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WORKING PAPER

Deriving values of the social rate of time preference

Chris Parker

New Zealand Treasury Working Paper 25/01

February 2025



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Te Kāwanatanga o Aotearoa New Zealand Government

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Abstract

This report estimates the 'social rate of time preference' (SRTP) for New Zealand public policy appraisal using a Ramsey equation. It uses the Weitzman (1998) method to derive a declining discount rate schedule of 2% (real) for years 1–30, 1.5% for years 31–100, and 1% for years 101 on. The report does not compare and contrast social discount rate methods and how to use them in practice.

JEL CLASSIFICATIONH43, H50KEYWORDSDiscounting; social discount rate; time preference, Government

Executive summary

Constant social rate of time preference

We derived a Ramsey equation based on a social welfare function of the consumption of goods and services over all future time. We required that the social rate of time preference (SRTP) is not less than the growth rate of consumption to ensure finiteness of net present values. To characterise our uncertainty about SRTP parameters we use triangular probability distributions of plausible values for the pure rate of time preference, annihilation risk, the elasticity of marginal social welfare, and the growth rate of real per capita consumption. These are set out below:

$r = \rho + \alpha + \mu g > g$	Triangular	distributio	on inputs	Comment
Parameters	Minimum	Mode	Max	
Pure rate time preference ρ	0.00%	0.25%	2.0%	Values used in major welfare economics studies
Annihilation risk α	0.00%	0.05%	0.20%	Extinction risk per century
Elasticity marginal social welfare with respect to consumption μ	0.25	0.75	1.5	Reasonably tolerable consumption loss from transfers
Growth rate of real consumption per capita <i>g</i>	0.75%	1.25%	2.0%	Historical per capita real consumption and projections

Resulting output

		<i>/••</i> • •		
		(Mean)		Range of restricted values for
SRTP r	0.8%	2.1% *	4.8%	SRTP. Rounded to one d.p.

* Between 1.2%–3.3%, with 95 percent confidence.

We use Monte Carlo simulation to derive a range of SRTP values centring around 2%, and between 1.2%–3.3% with 95 percent confidence. We discarded about 15% of the draws because they resulted in a SRTP less than the growth rate of consumption.

Declining SRTP

Using the Weitzman (1998) method we derived the following declining certainty equivalent SRTP schedule (using half percentage point increments):

Schedule of SRTPs	Years
2%	1-30
1.5%	31-100
1%	101+

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Deriving values of the social rate of time preference

1. Introduction

As part of the Treasury review of the Public Sector Discount Rate (Treasury Circular 2024/15, Treasury 2024a), this note provides an estimate of the range of plausible values for the social rate of time preference (SRTP) based on the following form of Ramsey equation:

$$r = \rho + \alpha + \mu g \tag{1}$$

The SRTP (r) estimate based on a Ramsey equation uses the parameters:

- pure rate time preference ρ
- annihilation risk α
- elasticity of marginal social welfare with respect to consumption μ
- growth rate of real consumption per capita *g*.

Appendix 1 provides a detailed mathematical derivation of the equation and parameters above. We review the literature and review alternative formulations for the Ramsey equation.

This note provides a range for an estimated constant SRTP, as well as a declining SRTP schedule based on a certainty-equivalent approach. This document does not canvass other approaches to setting the public sector discount rate.¹

Creedy and Passi (2017) and Creedy (2007) explain that a SRTP can be derived based on a mix of empirical estimates and value judgements of decision makers acting on behalf of society using a 'social welfare function'. They would assign 'welfare scores' (analogous to utility that an individual might assign to their own preferences) to different outcomes to determine which policy options are preferable to society, based on a set of ethical considerations.

¹ More can be found in Grimes (2023), Abelson and Dalton (2023), NZIER (2024a), Turk (2024), Groom et al (2022), and Creedy and Passi (2017).

2. Determining the ranges for parameters

For the general social welfare function of all future welfare we used for our analysis to be finite, the discount rate needs to be greater than the growth rate in perpetuity, ie, r > g, or that:²

$$\rho + \alpha + \mu g > g \tag{2}$$

We do not place restrictions directly on each parameter as a function of the other parameters, but rather on the total resulting SRTP value. We implicitly assume each parameter value is not independent by discarding certain combinations of parameter values. For example, if the pure rate of time preference ρ and the annihilation risk is thought to be very low, then we would require a somewhat higher value for marginal elasticity of marginal welfare μ .

Applying this restriction is not common in the literature, and nor is using Monte Carlo simulation to identify a range of possible outcomes whereby the possibility of r < g occuring is more apparent. While we are confident we should exclude combinations of values where r < g, we consider results in the analysis both with and without excluding draws where r < g.

2.1 Pure rate of time preference

The pure rate of time preference is highly contentious and normative; eg, some argue it should be zero (eg, Ramsey 1928) on the basis of "intergenerational equity". Creedy and Passi (2017) document ranges between 0.1% to 1.5% used by policy making authorities globally or by well-known welfare economics studies. Some other literature considers values of 0% (eg, HM Treasury 2024, Groom et al 2022). Sense Partners (2022) provides a Māori economic worldview from the iwi (tribe) Ngāi Tūhoe, suggesting the future should not be discounted.

Drupp et al (2018) surveyed 262 global experts and 38 percent thought it should be zero, but the responses were substantially right-skewed with a mode of zero, median of 0.5%, and a mean of 1.10%. Turk replicated the survey for New Zealand economists and within a group of 33 self-identified experts found a mode of 0.1%, median of 1.0%, and a mean of 1.7%. Based on these studies, we assume it lies in the range 0% to 2%, and assume a continuous triangular distribution for it with a mode of 0.25% (implying a substantial right-skew); this results in a mean estimate of 0.75%.

² That the discount rate needs to exceed the growth rate in perpetuity is described as an 'economically natural assumption' in Dixit (1993 p13) and Dixit and Pindyck (1994 p138).

2.2 Annihilation risk

To the pure rate of time preference we could add a annihilation risk, which Creedy and Passi (2017) note would be very small. (Sometimes they are grouped together.) If there were a 5% chance of annihilation by 100 years' time (eg, nuclear war, pandemic, climate collapse, comet strike), the annual risk would be 0.05%. Sandberg and Bostrom (2008) surveyed people's expectations of human extinction by 2100 at 19%, corresponding to about 0.2% annual chance. Rees (2003) argues around 50% chance of annihilation in the next 100 years, implying a 0.7% chance per annum. A 0.15% per annum chance corresponds to a 14% chance of annihilation within a century. Stern et al (2006) used 0.1%. HM Treasury (2024) uses 1%, although this is interpreted broader whereby 'catastrophe' risk relates to the stopping of an initiative's benefits and cost cashflows for reasons unconnected to its appraisal, and a small premium for 'systemic risk'.

We use a triangular distribution over the range 0% to 0.2%, with a mode of 0.05%, resulting in a mean estimate of 0.08%. Like Grimes (2023) we do not consider values of 1% per annum for cessation of all social welfare reasonable as it would imply too high a chance of extinction each 100 years.

2.3 The elasticity of marginal social value of consumption

The elasticity of marginal welfare μ governs the curvature of the increasing and concave social welfare function. The more curved it is, the more intolerant society is to consumption that is not averaged across people and/or across time.³

It is analogous — but quite distinct — to the concavity of an individual's utility function, which can represent their risk aversion.⁴ Weitzman (2007) suggests plausible values for individual's risk aversion is $\mu = 2 \pm 1$, and up to 4.

Evans (2005) used tax data across OECD countries to infer the value judgements of redistribution and estimated a value between 1.43–1.57 for New Zealand using a 'equal absolute sacrifice model' to estimate people's utility functions. Groom and Maddison (2013) estimated it for the United Kingdom finding it fluctuated considerably since World War Two, with a mean of 1.6. Such methods reveal results of democratic decision making, and are an alternative to experimental or survey methods of revealing inequality aversion.

Groom and Maddison (2019) undertake a meta-analysis of estimates of μ based on risk aversion (from insurance data), income tax schedules, revealed preferences from households saving and borrowing over time to smooth consumption, subjective wellbeing surveys, and revealed preference from consuming general goods and services assuming additively separable preferences. They estimate a value of $\mu = 1.53$ with a 95 percent confidence interval of 1.44–1.61.

³ See footnote 15 in the mathematical Appendix 1 for more.

⁴ At the individual level we would consider marginal utility, but at the society level we consider marginal welfare. We mathematically model these two concepts the same way, but there are material moral judgements required to transform and aggregate people's utility into social welfare. Estimates of marginal utility are not necessarily estimates of marginal welfare.

Creedy (2007) notes the tax data based estimates such as Evans (2005) and others based on equal absolute sacrifice as a policy objective necessary leads to a value of μ above one, which he argues is highly restrictive. He argues these methods generally do not reveal implicit value judgements suitable for a social welfare function.⁵ The application of people's individual risk aversion has a weaker normative basis for informing a social welfare function that aggregates people's utilities. (We do not mean to deny the relevance of people's revealed risk aversion to cost-benefit analysis: that information can be incorporated in an initiative's specific benefit and cost cashflows.)

Creedy (2007) takes more care than most with normative logic to avoid the 'Is-Ought' fallacy. He used a method from Atkinson (1970) to consider values for μ using a thought experiment of a 'leaky bucket' of how much loss of total consumption the decision maker is willing to tolerate to transfer resources from the wealthier to the poorer. One can use the implicit function theorem to equation (9) from appendix 1 holding *W* constant.⁶ This represents how much more consumption in period 0 needs to occur if \$1 is removed from another period '1' to hold the social welfare score constant:⁷

$$-\frac{dc_0}{dc_1} = \left(\frac{c_0}{c_1}\right)^{\mu} e^{(\rho+\alpha)(t_0-t_1)}$$
(3)

To allow us to focus not on time discounting for the moment but only on redistributions between unequal levels of consumption we can set ρ and α to zero, so that $-\frac{dc_0}{dc_1} =$

 $\left(\frac{c_0}{c_1}\right)^{\mu}$. Or we could suppose we are comparing consumption within a common period of time (ie, $t_0 = t_1$). We assume that intraperiod and interperiod inequality aversion are similar. Suppose c_1 were 1.5 times as great as c_0 , which could occur if total consumption grew at 2% per annum for about 20 years. Then $-\frac{dc_0}{dc_1} = 1.5^{-\mu}$. Table 1 shows if $\mu = 0.5$ then taking \$1 from c_1 gives 82 cents to consumption now, a leak of 18 cents. If $\mu = 1$, a leak of 33 cents is tolerated, and leaks of 46 cents and 56 cents are tolerated for μ of 1.5 and 2 respectively.

μ	0	0.25	0.5	0.75	1	1.25	1.5	2
$-\frac{dc_0}{dc_1}$	\$1.00	\$0.90	\$0.82	\$0.74	\$0.67	\$0.60	\$0.54	\$0.44
Leak	\$0.00	\$0.10	\$0.18	\$0.26	\$0.33	\$0.40	\$0.46	\$0.56

Table 1: Leaky bucket thought experiment, for 50% consumption difference

⁷ From equation (6) in the mathematical appendix 1, $\frac{dc_0}{dc_1} = -\frac{W_{c_1}}{W_{c_0}} = -\frac{c_1^{-\mu}e^{-(\rho+\alpha)t_1}}{c_0^{-\mu}e^{-(\rho+\alpha)t_0}} = -\left(\frac{c_0}{c_1}\right)^{\mu}e^{(\rho+\alpha)(t_0-t_1)}.$

⁵ He argues pp 11–12 such approaches to estimate μ ignore objectives raised by Edgeworth (1897) and others concerning the various interpretations of sacrifice theories. The approaches apply only to positive taxes and ignore a much broader set of taxes and transfers. He suggests the approaches are useful for their simplicity, but that tax structures have very complex links to value judgements and progression can arise with values of $\mu < 1$. Finally, even if the estimations plausibly represent social preferences, it is not immediately obvious what, for example, a value of $\mu = 0.5$ indicates about aversion to inequality. Creedy argues more direct demonstrations of what that implies is required, such as the leaky bucket thought experiment.

⁶ Whereby if $W(c_0(c_t), c_t) = \overline{W}$, such that c_0 is an implicit function of c_t to keep the function constant at \overline{W} , then $\frac{dc_0}{dc_1} = -\frac{W_{c_1}}{W_{c_0}}$, where W_{c_1} is short for dW/dc_1 .

A value of $\mu > 1$ represents strong aversion to inequality, or an acceptance to significantly shrink the size of the pie in order to redistribute. The usual tax and transfers system deals with redistributing income across people with several multiples difference of incomes.

There are two broad categories of cost or loss in the general tax and transfer system: administrative costs and 'deadweight losses' arising from distorted incentives to work and invest. The Treasury commissioned a review of administrative costs as part of the 2019 Tax Working Group (Evans 2018) that suggested 2-10% of revenue yield for compliance costs to taxpayers. Inland Revenue (2023 p65) report it costs them \$0.43 to collect \$100 in revenue. We have been unable to find applicable consensus estimates of the deadweight loss of tax, and it can vary depending on the context a tax occurs (with it being very low if it is an unavoidable land tax or even negative if a particular tax is Pigouvian and corrects for a negative externality). The Treasury has conventionally used 20% for 'deadweight losses'.

Table 2 shows the willingness to shrink the pie if consumption in scenario 1 were three times as great as scenario 0, ie, $c_1 = 3c_0$. Leaks of up to about 50 cents in the dollar are tolerable for values of μ of up to about 0.75. A leak of 75 cents per dollar is tolerable with a μ of 1.25, which means the benefit per dollar transferred would need to be at least a ratio of 3:1 to justify spending \$75 per \$25 of redistribution from someone three times as wealthy as the less wealthy.

μ	0	0.25	0.5	0.75	1	1.25	1.5	2
$-\frac{dc_0}{dc_1}$	\$1.00	\$0.76	\$0.58	\$0.44	\$0.33	\$0.25	\$0.19	\$0.11
Leak	\$0.00	\$0.24	\$0.42	\$0.56	\$0.67	\$0.75	\$0.81	\$0.89

Table 2: Leaky bucket thought experiment, for 3x consumption difference

Pirttilä and Uusitalo (2007) surveyed people on their inequality aversion and found a value of μ of about 0.5 for leaky bucket type questions of redistribution, but a much higher value of more than 3 about inequality of wage incomes. They found the latter might relate to people's opinions on the proper role of the welfare state.

That there could be so much diversity and context specificity in inequality aversion is supported by NZIER (2024b)'s survey of New Zealanders on their agreement with principles of distributional justice. People tend to favour non-consequentionist theories of distributive justice.⁸ NZIER found a high level of consensus among New Zealanders that fairness is about equal opportunities and reward for effort, individual rights and freedoms, and social, legal and political equality. These distributional justice theories do not lend themselves directly either to the social welfare function approach we are taking to derive a social discount rate for cost-benefit analysis, or to cost-benefit analysis generally.

⁸ Refer to footnote 12 for more discussion.

However, within our context of estimating a SRTP for cost-benefit analysis, and the more suitable leaky bucket interpretation of the social welfare function, the above analysis leads us to consider values of μ that are more likely below 1. For the purposes of the simulation we assume that μ varies according to a triangular distribution of minimum 0.25, mode of 0.75, and maximum of 1.5, resulting in a mean estimate of 0.83.⁹ The upper limit of 1.5 captures Evans (2005) often cited estimate for New Zealand.

2.4 Growth rate of consumption

New Zealand's annual growth rate of per-capita real private consumption has averaged 2.1% for the last 25 years. Ignoring one-off crashes and booms to look for longer term annual averages, the general range varies between 1% and 3%. Such a measure does not include degradations in natural environmental stocks, so the more general growth in per capita consumption of both market and non-market resources is probably lower.



Figure 1: Annual growth of per capita real private consumption (GDP)

Source: Real private consumption expenditure (GDP) from series GDE.Q.EC1.RA and population levels from Statistics New Zealand

The Treasury does projections for GDP per capita, but not specifically for consumption; see Figure 2 below. Fiscal projection models like the Fiscal Strategy Model (FSM) or Long-Term Fiscal Model (LTFM) are based on a labour force formula. This applies growth in total hours worked by the employed labour force and growth in labour force productivity to estimate average outcomes over the projected years. This is not assuming a change in real consumption's share of GDP beyond the end of the forecast period, which makes this effectively a projection of consumption. The estimate is about 1% per annum growth.

⁹ A value for μ of 1.25 implies an indifference with losing 94 cents in the dollar when transferring from someone ten times richer than the poorer (eg from someone earning \$400k to someone earning \$40k), which seems rather high. It suggests a required benefit cost ratio for each dollar taken of about 94/6=16, which is much higher than that required to justify investment in, say, infrastructure from general taxpayer funds (perhaps a BCR of 2 or 3). Similarly, a value of $\mu = 1.5$ implies a leak of 97 cents per dollar, requiring a BCR of about 30 (97/3=32).

Figure 2: Annual growth in real GDP per capita (percentage)



Source: The Treasury, PREFU 2023 forecast

We assume in future the growth rate of per capita real consumption g is distributed triangularly between 0.75% to 2% with a mode of 1.25%, resulting in a mean estimate of 1.33%.

3. Monte Carlo simulation of parameters

We used Monte Carlo simulation to estimate a range for the social rate of time preference based on the probability distributions for the input variables assumed above. The ranges of inputs outlined above are summarized in Table 3 below. We considered the impacts of including ("all draws") and excluding ("restricted set") combinations of draws from the probability distributions where the SRTP solution is less than the growth rate of consumption.

μ	Minimum	Mode	Мах	Implied mean
Pure rate time preference	0.00%	0.25%	2.0%	0.75%
Annihilation risk	0.00%	0.05%	0.20%	0.08%
Elasticity marginal social welfare with respect to consumption	0.25	0.75	1.5	0.83
Growth rate real consumption per capita	0.75%	1.25%	2.0%	1.33%

Table 3: Parameter ranges assumed, all triangular

The plot of outcomes for SRTP for the restricted set is illustrated in Figure 3. The deciles of values are plotted, and they are set out in Table 4 below. The average value is about 2.1% (to one decimal place, and illustrated in yellow), and it varies from 0.75% to about 4.85%; the median is about 2.0%. The 95 percent confidence interval for the restricted set is 1.2%-3.3% (to 1 decimal place¹⁰). The unrestricted set has a 95 percent confidence interval of 0.9%-3.2% and a mean of 1.9% (which can be deduced also by using the mean value of each parameter). Turk (2024)'s survey of New Zealand experts found 90% would recommend a social discount rate within 1.7%-3.0%, and here 80% of estimates are within 1.4%-2.8%, which is similar and slightly lower.

The Treasury's (2024) current long-run assumption for the annual nominal rate of return on the government 10-year bond is 4.3%. This is effectively a 2.3% real rate given an expected 2% CPI-based inflation (midpoint of the inflation targeting range of 1%-3%), which is near the centre of our range of SRTP estimates.

	Min	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	Мах
Restricted set	0.76%	1.40%	1.58%	1.73%	1.87%	2.01%	2.16%	2.33%	2.54%	2.82%	4.86%
All draws	0.30%	1.19%	1.40%	1.58%	1.74%	1.90%	2.06%	2.25%	2.46%	2.76%	4.86%

Table 4: Decile outputs for restricted and non-restricted cases

¹⁰ These are rounded because simulations vary slightly each time.





About 15% of the draws were discarded from the restricted set because the SRTP was less than the growth rate drawn, sometimes by as much as 1 percentage point. Discarding these led to the range of SRTP being higher than what one might suppose by considering the ranges of values for the individual variables in isolation. This is seen by comparing the two rows in Table 4 above.

4. Declining 'certainty equivalent' SRTP

The two Treasury-commissioned reports of Grimes (2023) and Abelson and Dalton (2023) reviewed different approaches to declining discount rates. These experts advised us (in these reports and in workshops) that we use the Weitzman (1998) approach to derive a certainty equivalent declining discount rate over a certain future cashflow, which is the approach taken here.¹¹ Weitzman's insight was if one is uncertain which discount rate to use, rather than deduce some average of the discount rates one should instead average the present values of a (certain) dollar of impact in the future (ie, the discount factors) from each potential discount rate. The equivalent discount rate that gives the same expected net present value is not the simple average of the potential rates but is weighted towards the lower of the potential rates. The further in time the discounted cash flow, the closer the certainty equivalent discount rate is to the lowest potential one considered. One intuition is the larger rates annhilate themselves quicker, leaving the lowest to have an increasingly larger influence. Thus the certainty equivalent discount rate declines over time.

This approach is suitable when there is permanent disagreement or uncertainty about ethical judgements that lead to heterogeneity in the appropriate SRTP to use and future benefit streams are certain. This, all else equal, serves to lower the hurdle rate for long-lived now or never projects.

Guthrie (2021) considers how the Weitzman result applies to investments that are irreversible, have timing flexibility, and whose benefit streams are uncertain. He finds variation in discount rates also increases the option value of delaying investment that offsets — somewhat or possibly more than — the decrease in the hurdle rate for long-lived assets. Furthermore, when the decision maker is calculating the optimal time to exercise an option they need to consider what they are likely to do in that future point in time to ensure decisions are 'dynamically consistent'. After considering the net effect of these matters, he suggests little would be lost in just ignoring discount rate heterogeneity for such investments. But if such heterogeneity were to be accounted for, he shows how to do it in an options valuation model that ensures investment policies are dynamically consistent.

The range of possible outcomes for the SRTP illustrated in Figure 3 means that uncertainty about the present value of constant expected benefits and costs increases with distance in time. Figure 4 illustrates the discount factor $1/(1 + r)^t$ for each discount rate r from Table 4 as a function of time horizon over a 200 year horizon. Lower discount rates result in flatter and higher discount factor curves.

¹¹ This approach was further clarified in Gollier and Weitzman (2010).

Figure 4: Discount factors



We take the simple average of the two sets of deciles (from the restricted and unrestricted sets) to find an average discount factor — the orange curve in Figure 5.



Figure 5: Average discount factor

The 'marginal discount rate' is the one that applies to the last year of cash flows (also known as the instantaneous discount rate, eg, Cropper et al 2014). Any declining discount rate (DDR) schedule should apply at the margin, rather than at the average, so that the discount factor curve is smooth and monotonically decreasing. For example, the HM Treasury's (2008) schedule of discount rates is 3.5% for years 1-30 and 3% for years 31-75. If a project had, say, 31 year life, only the 31st year is discounted at 3%. That is, the HM Treasury's marginal discount rate between years 31-75 is 3%.

Figure 6 illustrates both, with the (black) average 'certainty equivalent' SRTP being the single constant value that would give the average discount factor as at that year. The (blue) marginal certainty equivalent SRTP is calculated as the rate of change of the average discount factor with respect to time — it lies under the average SRTP since the average is reducing.

Rather than issue guidance on a continuously declining public sector discount rate we estimate a simple schedule of changes, much like the HM Treasury in the United Kingdom and increasingly other countries (Drupp et al 2018). This schedule aims to roughly approximate the blue marginal certainty equivalent SRTP, and is plotted as the grey curve, using fairly round numbers for both the SRTP and the period lengths for simplicity. If we start with the average of about 2% and keep that constant for 30 years and then reduce the marginal SRTP to 1.5% for years 31 to 100, then 1% thereafter, we find the resulting discount factor curve (the grey curve in Figure 7) is close enough to that required (the orange).

The declining discount rate schedule leads to slight under-discounting after year 30. However, two minor considerations we excluded from our analysis might have reduced the discount rates in Figure 6 by about 0.15%–0.2% (increasing inequality in future following Emmerling et al (2017) discussed in footnote 13 of maybe 0.16%, and the adjustment by Gollier (2013) discussed in Appendix 1 of maybe up to 0.05%). If these two adjustments were added then the declining discount rate schedule would be a very close approximation.



Figure 6: Marginal and average certainty equivalent SRTPs

Figure 7: Comparing discount factors



This declining SRTP schedule is summarised in Table 5. Note, most routine policy initiatives in practice may not have impacts after 30 years; only highly significant and transformative initiatives that do not have too much flexibility in waiting likely will be materially impacted by a declining SRTP.

Table 5: Declining SRTP schedule

Schedule for declining SRTP	Years
2%	1-30
1.5%	31-100
1%	101+

This declining SRTP leads to somewhat significant differences in the discount factors in the very long-term, to help better reflect intergenerational issues, so it is perhaps worth using for intergenerational issues. Figure 8 augments Figure 7 by including the discount factor with a constant SRTP of 2%. There is an increasing gap between the grey discount factor curve with a declining SRTP and blue discount factor curve with a constant SRTP; a discount factor of 0.1 is pushed out 50 years, from about year 120 to year 170. The sum of a uniform cashflow of \$1 over 200 years is about \$50 at a constant 2% discount rate, and about \$59 with a declining discount rate, which is a 19% increase. The impact of a declining SRTP would be proportionally much greater if an initiative had growing benefits over the long-term.



Figure 8: Discount factors with and without declining SRTP

References

- Abelson, Peter and Tim Dalton (2023) *Declining Discount Rates in Cost Benefit Analysis*, Applied Economics Pty Ltd report for the NZ Treasury, www.treasury.govt.nz/publications/commissioned-report/declining-discount-ratescost-benefit-analysis
- Atkinson, Anthony B. (1970) 'On the Measurement of Inequality', *Journal of Economic Theory*, 2, 244-263
- Creedy, John (2007) 'Policy Evaluation, Welfare Weights and Value Judgements: A Reminder', *Australian Journal of Labour Economics*, Vol. 10, No. 1, March 2007, pp 1-15
- Creedy, John and Passi, Hemant (2017) *Public sector discount rates: A comparison of alternative approaches*, New Zealand Treasury Working Paper 17/02
- Cropper, Maureen, Mark Freeman, Ben Groom and William Pizer (2014) "Declining Discount Rates", *American Economic Review*, 104 (5): 538–43
- Dixit, Avinash (1993) The Art of Smooth Pasting, Harwood Academic Publishers
- Dixit, Avinash, and Pindyck, Robert (1994) *Investment Under Uncertainty*, Princeton University Press
- Drupp, Moritz A., Mark C. Freeman, Ben Groom, and Frikk Nesje (2018) 'Discounting Disentangled', *American Economic Journal: Economic Policy*, 10(4): 109–134 https://doi.org/10.1257/pol.20160240
- Edgeworth, F.Y. (1897) 'The Pure Theory of Taxation', *Economic Journal*. Reprinted in Papers Relating to Political Economy, 2, 63-125. London: Macmillan
- Emmerling J, Groom B, Wettingfeld T. (2017) 'Discounting and the representative median agent', *Economics Letters*, 161:78–81
- Evans, Chris (2018) *The Compliance Costs Of Taxing Capital Gains*, report prepared for the Tax Working Group, Professor Chris Evans: The Compliance Costs of Taxing Capital Gains - 13 November 2018 - Information Release - Tax Working Group - New Zealand
- Evans, David (2005) 'The Elasticity of Marginal Utility of Consumption: Estimates for 20 OECD Countries', *Fiscal Studies*, 26, 197-224
- Gollier, Christian, and Martin Weitzman (2010) 'How should the distant future be discounted when discount rates are uncertain?' *Economics Letters* 107:350–353
- Gollier, Christian (2013) *Pricing the Planet's Future: The Economics of Discounting in an Uncertain World*. Princeton, NJ: Princeton University Press
- Grimes, Arthur (2023) *How Should the New Zealand Government Discount Future Payoffs?* Motu Economic and Public Policy Research report for the NZ Treasury, www.treasury.govt.nz/publications/commissioned-report/how-should-new-zealandgovernment-discount-future-payoffs

- Groom, Ben, and David Maddison (2013) *Non-Identical Quadruplets: Four New Estimates of the Elasticity of Marginal Utility for the UK.* Grantham Research Institute on Climate Change and the Environment Working Paper 121
- Groom, Ben, and David Maddison (2019) 'New Estimates of the Elasticity of Marginal Utility for the UK', *Environmental Resource Economics* 72: 1155–1182
- Groom, Ben, Moritz Drupp, Mark Freeman, Frikk Nesje (2022) The future, now: A review of social discounting. *Annual Review of Resource Economics*, 467-491
- Guthrie, Graeme (2021) 'Discounting, Disagreement, and the Option to Delay', *Environmental and Resource Economics*, volume 80, 95–133
- HM Treasury (2024) The Green Book Central Government Guidance on Appraisal and Evaluation www.gov.uk/government/publications/the-green-book-appraisaland-evaluation-in-central-government/the-green-book-2020
- Inland Revenue (2023) Annual Report 2023, Inland Revenue Annual Report, Te Tari Taake Pūrongo ā-Tau, 2022–23 (ird.govt.nz)
- NZIER (2024a) A Price On Time: Rethinking the Public Sector Discount Rate, NZIER Insight 114, www.nzier.org.nz/publications/a-price-on-time-rethinking-the-publicsector-discount-rate-nzier-insight-114
- NZIER (2024b) *Perceptions of fairness in New Zealand, Phase 1 report*, NZIER report to the New Zealand Treasury, August 2024, www.nzier.org.nz/hubfs/Public%20Publications/Client%20reports/Fairness%20Pha se%201%20Report.pdf
- Pirttilä, Jukka; Uusitalo, Roope (2007) *Leaky bucket in the real world: estimating inequality aversion using survey data*, CESifo Working Paper, No. 2026, Center for Economic Studies and ifo Institute (CESifo), Munich
- Ramsey, Frank (1928) 'A Mathematical Theory of Saving', *The Economic Journal*, Volume 38, Issue 152, 1 December 1928, 543–559
- Rees, Martin (2003) *Our Final Hour: A Scientist's Warning: How Terror, Error, and Environmental Disaster Threaten Humankind's Future In This Century—On Earth and Beyond*, Basic Books, New York City
- Sense Partners (2022) *Tūhoe economic worldview Mapping to an orthodox framework*, Final Report to the Ministry of Business, Innovation and Employment, 23 August 2022, www.mbie.govt.nz/business-and-employment/economicdevelopment/tuhoe-economic-worldview-mapping-to-an-orthodox-framework
- Stern, Nicholas, S Peters, V Bakhshi, A Bowen, C Cameron, S Catovsky, D Crane, S Cruickshank, S Dietz, N Edmonson, S-L Garbett, L Hamid, G Hoffman, D Ingram, B Jones, N Patmore, H Radcliffe, R Sathiyarajah, M Stock, C Taylor, T Vernon, H Wanjie and D Zenghelis (2006) *Stern review: the economics of climate change*. London: HM Treasury

- Thompson, Chris (2022) Equality, equity, and distributive justice. Background Paper to Te Tai Waiora: Wellbeing in Actearoa New Zealand 2022. Treasury Analytical Paper 22/03, www.treasury.govt.nz/sites/default/files/2022-12/ap22-03.pdf
- Turk, Zachary (2024) *Discounting Disentangled: Aotearoa*, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4830535
- Treasury (2024a) Treasury Circular 2024/15: Updated Public Sector Discount Rates for Cost Benefit Analysis, www.treasury.govt.nz/publications/circulars/treasurycircular-2024-15-updated-public-sector-discount-rates-cost-benefit-analysis
- Treasury (2024b) *Fiscal Strategy Model BEFU 2024*, www.treasury.govt.nz/publications/fsm/fiscal-strategy-model-befu-2024
- Weitzman, Martin (1998) 'Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate', *Journal of Environmental Economics and Management*, November, 36(3), 201-08
- Weitzman, Martin (2007) 'Subjective Expectations and Asset-Return Puzzles', *American Economic Review*, 97(4), 1102–1130

Appendix 1 – Mathematical basis for SRTP

There are two key dimensions we will abstract away from in the analysis that follows. The first is that goods and services could vary between market and non-market items, with a particular distinction between natural environmental attributes and general market consumables. The second is individual citizens can have vastly different bundles of goods and services that they consume also, with inequality between the rich and poor. Here we suppose we can simply add together the value of aggregate per capita consumption c_t in each period t for all kinds of goods and services for all people. This means aversion of the decision maker to within-period inequality is ignored, and we do not here consider different intertemporal treatments to natural environment versus other commodities.

These two simplications may not greatly affect what follows. If a particular natural species were to be increasingly threatened with extinction this could be represented through an escalating real price assigned to each item of that species, and so be implicitly captured in c_t . However, there could be a theoretical basis to consider having different discount rates across categories of consumption (Grimes 2023). If cross-sectional matters of inequality, inequity, and distributional justice were important (as outlined in Thompson 2022) then the decision maker may be able to weight the impacts of consumption across people differently when deriving the c_t term.¹² However, if matters of inequality, inequity, and distributional justice were both cross-sectional and occured over time (such as intergenerational poverty traps / privilege) then more general approaches may be required.¹³

A policy intervention may change the amount of total consumption over time, and the purpose of the judgement is whether that is preferrable to not intervening. The decision maker may compare the social welfare in each state to determine which scores higher.

¹² This is feasible for 'welfarist' or consequentionist theories of distributive justice, which assume that outcomes (and their distribution) are morally relevant. These theories include utilitarianism (maximising summed utilities), maxi-min (maximise the welfare of the worst-off person), prioritarianism (weighting the worse-off higher), and sufficientarianism (ensuring all have enough utility). Weighting would not address equity according to other theories of justice that take factors other than welfare and its distribution to be morally relevant. These other non-consequentionist theories include luck egalitarianism (equal opportunity rather than equal outcomes is important), relational egalitarianism (ensuring the moral equality of people), and libertarianism (just and fair processes and actions). These latter three seem to fit most with the ways New Zealanders think about fairness (NZIER 2024b).

¹³ For example, Emmerling et al (2017) adjusted the SRTP for growing intra-period inequality, which is akin to reducing the growth rate in consumption. New Zealand was not modelled, but for Australia they estimate the SRTP would be 0.16% points lower if $\mu = 1$ given the faster rate of growth of mean household income than median household income.

The concept of the 'social rate of time preference'

The social welfare score can be defined as follows, which is the sum into perpetuity¹⁴ of "social utility" $w(c_t)$ each period *t* measured in continuous time, with that social utility discounted with a "pure rate of time preference" ρ and adjusted for the risk of annihilation α that stops the cash flow stream:

$$W = \int_0^\infty w(c_t) e^{-(\rho+\alpha)t} dt \tag{4}$$

The above supposes there is no uncertainty or disagreement about the discount rate to use and that cash flows will be as expected. A common functional form used for social utility functions has a constant elasticity of marginal valuation of consumption, governed by $\mu > 0$:¹⁵

$$w(c_t) = \frac{c_t^{1-\mu}}{1-\mu}$$
(5)

for $\mu \neq 1$, and $w(c_t) = \ln c_t$ for $\mu = 1$.¹⁶ The parameter μ governs the curvature of the function (the rate of diminishing marginal value of consumption). This reflects the independent judges' aversion to variability of $w(c_t)$, or equivalently their preference to smooth $w(c_t)$. Substituting (5) into (4):

$$W = \int_0^\infty \frac{c_t^{1-\mu}}{1-\mu} e^{-(\rho+\alpha)t} dt$$
 (6)

If we consider policy interventions that causes only marginal (ie, infinitesimally small) deviations around a given forecast growth rate of consumption g (which is possibly not an optimised growth rate), then we can set $c_t = c_0 e^{gt}$, where c_0 is the initial period's consumption.

$$W = \int_0^\infty \frac{(c_0 e^{gt})^{1-\mu}}{1-\mu} e^{-(\rho+\alpha)t} dt$$
(7)

Distributing the exponents gives:

$$W = \int_0^\infty \frac{c_0^{1-\mu}}{1-\mu} e^{gt} e^{-(\rho+\alpha+\mu g)t} dt$$
 (8)

And finally:

$$W = \int_0^\infty w(c_0) e^{gt} e^{-rt} dt \tag{9}$$

where $r = \rho + \alpha + \mu g$ is the social rate of time preference (SRTP) that discounts consumption. The initial period's utility can be measured and scaled as per the usual approach in a cost-benefit appraisal (ie, the sum of willingness to pay), or $w(c_0) = c_0$. Actions can approximately be appraised by how they change the social welfare function:

$$\Delta W = \int_0^\infty \Delta c_t e^{-rt} dt \tag{10}$$

¹⁴ Turk (2024) surveyed 33 New Zealand self-identified experts on what they thought the appropriate time horizon for public policy planning should be in. About 3% said within 5 years; 9% within 30 years; 24% within 75 years; 30% within 150 years; 9% more than 150 years but not in perpetuity; and 27% in perpetuity.

¹⁵ The marginal valuation mv is $\frac{dw}{dc} = c^{-\mu}$. The elasticity of mv is $\frac{dmv}{dc}\frac{c}{mv} = -\mu c^{-\mu-1+1}c^{+\mu} = -\mu$.

¹⁶ The form (5) trends to $\ln c_t$ as $\mu \to 1$ from above or below.

If a policy initiative led to different rates of consumption growth (as the Stern Review considered for climate change) and/or the growth rate was expected to change over time, then rather than use (9) with a SRTP the decision maker might instead use (6) with a pure rate of time preference ρ .

If a policy altered the catastophe risk (eg, global treaties relating to existential threats such as warfare (nuclear weapons, bioweapons etc), artificial intelligence, greenhouse gas emissions) then the annihilation risk could be split out and time-indexed (α_t^1 with the policy and α_t^0 without), and the difference taken in a modified form of (6):

$$\Delta W = W^1 - W^0 = \int_0^\infty \frac{(c_t^1)^{1-\mu}}{1-\mu} e^{-(\rho + \alpha_t^1)t} dt - \int_0^\infty \frac{(c_t^0)^{1-\mu}}{1-\mu} e^{-(\rho + \alpha_t^0)t} dt$$
(6a)

Gollier (2013, chapter 3) expands this approach supposing the logarithm of consumption follows a continuous time random walk (geometric Brownian motion), normally distributed with a mean of g and a variance of s^2 . This leads to a lower SRTP of:

$$r = \rho + \alpha + \mu \bar{g} - \frac{1}{2}\mu(\mu + 1)s^2$$
(11)

The final term is a common type of adjustment with Brownian motion such as in the options literature (from Itô's Lemma, eg, see Dixit 1993), and the expected growth rate is adjusted upwards slightly to $\bar{g} = g + \frac{1}{2}s^2$.

We do not apply that approach here because we are more uncertain about the values of all the other parameters than we are about the growth rate of consumption, and we prefer to use the Weitzman (1998) approach to address this overall uncertainty or lack of clarity. It also does not seem to matter much for the range of parameters we consider. The variance s^2 of the growth rate in figure 1 is 0.04% (as the standard deviation *s* is about 2%), and so even with $\mu = 1.5$ — the maximum value we considered reasonable — this would reduce the SRTP by only 0.05%. At $\mu = 0.75$ — the most likely value we considered — the reduction in SRTP would be only 0.01%.¹⁷ Further adjustments can be made to account for autocorrelation of the growth rate of consumption, which reduces the discount rate more because the uncertainty of the discount rate persists longer. But we do not consider that refinement further because it is dwarfed by the uncertainty in the other parameters.

For (9) to be finite requires that the discount rate is greater than the growth rate both applied in perpetuity, ie, r > g,¹⁸ or that:

$$\rho + \alpha + \mu g > g \tag{2}$$

¹⁷ The Brownian motion SRTP is smaller than the original SRTP by $-\frac{1}{2}\mu s^2 + \frac{1}{2}\mu(\mu + 1)s^2$. The final term of (11) reduces the SRTP because of the 'prudence effect', but the adjustment to increase the expected growth rate offsets this.

¹⁸ Also see footnote 2. Although cost-benefit analysis compares just the incremental impact of an initiative on social welfare with and without the shock, if social welfare is infinitely large in both cases then the difference is indeterminant. Although any one initiative may have finite impact, it is this broader conceptualisation of total social welfare that we derive the SRTP from used to discount that initiative. But in case this is arguably not a concern we include the set of unrestricted SRTP estimates permitting r < g when deriving the declining discount rate schedule.

This is no problem if $\mu \ge 1$ for positive growth g; otherwise if it is less than one then it must be close to one if ρ is low, as (2) requires $\mu > \frac{g - (\rho + \alpha)}{g}$. Or if μ is low, then the pure rate of time preference needs to be close to the growth rate of consumption because (2) implies $\rho > g(1 - \mu) - \alpha$. We did not directly apply these restrictions to the individual parameters but rather to the resulting SRTP value, although more sophisticated treatment could have assumed correlations across the parameters.

Appendix 2 – Python code for Monte Carlo analysis

create a vector of samples randomly drawn from a triangular distribution

import numpy as np
SAMPLE_SIZE = 20000000
p = np.random.triangular(0.000, 0.0025, 0.02, SAMPLE_SIZE)
c = np.random.triangular(0.0, 0.0005, 0.002, SAMPLE_SIZE)
m = np.random.triangular(0.25, 0.75, 1.5, SAMPLE_SIZE)
g = np.random.triangular(0.0075, 0.0125, 0.02, SAMPLE_SIZE)

r = p+c+m*g

```
# this will create a vector of booleans indicating whether the value
is invalid
invalid = r < g</pre>
```

print("Fraction of invalid values: ", np.sum(invalid)/SAMPLE_SIZE)

```
# create a new vector that omit the invalid values (can just use the
boolean vector (negated) as an index)
new r = r[~invalid]
```

```
print("Average, excluding r<g: ", np.average(new_r))
print("Average, all: ", np.average(r))
print("Median, excluding r<g: ", np.median(new_r))
print("Median, all: ", np.median(r))
print("95 percent confidence interval, excluding r<g: ",
np.percentile(new_r,[2.5,97.5]))
print("95 percent confidence interval, all: ",
np.percentile(r,[2.5,97.5]))</pre>
```

```
# calculate the deciles of each distribution of r
deciles = np.percentile(new_r, np.arange(0, 101, 10))
deciles_all = np.percentile(r, np.arange(0, 101, 10))
```

```
# plot the distribution of r and indicate the deciles
import matplotlib.pyplot as plt
plt.hist(new_r, bins=300)
for d in deciles:
    plt.axvline(d, color='r')
    plt.axvline(np.average(new_r), color='y')
```

```
#also plot r-g (add an alpha to see through this histogram)
#plt.hist(r-g, bins=300, alpha=0.5)
```

```
#print(deciles)
print("Deciles, excluding r<g: ", deciles)
print("Deciles, all: ", deciles_all)</pre>
```

```
plt.savefig('tri_mc5.svg')
plt.show()
```

```
Fraction of invalid values: 0.14235995
Average, excluding r<g: 0.020712040782139805
Average, all: 0.019446737565499757
Median, excluding r<g: 0.02014444129797597
Median, all: 0.018977862594711997
95 percent confidence interval, excluding r<g: [0.01167424 0.03273402]
95 percent confidence interval, all: [0.00914517 0.03229159]
Deciles, excluding r<g: [0.00753732 0.01398635 0.01582565 0.01733453
0.01873825 0.02014444
0.0216454 0.02332529 0.02535559 0.02822808 0.0497272 ]
Deciles, all: [0.00303139 0.01189484 0.01404566 0.0157833 0.01738926
0.01897786
0.02063415 0.02245846 0.02462518 0.0276407 0.0497272 ]
```