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Abstract

The paper examines how the long-run inflation-unemployment tradeoff depends on the degree to which wage-price decisions are backward- versus forward-looking. When economic agents, facing time-contingent, staggered nominal contracts, have a positive rate of time preference, the current wage and price levels depend more heavily on past variables (e.g. past wages and prices) than on future variables. Consequently, the long-run Phillips curve becomes downward-sloping and, indeed, quite flat for plausible parameter values. This paper provides an intuitive account of how this long-run Phillips curve arises.

Keywords: Inflation-unemployment tradeoff, wage-price staggering, monetary policy, forward- and backward-looking wage-price behavior, traditional and New Phillips curve.

JEL Classifications: E2, E3, E5, J3.

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1. Introduction

The literature on the New Phillips curve[1] has shown that when economic agents face nominal frictions in the form of time-contingent staggered nominal contracts, then current nominal variables depend on past and future variables.[2] In the standard derivations of the New Phillips curve,[3] it is generally assumed that the weights on the backward- and forward-looking terms are equal[4] and thus there is no long-run inflation-unemployment tradeoff, i.e. the New Phillips curve is vertical in the long run. However, recent contributions to the microfoundations of wage-price setting under time-contingent staggered nominal contracts have shown that this assumption is incorrect.[5] In particular, when agents discount the future (viz., they have a positive rate of time preference), then the backward-looking variables are weighted more heavily than the forward-looking ones. Then, as shown below, the associated long-run Phillips cuve becomes downward-sloping. This result cannot be dismissed as a mere theoretical nicety without empirical relevance. On account of the nonlinear relation between the discount rate and the parameters of the Phillips curve, even reasonably small discount rates can lead to quite flat long-run Phillips curves. The purpose of this paper is to give an intuitive account of (i) why there is a long-run tradeoff between inflation and unemployment under these circumstances and (ii) how this tradeoff depends on backward- versus forward-orientation of wage-price decisions.

Our account involves an anatomy of the Phillips curve in terms of the backward- and forward-orientation of agents' wage-price decisions. It comes in three parts.

First, we show that when wage-price decisions are purely backward-looking, then the long-run Phillips curve is always downward-sloping. The underlying reason is straightforward. When the money supply grows and current nominal

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[1] It is often also called the "New Keynesian Phillips curve" or the "New Neoclassical Synthesis." Helpful recent surveys include Clarida, Gali and Gertler (1999), Gali (2002), Mankiw (2001), Roberts (1995) and Goodfriend and King (1997).

[2] Specifically, current wages and prices depend on their past values since some agents have not altered their wages or prices, and they also depend on their future values since the agents who currently change their wages or prices know that they may not be able to change them in the future.

[3] These are commonly based on the models of Taylor (1979, 1980a), Calvo (1983), or Rotemberg (1982).

[4] Specifically, in these derivations, a nominal variable X_t at time t depends linearly on past variables such as $X_{t-\tau}$, $\tau > 0$, and on future variables such as $X_{t+\tau}, \tau > 0$; and it is assumed that the weights on $X_{t-\tau}$ and $X_{t+\tau}$ are equal.

[5] For microfoundations of contracts with deterministic duration, see Ascari (2000), Graham and Snower (2002), Helpman and Leiderman (1990), Huang and Liu (2002), and others. The microfoundations of contracts with stochastic duration ("Calvo contracts") have been examined, among others, by Gali (2002) and Bernanke, Gertler and Gilchrist (2002).

variables depend only on past nominal variables, then wages and prices are continually lagging behind their moving targets. These targets - often called "desired" wages and prices in the New Keynesian literature[6] - are the wage and price levels that would obtain in the absence of nominal frictions. Since the money supply grows, the nominal targets grow as well. Furthermore, since wage-price decisions are backward-looking (dependent on past nominal variables), actual wages and prices never catch up with their growing targets. If there were a one-off increase in the money supply, full adjustment of wages and prices would eventually occur; but the money supply is growing continually, and thus the adjustment process is never completed. Each successive increase in the money supply starts a new adjustment process in which wages and prices lag behind their frictionless targets. Consequently, in the long-run equilibrium, actual wages and prices remain behind their moving targets and the distance between the two is constant. In this context, an increase in money growth - implying an increase in the growth of the wage-price targets - causes wages and prices to fall further behind their targets. Thus real money balances (the ratio of money to the price level) rise and unemployment falls. In this way, an increase in money growth raises inflation and reduces unemployment over the long run.

Second, we show that when wage-price decisions are purely forward-looking, the opposite happens and thus the long-run Phillips curve is upward-sloping. When current nominal variables depend only on future nominal variables and the money supply grows, then wages and prices are running ahead of their moving targets. In this context, an increase in money growth enables the money supply to catch up partially with actual wages and prices. Thus real money balances fall and unemployment rises.

Third, we turn to a standard time-contingent staggered nominal contract, in which current wage-price decisions depend on both backward- and forward-looking terms. We show that the movement of actual wages and prices relative to their targets depends on the degree to which agents' decisions are backward- versus forward-looking. When the weights on the backward- and forward-looking terms are equal, the lagging behavior of backward-looking wage-price decisions is exactly balanced by the leading behavior of forward-looking wage-price decisions, and in this special case the long-run Phillips curve turns out to be vertical. However, this special case does not hold when there is time discounting. Under discounting, the lagging behavior outweighs the leading behavior and thus the long-run Phillips curve turns out to be downward-sloping.

This conclusion holds even though money illusion is absent in our model. What the absence of money illusion implies is that *target* wages and prices move proportionally to the money supply; it does not ensure that *actual* wages and

[6] See, for example, Mankiw (2001).

prices do so. If the level of the money supply increases, then the actual wage and price levels will of course eventually catch up with their targets, so that money will be neutral in the long run. But if the money supply is growing, then the actual price level never catch up. On this account, the absence of money illusion does not guarantee a vertical long-run Phillips curve.

Although our analysis focuses on Taylor wage contracts, it is straightforward to show that the same reasoning and the same qualitative conclusions also apply to Calvo price contracts.

Our analysis is clearly at odds with both the traditional Keynesian, expectations-augmented Phillips curve (usually expressed simply as $\pi_t = \pi_{t-1} - b(u_t - u^n) + \varepsilon_t$, where π_t is inflation, u_t is unemployment, u^n is the NAIRU, and ε_t is white noise) and the standard New Phillips curve (commonly expressed as $\pi_t = E_t\pi_{t+1} - b(u_t - u^n) + \varepsilon_t$, where E_t stands for expectations formulated at time t). Although the traditional curve is backward-looking (current inflation depends on past inflation) whereas the New Phillips curve is forward-looking (current inflation depends on expected future inflation), both curves are vertical in the long-run. Our analysis is concerned with a different aspect of backward- versus forward-orientation - viz., the orientation of wage-price contracts - and this latter aspect turns out to be compatible with a long-run inflation-unemployment tradeoff.

Our macro model is presented in Section 2. In Section 3 we show how backward-looking wage-price decisions imply a downward-sloping long-run Phillips curve. Along the same lines, Section 4 shows how forward-looking wage-price decisions imply an upward-sloping long-run Phillips curve. The wage equations in Sections 3 and 4 will be specified in an ad hoc way; their purpose is simply to help us understand how the long-run inflation unemployment tradeoff is affected by the backward- versus forward-orientation of wage-price decisions. In Section 5, we show that our backward- and forward-looking wage equations can be interpretted as components of a microfounded Taylor wage contract equation. Putting these components together, the slope of the long-run Phillips curve becomes comprehensible as the resultant of backward- and forward-looking wage-price dynamics.

2. The Model

Our model is a particularly simple vehicle for conducting our anatomy of the Phillips curve.[7] It has the two central features: (i) frictions in wage-price setting, specified through Taylor wage contracts and (ii) a growing money supply. It also reflects the absence of money illusion, in that all equations are homogeneous of

[7] A microfounded dynamic general equilibrium model that underlies the equations of our macro model is given in Graham and Snower (2002).

degree zero in all nominal variables. All variables are in logs, except for the unemployment rate. All uninteresting constants are omitted.

The model is composed of three building blocks. The first block links unemployment to real money balances; the second specifies the money supply; and the third specifies how the wage and price levels respond to the money supply. To derive the inflation-unemployment tradeoff, we then show how changes in money growth affect inflation, and thereby real money balances and unemployment.

We begin with the first block. Aggregate product demand depends on real money balances: $Q_t^D = (M_t - P_t)$,where M_t is the money supply and P_t is the price level, and the coefficient on real money balances has been normalized to unity. (For brevity, we omit the standard derivation of this equation in terms of microfoundations.) The aggregate production function is $Q_t^S = N_t$. The product market clears: $Q_t^D = Q_t^S$. The labor supply is constant: $L_t = L$. The unemployment rate (not in logs) can be approximated as $u_t = L - N_t$. Thus we obtain the following unemployment equation:

$$u_t = L - (M_t - P_t) \tag{2.1}$$

i.e. unemployment depends inversely on real money balances.

To focus on the long-run Phillips curve, we need to consider permanent shocks to money growth, leading to permanent changes in inflation, that move the economy along this Phillips curve. Thus, in the second building block of the model, we let the growth rate of the money supply be a random walk:

$$\Delta M_t \equiv \mu_t = \mu_{t-1} + \varepsilon_t, \tag{2.2}$$

where M_t is the log of the money supply and ε_t is a white-noise error term. Economic agents with rational expectations are assumed to know the current money growth shock but not the future realizations of this shock.

In the final block, we describe wage and price setting. As noted, we will consider three wage-price setting equations: a backward-looking one, a forward-looking one, and finally a Taylor equation of staggered nominal wage contracts that incorporates both the backward- and forward-looking elements.

3. Backward-looking Wage-Price Setting and the Phillips Curve

In our backward-looking wage equation, the current nominal wage level depends on the past wage level and the current money supply:

$$W_t = aW_{t-1} + (1 - a) M_t, \tag{3.1}$$

where the "wage sluggishness parameter" a is a constant, $0 < a < 1$.

Under competitive price setting, the real wage is equal to the marginal product of labor and, given the production function above, this marginal product is unity, so that $P_t = W_t$. Thus the price equation is

$$P_t = aP_{t-1} + (1-a)\, M_t. \tag{3.2}$$

Substituting this equation into the unemployment equation (2.1) and defining the inflation rate as $\pi_t \equiv P_t - P_{t-1}$, we obtain the following inflation-unemployment tradeoff:[8]

$$\pi_t = \frac{1-a}{a}\,(L - u_t)\,. \tag{3.3}$$

Observe that changes in money growth move the economy along a downward-sloping long-run Phillips curve. The greater is the wage sluggishness parameter a, the flatter this long-run Phillips curve will be. Observe that this is so even though money illusion is absent in the macroeconomic system, (i.e. if all nominal variables are changed in the same proportion, then the real variables remain unchanged).

The intuition underlying this result is clarified in Figure 1. Here the initial price level P_0, and the price dynamics schedule PD_1 (equation (3.2)) relates the period-1 price P_1 to P_0. The period-1 money supply is M_1,[9] and thus the associated real money balances are $(M_1 - P_1)$. These real money balances are positively related to the unemployment rate (as described in equation (2.1)).

Suppose that the money supply grows ($\Delta M_t > 0$), so that the M_2 line lies above the M_1 line in the figure. Thus the price dynamics schedule shifts to PD_2. As result, the economy moves to point B in period 2, with price level rising to P_2 and the associated real money balances are $(M_2 - P_2)$.

Since we wish to focus on the long-run inflation-unemployment tradeoff, the figure depicts the economy in steady state, with a constant rate of money growth, constant inflation ($P_1 - P_0 = P_2 - P_1$) and constant real money balances ($M_1 - P_1 = M_2 - P_2$). Thus equilibrium output and unemployment remain unchanged from one period to the next as well.

It is now easy to see how the level of real money balances depends on the growth rate of the money supply. Suppose that the money supply had remained constant at M_1 forever, so that the price dynamics schedule had remained stationary at PD_1. Then the price dynamics would eventually have worked themselves out,

[8] Express the price equation as $(1-a)\,P_t = a\,(P_{t-1} - P_t) + (1-a)\,M_t$, which implies that $M_t - P_t = \frac{a}{1-a}\pi_t$. Substitute this equation into (2.1) to obtain (3.3).

[9] The money supply M_1 lies at the intersection of the price dynamics schedule PD_1 and the 45^o line because, by (3.2), if $P_t = P_{t-1}$, then $P_t = M_t$.

taking the economy to point A^* in Figure 1. The associated price level is P_1^*; this is the target price level, towards which the actual period-1 price is tending.

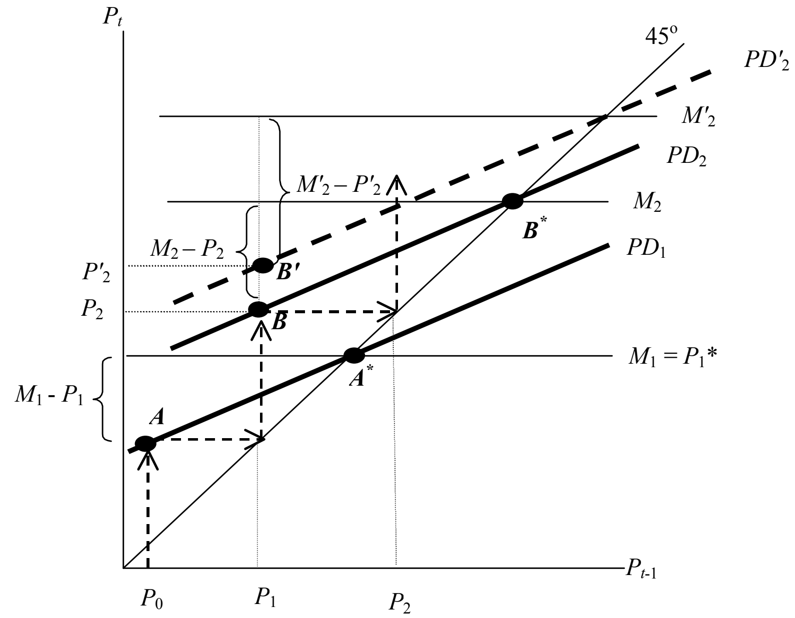

Fig. 1: Money Growth under Backward-Looking Wage Setting

When the money supply grows, however, the actual price level (at time t) never manages to catch up with its target level (at time t). Each time the money supply increases, raising the target price level in proportion, it initiates a prolonged process of price adjustment. But right after this process has begun, the money supply rises again.

The faster the money supply grows, the more the price level lags behind its target level, and thus the greater is the level of real money balances. This is also illustrated in Figure 1. Suppose that, starting from period 1 (and money supply M_1), the money supply grows faster than before, so that the period-2 money supply is M_2', where $M_2' > M_2$. Then the period-2 price dynamics schedule is higher than before (i.e. PD_2' lies above PD_2), and the economy moves to point B' (rather than B). Consequently, the price level lags further behind the money supply, so that the price increase (to P_2' rather than P_2) is less than proportional to the money supply increase (to M_2' rather than M_2). Thus real money balances are higher ($M_2' - P_2'$ rather than $M_2 - P_2$). In this way, an increase in money growth leads to a long-run rise in inflation and a long-run fall in unemployment.

4. Forward-looking Wage-Price Setting and the Phillips Curve

In our forward-looking wage equation, the current nominal wage level depends on the expected future wage level and the expected future money supply:

$$W_t = aW_{t+1}^e + (1-a)\,M_{t+1}^e, \tag{4.1}$$

where $X_{t+i}^e = E_t(X_{t+i})$ denotes the expectation of the variable X_{t+i} conditional on information available at time t, and the coefficient a (a constant, $0 < a < 1$) now becomes the "wage anticipation parameter"[10] .

Since $P_t = W_t$, the price equation is

$$P_t = aP_{t+1}^e + (1-a)\,M_{t+1}^e, \tag{4.2}$$

This equation, together with the unemployment equation (2.1), implies the following inflation-unemployment tradeoff:[11]

$$\pi_{t+1}^e = -(1-a)\left(L - u_{t+1}^e\right). \tag{4.3}$$

This long-run Phillips curve is upward-sloping. The greater is the wage anticipation parameter a, the flatter will be this relation.

Figure 2 provides the intuition for this Phillips curve. To focus on the long-run Phillips curve, we let the economy be in its steady state, in which money growth is constant and expectations are fulfilled each period. Starting from the initial price level P_0, the economy is expected (and does) move to point A on the period-1 price dynamics schedule [12] PD_1 ($P_1^e = \frac{1}{a}P_0 - \frac{1-a}{a}M_1^e$, by equation (4.2)), with period-1 price level P_1 and corresponding real money balances $(M_1 - P_1)$.

[10] Letting the wage anticipation parameter in this section be equal to the wage sluggishness parameter of the previous section provides a straightforward interpretation of the Taylor wage contract equation in the following section.

[11] Rewrite the price equation as $-\left(P_{t+1}^e - P_t\right) = (1-a)\,M_{t+1}^e - (1-a)\,P_{t+1}^e$, which implies $M_{t+1}^e - P_{t+1}^e = -\left(\frac{1}{1-a}\right)\pi_{t+1}^e$. Substitute this equation into the unemployment equation (2.1) to obtain (4.3).

[12] Observe that since $0 < a < 1$, this schedule is now steeper than the 45^o line. Thus the target price level - $P_1^* = M_1$, lying at the intersection between the PD_1 schedule and the 45^0 line - is unstable. As is well-known in the rational expectations literature, this implies that there is a unique rational expectations equilibrium at which the price level is proportional to the money supply, since all other positions imply explosive movements of real money balances through time.

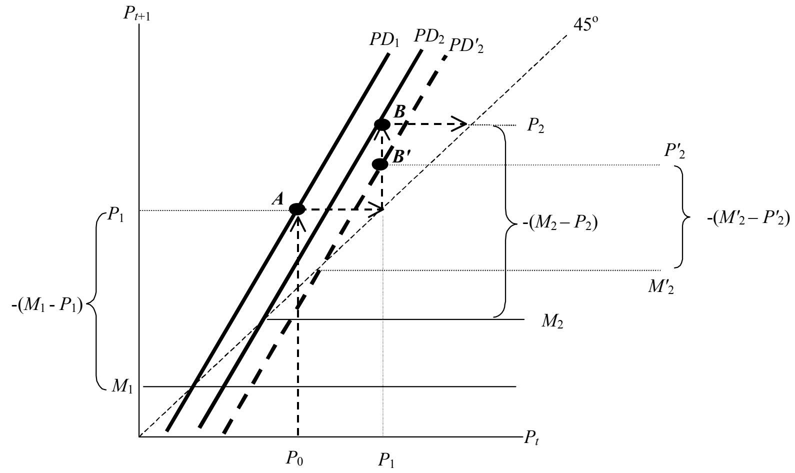

Fig. 2: Money Growth under Forward-Looking Wage Setting

In period 2 the money supply rises to M_2 and thus the price dynamics schedule shifts up to PD_2. Consequently, the economy moves to point B, with the associated real money balances of $M_2 - P_2$. Since the economy is in steady state, the growth of the money supply ($\Delta M_2 = \Delta M_1$) is constant through time, as are real money balances ($M_2 - P_2 = M_1 - P_1$) and the associated unemployment rate.

In this context, an increase in money growth means that the money supply comes closer to catching up with the actual price level. Thus real money balances fall and the unemployment rate rises. If the money supply grows faster in period 2 (to M_2' in Figure 2, where $M_2' > M_2$) so that the price dynamics schedule rises to PD_2' (instead of at PD_2), the economy moves to point B' (rather than B). Then real money balances are $M_2' - P_2'$ (instead of $M_2 - P_2$). This (comparative dynamic) fall in real money balances raises the unemployment rate.[13]

Under money growth, clearly, the critical difference between our backward- and forward-looking macro models is that in the backward-looking model, the actual price level lags behind its growing target, whereas in the forward-looking model the actual price level runs ahead of its target.[14] This has strong implications for monetary policy. In the backward-looking system, an increase in money growth causes the actual price level to fall further behind its target level, so that real money balances rise and unemployment falls. In the forward-looking system, by contrast, an increase in money growth causes the target price level to

[13]Expressing the period-1 forward-looking price equation as $P_1 = aP_2^e + (1-a)\,M_2^e$, we see that the greater is the expected period-2 money supply M_2^e, the lower is the expected period-2 price level P_2^e consistent with the current, period-1 price level P_1.

[14]Specifically, in Figure 2 the actual price level in period 1 is P_1, which exceeds the target price level $P_1^* = M_1$. Moving to period 2, the actual price level rises to P_2, which again exceeds the target price level $P_2^* = M_2$.

catch up partially with the actual price level, so that real money balances and unemployment rise.

5. Staggered Wage Contracts and the Phillips Curve

We now show that the Taylor wage contract equation is a convex combination of the backward- and forward-looking wage equations above. It turns out that if the weights on these equations are equal, then the long-run Phillips curve is vertical. But if - as the recent microfoundations of the Taylor contract equation imply - the weight on the backward-looking equation is greater than that on the forward-looking one, then the resulting long-run Phillips curve is downward-sloping.

Although our specification of staggered wage contracts is the same as Taylor's (1979, 1980a), we embed these contracts in a macro model that is quite different from his.[15] In Taylor's model, the money supply does not grow, and thus any monetary shock works its way through the staggered wage adjustment process and, once it has done so, the real variables return to their target rates. Furthermore, Taylor's wage contract equation gives equal weights to the backward- and forward-looking determinants.[16] Our model, as shown, contains a growing money supply and allows for different weights on the backward- and forward-looking determinants.

We make the standard assumptions that nominal wages are fixed for two periods and there are two contracts that are evenly staggered. The Taylor contract equation is

$$W_t = \alpha W_{t-1} + (1-\alpha) W_{t+1}^e + \gamma \left[c - \alpha u_t - (1-\alpha) u_{t+1}^e\right] + \omega_t, \tag{5.1}$$

where the contract wage W_t is set at the beginning of period t for periods t and $t+1$; $W_{t+1}^e = E_t(W_{t+1})$, α and γ are positive constants, $0 < \alpha < 1$, and ω_t is a white noise process. The variable u_t corresponds to what Taylor called "excess demand," i.e the difference between actual output ($Q_t = N_t$) and full-employment output ($Q_t^f = L$). We assume that the wage setters have knowledge of nominal wages and excess demands up to period t, and of the shock up to period $t-1$, so

[15] Specifically, the Taylor (1980, p.4-5) macro model has the following elements: (i) The contract wage depends on past and expected future contract wages, as well as current and future "excess labor demand." (All variables are in logs.) (ii) Excess labor demand is proportional to the output gap (the difference between actual and full-employment output). (iii) The output gap depends on detrended real money balances (viz., nominal money balances minus full-employment money balances minus the price level). (iv) Detrended nominal money balances (the difference between actual and full-employment money balances) are proportional to the price level.

[16] Taylor (1980a, p.5) explicitly notes that this symmetric weighting scheme does not allow for discounting effects.

that $E_t \omega_t = 0$. The discounting parameter α describes how backward- or forward-looking the contract is, the demand sensitivity parameter γ describes how strongly wages are influenced by demand, and the cost-push parameter c gives the upward pressure on wages in the absence of excess demand.

Since there are constant returns to labor, the price is a constant mark-up over the average wage:

$$P_t = \frac{1}{2}\left(W_t + W_{t-1}\right). \tag{5.2}$$

In sum, our macro model comprises the unemployment equation (2.1), the money supply equation (2.2), and the wage and price equations (5.1) and (5.2).

5.1. Interpretation

The wage contract equation (5.1) has an interesting interpretation. Substituting unemployment equation (2.1) and the price equation (5.2) into the wage contract equation (5.1) and taking expectations, we obtain:

$$\begin{aligned} W_t^e &= \alpha\left[\theta W_{t-1} + (1-\theta) M_t + (1-\theta)(c-L)\right] \\ &\quad + (1-\alpha)\left[\theta W_{t+1}^e + (1-\theta) M_{t+1}^e + (1-\theta)(c-L)\right], \end{aligned} \tag{5.3}$$

where $\theta = \frac{2-\gamma}{2+\gamma}$. Observe that this equation is a weighted average[17] of

- our backward-looking price equation (3.2) and
- our forward-looking price equation (4.2).

In this contract equation, the wage sluggishness parameter in the backward-looking term is equal to the wage anticipations parameter in the forward-looking term. Both are equal to θ, which depends on the demand sensitivity parameter γ. In what follows we assume that $0 < \gamma < 2$. Note that when $\gamma < 2$, it follows that $0 < \theta < 1$ which is in accordance with our assumption that the wage sluggishness (anticipation) parameter lies between zero and one in the backward- (forward-) looking equation of the previous section. [18]

Thus price setting under staggered contracts may be interpretted as a weighted average of the backward- and forward-looking determinants, where the relative weight is the discounting parameter α.

[17]Equation (5.3) also contains constant and labor force terms that were ignored in the backward- and forward-looking models, in order to generate the simplest possible intuitions.

[18]When $\gamma > 2$ then θ becomes negative and that would only imply that the roles of the backward- and forward- looking equations are reversed.

5.2. The Long-Run Phillips Curve

We now proceed to derive the long-run Phillips curve for our macro model. Assuming that W_t is dynamically stable, we use the money growth equation (2.2) to find the rational expectations solution of the wage contract equation (5.3):

$$W_t = (1 - \lambda_1) c + \lambda_1 W_{t-1} + (1 - \lambda_1) M_t + \kappa (1 - \lambda_1) \mu_t - (1 - \lambda_1) L + \omega_t, \quad (5.4)$$

where $\lambda_{1,2} = \frac{\phi_2 \mp \sqrt{\phi_2^2 - 4\phi_1\phi_3}}{2\phi_3}$, $\phi_1 = \alpha\left(1 - \frac{\gamma}{2}\right)$, $\phi_2 = 1 + \frac{\gamma}{2}$, $\phi_3 = (1 - \alpha)\left(1 - \frac{\gamma}{2}\right)$, and $\kappa = \frac{\lambda_2}{\lambda_2 - 1} - \alpha$. Note that λ_1 lies inside the unit circle, whereas λ_2 lies outside the unit circle (and so $\kappa > 0$).[19]

Substituting this equation into the price mark-up equation (5.2), we obtain a price dynamics equation, giving prices in terms of past prices, the money supply, and the labour force:

$$\begin{aligned} P_t = {} & (1 - \lambda_1) c + \lambda_1 P_{t-1} + (1 - \lambda_1) M_t + (1 - \lambda_1)\left(\kappa - \frac{1}{2}\right)\mu_t \\ & - (1 - \lambda_1) L - \frac{1}{2}\kappa (1 - \lambda_1) \varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned} \quad (5.5)$$

Thus real money balances are[20]

$$\begin{aligned} M_t - P_t = {} & - (1 - \lambda_1) c + \lambda_1 (M_{t-1} - P_{t-1}) + (1 - \lambda_1)\left(\frac{2\alpha - 1}{\gamma}\right)\mu_t \\ & + (1 - \lambda_1) L + \frac{1}{2}\kappa (1 - \lambda_1) \varepsilon_t - \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned} \quad (5.6)$$

Since the first difference of equation (5.5) is stable, we find that the inflation rate is equal to the money growth rate in the long run:[21]

$$\pi_t^{LR} = \mu_t^{LR}. \quad (5.7)$$

where $\pi_t = \Delta P_t$ is the inflation rate. Thus the long-run level of real money balances is:

$$M_t^{LR} - P_t^{LR} = \left(\frac{2\alpha - 1}{\gamma}\right)\mu_t^{LR} + L - c. \quad (5.8)$$

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[19] Note that the assumption of $\gamma < 2$ has the plausible implication that the autoregressive coefficient λ_1 in the wage dynamics equation (5.4) is positive.

[20] To obtain this equation, add and subtract M_t to the R.H.S. of the price dynamics eq. (5.5) and rearrange terms. Furthermore, it can be shown that $(1 - \lambda_1)\left(\frac{2\alpha-1}{\gamma}\right) = \left[\frac{1}{2}(1 + \lambda_1) - \kappa (1 - \lambda_1)\right]$.

[21] Since money growth follows a random walk, the long-run expected money growth rate (at time t) is equal to the current money growth rate (at time t), and thus the long-run expected inflation rate also bears the time subscript t.

Since aggregate demand is $Q_t = M_t - P_t$, the production function is $Q_t = N_t$, and unemployment is $u_t = L - N_t$, we obtain the long-run relationship between the unemployment rate and the growth rate of money supply:

$$u_t^{LR} = -\left(\frac{2\alpha - 1}{\gamma}\right)\mu_t^{LR} + c. \tag{5.9}$$

Finally, substituting (5.7) into (5.9), we obtain the long-run Phillips curve:

$$\pi^{LR} = -\left(\frac{\gamma}{2\alpha - 1}\right)u^{LR} + \left(\frac{\gamma}{2\alpha - 1}\right)c. \tag{5.10}$$

5.3. The Slope of the Long-Run Phillips Curve

Observe that the discounting parameter plays a crucial role in determining the long-run relation between inflation and unemployment. If $\alpha = 1/2$, the long-run Phillips curve is vertical; if $\alpha > 1/2$, it is downward-sloping; and if $\alpha < 1/2$, it is upward-sloping. The conventional assumption,[22] as noted, is that $\alpha = 1/2$. It is for this reason that the resulting New Keynesian Phillips curve becomes compatible with a unique NAIRU (or natural rate of unemployment). Some authors, however, have expressed misgivings about this assumption. For example, Blanchard and Fisher (1989) note that "even under lognormality of money and the price level (actually, even under certainty) the optimal rule is not [an equation with equal weights on the backward- and forward-looking terms]. For example, the subjective discount rate of price setters should enter the optimal rule" (p. 420, f. 27).

To derive the optimal rule, we require microfoundations of the Taylor contract equation. Ascari (2000), Ascari and Rankin (2002), Graham and Snower (2002), Helpman and Leiderman (1990), and others have provided microfoundations that explicitly consider the influence of time discounting. These studies find that $\alpha = \frac{1}{1+\delta}$, where δ is the households' discount factor ($\delta = \frac{1}{1+r}$, where r is the discount rate, equal to the real interest rate in the long-run equilibrium).[23] Thus, as long as agents discount the future ($\delta < 1$), the parameter α will always exceed 1/2, and thus the long-run Phillips curve is always downward-sloping.

The remaining question is whether this result is likely to be empirically important. It is impossible to resolve this issue on the basis of our current empirical

[22] See, for example, Blanchard and Fisher (1989), Roberts (1995), and Romer (1996).

[23] The discounting parameter in the model of Helpman and Leiderman (1990) also reduces to this when productivity is constant and there are constant returns to labor. This result is an approximation which holds exactly only when the steady state money supply is constant and the households' utility function is additively separable in real money balances. Thus our interpretation of the parameter α should be confined to sufficiently low rates of money growth.

knowledge. In particular, there is considerable disagreement about the relevant range of values for the demand sensitivity parameter γ. The estimates of Taylor (1980b) and Sachs (1980) extend from 0.05 to 0.1, but calibration of microfounded models assigns higher values. Nevertheless, it might be instructive to to note that if $\gamma = 0.05$, then the slope of the long-run Phillips curve is $s = -5.05$ when the discount factor is $r = 2\%$, and $s = -2.55$ when $r = 4\%$. Furthermore, when $\gamma = 0.1$, the slope is $s = -10.10$ when $r = 2\%$, and $s = -5.10$ when $r = 4\%$.

But although these long-run Phillips cruve are certainly far from vertical, it is important to emphasize that our model is obviously far too simplistic to yield reliable slope estimates. Our underlying theme has been that the inflation-unemployment tradeoff can be derived from the interaction between nominal frictions and money growth; but once this is accepted, it becomes clear that this tradeoff is also influenced by other types of growth (such as capital accumulation and productivity growth) and other frictions (such as inertia in employment and labor force participation). Thus a proper empirical assessment of the long-run Phillips curve requires a wider model. (See, for example, Karanassou, Sala, and Snower (2002).)

6. Concluding Thoughts

So, to summarize: (i) When the time discount rate is positive, the backward-looking determinants of wage formation have a stronger influence than the forward-looking ones. (ii) Hence an increase in money growth raises the inflation rate and reduces the unemployment rate in the long run, i.e. the long-run Phillips curve is downward-sloping.

To understand why the long-run Phillips curve is downward-sloping despite the money neutrality, it is important to focus on the interaction between money growth and price adjustment frictions. First, if there were no frictions - so that prices adjusted instantaneously to the money supply - then the absence of money illusion would ensure that any change in money growth leads to an equal change in inflation. In that event, changes in money growth have no real effects and the long-run Phillips curve is vertical. Second, in the presence of price adjustment frictions but no money growth (viz., a constant money supply), the absence of money illusion would ensure that the actual price level eventually catches up with the target price level. Thus any comparative static change in the money supply would eventually lead to a proportional comparative static change in the price level, so that - once again - there are no real effects. Third, however, when the economy is characterized by *both* money growth *and* time-contingent nominal contracts, then the absence of money illusion *no longer* guarantees a vertical Phillips curve. What the absence of money illusion guarantees, is that the target price level changes proportionately to the money supply. But changes in money growth influence how far the actual price level is from its target level. Thereby changes in money growth affect real money balances and the associated long-run unemployment rate.

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