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## Article — Published Version A note on the cost of capital with fixed payout ratios

Review of Quantitative Finance and Accounting

**Provided in Cooperation with:** Springer Nature

*Suggested Citation:* Diedrich, Ralf; Dierkes, Stefan; Gröger, Hans-Christian (2022) : A note on the cost of capital with fixed payout ratios, Review of Quantitative Finance and Accounting, ISSN 1573-7179, Springer US, New York, NY, Vol. 59, Iss. 4, pp. 1559-1575, https://doi.org/10.1007/s11156-022-01085-5

This Version is available at: https://hdl.handle.net/10419/311292

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**ORIGINAL RESEARCH** 



### A note on the cost of capital with fixed payout ratios

Ralf Diedrich<sup>1</sup> · Stefan Dierkes<sup>2</sup> · Hans-Christian Gröger<sup>3</sup>

Accepted: 14 June 2022 / Published online: 16 August 2022 © The Author(s) 2022

#### Abstract

The insights of Modigliani and Miller (Am Econ Rev 53:433–443, 1963) and Miles and Ezzell (15:719–730, https://doi.org/10.2307/2330405, 1980) on the cost of capital of firms rank among the most important results in financial theory. The underlying assumptions regarding the financial policy, however, can hardly be reconciled with empirical findings. We investigate the implications of an alternative approach that is characterized by a fixed payout ratio. By introducing additional assumptions about investment opportunities, we find relationships between the cost of equity of levered and unlevered firms. The results contribute to explaining empirical findings and open the possibility to base valuation techniques on realistic and yet practicable assumptions.

**Keywords** Cost of capital  $\cdot$  Dividend policy  $\cdot$  Payout ratio  $\cdot$  Financing policy  $\cdot$  Capital structure

JEL Classification  $G11 \cdot G32 \cdot G35$ 

#### 1 Introduction

The cost of capital is a core concept in financial theory, playing a fundamental role in valuation practice. Its most important function is mapping the operating and financial risks associated with the cash flow of a firm. While the investment program determines the operating risk, financial risk depends on the capital structure. A critical determinant of financial risk is the financing policy, defined as the set of rules that govern the firm's financing. The relationships between the cost of capital and differing financing policies have been examined in numerous studies. They are linked to the seminal contributions of Modigliani and

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Miller (1963) (MM), as well as of Miles and Ezzell (1980) (ME) and Harris and Pringle (1985).<sup>1</sup> MM assume the interest-bearing debt to be planned deterministically. Future tax shields are treated as certain, leading to a comparatively low cost of capital. ME and Harris and Pringle (1985) consider the case where debt is periodically resp. continuously adapted to a target capital structure (Appleyard and Strong 1989; Clubb and Doran 1991; Taggart 1991). Consequently, future tax shields are generally uncertain, resulting in a higher cost of capital. The different assumptions gain practical importance when applying the formulae for the levering and unlevering of beta factors (i.e., Hamada 1972; Miles and Ezzell 1985). Such formulae are basic elements in almost every textbook on corporate finance or valuation (i.e., Copeland et al. 2014; Berk and DeMarzo 2020; Brealey et al. 2020; Koller et al. 2022).

However, there are systematic deviations from the assumptions of MM and ME in reality. In contrast to the assumptions of ME, firms seem to have target zones instead of fixed debt-to-equity ratios (Graham and Harvey 2001; Fama and French 2005; Brounen et al. 2006; Leary and Roberts 2005; de Jong and Verwijmeren 2010; DeAngelo and Roll 2015), if at all. In a survey of 392 CFOs Graham and Harvey (2001) find that only 10% of firms have a strict target debt ratio. Another 34% have a "somewhat tight target or range" (p. 211). Brounen et al. (2006) and de Jong and Verwijmeren (2010) show comparable results for selected European countries and Canada. Furthermore, deviations from the target debt-to-equity ratio often occur, and adjustments only take place if the deviation exceeds a certain threshold. Fama and French (2002) find that the "mean reversion of leverage is, however, at a snail's pace" (p. 24). DeAngelo et al. (2011) develop a dynamic capital structure model, including transitory debt, to explain the low speed of adjustment to target debt ratios. Elsas and Florysiak (2011) find evidence that the adjustment speed is dependent on default risks. Other authors attribute these and other findings to model specifications and mismeasurement rather than to the characteristics of the adjustment process (Leary and Roberts 2005; Graham and Leary 2011; Frank and Shen 2014).

Further evidence on the debt-to-equity ratio focuses on the equity side of financing. For instance, Welch (2004) argues that a company's leverage ratio primarily varies with stock prices. Baker and Wurgler (2002) investigate how equity market timing affects the capital structure, concluding: "We believe the most realistic explanation for the results is that capital structure is largely the cumulative outcome of past attempts to time the equity market. In this theory, there is no optimal capital structure, so market timing financing decisions just accumulate over time into the capital structure outcome." (p. 29). Kayhan and Titman (2007) confirm "that history has a major influence on observed debt ratios, and that these effects at least partially persist for at least ten years." (p. 2). The conclusions of these and other studies are summarized by Grinblatt and Liu (2008) as follows: "The actual debt policies of firms tend to deviate from those specified by the MM and ME models. In these cases, the literature in finance offers little guidance on valuation." (p. 226).

Our paper contributes to overcoming this mismatch by elaborating on the cost of capital of a levered firm that prioritizes dividend policy by committing itself to a fixed payout ratio, defined as the quotient of the dividend and net income. In our framework, debt policy serves to implement this dividend policy. If necessary, debt is raised to enable the corresponding dividend payments. We abstract from the issuance of equity, following Welch

<sup>&</sup>lt;sup>1</sup> For the ongoing discussion on the relationship between the cost of capital and the financing policy, see for example, Fernández (2004), Fieten et al. (2005), Arzac and Glosten (2005), and Cooper and Nyborg (2006).

(2011), who proposes to treat financing activities and equity issuance separately. Under this prerequisite, the financing policy is entirely characterized by the financial requirements of investments and the fixed payout ratio. This is in line with the statement of Lambrecht and Myers (2012) that a dynamic theory of investment and payout determines a dynamic theory of capital structure. Consequently, the future level of debt and the resulting tax shields are stochastic and depend on the payout ratio. We derive the resulting relationship between the cost of capital of unlevered and levered firms using additional assumptions on the firms' investment opportunities. As MM and ME, we abstract from debt default risks.

Our interest in the consequences of fixed payout ratios for the cost of capital rests on the empirical findings on the dividend policy of firms. According to this evidence, dividend policy and debt are interlinked and influenced by the company's investment policy (Barclay and Smith 2005, p. 18). Lintner (1956) states in his pathbreaking article on dividend policy: "A prudent foresighted management will always do its best to plan ahead in all aspects of financial policy to avoid getting into such uncomfortable situations where dividends have to be cut substantially below those which the company's previous practice would lead stockholders to expect on the basis of current earnings." (p. 101). Further, according to Lintner (1956), dividend decisions are often guided by the idea of a target payout ratio and give high priority to meeting this quota. This conclusion has found support in many following studies (Marsh and Merton 1987, p. 3; DeAngelo and Roll 2015, p. 411). Brav et al. (2005) find that "dividend choices are made simultaneously with (or perhaps a bit sooner) than investment decisions" (p. 490) and "external funds would be raised before dividends would be cut." (pp. 521n). DeAngelo and Roll (2015) expand on this idea as follows: "For example, perhaps investment, payout, and equity issuance considerations govern the time path of leverage." (p. 411). Baker et al. (2001) found, in their survey of 188 CFOs, that adherence to a given dividend payout ratio is more important for the dividend policy than the leverage ratio (p. 29).

To derive the cost of capital of levered and unlevered firms with a fixed payout ratio we introduce assumptions about investment opportunities. In this respect, our approach differs from those of MM and ME, where net investments are given, albeit possibly being state-dependent. We differentiate two constellations—fixed and flexible investment. Fixed investments are characterized by a deterministic amount of net investment payments, for example, as part of an investment strategy that covers several periods. With flexible investments, investment opportunities and operating profits are affected by the same factors. Hence, net investment payouts are linked to operating profits. A fixed payout ratio leads to a lower cost of capital than the MM and ME models when investments are fixed. For flexible investments, the cost of capital is always lower than that of ME and can even be lower than that of MM, depending on the parameter constellation. These results contribute to explaining the above-mentioned empirical findings on dividend policy with a fixed payout ratio. Also, they indicate why compliance with a target payout ratio may be given a higher priority than that with a target capital structure.

Furthermore, our paper extends the literature on the consequences of alternative financing policies on the cost of capital. The approaches range here from combinations of the MM and ME models (i.e., Clubb and Doran 1995; Ruback 2002; Dierkes and Schäfer 2017; Arnold et al. 2018; Dierkes and de Maeyer 2020) to the development of new financing policies (i.e., Fernández 2007; Kruschwitz and Löffler 2020, pp. 114–136) or generalizations in which the financing policy remains variable (Grinblatt and Liu 2008). The latter approaches are of specific theoretical interest. However, it is questionable whether they are helpful for practice. Ansay (2010) states regarding the contribution of Grinblatt and Liu (2008): "While this paper certainly encompasses all the others, their results are definitely theoretically interesting but practically from little help as they use heavy mathematics and some abstract parameters that do not yield a straightforward expression for the tax shield discount rate." (p. 26). In contrast, our considerations lead to relatively simple relationships between the costs of equity of levered and unlevered firms. Moreover, no quantities are used that are not directly related to the assumed financing policy or the assumptions concerning capital expenditures. Thus, the results of this paper can directly be applied for practical valuation purposes if the underlying assumptions are met.

The remainder of this paper proceeds as follows. In Sect. 2, we present the underlying valuation model and derive a general expression for the cost of capital of a levered firm. This model is adapted to the case of a firm with a fixed payout ratio in Sect. 3. The subsequent Sects. 4 and 5 introduce the distinction between fixed and flexible investments and elaborate on the cost of capital of firms with fixed payout ratios. Finally, we summarize our conclusions in Sect. 6.

#### 2 Model and valuation approach

Let  $(\Omega, A, P)$  be a probability space endowed with filtration  $\mathbb{F} = \{F_{\theta}\}_{\theta \in N}$ . All quantities considered are modeled by integrable stochastic processes adapted to  $\mathbb{F}$ .  $\mathbb{E}_{\theta}[x_t] = \mathbb{E}[x_t|F_{\theta}]$ denotes the expected value of x at time t = 0, 1, 2, ..., conditioned on the information in  $0 \le \theta \le t$ . If there is no explicit reference to information, the quantity considered refers to information in  $\theta = 0$ . Here,  $x_t$  denotes the free cash flow as the amount available for payments to shareholders when the firm is unlevered. In the case of a levered firm,  $\Delta D_t \ge 0$ denotes the amount of debt available during period t. The increase in debt at time t is  $\Delta D_t = D_t - D_{t-1}$ . For simplicity, we assume that debt is risk-free and yields the risk-free interest rate i. i is constant over time and not subject to interest rate risk.  $\tau \in (0, 1)$  symbolizes the tax rate of the firm, which is also constant over time and deterministic. The tax shield  $ts_t = i \cdot \tau \cdot D_{t-1}$  records the deductibility of interest from the corporate tax base. Taking into account changes in debt, as well as interest payments to debtholders and tax shields, the flow to equity  $x_t^{FTE}$  of a levered firm is:

$$x_t^{FTE} = x_t - i \cdot (1 - \tau) \cdot D_{t-1} + \Delta D_t.$$

$$\tag{1}$$

We use a standard valuation approach based on the cost of capital as a measuring device. The market value of the unlevered firm  $V_{t-1}^u$  and the market value of equity of the levered firm  $E_{t-1}^\ell$  result from:

$$E_{\theta}[V_{t-1}^{u}] = \frac{E_{\theta}[x_{t}] + E_{\theta}[V_{t}^{u}]}{1 + \rho_{\theta,t}^{u}}, E_{\theta}[E_{t-1}^{\ell}] = \frac{E_{\theta}[x_{t}^{FTE}] + E_{\theta}[E_{t}^{\ell}]}{1 + \rho_{\theta,t}^{\ell}}.$$
 (2)

 $\rho_{\theta,t}^{u} > i$  denotes the cost of capital when the firm is unlevered over all periods  $t > \theta$ .  $\rho_{\theta,t}^{\ell}$  represents the cost of equity of the levered firm. The notation considers the possibility that the cost of capital is a period-specific stochastic quantity.<sup>2</sup> Note that the definition of the cost of capital inherent in (2) is sufficiently general to cover a large variety of possible specifications.

<sup>&</sup>lt;sup>2</sup> For the concept of stochastic discount rates see Fama (1996), Brennan (1997), and Ang and Liu (2004).

The relationship between  $\rho_{\theta,t}^{u}$  and  $\rho_{\theta,t}^{\ell}$  reflects the one between  $E_{\theta}[V_{t-1}^{u}]$  and  $E_{\theta}[E_{t-1}^{\ell}]$ . If the capital market is arbitrage-free, that is,

$$E_{t-1}^{\ell} + D_{t-1} = V_{t-1}^{u} + V_{t-1}^{TS},$$
(3)

the value of the levered firm is the sum of the market values of equity and debt. It exceeds the value of the unlevered firm by  $V_{t-1}^{TS}$ , the value of the tax shield of debt financing (Myers 1974). Equation (3) implies:<sup>3</sup>

**Proposition 1** *The cost of equity of a levered firm is:* 

$$\rho_{\theta,t}^{\ell} = \rho_{\theta,t}^{u} + (\rho_{\theta,t}^{u} - i \cdot (1 - \tau)) \cdot \frac{\mathrm{E}_{\theta}[D_{t-1}]}{\mathrm{E}_{\theta}[E_{t-1}^{\ell}]} + \frac{\mathrm{E}_{\theta}[V_{t}^{IS}] - (1 + \rho_{\theta,t}^{u}) \cdot \mathrm{E}_{\theta}[V_{t-1}^{IS}]}{\mathrm{E}_{\theta}[E_{t-1}^{\ell}]}.$$
 (4)

Proposition 1 can be used to determine the cost of equity of alternative financing policies.

To enable a comparison of the cost of capital in what follows, the implications of MM and ME for the cost of equity are outlined. MM assume an autonomous determination of debt; hence, the tax shield is a deterministic quantity. The payout ratio must be adjusted in each period so that the funds required for net investments are available. Furthermore, MM assume that debt remains constant over time, whereas in Proposition 1, a change in debt is permitted to enable a comparison of the cost of capital between different financing policies.<sup>4</sup> Due to the deterministic development of tax shields,  $V_{t=1}^{TS,MM}$  is calculated using the risk-free interest rate (Modigliani and Miller 1963; Graham 2003, p. 1079):

$$V_{t-1}^{TS,MM} = \frac{i \cdot \tau \cdot D_{t-1}^{MM} + V_t^{TS,MM}}{1+i}$$

Hence, Proposition 1 implies the following cost of equity:

$$\rho_{\theta,t}^{\ell,MM} = \rho_{\theta,t}^{u} + (\rho_{\theta,t}^{u} - i) \cdot \frac{D_{t-1}^{MM} - V_{t-1}^{TS,MM}}{\mathbf{E}_{\theta}[E_{t-1}^{\ell,MM}]}.$$
(5)

When  $D_s^{MM} = D_{t-1}^{MM}$  for all  $s \ge t$  is assumed, (5) yields the well-known MM theorem concerning the cost of capital of levered and unlevered firms, with  $V_{t-1}^{TS,MM} = \tau \cdot D_{t-1}^{MM}$ .

In contrast to MM, ME assume an adjustment of debt at the beginning of each period so that a target debt-to-equity ratio  $L_{t-1} = E_{\theta}[D_{t-1}]/E_{\theta}[E_{t-1}^{\ell}]$  is achieved.<sup>5</sup> Again, the payout ratio serves as a device for balancing financial funds with financial needs. As the future market value of the firm is a stochastic quantity, debt follows a stochastic process. According to ME and the assumption of risk-free debt, the tax shield of the current period is known with certainty, whereas the tax shield of subsequent periods is subject to the same risk as the unlevered firm. Consequently, the value of the tax shields results from (Miles and Ezzell 1980, pp. 722n; Graham 2003, p. 1079):

<sup>&</sup>lt;sup>3</sup> See Appendix 1.

<sup>&</sup>lt;sup>4</sup> Modigliani and Miller (1963, p. 436), state that the assumption of constant debt is not necessary for their analysis.

<sup>&</sup>lt;sup>5</sup> Miles and Ezzell (1985) assume a constant leverage ratio. The generalization to a period-specific leverage ratio is straightforward; see, for example, Löffler (2001).

$$\mathbf{E}_{\theta}[V_{t-1}^{TS,ME}] = \frac{i \cdot \tau \cdot \mathbf{E}_{\theta}[D_{t-1}^{ME}]}{1+i} + \frac{\mathbf{E}_{\theta}[V_{t}^{TS,ME}]}{1+\rho_{\theta,t}^{u}}.$$

Proposition 1 yields:

$$\rho_{\theta,t}^{\ell,ME} = \rho_{\theta,t}^{u} + (\rho_{\theta,t}^{u} - i) \cdot \frac{1 + i \cdot (1 - \tau)}{1 + i} \cdot L_{t-1}^{ME}.$$
(6)

If the specifications for the debt-to-equity ratio  $L_{t-1}^{ME}$  and debt  $D_{t-1}^{MM}$  result in the same expected debt for each future period, the cost of equity according to ME exceeds that according to MM, that is,  $\rho_{\theta,t}^{\ell,f,MM} < \rho_{\theta,t}^{\ell,f,ME}$ .

#### 3 Cost of equity with fixed payout ratios and unspecified investments

To consider the impact of a fixed payout ratio on the cost of capital, we need to disaggregate the quantities used in the last section. The free cash flow is decomposed into operating profit after taxes  $op_t$  and net investments  $ni_t$  (Koller et al. 2020, pp. 48n):

$$x_t = op_t - ni_t. ag{7}$$

Based on the operating profit, the net income  $g_t$  of period *t* is calculated. It corresponds to the operating profit after taxes less interest payments, lowered by the tax shield of debt financing:

$$g_t = op_t - i \cdot (1 - \tau) \cdot D_{t-1}.$$
(8)

In accordance to the assumption of risk-free debt, we concentrate on profitable firms with  $E_{\theta}[g_t] \ge 0$  for all  $t > \theta$ .

Our key parameter, the payout ratio  $q_t$ , is defined as the portion of net income distributed to shareholders. Hence, the flow to equity equals the product of net income and payout ratio, i. e.,  $x_t^{FTE} = q_t \cdot g_t$ . Note that the issuance of shares and share repurchases are not considered. This follows the idea of separating regular financing activities from equity issuance, introduced by Welch (2011), and is related to the assumption that market expectations are focused on the former.

Taken together, the above relationships and  $x_t^{FTE} = q_t \cdot g_t$  imply the following evolution of debt:<sup>6</sup>

$$D_{t} = (1 + i \cdot (1 - q_{t}) \cdot (1 - \tau)) \cdot D_{t-1} + q_{t} \cdot ni_{t} - (1 - q_{t}) \cdot x_{t}.$$
(9)

As *i* and  $\tau$  are deterministic quantities, whereas the free cash flow  $x_t$  is subject to risk, a deterministic development of debt  $D_t$  is only possible if all random factors influencing on  $x_t$  are compensated by a state-dependent payout ratio  $q_t$  or net investments  $ni_t$ . In particular, an unlevered firm will only remain unlevered with certainty if its net investments or dividends are adjusted in such a way that  $q_t \cdot ni_t = (1 - q_t) \cdot x_t$  for all *t*.

We are interested in the consequences of a predetermined fixed payout ratio, hence, assume that the payout ratio  $q_t$  is a deterministic quantity constant over time; the period index is omitted. Since the firm cannot permanently forgo dividends or pay dividends

<sup>&</sup>lt;sup>6</sup> See Appendix 2.

exceeding net income,  $q \in (0, 1]$  is presupposed. Like ME, we assume debt is adjusted periodically. The aim of the adjustment, however, is not to maintain a target capital structure but to enable dividends to be paid in line with the prespecified payout ratio while at the same time covering the financing needs for investments. Consequently, differing from MM and ME, the dividend policy determines the capital structure in our approach.

Based on (9), expected debt can be derived from the information available at  $\theta$  by taking expectations and solving the resulting relationship as a stochastic difference equation. Starting from debt at time  $\theta$ , this results in:

$$\mathbf{E}_{\theta}[D_{t}] = (1+\mu)^{t-\theta} \cdot D_{\theta} + \sum_{s=\theta+1}^{t} (1+\mu)^{t-s} \cdot \left(q \cdot \mathbf{E}_{\theta}[ni_{s}] - (1-q) \cdot \mathbf{E}_{\theta}[x_{s}]\right), \quad (10)$$

with  $\mu = i \cdot (1 - q) \cdot (1 - \tau)$ . For q = 1, the expected increase in debt corresponds to expected net investments, i. e.  $E_{\theta}[\Delta D_t] = E_{\theta}[n_{it}]$ . All net investments are financed by additional debt. For q < 1, the expected net investments are financed by equity and debt. The contribution of equity results from retained net income; additional debt covers the remaining net investments.

Using (10), Proposition 1 can be specified to the case of a fixed payout ratio as follows:

**Proposition 2** *The cost of equity of a levered firm with a fixed payout ratio is:* 

$$\rho_{\theta,t}^{\ell} = \rho_{\theta,t}^{u} + (\rho_{\theta,t}^{u} - i) \cdot \frac{q \cdot (1 - \tau)}{\tau + q \cdot (1 - \tau)} \cdot \frac{E_{\theta}[D_{t-1}]}{E_{\theta}[E_{t-1}^{\ell}]}$$
$$- (\rho_{\theta,t}^{u} - \kappa_{\theta,t}) \cdot \frac{q \cdot \tau}{\tau + q \cdot (1 - \tau)} \cdot \frac{E_{\theta}[K_{t-1}]}{E_{\theta}[E_{t-1}^{\ell}]}, \tag{11}$$
$$\text{with } E_{\theta}[K_{t-1}] = \sum_{s=t}^{\infty} E_{\theta}[ni_{s}] \cdot \prod_{t=t}^{s} (1 + \kappa_{\theta,t})^{-1}.$$

The proof of Proposition 2 is given in Appendix 3. Here,  $\kappa_{\theta,t}$  denotes the cost of capital associated with the net investment payouts. To check the consistency of (11), note that according to (10) a firm unlevered at time  $\theta$  is expected to remain unlevered only if expected net investments  $E_{\theta}[ni_t]$  are a fixed fraction of the expected free cash flow  $E_{\theta}[x_t]$ for all  $t > \theta$ . In this case, we have  $E_{\theta}[D_{t-1}] = 0$ ,  $\kappa_{\theta,t} = \rho_{\theta,t}^u$ , and, consequently,  $\rho_{\theta,t}^{e} = \rho_{\theta,t}^{u}$ .

#### 4 Cost of equity with fixed payout ratios and fixed investments

To further specify the link between the cost of capital of levered and unlevered firms according to Proposition 2, more information about  $\kappa_{\theta,t}$ , the cost of capital associated with net investment payouts, is needed. More specifically, the expectations of market participants regarding net investments must be considered. In the simplest case, the firm has a fixed long-term investment plan that has been communicated to the market. If there is no doubt about the ability of the firm to exert this plan, the expectations of market participants concerning net investment payouts are deterministic. Consequently, net investment payouts are disentangled from the evolution of profits. Capital expenditures may be high even when ex-ante profits are low, and vice versa. In the ex-ante perspective, the operating profit is subject to risk, whereas there is no risk concerning net investment payouts. Consequently, these payouts must be discounted with the risk-free interest rate, i. e.  $\kappa_{\theta,t} = i$ .

With fixed investments in this sense, Proposition 2 yields:

**Proposition 3** With fixed investments, the cost of equity of a levered firm with a fixed payout ratio is:

$$\rho_{\theta,t}^{\ell f} = \rho_{\theta,t}^{u,f} + (\rho_{\theta,t}^{u,f} - i) \cdot \frac{q}{\tau + q \cdot (1 - \tau)} \cdot \left( (1 - \tau) \cdot \frac{\mathcal{E}_{\theta}[D_{t-1}^{f}]}{\mathcal{E}_{\theta}[\mathcal{E}_{t-1}^{\ell,f}]} - \tau \cdot \frac{K_{t-1}^{f}}{\mathcal{E}_{\theta}[\mathcal{E}_{t-1}^{\ell,f}]} \right),$$
(12)  
with  $K_{t-1}^{f} = \sum_{s=t}^{\infty} ni_{s} \cdot (1 + i)^{t-s-1}.$ 

Superscript *f* indicates fixed investments.  $K_{t-1}^f$  corresponds to the present value of the net investment payouts, calculated using the risk-free interest rate. Since the free cash flow  $x_t$  is subject to risk, (10) implies that an unlevered firm with fixed investments is expected to remain unlevered ( $E_{\theta}[D_{t-1}^f] = 0$ ) if net income is fully distributed (q = 1) and no net investments are made ( $K_{t-1}^f = 0$ ). Consequently, (12) yields  $\rho_{\theta,t}^{\ell,f} = \rho_{\theta,t}^{u,f}$ . The scope of Proposition 3 can be extended when the distinction between systematic and

The scope of Proposition 3 can be extended when the distinction between systematic and unsystematic risk is considered. Only systematic risk affects the cost of capital as market participants avoid unsystematic risk by diversifying. If there is no systematic risk, the risk-free interest rate must be used for discounting. Hence, for Proposition 3, it is sufficient to assume that the level of net investments is not affected by systematic risk. In other words: Capital expenditures can be subject to extraordinary stochastic events that lead, for example, to unforeseen investment opportunities, as long as the possibility of those events does not cause a systematic risk. This is true whether or not operating profits are also affected by these events.

The consequences of (12) can be compared with (5) and (6) resp. MM and ME if the same investments are is assumed and the specifications for q,  $D_{t-1}^{MM}$ , and  $L_{t-1}^{ME}$  are chosen such that the same debt is expected in future periods.<sup>7</sup> Under these presuppositions, (5) and (12) yield the same cost of capital, i. e.  $\rho_{\theta,t}^{\ell,f} = \rho_{\theta,t}^{\ell,f,MM}$ , when net income is fully distributed in each period (q = 1). In this case, there is de facto no difference between the assumptions of a fixed payout ratio and autonomous debt, since all (fixed) net investments are financed by additional (autonomous) debt ( $ni_s = \Delta D_s^{MM}$  for all  $s \ge t$ ). In the case of a lower payout ratio (q < 1), however, net investments are only partially financed by additional debt.  $ni_s \ge \Delta D_s^{MM}$  for all  $s \ge t$  implies  $\tau \cdot K_{t-1}^f \ge VTS_{t-1}^{f,MM} - \tau \cdot D_{t-1}^{f,MM}$  and, consequently, the cost of capital is lower under a fixed payout ratio than under autonomous debt, i. e.  $\rho_{\theta,t}^{\ell,f} < \rho_{\theta,t}^{\ell,f,MM}$ . This contradicts the conventional view that MM's debt policy leads to the lowest cost of capital. Rather, with fixed investments and autonomous debt, the need to adjust the payout ratio in each period to meet financing demands entails a specific risk for shareholders, leading to an increasing cost of capital. This risk outweighs the uncertainty of future tax shields caused by the commitment towards a fixed payout ratio. This result is in line with the empirical findings cited in the introduction, and provides a simple explanation for why some firms set a target payout ratio. Finally, note that the cost of capital according to (12) is generally lower than that of ME. This follows from  $\rho_{\theta,t}^{\ell,f,MM} < \rho_{\theta,t}^{\ell,f,ME}$ , which was derived in Sect. 2, and the conclusions above.

<sup>&</sup>lt;sup>7</sup> Moreover, it is necessary to assume that the intended investments are compatible with the financing policies under consideration. Regarding MM, for example, this would not be the case if net income were insufficient to cover the difference between net investments and autonomous debt. In this case, the intended net investment payments would not be possible even if all net income was retained.

<sup>&</sup>lt;sup>8</sup> See Appendix 4.

#### 5 Cost of equity with fixed payout ratios and flexible investments

The assumption that future net investments follow a fixed investment plan is certainly adequate in some cases. In most cases, however, future net investments will depend on information not yet available at the time of valuation. According to the neoclassical theory of the firm, the management evaluates investment projects based on expected cash inflows and outflows, resp. the corresponding operating profits. Hence, from an ex-ante perspective, future net investments are driven by the same factors as expectations about subsequent operating profits. In a stable environment, these factors regularly affect present and future operating profits, which is the case, for example, when operating profit is highly dependent on raw material prices and current prices reflect expectations about future price developments. If such a constellation can be assumed, there is a link between net investments and operating profits that can be used to specify Proposition 2 in an alternative way. In the simplest case, the portion of operating profit that is used for net investments is deterministic. According to (7), net investments must then be discounted with the cost of capital of the unlevered firm. Since future capital expenditures are tied to an ex-ante unknown quantity, we term this the case of flexible investments. Proposition 2 yields:

**Proposition 4** With flexible investments, the cost of equity of a levered firm with a fixed payout ratio is:

$$\rho_{\theta,t}^{\ell,\nu} = \rho_{\theta,t}^{u,\nu} + (\rho_{\theta,t}^{u,\nu} - i) \cdot \frac{q \cdot (1 - \tau)}{\tau + q \cdot (1 - \tau)} \cdot \frac{\mathsf{E}_{\theta}[D_{t-1}^{\nu}]}{\mathsf{E}_{\theta}[E_{t-1}^{\ell,\nu}]}.$$
(13)

Superscript v indicates flexible investments. Again, the scope of Proposition 4 can be extended concerning the distinction between systematic and unsystematic risk. For (13) to hold, it is not necessary to assume that net investment is a deterministic part of operating profits in each future period. Instead, it is sufficient to assume that net investment and operating profits are subject to the same systematic risk. As a result of unsystematic risk, extraordinary events may affect operating profits, while long-term profit expectations and net investments remain unaffected. Hence, even if the preconditions for the case of flexible investments are met, a decoupling of capital expenditures and operating profits should not come as a surprise.

To compare (13) with the cost of capital according to MM or ME, the same assumptions as in the last section must be met. This provided, the cost of capital (13) is equal to that in the MM case, i. e.  $\rho_{\theta,t}^{\ell,v} = \rho_{\theta,t}^{\ell,v,MM}$ , when all net income is distributed (q = 1), no net investments are made, and, hence, no additional autonomous debt is required ( $\Delta D_s^{MM} = 0$  for all  $s \ge t$ ). In this case, (13) converts to the classical MM formula for the cost of equity of a levered firm with  $D_s^{MM} = D_{t-1}^{MM}$  for all  $s \ge t$ . For a smaller payout ratio (q < 1), the cost of equity increases as a consequence of an increasing debt-to-equity ratio. This effect, however, is mitigated by the equity risk-reducing effect of the fixed payout ratio. On the other hand, in the MM case, the higher debt-to-equity ratio caused by the increase in autonomous debt is partially offset by the benefits of deterministic tax shields. The costs of equity remain the same in both cases as long as the retention ratio (1 - q) is equal to the share of the operating profit used for net capital expenditures.<sup>9</sup> However, with high free cash flows,  $\rho_{\theta,t}^{\ell,v} < \rho_{\theta,t}^{\ell,v,MM}$ becomes possible when the fixed payout ratio and net investments are relatively low. As

<sup>&</sup>lt;sup>9</sup> Under this condition, the sum expression in (10) vanishes and thus expected tax shields have equal value in both cases.

with fixed investments, the equity risk-reducing effect of the fixed payout ratio then outweighs the benefits of deterministic tax shields. Compared to the debt policy of ME, a fixed payout ratio always leads to a lower cost of capital for flexible investments.  $\rho_{\theta,t}^{\ell,v} < \rho_{\theta,t}^{\ell,v,ME}$ follows from (6) and (13) and the specifications for q,  $\tau$  and i.<sup>10</sup> These results also provide an explanation of why compliance with a target payout ratio is sometimes given a higher priority than adherence to a target capital structure (Baker et al. 2001, p. 29).

Compared to (11) and (12), (13) entails a considerable simplification regarding the cost of equity of levered firms. In particular, it refers only to the predetermined payout ratio representing the assumed financing policy and not to other quantities, such as expected investment payouts. Therefore, (13) seems to be a suitable alternative to the formulae of MM and ME in practical applications. However, caution must be exercised concerning the relationship between the payout ratio and the debt-to-equity ratio. According to (9), a reduction of the payout ratio leads to a decrease in expected debt and the debt-to-equity ratio. Hence, varying the payout ratio in (13) without considering the consequences on debt would lead to incorrect results. The fact that the cost of equity according to (13) will regularly lie between those resulting from the MM and ME formulae can be helpful for valuation practice, as it opens up the possibility for a well-founded compromise when conflicts over the application of these formulae arise.

#### 6 Conclusions

Frictionless capital markets imply that the cost of capital of a firm depends on its financing policy. Consequently, an application of this concept in corporate finance or valuation ought to be based on appropriate assumptions on the financing policy. The two most prominent alternatives, resulting from the works of MM and ME, prioritize debt policy over dividend policy. Neither of them is unambiguously confirmed by empirical findings. As a result, it is unclear which of them or whether any should be used. This is particularly true if the firm prioritizes its dividend policy. As this paper shows, the problem can be mitigated by resorting to the assumption of a fixed payout ratio, combined with assumptions concerning future investments. The idea of a fixed payout ratio has comparatively good empirical support and assumptions on investments are essential in valuation anyway.

If the company pursues a strategy whose implementation requires successive investments, it is plausible to assume fixed investments. Flexible investments appear more plausible if the company operates in volatile markets so that net investments are adjusted to market developments in the short term. Moreover, the idea of flexible investments is especially relevant for the distant future, for which fixed investments make less sense. In valuation contexts, it is regularly assumed that a firm will achieve a steady-state, in which all relevant parameters grow at the same rate. Typically, this assumption rests on the idea that a fixed fraction of the operating profit is used for net investments (Gordon and Shapiro 1956; Koller et al. 2020, p. 49). This assumption constitutes flexible investments in our framework. Yet, the scope of our results can be extended by distinguishing between systematic and unsystematic risk.

<sup>&</sup>lt;sup>10</sup> Note that  $\frac{1+i\cdot(1-\tau)}{1+i} > \frac{q\cdot(1-\tau)}{\tau+q\cdot(1-\tau)}$  holds for all permissible  $q, \tau$  and i. Thus, since the expected flow to equity is kept identical throughout the comparison,  $E_{\theta}[E_{t-1}^{\ell,\nu,ME}]$  cannot exceed  $E_{\theta}[E_{t-1}^{\ell,\nu}]$  without causing a contradiction.

The main results of our paper are relationships between the cost of capital of levered and unlevered firms that give priority to dividend policy over debt policy. From a theoretical point of view, these relationships reflect the implications of fixed and flexible investments and fixed payout ratios in arbitrage-free markets. Remarkably, for fixed investments, the cost of capital is lower than in the case of MM. This contradicts the conventional view that financing according to MM leads to the lowest cost of capital of levered firms. For flexible investments, a fixed payout ratio generally leads to a lower cost of capital compared to ME; compared to MM, a lower cost of capital is possible depending on the valuation parameter. Under empirical aspects, these results contribute to explaining empirical findings on the specification of a target payout ratio and its relative importance compared to a target capital structure. From a practitioner's point of view, the inferred relationships can help mitigate valuation issues when debt policy is subordinate to dividend policy.

#### Appendix 1: Proof of Proposition 1

The expected flow to equity is:

$$\mathbf{E}_{\theta}[x_t^{FTE}] = \mathbf{E}_{\theta}[x_t] - (1 - \tau) \cdot i \cdot \mathbf{E}_{\theta}[D_{t-1}] - (\mathbf{E}_{\theta}[D_{t-1}] - \mathbf{E}_{\theta}[D_t]).$$

Rearranging the formula for the market value of the unlevered firm (2) gives:

$$\mathbf{E}_{\theta}[x_t] = \mathbf{E}_{\theta}[V_{t-1}^u] \cdot (1 + \rho_{\theta,t}^u) - \mathbf{E}_{\theta}[V_t^u].$$

Proposition 1 follows by substituting  $E_{\theta}[x_t^{FTE}]$  and  $E_{\theta}[x_t]$  in the second part of (2), also taking into account (3).

#### Appendix 2: Derivation of Eq. (9)

Rearranging the relationship between net investments and the change in debt yields:

$$D_t = ni_t - (1 - q_t) \cdot g_t + D_{t-1}.$$

Equation (9) follows from the definitions:

$$\begin{split} D_t &= ni_t - (1 - q_t) \cdot (op_t - i \cdot (1 - \tau) \cdot D_{t-1}) + D_{t-1} \\ &= (1 + i \cdot (1 - q_t) \cdot (1 - \tau)) \cdot D_{t-1} + ni_t - (1 - q_t) \cdot op_t \\ &= (1 + i \cdot (1 - q_t) \cdot (1 - \tau)) \cdot D_{t-1} + q_t \cdot ni_t - (1 - q_t) \cdot (op_t - ni_t) \\ &= (1 + i \cdot (1 - q_t) \cdot (1 - \tau)) \cdot D_{t-1} + q_t \cdot ni_t - (1 - q_t) \cdot x_t. \end{split}$$

#### Appendix 3: Proof of Proposition 2

The proof of Proposition 2 starts with (10):

$$(10) \mathcal{E}_{\theta}[D_{t}] = (1+\mu)^{t-\theta} \cdot D_{\theta} + \sum_{s=\theta+1}^{t} (1+\mu)^{t-s} \cdot \left(q \cdot \mathcal{E}_{\theta}[ni_{s}] - (1-q) \cdot \mathcal{E}_{\theta}[x_{s}]\right)$$
$$t = \theta, \theta + 1, ..., \quad \theta = 0, 1, ...$$

The expected tax shield is:

$$\begin{split} \mathbf{E}_{\theta}[ts_{t+1}] &= i \cdot \tau \cdot \mathbf{E}_{\theta}[D_{t}] \\ &= i \cdot \tau \cdot (1+\mu)^{t-\theta} \cdot D_{\theta} \\ &+ \sum_{s=\theta+1}^{t} i \cdot \tau \cdot q \cdot (1+\mu)^{t-s} \cdot \mathbf{E}_{\theta}[ni_{s}] - \sum_{s=\theta+1}^{t} i \cdot \tau \cdot (1-q) \cdot (1+\mu)^{t-s} \cdot \mathbf{E}_{\theta}[x_{s}] \\ &= ts_{\theta,t+1}^{D} + \sum_{s=\theta+1}^{t} \mathbf{E}_{\theta}[ts_{s,t+1}^{NI}] - \sum_{s=\theta+1}^{t} \mathbf{E}_{\theta}[ts_{s,t+1}^{FCF}] \\ t = \theta, \theta + 1, \dots, \quad \theta = 0, 1, \dots \end{split}$$

Hence, the tax shield can be decomposed into three parts, which depend on the debt available at time  $\theta$  and net investments and the free cash flow from  $\theta + 1$  to t. The first part of the expected tax shield in t + 1 depends on debt at time  $\theta$  only:

$$ts^{D}_{\theta,t+1} = i \cdot \tau \cdot (1+\mu)^{t-\theta} \cdot D_{\theta}t = \theta, \theta+1, ..., \quad \theta = 0, 1, ...$$

Since this part of the tax shield is known with certainty at the time of the valuation, its market value is determined by the risk-free interest rate. A recursive analysis yields the following result:

$$V_{\theta,t}^{TS,D} = \frac{ts_{\theta,t+1}^{D} + V_{\theta,t+1}^{TS,D}}{1+i}t = \theta, \theta + 1, ..., \quad \theta = 0, 1, ...$$

Since the value of the tax shield increases at rate  $\mu$ , we have:

$$V_{\theta,t}^{TS,D} = (1+\mu)^{t-\theta} \cdot \frac{i \cdot \tau}{i-\mu} \cdot D_{\theta}t = \theta, \theta+1, ..., \quad \theta = 0, 1, ...$$

The second part of the expected tax shield in t + 1 depends on the net investments of period *s* as follows:

$$\begin{split} \mathbf{E}_{\theta}[ts_{s,t+1}^{NI}] &= i \cdot \tau \cdot q \cdot (1+\mu)^{t-s} \cdot \mathbf{E}_{\theta}[ni_s] \\ s &= \theta+1, \theta+2, ..., \quad t=s,s+1, ..., \quad \theta=0,1, ... \end{split}$$

For  $t \ge s$ , net investments at time *s* are known at time *t*. Hence, the value contribution of these tax shields at time *t* is determined by the risk-free interest rate:

$$\begin{split} V^{TS,NI}_{s,t} &= \frac{ts^{NI}_{s,t+1} + V^{TS,NI}_{s,t+1}}{1+i} \\ s &= \theta + 1, \theta + 2, ..., \quad t = s, s+1, ..., \quad \theta = 0, 1, ... \end{split}$$

It follows that:

$$V_{s,t}^{TS,NI} = (1+\mu)^{t-s} \cdot \frac{i \cdot \tau}{i-\mu} \cdot q \cdot ni_s$$
  
$$s = \theta + 1, \theta + 2, \dots, \quad t = s, s+1, \dots, \quad \theta = 0, 1, \dots$$

For t < s, net investments at time s are still in the future. The market value at time s is therefore calculated using a cost of capital  $\kappa_{\theta,t}$  adjusted to the risk of the net investment payouts:

$$\begin{split} \mathbf{E}_{\theta}[V_{s,t}^{TS,NI}] &= \frac{\mathbf{E}_{\theta}[V_{s,t+1}^{TS,NI}]}{1+\kappa_{\theta,t+1}} \\ t &= \theta, \theta+1, ..., s-1, \quad s = t+1, t+2, ..., \quad \theta = 0, 1, ... \end{split}$$

It follows that:

$$\begin{split} \mathbf{E}_{\theta}[V_{s,t}^{TS,NI}] &= \mathbf{E}_{\theta}[V_{s,s}^{TS,NI}] \cdot \prod_{r=t+1}^{s} (1+\kappa_{\theta,r})^{-1} = \frac{i \cdot \tau}{i-\mu} \cdot q \cdot \mathbf{E}_{\theta}[ni_{s}] \cdot \prod_{r=t+1}^{s} (1+\kappa_{\theta,r})^{-1} \\ t &= \theta, \theta+1, ..., s-1, \quad s=t+1, t+2, ..., \quad \theta = 0, 1, ... \end{split}$$

The third part of the tax shield expected in t + 1 depends on the free cash flow of period s:

$$\begin{split} \mathbf{E}_{\theta}[ts_{s,t+1}^{FCF}] &= i \cdot \tau \cdot (1-q) \cdot (1+\mu)^{t-s} \cdot \mathbf{E}_{\theta}[x_s] \\ s &= \theta + 1, \theta + 2, ..., \quad t = s, s+1, ..., \quad \theta = 0, 1, ... \end{split}$$

Similar to above, for  $t \ge s$ , the value contribution of these tax shields at time *t* is calculated using the risk-free interest rate:

$$\begin{aligned} V_{s,t}^{TS,FCF} &= (1+\mu)^{t-s} \cdot \frac{i \cdot \tau}{i-\mu} \cdot (1-q) \cdot x_s \\ s &= \theta + 1, \theta + 2, ..., t = s, s+1, ..., \theta = 0, 1, ... \end{aligned}$$

For t < s, the value contribution at time s is discounted using the cost of capital of the unlevered firm:

$$\mathbf{E}_{\theta}[V_{s,t}^{TS,FCF}] = \frac{\mathbf{E}_{\theta}[V_{s,t+1}^{TS,FCF}]}{1 + \rho_{\theta,t+1}^{u}} \\ t = \theta, \theta + 1, ..., s - 1, \quad s = t + 1, t + 2, ..., \quad \theta = 0, 1, ...$$

It follows that:

$$\begin{split} \mathbf{E}_{\theta}[V_{s,t}^{TS,FCF}] &= \frac{i \cdot \tau}{i - \mu} \cdot (1 - q) \cdot \mathbf{E}_{\theta}[x_s] \cdot \prod_{r=t+1}^{s} (1 + \rho_{\theta,r}^{u})^{-1} \\ t &= \theta, \theta + 1, ..., s - 1, \quad s = t + 1, t + 2, ..., \quad \theta = 0, 1, ... \end{split}$$

Hence, the total market value of the tax shields is:

$$\begin{split} \mathbf{E}_{\theta}[V_{t}^{TS}] &= V_{\theta,t}^{TS,D} + \sum_{s=\theta+1}^{\infty} \mathbf{E}_{\theta}[V_{s,t}^{TS,NI}] - \sum_{s=\theta+1}^{\infty} \mathbf{E}_{\theta}[V_{s,t}^{TS,FCF}] \\ t &= \theta, \theta+1, ..., \quad \theta = 0, 1, ... \end{split}$$

Inserting (10) yields:

$$\begin{aligned} \mathbf{E}_{\theta}[V_t^{TS}] &= \frac{i \cdot \tau}{i - \mu} \cdot \mathbf{E}_{\theta}[D_t] + \sum_{s=t+1}^{\infty} \mathbf{E}_{\theta}[V_{s,t}^{TS,NI}] - \sum_{s=t+1}^{\infty} \mathbf{E}_{\theta}[V_{s,t}^{TS,FCF}] \\ t &= \theta, \theta + 1, \dots \quad \theta = 0, 1, \dots \end{aligned}$$

When considering (9), the result becomes:

$$\begin{aligned} \mathsf{E}_{\theta}[V_{t}^{TS}] &- (1 + \rho_{\theta,t}^{u}) \cdot \mathsf{E}_{\theta}[V_{t-1}^{TS}] \\ &= -\frac{i \cdot \tau}{i - \mu} \cdot \left( (\rho_{\theta,t}^{u} - \mu) \cdot \mathsf{E}_{\theta}[D_{t-1}] + (\rho_{\theta,t}^{u} - \kappa_{\theta,t}) \cdot q \cdot \mathsf{E}_{\theta}[K_{t-1}] \right) \\ &\text{with} \end{aligned}$$

$$E_{\theta}[K_{t-1}] = \sum_{s=t}^{\infty} E_{\theta}[ni_s] \cdot \prod_{r=t}^{s} (1 + \kappa_{\theta,r})^{-1} t = \theta, \theta + 1, \dots \quad \theta = 0, 1, \dots$$

Inserted in (4), this results in Proposition 2.

Appendix 4: Proof of 
$$ho_{ heta,t}^{\ell,f} < 
ho_{ heta,t}^{\ell,f,MM}$$
 for  $q < 1$ 

Using (5) and (12), we show that

$$\begin{split} \rho_{\theta,t}^{u,f} + (\rho_{\theta,t}^{u,f} - i) \cdot \frac{q}{\tau + q \cdot (1 - \tau)} \cdot \left( (1 - \tau) \cdot \frac{\mathsf{E}_{\theta}[D_{t-1}^{f}]}{\mathsf{E}_{\theta}[E_{t-1}^{\ell,f}]} - \tau \cdot \frac{K_{t-1}^{f}}{\mathsf{E}_{\theta}[E_{t-1}^{\ell,f}]} \right) \\ < \rho_{\theta,t}^{u,f} + (\rho_{\theta,t}^{u,f} - i) \cdot \frac{D_{t-1}^{f,MM} - V_{t-1}^{TS,f,MM}}{\mathsf{E}_{\theta}[E_{t-1}^{\ell,f,MM}]}, \end{split}$$

with

$$\begin{split} K_{t-1}^{f} &= \sum_{s=t}^{\infty} ni_{s} \cdot (1+i)^{t-s-1}, \quad \mathbf{E}_{\theta}[D_{t-1}^{f}] = D_{t-1}^{f,MM} \quad \text{and} \\ V_{t-1}^{TS,f,MM} &= \sum_{s=t}^{\infty} i \cdot \tau \cdot D_{s-1}^{f,MM} \cdot (1+i)^{t-s-1} \\ &= \tau \cdot D_{t-1}^{f,MM} + \tau \cdot \sum_{s=t}^{\infty} \Delta D_{s}^{f,MM} \cdot (1+i)^{t-s-1}. \end{split}$$

According to Appendix 3, the value of the tax shield can be decomposed into three parts:

$$\mathbf{E}_{\theta}[V_{t}^{TS}] = V_{\theta,t}^{TS,D} + \sum_{s=\theta+1}^{\infty} \mathbf{E}_{\theta}[V_{s,t}^{TS,NI}] - \sum_{s=\theta+1}^{\infty} \mathbf{E}_{\theta}[V_{s,t}^{TS,FCF}]$$

Note that this decomposition can be done in the MM case, as well as in the case of a fixed payout ratio. Due to the assumptions, the expected tax shields underlying all three parts are identical. In the MM case, the overall value of the tax shields is calculated by the risk-free interest rate, as the payout ratio is adjusted according to the autonomous debt policy. In the case of a fixed payout ratio with fixed investments, this only holds for the first and second parts. The expected tax shields corresponding to the third part are however discounted with the cost of capital of the unlevered firm, as shown in Appendix 3. Because of the negative sign, the value effect is positive. Hence,  $E_{\theta}[V_{t-1}^{TS,f}] \ge V_{t-1}^{TS,f,MM}$  follows. Equation (3) results in  $E_{\theta}[E_{t-1}^{\ell,f}] \ge E_{\theta}[E_{t-1}^{\ell,f,MM}]$ . Consequently, considering  $\frac{1}{\tau+q\cdot(1-\tau)} < 1$ ,

$$(1 - \tau) \cdot D_{t-1}^{f,MM} - \tau \cdot K_{t-1}^{f} \le D_{t-1}^{f,MM} - V_{t-1}^{TS,f,MM}$$

resp.

$$\tau \cdot K_{t-1}^f \ge V_{t-1}^{TS,f,MM} - \tau \cdot D_{t-1}^{f,MM}$$

suffices to demonstrate the intended result.  $ni_s \ge \Delta D_s^{f,MM}$  for all  $s \ge t$  implies

$$\tau \cdot K_{t-1}^{f} = \tau \cdot \sum_{s=t}^{\infty} ni_{s} \cdot (1+i)^{t-s-1} \ge \tau \cdot \sum_{s=t}^{\infty} \Delta D_{s}^{f,MM} \cdot (1+i)^{t-s-1} = V_{t-1}^{TS,f,MM} - \tau \cdot D_{t-1}^{f,MM}.$$

Funding Open Access funding enabled and organized by Projekt DEAL.

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