might not be sufficient if consumer's subjective evaluations are heavily dominated by a rather high degree of loss aversion. However, in view of the results for a slightly lower loss aversion, it seems promising to take measures to help consumers reduce their loss aversion at least to some degree and combine these measures with product offerings that constitute a suitable compromise for consumers with a slightly lower degree of loss aversion.<sup>21</sup>

So to sum up this first example, the advantages of our approach can be seen particularly in the case  $\lambda = 1.75$ , where the favorable compromise combining a stock ratio of 100% (as for the optimal EUT product) and a guarantee (as for the preferable MCPT product) is almost as attractive as the most attractive product, but provides a significantly higher EUT CE. However, a heavy loss aversion can cause the favorable compromise to coincide with the most attractive product under MCPT. This shows the limitations of the approach and leads to the suggestion, that it should be used in combination with approaches that help consumers to overcome their loss aversion at least to some extent.

Next, we analyze the favorable compromise for different values of risk aversion (and attitude) as well as loss aversion. Table 1 displays the optimal product under EUT, the preferred product under MCPT and the favorable compromise under the K-measure for different levels of risk aversion and attitude as well as loss aversion.

We observe that the favorable compromise is always a cliquet or ratch-up product, since these products protect annual gains, which is important under MCPT, cf., also Fig. 1. When risk aversion or loss aversion (or both) are rather low (red cells in

<sup>&</sup>lt;sup>21</sup> For example, by framing in an appropriate manner, cf., Tversky and Kahneman [61].



**Fig. 1** The certainty equivalent returns of MCPT and EUT for different levels of loss aversion. The diamond  $(\diamondsuit)$  corresponds to ratch-up, the star (\*) to cliquet, the plus (+) to roll-up and the circle ( $\circ$ ) to constant mix products, where the symbols are bold if the corresponding product lies on the efficient frontier. The black triangle denotes a pure investment in the risk-free asset. The red symbol denotes the favorable compromise based on the function *K*. The dashed vertical line represents the highest EUT CE, while the dashed horizontal line represents the highest MCPT CE

Table 1) the favorable compromise has a stock ratio of 100% (like the EUT optimal product) and a low annual guarantee (similar to the preferable MCPT product).

For a higher loss aversion and a medium risk aversion (blue cells in Table 1) the favorable compromise is a cliquet product with an annual guaranteed rate slightly above 0 and a stock ratio of 50%. If risk aversion becomes lower, the guaranteed rate becomes negative and the stock ratio increases. This is mainly driven by the fact that for the MCPT preferable product the guaranteed rate decreases and the stock ratio increases as the upside potential is valued higher. On the other hand, if risk aversion becomes higher, the stock ratio decreases and guarantees increase. For very high risk aversion (green cells in Table 1) the favorable compromise is a cliquet product with a guaranteed rate above 1% (1.43% and 1.14%) and a stock ratio of roughly 40%, independent of loss aversion. The compromise has a slightly higher EUT CE than the preferable MCPT product, since the guarantee is lower.

All in all, we observe that the stock ratio (guarantee) of the favorable compromise is decreasing (increasing) for an increasing loss aversion or increasing risk aversion. Further, for a high loss aversion the compromise is very similar or equal to the

favorable compromise / preferable MCPT	EUT optimal
area the favorable compromise has a stock ratio of 50% and a annual guarantee slightly above 0	ow annual guarantee. In the blue
rea the favorable compromise has a annual guarantee above 1%. In the red area the favorable compromise has a stock ratio of 100% and a	stant mix products. In the green
ven as a range due to different loss aversions. The diamond ( $\Diamond$ ) corresponds to ratch-up, the star (*) to cliquet and the circle ( $\circ$ ) to con-	EUT product the MCPT CE is g
antee, the EUT CE and the MCPT CE are given in parenthesis, ( $\alpha$ in $\%$ , $\theta$ in $\%$ , $\beta$ in $\%$ , EUT CE in $\%$ , MCPT CE in $\%$ ). For the optimal	premium, the stock ratio, the gui
e based on the combined preference function K for different combination of risk aversion, risk attitude and loss aversion. The investmen	Table 1         The favorable comprom

	0				<i>c</i>	<i>r</i> - o	
	EUT optimal	favorabi	le compromise / preferable M <sup>i</sup>	CPT			
$\gamma = \gamma$		Y	1	1.25	1.75	2.25	2.75
0		К	*(60, 40, 1.43, 3.26, 3.65)	*(60, 40, 1.43, 3.26, 3.65)	*(60, 40, 1.43, 3.26, 3.65)	*(60, 40, 1.43, 3.26, 3.65)	*(60, 40, 1.43, 3.26, 3.65)
0.0	o(100, 82.9, -, 4.29, [1.47, -1.71])	MCPT	*(60, 37.5, 1.7, 3.22, 3.67)	*(60, 37.5, 1.7, 3.22, 3.67)	*(60, 37.5, 1.7, 3.22, 3.67)	*(60, 37.5, 1.7, 3.22, 3.67)	*(60, 37.5, 1.7, 3.22, 3.67)
10 0		K	$\Diamond(75, 100, -5.54, 4.08, 3.19)$	*(60, 42.5, 1.14, 3.3, 3.75)	*(60, 42.5, 1.14, 3.3, 3.75)	*(60, 42.5, 1.14, 3.3, 3.75)	*(60, 42.5, 1.14, 3.3, 3.75)
60.0	o(100, 90,, 4.43, [z.z9,z.01])	MCPT	*(60, 40, 1.43, 3.26, 3.77)	*(60, 40, 1.43, 3.26, 3.77)	*(60, 40, 1.43, 3.26, 3.77)	*(60, 40, 1.43, 3.26, 3.77)	*(60, 40, 1.43, 3.26, 3.77)
5	(100 100 1 EE [9 05 9 99])	K	$\diamond(75, 100, -5.54, 4.21, 4.04)$	*(60, 45, 0.82, 3.34, 3.88)	*(60, 45, 0.82, 3.34, 3.88)	*(60, 45, 0.82, 3.34, 3.88)	*(60, 45, 0.82, 3.34, 3.88)
3	o(100,100, -, -,00, [3.20, -3.33])	MCPT	$\diamond(75, 100, -5.54, 4.21, 4.04)$	*(60, 42.5, 1.14, 3.3, 3.91)	*(60, 42.5, 1.14, 3.3, 3.91)	*(60, 42.5, 1.14, 3.3, 3.91)	*(60, 42.5, 1.14, 3.3, 3.91)
11	(los c co la se la cort cort).	К	*(90, 100, -29.15, 4.71, 4.65)	$\Diamond(75, 100, -5.54, 4.35, 3.78)$	*(60, 47.5, 0.49, 3.39, 4.05)	*(60, 47.5, 0.49, 3.39, 4.05)	*(60, 47.5, 0.49, 3.39, 4.05)
e7.0	o(100, 100, -, 4.00, [4.20, -0.04])	MCPT	$\diamond(75, 100, -5.54, 4.35, 4.94)$	*(60, 45, 0.82, 3.35, 4.08)	*(60, 45, 0.82, 3.35, 4.08)	*(60, 45, 0.82, 3.35, 4.08)	*(60, 45, 0.82, 3.35, 4.08)
90	(TEC & OC 2] F 2 OOL OOL/-	К	*(90, 100, -29.15, 4.88, 5.78)	$\Diamond(75, 100, -5.54, 4.48, 4.71)$	*(60, 50, 0.14, 3.44, 4.28)	*(60, 50, 0.14, 3.44, 4.28)	*(60, 50, 0.14, 3.44, 4.28)
0.0	o(100, 100, -, 5.1, [3.20, -4.31])	MCPT	*(75, 100, -16.36, 4.59, 5.89)	$\diamond(75, 100, -5.54, 4.48, 4.71)$	*(60, 47.5, 0.49, 3.4, 4.31)	*(60, 47.5, 0.49, 3.4, 4.31)	*(60, 47.5, 0.49, 3.4, 4.31)
200	~/100 100 E 22 [2 27 4 70])	K	*(90, 100, -29.15, 5.06, 6.95)	$\diamond(75, 100, -5.54, 4.62, 5.68)$	*(60, 50, 0.14, 3.45, 4.6)	*(60, 50, 0.14, 3.45, 4.6)	*(60, 50, 0.14, 3.45, 4.6)
0.00	o(100, 100, -, 0.00, [0.01, -4.18])	MCPT	*(80, 100, -19.58, 4.83, 7.06)	*(60, 100, -9.38, 4.37, 5.79)	*(60, 50, 0.14, 3.45, 4.6)	*(60, 50, 0.14, 3.45, 4.6)	*(60, 50, 0.14, 3.45, 4.6)
00 0	~(100 100 E 46 [7 0E E 06])	К	*(90, 100, -29.15, 5.17, 7.66)	*(75, 100, -16.36, 4.8, 6.26)	*(60, 100, -9.38, 4.43, 4.32)	*(60, 50, 0.14, 3.46, 4.79)	*(60, 50, 0.14, 3.46, 4.79)
0010	o(100, 100, -, 0.40, [r.00, -0.00])	MCPT	*(80, 100, -19.58, 4.92, 7.76)	*(60, 100, -9.38, 4.43, 6.49)	*(60, 50, 0.14, 3.46, 4.79)	*(60, 50, 0.14, 3.46, 4.79)	*(60, 50, 0.14, 3.46, 4.79)
00	1001 001 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	К	*(95, 100, -38.05, 5.39, 7.96)	*(80, 100, -19.58, 4.98, 6.6)	*(60, 100, -9.38, 4.47, 4.84)	*(60, 50, 0.14, 3.46, 4.91)	*(60, 50, 0.14, 3.46, 4.91)
л. С	o(100, 100, -, 3:33, [7:3, -3:24])	MCPT	*(80, 100, -19.58, 4.98, 8.23)	*(60, 100, -9.38, 4.47, 6.95)	*(60, 50, 0.14, 3.46, 4.91)	*(60, 50, 0.14, 3.46, 4.91)	*(60, 50, 0.14, 3.46, 4.91)
0.05	(100 1 00 1 20 10 10 10 10 10 10 10 10 10 10 10 10 10	K	*(95, 100, -38.05, 5.59, 9.13)	*(80, 100, -19.58, 5.12, 7.84)	*(60, 100, -9.38, 4.56, 6.12)	*(60, 72.5, -3.64, 3.95, 4.91)	*(60, 52.5, -0.22, 3.52, 5.23)
0.90	o(100, 100, -, 5.75, [5.04, -5.09])	MCPT	*(80, 100, -19.58, 5.12, 9.38)	*(60, 100, -9.38, 4.56, 8.08)	*(60, 100, -9.38, 4.56, 6.12)	*(60, 52.5, -0.22, 3.52, 5.27)	*(60, 52.5, -0.22, 3.52, 5.23)
÷	~/100 100 E [0 76 E 04])	K	*(95, 100, -38.05, 5.79, 10.28)	*(85, 100, -23.64, 5.44, 8.89)	*(60, 100, -9.38, 4.66, 7.39)	*(60, 100, -9.38, 4.66, 5.43)	*(60, 57.5, -1, 3.64, 5.54)
+	o(100,100,,0,[a.00,001])	MCPT	*(85, 100, -23.64, 5.44, 10.51)	*(65, 100, -11.39, 31.81, 9.21)	*(60, 100, -9.38, 4.66, 7.39)	*(60, 70, -3.17, 3.93, 5.8)	*(60, 52.5, -0.22, 3.54, 5.64)

preferable product of the subjective preference function, because objectively more attractive products come with no or only weak guarantees, which is heavily penalized by a high loss aversion.

#### 3.5.2 Sensitivity analysis

In this section, we present results of various sensitivity analyses.

**3.5.2.1 Risk aversion and attitude** We will now have a closer look at different combinations of risk aversion ( $\gamma$ ) and risk attitude (*a*) on the favorable compromise by varying *a* and  $\gamma$  independently. In the base case, for simplicity we had assumed  $\gamma = a$ . However, while the risk aversion parameter  $\gamma$  in EUT is related to the total payoff, the risk attitude *a* in MCPT is related to interim value changes. Further, the risk attitude captures risk aversion as well as risk affinity and depends on the reference point, the loss aversion, as well as on the probability weighting. Hence, applying the same values for both does not make them automatically comparable, cf., Harrison and Rutström [29] for a discussion. Under EUT a risk aversion parameter  $\gamma = 0.88$  relates to a constant relative risk aversion of 0.12 which is at the lower end of typical estimates for the relative risk aversions (i.e., smaller values of  $\gamma$ ) in the sensitivity analysis. Again, we have performed the analysis for different levels of loss aversion, however, in the remainder we focus on the case of  $\lambda = 1.75$  where the results are most interesting. The results are shown in Table 2.

Firstly, we observe that the range of the optimal stock ratio under EUT varies between 37.5% ( $\gamma = 0.1$ ) and 100% ( $\gamma = 0.7$ ) depending on the risk aversion. Also, we observe that the favorable compromise is always a cliquet product with  $\alpha = 0.6$ with different stock ratios. While stock ratios are increasing in *a*, we find that the impact of the risk aversion on the favorable compromise is rather low (at least for  $\gamma \ge 0.1$ ). We observe for nearly all considered combinations of risk aversion and risk attitude, that the favorable compromise is the MCPT preferable product or a very similar product (only with a slightly higher stock ratio) since products with a higher EUT CE return have a significantly lower MCPT CE return. Only for a = 0.88 and  $\gamma = 0.7$  respectively 0.5, the favorable product is, as in the base case ( $a = \gamma = 0.88$ ), a product with a stock ratio of 100% respectively 72.5% (similar to the EUT optimal product) and the maximum possible guaranteed rate (similar to the MCPT preferred product).

**3.5.2.2 Preference weighting** Next, we have analyzed the effect of preference weighting  $(\omega)$ . We consider the base case with  $a = \gamma = 0.88$  and vary  $\lambda$  between 1.25 and 2.75 as well as  $\omega$  from 0 (pure EUT) to 1 (pure MCPT) for each level of loss aversion, cf., Fig. 2. Varying the preference weighting changes the indifference curves which results in different favorable compromises. By increasing  $\omega$ , the favorable compromise changes along the efficient frontier starting on the vertical line for  $\omega = 0$ .

We observe that with increasing loss aversion, the value of  $\omega$  above which the MCPT preferable product constitutes the favorable compromise decreases.

ment prei CE in %).	mium, the stock . The star (*) coi	ratio, the guarantee, the EUT CE an tresponds to cliquet and the circle (0)	d the MCPT CE of the products are gi to constant mix products	jiven in parenthesis, i.e., ( $\alpha$ in %, $\theta$ in	%, <i>g</i> in %, EUT CE in %, MCPT
a	favorable co	ompromise / preferable MCPT / EUT	optimal		
	X	0.1	0.3	0.5	0.7
0.65	K	*(60,42.5,1.14,3.23,3.75)	*(60,42.5,1.14,3.26,3.75)	*(60,42.5,1.14,3.28,3.75)	*(60,42.5,1.14,3.3,3.75)
	MCPT	* (60, 40, 1.43, 3.21, 3.77)	* (60, 40, 1.43, 3.23, 3.77)	* (60, 40, 1.43, 3.25, 3.77)	* (60, 40, 1.43, 3.27, 3.77)
	EUT	$\circ(100, 37.5, -, 3.56, 0.04)$	$\circ(100, 47.5, -, 3.71, -0.03)$	◦(100, 67.5, −, 4, −0.2)	o(100, 100, -, 4.65, -0.55)
0.75	K	*(60, 45, 0.82, 3.25, 4.08)	*(60,47.5,0.49,3.31,4.05)	*(60, 47.5, 0.49, 3.34, 4.05)	*(60,47.5,0.49,3.38,4.05)
			(00 / 00 C L0 U Z/ UZ/ -	(00 / 10 0 00 2/ 03/ ·	

**Table 2** The favorable compromise based on the combined preference function K for different combination of risk aversion and risk attitude and for  $\lambda = 1.75$ . The invest-

a	favorable co	ompromise / preferable MCPT / EUT o	ptimal		
	٢	0.1	0.3	0.5	0.7
0.65	Κ	*(60,42.5,1.14,3.23,3.75)	*(60,42.5,1.14,3.26,3.75)	*(60,42.5,1.14,3.28,3.75)	*(60,42.5,1.14,3.3,3.75)
	MCPT	* (60, 40, 1.43, 3.21, 3.77)	* (60, 40, 1.43, 3.23, 3.77)	* (60, 40, 1.43, 3.25, 3.77)	* (60, 40, 1.43, 3.27, 3.77)
	EUT	$\circ(100, 37.5, -, 3.56, 0.04)$	o(100, 47.5, -, 3.71, -0.03)	$\circ(100, 67.5, -, 4, -0.2)$	o(100, 100, -, 4.65, -0.55)
0.75	K	*(60, 45, 0.82, 3.25, 4.08)	*(60, 47.5, 0.49, 3.31, 4.05)	*(60, 47.5, 0.49, 3.34, 4.05)	*(60,47.5,0.49,3.38,4.05)
	MCPT	* (60, 45, 0.82, 3.25, 4.08)	* (60, 45, 0.82, 3.28, 4.08)	* (60, 45, 0.82, 3.31, 4.08)	* (60, 45, 0.82, 3.34, 4.08)
	EUT	$\circ(100, 37.5, -, 3.56, 0.22)$	◦(100, 47.5, −, 3.71, 0)	$\circ(100, 67.5, -, 4, -0.21)$	$\circ(100, 100, -, 4.65, -0.55)$
0.88	K	*(60, 50, 0.14, 3.29, 4.79)	*(60,50,0.14,3.33,4.79)	*(60,72.5,-3.64,3.62,4.54)	*(60,100,-9.38,4.1,4.32)
	MCPT	* (60, 50, 0.14, 3.29, 4.79)	* (60, 50, 0.14, 3.33, 4.79)	* (60, 50, 0.14, 3.38, 4.79)	* (60, 50, 0.14, 3.42, 4.79)
	EUT	$\circ(100, 37.5, -, 3.56, 0.75)$	$\circ(100, 47.5, -, 3.71, 0.36)$	$\circ(100, 67.5, -, 4, -0.07)$	$\circ(100, 100, -, 4.65, -0.55)$
0.95	K	*(60,92.5,-7.72,3.19,5.98)	*(60,100,-9.38,3.41,6.12)	*(60,100,-9.38,3.75,6.12)	*(60, 100, -9.38, 4.1, 6.12)
	MCPT	* (60, 100, -9.38, 3.08, 6.12)	* (60, 100, -9.38, 3.41, 6.12)	* (60, 100, -9.38, 3.75, 6.12)	* (60, 100, -9.38, 4.1, 6.12)
	EUT	$\circ(100, 37.5, -, 3.56, 1.16)$	$\circ(100, 47.5, -, 3.71, 0.78)$	o(100, 67.5, -, 4, 0.28)	$\circ(100, 100, -, 4.65, -0.55)$



**Fig. 2** The certainty equivalent returns of MCPT and EUT for different level of loss aversion. The diamond  $(\diamondsuit)$  corresponds to ratch-up, the star (\*) to cliquet, the plus (+) to roll-up and the circle ( $\circ$ ) to constant mix products, where the symbols are bold if the corresponding product lies on the efficient frontier. The red symbol denotes the favorable compromise for different values of  $\omega$ . The dashed vertical line represents the highest EUT CE, while the dashed horizontal line represents the highest MCPT CE

Moreover, the favorable compromise comes with lock-in features in most cases (except for  $\lambda = 1.25$  and  $\omega \le 0.15$ ), since loss aversion heavily reduces the attractiveness of products with terminal guarantees only (due to evaluation of annual changes in MCPT). In case of low loss aversion, the favorable compromise always has a stock ratio of 100%. If loss aversion is higher, the stock ratio decreases.

**3.5.2.3** Parameters of the combined preference function We have also analyzed the effect of different exponents  $\beta$  in the combined preference function *K*, which also influences the shape of the indifference curves. We find that reasonable choices of  $\beta$  do not significantly influence the favorable compromise.

Moreover, we find that for reasonable choices of the minimum constraint b the favorable compromises do not or only slightly change, i.e., only the stock ratio or the guarantee level of the favorable compromise might be slightly higher or lower.

**3.5.2.4 Probability distortion** Further, we have considered different values for the probability distortion parameter  $\zeta$ . If  $\zeta = 0.5$ , the favorable compromise is very similar to the base case, i.e., for medium and low risk aversion and attitude, there is a tendency for higher stock ratios (due to the strong overweighting of high gains). As a

consequence, the favorable compromises have a higher EUT CE than in the base case. If  $\zeta = 0.8$ , the favorable compromise is a pure stock investment in case of no loss aversion and a rather low risk aversion and attitude (which is also the optimal EUT product). For other loss aversion as well as combinations of risk aversion and attitude, the favorable compromise is similar as in the base case (only the stock ratio is slightly lower and the guarantee is slightly higher). The reason is that the probability of large gains is less overweighted and the probability of medium losses is less underweighted, which makes higher guarantees and lower stock ratios more attractive.

If there is no probability distortion, the favorable compromise for very risk averse consumers is a product without guarantee and a very low stock ratio (5% or 7.5%) for all levels of loss aversion, because the probability of a loss is close to zero. In this case, the favorable compromise comes with no guarantee (as the optimal EUT product) and a low stock ratio (as the preferable MCPT product). The favorable compromise is a pure stock investment in case of no or low loss aversion and rather low risk aversion and attitude (coinciding with the optimal EUT product). Otherwise, the favorable compromise is a cliquet product with similar guarantee rates (close to zero) as in the base case, but with a lower stock ratio, since the probabilities of large gains are not overweigthed. Again, the guarantees of the favorable compromises are lower than the guarantees of the preferable MCPT products.

**3.5.2.5 Financial market** Next, we have performed sensitivity analyses with respect to the financial market parameters  $\mu$ ,  $\sigma$  and r. If we change  $\sigma$  from 0.3 to 0.1, the favorable compromise is a cliquet product or a product without guarantee. For consumers with loss aversion, the K-measure suggests cliquet products with a stock ratio of 100% and a guaranteed rate between -6.6% and 0.5%, where the guaranteed rate is increasing for an increasing loss aversion. The stock ratios of the favorable compromise are higher than in the base case, because guarantees are less expensive as the volatility of the stock market is lower. Moreover, for high loss aversion, the favorable compromises are similar to the preferable MCPT product. For no loss aversion a pure stock investment (optimal EUT product) is favorable, since the probability of large losses is reduced by the low stock market volatility.

If the risk-free interest rate and the drift of the stock market are lower (r = 1% and  $\mu = 4\%$ ), as in the base case only cliquet and ratch-up products are favorable. For a low loss aversion and a high risk aversion the favorable compromise is a ratch-up with a stock ratio close to 100% and a low guarantee, i.e., if guarantees are more expensive, only protection against high annual losses is desired. Similarly, if loss aversion remains low and risk aversion is only medium or low, the stock ratio increases up to 100% and guarantees are further reduced to levels between -40.05% and -21.58%. For a high loss aversion, the K-measure suggests a cliquet product with a stock ratio of roughly 30% and a guarantee close to 0. Here, the loss aversion results in guarantees close to zero which can now only be afforded for low stock ratios.

**3.5.2.6 Time horizon** Next, we extended the time horizon of the investment period (T) to 15 years and computed the favorable compromises. We found a similar structure as for the base case: for low and medium loss aversion, the approach works well

and identifies suitable compromises with similar characteristics as in the base case whereas for a high loss aversion, the favorable compromise is the product with the highest subjective attractiveness.

**3.5.2.7 Combined preference function** We also analysed the effect of a different combined preference function K which considers the time value of money. More precisely, we consider the following K-function:

$$K(c_i) := \frac{1}{2}\sqrt{\left(1+r_i^{P_1}\right)^T - 1} + \frac{1}{2}\sqrt{\left(1+r_i^{P_2}\right)^T - 1}.$$
 (2)

Note that also this function fulfills the desired properties (1) - (5) and the proofs are similar as for the *K*-function in the base case. Based on this *K*-function, we have performed the same analysis as in Sect. 3.5.1 and we found that the favorable compromises are very similar as in the base case setting. This holds for all combinations of risk aversion, risk attitude and loss aversion.

**3.5.2.8 Subjective preference function** Finally, we changed the subjective preference function from MCPT to CPT, where only the terminal value is considered (cf., Appendix A). As in the base case, we investigate which product constitutes the most favorable compromise in the base case with  $a = \gamma = 0.88$  and  $\lambda$  between 1 and 2.25. Figure 3 shows the CPT and EUT CE returns of the products under consideration for different levels of loss aversion similar to Fig. 1.<sup>22</sup>

In case of no or low loss aversion ( $\lambda = 1$  or 1.25), the optimal product under EUT (pure stock investment) and the preferred product under CPT (roll-up with a stock ratio of 100% and only a very weak guaranteed benefit of 0.62) are very similar (and hence differ only marginally in their CE returns). As a consequence, also the favorable compromise is a pure stock investment.

For a medium loss aversion ( $\lambda = 1.75$ ), the favorable compromise still comes with a stock ratio of 100%, but now includes a roll-up feature with a guaranteed benefit of 0.62. Hence it combines features of the optimal EUT product (100% stock ratio) and the preferable CPT product (albeit with a lower guaranteed roll-up benefit than for the preferable CPT product). This again illustrates the benefit of the approach. We find that the objectively optimal product (pure stock investment) is significantly less attractive than the preferred product (CPT CE of 5.42% instead of 6.43%). Consequently, a consumer would very likely avoid this product (even if it is recommended by an advisor). On the other hand, the preferred product under CPT reduces the expected utility heavily compared to the optimal product (EUT CE of 4.05% instead of 5.46%). The favorable compromise product, however, is subjectively more attractive than the EUT-optimal product (CPT CE of 6.06% as opposed to 5.42%) while providing a significantly higher expected utility than the preferred product under CPT (EUT CE of 5.16% as opposed to 4.05%). Hence, promoting this product could help consumers to make an objectively better (although not optimal) choice.

 $<sup>^{22}</sup>$  Additionally, we have also analyzed the favorable compromise for different values of risk aversion (and attitude) as well as loss aversion in the CPT case. The results are displayed in Appendix C in Table 3 (in the same fashion as Table 1 in the MCPT case).

If loss aversion is even higher ( $\lambda = 2.25$ ), the subjective attractiveness under CPT of most products with low or without guarantees, including the EUT optimal product, is reduced heavily. As a consequence the preferred product under CPT coincides with the favorable compromise in this setting. This is a roll-up product with a stock ratio of 100% and a guaranteed benefit above the reference point. The EUT CE of this product is 4.05% and the CPT CE is 6.43%.<sup>23</sup>

To sum up, the proposed combined preference function can in most cases find a favorable compromise which is different to the most attractive product in the setting of retirement saving products. Also, it is possible to adjust the preference theories which describe the normative or descriptive model and still find compromises. Moreover, the results are robust to reasonable parameter changes of the proposed combined preference function  $(b, \beta, \text{ and } \omega)$ . Hence, the exact specification of *K* (which might be rather difficult to estimate) has less influence on the compromise than the (objective and subjective) preference formulations on which there is a large amount of literature.

# 4 Conclusion and outlook

In this paper, we have proposed an approach designed to identify choices that constitute a suitable compromise between a theoretically optimal choice (which might be rejected by consumers due to behavioral biases) and the subjectively most attractive choice (which might come with a rather low objective utility). Ideally, such a compromise should be subjectively more attractive than the objectively optimal choice while providing a higher objective utility than the subjectively preferred choice. We have stated desirable properties of such compromises and - based on these properties - proposed a methodology that can be applied using two arbitrary preference functions.

We have illustrated our approach in an applications to retirement savings products where EUT was used as an objective and (M)CPT as a subjective preference function and identified "favorable compromise products". The results of the applications show that the approach can in many cases identify suitable compromises which fit consumers needs while at the same time being subjectively attractive.

Our results indicate that the degree of loss aversion has a particularly high impact on the identified compromise product. For individuals with very low loss aversion, the suggested compromise is identical or very similar to the EUT optimal product. In such cases, no compromise might be required in the first place and thus no actions are necessary to improve the consumer's decision. For moderate values of loss aversion, the approach seems to work particularly well, identifying choices that combine characteristics of the objectively optimal product and the subjectively preferred product. As a consequence, the suggested

<sup>&</sup>lt;sup>23</sup> Since the guaranteed benefit of this product is greater than 1, no losses can occur and hence loss aversion has no impact on this product's CE return.



**Fig. 3** The certainty equivalent returns of CPT and EUT for different levels of loss aversion. The diamond  $(\diamondsuit)$  corresponds to ratch-up, the star (\*) to cliquet, the plus (+) to roll-up and the circle ( $\circ$ ) to constant mix products, where the symbols are bold if the corresponding product lies on the efficient frontier. The black triangle denotes a pure investment in the risk-free asset. The red symbol denotes the favorable compromise based on the function *K*. The dashed vertical line represents the highest EUT CE, while the dashed horizontal line represents the highest CPT CE

compromise products are subjectively more attractive than the objectively optimal choice while providing a higher objective utility than the subjectively preferred choice. This is an important result as it shows that we can find suitable compromises for many individuals. Hence, for individuals with reasonable degree of loss aversion, marketing the favorable compromise might already be sufficient to significantly improve consumers' decisions. For high loss aversion, however, the "favorable compromise" is given by the subjectively preferred product. Hence, for such individuals, the willingness to compromise might not be sufficient. We therefore propose to combine our approach with other measures which are suitable to reduce the degree of loss aversion at least to some extent. These measures may consist of appropriate financial advise/education or suitable framing which can reduce the fear of (interim) losses, or even of incentives (government subsidies, etc.) followed by marketing of products which are suitable compromises for consumers with more moderate loss aversion. Due to the demographic change in many countries, private old-age provision will be increasingly important to maintain a desired standard of living. Consequently, many countries promote private old-age provision (including occupational old-age provision) e.g., by government subsidized schemes or tax advantages. In this context, the development of retirement savings products that are accepted by consumers and at the same time objectively fit their needs is of high relevance. Hence our findings should be of importance to legislators, product providers, as well as financial advisors.

The paper provides numerous suggestions for further research: Firstly, it is important to investigate under which assumptions (e.g., with respect to the underlying preference formulations and their weights) and to what extent people will actually accept suitable compromises. In this context, it would also be interesting to investigate which additional actions can improve individuals' decision making. Also, it seems worthwhile to analyze whether the strong impact of loss aversion can be confirmed empirically. Secondly, future research should identify suitable compromises based on more realistic product designs (e.g., including products with collective savings elements that play an important role in many countries) and particularly including the decumulation period (i.e., the annuitization decision and the design of annuity products in the payout phase).

## Appendix A. (Multi) Cumulative Prospect Theory

In Cumulative Prospect Theory (CPT) an investment A with (random) final outcomes E is valued with an S-shaped value function v and with respect to a given reference point  $\chi$ . The gains and losses are described by the random variable  $X := E - \chi$ . Then the CPT utility is defined as

$$CPT(X) = \int_{-\infty}^{0} v(x)d(w(F(x))) + \int_{0}^{\infty} v(x)d(-w(1 - F(x))),$$
(3)

where  $F(x) = \mathbb{P}(X \le x)$  and v is the investor's value-function which is defined as  $v(x) := x^a \mathbb{1}\{x \ge 0\} - \lambda |x|^a \mathbb{1}\{x < 0\}$  where  $\lambda > 0$  is the loss aversion parameter and  $a \in \mathbb{R}_+$  controls the risk appetite. The probability distortion function is given by  $w(p) := \frac{p^{\zeta}}{\left(p^{\zeta} + (1-p)^{\zeta}\right)^{\frac{1}{\zeta}}}$  with  $\zeta \in (0.28, 1]$ , where the lower boundary for  $\zeta$  is chosen such that w is strictly monotonically increasing for  $p \in [0, 1]$ .

In Multi Cumulative Prospect Theory (MCPT) the annual gains and losses  $(X_t)$  of an investment A are taken into account, i.e.,  $X_t := A_t - \chi_t$ , where  $t \in \{1, ..., T\}$ , T is the maturity of the investment,  $A_t$  is the account value at time t and  $\chi_t$  is the reference point at time t. The MCPT value of investment A is then defined by

$$MCPT(A) := \sum_{t=1}^{T} \eta^t CPT(X_t)$$
(4)

with a discounting parameter  $\eta \in \mathbb{R}_+$  and with CPT(X) as defined in (3).

# Appendix B. The properties of the combined preference function

In what follows, we show that the proposed combined preference function fulfills the five properties from Section 2.3 for all choices which are in the consideration set C. The first two properties hold, since the real numbers are ordered (and  $K(c_i) \ge 0$  for all  $c_i \in C$ ). The validation of the second statement of Property 3 is staightforward (i.e., if  $r_i^{P_1} = r_j^{P_1}$  and  $r_i^{P_2} = r_j^{P_2}$  then  $K(c_i) = K(c_j)$ ). Further, it holds that if  $c_i$  dominates  $c_j$  then  $r_i^{P_1} > r_j^{P_1} \ge b$  and  $r_i^{P_2} > r_j^{P_2} \ge b$ , since K is increasing in both dimensions as

$$\frac{\partial K}{\partial r_i^{P_1}} = (1-\omega)\beta(r_i^{P_1}-b)^{\beta-1}$$
(5)

$$\frac{\partial K}{\partial r_i^{P_2}} = \omega \beta (r_i^{P_2} - b)^{\beta - 1}.$$
(6)

It follows that  $K(c_i) > K(c_j)$ . For property 4, we know that choices on an indifference line have the same K-value, this means that if *K* is fixed then  $r_i^{P_2}$  is a function of  $r_i^{P_1}$ , i.e.,

$$\begin{split} r_i^{P_2} &= f(r_i^{P_1}) = \left(\frac{K - (1 - \omega)(r_i^{P_1} - b)^{\beta}}{\omega}\right)^{1/\beta} + b \\ f'(r_i^{P_1}) &= -\omega^{-1/\beta} \left(K - (1 - \omega)(r_i^{P_1} - b)^{\beta}\right)^{1/\beta - 1} (1 - \omega)(r_i^{P_1} - b)^{\beta - 1} \\ f''(r_i^{P_1}) &= -\frac{1 - \omega}{\omega^{1/\beta}} \left[ -\frac{1 - \beta}{\beta} \left(K - (1 - \omega)(r_i^{P_1} - b)^{\beta}\right)^{1/\beta - 2} \beta (1 - \omega) \left((r_i^{P_1} - b)^{\beta - 1}\right)^2 \\ &+ \left(K - (1 - \omega)(r_i^{P_1} - b)^{\beta}\right)^{1/\beta - 1} (\beta - 1)(r_i^{P_1} - b)^{\beta - 2} \right] \\ &= \frac{1 - \omega}{\omega^{1/\beta}} \left[ (1 - \beta) \left(K - (1 - \omega)(r_i^{P_1} - b)^{\beta}\right)^{1/\beta - 2} (1 - \omega) \left((r_i^{P_1} - b)^{\beta - 1}\right)^2 \\ &+ \left(K - (1 - \omega)(r_i^{P_1} - b)^{\beta}\right)^{1/\beta - 1} (1 - \beta)(r_i^{P_1} - b)^{\beta - 2} \right] > 0, \end{split}$$

since  $r_i^{P_1} - b > 0$ ,  $K - (1 - \omega)(r_i^{P_1} - b)^{\beta} > 0$  and  $1 - \beta > 0$ . If we plug in our values  $b = 0, \omega = 0.5$  and  $\beta = 0.5$ , we get:

$$\begin{aligned} r_i^{P_2} &= f(r_i^{P_1}) = \left(2K - (r_i^{P_1})^{0.5}\right)^2 = 4K^2 - 4K(r_i^{P_1})^{0.5} + r_i^{P_1} \\ f'(r_i^{P_1}) &= -2K(r_i^{P_1})^{-0.5} + 1 \\ f''(r_i^{P_1}) &= K(r_i^{P_1})^{-1.5} > 0 \end{aligned}$$

Lastly, the definition of the K function as well as the underlying preference functions are independent of the set C and thus Property 5 holds.

**Appendix C. Additional numerical results** 

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ment premium, the stock ratio, the guaranteed terminal value, the EUT CE and the CPT CE are given in parenthesis, i.e., ( $\alpha$  in %,  $\theta$  in %, exp(gT), EUT CE in %, CPT CE Table 3 The favorable compromise based on the combined preference function K for different combination of risk aversion, risk attitude and loss aversion. The investin %). For the optimal EUT product the CPT CE is given as a range due to different loss aversions. The plus (+) corresponds to roll-up and the circle (o) to constant mix products. In the blue area the favorable compromise is a pure stock investment. In the red area the favorable compromise is a roll-up with 100% stock ratio and terminal guarantee of 1.04

	EUT optimal	favora	ble compromise / preferab	le CPT			
$a = \gamma$		۲	1	1.25	1.75	2.25	2.75
00		К	+(65, 100, 1.04, 3.61, 4.34)	+(65, 100, 1.04, 3.61, 4.34)	+(65, 100, 1.04, 3.61, 4.34)	+(65, 100, 1.04, 3.61, 4.34)	+(65, 100, 1.04, 3.61, 4.34)
0.0	o(100, 82.3, -, 4.23, [2.43, -0.09])	CPT	+(60, 100, 1.07, 3.51, 4.44)	+(60, 100, 1.07, 3.51, 4.44)	+(60, 100, 1.07, 3.51, 4.44)	+(60, 100, 1.07, 3.51, 4.44)	+(60, 100, 1.07, 3.51, 4.44)
2 0 0		К	+(65, 100, 1.04, 3.65, 4.64)	+(65, 100, 1.04, 3.65, 4.64)	+(65, 100, 1.04, 3.65, 4.64)	+(65, 100, 1.04, 3.65, 4.64)	+(65, 100, 1.04, 3.65, 4.64)
co.0	o(100, 90, -, 4.43, [3.47, -0.07])	CPT	+(60, 100, 1.07, 3.54, 4.67)	+(60, 100, 1.07, 3.54, 4.67)	+(60, 100, 1.07, 3.54, 4.67)	+(60, 100, 1.07, 3.54, 4.67)	+(60, 100, 1.07, 3.54, 4.67)
t C		Х	$\circ(100, 100, -, 4.65, 4.51)$	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)
	o(100, 100, -, 4.00, [4.01, -0.00])	CPT	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)	+(65, 100, 1.04, 3.7, 4.96)
i t c		К	$\circ(100, 100, -, 4.88, 5.41)$	+(95, 100, 0.62, 4.69, 4.81)	+(70, 100, 1.01, 3.88, 5.22)	+(70, 100, 1.01, 3.88, 5.22)	+(70, 100, 1.01, 3.88, 5.22)
e /	o(100, 100,, 4.66, [0.41, 0.09])	CPT	+(95, 100, 0.62, 4.69, 5.49)	+(65, 100, 1.04, 3.74, 5.3)	+(65, 100, 1.04, 3.74, 5.3)	+(65, 100, 1.04, 3.74, 5.3)	+(65, 100, 1.04, 3.74, 5.3)
0		К	$\circ(100, 100, -, 5.1, 6.31)$	+(95, 100, 0.62, 4.87, 5.74)	+(70, 100, 1.01, 3.95, 5.68)	+(70, 100, 1.01, 3.95, 5.68)	+(70, 100, 1.01, 3.95, 5.68)
0.0	o(100, 100, -, 3.1, [0.31, 0.00])	CPT	+(95, 100, 0.62, 4.87, 6.37)	+(95, 100, 0.62, 4.87, 5.74)	+(70, 100, 1.01, 3.95, 5.68)	+(70, 100, 1.01, 3.95, 5.68)	+(70, 100, 1.01, 3.95, 5.68)
10 O	(Frank Official Control of Control	К	$\circ(100, 100, -, 5.33, 7.19)$	$\circ(100, 100, -, 5.33, 6.41)$	+(95, 100, 0.62, 5.05, 5.46)	+(70, 100, 1.01, 4.01, 6.15)	+(70, 100, 1.01, 4.01, 6.15)
0.00	o(100, 100, -, 0.00, [.119, 1.04])	CPT	+(95, 100, 0.62, 5.05, 7.24)	+(95, 100, 0.62, 5.05, 6.65)	+(70, 100, 1.01, 4.01, 6.15)	+(70, 100, 1.01, 4.01, 6.15)	+(70, 100, 1.01, 4.01, 6.15)
00 0		К	$\circ(100, 100, -, 5.46, 7.72)$	$\circ(100, 100, -, 5.46, 6.97)$	+(95, 100, 0.62, 5.16, 6.06)	+(70, 100, 1.01, 4.05, 6.43)	+(70, 100, 1.01, 4.05, 6.43)
0.00	o(100, 100, -, 3.40, [2, 2.1/])	CPT	+(95, 100, 0.62, 5.16, 7.74)	+(95, 100, 0.62, 5.16, 7.19)	+(70, 100, 1.01, 4.05, 6.43)	+(70, 100, 1.01, 4.05, 6.43)	+(70, 100, 1.01, 4.05, 6.43)
0	(100 0 00 0 2 2 0 00 0 001)	Х	$\circ(100, 100, -, 5.55, 8.06)$	$\circ(100, 100, -, 5.55, 7.34)$	+(95, 100, 0.62, 5.23, 6.45)	+(85, 100, 0.84, 4.72, 6.03)	+(70, 100, 1.01, 4.08, 6.62)
л. С	o(100, 100, -, 3.33, [s.00, 2.02])	CPT	+(95, 100, 0.62, 5.23, 8.07)	+(95, 100, 0.62, 5.23, 7.54)	+(85, 100, 0.84, 4.72, 6.64)	+(70, 100, 1.01, 4.08, 6.62)	+(70, 100, 1.01, 4.08, 6.62)
200		К	$\circ(100, 100, -, 5.78, 8.9)$	$\circ(100, 100, -, 5.78, 8.23)$	+(95, 100, 0.62, 5.42, 7.41)	+(90, 100, 0.75, 5.12, 6.74)	+(80, 100, 0.91, 4.59, 6.69)
0.50	o(100, 100, -, o. (o, (o.9, o.(9))	CPT	$\circ(100, 100, -, 5.78, 8.9)$	+(95, 100, 0.62, 5.42, 8.4)	+(90, 100, 0.75, 5.12, 7.49)	+(75, 100, 0.96, 4.36, 7.09)	+(70, 100, 1.01, 4.14, 7.08)
÷		Х	$\circ(100, 100, -, 6, 9.71)$	$\circ(100, 100, -, 6, 9.1)$	+(95, 100, 0.62, 5.6, 8.33)	+(95, 100, 0.62, 5.6, 7.41)	+(90, 100, 0.75, 5.27, 6.99)
1	o(100, 100, -, 0, [9.71, 0])	CPT	$\circ(100, 100, -, 6, 9.71)$	+(95, 100, 0.62, 5.6, 9.21)	+(90, 100, 0.75, 5.27, 8.35)	+(85, 100, 0.84, 4.97, 7.78)	+(75, 100, 0.96, 4.44, 7.6)

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