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## Endogenous limits on proposal power

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# Endogenous Limits on Proposal Power<sup>1</sup>

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## **Abstract**

We present a dynamic model of legislative bargaining in which policy making proceeds until the proposer has no more incentive to make a new proposal to replace the previously approved policy. We characterize stationary Markov perfect equilibria for the game and show that in all pure-strategy equilibria, a majority of voters without proposal power have an incentive to protect each others' benefits to secure their own long-term bargaining positions in the legislature. As a consequence, the value of proposal power is constrained.

**JEL Classification Numbers:** C72, D72, D78.

**Keywords:** Legislative bargaining, proposal power, reconsideration, evolving default, distributive politics.

# 1 Introduction

Legislative decision-making is one of the most fundamental political institutions. In recent years it has been commonly modeled as a multilateral bargaining process with some majoritarian voting rule as the decision rule. The basic idea, developed by Baron and Ferejohn (1989a), is that a policy is made upon in a sequence of proposal making and voting stages by members of a legislature until a proposal is accepted by any decisive coalition. Perhaps one most fundamental insight of this literature is the importance of the "power to propose" (Romer and Rosenthal, 1978; Baron and Ferejohn, 1989a,b), that is, the ability of a designated proposer to take considerably more of the benefits than any other legislator. For example, in the baseline model of Baron and Ferejohn (1989a) with a closed amendment rule, any proposer makes a policy proposal to satisfy a bare majority of the "cheapest" voters necessary to ensure acceptance, fully expropriating the voters excluded from this minimal winning coalition. As a consequence, the distribution of benefits in equilibrium is highly unequal, with the proposer taking a disproportionately large share.

A key assumption of the Baron-Ferejohn model is that decision-making on a given issue effectively ends when a majority approves a proposal. However, in real-life politics legislators may be able to ask to reconsider a passed bill. In particular, the passage of a bill does not prevent the legislature from coming back to the same policy issue at a later date. Rather, it changes the default for subsequent policy making.

This feature of sequential bargaining was recently pointed out by Bernheim, Rangel and Rayo (2006), who proposed a distributive model of legislative bargaining with the possibility of reconsideration. They assume fixed and finite rounds of proposal making and common knowledge of the sequence of proposers before any legislative action begins. Given that sufficiently many legislators have a chance to make

a policy proposal, in the unique equilibrium the last proposer is able to implement his ideal policy. The model of Bernheim et al. thus yields an extreme form of proposal power, generated by the privileged position of the last proposer. This suggests that reconsideration would only strengthen proposal power and lead to fully unequal allocation of public resources.

In this paper, we modify a critical element of Bernheim et al. (2006). Specifically, in our analytical framework the last proposal round is endogenously determined rather than exogenously given. In other words, policy making is finalized only after the legislator with proposal power has no more incentive to replace the previously approved policy. This apparently minor modification nevertheless leads to radically different implications, as there arise incentives among the non-proposing legislators, or voters, to limit the ability of the proposer to expropriate others. To make this point as clear as possible, we model the pure case where only one legislator is persistently conferred the power to make policy proposals and all other legislators have no proposal power at all. The assumption of a single persistent proposer serves the theoretical purpose to isolate endogenous constraints on proposal power.<sup>1</sup>

As we show, the possibility of reconsideration induces a group of voters to "defend" the benefits for one another and implicitly coordinate their behavior against the sole powerful proposer. In particular, self-interested voters may decline any policy proposal when some other voters are substantially expropriated. In equilibrium, voters protect others as a means to prevent the proposer from playing off the voters against each other in the future. Intuitively, voter A protects voter B so that proposer C cannot use the low reservation value of B to exploit A. The voters' incentive to

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<sup>1</sup>Some legislative systems, however, come quite close to this idealized case. For example, comparative scholars have long observed that the constitutional features of parliamentary systems lead to high levels of agenda control by the executive, i.e. the cabinet (Döring, 1995). In the U.S. Congress, the proposal power over some policy issues is effectively held by members of the associated House Committees (Knight, 2005).

protect counterbalances the proposer's ability to exploit, which leads to a more equal allocation of benefits. The *ex post* value of proposal power is therefore substantially limited. Our theory applies to risk-neutral, purely self-interested agents. It does not depend on risk-aversion or any form of fairness concerns. The underlying mechanism can be illustrated by two examples below.

**Example 1** Consider a legislature with three players. The first player is assumed to be the proposer. The legislature must divide 6 dollars, where each dollar is indivisible. Suppose the initial default is  $x^0 = (1, 2, 3)$ , where the  $i$ -th element of  $x^0$  refers to the amount that goes to the  $i$ -th player. A default is the policy that will be implemented at the end of the legislative session if no new policy is made in the rest of the session. The proposer has sole power to make proposals and initiate reconsiderations. A proposal, once made, is immediately voted in the legislature by majority rule against the default. A policy proposal, once approved in the legislature, serves as the new default and may be subject to reconsideration if the proposer chooses to make a new proposal to replace it. The policy that (as default) survives until the end of the legislative session is the final policy outcome.

It is easy to see that in a one-round closed-rule legislative bargaining game, the proposer would propose  $y = (4, 2, 0)$ . This policy would be approved by the players who receive positive benefits and rejected by the player who receives nothing. Notice that the last player is fully expropriated since his vote is not necessary for the approval of the proposal. Now consider the case where the proposer is likely to have a chance to reconsider the policy issue and make another proposal. Then the second player would not accept policy  $y$  in equilibrium, even though  $y$  yields her exactly the same amount of benefits as the default. To see why, consider counterfactually what would happen if this player accepted  $y$ . In this situation, the proposer would have an incentive to

reconsider the policy issue and propose  $(6, 0, 0)$ , which would not be vetoed by the last player, who is indifferent. This implies that the second player would be eventually fully expropriated if she voted for  $y$  in the first place.

By the same logic, the proposer is not able to pass any policy that gives the last player, whose vote is not needed, any amount less than 2 dollars. The equilibrium policy outcome is thus  $(2, 2, 2)$ , an egalitarian division of the 6 dollars. In this equilibrium, the second player wants to protect the benefits for the last player, since by doing so the second player secures her long-term bargaining position in the legislature. Note that the value of proposal power is substantially reduced compared to a case without reconsideration.

**Example 2** Consider a legislature with five players. Again, the first player is assumed to be the sole proposer. The legislature must divide 10 dollars, where each dollar is indivisible. Suppose the initial default is  $(0, 1, 2, 3, 4)$ . Assume the political process is as given in the previous example. As before, in one-round closed-rule legislative bargaining, the proposer would propose  $z = (7, 1, 2, 0, 0)$ . The last two players are fully expropriated since their votes are not necessary for the approval of the proposal. However, if under some institution the proposer is likely to have a chance to reconsider the policy issue and make another proposal, players 2 and 3 would not accept policy  $z$  in equilibrium. With  $z$  as the new default policy, they would be fully expropriated and eventually given nothing.

One possible policy proposal players 2 and 3 would accept in an equilibrium is  $x^* = (4, 2, 2, 2, 0)$ . With  $x^*$  as the new default, the proposer would not be able to pass any new policy that gives himself more than 4 dollars. This is because players 2, 3 and 4 all have incentives to protect the benefits for one another. If any player  $i \in \{2, 3, 4\}$  was expropriated and given less than 2 dollars, the proposer would have an incentive

to reconsider the policy and ally with both the last player and the expropriated one to increase his own benefit entitlement.

In this equilibrium, some voter, player 4 in the example, is not fully expropriated even if his vote is not needed. Players 2 and 3 vote for  $x^*$  against the default, and player 2 is even given more than what he would receive from the default. The value of proposal power is substantially reduced. Out of fear that the proposer may use his proposal power to exploit those players with lower reservation values in the future, a group of players take care of one another even if they are self-interested.

The purpose of this paper is primarily theoretical. We develop a general distributive model that identifies clearly the effects illustrated in the above examples, and show that the endogenous limits to proposal power, imposed by the voters, are robust in all pure-strategy stationary equilibria if the probability for reconsideration is sufficiently high.

We also identify a second class of stationary equilibria characterized by mixed proposal strategies. In those equilibria, the proposer randomizes among possible winning coalitions and strategically designs a series of policy proposals so that he eventually captures almost all of the benefits. The pattern of these equilibria is similar to that of Kalandrakis (2004, 2007), who constructs a mixed-strategy stationary equilibrium for legislative bargaining in a dynamic environment where after periods of transition every proposer takes all the benefits. However, we argue that if the legislature needs to make a collective decision on whether to discuss a policy, i.e. to put it on the agenda, these mixed-strategy equilibria disappear and only the pure-strategy equilibria survive.

It is important to distinguish our theory from one that is based on weaker forms of proposal protocols in legislative bargaining. Baron and Ferejohn (1989a) argue that

agendas that allow for (possibly nested) amendments, the so-called "open amendment rules," may effectively lead to weaker proposer premiums and more egalitarian allocation of the benefits. As a proposer gives away more benefits to more voters in the legislature, he could reduce the probability that some other legislator submits an amendment and therefore his proposal is more likely to be approved. A limited proposal power in this case thus results from voluntary choices by the proposer due to a trade-off between the approval probability and the extent of exploitation.

The endogenous constraints on proposal power in our theory, on the other hand, does not rely on a more dispersed allocation of proposal power. Instead, it arises from equilibrium behavior of the non-proposers even in the closed-rule case. Our analytical framework thus offers a novel yet complementary mechanism of sequential majoritarian bargaining that, while potentially of equal importance, has been overlooked. With the possibility of reconsideration, the voters are induced to take care of some of the others, so that the proposer is limited to make a policy proposal from a smaller set of policies that allocate the benefits more equally. Counterintuitively, this effect is especially strong when there is only one single legislator that persistently holds proposal power. Importantly, a concentration of proposal power through procedure does not necessarily lead to valuable proposal power in outcomes.

Motivated by recent empirical literature that questions the extent of proposal power implied by the model of Baron and Ferejohn (1989a, 1989b), various other explanations for the limits of proposal power have been proposed, ranging from different proposal protocols (Baron and Ferejohn, 1989a; Morelli, 1999), risk aversion (Battaglini and Palfrey, 2007; Bowen and Zahran, 2007), to dynamic risk-sharing (Dixit, Grossman and Gul, 2000), and even fairness and entitlement concerns (Diermeier and Gailmard, 2006).<sup>2</sup> Of course, we provide an alternative account for some

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<sup>2</sup>As various distributive models of legislative bargaining have been tested in laboratory exper-

stylized facts, e.g. why a proposer may allocate positive benefits to more than a minimum winning coalition of legislators. More importantly, our theory leads to new empirical implications, potentially testable, that the other available theories cannot explain. For example, in an extension in which the proposal power is controlled by a committee of legislators instead of a single legislator, our theory identifies a positive correlation between the committee size and the size of a supermajority of legislators who receive positive benefits in equilibrium. Such a positive correlation is consistent with the data set of Knight (2005).

There is a growing literature, based on dynamic extensions of the Baron-Ferejohn model, that explores policy dynamics in an environment where the status quo in one period is endogenously determined by the policy chosen in the previous period (Baron 1996; Kalandrakis, 2004, 2007; Bowen and Zahran, 2007; Battaglini and Coate, 2007, 2008). A common feature in our model and this literature is that the status quo, or the default policy, endogenously evolves. However, our model is not a dynamic extension of the Baron-Ferejohn model, but instead it should be interpreted as an alternative model of policy choice for the stage game in a dynamic setup. For example, Diermeier and Fong (2008a, 2008c) apply the model presented in this paper to study, respectively, the ratchet effect of government spending in multiparty parliamentary countries and the policy dynamics in new democracies.

Our paper is also linked to the literature on stationary equilibria in dynamic games. Notice that a stationary equilibrium for dynamic legislative bargaining with an endogenous default need not exist, and, if it does, it is usually associated with mixed strategies as is the case in, e.g., Kalandrakis (2004; 2007), Fong (2006), Battaglini and

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iments (McKelvey, 1991; Fréchette, Kagel, and Lehrer, 2003; Diermeier and Gailmard, 2006; and Battaglini and Palfrey, 2007) or using field data from the U.S. Congress (Knight, 2005), a common finding in these studies is that proposal power is indeed valuable but not as much as predicted by the theory.

Palfrey (2007), and Bowen and Zahran (2007). Duggan and Kalandrakis (2007) prove general existence of a pure-strategy stationary equilibrium for this class of dynamic games, but only with some suitably assumed randomness on preferences and the dynamic process of the status quo. Although our model does not satisfy the sufficient conditions of Duggan and Kalandrakis (2007), we show that a class of pure-strategy stationary equilibria still exists if the probability that a passed bill can be reconsidered is sufficiently high. Therefore, our model is tractable for general characterization and can be easily applied to dynamic models or embedded in a specific economic context.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 defines a stationary Markov perfect equilibrium. Section 4 illustrates the key mechanism and intuition of the model in a special case with three players. Section 5 provides general characterization of all pure-strategy stationary equilibria, and show that the results for a game with three players can be generalized for a game with an arbitrary odd number of players. Section 6 identifies a different class of mixed-strategy equilibria and discusses the issue of equilibrium selection. Section 7 presents an extension of the model, in which a committee of legislators, instead of a single proposer, is granted proposal power. Section 8 concludes and finally, the Appendix includes all the proofs.

## 2 The Model

Let  $N = \{1, 2, \dots, n\}$  be a set of  $n$  players in the legislature, where  $n = 2m + 1$  and  $m \in \mathbb{N}$ . The legislature must decide on how to divide  $\pi$  units of total benefits among the players, where  $\pi \in \mathbb{N}$  is exogenously given. The policy space is finite and denoted by  $X = \Delta_{\pi}^n \equiv \{x \in \mathbb{Z}_{+}^n : \sum_{\ell=1}^n x_{\ell} = \pi\}$ .<sup>3</sup> Given any policy  $x = (x_1, x_2, \dots, x_n) \in X$ ,

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<sup>3</sup>Recently, the assumption of a discrete policy space has become more popular in political economy models of coalition formation. For examples, see Bernheim, Rangel and Rayo (2006), Dekel, Jackson

player  $\ell \in N$  receives  $x_\ell$  units of benefits and derives a utility of  $u_\ell(x) = x_\ell$ .

There is one *proposer* in the legislature. Assume this position is occupied by player 1. The proposer is conferred the sole power to make policy proposals from the policy space during the legislative session. All other players,  $i \in N \setminus \{1\}$ , are referred to as *voters*.

The legislature selects a policy over the course of potentially infinite rounds of proposal making, where the number of rounds depends on exogenous factors and the decision made by the proposer. Activities prior to each round  $t \in \mathbb{N}$  establish a default policy  $x^{t-1} \in X$ . In each round  $t$ , once reached, the proposer can choose to make a proposal  $y^t \in X$  or pass the proposal round. A "pass" means inaction by the proposer and, for mathematical convenience, is modeled as a proposal  $y^t = x^{t-1}$ . The proposal, once made, is then put to an immediate vote against  $x^{t-1}$ . If it is approved by majority rule, it replaces  $x^{t-1}$  as the default policy and  $x^t = y^t$ . If it is not approved, the default policy remains the same and  $x^t = x^{t-1}$ . As the legislative session commences, an initial default  $x^0 \in X$  is exogenously given. The initial default is interpreted as the policy that has been enacted prior to the legislative session. The policy that survives as default till the end of the legislative session is implemented.

How the legislative session would end is described below. Whereas Bernheim et al. (2006) assume a fixed number of proposal rounds, in the model considered here the last proposal round is not predetermined. We say the legislative session ends endogenously after proposal round  $t$ , if the default  $x^t$  established by the first  $t$  rounds of proposal making and voting is such that the proposer will choose to pass any possible proposal

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and Wolinsky (2008), and Acemoglu, Egorov and Sonin (2008a,b). This modeling choice is usually motivated by technical convenience. Substantively, a discrete policy space limits the extent to which utilities are transferable among the players. From the perspective of modeling real-life distributive politics, the assumption seems innocuous, as, e.g. entitlement programs and the allocation of local transportation and infrastructure projects, usually involve a minimal spending unit, even if it is very small, say a dollar.

round  $t' > t$ . In addition, after any proposal round the legislative session may be terminated exogenously with probability  $1 - \delta$ , where  $\delta \in [0, 1)$  is the probability that the proposer will have an opportunity to reconsider the policy that emerges from the current round. We interpret  $\delta$  as a parameter of the legislative institution, since various congressional rules, unmodeled here, may affect the likelihood of chances for reconsideration.<sup>4</sup> For example, the case of  $\delta = 0$  is associated with one-round closed-rule legislative bargaining (Romer and Rosenthal, 1978; Baron and Ferejohn, 1989). This paper focuses on institutions in the limiting case with  $\delta \in (0, 1)$  sufficiently large.

Diermeier and Fong (2008b) presents a variant of the model with a continuous-time framework where a policy, once enacted, is in effect until it is reformed through the political process. With a single, permanent proposer that model shares the same mathematical structure as the one presented here.

### 3 Equilibrium Definition

We focus the analysis on stationary Markov perfect equilibria.<sup>5</sup> In any proposal round  $t \in \mathbb{N}$ , the only payoff-relevant state variable is the prevailing default, which will be the policy outcome if it is not replaced by any new proposal in the rest of the legislative session. The legislature thus faces an identical collective choice problem in proposal rounds  $t_1$  and  $t_2 \neq t_1$ , if the prevailing defaults in those two rounds are identical. Stationarity implies that the players condition their strategies only on the prevailing default. From now on, we drop the subscript  $t$  for the proposal round from the notations.

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<sup>4</sup>In principle, the probability of session continuation needs not be stationary over proposal rounds. For example, in the finite-period model of Bernheim et al. (2006),  $\delta_t = 1$  for all  $t < T$ , and  $\delta_t = 0$  for all  $t \geq T$ , where  $T$  is the fixed number of proposal rounds.

<sup>5</sup>See Baron and Ferejohn (1989) and Austen-Smith and Banks (2005) for justifications of stationary equilibria in legislative bargaining games. See Maskin and Tirole (2001) for more general discussions.

Let  $P(X)$  be the space of probability measures over the policy space  $X$ , and  $\sigma : X \rightarrow P(X)$  be the proposal strategy of the proposer.<sup>6</sup> In particular,  $\sigma(x, y)$  denotes the probability that the proposer proposes a policy  $y \in X$  when the prevailing default is  $x \in X$ .

Let  $U_\ell(x)$  be the expected utility of player  $\ell$  if the prevailing default is  $x$ . With probability  $1 - \delta$  the legislative session is exogenously terminated after the current proposal round and this player receives a utility of  $u_\ell(y)$ . With probability  $\delta$  the proposer has a chance to reconsider the approved policy and makes a new proposal according to policy rule  $f$ . In this case, player  $\ell$  receives a continuation utility of  $U_\ell(f(x))$ . Thus,

$$U_\ell(x) = (1 - \delta) u_\ell(x) + \delta U_\ell(f(x)). \quad (1)$$

We refer to  $U_\ell : X \rightarrow \mathbb{R}$  as the value function of player  $\ell$ .

We make two technical assumptions regarding how the players break indifference. First, any player votes against a policy proposal if and only if passage of the proposal makes him strictly worse off. Second, the proposer never proposes any shift in policy that is destined to be vetoed by a majority of voters. None of our qualitative results depend on the first assumption. The second one simplifies the notation, but is otherwise innocuous since making a losing proposal is equivalent to remaining at the prevailing default.

In each proposal round, the proposer selects a policy proposal to maximize his expected utility. A policy is politically feasible if, as a proposal it would be approved by a majority of players. By the second assumption above, the maximization problem of the proposer is subject to the constraint of political feasibility. By the first

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<sup>6</sup> A pure-strategy equilibrium can be a focal point for the game considered here due to its simplicity. See Diermeier and Fong (2008a) for characterization and comparison of pure-strategy and mixed-strategy equilibria for related game with distributive politics.

assumption above, the feasibility constraint is equivalent to an incentive compatibility constraint that requires that a majority of players are weakly better off with the proposal policy than with the prevailing default. To sum up, given any default  $x \in X$ , any policy  $y \in X$  that the proposer may propose with a strictly positive probability must solve

$$\begin{aligned} \max_{y \in X} \quad & U_1(y) \\ \text{s.t.} \quad & |\{\ell \in N : U_\ell(y) \geq U_\ell(x)\}| \geq m + 1, \end{aligned} \tag{2}$$

where, for any finite set  $A$ ,  $|A|$  denotes the number of its elements. We are now ready to define an equilibrium.

**Definition 1** A *stationary Markov perfect equilibrium* is a proposal strategy  $\sigma$  and a set of value functions  $\{U_\ell\}_{\ell=1}^n$  such that:

1. Given  $\sigma$ ,  $\{U_\ell\}_{\ell=1}^n$  solve the equation system defined by (??).
2. Given  $\{U_\ell\}_{\ell=1}^n$ ,  $\sigma(x, y) > 0$  only if  $y \in X$  solves problem (2) of the proposer with a default  $x \in X$ .

Taking any stationary Markov perfect equilibrium with proposal strategy  $\sigma$ , we introduce some useful notations and impose additional requirements on the equilibria we characterize in this paper. A policy  $x \in X$  is stable in equilibrium if  $\sigma(x, x) = 1$ , i.e., it persists as default with probability one. Let  $S \equiv \{x \in X : \sigma(x, x) = 1\}$  be the set of all stable policies, or the stable set. Denote the ideal policy of player  $\ell \in N$  by  $s^\ell \in X$ , where  $s_\ell^\ell = \pi$  and  $s_i^\ell = 0$  for all  $i \in N \setminus \{\ell\}$ . As the proposer would never make a new proposal to replace his ideal policy as default, the stable set is nonempty and  $s^1 \in S$ . We impose a notion of symmetry on the solution concept and restrict attention to equilibria where *the stable set is symmetric with respect to all voters*, i.e.,

if  $x \in S$  and  $y \in X$  is such that, for some  $i, j \in N \setminus \{1\}$ ,  $y_i = x_j$ ,  $y_j = x_i$ , and  $y_\ell = x_\ell$  for all  $\ell \in N \setminus \{i, j\}$ , then  $y \in S$ .

Let  $\{x^t\}_{t=0}^\infty$  be any sequence of policies such that  $\sigma(x^{t-1}, x^t) > 0$  for all  $t \in \mathbb{N}$ . This policy path traces the evolution of default along some equilibrium path starting from an initial default  $x^0 \in X$ . We further restrict attention to equilibria where *any policy path induced by  $\sigma$  converges*, i.e., for all  $x^0 \in X$ , there exists  $T(x^0) \in \mathbb{N}$  such that  $\sigma(x^t, x^t) = 1$  for all  $t \geq T(x^0)$  and all equilibrium policy paths  $\{x^t\}_{t=0}^\infty$ . With the policy space considered here, there exist equilibria where the policy path does not converge for some initial default.<sup>7</sup> Along those equilibrium paths, in every proposal round the proposer makes a new policy proposal with positive probability and policy making continues until the legislative session is exogenously terminated. Given that we assume a single persistent proposer, it is natural to rule out the equilibria with nonconvergent policy paths.

**Definition 2** A *legislative equilibrium* is a stationary Markov perfect equilibrium where the stable set is symmetric with respect to voters and every policy path induced by the equilibrium proposal strategy converges.

The main contribution of this paper is based on full characterization of pure-strategy legislative equilibria for all  $\delta \in (0, 1)$  sufficiently large. A proposal strategy  $\sigma$  is a pure strategy if there exists a policy rule,  $f : X \rightarrow X$ , such that  $\sigma(x, f(x)) = 1$  for all  $x \in X$ . The existence of a pure-strategy stationary Markov perfect equilibrium for the game considered here is not specific to the particular distributive policy space assumed in this paper. Diermeier and Fong (2008b) prove the existence of, and pro-

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<sup>7</sup>For example, assume  $n = 3$  and  $\pi = 1$ . The policy space thus is  $\Delta_1^3 = \{s^1, s^2, s^3\}$ . There exists a stationary Markov perfect equilibrium with a proposal strategy  $\sigma$  such that  $\sigma(s^1, s^1) = 1$ ,  $\sigma(s^2, s^3) = 1$ , and  $\sigma(s^3, s^2) = 1$ . Suppose  $x^0 = s^2$ . Then the policy alternates between  $s^3$  in odd periods and  $s^2$  in even periods until the legislative session is exogenously terminated.

pose an algorithm to construct, a pure-strategy stationary Markov perfect equilibrium for the game with a single proposer and the possibility of reconsideration, given any *finite* policy space and any  $\delta \in (0, 1)$  sufficiently large.<sup>8</sup>

In any legislative equilibrium with a policy rule  $f$ , the stable set is

$$S = \{x \in X : f(x) = x\}.$$

For any  $x^0 \in X$ , let  $\{f^t(x^0)\}_{t=0}^\infty$  be such that  $f^0(x^0) = x^0$  and  $f^t(x^0) = f(f^{t-1}(x^0))$  for all  $t \in \mathbb{N}$ . Since any policy path induced by  $f$  converges in a legislative equilibrium, let  $f^\infty(x^0) \equiv \lim_{t \rightarrow \infty} f^t(x^0)$ . By definition,  $f^\infty(x^0) \in S$ . We will call  $f^\infty(x^0)$  the *final policy outcome* if the legislative session lasts for sufficiently many proposal rounds so that in realization the stable set is reached.

A preliminary analysis shows that, except for rare cases, the policy outcome in any equilibrium for a one-round closed-rule legislative bargaining game cannot be the final policy outcome in a pure-strategy legislative equilibrium for the game that allows the possibility of reconsideration. Intuitively, this means that allowing reconsideration fundamentally changes the set of equilibria compared to the model of Baron and Ferejohn (1989a). This observation is summarized in the first proposition. The proof is presented in the Appendix.

**Proposition 1** Let  $\hat{f}(x)$  be the policy rule in any legislative equilibrium for the game with  $\delta = 0$ . Assume  $\delta > 0$  and take any  $x^0 \in X$  such that  $\hat{f}(x^0) \neq s^1$ . Then in any legislative equilibrium with policy rule  $f$ ,  $f^\infty(x^0) \neq \hat{f}(x^0)$ .

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<sup>8</sup>A pure-strategy stationary Markov perfect equilibrium, in general, needs not exist in dynamic games that involve coalition formation. For example, see Fong (2006) and Kalandrakis (2004; 2007) for dynamic legislative bargaining games where the status quo is endogenously determined and no pure-strategy stationary equilibrium exists.

## 4 A Legislature with Three Players

This section considers a legislature with three players. This special case illustrates the key mechanism of the model with transparency. General results are presented in later sections. Proposition 2 constructs and characterizes a class of legislative equilibria for a three-player legislature. The proof is presented in the Appendix.

**Proposition 2** Assume  $n = 3$ ,  $X = \Delta_\pi^3$ , and  $\delta \in (0, 1)$  sufficiently large. For all functions  $e : X \rightarrow \mathbb{Z}_+$  such that, for all  $x \in X$ ,  $\min\{x_2, x_3\} \leq e(x) \leq \frac{1}{2}(x_2 + x_3)$ , there exists a legislative equilibrium with policy rule  $f$  such that, for all  $x \in X$ ,  $f_1(x) = \pi - 2e(x)$  and  $f_2(x) = f_3(x) = e(x)$ .

This proposition has several important implications. First, in equilibrium the proposer receives no less than his benefit entitlement by default. In other words,  $f_1(x) \geq x_1$ . Second, in equilibrium a policy is stable if and only if both voters receive the same amount of benefits from the policy. In other words,  $S = \{x \in X : x_2 = x_3\}$ . Third, the proposer always proposes a stable policy immediately, and reconsideration never occurs on the equilibrium path. In other words,  $f(x) \in S$  for all  $x \in X$ . The possibility of reconsideration, however, changes equilibrium proposal making and voting. Fourth, the two voters receive the same amount of benefits in equilibrium, and this amount is no less than the default benefit entitlement of the voter who receives less benefits from the default among the two voters. In other words,  $f_2(x) = f_3(x) \geq \min\{x_2, x_3\}$ . Crucially, this implies that the voter whose vote is not needed to pass the new policy is not fully expropriated by the proposer. In fact, the proposer can expropriate at most  $\max\{x_2, x_3\} - \min\{x_2, x_3\}$  units of the benefits. As a consequence, in equilibrium the benefit level received by the proposer is bounded above by  $\pi - 2\min\{x_2, x_3\}$ , and this is smaller than  $\pi - \min\{x_2, x_3\}$ , the proposer's

benefits in the case of legislative bargaining where reconsideration is not allowed.

The equilibrium proposal strategy can intuitively be described as follows: the proposer seeks voting support from the voter with a lower reservation value and expropriates the other voter to the extent that the two voters receive the same amount of benefits. In a dynamic setup with an evolving default, however, it is not necessarily trivial to assess which voter is the cheaper one to satisfy.<sup>9</sup> In our case the critical voter is the one who receives less from the default. To see why, consider any  $x \in X$  such that  $x_2 < x_3$ . Since  $x_2 \leq e(x) < x_3$ , the reservation values of players 2 and 3 are calculated as  $U_2(x) = (1 - \delta)x_2 + \delta e(x) \in (e(x) - 1, e(x)]$  and  $U_3(x) = (1 - \delta)x_3 + \delta e(x) > e(x)$ , given that  $\delta \in (0, 1)$  is sufficiently large. In equilibrium, the proposer offers  $e(x)$  units of benefits to both voters. Among all stable policies, this is the one that just satisfies player 2 by her reservation value and maximizes the expected utility of the proposer.

Why does the proposer have to offer both voters an equal amount of benefits? Suppose that the proposer offers  $e(x)$  units of benefits to player 2 but only some  $k < e(x)$  units to player 3. It is obvious that player 3 will vote against the proposal since her benefit level is reduced. But so will player 2. To understand why, consider counterfactually, what would happen if player 2 approved the policy  $y = (\pi - e(x) - k, e(x), k)$ . With probability  $1 - \delta$  the legislative session would end immediately and  $y$  would be implemented. With probability  $\delta$ , however, the proposer would have a chance to reconsider the policy issue and propose a new policy  $f(y) = (\pi - 2e(y), e(y), e(y))$  according to his equilibrium strategy. This policy makes player 3 at least as well off as with  $y$  and therefore would be approved by majority voting. Since  $e(y) \leq \frac{1}{2}(e(x) + k) < e(x)$ , by voting for policy  $y$  player

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<sup>9</sup>For example, Bernheim, Rangel and Rayo (2006) use backward induction to identify the set of voters from whom an agenda setter optimally seeks voting support.

2 would be worse off when the policy is reconsidered. Anticipating such an adverse consequence, player 2 will always vote against the proposal of  $y$ , even though she receives  $e(x) \geq x_2$  units of benefits from this policy. By this argument, player 2 will not allow the proposer to expropriate the other voter too much so that, in the subsequent proposal rounds, the other voter will have a lower reservation value than hers and look more attractive for the proposer to ally with. As a consequence, the best the proposer can achieve is to offer both voters equal amount of benefits and just satisfy the voter who is given less by the default.

Although the voters derive utilities only from the benefits they receive, in equilibrium they have indirect preferences over the *distribution of benefits*. In the above example, player 2 strictly prefers  $(\pi - 2e(x), e(x), e(x))$  to  $(\pi - e(x) - k, e(x), k)$ , where  $k < e(x)$ , even though either policy, if realized, gives her  $e(x)$  units of benefits. Through the dynamic link of an evolving default, the allocation of benefits affects the distribution of bargaining power in the rest of the legislative session.

Therefore, the two voters effectively demand a more egalitarian allocation of resources between them. In particular, any voter does not allow the other voter to be sufficiently expropriated by the proposer. This demand for "fairer allocations" results from self-interested voters who want to improve their long-term bargaining positions. It does not depend on primitive preferences for fair allocations or risk aversion.<sup>10</sup>

On the other hand, the proposer has an incentive to expropriate as much as possible. The proposer proposes less benefits for himself compared to the case without reconsideration because mutual protection between the voters imposes endogenous constraints on the set of policies that can be approved by majority voting in equilibrium. As a consequence, the proposer has limited ability to expropriate the voter

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<sup>10</sup>In a model of collective decision over legislative procedures, this insight may have implications for the existence of minority rights and benefits in legislatures.

whose vote is not needed.

Notice also that, compared to the case without reconsideration, granting the proposer the power to reconsider *reduces* his equilibrium payoff. Therefore, a proposer would have an incentive to commit not to reconsider a passed bill. In the model considered here such a promise would not be credible.

Depending on the initial default, the proposer may *not* be the one who receives the most benefits in equilibrium. Indeed, he may be the player who gets the *least* amount. This, for example, happens if the proposer already receives the least benefits by the initial default. For example, assume  $\pi = 10$  and the initial default is  $(1, 4, 5)$ . The unique equilibrium policy outcome then is  $(2, 4, 4)$ .

Proposition 2 also implies the existence of multiple legislative equilibria. In particular, for any  $x \in X$  such that  $|x_2 - x_3| \geq 2$ , different values of  $e(x)$  are permissible. Consider a numerical example with  $\pi = 6$ . Assume the initial default is  $x = (2, 1, 3)$  and discuss two possibilities. First, suppose everyone believes that, if the default  $x = (2, 1, 3)$  remains and the legislative session continues, policy  $y^1 = (4, 1, 1)$  will be proposed and approved in the next proposal round. Given this expectation, the reservation values of players 2 and 3 are calculated as  $U_2(x) = (1 - \delta) + \delta = 1$  and  $U_3(x) = 3(1 - \delta) + \delta > 1$ . The proposer then has to offer both voters 1 unit of benefits to obtain a vote from player 2, and  $y^1$  is the optimal choice for him. The fact that the proposer proposes  $y^1$  when facing the default  $x$  is consistent with the common belief of the players. Second, suppose instead everyone believes that, if the default  $x = (2, 3, 1)$  remains and the legislative session continues, policy  $y^2 = (2, 2, 2)$  will be proposed and approved in the next proposal round. Given this expectation, the reservation values of players 2 and 3 are calculated as  $U_2(x) = (1 - \delta) + 2\delta \in (1, 2)$  and  $U_3(x) = 3(1 - \delta) + 2\delta \in (2, 3)$ . The proposer then has to offer both voters 2 units of benefits to obtain a vote from player 2, and  $y^2$  is the optimal choice for him.

The equilibrium proposal strategy thus is consistent with the common belief of the players.

The two possible equilibrium policy outcomes in this example are  $y^1 = (4, 1, 1)$  and  $y^2 = (2, 2, 2)$ . In the former equilibrium, the proposer expropriates 2 units of benefits from the voter whose vote is not needed to pass the equilibrium policy, whereas in the latter he is not able to increase his benefit entitlement. In both cases the value of proposal power is constrained compared to that implied by the closed-rule legislative bargaining game.

The existence of multiple equilibria is driven by self-fulfilling expectations.<sup>11</sup> How beliefs are coordinated goes beyond the scope of this paper. In principle, legislators may have channels to communicate with one another through policy deliberation or in private occasions before the official legislative procedures.

The multiplicity of equilibria is inherent in the discrete policy space. All the equilibria characterized in Proposition 2 are stable and robust to trembles. Moreover, the mechanism of mutual protection by the voters relies on the condition that there is not a predetermined last proposal round, so none of the legislative equilibria in the proposition can be supported in a model with finite proposal rounds. Therefore, a finite-round model cannot be applied to refine the equilibria.

The next proposition focuses on the final policy outcome in equilibrium and characterizes necessary conditions for all pure-strategy legislative equilibria, including the ones where reconsideration actually occurs.<sup>12</sup> The proof is presented in the Appendix.

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<sup>11</sup>See Hassler et al. (2003) for another example of self-fulfilling multiple equilibria in a political economy model with a redistributive policy and repeated voting.

<sup>12</sup>Here is one example in which a passed policy may be reconsidered along some equilibrium path. Assume  $n = 3$ ,  $\pi = 2$  and  $X = \Delta_2^3$ . For all distinct  $i, j \in N$ , let  $s^{ij} \in X$  be such that  $s_i^{ij} = s_j^{ij} = 1$ . Also recall that, for all  $\ell \in N$ ,  $s^\ell$  is the ideal policy of player  $\ell$  so  $s_\ell^\ell = 2$ . There exists a legislative equilibrium with policy rule  $f$  such that  $f(s^1) = f(s^2) = f(s^3) = f(s^{13}) = s^1$ ,  $f(s^{12}) = s^{13}$ , and  $f(s^{23}) = s^{23}$ . Consider the initial default  $x^0 = s^{12}$ . Then it takes two proposal rounds to reach the final policy outcome. Formally,  $f^\infty(x^0) = f^2(x^0) = s^1 \neq f(x^0) = s^{13}$ .

**Proposition 3** Assume  $n = 3$ ,  $X = \Delta_\pi^3$ , and  $\delta \in (0, 1)$  sufficiently large. Any legislative equilibrium with policy rule  $f$  must be such that, for all  $x \in X$ ,  $f_2^\infty(x) = f_3^\infty(x) \geq \min\{x_2, x_3\}$  and  $x_1 \leq f_1^\infty(x) \leq x_1 + \max\{x_2, x_3\} - \min\{x_2, x_3\}$ .

Implied by Proposition 3, the endogenous constraint on proposal power imposed by mutual protection of the voters is characteristic of *all* pure-strategy legislative equilibria. Even in the equilibrium where proposal power has maximal strength, i.e.,  $f_1^\infty(x) = x_1 + \max\{x_2, x_3\} - \min\{x_2, x_3\}$ , the proposer still receives strictly less than what he would do in closed-rule legislative bargaining, i.e.,  $x_1 + \max\{x_2, x_3\}$ . The insight holds no matter which equilibrium occurs.

Proposition 3 also identifies the bounds on the final policy outcome resulting from any initial default. Observe that any possible final policy outcome can be supported in some pure-strategy legislative equilibrium described in Proposition 2. Therefore, Proposition 2 presents *all* pure-strategy legislative equilibria where no reconsideration occurs. With the focus on the equilibrium allocation of the benefits and the value of proposal power, a full characterization of all those equilibria is sufficient.

## 5 General Results

This section presents results for the game with an arbitrary odd number of players. Some additional notations facilitate the exposition. Fix any  $x \in \Delta_\pi^n$ . For any  $k \in \mathbb{Z}_+$ , let  $Q(x, k) \equiv \{i \in N \setminus \{1\} : x_i = k\}$  be the collection of all voters who receive exactly  $k$  units of benefits from policy  $x$ . For any  $K \in \mathbb{Z}_+$ , the subset  $\bigcup_{k=0}^K Q(x, k)$  of players thus includes all and only voters who receive no more than  $K$  units of benefits from policy  $x$ . For any  $r \in \{1, 2, \dots, n-1\}$ , let  $z_r(x) \equiv \min K \in \mathbb{Z}_+ \text{ s.t. } \left| \bigcup_{k=0}^K Q(x, k) \right| \geq r$ . In words,  $z_r(x)$  denotes the benefits received from policy  $x$  by the voter who is in the  $r$ -th place from bottom if all voters are ranked according to

their benefit entitlement.

The first theorem establishes the existence of a pure-strategy legislative equilibrium and characterizes a class of equilibria where no reconsideration occurs. The proof is presented in the Appendix.

**Theorem 1** Assume  $\delta \in (0, 1)$  sufficiently large. For any function  $e : X \rightarrow \mathbb{Z}_+$  such that  $z_m(x) \leq e(x) \leq \frac{\pi - x_1}{m+1}$  for all  $x \in X$ , there exists a legislative equilibrium in which:

1. The policy rule  $f$  can be constructed by the following algorithm. Step A. For all  $x \in X$ , take a coalition  $M_f(x)$  of voters such that  $|M_f(x)| = m$  and  $M_f(x) \subseteq \bigcup_{k=0}^{e(x)} Q(x, k)$ . Such a coalition exists since  $e(x) \geq z_m(x)$ . Step B. Take any voter  $i_f(x) \in N \setminus (M_f(x) \cup \{1\})$  and let  $M_f^+(x) = M_f(x) \cup \{i_f(x)\}$ . Step C. Let

$$f_\ell(x) = \begin{cases} \pi - (m+1)e(x), & \text{if } \ell = 1, \\ e(x), & \text{if } \ell \in M_f^+(x), \\ 0, & \text{otherwise.} \end{cases}$$

2.  $S = \bigcup_{k=0}^{\infty} S(k)$ , where  $S(0) \equiv \{s^1\}$  and, for all  $k \in \mathbb{N}$ ,

$$S(k) \equiv \{x \in X : Q(x, k) = m+1 \text{ and } Q(x, 0) = m-1\}.$$

3. For all  $x \in X$ ,  $f(x) \in S$ .
4. For all  $x \in X$  and for all  $i \neq 1$  such that  $U_i(f(x)) \geq U_i(x)$ ,  $f_i(x) \geq z_m(x)$ .
5. For all  $x \in X$ ,  $x_1 \leq f_1(x) \leq \pi - (m+1)z_m(x)$ .

Part 1 of the theorem presents the form of an equilibrium policy rule. Recall that the legislature consists of  $2m + 1$  players, and the proposer only needs  $m$  votes from the voters to pass a new policy. However, in equilibrium a group of  $m + 1$  voters receive positive amounts of benefits. This implies that one voter whose vote is not necessary for approval of the policy is not fully expropriated by the proposer. Moreover, all these  $m + 1$  voters receive the same amount of benefits in equilibrium, despite their possibly differentiated benefit allocations in the initial default.

The algorithm to construct a policy rule also implies the existence of multiple legislative equilibria. The multiplicity is due to self-fulfilling expectations, as explained in the case with three players. For a numerical example, assume  $n = 5$  ( $m = 2$ ) and  $\pi = 10$ . Consider an initial default  $x^0 = (0, 1, 2, 3, 4)$ . The possible equilibrium outcomes then include  $(4, 2, 2, 2, 0)$ ,  $(4, 2, 2, 0, 2)$ ,  $(1, 3, 3, 3, 0)$ ,  $(1, 3, 3, 0, 3)$ ,  $(1, 3, 0, 3, 3)$  and  $(1, 0, 3, 3, 3)$ . These equilibria are associated with  $e(x^0) \in \{2, 3\}$ , and the proposer receives 4 units or 1 unit of benefits, respectively.

Part 2 of the theorem characterizes the stable set in equilibrium. The ideal policy of the proposer,  $s^1$ , is obviously stable. For any other stable policy, a group of  $m + 1$  voters receive an equal and positive amount of the benefits whereas the other  $m - 1$  voters receive nothing. The intuition for these policies to be stable is crucial to understand the mechanism of the model.

Consider an initial default  $x^0 \in S(K)$  for some  $K \in \mathbb{N}$ . By definition of  $S(K)$ , there is a group  $Q(x^0, K)$  of  $m + 1$  voters who receive  $K$  units of benefits by default and all the other  $m - 1$  voters get nothing. In order to pass any new policy, the proposer must obtain at least one vote from voters in  $Q(x^0, K)$ . A critical observation is that any voter  $i \in Q(x^0, K)$  does not accept any policy proposal such that the proposer increases his own benefit entitlement but some other voter  $j \in Q(x^0, K)$  is expropriated and given strictly less than  $K$  units of benefits. Suppose counterfactually

that a proposal  $x^1 \in X$  was approved in the first proposal round such that  $x_1^1 > x_1^0$  and  $x_j^1 < x_j^0 = K$ . Then in the second proposal round, once reached, the proposer would obtain enough votes from a group of  $m$  voters in  $\bigcup_{k=0}^{z_m(x^1)} Q(x^1, k)$  and pass a policy  $x^2 = f(x^1)$  that permanently reduces benefit entitlement of any voter  $i \in Q(x^0, K)$  from the level of  $K$  units. This is true since  $x_i^2 \in \{0, e(x^1)\}$  for all  $i \in Q(x^0, K)$ , and  $e(x^1) \leq \frac{\pi - x_1^1}{m+1} < \frac{\pi - x_1^0}{m+1} = K$ . As a consequence, in the very first proposal round no voter  $i \in Q(x^0, K)$  would allow the proposer to expropriate any other voter in the same group. Thus, the proposer cannot do better than allow the initial default to persist. In equilibrium, all voters in the coalition of  $Q(x^0, K)$  protect the benefits for one another.

The same intuition applies for any initial default. Any voter  $i$  accepts a policy  $x^* \in X$  only if, with  $x^*$  as default, the number of voters who look more attractive for the proposer as potential future allies is less than  $m$ . This follows because in the future the proposer may use his power to exploit  $i$  along with the voters with lower reservation values than  $i$ .

Part 3 of the theorem indicates that the proposer always proposes a stable policy and reconsideration never occurs in these equilibria. Parts 4 says that every voter who votes for the equilibrium policy must receive no less than the benefit entitlement of the voter who is ranked the  $m$ -th from the bottom according default benefit entitlement. Part 5 shows the bounds on the equilibrium benefit entitlement of the proposer.

In a legislative equilibrium described in Theorem 1, a policy proposal is usually not symmetric to the voters. In other words, voters who receive the same from the default may be treated unequally by the proposer. This is evident in the following numerical example. Assume  $n = 5$  and  $\pi = 50$ . Consider a default  $x = (10, 10, 10, 10, 10)$ . The proposer needs two votes from the voters to pass a new policy but cannot fully expropriate the other two. That said, the proposer has to offer three out of four

voters 10 units of benefits and expropriates the fourth. Therefore, he can play a pure strategy and propose any policy  $y^i \in X$  for some  $i \neq 1$  such that  $y_1^i = 20$ ,  $y_i^i = 0$ , and  $y_\ell^i = 10$  for all  $j \in N \setminus \{1, i\}$ . With any of these policies, voter  $i$ , whose identity is common knowledge, is targeted for expropriation. Intuitively, imagine that in equilibrium a group of voters, for example players 2, 3, and 4, form a coalition and commit to protect the benefits for one another. They will not accept any policy with which as default any of their coalition members would be eventually offered less than 10 units of benefits. Given this, the proposer can only expropriate player 5. The commitment is credible since voters in this formed coalition are weakly better off keeping their promises given any best response of the proposer.

One may ask whether randomization of policies  $\{y^i\}_{i=2}^5$  is also a possible equilibrium strategy. Such randomization treats all voters equally and anonymously, but it cannot constitute an equilibrium strategy. Suppose all players anticipate that if the default  $x$  remains the proposer will play a mixed strategy and randomize the policies  $\{y^i\}_{i=2}^5$  in the subsequent proposal round. Then the reservation value of any voter  $j \neq 1$  is  $U_j(x) = 10(1 - \delta) + \delta\left(\frac{3}{4}(10)\right)$  and  $7 < U_j(x) < 8$  for  $\delta \in (0, 1)$  sufficiently large. Therefore, the proposer can pass any policy  $z^i$  for some  $i \neq 1$  such that  $z_1^i = 26$ ,  $z_i^i = 0$ , and  $z_j^i = 8$  for all  $j \in N \setminus \{1, i\}$ . The expectations of the players are thus not consistent with the equilibrium strategy. Similarly, if all players anticipate that the proposer will randomize the policies  $\{z^i\}_{i=2}^5$  in the next proposal round as the default  $x$  remains, then the reservation values of all voters would turn out to be even smaller so that the proposer could pass a more expropriating policy than any of  $\{z^i\}_{i=2}^5$ . This analysis hints a possibility of a mixed-strategy legislative equilibrium in which the proposer is able to capture a substantial portion of the total benefits in equilibrium. A characterization of such an equilibrium along with the issue of equilibrium selection is provided in Section 6.

The next theorem characterizes the necessary conditions in *any* pure-strategy legislative equilibrium and focuses on the final policy outcome. The proof is presented in the Appendix.

**Theorem 2** Assume  $\delta \in (0, 1)$  sufficiently large. Any legislative equilibrium with policy rule  $f$  must be such that, for all  $x \in X$ ,  $f^\infty(x) \in \bigcup_{k=0}^\infty S(k)$  and  $x_1 \leq f_1^\infty(x) \leq \pi - (m+1)z_m(x)$ .

The theorem identifies the bounds on the final policy outcome resulting from any initial default. In cases in which proposal power has its maximal strength, i.e.,  $f_1^\infty(x) = \pi - (m+1)z_m(x)$ , the proposer still receives unambiguously less than what he would get in closed-rule legislative bargaining. Therefore, in *all* pure-strategy legislative equilibria a group of voters implicitly coordinate to support each other against the sole proposer. Also observe that any possible final policy outcome can be supported in some pure-strategy legislative equilibrium described in Theorem 1. Therefore, Theorem 1 presents *all* pure-strategy legislative equilibria where no reconsideration occurs. With the focus on the equilibrium benefit allocation, a full characterization of all those equilibria thus is sufficient.

## 6 Mixed-Strategy Legislative Equilibrium

In addition to the pure-strategy legislative equilibria, there exists a class of legislative equilibria with mixed proposal strategies. In those equilibria, if the legislative session continues for at least two proposal rounds, it is guaranteed that the proposer takes all the benefits and leaves nothing to the others, regardless of the initial default. The sole proposer has nearly dictatorial power, and the equilibrium policy outcome is reminiscent of those in Bernheim et al. (2006) and Kalandrakis (2004, 2007). The

next proposition presents results for a legislature with three players. The proof is presented in the Appendix.

**Proposition 4** Assume  $n = 3$ ,  $X = \Delta_\pi^3$  and  $\delta \in (0, 1)$  sufficiently large. There exists a legislative equilibrium  $(\sigma, \{U_\ell\}_{\ell=1}^3)$  such that:

1. For all  $x \in X$  such that  $x_2 x_3 = 0$ ,

$$\sigma(x, y) = \begin{cases} 1, & \text{if } y = s^1, \\ 0, & \text{if } y \neq s^1. \end{cases}$$

2. For any  $x \in X$  such that  $x_2 x_3 \neq 0$ ,

$$\sigma(x, y) = \begin{cases} \frac{1}{2} - \frac{x_2 - x_3}{2\delta(x_2 + x_3)}, & \text{if } y = (x_1, \pi - x_1, 0), \\ \frac{1}{2} + \frac{x_2 - x_3}{2\delta(x_2 + x_3)}, & \text{if } y = (x_1, 0, \pi - x_1), \\ 0, & \text{otherwise,} \end{cases}$$

and  $U_i(x) < x_i$  for all  $i \neq 1$ .

This proposition describes the mixed proposal strategy. In words, the proposer is able to pass his ideal policy right away if at least one voter receives nothing by default. Otherwise, the proposer manipulates the sequence of proposals so that he is able to pass his ideal policy in the second proposal round, if the legislative session reaches it. In the interim proposal round, i.e., the first round, the proposer takes all the benefits from one voter to the other. This is done with randomization so that, with positive probability, each voter may receive nothing according to the approved policy in the first proposal round.

As an example, assume  $\pi = 6$  and consider a default  $(2, 3, 1)$ . In the equilibrium characterized in Proposition 4, with probability  $\frac{1}{2} \left(1 - \frac{1}{2\delta}\right) \simeq \frac{1}{4}$ , the proposer seeks

voting support from player 2, who is the voter favored by the default, and with probability  $\frac{1}{2} \left(1 + \frac{1}{2\delta}\right) \simeq \frac{3}{4}$ , he seeks voting support from player 3, who is the voter disadvantaged by the default. The proposer gives 2 units of the benefits to himself, and the other 4 units to whoever he chooses to ally with. The policy outcome at the end of the first round is either  $(2, 4, 0)$  or  $(2, 0, 4)$ . With probability  $\delta$  the legislative session continues and the proposer can pass his ideal policy  $(6, 0, 0)$ . Observe that, if both voters anticipate the mixed proposal strategies in the very beginning of the session, their reservation values would be the same as  $2(1 - \delta^2)$ . This sets the ground for the proposer to randomize between the two voters.

The mixed-strategy equilibrium again is a result of self-fulfilling expectations. In equilibrium, regardless of the initial default, all players anticipate that, if the default remains, the proposer will play mixed strategies and eventually implement his ideal policy. Given such a common belief, both voters must have very small reservation values so that they are willing to accept a policy proposal that delivers very small expected utilities to them and eventually carries out their expectation. This equilibrium thus exhibits an extreme form of the "power to propose".

Part 2 of Proposition 4 also implies that, except for rare cases in which the proposer can immediately pass his ideal policy, both voters would be strictly better off if the legislative bargaining game was not played and the initial default was directly implemented. In the previous example, the value of the game for player  $i \neq 1$  is  $U_i(x) = 2(1 - \delta^2) < x_i$  given that  $\delta \in (0, 1)$  is sufficiently large. This property plays a crucial role when equilibrium selection is discussed later.

Mixed-strategy equilibria also exist, in a similar fashion, for a legislature with five or more than five players.

**Proposition 5** Assume odd  $n \geq 5$ ,  $X = \Delta_\pi^n$  and  $\delta \in (0, 1)$  sufficiently large. There

exists a legislative equilibrium  $(\sigma, \{U_\ell\}_{\ell=1}^n)$  such that the policy converges to the ideal policy of the proposer after a transition of at most one proposal round. In particular:

1. For all  $x \in X$  such that  $|Q(x, 0)| \geq m$ ,  $\sigma(x, s^1) = 1$ .
2. For all  $x \in X$  such that  $|Q(x, 0)| < m$ ,  $\sigma(x, y) > 0$  only if  $\sigma(y, s^1) = 1$ .

Moreover,  $\{i \neq 1 : U_i(x) < x_i\} \geq m + 1$ .

The same pattern of policy dynamics is identified in a mixed-strategy stationary equilibrium constructed by Kalandrakis (2007) for a dynamic legislative bargaining model with an evolving default, an arbitrary odd  $n \geq 5$  players, an arbitrary  $\delta \in (0, 1)$ , an arbitrary probability for proposer selection, and a *continuous* distributive policy space. In fact, the technique in the proof of his Proposition 2 can be applied to the model considered here with a finite policy space. The proof of our Proposition 5 is thus omitted.

Whereas the class of equilibria first identified in the continuous model of Kalandrakis (2004, 2007) is also prevalent in our model with a discrete policy space, the reverse is not true. The pure-strategy equilibria with mutual protection among the voters *cannot* be supported as a stationary equilibrium in the model of Kalandrakis.

So far two classes of legislative equilibria have been established. In one class of legislative equilibria, the proposer plays a pure proposal strategy and a group of voters protect the benefits for one another in order to secure their bargaining positions in the subsequent proposal rounds. The value of proposal power is thus constrained. In the other class of legislative equilibria, the proposer plays mixed proposal strategies and has nearly dictatorial power. These equilibria display contrasting features.

The obvious next question is: Whether and on what grounds can we select between the two classes of equilibria?

We argue that the pure-strategy legislative equilibria should be the ones to be observed. In legislatures the policy making process is typically proceeded by a procedural stage where the legislature decides on whether to deliberate on a given policy issue. If a majority of legislators believe that they would be strictly worse off once legislative actions are taken on a certain policy issue, they can block the legislation on this policy in the procedural stage. This is the case if they expect a mixed-strategy legislative equilibrium in the legislative bargaining game.

Formally, consider an augmented game with one additional stage before legislative bargaining begins. In that pre-bargaining stage, an initial default  $x^0 \in X$  is exogenously given and the identity of proposer (e.g., player 1) in the bargaining stage has been known to the legislature. The legislature must decide by majority rule whether or not to enter the legislative bargaining process described in Section 2. The choice is denoted by  $\theta \in \{0, 1\}$ . If  $\theta = 1$ , then the legislative bargaining game commences. Instead if  $\theta = 0$ , then the legislative session ends immediately and the initial default  $x^0$  is implemented. Propositions 4 and 5 assert that, in the mixed-strategy equilibria where the proposer has nearly dictatorial power, more than one-half of the voters are better off with the initial default than with the equilibrium policy outcome of the legislative bargaining game. Therefore, if the players anticipate that the proposer will play expropriating mixed strategies,  $\theta = 0$  will be chosen by majority rule in the pre-bargaining stage. In other words, conditional on the occurrence of legislative bargaining over some policy issue, the final policy outcome should be consistent with the prediction of some pure-strategy equilibrium but not a mixed-strategy one.

## 7 Committees as Proposers

In practice, a policy proposal may be made by a committee instead of a single legislator. This is well known in the case of the U.S. Congress. Knight (2005), for example, describes how the House Committee of Transportation and Infrastructure effectively controls agenda setting on certain bills that allocate the federal budget over local transportation projects.

Our model can be extended to account for committees as proposers. Moreover, we can address the question about how the size of a proposal committee may affect the allocation of benefits and the endogenous constraints on proposal power.

Let  $P \subset N$  be the set of players in the proposal committee. So that the committee members alone are not decisive in majority voting, assume  $p = |P| \leq m$ . All the other players are referred to as voters. The policy is made through a sequence of proposal making and voting analogous to the one described in Section 2, with the only exception that every proposal must be made by the proposal committee.

Proposal making itself is a collective decision in the committee. To simplify the analysis, we ignore the distributional issue among committee members and treat the whole committee as a single political actor that controls  $p$  votes and proposal power. In other words, we assume that all committee members vote for any policy proposed by the committee.<sup>13</sup> After all, in a distributive setting all committee members have aligned interests in expropriating the rest of the legislature to a maximal extent. This assumption may be a good approximation for the data set of Knight (2005), where almost every representative in the House Committee of Transportation and Infrastructure receives a positive allocation of the federal budget for transportation

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<sup>13</sup>Our qualitative results do not rely on this extreme simplification. The key insight holds as long as the number of committee members who support the proposed policy is increasing in the committee size, which is true if the proposal decision is made by a voting rule, e.g. simple majority.

projects in his district.

The intuition developed in the baseline case with a single proposer can be carried over to this extension with a proposal committee. Again, the focus is the voters' incentives. In order to pass any new policy, the committee, which already controls  $p$  votes, requires  $m + 1 - p$  additional votes from the voters. With  $\delta \in (0, 1)$  sufficiently large, any voter would not vote for a proposal if some other  $m + 1 - p$  voters are substantially expropriated by this proposal, because the expropriated voters in this round would become the cheaper coalition partners for the committee members to ally with in potential future proposal rounds. Therefore, the committee can only fully expropriate up to  $m - p$  voters, and a group of at least  $p$  voters whose votes are not needed to approve the proposal must be offered positive benefits by the new policy.

The next theorem formalizes this insight. To facilitate the exposition, let  $Q^P(x, k) \equiv \{i \in N \setminus P : x_i = k\}$  be the collection of all voters who receive exactly  $k$  units of benefits from policy  $x$ . For any  $r \in \{1, 2, \dots, n - p\}$ , let  $z_r^P(x) \equiv \min K \in \mathbb{Z}_+ \text{ s.t. } \left| \bigcup_{k=0}^K Q^P(x, k) \right| \geq r$  denote the benefits received from policy  $x$  by the voter who ranked the  $r$ -th from the bottom if all voters are ranked according to their benefit entitlement. The proof is presented in the Appendix.

**Theorem 3** Assume  $X = \Delta_\pi^n$ ,  $\delta \in (0, 1)$  sufficiently large, and a proposal committee  $P$  with  $p$  members, where  $1 \leq p \leq m$ . A pure-strategy legislative equilibrium exists. Moreover, in any pure-strategy legislative equilibrium, for any default  $x \in X$ , the final policy outcome  $x^*$  is such that  $\sum_{i \in P} x_i \leq \sum_{i \in P} x_i^* \leq \pi - (m + 1) z_{m+1-p}^P(x)$ ,  $|Q^P(x^*, 0)| = m - p$ , and  $|Q^P(x^*, e^*)| = m + 1$  for some  $e^* \in \mathbb{Z}_+$  such that  $z_{m+1-p}^P(x) \leq e^* \leq \frac{1}{m+1} (\pi - \sum_{i \in P} x_i)$ . Notably, the number of players who receive a positive amount of benefits in equilibrium is  $|\bigcup_{k=1}^\infty Q^P(x^*, k) \cup P| = m + 1 + p$ , which is increasing in the size of the proposal committee.

Theorem 3 implies that more than a bare majority of players receive positive benefits in equilibrium and the size of this supermajority can be large. Moreover, there is a positive correlation between committee size  $p$  and the number of players who receive a positive amount of benefits in equilibrium. This is because, counterintuitively, a larger proposal committee is endogenously constrained to offer positive benefits to a larger fraction of the legislature. These features are absent in games with closed-rule legislative bargaining, where regardless of the committee size, only a bare majority of players, including all committee members, receive positive benefits and all other players get nothing.

The empirical implications of our extended model are potentially testable with congressional data. For example, the positive correlation between committee size and the number of players that receive positive benefits is consistent with the observation by Knight (2005). In 1991, the committee that controls proposal power includes 55 representatives and 46% of the districts receive a positive allocation of federal budget earmarked for local transportation projects. In contrast, in 1998 the committee includes 72 representatives and 82% of the districts receive a positive allocation. We leave related empirical work for future research.

## 8 Concluding Remarks

This paper has proposed a new analytical framework of legislative bargaining. A central idea is that policy making is finalized only after all players with agenda control have no more incentive to replace the default policy. The setup, we believe, captures important features of legislative decision-making, such as an endogenous default and the possibility of reconsideration without a predetermined last proposal round. We characterize a class of pure-strategy equilibria in which a group of voters protect

the benefits for one another against the proposer. Thus the value of proposal power is substantially constrained. The model is tractable and can be easily applied to dynamic models or embedded in a specific economic context, e.g. from public finance.

There are various natural extensions of the model. An immediate next step of research is to allow turnover of proposal power. This introduces a nontrivial trade-off between the incentive to protect and the incentive to exploit in a model with the possibility of reconsideration. On one hand a voter has an incentive to defend the benefits of the other voters so as to secure her bargaining position as a future voter. On the other hand, she may well have an opposite incentive to allow the current proposer expropriating the other voters as much as possible since this will improve her bargaining position as a future proposer. The former effect is the focus of this paper whereas the latter is highlighted by Kalandrakis (2004; 2007). The interplay of these effects can be studied in a more general model, possibly with random selection of proposers. We conjecture that in such a model with a large number of legislators the results presented in this paper would be likely to hold. As the probability for a legislator to be chosen as proposer is decreasing in the size of the legislature, the incentive to exploit would be dominated by the incentive to protect.

Another extension would be to incorporate a richer institutional structure into the legislative process, e.g. the existence of a gatekeeper. We define a *gatekeeper* as the player who is conferred the veto right to block any policy proposal made by some others and at the same time able to propose a new policy in some situations. The sequence of events in the game can be modified as follows. There is an initial default and one player is randomly assigned to be a gatekeeper. The players then are able to make policy proposals in turn. A player can choose to pass his turn if proposing a policy does not make himself better off. Once a proposal is made, it has to be approved by the gatekeeper and then voted on against the default by majority rule.

A passed proposal becomes the new default in future proposal rounds. Legislative interaction ceases after all players pass their proposal turns. The final default policy is then implemented. In a model like this, we would be able to compare the respective values of proposal power and gatekeeping power. These institutional variations are potentially important for a comparative study of legislative decision-making, as the sole-proposer case is reminiscent of a cabinet in a parliamentary democracy while universal proposing with a gatekeeper may be a good representation of decision-making in the U.S. Congress.<sup>14</sup>

The analytical framework proposed in this paper could also be incorporated into fuller developed models of public finance and macroeconomic policy choice.<sup>15</sup> As recent empirical studies on political economy and comparative constitutions have established various stylized facts and raised new questions about how political institutions shape the dynamics of policy, we expect fruitful insights from such an approach.

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<sup>14</sup>See Diermeier and Myerson (1994, 1999) for some early development of comparative models in this direction.

<sup>15</sup>See Diermeier and Fong (2008a), for example, for an application of the model that accounts for the patterns of government spending dynamics in multi-party parliamentary countries.

## Appendix

Throughout the appendix we use the two additional notations defined and explained in Section 5. Fix any  $x \in X$ . For all  $k \in \mathbb{Z}_+$ ,  $Q(x, k) \equiv \{i \in N \setminus \{1\} : x_i = k\}$ , and for all  $r \in \{1, 2, \dots, n-1\}$ , let  $z_r(x) \equiv \min K \in \mathbb{Z}_+$  s.t.  $\left| \bigcup_{k=0}^K Q(x, k) \right| \geq r$ .

**Proof of Proposition 1.** Step A. For  $\delta = 0$ , in any legislative equilibrium with policy rule  $\hat{f}$ , for any  $x \in X$  there exists  $M(x) \subset N$  such that  $|M(x)| = m$ ,  $\bigcup_{k=0}^{z_m(x)-1} Q(x, k) \subset M(x)$ ,  $M(x) \setminus \left( \bigcup_{k=0}^{z_m(x)-1} Q(x, k) \right) \subset Q(x, z_m(x))$ , and

$$\hat{f}_\ell(x) = \begin{cases} \pi - \sum_{i \in M(x)} x_i, & \text{if } \ell = 1, \\ x_\ell, & \text{if } \ell \in M(x), \\ 0, & \text{otherwise.} \end{cases}$$

This is a standard result proved by Baron and Ferejohn (1989a). Step B. For any  $\delta \in [0, 1)$ , in any legislative equilibrium with policy rule  $f$ ,  $f(s^1) = s^1$  since  $U_1(x) < U_1(s^1)$  for all  $x \neq s^1$ . Step C. Assume  $\delta > 0$  and consider any legislative equilibrium with policy rule  $f$ . Suppose that there exists some  $x \in X$  such that  $\hat{f}(x) \neq s^1$  and  $f^\infty(x) = \hat{f}(x)$ . By definition of  $f^\infty$ ,  $f^\infty(\hat{f}(x)) = \hat{f}(x)$ . Then for all  $i \in Q(\hat{f}(x), 0)$ ,  $U_i(\hat{f}(x)) = 0$ . By Step A,  $\left| Q(\hat{f}(x), 0) \right| = m$ . Observe that  $U_1(s^1) > U_1(\hat{f}(x))$  by Step B and  $U_i(s^1) = U_i(\hat{f}(x))$  for all  $i \in Q(\hat{f}(x), 0)$ . In other words, in any proposal round with  $\hat{f}(x)$  as default,  $s^1$  is politically feasible and strictly preferred to  $\hat{f}(x)$  by the proposer. This contradicts the supposition that  $f^\infty(x) = \hat{f}(x)$ . ■

**Proof of Proposition 2.** This proposition is a special case of Theorem 1 with  $m = 1$ . Consider the algorithm in Part 1 of Theorem 1. For any  $x \in X$ ,  $M_f^+(x) = \{2, 3\}$  is uniquely determined and  $z_1(x) = \min\{x_2, x_3\}$ . ■

**Proof of Proposition 3.** This proposition is a special case of Theorem 2 with  $m = 1$ . For any  $k \in \mathbb{N}$ , if  $Q(x, k) = m + 1$  then  $x_2 = x_3$ . Therefore, the equilibrium stable set is  $S = \{x \in X : x_2 = x_3\}$ . ■

Lemma 1 is useful for proofs of Theorems 1 and 2.

**Lemma 1** Consider any legislative equilibrium  $(f, \{U_\ell\}_{\ell=1}^n)$ . For all  $x \in X = \Delta_\pi^n$  and all  $\ell \in N$ :

1.  $U_\ell(f(x)) \geq U_\ell(x) \Leftrightarrow U_\ell(f(x)) \geq x_\ell$ .
2. For  $\delta \in (0, 1)$  sufficiently large,  $f_\ell^\infty(x) < x_\ell \Rightarrow U_\ell(f(x)) < x_\ell$ .

**Proof of Lemma 1.** Take any  $x \in X$  and any  $\ell \in N$ . Part 1 is obviously true, since  $U_\ell(x) = (1 - \delta)x_\ell + \delta U_\ell(f(x))$  by definition. To prove Part 2, let  $T \in \mathbb{N}$  be such that  $f^t(x) = f^\infty(x)$  for all  $t \geq T$ , and either  $T = 1$  or  $f^{t+1}(x) \neq f^t(x)$  for all  $t \leq T - 1$ . Then  $U_\ell(f(x)) = (1 - \delta) \sum_{t=1}^{T-1} \delta^{t-1} f_\ell^t(x) + \delta^{T-1} f_\ell^\infty(x)$ . Suppose that  $f_\ell^\infty(x) < x_\ell$ . Given that  $\delta \in (0, 1)$  is sufficiently large,  $U_\ell(f(x))$  is sufficiently close to  $f_\ell^\infty(x)$  so that  $U_\ell(f(x)) < x_\ell$ . ■

**Proof of Theorem 1.** Parts 2-5 are immediate implications of Part 1, which we prove by construction and verification of an equilibrium.

Step A. Given the policy rule described in Part 1 of the theorem, we construct the reservation value functions that satisfy the equation system defined by (??). In particular, for all  $x \in S$  and all  $\ell \in N$ ,  $U_\ell(x) = x_\ell$ ; for all  $x \notin S$ ,

$$U_\ell(x) = \begin{cases} (1 - \delta)x_1 + \delta(\pi - (m + 1)e(x)), & \text{if } \ell = 1, \\ (1 - \delta)x_\ell + \delta e(x), & \text{for all } \ell \in Q(f(x), e(x)), \\ (1 - \delta)x_\ell, & \text{for all } \ell \in Q(f(x), 0). \end{cases}$$

Step B. Given the reservation value functions constructed in Step A, we then verify that, for any  $x \in X$ , the policy rule described in Part 1 of the theorem solves problem (2) of the proposer. Taking any  $x \in X$ , we prove the optimality of  $f(x)$  through a series of claims. Claim 1 shows that at least  $m$  voters weakly prefer  $f(x)$  to  $x$ , so  $f(x)$  can be approved by majority voting if it is preferred and proposed by the proposer. Claim 2 is an intermediate result useful in the subsequent claims. Claim 3 shows that the proposer also weakly prefers  $f(x)$  to  $x$ . Claim 4 shows that the proposer cannot be strictly better off by proposing any policy other than  $f(x)$ .

CLAIM 1. There exists a coalition  $M$  of voters such that  $|M| = m$  and  $U_i(f(x)) \geq U_i(x)$  for all  $i \in M$ .

PROOF. This claim is trivial if  $x \in S$  so assume that  $x \notin S$ . Notice that, by the algorithm that constructs the policy rule,  $M_f(x) \subset Q(f(x), e(x)) \cap \left( \bigcup_{k=0}^{e(x)} Q(x, k) \right)$  and  $|M_f(x)| = m$ . Since  $f(x) \in S$  and  $M_f(x) \subset Q(f(x), e(x))$ ,  $U_i(f(x)) = f_i(x) = e(x)$  for all  $i \in M_f(x)$ . Since  $M_f(x) \subset \bigcup_{k=0}^{e(x)} Q(x, k)$ ,  $x_i \leq e(x)$  for all  $i \in M_f(x)$ . Thus  $U_i(f(x)) = e(x) \geq (1 - \delta)x_i + \delta e(x) = U_i(x)$  for all  $i \in M_f(x)$ .

CLAIM 2. Suppose that  $x \in S(k)$  for some  $k \in \mathbb{Z}_+$ . Then  $U_1(x) = \pi - (m + 1)k$ .

PROOF. This claim directly follows the definition of  $S(k)$  and the construction of  $U_1(x)$ .

CLAIM 3.  $U_1(f(x)) \geq U_1(x)$ .

PROOF. This claim is trivial if  $x \in S$  so assume that  $x \notin S$ . Since  $f(x) \in S(e(x))$ ,  $U_1(f(x)) = \pi - (m + 1)e(x)$  by Claim 2. Since  $e(x) \leq \frac{\pi - x_1}{m + 1}$ ,  $U_1(f(x)) \geq x_1$ . By Lemma 1,  $U_1(f(x)) \geq U_1(x)$ .

CLAIM 4. For all  $y \in X$ , either  $U_1(y) \leq U_1(f(x))$ , or there exists a coalition  $M_+$  of voters, such that  $|M_+| = m + 1$  and  $U_i(y) < U_i(x)$  for all  $i \in M_+$ .

PROOF. We discuss three cases. Case 1. Take any  $y \in X$  such that  $e(y) > e(x)$ . Since  $f(x) \in S(e(x))$ ,  $U_1(f(x)) = \pi - (m+1)e(x)$  by Claim 2. Since  $f(y) \in S(e(y))$ , similarly  $U_1(y) = (1-\delta)y_1 + \delta(\pi - (m+1)e(y))$ . Since  $e(y) > e(x)$ ,  $U_1(f(x)) > U_1(y)$  given that  $\delta \in (0, 1)$  is sufficiently large. Case 2. Take any  $y \in X$  such that  $e(y) < e(x)$ . This implies that  $e(x) \geq 1$ . Let  $M_+ = Q(f(x), e(x))$ . Then  $|M_+| = m+1$  since  $f(x) \in S(e(x))$ . By the reservation value functions constructed in Step A, for all  $i \in M_+$ ,  $U_i(y) = (1-\delta)y_i + \delta f_i(y)$  and  $U_i(x) = (1-\delta)x_i + \delta e(x)$ . Since  $f(y) \in S(e(y))$ ,  $f_j(y) \in \{0, e(y)\}$  for all  $j \neq 1$ . Therefore,  $f_i(y) < e(x)$  for all  $i \in M_+$ . As a consequence,  $U_i(x) > U_i(y)$  for all  $i \in M_+$ , given that  $\delta \in (0, 1)$  is sufficiently large. Case 3. Take any  $y \in X$  such that  $e(y) = e(x)$ . By Claim 2,  $U_1(f(x)) = \pi - (m+1)e(y)$  and  $U_1(y) = (1-\delta)y_1 + \delta(\pi - (m+1)e(y))$ . Since  $e(y) \leq \frac{\pi - y_1}{m+1}$ ,  $y_1 \leq \pi - (m+1)e(y)$ . Then  $U_1(y) \leq U_1(f(x))$ . ■

**Proof of Theorem 2.** This theorem breaks into three parts: (1)  $f_1^\infty(x) \geq x_1$ ; (2)  $f_1^\infty(x) \in \bigcup_{k=1}^\infty S(k)$ ; and (3)  $f_1^\infty(x) \leq \pi - (m+1)z_m(x)$ . They are proved by Steps A, C, and D, respectively. Step B provides an instrumental result useful in Steps C and D.

Step A. Suppose that  $f_1^\infty(x) < x_1$ . Then by Lemma 1, for all  $\delta \in (0, 1)$  sufficiently large,  $U_1(f(x)) < x_1$  and  $U_1(f(x)) < U_1(x)$ . This contradicts the optimality of  $f(x)$  for the proposer. Therefore,  $f_1^\infty(x) \geq x_1$ .

Step B. For all  $K \in \mathbb{Z}_+$ , let

$$R(K) \equiv \left\{ x \in X \setminus S(K) : \left| \bigcup_{k=0}^K Q(x, k) \right| \geq m \text{ and } \left| \bigcup_{k=0}^{K-1} Q(x, k) \right| < m \right\}.$$

Notice that  $\{S(k), R(k)\}_{k=0}^\infty$  form a partition of the policy space  $X$ . Consider any legislative equilibrium  $(f, \{U_\ell\}_{\ell=1}^n)$ . In this step we want to show that  $f(x) = x$  for

all  $x \in \bigcup_{k=0}^{\infty} S(k)$  and  $f(x) \neq x$  for all  $x \in \bigcup_{k=0}^{\infty} R(k)$ . In particular, we show that the following two statements are true for all  $k \in \mathbb{Z}_+$ :

S1.  $x \in S(k) \Rightarrow f(x) = x$ .

S2.  $x \in R(k) \Rightarrow f(x) \neq x$ .

The proof proceeds by math induction through a series of claims. Claim 1 shows that S1 is true of  $k = 0$ . Claim 2 is an instrumental result useful in the rest of the induction. Claim 3 shows that if S1 is true for some  $k = K \in \mathbb{Z}_+$  then S2 is also true for  $k = K$ . Finally, Claim 4 shows that if both S1 and S2 are true for all  $k$  up to some  $K \in \mathbb{Z}_+$  then S1 is true for  $k = K + 1$ .

CLAIM 1. S1 is true for  $k = 0$ .

PROOF. Notice that  $S(0) = \{s^1\}$  and suppose that  $f(s^1) \neq s^1$ . Then  $f_1(s^1) < \pi$  and  $U_1(f(s^1)) < \pi$ . As a consequence,  $U_1(s^1) = (1 - \delta)\pi + \delta U_1(f(s^1)) > U_1(f(s^1))$ . This contradicts the optimality of  $f(s^1)$  for the proposer.

CLAIM 2. Suppose  $x \in R(K)$  for some  $K \in \mathbb{Z}_+$ . Then  $x_1 < \pi - (m + 1)K$ .

PROOF. Take any  $x \in R(K)$ . If  $K = 0$  then obviously  $x_1 < \pi$  since  $x \neq s^1$ . Suppose  $K > 0$  for the rest of the proof. By definition of  $R(K)$ ,  $\left| \bigcup_{k=0}^{K-1} Q(x, k) \right| < m$ , which is equivalent to the condition that  $|\bigcup_{k=K}^{\infty} Q(x, k)| \geq m + 1$ . Let  $M_+ = \bigcup_{k=K}^{\infty} Q(x, k)$  and discuss three cases below. Case 1. Suppose that there exists  $j_1 \in M_+$  such that  $x_{j_1} \geq K + 1$ . Then  $x_1 \leq \pi - mK - (K + 1)$ . Case 2. Suppose that  $|M_+| \geq m + 2$ . Then  $x_1 \leq \pi - (m + 2)K$ . Case 3. Suppose that  $|M_+| = m + 1$  and  $x_i = K$  for all  $i \in M_+$ . Then there must be some  $j_2 \notin (M_+ \cup \{1\})$  such that  $x_{j_2} > 0$ . If this is not true, then  $x \in S(K)$ , which contradicts the supposition that  $x \in R(K)$ . Again,  $x_1 = \pi - (m + 1)K - x_j < \pi - (m + 1)K$ .

CLAIM 3. Suppose that S1 is true for some  $k = K \in \mathbb{Z}_+$ . Then S2 is true for  $k = K$ .

PROOF. Take any  $x \in R(K)$  and suppose that  $f(x) = x$ . Then  $U_\ell(x) = x_\ell$  for all  $\ell \in N$ . By definition of  $R(K)$ ,  $\left| \bigcup_{k=0}^K Q(x, k) \right| \geq m$ , so let  $M \subset \bigcup_{k=0}^K Q(x, k)$  be such that  $|M| = m$ . Take any  $y \in S(K)$  such that  $M \subset Q(y, K)$ . By S1 for  $k = K$ ,  $U_\ell(y) = y_\ell$  for all  $\ell \in N$ . Then  $U_i(y) = K \geq U_i(x)$  for all  $i \in M$ . Moreover,  $U_1(y) = \pi - (m+1)K > x_1 = U_1(x)$ , where the strict inequality is established by Claim 2. We have shown that the proposer strictly prefers  $y$  to  $x$  and at least  $m$  voters weakly prefer  $y$  to  $x$ . This contradicts the supposition that  $f(x) = x$ .

CLAIM 4. Suppose that S1 and S2 are true for all  $k$  up to some  $K \in \mathbb{Z}_+$ . Then S1 is true for  $k = K + 1$ .

PROOF. Take any  $x \in S(K+1)$  and suppose that  $f(x) \neq x$ . We discuss different possibilities for  $f^\infty(x)$  and look for a contradiction. First, suppose that  $f^\infty(x) \in \bigcup_{k=1}^K R(k)$ . Then  $f(f^\infty(x)) = f^\infty(x)$ . This, however, contradicts the fact that S2 is true for all  $k \leq K$ . Second, suppose that  $f^\infty(x) \in S(k)$  for some  $k \leq K$ . Then for all  $i \in Q(x, K+1)$ ,  $f_i^\infty(x) \in \{0, k\}$  so  $f_i^\infty(x) \leq k < K+1 = x_i$ . By Lemma 1, for  $\delta \in (0, 1)$  sufficiently large,  $U_i(f(x)) < x_i$  and  $U_i(f(x)) < U_i(x)$  for all  $i \in Q(x, K+1)$ . By definition of  $S(K+1)$ ,  $Q(x, K+1)$  includes  $m+1$  voters. Therefore,  $f(x)$  is not politically feasible when the default is  $x$ . This is a contradiction. Third, suppose that  $f^\infty(x) = \hat{x} \in S(K+1) \setminus \{x\}$ . By definition of  $f^\infty$ ,  $f(\hat{x}) = f^\infty(\hat{x})$ . Observe that all elements of  $S(K+1)$  are symmetric with respect to the voters. By the requirement for symmetric stable sets in the definition of a legislative equilibrium,  $f(x) = x$  as well. This is a contradiction. Fourth, suppose that  $f^\infty(x) \notin \bigcup_{k=1}^K R(k) \cup_{k=1}^{K+1} S(k)$  and consider two cases. Case 1. If  $f^\infty(x) \in S(k)$  for some  $k \geq K+2$ , then obviously  $f_1^\infty(x) = \pi - (m+1)(K+2) < x_1$ . Case 2. If  $f^\infty(x) \in R(k)$  for some

$k \geq K + 1$ , then by Claim 2,  $f_1^\infty(x) < \pi - (m + 1)(K + 1) = x_1$ . In both cases,  $f_1^\infty(x) < x_1$ . By Lemma 1, for  $\delta \in (0, 1)$  sufficiently large,  $U_1(f(x)) < x_1$  and  $U_1(f(x)) < U_1(x)$ . This contradicts the optimality of  $f(x)$  for the proposer. Finally, the only remaining possibility is that  $f^\infty(x) = x$ . This, however, contradicts the supposition that  $f(x) \neq x$ .

Step C. Implied by Step B,  $f(x) = x$  if and only if  $x \in \bigcup_{k=1}^\infty S(k)$ . By definition of  $f^\infty$ ,  $f^\infty(x) \in \bigcup_{k=1}^\infty S(k)$  for all  $x \in X$ .

Step D. It remains to show that  $f_1^\infty(x) \leq \pi - (m + 1)z_m(x)$ . Suppose to the contrary that  $f_1^\infty(x) > \pi - (m + 1)z_m(x)$ . By Step B, there exists  $K \in \mathbb{Z}_+$  such that  $f^\infty(x) \in S(K)$ . Then  $Q(f^\infty(x), K) = m + 1$  and  $Q(f^\infty(x), 0) = n - m - 2$ . The assumed lower bound of  $f_1^\infty(x)$  implies that  $K < z_m(x)$ . Let  $M'_+ = \bigcup_{k=z_m(x)}^\infty Q(x, k)$ . Observe that, for all  $i \in M'_+$ ,  $f_i^\infty(x) \in \{0, K\}$  so  $f_i^\infty(x) \leq K < z_m(x) \leq x_i$ . By Lemma 1, for  $\delta \in (0, 1)$  sufficiently large,  $U_i(f(x)) < x_i$  and  $U_i(f(x)) < U_i(x)$  for all  $i \in M'_+$ . By definition of  $z_m(x)$ ,  $\left| \bigcup_{k=0}^{z_m(x)-1} Q(x, k) \right| < m$  so  $|M'_+| = \left| \bigcup_{k=z_m(x)}^\infty Q(x, k) \right| \geq m + 1$ . This implies that  $f(x)$  is not politically feasible when the default is  $x$ , which is a contradiction. ■

**Proof of Proposition 4.** Step A. Given the mixed strategy  $\sigma$  described the proposition, we first construct the value functions according to the equation system defined by (??). In particular, for all  $x \in X$  such that  $x_2x_3 = 0$ ,

$$U_\ell(x) = \begin{cases} (1 - \delta)x_1 + \delta\pi, & \text{if } \ell = 1, \\ (1 - \delta)x_\ell, & \text{if } \ell \neq 1, \end{cases}$$

and for any  $x \in X$  such that  $x_2 x_3 > 0$ ,

$$U_\ell(x) = \begin{cases} (1 - \delta^2) x_1 + \delta^2 \pi, & \text{if } \ell = 1, \\ \frac{1}{2} (1 - \delta) (\pi - x_1), & \text{if } \ell \neq 1. \end{cases}$$

Step B. Given the value functions constructed in Step A, we then prove part 1 of the proposition. Without loss of generality, take any  $x \in X$  such that  $x_2 \geq x_3 = 0$ . Then  $U_3(s^1) = U_3(x) = 0$  and  $U_1(s^1) = \pi > U_1(z)$  for all  $z \neq s^1$ . Therefore  $\sigma(x, s^1) = 1$ .

Step C. Given the value functions constructed in Step A, we now prove part 2 of the proposition. Without loss of generality, take any  $x \in X$  such that  $x_2 \geq x_3 > 0$ . We want to show that  $(x_1, \pi - x_1, 0)$  and  $(x_1, 0, \pi - x_1)$  both solve problem (2) of the proposer. Notice that  $U_2(x) = U_3(x)$  even if  $x_2 \neq x_3$ . Therefore, it is without loss of generality to consider only  $y = (x_1, \pi - x_1, 0)$ . The proof proceeds with two claims, where Claim 1 says that  $y$  is politically feasible, and Claim 2 says that no other politically feasible policy yields the proposer a strictly higher expected utility than  $y$ .

CLAIM 1.  $U_1(y) \geq U_1(x)$  and  $U_i(y) \geq U_i(x)$  for some  $i \neq 1$ .

PROOF. Applying the constructed value functions, we have  $U_1(y) - U_1(x) = \delta(1 - \delta)(\pi - x_1) > 0$  and  $U_2(y) - U_2(x) = \frac{1}{2}(1 - \delta)^2(\pi - x_1) > 0$ .

CLAIM 2. For all  $z \in X$ , either  $U_1(z) \leq U_1(y)$  or  $U_i(z) < U_i(x)$  for all  $i \neq 1$ .

PROOF. Without loss of generality, take any  $z \in X$  such that assume  $z_2 \geq z_3$ . We discuss the four cases below. Case 1. Suppose that  $z_2 + z_3 \leq \pi - x_1 - 1$  and  $z_3 \geq 1$ . Then for all  $i \neq 1$ ,  $U_i(z) - U_i(x) = -\frac{1}{2}(1 - \delta^2) < 0$ . Case 2. Suppose that  $z_2 + z_3 \leq \pi - x_1 - 1$  and  $z_3 = 0$ . Then obviously  $U_3(z) = 0 < U_3(x)$ . Moreover,

$U_2(z) - U_2(x) = \frac{1}{2}(1 - \delta)((1 - \delta)(\pi - x_1) - 2) < 0$ , where the strictly inequality is true given that  $\delta \in (0, 1)$  is sufficiently large. Case 3. Suppose that  $z_2 + z_3 \geq \pi - x_1$  and  $z_3 \geq 1$ . Then  $z_1 \leq x_1$  and  $U_1(z) - U_1(y) = -(1 - \delta)(x_1 - z_1 + \delta(\pi - z_1)) \leq 0$ . Case 4. Suppose that  $z_2 + z_3 \geq \pi - x_1$  and  $z_3 = 0$ . Then  $z_1 \leq x_1$  and  $U_1(z) - U_1(y) = -(1 - \delta)(x_1 - z_1) \leq 0$ .

Finally, it is straightforward to verify that  $\frac{1}{2} \pm \frac{x_2 - x_3}{2\delta(x_2 + x_3)} \in [0, 1]$  and  $U_i(x) < x_i$  for all  $i \neq 1$ , given that  $\delta \in (0, 1)$  is sufficiently large. ■

**Proof of Theorem 3.** Without loss of generality, assume  $P = \{1, 2, \dots, p\}$ . Since we do not care about the distribution of benefits among committee members, we transform the original game into an equivalent game with  $n^p \equiv 2m - p + 2$  players, that consist of one single proposer and  $2m - p + 1$  voters, and a quota voting rule that requires  $m^p \equiv m - p + 1$  votes from the voters to pass any new policy. The equivalent policy space thus is  $\Delta_\pi^{n^p}$ , where for any  $x \in \Delta_\pi^n$ , there exists  $x' \in \Delta_\pi^{n^p}$  such that  $x'_1 = \sum_{i=1}^p x_i$  and  $x'_j = x_{j+p-1}$  for all  $j \in \{2, \dots, n^p\}$ . The policy rule and all reservation value functions for the equivalent game can be defined over the equivalent policy space  $\Delta_\pi^{n^p}$ . A pure-strategy legislative equilibrium for the equivalent game can then be defined analogous to the definition in Section 3. Finally, the proof of Theorem 2 also serves to characterize an equilibrium for the equivalent game if we substitute  $n$  and  $m$  by  $n^p$  and  $m^p$ , respectively, throughout the proof. ■

## References

- [1] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. 2008a. "Coalition Formation in Nondemocracies." *Review of Economic Studies*, forthcoming.
- [2] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. 2008b. "Dynamics and Stability of Constitutions, Coalitions, and Clubs." Unpublished.
- [3] Austen-Smith, David and Jeffrey S. Banks. 2005. *Positive Political Theory II: Strategy and Structure*. Ann Arbor, MI: University of Michigan Press.
- [4] Baron, David P. 1996. "A Dynamic Theory of Collective Goods Programs." *American Political Science Review*, 90(2): 316-30.
- [5] Baron, David P. and John Ferejohn. 1989a. "Bargaining in Legislatures." *American Political Science Review*, 83(4): 1181-1206.
- [6] Baron, David P. and John Ferejohn. 1989b. "The Power to Propose." In *Strategic Choice in Politics*, ed. Peter C. Ordeshook. Ann Arbor, MI: University of Michigan Press, pp.343-366.
- [7] Battaglini, Marco and Stephen Coate. 2007. "Inefficiency in Legislative Policy-Making: A Dynamic Analysis." *American Economic Review*, 97(1): 118-149.
- [8] Battaglini, Marco and Stephen Coate. 2008. "A Dynamic Theory of Public Spending, Taxation and Debt." *American Economic Review*, 98(1): 201-236.
- [9] Battaglini, Marco and Thomas R. Palfrey. 2007. "The Dynamics of Redistributive Politics." Unpublished.
- [10] Bernheim, Douglas, Antonio Rangel, and Luis Rayo. 2006. "The Power of the Last Word in Legislative Policy Making." *Econometrica*, 74(5): 1161-1190.

- [11] Bowen, T. Renee and Zaki Zahran. 2007. "On Dynamic Compromise." Unpublished.
- [12] Dekel, Eddie, Matthew O. Jackson, and Asher Wolinsky. 2008. "Vote Buying: Legislatures and Lobbying." Unpublished.
- [13] Diermeier, Daniel and Pohan Fong. 2008a. "Policy Persistence in a Multi-party Parliamentary Democracy." In *Political Institutions and Economic Performance*, ed. Elhanan Helpman. Cambridge, MA: Harvard University Press.
- [14] Diermeier, Daniel and Pohan Fong. 2008b. "Legislative Bargaining with Reconsideration: An Algorithm." Unpublished.
- [15] Diermeier, Daniel and Pohan Fong. 2008c. "Policy Dynamics in Transitions to Democracy." Unpublished.
- [16] Diermeier, Daniel and Sean Gailmard. 2006. "Self-Interest, Inequality, and Entitlement in Majoritarian Decision-Making." *Quarterly Journal of Political Science*, 1(4): 327-350.
- [17] Diermeier, Daniel and Roger B. Myerson. 1994. "Bargaining, Veto Power, and Legislative Committees." Northwestern University CMSEMS Working Paper No.1089.
- [18] Diermeier, Daniel and Roger B. Myerson. 1999. "Bicameralism and Its Consequences for the Internal Organization of Legislatures." *American Economic Review*, 89(5):1182-1196.
- [19] Dixit, Avinash, Gene M. Grossman, and Faruk Gul. 2000. "The Dynamics of Political Compromise." *Journal of Political Economy*, 108(3): 531-568.

- [20] Döring, Herbert. 1995. "Time as a Scarce Resource: Government Control of the Agenda." In *Parliaments and Majority Rule in Western Europe*, ed. Herbert Döring. New York: St. Martin's Press.
- [21] Duggan, John and Tasos Kalandrakis. 2007. "Dynamic Legislative Policy Making." Wallis Institute of Political Economy Working Paper No.45.
- [22] Fong, Pohan. 2006. "Dynamics of Government and Policy Choice." Unpublished.
- [23] Fréchette, Guillaume, John H. Kagel, Steven F. Lehrer. 2003. "Bargaining in Legislatures: An Experimental Investigation of Open versus Closed Amendment Rules." *American Political Science Review*, 97(2): 221-232.
- [24] Hassler, John, Kjetil Storesletten, José V. Rodríguez Mora and Fabrizio Zilibotti. 2003. "The Survival of the Welfare state." *American Economic Review* 93(1): 87-112.
- [25] Kalandrakis, Tasos. 2004. "A Three-Player Dynamic Majoritarian Bargaining Game." *Journal of Economic Theory*, 16(2): 294-322.
- [26] Kalandrakis, Tasos. 2007. "Majority Rule Dynamics with Endogenous Status Quo." Wallis Institute of Political Economy Working Paper No.46.
- [27] Knight, Brian. 2005. "Estimating the Value of Proposal Power." *American Economic Review*, 95(5): 1639-1652.
- [28] Maskin, Eric and Jean Tirole. 2001. "Markov Perfect Equilibrium I. Observable Actions." *Journal of Economic Theory*, 100(2): 191-219.
- [29] McKelvey, Richard D. 1991. "An Experimental Test of a Stochastic Game Model of Committee Bargaining." In *Laboratory Research in Political Economy*, ed. Thomas R. Palfrey. Ann Arbor, MI: University of Michigan Press.

- [30] Morelli, Massimo. 1999. "Demand Competition and Policy Compromise in Legislative Bargaining." *American Political Science Review*, 93(4): 809-820.
- [31] Romer, Thomas and Howard Rosenthal. 1978. "Political Resource Allocation, Controlled Agendas, and the Status Quo." *Public Choice*, 33(4): 27-43.