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A note on simulating the effect of monetary policy changes using only forward curves as inputs

by Ansgar Rannenberg





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Abstract

I show that in linear rational expectation models, the effect of a monetary tightening can be simulated using contemporaneous and anticipated monetary policy shocks that replicate the forward curves observed during the period of interest, normalized with the forward curve observed in the quarter before the tightening period of interest begins. In particular, the shocks in response to which the tightening occurs are irrelevant. All required information is incorporated in the normalized forward curves. I confirm this result via simulations and a formal proof. Then I use it to assess the effects of the recent monetary tightening in the Euro Area.

Keywords: policy counterfactuals, monetary policy, interest rate expectations. JEL Codes: E52, E43.

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The views expressed in this paper are those of the author and do not necessarily reflect the views of the National Bank of Belgium, the Eurosystem, or any other institution with which the author is affiliated.

Non-technical summary

Evaluating the effect of changes in monetary policy using linearized rational expectation macroeconomic models is a staple in the work of central bank macro modelers and other researchers. This note shows that and how the effects of a monetary policy tightening (or loosening) relative to a counterfactual can be simulated using an interest rate peg informed purely by the information embedded in Overnight Index Swap (OIS) rates. For the simulation method I propose to be valid, it is necessary to assume a specific linear model (or a set of models), and to specify a beginning and end date of the period of monetary accommodation under the counterfactual. By contrast, it is irrelevant whether the tightening whose macroeconomic effects are of interest was an endogenous response to non-policy shocks via the monetary policy rule of the model, or to exogenous policy shocks.

Specifically, I show that a simulation that captures the effect of a tightening relative to a counterfactual specified in the way just described has to proceed as follows. First, one needs to define the beginning and the end of the period over which monetary policy remains accommodative under the non-tightening counterfactual policy. The non-tightening counterfactual interest rate trajectory then is the interest rate trajectory stretching from the beginning to the end of that period as expected by agents during the quarter preceding that period. Secondly, for each quarter of that period, one then normalizes the actual forward curve observed in that respective quarter (stretching from that quarter to the final quarter of the period of monetary accommodation) by subtracting the non-tightening counterfactual interest rate trajectory. Using this information, I then conduct a simulation where in each quarter, I replicate the contemporaneous and contemporaneously expected interest rate values from these normalized forward curves using an interest rate peg.

I show that this approach is valid first via simulations of a stylized New Keynesian model. In particular, I subject the model economy to a sequence of supply and demand shock innovations to generate an inflationary scenario. I perform this simulation under two alternative assumptions regarding the monetary policy rule. In the first simulation I assume that the policy rate simply follows the interest feedback rule of the model. The second simulation is the non-tightening counterfactual just described. I then compute the effect of the tightening by subtracting the results of the second simulation from those of the first. Apart from the effect of the tightening on variables of interest like output and inflation, this subtraction also yields the model counterpart of the normalized forward curves mentioned above. I then perform a separate simulation where I use an interest rate peg in order to replicate the normalized forward curves in the model. The resulting paths of inflation, output etc. are identical to the values computed based on the two simulations of the adverse supply shocks.

Following this step, I provide a formal proof of this result, first in the context of the simple New Keynesian model, and then in the context of a general quasi-linear rational expectations model. Finally, I employ the result to simulate the macroeconomic effects of the recent monetary tightening in the Euro Area, using the model of Rannenberg (2024).

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1 Introduction

Evaluating the effect of changes in monetary policy using linearized rational expectation macroeconomic models is a staple in the work of central bank macro modelers and other researchers. This note shows that and how the effects of a monetary policy tightening (or loosening) relative to a counterfactual can be simulated using an interest rate peg informed purely by the information embedded in Overnight Index Swap (OIS) rates. For the simulation method I propose to be valid, it is necessary to assume a specific linear model (or a set of models), and to specify a beginning and end date of the period of monetary accommodation under the counterfactual, i.e. the quarter after which monetary policy is again determined endogenously by the interest feedback rule of the model. By contrast, it is irrelevant whether the tightening whose macroeconomic effects are of interest was an endogenous response to non-policy shocks via the monetary policy rule of the model, or to exogenous policy shocks.

Specifically, I show that a simulation that captures the effect of a tightening relative to a counterfactual specified in the way just described has to proceed as follows. First, one needs to define the beginning and the end of the period over which monetary policy remains accomodative under the non-tightening counterfactual policy. The non-tightening counterfactual interest rate trajectory stretching from the beginning to the end of that period as expected by agents during the quarter preceding that period. Secondly, for each quarter of that period, one then normalizes the actual forward curve observed in that respective quarter (stretching from that quarter to the final quarter of the period of monetary accomodation) by subtracting the non-tightening counterfactual interest rate trajectory. Using this information, I then conduct a simulation where in each quarter, I replicate the contemporaneous and contemporaneously expected interest rate values from these normalized forward curves using an interest rate peg.

I show that this approach is valid first via simulations of a stylized New Keynesian model (see Section 2). In particular, I subject the model economy to a sequence of supply and demand shock innovations to generate an inflationary scenario. I perform this simulation under two alternative assumptions regarding the monetary policy rule. In the first simulation I assume that the policy rate simply follows the interest feedback rule of the model. The second simulation is the nontightening counterfactual just described. I then compute the effect of the tightening by subtracting the results of the second simulation from those of the first. Apart from the effect of the tightening on variables of interest like output and inflation, this subtraction also yields the model counterpart of the normalized forward curves mentioned above. I then perform a separate simulation where I use an interest rate peg in order to replicate the normalized forward curves in the model. The resulting paths of inflation, output etc. are identical to the values computed based on the two simulations of the adverse supply shocks.

Following this step, I provide a formal proof of this result, first in the context of the simple New Keynesian model, and then in the context of a general quasi-linear rational expectations model. Finally, I employ the result to simulate the macroeconomic effects of the recent monetary tightening in the Euro Area, using the model of Rannenberg (2024).

The method I propose is related in spirit to the contributions of McKay and Wolf (2023), Hebden and Winkler (2021) and De Groot et al. (2021) in that these contributions also construct policy counterfactuals in quasi linear models without relying on the knowledge of the non-policy shock processes to which policy responds, using only the response of output, inflation and the short term interest rate to contemporaneous and anticipated monetary policy shocks. Their contributions also rely on the fact that policy affects private sector behavior only via the current and expected path of the policy instruments. It is irrelevant whether this path is the result of shocks to the monetary policy rule, or the systematic component of monetary policy, or which shocks drive the evolution of the systematic component (McKay and Wolf (2023)). My contribution adds to this literature by showing that, conditional on a model and the specification of a non-tightening counterfactual period, one can simulate the effect of an empirically observed monetary policy rate in order to the normalized forward curves.

The capacity of the method proposed here to produce "sensible" results is naturally limited by the properties of the models used to apply it and the specifics of the accomodative policy counterfactual. For instance, if the specified counterfactual period and thus the interest rate peg that needs to be simulated are long, and the model is subject to the forward guidance puzzle (Del Negro et al. (2017), Carlstrom et al. (2015)), the simulated effects on output and inflation may be (very) large relative to the simulated interest rate trajectory. Furthermore, a model that is subject to the "reversal puzzle" of Carlstrom et al. (2015) for the peg that would have to be simulated, i.e. the finding as the length of the interest rate peg becomes sufficiently long, the sign of the simulated effects on output and inflation may switch, is arguably also not suitable. The model I use to simulate the effects of the recent Euro Area monetary tightening is not subject to either of these puzzles. Finally, the method

is applicable only to linearized models, as it is this quality that ensures that the differential effects of the tightening can be solved for without recourse to the paths of the non-policy variables in the baseline scenario.

In the following Section, I illustrate the main result using simulations of a simple model. Section 3 formally proofs this result in the context of a simple New Keynesian model, while Appendix A contains a proof in the context of a general linear rational expectations model.

2 Simulations

I first illustrate the result using simulations of a the simple, purely forward looking New Keynesian model:

$$\ddot{\Pi}_t = \kappa \dot{Y}_t + E_t \ddot{\Pi}_{t+1} + u_{\Pi,t} \tag{1}$$

$$\hat{Y}_{t} = E_{t}\hat{Y}_{t+1} - \left(\hat{R}_{t} - E_{t}\hat{\Pi}_{t+1}\right) + u_{Y,t}$$
⁽²⁾

$$\hat{R}_t = \phi_\Pi \hat{\Pi}_t + \phi_y \hat{Y}_t \tag{3}$$

$$u_{\Pi,t} = \rho_{u\Pi} u_{\Pi,t-1} + \eta_{\Pi,t} \tag{4}$$

$$u_{Y,t} = \rho_{uY} u_{Y,t-1} + \eta_{Y,t} \tag{5}$$

where $u_{\Pi,t}$ and $u_{Y,t}$ represent exogenous cost-push and demand shocks with innovations $\eta_{\Pi,t}$ and $\eta_{Y,t}$, respectively. For the calibration, I assume $\kappa = 0.05$, $\phi_{\Pi} = 1.5$, $\phi_y = \frac{0.5}{4}$ and $\rho_{u\Pi} = \rho_{uY} = 0.95$. In a first step, I simulate the response of the model to inflationary cost-push and demand shocks. I assume that over the course of 5 quarters, each quarter a new unexpected shock innovation arrives. I refer to the quarter just before the first shock arrives as quarter zero. All simulations in this paper are conducted using Dynare 6.1 (Adjemian et al. (2024)). As can be obtained from Figure 1 (solid line labeled "Baseline"), the shocks increase inflation which causes a strong rise of the policy interest rate and a strong decline in GDP. This tightening is reflected both in the increase in the policy interest rate and, importantly, in the successive upward shifts of the forward curves (see the lower right panel).

My goal is to compute the effect of the monetary tightening compared to a counterfactual where monetary policy does not immediately respond to the paths of output and inflation triggered by the supply and demand shocks. Instead, during the quarters 1 to 3, it pegs the interest rate at the path expected in quarter 0, before the arrival of the shocks in response to which monetary policy tightens. This path is denoted as \hat{R}_t^{acc} . Agents fully anticipate the length of the interest rate peg under monetary accomodation. For simplicity, I assume that in period 0, the economy is in the steady-state of, implying that $\hat{R}_t^{acc} = 0$. As will be shown in the next section, this simplifying assumption is in fact not necessary. Formally, the accomodative policy is described by

$$\hat{R}_{t} = \begin{cases} 0 & \text{for } t = 1, 2, 3\\ \phi_{\Pi} \hat{\Pi}_{t} + \phi_{y} \hat{Y}_{t} & \text{for } t > 3 \end{cases}$$
(6)

As can be obtained from Figure 1 (black dotted line), for the duration of the peg, the interest rate peg implies higher output and inflation than under the baseline policy. Starting from period 4, the paths of output and inflation are identical in both scenarios, due to the fact that the model is purely forward looking, i.e. there are no lags of endogenous variables. However, the results hold also under less restrictive assumptions, as shown in the following Section.

I define the effect of the tightening as the difference between the value of a variable without monetary accomodation X_t^{base} and its value under monetary accomodation X_t^{acc} as $\Delta X_t \equiv X_t^{base} - X_t^{acc}$. ΔX_t is displayed in Figure 2 as the black solid line labeled "True effect". The lower right panel of Figure 2 displays the result of this computation for the forward curves observed during the first three quarters. Formally, these are the $E_k \Delta \hat{R}_j$ for each period j for which $k \leq j \leq 3$ and each k = 1, 2, 3. I refer to the $E_k \Delta \hat{R}_1, E_k \Delta \hat{R}_2, E_k \Delta \hat{R}_3$ sequence as the normalized forward curve of quarter k.

I then conduct two additional simulations featuring only monetary policy shocks and no nonpolicy shocks. In the first simulation simulation, labeled staggered foresight, I peg the interest rate such that in the first three quarters, the simulation replicates the respective normalized forward curve. Formally, in each quarter k = 1 - 3, I peg $E_k \hat{R}_j = E_k \Delta \hat{R}_j$, for each period j for which $k \leq j \leq 3.^1$ For each quarter, the lower right panel of Figure 2 displays the respective expected interest rate trajectory induced by this peg as the solid lines.² For j > 3, monetary policy follows the interest feedback rule of the model $\hat{R}_t = \phi_{\Pi} \hat{\Pi}_t + \phi_y \hat{Y}_t$, and that is fully anticipated.

The results of this simulation, displayed as the red dotted diamond line, exactly replicate the effect of the tightening computed from the results of the two simulations of the adverse supply shocks (the black solid line).

In the second simulation, labeled "Simulation as unanticipated monetary policy shocks", I also replicate the path of $\Delta \hat{R}_k$ for k = 1 - 3, but not the normalized forward curve observed at each point in time. Specifically, in each quarter k agents assume that beginning in k + 1, the interest rate will follow the model's interest feedback rule, which in this model means that it returns to the steady-state. As can be obtained from Figure 2, this simulation strongly understates the true effect of the tightening. The lower right panel of Figure 2 suggests how this result arises: While the simulation replicates the respective contemporaneous $\Delta \hat{R}_k$ in each quarter k, the respective expected interest rate trajectory for the following quarters is systematically below the normalized forward curve.

Hence, without knowing the shocks that caused the tightening, the effect of the monetary tightening compared to a monetary accomodation counterfactual where the interest rate remains constant for 3 quarters can be computed by feeding in each quarter the respective deviation of the forward curve from the non-tightening counterfactual into the model. Recall that the forward curve under the non-tightening counterfactual is the interest rate trajectory expected in quarter 0 (just before the tightening starts) for the subsequent 3 quarters. Hence $E_k \Delta \hat{R}_j$ can be easily estimated from financial market data as $E_k \Delta \hat{R}_j = E_k \tilde{R}_j - E_0 \tilde{R}_j$, for each each j for which $k \leq j \leq 3$, and for each quarter k = 1, 2, 3. The $E_0 \tilde{R}_j$ sequence denotes the quarter 0 forward curve derived from quarter 0 OIS rates. The $E_k \tilde{R}_j$ sequences denote the forward curves observed in each quarter k = 1, 2, 3.

In the following section I provide a formal proof in the context of a simple New Keynesian model, while a proof for the case of a more general linear RE model is relegated to Appendix A.

¹Obviously, for j = k, $E_k \hat{R}_j$ is simply the actual interest rate in quarter k and thus the expectation operator is redundant in that particular case.

²For instance, the blue solid line displays $E_1 \Delta \hat{R}_1$, $E_1 \Delta \hat{R}_2$ and $E_1 \Delta \hat{R}_3$



Figure 1: Effect of inflationary shocks under the baseline and with monetary accomodation over 3 quarters

Note: This graph displays the results of simulations under two alternative assumptions about monetary policy. Over the course of 5 quarters, each quarter a new unexpected shock innovation arrives. The resulting paths of the shocks are displayed in the lower left panel. The first three panels display the effect of a sequence of inflationary demand and cost push shocks on inflation, output and the policy interest rate. The black solid line represents the baseline scenario, where monetary policy always follows the interest feedback rule of the model. The black dotted line displays the results for the monetary accomodation scenario, see equation (6). The lower right panel displays the forward curves observed in quarters 1-3, including the respective contemporaneous interest rate. I implement the interest rate peg under the accomodative policy using Dynare's "Perfect foresight with expectation errors" solver. Specifically, I replace equation 3 with $\hat{R}_t =$ $(1 - DUM_t) \left(\phi_{\Pi} \hat{\Pi}_t + \phi_y \hat{Y}_t\right) + DUM_t \hat{R}_t^{peg}$. DUM_t denotes an exogenous dummy variable that equals 1 during the first three quarters, and is expected to do so by agents. It's path is implemented in Dynare using the standard "shocks" block.



Note: The black solid line labeled "True effect" in the first row of the figure displays the effect of the tightening calculated as $\Delta X_t \equiv X_t^{base} - X_t^{acc}$. X_t^{base} denotes the response of the respective variables to the sequence of adverse supply shocks in the baseline scenario, as displayed in Figure 1 as the black solid lines. X_t^{acc} denotes the response with monetary accomodation, displayed in Figure 1 as the black dotted line. The red diamond line labeled "Staggered foresight simulation" displays the results of a simulation where in each quarter k = 1 - 3, I peg $E_k \hat{R}_j = E_k \Delta \hat{R}_j$, for each period j for which $k \leq j \leq 3$, as displayed in the lower right panel. The "Simulation as unanticipated monetary policy shocks" (red solid line) replicates the path of $\Delta \hat{R}_k$ for k = 1 - 3, but assumes that in each quarter k, agents expect that beginning in k + 1, the interest rate follows the model's interest feedback rule.

I implement the staggered foresight simulation using Dynare's "Perfect foresight with expectation errors" solver. Specifically, I replace equation (3) with $\hat{R}_t = (1 - DUM_t) \left(\phi_{\Pi} \hat{\Pi}_t + \phi_y \hat{Y}_t \right) + DUM_t \hat{R}_t^{peg}$. DUM_t denotes an exogenous dummy variable that equals 1 during the first three quarters, and is expected to do so by agents. It's path is implemented in Dynare using the standard "shocks" block. By contrast, \hat{R}_t^{peg} is an exogenous variable whose value can be different from what agents expect in advance. Specifically, in quarter one, agents expect that $\hat{R}_1^{peg} = \Delta \hat{R}_1, \hat{R}_2^{peg} = E_1 \Delta \hat{R}_2, \hat{R}_3^{peg} = E_1 \Delta \hat{R}_3$. Once quarter two arrives, the expected \hat{R}_t^{peg} trajectory is updated, and we have $\hat{R}_2^{peg} = \Delta \hat{R}_2, \hat{R}_3^{peg} = E_2 \Delta \hat{R}_3$. In quarter three, we have $\hat{R}_3^{peg} = \Delta \hat{R}_3$. The paths of \hat{R}_t^{peg} expected in each quarter are implemented using three "shocks(learnt in)" blocks.

3 Proof in a simple New Keynesian model

I now proof the above result formally in the context of a still simple but more general New Keynesian model, now featuring lags of the endogenous variables. The model is given by:

$$\hat{\Pi}_t = \kappa \hat{Y}_t + b_{\Pi} E_t \hat{\Pi}_{t+1} + (1 - b_{\Pi}) \hat{\Pi}_{t-1} + u_{\Pi,t}$$
(7)

$$\hat{Y}_{t} = b_{Y} E_{t} \hat{Y}_{t+1} + (1 - b_{Y}) \hat{Y}_{t-1} - \frac{1}{\sigma} \left(\hat{R}_{t} - E_{t} \hat{\Pi}_{t+1} \right) + u_{Y,t}$$
(8)

$$\hat{R}_t = (1 - DUM_t) \left((1 - \rho_R) \left(\phi_\Pi \hat{\Pi}_t + \phi_y \hat{Y}_t \right) + \rho_R \hat{R}_{t-1} \right) + DUM_t \hat{R}_t^{peg}$$
(9)

$$u_{\Pi,t} = \rho_{u\Pi} u_{\Pi,t-1} + \eta_{\Pi,t} \tag{10}$$

$$u_{Y,t} = \rho_{uY} u_{Y,t-1} + \eta_{Y,t} \tag{11}$$

Over period 1 to T, this economy is subject to unexpected shock innovations $\eta_{.,t}$. I want to derive the difference in the trajectory of output and inflation between a scenario where monetary policy tightens in response to these shocks and an alternative accomodative scenario where monetary policy lets the interest rate follow the trajectory of \hat{R}_t^{acc} over quarters 1 to T. \hat{R}_t^{acc} denotes the interest rate trajectory that was expected in period zero for the subsequent T quarters. Without monetary accomodation, the interest rate either adjusts endogenously $(DUM_t = 0)$ or is pegged at a to be determined rate $\hat{R}_t^{peg}(DUM_t = 1)$.

If $DUM_t = 0$ and is expected to remain at this value for ever, and assuming the Blanchard and Kahn (1980) conditions are met, the minimum state variable solution (MSV) of this model can be found and has the form:

$$\hat{\Pi}_{t} = a_{\Pi\Pi}\hat{\Pi}_{t-1} + a_{Y\Pi}\hat{Y}_{t-1} + a_{R\Pi}\hat{R}_{t-1} + a_{u_{\Pi}\Pi}u_{\Pi,t} + a_{u_{Y}\Pi}u_{Y,t}$$
(12)
$$\hat{Y}_{t} = \hat{\Pi}_{t}\hat{\Pi}_{t-1} + \hat{U}_{t-1}\hat{Y}_{t-1} + \hat{U}_{t-1}\hat{Y$$

$$\hat{Y}_{t} = a_{\Pi Y}\hat{\Pi}_{t-1} + a_{YY}\hat{Y}_{t-1} + a_{RY}\hat{R}_{t-1} + a_{u_{\Pi}Y}u_{\Pi,t} + a_{u_{Y}Y}u_{\Pi,t}$$
(13)

$$\hat{R}_t = a_{\Pi R} \hat{\Pi}_{t-1} + a_{YR} \hat{Y}_{t-1} + a_{RR} \hat{R}_{t-1} + a_{u_{\Pi}R} u_{\Pi,t} + a_{u_YR} u_{\Pi,t}$$
(14)

where the $a_{..}$ coefficients are non-linear functions of the deep model parameters. Note for the special case of the simplest, purely forward looking New Keynesian model ($b_{\Pi} = b_Y = 1$, $\rho = 0$), all coefficients except those on the exogenous drivers $u_{\Pi,t}$ and $u_{Y,t}$ are zero.

Equations (7), (8), (10) and (11) hold both with and without monetary accomodation, while (9) becomes $\hat{R}_t = \hat{R}_t^{acc}$ in the monetary accomodation scenario. Hence subtracting the system under monetary accomodation from the system without yields

$$\Delta \hat{\Pi}_t = \kappa \Delta \hat{Y}_t + b_\Pi E_t \Delta \hat{\Pi}_{t+1} + (1 - b_\Pi) \Delta \hat{\Pi}_{t-1}$$
(15)

$$\Delta \hat{Y}_t = b_Y E_t \Delta \hat{Y}_{t+1} + (1 - b_Y) \Delta \hat{Y}_{t-1} - \frac{1}{\sigma} \left(\Delta \hat{R}_t - E_t \Delta \hat{\Pi}_{t+1} \right)$$
(16)

$$\Delta \hat{R}_t = (1 - DUM_t) \left((1 - \rho_R) \left(\phi_{\Pi} \hat{\Pi}_t^{base} + \phi_y \hat{Y}_t^{base} \right) + \rho_R \hat{R}_{t-1}^{base} \right) + DUM_t \hat{R}_t^{peg} - \hat{R}_t^{acc}$$
(17)

where the superscript *base* indicates the path of the respective variable under the baseline scenario. Note that the exogenous driving processes are canceled out by this operation. Furthermore, using $\hat{R}_t^{acc} = DUM_t \hat{R}_t^{acc} + (1 - DUM_t) \hat{R}_t^{acc}$, the policy rule can be rewritten as

$$\Delta \hat{R}_t = (1 - DUM_t) \left((1 - \rho_R) \left(\phi_{\Pi} \hat{\Pi}_t^{base} + \phi_y \hat{Y}_t^{base} \right) + \rho_R \hat{R}_{t-1}^{base} - \hat{R}_t^{acc} \right) + DUM_t \left(\hat{R}_t^{peg} - \hat{R}_t^{acc} \right)$$

To show how the model can be solved without any reference to the monetary policy rule if the path of interest rate expectations given, consider the first period after the end of monetary accomodation, T + 1. Both with and without monetary accomodation, from period T+1 onwards the variables are determined by (12)-(14). Hence for period T+1, we can subtract the system under monetary accomodation in the same way:

$$\Delta \hat{\Pi}_{T+1} = a_{\Pi\Pi} \Delta \hat{\Pi}_T + a_{Y\Pi} \Delta \hat{Y}_T + a_{R\Pi} \Delta \hat{R}_T \tag{18}$$

$$\Delta \hat{Y}_{T+1} = a_{\Pi Y} \Delta \hat{\Pi}_T + a_{YY} \Delta \hat{Y}_T + a_{RY} \Delta \hat{R}_T \tag{19}$$

$$\Delta \hat{R}_{T+1} = a_{\Pi R} \Delta \hat{\Pi}_T + a_{YR} \Delta \hat{Y}_T + a_{RR} \Delta \hat{R}_T \tag{20}$$

Note how the exogenous driving processes again cancel out. Furthermore, without endogenous persistence, $\Delta \hat{\Pi}_{T+1} = \Delta \hat{Y}_{T+1} = \Delta \hat{R}_{T+1} = 0.$

Consider next the equations (15) and (16) for t = 1. Since in period 0, the values of the variables with and without monetary accomodation in period 1 to T are still identical, we have $\Delta \hat{Y}_0 = \Delta \hat{\Pi}_0 = 0$. Hence

$$\Delta \hat{\Pi}_1 = \kappa \Delta \hat{Y}_1 + b_{\Pi} E_t \Delta \hat{\Pi}_2$$
$$\Delta \hat{Y}_1 = b_Y E_t \Delta \hat{Y}_2 - \frac{1}{\sigma} \left(\Delta \hat{R}_1 - E_t \Delta \hat{\Pi}_2 \right)$$

Next I stack the model equations over periods T to 1. Furthermore, I take expectations as of period 1. Then, for periods T+1 and T, there are 6 equations

$$E_1 \Delta \hat{\Pi}_{T+1} = a_{\Pi\Pi} E_1 \Delta \hat{\Pi}_T + a_{Y\Pi} \Delta E_1 \hat{Y}_T + a_{R\Pi} E_1 \Delta \hat{R}_T$$
(21)

$$E_1 \Delta \hat{Y}_{T+1} = a_{\Pi Y} E_1 \Delta \hat{\Pi}_T + a_{YY} E_1 \Delta \hat{Y}_T + a_{RY} E_1 \Delta \hat{R}_T$$

$$\tag{22}$$

$$E_1 \Delta \hat{R}_{T+1} = a_{\Pi R} E_1 \Delta \hat{\Pi}_T + a_{YR} E_1 \Delta \hat{Y}_T + a_{RR} E_1 \Delta \hat{R}_T \tag{23}$$

$$E_1 \Delta \hat{\Pi}_T = \kappa E_1 \Delta \hat{Y}_T + b_\Pi E_1 \Delta \hat{\Pi}_{T+1} + (1 - b_\Pi) E_1 \Delta \hat{\Pi}_{T-1}$$
(24)

$$E_{1}\Delta\hat{Y}_{T} = b_{Y}E_{1}\Delta\hat{Y}_{T+1} + (1-b_{Y})E_{1}\Delta\hat{Y}_{T-1} - \frac{1}{\sigma}\left(E_{1}\Delta\hat{R}_{T} - E_{1}\Delta\hat{\Pi}_{T+1}\right)$$
(25)

$$E_{1}\Delta\hat{R}_{T} = (1 - DUM_{T})\left((1 - \rho_{R})\left(\phi_{\Pi}E_{1}\hat{\Pi}_{T}^{base} + \phi_{y}E_{1}\hat{Y}_{T}^{base}\right) + \rho_{R}E_{1}\hat{R}_{T-1}^{base} - \hat{R}_{T}^{acc}\right) + DUM_{T}\left(E_{1}\hat{R}_{T}^{peg} - \hat{R}_{T}^{acc}\right)$$
(26)

For periods 1 < t < T, there are 3 X (T-2) equations:

$$E_{1}\Delta\hat{\Pi}_{t} = \kappa E_{1}\Delta\hat{Y}_{t} + b_{\Pi}E_{1}\Delta\hat{\Pi}_{t+1} + (1 - b_{\Pi})E_{1}\Delta\hat{\Pi}_{t-1}$$
(27)

$$E_{1}\Delta\hat{Y}_{t} = b_{Y}E_{1}\Delta\hat{Y}_{t+1} + (1 - b_{Y})E_{1}\Delta\hat{Y}_{t-1} - \frac{1}{\sigma}\left(E_{1}\Delta\hat{R}_{t} - E_{t}\Delta\hat{\Pi}_{t+1}\right)$$
(28)

$$E_{1}\Delta\hat{R}_{t} = (1 - DUM_{t})\left((1 - \rho_{R})\left(\phi_{\Pi}E_{1}\hat{\Pi}_{t}^{base} + \phi_{y}E_{1}\hat{Y}_{t}^{base}\right) + \rho_{R}E_{1}\hat{R}_{t-1}^{base} - \hat{R}_{t}^{acc}\right) + DUM_{t}\left(E_{1}\hat{R}_{t}^{peg} - \hat{R}_{t}^{acc}\right)$$
(29)

• • • •

And for period 1 there are 3 equations:

$$\Delta \hat{\Pi}_1 = \kappa \Delta \hat{Y}_1 + b_{\Pi} E_t \Delta \hat{\Pi}_2 \tag{30}$$

$$\Delta \hat{Y}_1 = b_Y E_t \Delta \hat{Y}_2 - \frac{1}{\sigma} \left(\Delta \hat{R}_1 - E_t \Delta \hat{\Pi}_2 \right) \tag{31}$$

$$\Delta \hat{R}_{1} = (1 - DUM_{1}) \left((1 - \rho_{R}) \left(\phi_{\Pi} \hat{\Pi}_{1}^{base} + \phi_{y} \hat{Y}_{1}^{base} \right) + \rho_{R} \hat{R}_{0}^{base} - \hat{R}_{1}^{acc} \right) + DUM_{1} \left(\hat{R}_{1}^{peg} - \hat{R}_{1}^{acc} \right)$$
(32)

Hence in total there are $(T+1) \times 3$ equations. If we consider each variable-period combination as an unknown, for the case with endogenous tightening (i.e. $DUM_1 = \dots = DUM_t = DUM_T = 0$), there are in total (T+1) X 3+T X 2 unknowns. (T+1) X 3 unknowns arise from the presence of $E_1 \Delta \hat{\Pi}_t, E_1 \Delta \hat{Y}_t$ and $E_1 \Delta \hat{R}_t$ over T+1 periods, and the additional TX2 unknowns originate from the presence of the expected *level* of output and inflation $E_1 \hat{Y}_t^{base}$ and $E_1 \hat{\Pi}_t^{base}$ (as opposed to the deviation Δ from the accomodation scenario) in the monetary policy rule over periods 1 to T. To obtain the paths of $E_1 \hat{Y}_t^{base}$ and $E_1 \hat{\Pi}_t^{base}$, one would have to simulate the model (under the baseline scenario and endogenous monetary policy), which in turn would require knowledge of the expected paths of the exogenous drivers $u_{\Pi,t}$ and $u_{Y,t}$, or one would have to obtain them from another source. However, note that $E_1 \hat{Y}_t^{base}$ and $E_1 \hat{\Pi}_t^{base}$ appear only in the monetary policy rule, and thus affect the values of $E_1 \Delta \hat{Y}_t$ and $E_1 \Delta \hat{\Pi}_t$ only via their effect on $E_1 \Delta \hat{R}_t$. Hence, if we could obtain the values of $E_1 \Delta \hat{R}_t$ from an alternative source, and thus disregard the monetary policy rule during the first t periods, we could solve for the $E_1 \Delta \hat{Y}_t$ and $E_1 \Delta \hat{\Pi}_t$ sequences. In that case we would have (T+1)X2 equations and unknowns. In particular, it is irrelevant whether this path is the result of shocks to the monetary policy rule, or the systematic component of monetary policy, or which shocks drive the evolution of the systematic component (McKay and Wolf (2023)). Knowledge of the particular dynamics of the exogenous drivers $u_{\Pi,t}$ and $u_{Y,t}$ underlying the dynamics of $E_1 \hat{Y}_t^{base}$ and $E_1 \hat{\Pi}_t^{base}$ becomes unnecessary. All the information the model requires are incorporated in the T values of $E_1 \Delta \hat{R}_t$.

As mentioned above, $E_1 \Delta \hat{R}_t$ can be estimated from financial market (OIS) data, and I denoted that estimate as $E_1 \Delta \tilde{\hat{R}}_t$. Accordingly, values of $\Delta \hat{Y}_1$ and $\Delta \hat{\Pi}_1$ can then be simulated by setting $DUM_1 = \dots = DUM_t = DUM_T = 1$ (thus removing $E_1 \hat{Y}_t^{base}$ and $E_1 \hat{\Pi}_t^{base}$ from the system) and setting $E_1 \hat{R}_t^{peg}$ such that $E_1 \hat{R}_t^{peg} - \hat{R}_t^{acc}$ equals the estimates of $\Delta E_1 \tilde{\hat{R}}_1, \Delta E_1 \tilde{\hat{R}}_2, \dots, \Delta E_1 \tilde{\hat{R}}_T$ obtained from the data. To compute $\Delta \hat{Y}_2$ and $\Delta \hat{\Pi}_2$, that simulation can be repeated using $\Delta E_1 \tilde{\hat{R}}_2, \Delta E_1 \tilde{\hat{R}}_3, \dots, \Delta E_1 \tilde{\hat{R}}_T$, and so on and so forth.



4 An application to the recent monetary tightening in the Euro Area

In this Section, I use the result developed above in order to assess the macroeconomic effects of the recent monetary tightening in the Euro Area. I simulate the effects during the 2022Q1-2023Q4 period, as the euro short-term rate (&STR) peaks in 2023Q4.³ I specify the beginning and end date of the period of monetary accommodation under the counterfactual as 2022Q1 and 2025Q5, respectively. The interest rate trajectory under counterfactual scenario is correspondingly the one expected in 2021Q4 for the 2022Q1 to 2025Q5 period. The simulation replicates 8 normalized forward curves, obtained from &STR OIS rates, which are reported in the lower right panel of Figure 3.

For the simulation I use the Euro Area model of Rannenberg (2024). The model is particularly suitable for a simulation of an interest rate peg over a larger number of quarters since it is subject to neither to the forward guidance puzzle nor to the "reversal puzzle" highlighted by Carlstrom et al. (2015), due to the presence of Preferences Over Safe Assets (POSA). Furthermore, Rannenberg (2024) includes OIS rates over horizons of 1 to 12 quarters as measures of interest rate expectations in the estimation of the model. As can be obtained from Figure 3, the model predicts that by 2023Q4 the monetary tightening reduced output by almost 11% relative to the accompatible to the simulation, a bit more than half of the output contraction was driven by investment, while the remainder is

³The \mathfrak{CSTR} is the overnight at which Euro Area banks borrow and lend from each other.

driven by consumption.

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A General proof in a linear rational expectations model with n variables

Consider a linear rational expectation model with n endogenous, non-policy variables variables $Y_{i,t}$, and n non-policy equations:

$$0 = \sum_{i=1}^{n} \left(a_{1,i,-1}Y_{i,t-1} + a_{1,i,0}Y_{i,t} + a_{1,i,1}E_{t}Y_{i,t+1} \right) + \sum_{k=1}^{n_{shocks}} \left(a_{1,k}^{u}u_{k,t} \right) + a_{1,0}^{R}\hat{R}_{t} + a_{1,-1}\hat{R}_{t-1}$$
...
$$0 = \sum_{i=1}^{n} \left(a_{j,i,-1}Y_{i,t-1} + a_{j,i,0}Y_{i,t} + a_{j,i,1}E_{t}Y_{i,t+1} \right) + \sum_{k=1}^{n_{shocks}} \left(a_{j,k}^{u}u_{k,t} \right) + a_{j,0}^{R}\hat{R}_{t} + a_{j,-1}\hat{R}_{t-1}$$
...
$$0 = \sum_{i=1}^{n} \left(a_{n,i,-1}Y_{i,t-1} + a_{n,i,0}Y_{i,t} + a_{n,i,1}E_{t}Y_{i,t+1} \right) + \sum_{k=1}^{n_{shocks}} \left(a_{n,k}^{u}u_{k,t} \right) + a_{n,0}^{R}\hat{R}_{t} + a_{n,-1}\hat{R}_{t-1}$$

There are n_{shocks} endogenous shocks $u_{k,t}$ following AR(1) or ARMA(1,1) processes. Furthermore, there is a policy instrument \hat{R}_t , which in the absence of an interest rate peg is determined by a policy rule featuring the endogenous non-policy variables and its own lag:

$$\hat{R}_{t} = (1 - DUM_{t}) \left(\sum_{i=1}^{n} (p_{i,-1}Y_{i,t-1} + p_{i,0}Y_{i,t} + p_{i,1}E_{t}Y_{i,t+1}) + \rho_{R}\hat{R}_{t-1} \right) + DUM_{t}\hat{R}_{t}^{peg}$$

By setting $DUM_t = 1$, the effect of the non-policy variables on \hat{R}_t can be switched off in favor of pegging \hat{R}_t to a value \hat{R}_t^{peg} , to be determined below.

Assume that over period 1 to T, the model is subject to unexpected innovations to the exogenous drivers $u_{k,t}$. I want to compute the difference in the trajectory of $Y_{i,t}$ between a scenario where monetary policy tightens in response to these shocks and an alternative accomodative scenario where monetary policy lets the interest follow the trajectory of \hat{R}_t^{acc} over periods 1 to T. \hat{R}_t^{acc} denotes the interest rate trajectory that was expected in period zero, before the arrival of the shocks. In the baseline scenario, the interest rate either adjusts endogenously $(DUM_t = 0)$ or is pegged at $\hat{R}_t^{peg}(DUM_t = 0)$.

If $DUM_t = 0$ and is expected to remain at this value for ever after, the minimum state variable solution of this model can be found and is given by the following n+1 equations :

$$\begin{split} Y_{1,t} &= \sum_{i=1}^{n} b_{1,i} Y_{i,t-1} + \sum_{k=1}^{n_{shocks}} \left(b_{1,k}^{u} u_{k,t-1} \right) + b_{1}^{R} \hat{R}_{t-1} \\ & \dots \\ Y_{j,t} &= \sum_{i=1}^{n} b_{j,i} Y_{i,t-1} + \sum_{k=1}^{n_{shocks}} \left(b_{j,k}^{u} u_{k,t-1} \right) + b_{j}^{R} \hat{R}_{t-1} \\ & \dots \\ Y_{n,t} &= \sum_{i=1}^{n} b_{n,i} Y_{i,t-1} + \sum_{k=1}^{n_{shocks}} \left(b_{n,k}^{u} u_{k,t-1} \right) + b_{n}^{R} \hat{R}_{t-1} \\ & \hat{R}_{t} &= \sum_{i=1}^{n} b_{R,i} Y_{i,t-1} + \sum_{k=1}^{n_{shocks}} \left(b_{R,k}^{u} u_{k,t-1} \right) + b_{R}^{R} \hat{R}_{t-1} \end{split}$$

with the $b_{j,i}$, $b_{R,i}$, b_j^R and b_R^R coefficients can be found using the method of undetermined coefficients if the Blanchard and Kahn (1980) conditions are satisfied.

The non-policy equations hold both in the baseline scenario and with monetary accomodation, and the values of the exogenous driver $u_{k,t}$ are identical as well, while the policy rule (9) becomes $\hat{R}_t = \hat{R}_t^{acc}$ in the monetary accomodation scenario. Hence substracting the system under monetary accomodation from the system without yields:

$$\begin{aligned} 0 &= \sum_{i=1}^{n} \left(a_{1,i,-1} \Delta Y_{i,t-1} + a_{1,i,0} \Delta Y_{i,t} + a_{1,i,1} E_{t} \Delta Y_{i,t+1} \right) + a_{1,0}^{R} \Delta \hat{R}_{t} + a_{1,-1}^{R} \Delta \hat{R}_{t-1} \\ & \dots \\ 0 &= \sum_{i=1}^{n} \left(a_{j,i,-1} \Delta Y_{i,t-1} + a_{j,i,0} \Delta Y_{i,t} + a_{j,i,1} E_{t} \Delta Y_{i,t+1} \right) + a_{j,0}^{R} \Delta \hat{R}_{t} + a_{j,-1}^{R} \Delta \hat{R}_{t-1} \\ & \dots \\ 0 &= \sum_{i=1}^{n} \left(a_{n_{nvar},i,-1} \Delta Y_{i,t-1} + a_{n,i,0} Y_{i,t} + a_{n,i,1} E_{t} \Delta Y_{i,t+1} \right) + a_{n,0}^{R} \Delta \hat{R}_{t} + a_{j,-1} \Delta \hat{R}_{t-1} \\ & \Delta \hat{R}_{t} &= \left(1 - DUM_{t} \right) \left(\sum_{i=1}^{n} \left(p_{i,-1} Y_{i,t-1}^{base} + p_{i,0} Y_{i,t}^{base} + p_{i,1} E_{t} Y_{i,t+1}^{base} \right) + \rho_{R} \hat{R}_{t-1}^{base} - \hat{R}_{t}^{acc} \right) + DUM_{t} \left(\hat{R}_{t}^{peg} - \hat{R}_{t}^{acc} \right) \end{aligned}$$

Note that the exogenous driving processes $u_{k,t}$ are canceled out by this operation.

Now consider the first period after the end of monetary accomodation, T + 1. Both with and without monetary accomodation, from period T+1 onwards the path of the variables is given by the Minimum State Variable solution. Hence for period T+1, we can make the same subtraction for those equations:

$$E_T \Delta Y_{1,T+1} = \sum_{i=1}^n b_{1,i} \Delta Y_{i,T} + b_1^R \Delta \hat{R}_T$$
....
$$E_T \Delta Y_{j,T+1} = \sum_{i=1}^n b_{j,i} \Delta Y_{i,T} + b_j^R \Delta \hat{R}_T$$
....
$$E_T \Delta Y_{n,T+1} = \sum_{i=1}^n b_{n,i} \Delta Y_{i,T} + b_n^R \Delta \hat{R}_T$$

$$E_T \Delta \hat{R}_{T+1} = \sum_{i=1}^n b_{R,i} \Delta Y_{i,T} + b_R^R \Delta \hat{R}_T$$

Note that the exogenous driving processes again cancel out.

Consider next the non-policy equations for the case of t = 1. Since in period 0, the values of the variables with and without monetary accomodation in period 1 to T are still identical, we have

 $\Delta Y_{i,0}=0$. Hence

$$0 = \sum_{i=1}^{n} (a_{1,i,0} \Delta Y_{i,1} + a_{1,i,1} E_1 \Delta Y_{i,2}) + a_{1,0}^R \Delta \hat{R}_1$$

...
$$0 = \sum_{i=1}^{n} (a_{j,i,0} \Delta Y_{i,1} + a_{j,i,1} E_1 \Delta Y_{i,2}) + a_{j,0}^R \Delta \hat{R}_1$$

...
$$0 = \sum_{i=1}^{n} (a_{n,i,0} Y_{i,1} + a_{n,i,1} E_1 \Delta Y_{i,2}) + a_{n,0}^R \Delta \hat{R}_t + a_{j,-1} \Delta \hat{R}_{t-1}$$

Next I stack the model equations over periods T to 1. Furthermore, we take expectations as of period 1. Then, for periods T+1 and T, we have 2 X (n+1) equations:

$$\begin{split} E_{1}\Delta Y_{1,T+1} &= \sum_{i=1}^{n} b_{1,i}\Delta E_{1}Y_{i,T} + b_{1}^{R}\Delta E_{1}\hat{R}_{T} \\ & \dots \\ E_{1}\Delta Y_{j,T+1} &= \sum_{i=1}^{n} b_{j,i}\Delta E_{1}Y_{i,T} + b_{j}^{R}\Delta E_{1}\hat{R}_{T} \\ & \dots \\ E_{1}\Delta Y_{n,T+1} &= \sum_{i=1}^{n} b_{n,i}\Delta E_{1}Y_{i,T} + b_{n}^{R}\Delta E_{1}\hat{R}_{T} \\ E_{1}\Delta \hat{R}_{T+1} &= \sum_{i=1}^{n} b_{n,i}\Delta E_{1}Y_{i,T} + b_{n}^{R}\Delta E_{1}\hat{R}_{T} \\ & E_{1}\Delta \hat{R}_{T+1} &= \sum_{i=1}^{n} b_{R,i}\Delta E_{1}Y_{i,T} + b_{n}^{R}\Delta E_{1}\hat{R}_{T} \\ 0 &= \sum_{i=1}^{n} (a_{1,i,-1}\Delta E_{1}Y_{i,T-1} + a_{1,i,0}\Delta E_{1}Y_{i,T} + a_{1,i,1}\Delta E_{T}Y_{i,T+1}) + a_{1,0}^{R}\Delta \hat{R}_{T} + a_{1,-1}^{R}E_{1}\Delta \hat{R}_{T-1} \\ & \dots \\ 0 &= \sum_{i=1}^{n} (a_{j,i,-1}\Delta E_{1}Y_{i,T-1} + a_{j,i,0}\Delta E_{1}Y_{i,T} + a_{j,i,1}\Delta E_{1}Y_{i,T+1}) + a_{j,0}^{R}\Delta \hat{R}_{T} + a_{j,-1}^{R}E_{1}\Delta \hat{R}_{T-1} \\ & \dots \\ 0 &= \sum_{i=1}^{n} (a_{n_{nvar},i,-1}\Delta E_{1}Y_{i,T-1} + a_{n,i,0}\Delta E_{1}Y_{i,T} + a_{n,i,1}E_{T}\Delta Y_{i,T+1}) + a_{n,0}^{R}\Delta \hat{R}_{T} + a_{n,-1}^{R}E_{1}\Delta \hat{R}_{T-1} \\ & \dots \\ 0 &= \sum_{i=1}^{n} (a_{n_{nvar},i,-1}\Delta E_{1}Y_{i,T-1} + a_{n,i,0}\Delta E_{1}Y_{i,T} + a_{n,i,1}E_{T}\Delta Y_{i,T+1}) + a_{n,0}^{R}\Delta \hat{R}_{T} + a_{n,-1}^{R}E_{1}\Delta \hat{R}_{T-1} \\ & \Delta \hat{R}_{T} &= (1 - DUM_{T}) \left(\sum_{i=1}^{n} (p_{i,-1}E_{1}Y_{i,T-1}^{base} + p_{i,0}E_{1}Y_{i,t+1}^{base} + p_{i,1}E_{1}Y_{i,t+1}^{base}) + \rho_{R}E_{1}\hat{R}_{T-1}^{base} - E_{1}\hat{R}_{T}^{acc} \right) \\ &+ DUM_{i} \left(E_{1}\hat{R}_{T}^{peg} - E_{1}\hat{R}_{T}^{acc} \right) \end{split}$$

••••

For periods 1 < t < T, there are (n+1) X (T-2) equations:

$$0 = \sum_{i=1}^{n} \left(a_{1,i,-1} \Delta E_{1} Y_{i,t-1} + a_{1,i,0} \Delta E_{1} Y_{i,t} + a_{1,i,1} E_{1} \Delta Y_{i,t+1} \right) + a_{1,0}^{R} \Delta \hat{R}_{t} + a_{1,-1}^{R} E_{1} \Delta \hat{R}_{t-1}$$
...
$$0 = \sum_{i=1}^{n} \left(a_{j,i,-1} \Delta E_{1} Y_{i,t-1} + a_{j,i,0} \Delta E_{1} Y_{i,t} + a_{j,i,1} E_{1} \Delta Y_{i,t+1} \right) + a_{j,0}^{R} \Delta \hat{R}_{t} + a_{j,-1}^{R} E_{1} \Delta \hat{R}_{t-1}$$
...
$$0 = \sum_{i=1}^{n} \left(a_{n,i,-1} \Delta E_{1} Y_{i,t-1} + a_{n,i,0} E_{1} Y_{i,t} + a_{n,i,1} E_{1} \Delta Y_{i,t+1} \right) + a_{n,0}^{R} \Delta E_{1} \hat{R}_{t} + a_{n,-1}^{R} E_{1} \Delta \hat{R}_{t-1}$$

$$\Delta \hat{R}_{t} = \left(1 - DUM_{t} \right) \left(\sum_{i=1}^{n} \left(p_{i,-1} E_{1} Y_{i,t-1}^{base} + p_{i,0} E_{1} Y_{i,t}^{base} + p_{i,1} E_{1} Y_{i,t+1}^{base} \right) + \rho_{R} \Delta E_{1} \hat{R}_{t-1} - E_{1} \hat{R}_{t}^{acc} \right)$$

$$+ DUM_{t} \left(E_{1} \hat{R}_{t}^{peg} - E_{1} \hat{R}_{t}^{acc} \right)$$

And for period 1 there are n+1 equations:

 E_1

.

$$\begin{split} 0 &= \sum_{i=1}^{n} \left(a_{1,i,0} \Delta Y_{i,1} + a_{1,i,1} E_{1} \Delta Y_{i,2} \right) + a_{1,0}^{R} \Delta \hat{R}_{1} \\ & \dots \\ 0 &= \sum_{i=1}^{n} \left(a_{j,i,0} \Delta Y_{i,1} + a_{j,i,1} E_{1} \Delta Y_{i,2} \right) + a_{j,0}^{R} \Delta \hat{R}_{t} \\ & \dots \\ 0 &= \sum_{i=1}^{n} \left(a_{n,i,0} \Delta Y_{1,1} + a_{n,i,1} E_{1} \Delta Y_{i,2} \right) + a_{n,0}^{R} \Delta \hat{R}_{t} \\ \Delta \hat{R}_{1} &= \left(1 - DUM_{1} \right) \left(\sum_{i=1}^{n} \left(p_{i,-1} Y_{i,0}^{base} + p_{i,0} Y_{i,1}^{base} + p_{i,1} E_{1} Y_{i,1}^{base} \right) + \rho_{R} \hat{R}_{0}^{base} - \hat{R}_{1}^{acc} \right) + DUM_{t} \left(\hat{R}_{1}^{peg} - \hat{R}_{1}^{acc} \right) \end{split}$$

If we consider each variable-period combination as an unknown, for the case with endogenous tightening (i.e. $DUM_1 = ... = DUM_t = DUM_T = 0$), there are in total (T+1) X (n+1)+T X n unknowns. (T+1) X (n+1) unknowns arise from the presence of $E_1Y_{i,t}$ over T+1 periods, and the additional TXn unknowns originate from the presence of the expected *level* of the endogenous non-policy variables $E_1Y_{i,t}$ (as opposed to the deviation Δ from the accomodation scenario) in the monetary policy rule over periods 1 to T. To obtain the paths of the $E_1Y_{i,t}$ variables, one would have to simulate the model (under the baseline scenario and endogenous drivers $u_{k,t}$, or one would have to obtain them from an alternative source source. However, note that the $E_1Y_{i,t}$ variables appear only in the monetary policy rule, and thus affect the values of $\Delta E_1Y_{i,t}$ only via their effect on $E_1\Delta \hat{R}_t$. Hence, if we could obtain the values of $E_1\Delta \hat{R}_t$ from an alternative source we could disregard the monetary policy rule and solve for the $\Delta E_1Y_{i,t}$ sequences. In that case we would have (T+1)Xn equations and unknowns. In particular, knowledge of the particular dynamics of the

exogenous drivers $u_{k,t}$ underlying the dynamics of $E_1Y_{i,t}$ becomes redundant. All the information relevant for the model are incorporated in the T values of $E_1\Delta \hat{R}_t$.

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