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Reimagining Risk Beyond Normality: Managing Catastrophic Events and Higher-Order Moments

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Abstract

This paper proposes a new framework that enhances traditional risk assessment by incorporating higher-order statistical moments—specifically skewness and kurtosis—through the Cornish-Fisher expansion. Standard risk models, which rely primarily on mean and variance, often underestimate the financial buffers required to hedge against extreme, low-probability shocks. To address this limitation, this study applies the Cornish-Fisher expansion to develop a more comprehensive framework that quantifies the economic impact of extreme events, with direct implications for financial institutions and policymakers. Utilizing a two-period intertemporal consumption model, this study employs Monte Carlo simulations to assess how catastrophic shocks—such as the COVID-19 pandemic—affect key economic indicators. The results demonstrate that negative skewness amplifies downside risk, necessitating stronger precautionary savings and financial reserves. Furthermore, the persistent underinvestment in global preparedness—particularly in pandemic risk management—can be attributed to conventional risk models that fail to capture the full severity of extreme events. The perspective presented in this paper not only enhances theoretical models but also has critical implications for practical applications, particularly in risk management and policy design, where extreme outcomes must be carefully accounted for.

Keywords: Catastrophic Risk Management; Extreme Event Modeling; Financial Buffers

JEL Codes: G32; D81; C15

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1. Introduction: Understanding Global Shocks: Lessons from COVID-19 and Beyond

Since the onset of COVID-19, the global economy has repeatedly faced large-scale external shocks, including pandemics, geopolitical conflicts (e.g., U.S.-China trade tensions, Russia's invasion of Ukraine), and environmental crises. This persistent wave of disruptions has led former British Prime Minister Gordon Brown to describe the current state of affairs as a "permacrisis." Given the high degree of global economic interconnectivity, even low-probability events can trigger severe and widespread financial consequences (Brown et al., 2023).

A fundamental challenge in managing these crises is that they are exogenous shocks, making them difficult to predict using conventional economic models. While theoretical frameworks have long considered pandemics and climate-related risks, their real-world impacts often exceed expectations due to non-linear economic interactions (Stern, 2007; Barro & Urúa, 2012; WHO, 2022; Mazzucato & Ghebreyesus, 2024). Existing risk management models, which predominantly rely on mean-variance approaches, tend to underestimate both the severity and frequency of these extreme events (Kalyon, 1971; Fabozzi et al., 2011; Lee & Lee, 2010; Taleb, 2010).

This paper proposes a framework that enhances traditional risk assessment by incorporating higher-order statistical moments—specifically skewness and kurtosis—through the Cornish-Fisher expansion.² It provides a more accurate framework for evaluating catastrophic risks and addressing the biases inherent in traditional mean-variance models. The findings highlight the policy and financial implications of negative skewness, demonstrating that it amplifies downside risks, which in turn necessitates larger financial buffers to mitigate future economic shocks.

While this enhanced modeling approach offers valuable insights, it faces certain limitations. The inherent uncertainty in estimating both the magnitude and probability of catastrophic events makes it challenging to design purely quantitative solutions. Any attempt to model these risks inevitably relies on assumptions, historical benchmarks, and scenario-based forecasting, reducing practical applicability.

Additionally, even if an improved quantitative model were achievable, the financial buffers required to hedge against such risks might be unrealistically large. This limitation underscores the need to rethink risk management strategies by complementing financial buffers with policy interventions that directly reduce risk exposure. Two approaches include: i). developing proactive policy tools that mitigate the probability and impact of catastrophic events (e.g., investments in healthcare infrastructure, climate adaptation strategies); and ii) integrating

² The Cornish-Fisher expansion is a statistical method used to approximate the quantiles of a probability distribution by incorporating higher-order moments, specifically skewness and kurtosis. It is particularly useful in cases where distributions deviate from normality, as it provides a more accurate estimation of extreme values compared to standard normal approximations. The expansion is derived as a modification of the Taylor series, where the transformation of quantiles is expressed as a polynomial expansion in terms of cumulants (central moments of the distribution). This allows for an adjusted mapping of the standard normal quantiles to better reflect asymmetry (skewness) and tail behavior (kurtosis) in real-world data distributions. The Cornish-Fisher expansion is widely applied in risk assessment, economic modeling, and financial decision-making, where capturing the impact of extreme events is crucial

strategic foresight techniques to navigate the uncertainties surrounding global risk, rather than relying solely on financial hedging mechanisms

While these solutions extend beyond the scope of this paper, the findings provide a foundation for future research in refining risk models and shaping policy frameworks that address the growing complexity of global catastrophes.

2. Exploring Uncertainty: A Literature Review

Catastrophic risks refer to low-probability events that have a substantial impact on individuals, economies, and global systems—such as natural disasters, global pandemics, and market crashes. Bostrom and Cirkovic (2008) state that these events inflict acute damage on a global scale. Catastrophic risks pose unique challenges for mitigation and management due to their rare and unprecedented nature (Currie & Ó hÉigeartaigh, 2018). This underscores the necessity of financial management in preparing for and responding to such risks.

According to Willis et al. (2024), these risks have long-term implications for environmental and economic stability, triggering market shifts and influencing governance systems. Brown et al. (2023) introduce the concept of "permacrisis", in which global economic interdependence exacerbates the severity and widespread consequences of disruptions.

Currently, financial management of catastrophic risks focuses on developing contingency plans to mitigate potential disruptions. The core objective is to ensure that sufficient funds are available for disaster relief efforts. Effective risk management requires countries to assess their exposure, the probability of catastrophic events, and their potential impact. Catastrophe models, which estimate average and maximum losses based on potential event parameters, play a key role in shaping financial management systems, particularly within insurance markets (World Bank Group, 2024; OECD, 2021).

However, risk prevention presents significant economic and statistical challenges. Current approaches rely heavily on the Value at Risk (VaR) methodology to quantify and manage portfolio risk (Samunderu & Murahwa, 2021). While VaR provides a probabilistic estimate of catastrophic losses, its predictive accuracy is limited by the exogenous and unpredictable nature of these events, often leading to underestimations of their true impact (Stern, 2007).

Another conventional approach is the Capital Asset Pricing Model (CAPM), which assumes that risk and returns follow a normal distribution based on a risk premium and a risk-free interest rate. However, in real-world scenarios, risk and returns often deviate from a Gaussian distribution, as markets frequently encounter higher-order moments. These factors significantly affect risk management and should be incorporated into decision-making models (Fama & French, 2004; Lee & Lee, 2010; Samunderu & Murahwa, 2021).

Furthermore, traditional mean-variance models fail to account for the non-linear dynamics of catastrophic events, leading to a systematic underestimation of their probability and impact (Kalyon, 1971; Fabozzi, 2011). Barro (2012) suggests that more complex models of disaster dynamics are essential to smooth the transition between disaster and recovery. A robust risk

assessment framework is crucial for informed decision-making and the effective allocation of resources for disaster preparedness and response.

Literature highlights skewness and kurtosis as critical higher-order moments that enhance risk models by capturing the likelihood and impact of extreme events. Kurtosis risk, often referred to as heavy-tail risk, provides insights into the probability of extreme events, while skewness helps anticipate severe losses and assess model reliability (Hatem, 2022; Dal Moro, 2013). Catastrophic shocks tend to introduce negative skewness, indicating an increased likelihood of severe economic downturns, which necessitates larger financial buffers to absorb potential losses.

The four statistical moments—mean, variance, skewness, and kurtosis—enhances VaR methodologies by incorporating non-normal distributions. This approach improves risk assessments by accounting for biases and heavy tails caused by skewness and kurtosis (Amédée-Manesme et al., 2015).

Modeling catastrophic risks requires assumptions and historical benchmarks, making precise predictions difficult. Moreover, the financial buffers necessary to hedge against catastrophic risks may be prohibitively large, making them impractical for most governments and institutions. These limitations highlight the need to shift focus toward mitigating event probability and preparing for their economic impact, as proposed by Duran-Fernandez (2024).

Beyond parametric risk management, Knightian uncertainty presents an additional challenge for decision-makers. Knightian uncertainty arises when future risks are inherently unquantifiable, making traditional probabilistic models ineffective in predicting extreme events (Scoblic, 2020).

Two quantitative techniques commonly used to address uncertainty are the info-gap robust satisficing approach and the min-max approach. The info-gap approach balances confidence and performance by estimating an acceptable level of robustness for a desired outcome. The min-max approach, in contrast, seeks to minimize the occurrence of the worst possible outcome. Both non-probabilistic alternatives integrate Bayesian and Frequentist statistical methods to build more resilient risk models that can accommodate Knightian uncertainty (Ben-Haim & Demertzis, 2016).

A powerful tool for dealing with uncertainty is strategic foresight. As Wilkinson (2017) argues, data-driven forecasting has limitations when applied to "TUNA" challenges—Turbulent, Uncertain, Novel, and Ambiguous events. Strategic foresight involves a systematic exploration of potential future scenarios to support better strategic decision-making, ensuring that current policies align with long-term risks and opportunities (Cook et al., 2014).

3. Methodological Framework

The objective of this paper is to investigate how central moments beyond the mean and variance influence consumption smoothing decisions under catastrophic shocks. Consumption smoothing can be understood as an intertemporal transfer of income from the present to the future across different potential states of nature.

Our approach is as follows: First, we start with the optimization conditions of a two-period intertemporal consumption problem under uncertainty, where second-period income follows a stochastic process. We examine the Euler equation for this problem and apply the Cornish-Fisher expansion to linearize it. Specifically, we estimate the first, second, third, and fourth-order expansions to express the Euler condition in terms of central moments: mean, variance, skewness, and kurtosis.

Next, we propose a stochastic process to model catastrophic shocks. We assume that an economic variable (in this case, income) follows a normal distribution but is subject to a catastrophic shock with a given probability. The central moments of the resulting distribution, as well as the probability distribution itself, are estimated using Monte Carlo simulation.

We calibrate the model to replicate the distribution of global GDP growth, using the observed mean and variance of this variable. We then introduce the assumption that this variable is subject to a catastrophic shock and estimate the resulting probability distribution and central moments. To model the shock, we incorporate a scenario similar to the COVID-19 shock, as estimated by the IMF for 2020 (Duran-Fernandez, 2024) and consider probability scenarios akin to the 1918 Spanish Flu pandemic, based on the estimates of Marani et al. (2021).

To isolate any further effects, we set the discount factor and interest rate to one and zero, respectively, thereby removing any consumption smoothing effects due to these factors.

Finally, we use the central moments estimated for GDP to calculate income and consumption transfers between periods one and two, applying the first, second, third, and fourth-order expansions of the Cornish-Fisher expansion. This methodology allows us to estimate how the income distribution shifts when subjected to a catastrophic shock and assess whether income transfer strategies to hedge against catastrophic risk are sensitive to variance, skewness, and kurtosis.

3.1. Uncertainty and the Cornish-Fisher expansion

In this section, we explore decision-making under uncertainty, focusing on the limitations of the traditional mean-variance framework. While economic models often reduce uncertainty to a relationship between mean and variance, this simplification can be overly reductive and fail to capture the full complexity of decision-making under risk.

The aim of this section is to analytically demonstrate that decision-making under uncertainty is significantly more nuanced and is influenced by higher-order moments such as skewness and

kurtosis. We employ a simple two-period consumption model under uncertainty, utilizing the Cornish-Fisher expansion—a commonly used tool in economics and finance for such purposes. While alternative analytical methods are available, we favor this approach due to its simplicity, which allows for the identification of clear, stylized facts.

Assume a consumer seeks to maximize their lifetime utility, which depends on consumption in two periods. The consumer decides how much to consume in each period, given income Y in period 1 and income Y_2 in period 2. Income in the first period is constant, while income in the second period is a stochastic variable with a distribution function $f(Y_2)$. The consumer's optimization problem is formalized in Equation 1.

$$\text{Eq.1} \quad u(x_1) + \beta E[u((1+r)(Y_1 - x_1) + Y_2)]$$

The first-order conditions of Equation 1 are obtained by differentiating with respect to x_1 , as presented in Equation 2.

$$\text{Eq.2} \quad u'(x_1) = \beta(1+r)E[u'((1+r)(Y_1 - x_1) + Y_2)]$$

To approximate the right-hand side of Equation 2, where Y_2 is a random variable, we apply the Cornish-Fisher expansion to the utility function. The objective is to linearize the expected value in terms of the central moments of Y_2 . Equation 3 presents the Cornish-Fisher expansion of the expected value of marginal utility in Equation 2, around $z = (1+r)(Y_1 - x_1) + \mu$. The expansion is carried out up to the fourth term, where μ , σ^2 , γ_1 and γ_2 are the mean, variance, skewness and kurtosis of Y_2 respectively.

$$\text{Eq.3} \quad E[u'((1+r)(Y_1 - x_1) + Y_2)] \approx A(1 + B + C + D)$$

$$A = u'(z)$$

$$B = \frac{1}{2}\sigma^2 u''(z)$$

$$C = \frac{1}{6}\gamma_1 \sigma^3 u'''(z)$$

$$D = \frac{1}{24}(\gamma_2 - 3)\sigma^4 u''''(z)$$

$$z = (1+r)(Y_1 - x_1) + \mu$$

Assume that the utility function is a logarithmic transformation of a symmetric Cobb-Douglas function, with the parameter for both periods set to 0.5. By substituting the derivatives of the utility function into Equations 2 and 3, the Cornish-Fisher expansion of the Euler equation is obtained and presented in Equation 4.

Eq.4

$$\frac{1}{x_1} \approx \frac{\beta}{(1+r)(Y_1 - x_1) + \mu} \left[1 - \frac{1}{2} \frac{\sigma^2}{((1+r)(Y_1 - x_1) + \mu)^2} + \frac{1}{3} \frac{\gamma_1 \sigma^3}{((1+r)(Y_1 - x_1) + \mu)^3} - \frac{1}{4} \frac{(\gamma_2 - 3)\sigma^4}{((1+r)(Y_1 - x_1) + \mu)^4} \right]$$

3.2 Simulating Catastrophic Events: A Novel Risk Assessment Framework

Catastrophic events, though infrequent, can have profound effects on economic indicators. Understanding these impacts is critical for effective risk management. This analysis aims to model the influence of such events by examining the consequences of varying magnitudes and probabilities, with the goal of quantifying their impact on the distribution of an economic variable.

The analysis presented in this section is based on a simulation framework designed to capture the dynamic interaction between baseline variability and the occurrence of catastrophic events. The methodological approach is outlined in the following steps:

Baseline variability modelling. The baseline metric, denoted as x is modelled as a random variable following a standard normal distribution with mean $\mu=0$ and variance $\sigma^2=1$. This serves as the reference scenario in the absence of catastrophic events. This metric can represent GDP, household income, revenues, profits or any other related economic variable.

Probability of catastrophic events. A uniformly distributed random variable p with values in the interval $[0, 1]$ is generated to simulate the likelihood of a catastrophic event occurring.

Impact modelling. The impact magnitude v represents the severity of the catastrophic event, taking negative values to reflect adverse impacts. The event occurs when the probability p falls below a specified threshold q resulting in a direct modification of the baseline metric.

Overall metric outcome. The overall metric outcome, z , is computed as the sum of the baseline metric x and the impact, where $\varphi = v$ if the event occurs, and $\varphi = 0$ otherwise. This formulation allows for the analysis of the cumulative effect of both the baseline variability and the catastrophic event.

Simulation and statistical analysis. The study conducts 10,000 simulations for each combination of impact magnitude v and probability threshold q . The resulting distribution of z is analyzed to derive key statistical metrics including mean, standard deviation, skewness, kurtosis, and median. Additionally, histograms are used to visualize the distributional shifts across different scenarios.

The simulation results provide a detailed account of how catastrophic events influence the overall metric outcome. Key findings include:

Mean. As expected, the mean of the metric outcome z decreases with both the increasing severity and likelihood of catastrophic events.

Variance. The variance of z rises in scenarios with higher impact magnitudes and probabilities, indicating greater variability in outcomes.

Skewness and Kurtosis. Represented by γ_1 and γ_2 respectively, reveal that more severe and likely catastrophic events induce significant deviations from normality, with distributions becoming increasingly skewed and leptokurtic.

Table 1 presents a summary of the statistics of z across varying magnitudes v and probabilities q , while Figure 1 displays the corresponding histograms. The empirical results demonstrate that as the magnitude and probability of catastrophic events increase, there is a significant deterioration in the overall metric outcome. These findings suggest that catastrophic events not only reduce the mean outcome but also amplify uncertainty, as reflected by increased standard deviation, skewness, and kurtosis.

3.3. Quantifying Economic Sensitivity to Catastrophic Shocks

A catastrophic event, for the purposes of this analysis, is defined as a large negative shock to income that can occur at any given moment with probability p . We assume that income follows an underlying normal distribution, and thus, the catastrophic event is treated as an exogenous shock. As demonstrated by recent global events, such shocks can stem from extreme climate-related incidents or global pandemics.

In this section, we present a calibration exercise to replicate the distribution of global GDP growth. Table 2 presents the mean, variance, skewness, and kurtosis of world GDP as described in Equation 5, covering the period from 1960 to 2023.

Eq.5
$$G_{t+1} = \frac{Y_{t+1}}{Y_t}$$

Figure 2 shows the histogram of this variable. The skewness and kurtosis suggest that this variable does not follow a normal distribution. A Jarque-Bera normality test further rejects the hypothesis of normality. The negative skewness indicates that economic crises are more frequent than economic booms. Interestingly, during this period, only two global recessions were recorded—in 2009 during the Great Recession and in 2023 due to the COVID-19 pandemic—suggesting that the negative skew can be attributed more to local and regional events rather than global shocks.

We aim to introduce an idiosyncratic catastrophic global shock into this distribution. To do so, we use parameters that describe the economic impact of a global pandemic. The IMF estimates the economic cost of the COVID-19 pandemic at USD 13.8 trillion, equivalent to a 15% drop in global GDP (WHO, 2024). However, global GDP only decreased by 2.9% in 2020, as this is the net effect after fiscal, monetary, and other countercyclical measures were applied worldwide.

We use the 15% drop as the base scenario for a catastrophic event, but we also evaluate other catastrophic events using this benchmark. Specifically, we analyze shocks that are one-third, one-half, and twice this drop. Regarding the probability of experiencing a pandemic shock, we examine scenarios where the probability is one-third, one-half, equal to, and twice the estimated probability of experiencing a 1918 Spanish Flu-like pandemic, as reported by Marani et al. (2021) (Table 2).

We estimate the shock in a two-period framework, normalizing income in period 1 to 100. We assume that growth follows the mean and variance of Equation 5, where Y represents world GDP. We conducted a Monte Carlo simulation with 10,000 repetitions to estimate the distribution of GDP under the global catastrophic shock, in this case, a global pandemic with an economic impact similar to that of COVID-19 and a probability aligned with the 1918 Spanish Flu pandemic presented in Table 2. The catastrophic shock follows the process described in the previous section.

Figure 3 shows the estimated probability distributions for each combination of catastrophic shock magnitudes and pandemic probabilities. Table 3 presents the central moments and median.

3.4. How Catastrophic Events Influence Income Transfers and Savings

We recall the Cornish-Fisher expansion from Equation 4 and utilize the central moments estimated in the simulation presented in the previous section (Table 4). The objective of this analysis is to investigate how income transfer decisions vary when estimated using different approximations of the Cornish-Fisher expansion.

Table 5 presents the estimation of consumption in period 1 (x_1) and the net income transferred from period 1 to period 2 ($y_1 - x_1$). Columns 1 and 2 in Table 4 show consumption and income transfer estimated by the Cornish-Fisher expansion using only the linear term, omitting terms B, C, and D from Equation 4. This approximation only considers the mean (μ) and is equivalent to assuming that income in period 2 equals the mean (non-stochastic), or that preferences are risk neutral.

Columns 4 and 5 in Table 4 present consumption in period 1 (x_1) and income transfer ($y_1 - x_1$) using the linear and quadratic terms of the Cornish-Fisher expansion, omitting C and D from Equation 4. This approximation assumes that income in period 2 (Y_1) is normally distributed with zero skewness (γ_1) and no excess kurtosis ($3 - \gamma_2$). As shown by the simulations, catastrophic events induce skewness and excess kurtosis; therefore, this approximation implicitly assumes that the impact of such events is negligible.

Columns 7, 8, 10, and 11 in Table 4 present consumption in period 1 (x_1) and income transfer ($y_1 - x_1$) using the cubic and fourth-degree polynomial approximations of the Cornish-Fisher expansion, respectively. These approximations take into account the skewness of the distribution and kurtosis (γ_1 and γ_2).

The fourth-degree polynomial approximation (Columns 10 and 11 in Table 4) should provide a more accurate estimate compared to the lower-order approximations. Columns 3, 6, and 9 in Table 4 show the percentage difference in income transfer between the linear, quadratic, and cubic approximations, relative to the fourth-degree approximation. These estimates serve as a proxy for the estimation error of each approximation and can be interpreted as the underestimation or overestimation of income transfers.

As seen in Equation 4, each successive term is divided by increasingly higher values, meaning that, although higher-order moments are not zero in principle, their contribution tends to diminish.

4. Discussion

4.1. Revisiting the Cornish-Fisher Expansion for Risk Modeling

Consider two special cases: first, the non-stochastic case, and second, the case where Y_2 follows a normal distribution with mean μ and σ^2 .

For the non-stochastic case, the value of x_1 is estimated as in Equation 6. Notice that when $Y_1=Y_2$, when $r=0, \beta=1$, consumption in each period is equal to income, implying that no intertemporal income transfer exists. This indicates that, in the absence of uncertainty, intertemporal income transfers are driven solely by interest rates and the discount factor.

In the simulation analysis, $Y_2 = Y_1(1+g)$ and since $E[(1+g)] > 0$, it is expected that income in period 2 will be higher than in period 1. Therefore, even with a zero interest rate and a discount factor of 1, we anticipate observing positive income transfers from the present to the future, regardless of the values of other central moments. According to Equation 6, it is expected that consumption will be smoothed, resulting in consumption in each period being equal to the average income over the entire time horizon ($x_1 = (Y_1 + Y_2)/2$). This can be considered the baseline in the absence of uncertainty, and any introduction of uncertainty will move the economy away from this baseline.

Eq.6

$$x_1 \approx \frac{(1+r)Y_1 + Y_2}{(1+r)(1+\beta)}$$

For the case where Y_2 follows a normal distribution with mean μ and σ^2 , the third and fourth terms of the Cornish-Fisher approximation of the Euler Equation are presented in Equation 6.

Notice that even when $Y_1 = \mu$, and $r=0$ and $\beta=1$ intertemporal consumption adjustments still arise. The larger the σ^2 , the lower the consumption in period 1. Intuitively, if the expected income remains the same, uncertainty incentivizes consumers to save as a risk hedging strategy (Figure 4)

Figure 5 illustrates the relationship between present consumption and variance. As variance increases, consumption in period 1 decreases, leading to a greater income transfer from the

present to the future. This reflects a classic risk management strategy, where agents build financial buffers through savings to hedge against future risks.

$$\text{Eq.7} \quad \frac{1}{x_1} \approx \frac{\beta}{(1+r)(Y_1 - x_1) + \mu} \left[1 - \frac{1}{2} \frac{\sigma^2}{((1+r)(Y_1 - x_1) + \mu)^2} \right]$$

In general, for non-normal cases, the third and fourth terms of the Cornish-Fisher expansion can differ from those in the normal case.

Figure 6 illustrates the sensitivity of consumption to changes in skewness. As skewness increases, present consumption rises, and intertemporal income transfers shift from the future to the present. Conversely, as skewness becomes more negative—indicating a higher probability of catastrophic shocks in the future—present consumption decreases, as agents transfer more income to the future in anticipation of potential negative events. Negative skewness prompts a risk management strategy similar to that observed with an increase in variance. On the other hand, positive skewness reduces risk aversion, and in extreme cases, agents may increase present consumption. Thus, financial buffers are highly sensitive to skewness.

In contrast, Figure 7 shows that kurtosis is negatively correlated with present consumption. As kurtosis increases—implying a greater likelihood of extreme outcomes—income is shifted from the future to the present. This result is less intuitive, as kurtosis does not have the same immediate interpretation as other moments. However, higher kurtosis indicates a greater probability of both negative and positive extreme events. Since these events are symmetric, their impact on consumption is similar to that of increased variance: agents tend to save more and build buffers to hedge against future risks.

This analysis underscores the critical importance of considering central moments in decision-making under uncertainty. Our findings suggest that these higher-order moments can significantly influence agents behavior and economic outcomes, particularly in scenarios where the traditional mean-variance framework proves inadequate. For example, in situations involving catastrophic income shocks or skewed income distributions, relying solely on mean and variance may lead to incomplete or misleading conclusions. By incorporating these higher-order moments into our analysis, we provide a more comprehensive understanding of how uncertainty shapes consumption decisions.

4.2. Understanding the Economic Impact of Catastrophic Events

The results of the Monte Carlo simulation suggest some straightforward stylized facts. As the probability of catastrophic events increases, the mean decreases, variance increases, skewness becomes negative, and kurtosis rises. Similarly, as the magnitude of the catastrophic shock increases, the mean decreases, variance increases, skewness becomes more negative, and kurtosis rises. Both higher probability and greater magnitude of the shock generate increased downside risk for the underlying economic variable (Table 1).

In terms of the probability distribution, the shock creates a "bump" to the left of the underlying normal distribution. Interestingly, this "bump" appears to be symmetric, but when combined with

the underlying normal distribution, it shifts the mean and median to the right, creating negative skewness and higher excess kurtosis. As the risk increases, the "bump" becomes larger, and the overall distribution diverges further from a normal distribution (Figure 3).

Interestingly, the actual shape of the world GDP growth distribution does not exhibit this "bump"; instead, the entire distribution appears to be shifted to the right. As noted, global shocks are rare in the actual distribution and cannot solely account for the negative skewness. It seems that other processes at the national or regional level are contributing to this skewness and excess kurtosis, rather than a global catastrophic risk process (Figure 2)

This does not imply that modeling catastrophic risk is irrelevant. The global economy has never been as complex and interconnected as it is today, and the types of global risks we face are new and unique to the present time. The recent succession of global crises suggests that new processes might be emerging in the global economy, which are worth studying. As anecdotal evidence, it is noteworthy that in a 63-year time series, the only global recessions have occurred within the last 15 years.

4.3. Biases in Risk Management: Lessons from Catastrophic Events

The estimation of consumption and intertemporal income transfers under different approximations of the Cornish-Fisher expansion yields the following results:

For the actual probability distribution of global growth, the quadratic approximation is equal to the fourth-degree approximation up to the third decimal place, suggesting that for historical data, relying on variance alone is sufficient for decision-making under uncertainty. Skewness and kurtosis appear to be negligible. Moreover, the linear and quadratic approximations are identical up to the second decimal place, indicating that even a linear approximation, relying exclusively on the mean, is accurate enough. This suggests that a non-stochastic approach, which assumes future income will match historic growth, is sufficiently precise to approximate the complete distribution.

However, the situation changes when a catastrophic event is introduced. The larger the shock and its probability, the greater the estimation error in the linear and quadratic approximations. The cubic approximation, which accounts for skewness, is very similar to the fourth-degree expansion unless the shock is extremely large.

In the central scenario, which models a shock similar to the COVID-19 pandemic ($v=-15.6\%$ and $p=1.9\%$) with a probability similar to the upper bound of a 1918 Spanish Flu event, the linear estimation of income transfer is 0.02% lower than the fourth-degree estimation. Even the quadratic approximation results in an estimation error of 0.001% (Table 4).

Although these magnitudes may seem insignificant, these errors refer to a percentage of the global economy. Table 5 estimates the under-savings relative to the global economy in 2021. For example, under the central scenario, a risk-hedging strategy relying exclusively on the linear approximation would fall short by USD 21.85 billion compared to the more accurate fourth-degree expansion estimation.

Nevertheless, the real issue arises when comparing savings under observable parameters with extreme cases. For example, if the real impact of a pandemic were twice that of the COVID-19 shock and the probability were three times the upper bound of a 1918 Spanish Flu-like event, the world should be saving 0.75% of its GDP to prepare for such an event, amounting to USD 78 billion per year. Using a discount rate of 12%, this implies that a pandemic rainy-day fund should be at least USD 656.3 billion, larger than any existing reserve nowadays.

4.4. Policy insights

The results of this paper suggest that persistent underinvestment in global preparedness, particularly in areas such as pandemic risk mitigation, climate adaptation, and financial market stability, remains a critical challenge. The findings indicate that a fundamental shift in risk management strategies is required—one that moves beyond standard financial hedging techniques and incorporates broader policy-driven interventions.

The paper suggests that building financial buffers to hedge risk might be unfeasible, an insight that reflects real-life situations. For example, the World Bank and WHO estimate that a comprehensive financial response for pandemic preparedness and response (PPR) should amount to USD 10 billion per year. However, between 2022 and 2024, the World Bank's Pandemic Fund managed to raise only USD 1.6 billion.

The failure to account for higher-order central moments induced by catastrophic events may be one of several reasons why financing PPR has been so challenging. A comprehensive risk management strategy for this case should emphasize prevention by reducing the probability of extreme events, rather than focusing solely on building enormous financial reserves. For instance, investment in primary healthcare infrastructure and universal health coverage could reduce systemic vulnerabilities, thereby decreasing both the probability and severity of future shocks (Duran-Fernandez et al., 2024).

Another key policy insight is the limitations of purely quantitative risk management approaches. The complete characterization of a catastrophic event extends beyond understanding just the mean and variance, and in practice, it would be extremely difficult to quantify all necessary parameters. This suggests that risk management strategies should embrace a hybrid approach, combining quantitative modeling with qualitative methods such as strategic foresight.

Strategic foresight techniques can enable policymakers and financial institutions to anticipate and adapt to highly uncertain, non-linear risks (Cook et al., 2014; Gariboldi et al., 2021). Unlike traditional quantitative models, which rely on historical data and probability estimates that may not accurately capture unprecedented catastrophic shocks, strategic foresight employs qualitative and adaptive tools such as scenario planning to explore multiple plausible futures. By mapping out alternative risk scenarios, assessing potential cascading effects, and identifying key early-warning signals, decision-makers can develop more flexible and resilient policy responses.

This approach helps overcome the inherent limitations of statistical risk models, which often fail to account for deep uncertainty, systemic interactions, and behavioral responses to crises.

4.5. Limitations of Current Models in Addressing Catastrophic Risks

We identify four significant limitations of this approach that need to be addressed.

The first limitation is the use of a two-period model. A more robust approach would involve analyzing a multi-period framework, allowing agents to spread the impact of a catastrophic shock over a longer horizon. While a longer time horizon increases the likelihood of observing multiple catastrophic events, these events are low-probability occurrences, making their impact difficult to model precisely. A multi-period model would enable agents to transfer more income into an intertemporal "rainy day" fund, thereby smoothing consumption over time without large disruptions in any single period. While the general patterns observed in this analysis would likely persist, the magnitude of the effects could be smaller in a more dynamic framework.

One justification for this approach is that if linear and quadratic estimations had been identical to those using higher-order moments, we could have ruled out any significant bias in catastrophic risk estimation. However, since higher-order moments do introduce notable effects, further exploration of these phenomena in a multi-period setting is warranted.

Another important factor, as noted by Duran-Fernandez (2004), is the tendency toward short-term planning horizons driven by political cycles, which may contribute to the global underinvestment in PPR. Establishing financial buffers, such as "rainy day" funds, can be seen as a public good—and in the case of global pandemics, a global public good—requiring government intervention and public-private collaboration. In this context, a two-period model may serve as a reasonable proxy for the short-term planning that often characterizes political decision-making, but further investigation is needed to assess how multi-period dynamics influence preparedness decisions.

A second limitation relates to the definition of income transfers. In an intertemporal consumption model without production, income transfers can take the form of financial savings or buffers composed of consumption goods. In the standard model, goods are perishable, and production must be fully consumed in each period. For a global economy, which operates as a closed system, net financial savings should equal zero, making it impossible to transfer income from the present to the future or vice versa.

However, income transfers can occur if goods are durable and storable. In a production model, output can be transformed into real assets, which can be consumed during a negative shock scenario. This suggests that buffers in the case of a global pandemic—or other catastrophic shock—are not necessarily financial but may take the form of physical stockpiles. Effective risk management in such a scenario is more complex, as it involves saving specific assets or durable goods and understanding how these can be converted into consumption. Rigidities in this process make income transfers harder to model accurately.

This is not merely theoretical. During the COVID-19 pandemic, global supply chains experienced significant disruptions, and the creation of buffers often involved restricting trade and developing local manufacturing capacities to mitigate risks posed by distant suppliers. Reshoring and nearshoring efforts, promoted by new industrial policies in several countries, are

direct responses to these challenges, underscoring the complexity of managing catastrophic risks beyond merely building financial buffers.

The third limitation concerns the magnitude and probability of global catastrophic shocks. The parameters used in this paper serve as benchmarks, but real-world values could differ significantly, making it impossible to estimate them with certainty. The true probability and magnitude of future global pandemics or catastrophic climate events in an interconnected world remain highly uncertain. The Stern Report (2006), for example, estimated that climate change could have a permanent negative impact on consumption of -16%, far exceeding the one-time USD 13.8 trillion cost of the COVID-19 pandemic. More research is needed to improve catastrophe modeling, refine uncertainty estimates, and enhance our understanding of long-term economic risks.

A fourth limitation is that this paper is theoretical in nature, relying on modeling assumptions and parameters benchmarked against COVID-19. While the findings offer valuable insights, their practical application and policy implications require further empirical validation. The economic estimates presented here are based on historical data and simulations, but real-world implementation of financial buffers, and policy interventions, necessitates deeper engagement with policymakers and industry stakeholders. Future research should focus on bridging the gap between theoretical models and actionable policy recommendations, including case studies, empirical testing, and scenario-based policy simulations to assess the effectiveness of different risk mitigation strategies in practice.

Addressing these limitations will be critical for advancing catastrophe risk management and ensuring that risk models can effectively guide financial institutions, governments, and international organizations in designing more resilient economic policies.

5. Concluding Remarks

This paper provides an in-depth analysis of global catastrophe risk management within a neoclassical framework, demonstrating the limitations of traditional risk models—particularly those based on linear risk-neutral preferences and mean-variance analysis—in adequately hedging against extreme shocks. By incorporating higher-order moments such as skewness and kurtosis, this study offers a more nuanced understanding of risk dynamics, emphasizing how conventional models underestimate downside risks and fail to capture the true severity of catastrophic events. The findings underscore the necessity of larger financial buffers and more adaptive policy interventions to mitigate systemic vulnerabilities.

To create more robust catastrophic financial plans, policymakers should prioritize policies that reduce the probability of a catastrophe rather than relying solely on reactive financial buffers. This includes investing in risk prevention strategies, such as strengthening public health systems, climate adaptation measures, and resilient infrastructure. Financial institutions, in turn, must incorporate non-linear risk assessments into portfolio management and explore innovative financial instruments tailored to extreme event scenarios. The integration of strategic foresight into decision-making processes will be critical for anticipating and mitigating emerging risks in an increasingly volatile world.

Additionally, future research should adopt interdisciplinary approaches, integrating insights from epidemiology, environmental science, artificial intelligence, and behavioral economics to develop holistic risk mitigation strategies. By moving beyond traditional economic models and embracing collaborative, data-driven, and cross-sectoral research, risk management frameworks can evolve to better anticipate, mitigate, and respond to catastrophic risks.

By integrating these perspectives, policymakers, financial institutions, and researchers will be better equipped to build a more resilient global economy capable of withstanding future crises and systemic shocks.

6. References

- [1] Amédée-Manesme, C., Barthélémy, F. & Keenan, D. (2015). Cornish-Fisher Expansion for Commercial Real Estate Value at Risk. *J Real Estate Finan Econ* 50, 439–464. <https://doi.org/10.1007/s11146-014-9476-x>
- [2] Barro, R. J., & Ursúa, J. F. (2012). Rare macroeconomic disasters. *Annu. Rev. Econ.*, 4(1), 83-109.
- [3] Ben-Haim, Y., Demertzis, M. (2016). Decision Making in Times of Knightian Uncertainty: An Info-Gap Perspective. *Economics: The Open-Access, Open-Assessment E-Journal*, 10 (2016-23): 1—29. <http://dx.doi.org/10.5018/economics-ejournal.ja.2016-23>
- [4] Bostrom, N., & Cirkovic, M. (2005). *Global catastrophic risks*. Oxford University Press.
- [5] Brown, G., El-Erian, M., Spence, M., & Lidow, R. (2023). *Permacrisis: A plan to fix a fractured world*. Simon and Schuster.
- [6] Cook, C. N., Inayatullah, S., Burgman, M. A., Sutherland, W. J., & Wintle, B. A. (2014). Strategic foresight: how planning for the unpredictable can improve environmental decision-making. *Trends in ecology & evolution*, 29(9), 531-541.
- [7] Currie, A., & Ó hÉigearthaigh, S. (2018). Working together to face humanity’s greatest threats: Introduction to the future of research on catastrophic and existential risk. *Futures*, 102, 1-5. <https://doi.org/10.1016/j.futures.2018.07.003>
- [8] Dal Moro, E. (2013). Kurtosis and Skewness Estimation for Non-Life Reserve Risk Distribution. *ASTIN Colloquium 2013*. <https://ssrn.com/abstract=2281902>
- [9] Duran-Fernandez, R. (2024) “Exploring innovative financing solutions for pandemic preparedness and response” The WHO Council on the Economics of Health for All. *Insight Council Insight No. 7*. Geneva: World Health Organization; 2024 <https://www.who.int/publications/m/item/exploring-innovative-financing-solutions-for-pandemic-preparedness-and-response---council-insight-no.-7>
- [10] Duran-Fernandez, R., Bernal-Serrano, D., Garcia-Huitron, J. A., & Hutubessy, R. (2024). Financing for pandemic preparedness and response measures: a systematic scoping review. *Bulletin of the World Health Organization*, 102(5), 314.
- [11] Fabozzi, F. J., Markowitz, H. M., & Gupta, F. (2011). Portfolio selection. *The Theory and Practice of Investment Management*, 45-78.
- [12] Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: Theory and evidence. *The Journal of Economic Perspectives*, 18(3), 25–46. <https://doi.org/10.1257/0895330042162430>
- [13] Gariboldi, M. I., Lin, V., Bland, J., Auplish, M., & Cawthorne, A. (2021). Foresight in the time of COVID-19. *The Lancet Regional Health–Western Pacific*, 6.
- [14] Hatem, G. (2022). Normality testing methods and the importance of skewness and kurtosis in statistical analysis. *BAU Journal – Science and Technology*. Vol. 3: Iss. 2, Article 7. <https://doi.org/10.54729/KTPE9512>
- [15] Kalymon, B. A. (1971). Estimation risk in the portfolio selection model. *Journal of Financial and Quantitative Analysis*, 6(1), 559-582.
- [16] Lee, A. C., & Lee, J. C. (2010). *Handbook of quantitative finance and risk management* (Vol. 1). C. F. Lee (Ed.). New York: Springer.
- [17] Marani, M., Katul, G. G., Pan, W. K., & Parolari, A. J. (2021). Intensity and frequency of extreme novel epidemics. *Proceedings of the National Academy of Sciences*, 118(35), e2105482118.
- [18] Mazzucato, M., & Ghebreyesus, T. A. (2024). Advancing the economics of health for all. *The Lancet*.
- [19] OECD (2021), *Financial Management of Catastrophe Risks: Approaches to Building Financial Resilience* https://www.oecd.org/content/dam/oecd/en/publications/reports/2021/11/financial-management-of-catastrophe-risks_413d99f6/92106475-en.pdf
- [20] Samunderu, E., & Murahwa, Y. T. (2021). Return Based Risk Measures for Non-Normally Distributed Returns: An Alternative Modelling Approach. *Journal of Risk and Financial Management*, 14(11), 540. <https://doi.org/10.3390/jrfm14110540>
- [21] Scoblic, J. P. (2020). Strategic foresight as dynamic capability: A new lens on Knightian uncertainty. Harvard Business School, Boston, MA, USA. Working Paper 20-093. Retrieved from https://www.hbs.edu/ris/Publication%20Files/20-093_7e70d4a3-aab8-449c-82e9-62cf143d6413.pdf
- [22] Stern, N. (2007). *The economics of climate change: The Stern review*. Cambridge University Press.

- [23] Taleb, N. N. (2010). *The Black Swan: The Impact of the Highly Improbable: With a new section: " On Robustness and Fragility"* (Vol. 2). Random house trade paperbacks.
- [24] WHO (2022) *Health for all: transforming economies to deliver what matters*. The WHO Council on the Economics of Health for All. Geneva: World Health Organization; 2022
(<https://www.who.int/publications/i/item/9789240080973>, accessed on 11 September 2024)
- [25] Wilkinson, A. (2017). *Strategic foresight primer*. European Political Strategy Centre. Retrieved from https://cor.europa.eu/Documents/Migrated/Events/EPSC_strategic_foresight_primer.pdf
- [26] Willis et al. (2024). Understanding and managing global catastrophic risk. RAND
https://www.rand.org/pubs/research_briefs/RBA2981-1.html
- [27] World Bank Group. (2024). *Disaster Risk Financial Management: strengthening public finances and promoting resilience*. In *World Bank*. <https://www.worldbank.org/en/region/lac/brief/gestion-financiera-riesgo-desastres-america-central-republica-dominicana>

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Figure 1 Histograms of z

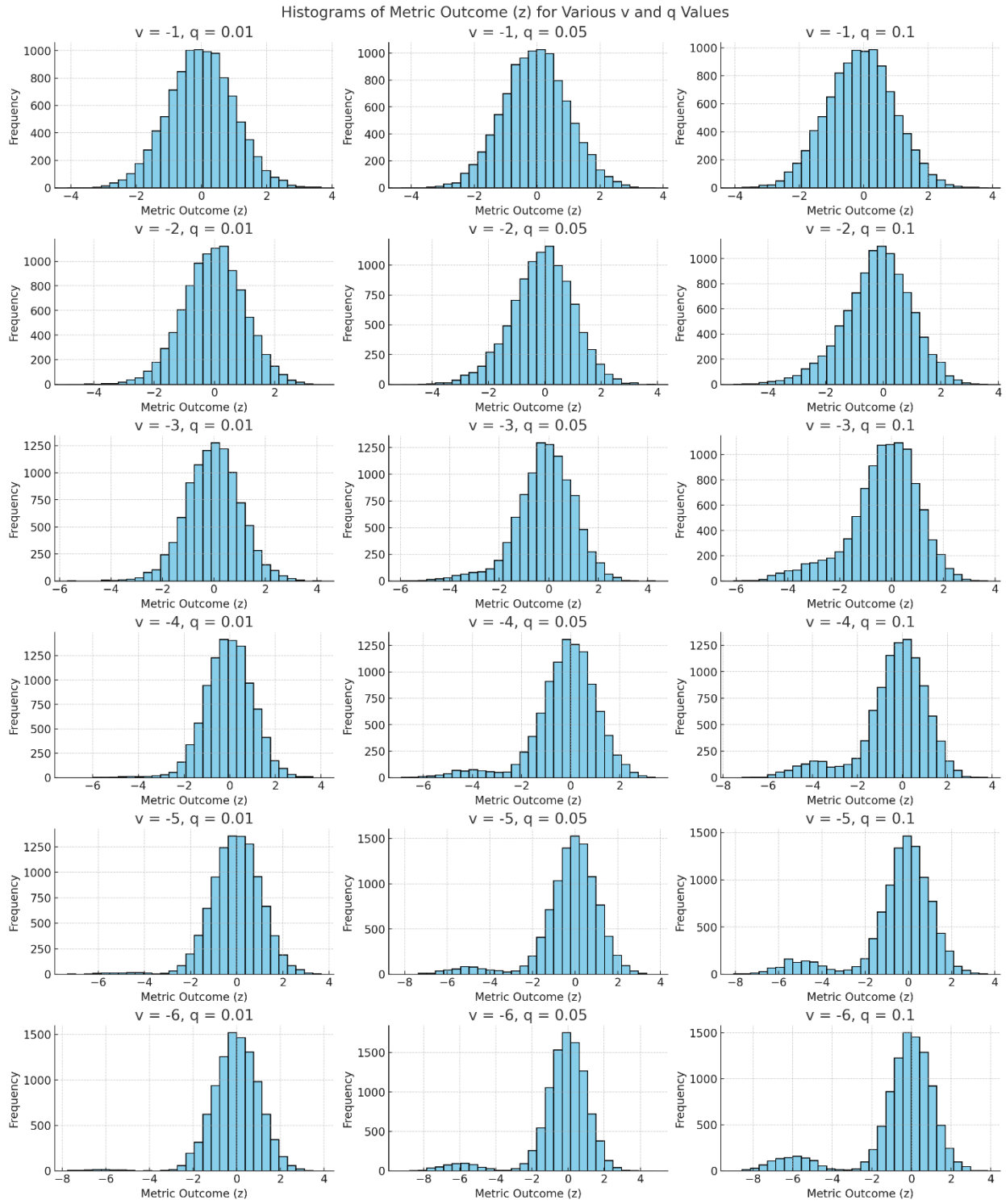


Figure 2 Histograms of $G=1+g$

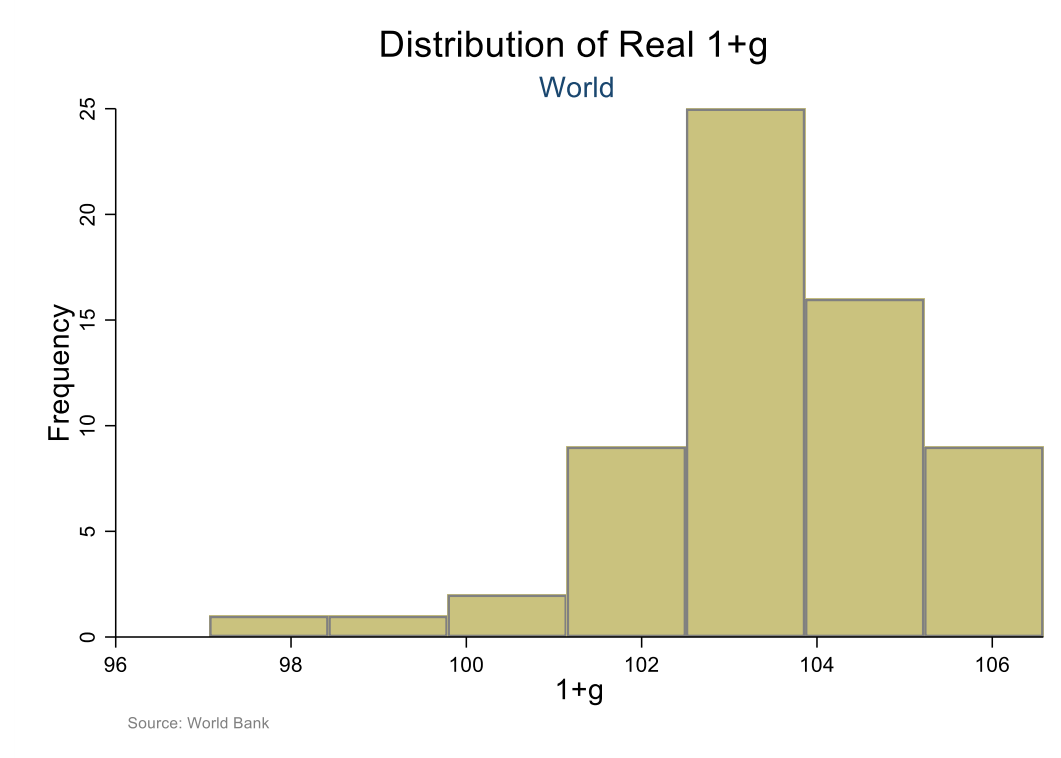


Figure 3 Distribution of World GDP with catastrophic shock

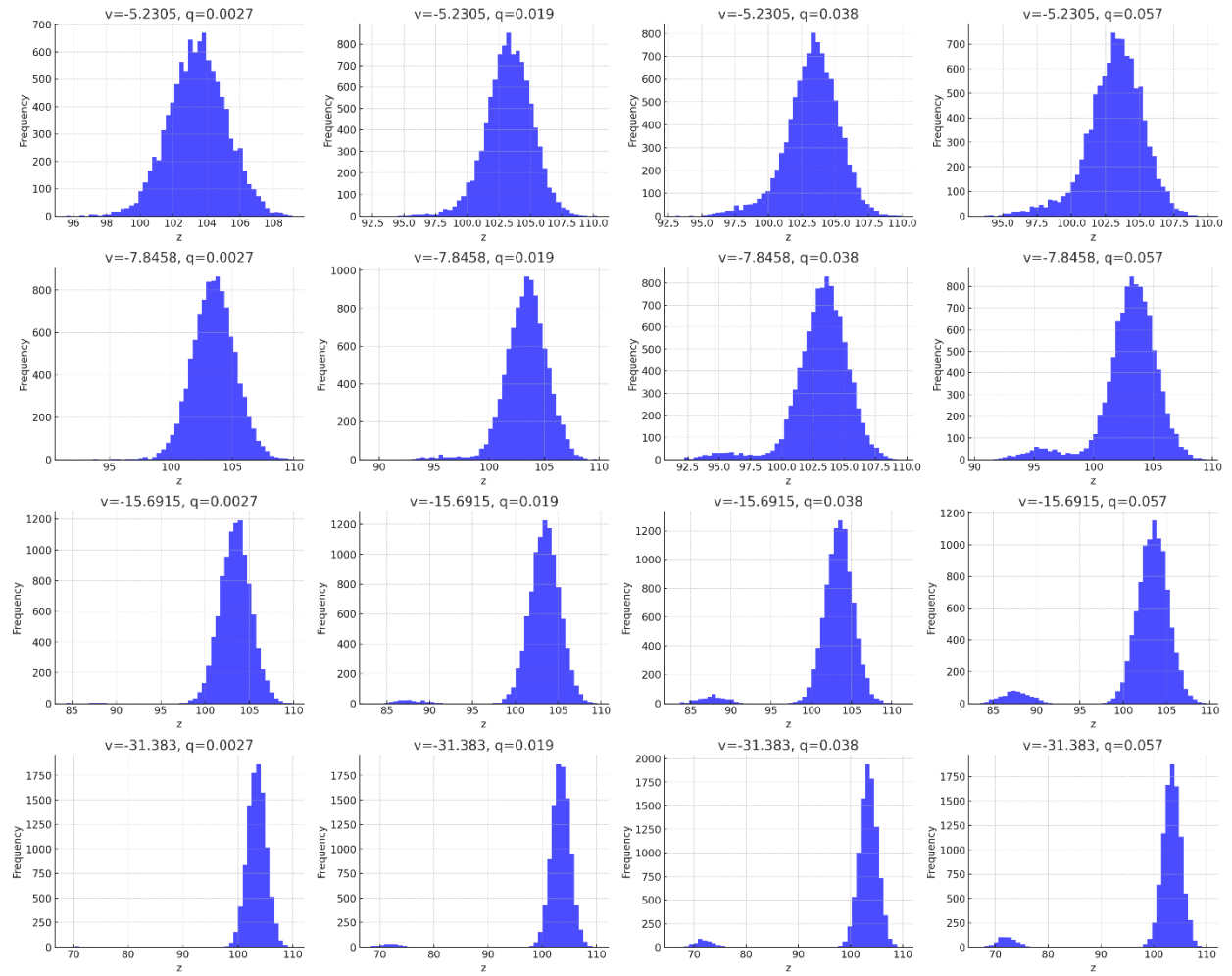


Figure 4 Variance vs Consumption

$$Y_I = \mu = 100, r = 0, \beta = 1$$

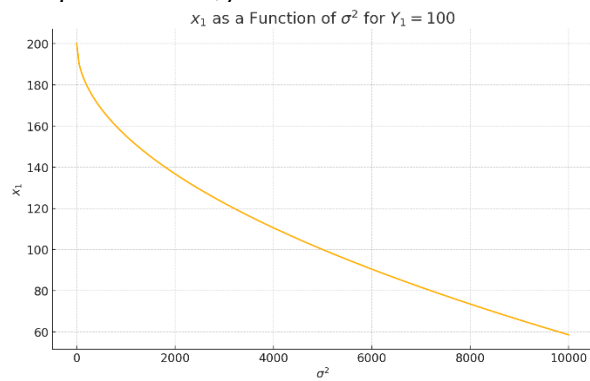


Figure 5 Skewness vs Consumption

$$Y_I = \mu = 100, r = 0, \beta = 1, \sigma^2 = 1$$

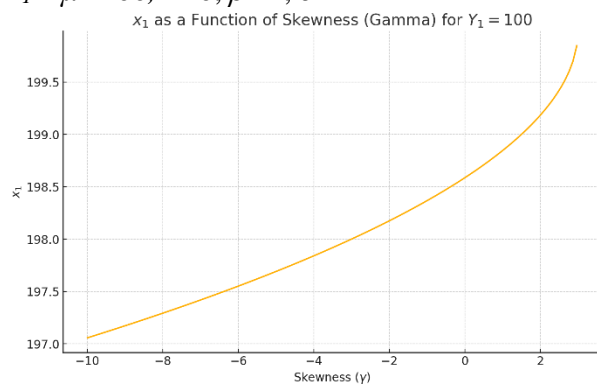


Figure 6 Excess Kurtosis vs Consumption

$$Y_I = \mu = 100, r = 0, \beta = 1, \sigma^2 = 1, \gamma_I$$

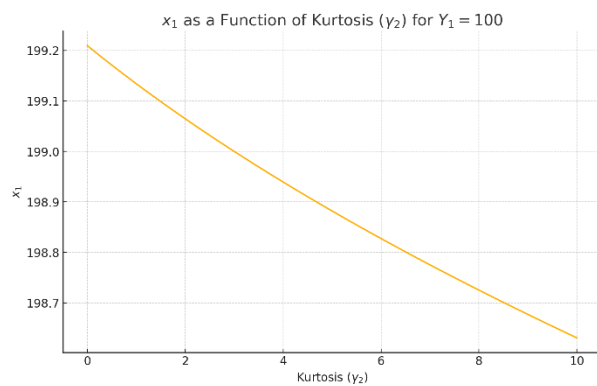


Table 1. Central moments or a distribution with catastrophic shock

				Skewness	
q	Median	Mean μ	Variance σ^2	γ_1	Kurtosis γ_2
$\nu=1$					
0.010	0.002	0.003	1.008	0.009	0.077
0.050	-0.048	-0.059	1.023	-0.028	0.011
0.100	-0.095	-0.104	1.047	-0.001	0.015
$\nu=2$					
0.010	0.010	-0.007	1.012	-0.047	0.108
0.050	-0.058	-0.114	1.099	-0.311	0.401
0.100	-0.145	-0.210	1.163	-0.352	0.376
$\nu=3$					
0.010	-0.040	-0.048	1.046	-0.198	0.510
0.050	-0.074	-0.152	1.190	-0.633	1.306
0.100	-0.139	-0.295	1.355	-0.768	1.004
$\nu=4$					
0.010	-0.032	-0.050	1.064	-0.487	1.861
0.050	-0.063	-0.196	1.321	-1.218	2.964
0.100	-0.144	-0.409	1.585	-1.191	1.678
$\nu=5$					
0.010	-0.029	-0.065	1.121	-0.886	3.785
0.050	-0.066	-0.245	1.467	-1.645	4.508
0.100	-0.112	-0.472	1.785	-1.546	2.519
$\nu=6$					
0.010	-0.030	-0.069	1.165	-1.265	6.458
0.050	-0.060	-0.305	1.643	-2.114	6.172
0.100	-0.141	-0.591	2.041	-1.783	3.060

Source: 10,000 repetition simulation

Table 2. Statistics and estimation of parameters

Parameter	Value
World Real GDP Growth Statistic ^{/1}	
Mean μ	103.447800
Variance σ^2	2.928913
Skewness γ_1	-0.974012
Kurtosis γ_2	5.420554
Jarque Bera Normality Test ^{/2}	
$\chi(2)$	2534.00%
p-value	0.00%
Pandemic Shock ^{/3}	
$\frac{1}{3} x$	-5.23%
$\frac{1}{2} x$	-7.85%
1x	-15.69%
2 x	-31.38%
Probability of Pandemic Shock ^{/4}	
Lower Bound	0.27%
Upper Bound	1.90%
2x Upper Bound	3.80%
3x Upper Bound	5.70%

/1 Statistics are for $1+g$, where g is annual growth

/2 Jarque Bera Test is rejected

/3 GDP growth IMF Estimation for 2020

/4 Probability of experiencing a Spanish Flu 1918 like scenario in any given year

Source: World GDP World Bank, Pandemic Shock IMF; Probability of Pandemic Shock Marani, et al. (2021)

Table 3. World GDP Growth Central moments with catastrophic shock

q	Mean μ	Variance σ^2	Skewness γ_1	Kurtosis γ_2	Median
Calibrated	103.45	2.93	-0.97	5.42	103.55
Catastrophic shock (neg)					
v=5.2305					
0.0027	103.43	3.01	-0.06	3.20	103.44
0.0190	103.36	3.49	-0.39	4.05	103.41
0.0380	103.26	4.01	-0.63	4.33	103.38
0.0570	103.13	4.49	-0.81	4.57	103.31
v=7.8458					
0.0027	103.43	3.23	-0.30	4.28	103.45
0.0190	103.28	4.20	-1.14	7.16	103.40
0.0380	103.18	5.09	-1.44	7.33	103.39
0.0570	102.99	6.45	-1.50	6.65	103.34
v=15.6915					
0.0027	103.41	3.56	-1.24	13.19	103.42
0.0190	103.14	7.89	-3.42	20.74	103.40
0.0380	102.92	11.41	-3.21	15.80	103.39
0.0570	102.51	16.72	-2.76	10.93	103.31
v=31.3830					
0.0027	103.37	5.84	-6.40	85.43	103.44
0.0190	102.87	20.94	-5.71	39.36	103.40
0.0380	102.26	38.89	-4.31	21.38	103.35
0.0570	101.63	56.88	-3.49	14.04	103.31

Source: 10,000 repetition simulation

Table 4. Consumption and savings under simulated catastrophic shock

q	Cornish-Fisher 1 terms			Cornish-Fisher 2 terms			Cornish-Fisher 3 terms			Cornish-Fisher 4 terms	
	x_I	$Y_I - x_I$	Error	x_I	$Y_I - x_I$	Error	x_I	$Y_I - x_I$	Error	x_I	$Y_I - x_I$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	101.7239	-1.7239	0.01%	101.7311	-1.7311	0.00%	101.73118	-1.73118	0.000%	101.73118	-1.73118
Catastrophic Scenarios											
$v = -5.2305$											
0.00270	101.71253	-1.71253	0.00740%	101.71992	-1.71992	0.00001%	101.71993	-1.71993	0.00000%	101.71993	-1.71993
0.01900	101.68219	-1.68219	0.00863%	101.69078	-1.69078	0.00004%	101.69082	-1.69082	0.00000%	101.69082	-1.69082
0.03800	101.62854	-1.62854	0.00995%	101.63840	-1.63840	0.00009%	101.63849	-1.63849	0.00000%	101.63849	-1.63849
0.05700	101.56261	-1.56261	0.01117%	101.57365	-1.57365	0.00013%	101.57377	-1.57377	0.00001%	101.57378	-1.57378
$v = -7.8458$											
0.00270	101.71715	-1.71715	0.00796%	101.72508	-1.72508	0.00003%	101.72511	-1.72511	0.00000%	101.72511	-1.72511
0.01900	101.64151	-1.64151	0.01051%	101.65185	-1.65185	0.00017%	101.65201	-1.65201	0.00001%	101.65202	-1.65202
0.03800	101.59092	-1.59092	0.01282%	101.60346	-1.60346	0.00028%	101.60373	-1.60373	0.00001%	101.60374	-1.60374
0.05700	101.49437	-1.49437	0.01631%	101.51026	-1.51026	0.00042%	101.51066	-1.51066	0.00002%	101.51068	-1.51068
$v = -15.6915$											
0.00270	101.70518	-1.70518	0.00891%	101.71395	-1.71395	0.00014%	101.71408	-1.71408	0.00001%	101.71409	-1.71409
0.01900	101.57236	-1.57236	0.02072%	101.59180	-1.59180	0.00128%	101.59295	-1.59295	0.00013%	101.59308	-1.59308
0.03800	101.46051	-1.46051	0.04054%	101.49949	-1.49949	0.00156%	101.50067	-1.50067	0.00038%	101.50105	-1.50105
0.05700	101.25376	-1.25376	0.04468%	101.29510	-1.29510	0.00334%	101.29818	-1.29818	0.00026%	101.29844	-1.29844
$v = -31.3830$											
0.00270	101.68280	-1.68280	0.01616%	101.69717	-1.69717	0.00179%	101.69863	-1.69863	0.00033%	101.69896	-1.69896
0.01900	101.43371	-1.43371	0.06196%	101.48539	-1.48539	0.01028%	101.49428	-1.49428	0.00139%	101.49567	-1.49567
0.03800	101.12921	-1.12921	0.11698%	101.22563	-1.22563	0.02056%	101.24280	-1.24280	0.00339%	101.24619	-1.24619
0.05700	100.81328	-0.81328	0.17091%	100.95495	-0.95495	0.02924%	100.97977	-0.97977	0.00442%	100.98419	-0.98419

1 Estimation error respect to the Cornish-Fisher expansion with 4 terms

Source: 10,000 repetition simulation

Table 5. Under-savings for different estimations of a catastrophic scenario USD billion

q	Cornish-Fisher 1 terms		Cornish-Fisher 2 terms		Cornish-Fisher 3 terms	
	Annual	NPV	Annual	NPV	Annual	NPV
Calibrated						
Catastrophic Scenarios						
$v=-5.2305$						
0.0027	7.80	65.02	0.01	0.09	0.00	0.00
0.0190	9.10	75.83	0.04	0.35	0.00	0.00
0.0380	10.49	87.42	0.09	0.79	0.00	0.00
0.0570	11.78	98.14	0.14	1.14	0.01	0.09
$v=-7.8458$						
0.0027	8.39	69.94	0.03	0.26	0.00	0.00
0.0190	11.08	92.34	0.18	1.49	0.01	0.09
0.0380	13.52	112.64	0.30	2.46	0.01	0.09
0.0570	17.20	143.30	0.44	3.69	0.02	0.18
$v=-15.6915$						
0.0027	9.39	78.29	0.15	1.23	0.01	0.09
0.0190	21.85	182.05	1.35	11.25	0.14	1.14
0.0380	42.74	356.19	1.64	13.71	0.40	3.34
0.0570	47.11	392.57	3.52	29.35	0.27	2.28
$v=-31.3830$						
0.0027	17.04	141.99	1.89	15.73	0.35	2.90
0.0190	65.33	544.40	10.84	90.32	1.47	12.21
0.0380	123.34	1,027.82	21.68	180.65	3.57	29.79
0.0570	180.20	1,501.66	30.83	256.91	4.66	38.84

Source: Own estimation based on simulation