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## Does the Fiscal Theory of the Price Level explain US postwar behaviour?\*

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#### Abstract

We implement a quantitative empirical test of the fiscal theory of the price level (FTPL) model via indirect inference, comparing it to a standard New Keynesian model. The FTPL alternative creates a serious instability problem because it triggers a 'doom loop' in which inflation pushes up interest rates which in turn pushes up deficits and debt and so inflation. Without some sort of endogenous feedback response this instability prevents the model from solving in finite space; a fiscal 'whatever it takes' commitment to prevent unstable paths fails to create stability because no steady state inflation solution exists. This is the case whether we embed FTPL in an otherwise conventional New Keynesian model or in a classical RBC model. We then went on to look for endogenous responses of government spending and tax to the economy — a Fiscal Rule — which might render the FTPL model sufficiently stable to be testable. We found such a Rule: in it spending stabilises the output gap while tax responds to inflation, with an inflation cap ('tax reform') — such that if inflation exceeds some high rate it overrides the FTPL terminal condition by inserting whatever terminal surplus will cap inflation at this rate. With this rule in place we can solve an RBC version of the model without triggering intolerable volatility; this version is on the test rejection borderline on our full postwar sample whereas the standard New Keynesian model passes the test easily.

<sup>\*</sup>We are grateful for comments on previous versions of this paper from Peter Spencer, Jose Torres, Fabio Canova, John Cochrane, and participants at conferences in York in March 2024 and in Kathmandu in September 2024. We are responsible for any remaining problems and inaccuracies in this version.

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#### 1 Introduction: testing the Fiscal Theory of the Price Level

The Fiscal Theory of the Price Level (FTPL) asserts that inflation is determined by fiscal policy. Under FTPL the price level, or inflation, is determined by the need to impose fiscal solvency by equating the market value of outstanding government debt to the expected present value of exogenous future primary surpluses, a situation that is also termed 'fiscal dominance'. The FTPL is a necessary consequence for fiscal solvency of exogenous primary government surpluses, also called 'non-Ricardian' fiscal policy. Orthodox theory asserts that inflation is determined by monetary policy as expressed, for example, by a Taylor Rule (Taylor 1993). The government must then run a primary fiscal surplus to ensure solvency. This policy is known as 'Ricardian' and involves 'monetary dominance'. Notice that both theories assume that governments maintain solvency; it is simply the means to do so that differs.

A large literature has debated the FTPL assertion both from the viewpoint of theory and empirical evidence. Recently, Cochrane (2022, 2023), a longtime FTPL protagonist, has contributed further to this with a new book and an accompanying paper. He suggests that the FTPL explains US inflation history, notably since the great financial crisis, when interest rates were forced to the zero lower bound.

We are therefore faced with two rival theories of what causes inflation. The debate over them goes back at least three decades, and is still unresolved. There is a large literature attempting to test them in a variety of ways, which we survey below. Yet it has proved impossible to settle the debate through these tests because they cannot effectively distinguish between the two theories. This is surprising, considering how radically they differ on the underlying cause of inflation and therefore on the general behaviour of the economy this implies. In this paper we carry out a new simulation-based test of the two theories which we think offers a way of both distinguishing the two models' behaviour and formally checking which of them, if either, can match the US postwar data.

For the orthodox Ricardian model, we employ a standard New Keynesian model developed in a series of papers by Le et al. (2011, 2016b and 2021) which we detail below. For the FTPL theory, we embed it in two models, in each of which it is assumed to be operating to determine inflation and either of which gives a coherent account of its implied behaviour: first, in an appropriately modified standard New Keynesian model and second, in a classical Real Business Cycle (RBC) model; their different specifications can then be tested. The equation listing of the standard New Keynesian model is in Appendix A. The required modifications for FTPL to hold are described below. For the orthodox New Keynesian model fiscal policy must commit to providing a surplus sufficient to pay interest on the debt in the long run — 'Ricardian policy'. For the FTPL, there is no widely agreed specification. In the basic version of FTPL the price level jumps at some point to ensure solvency. This in turn implies that inflation must proceed at a sufficient rate to cause this rise in the price level — so turning FTPL into a theory of inflation. In our suggested specification we impose this implied inflation rate on the rest of the model which may follow one of the two versions noted above, either assuming price-rigidity as in the New Keynesian approach or assuming full price flexibility as in the RBC approach.

We use a method widely used to test structural macroeconomic models: the method of indirect inference. In this test the structural model is specified in full, including its error and other exogenous variable processes, as implied by the data and the model over the chosen sample period. The model is then simulated over the sample period many times, via repeated bootstrapping of the innovations in these processes, to create many 'potential histories' of what could have happened over the period according to this model. The behaviour of a few selected endogenous variables is estimated for each history in a descriptive summary or 'auxiliary model' which can take a wide variety of forms, including moments, scores, impulse response functions, and, as in this paper, the coefficients of a vector autoregression (VAR). Using Monte Carlo experiments (Meenagh et al, 2019) it is found that the test performs with approximately equal power whatever auxiliary model is chosen. The test consists of comparing the simulated auxiliary model with that estimated on the sample data. The simulated values of the structural model provide a joint probability distribution of the model coefficients. This can be used to determine whether the data-based coefficients lie within this distribution with a joint probability of less than some threshold, e.g. 5%. The data simulated from the structural model is obtained using the Fair and Taylor (1983) nonlinear algorithm, a method widely used from the 1980s onwards to solve rational expectations models. The algorithm solves the model forwards up to a distant terminal date, T, at which point the model reaches a steady state — the distance ahead of T can be varied until the solution at t is no longer sensitive; with large T, here 50 quarters ahead, sensitivity typically disappears<sup>1</sup>. The terminal condition for the FTPL model, in which the primary surplus is exogenous, differs clearly from that of the Orthodox New Keynesian model, where inflation reaches the target in the monetary policy rule and the primary fiscal surplus pays the real debt interest from then on. For FTPL, the terminal price level is set by the need to devalue the accumulated debt from prior primary surpluses or deficits to a level at which real debt interest can then be paid by the terminal and future primary surplus; the implied inflation rate from t up to T is then whatever will produce this price level; this inflation rate then becomes the expected rate at T, and actual inflation is then found via solving the model backwards to t. Similar considerations apply to using the RBC model.

We noted earlier that FTPL requires the price level to rise in order to achieve fiscal solvency. In obtaining the simulated data for our test procedure it is assumed that the distribution of disturbances drawn from is the same as in the past. The FTPL model specifies that past fiscal deficits give rise to future price rises in a way that devalues the debt that will be accumulated. In effect, it requires that such rises have occurred in the data period used to estimate the models; thus the data behaviour should reflect such rises in the past, accompanying past deficits. It is this pattern of past price behaviour accompanying past deficits that enables the power of our tests.

In both types of basic model that we employ — New Keynesian and RBC — we found the FTPL version gave strikingly different simulation behaviour. Given that the nominal primary surplus process was nonstationary, it generated a wide spread of terminal debt and hence inflation; this terminal inflation in turn was reflected in nominal interest rates from t to T, which contributed to yet larger terminal debt and so inflation. The resulting iteration between inflation and debt interest created a 'doom loop' which prevented the model from solving for finite values. This 'doom loop' is caused in the model by monetary policy embodying an estimated interest rate response to inflation of unity or more, sufficient to maintain a real interest rate that ensures output convergence to equilibrium; this leads to debt rising due to increasingly large debt service payments. This 'loop' embodies an above-unit root, the presence of which in rational expectations models is well-known to prevent a solution — it would also do so, were we using the Dynare solution method (e.g. Collard and Juilliard, 2001), also widely used.

Our standard solution algorithm thus fails to find a solution because of the FTPL model's fundamental instability; the algorithm requires that the solution is stable, and this is usually achieved by eliminating the unstable solution paths by some restriction on the model. Thus in the standard Ricardian model, the Taylor Rule prevents any exploding inflation path by raising interest rates more rapidly so that output falls until the inflation is stopped; thus the path cannot be a solution because of this transversality condition enforced by monetary policy. However in the FTPL model, as fast as the Taylor Rule raises interest rates fiscal policy raises debt and so inflation; in effect fiscal policy prevents monetary policy from stopping the path, so that no stable path is allowed and the unstable path cannot be prevented. Thus there is no viable solution. We show that there would be if the Taylor Rule responded without raising interest rates. However, this would imply the solution would fail to converge on output equilibrium and so the model would not have a steady state, so being misspecified. Notice that these problems can be avoided by the sort of regime switching suggested by Leeper et al. (e.g. Leeper and Leith, 2016); this is because the FTPL regime episode is followed by another eventually where the model converges. In a very recent paper Smets and Wouters (2024) have investigated a switching model for the postwar period, in which these problems are thereby avoided. However, we are testing the proposition advanced by Cochrane that the model applies across the whole sample period, so no future switch can produce convergence. In a final attempt to solve the FTPL model for a stable path, we looked for a stable solution enforced by a fiscal transversality condition in which any unstable path would be closed down by a 'whatever it takes' response of fiscal contraction. However, we found that no resulting stable path would solve the model because the exogenous primary deficits required higher stady state inflation than this permitted. We found this for both our New Keynesian version of the FTPL (replacing the Taylor Rule with a simple real interest rate response to the output gap, the minimum needed to ensure output convergence as noted above) and for our RBC version.

<sup>&</sup>lt;sup>1</sup>It is also possible to solve these models with Dynare (see e.g. Juilliard, 1996; Collard and Juilliard, 2001). This computes the model steady state and solves for the model effects of shocks around this by taking an  $n^{th}$  order Taylor series expnasion. This method is routinely applied to the orthodox model which has a defined steady state inflation rate. However, under FTPL with a nonstationary nominal primary surplus as found in the data, the steady state nominal surplus, and so also the steady state level of debt and its growth rate are all moving over time so there is a constantly changing solution for inflation, implying that inflation and interest rates have a unit root. Dynare therefore cannot be used.

This failure of the FTPL regime to generate a stable solution over the whole period led us to search for versions of FTPL where fiscal policy responds endogenously to inflation and output rather than being simply exogenous as in the 'basic' FTPL theory. We found a version embedded within the RBC model that could achieve reasonable stability, but this turned out to be on the borderline of rejection by our test, whereas the standard New Keynesian model was easily accepted. We concluded by reviewing some recent work on fiscal policy well short of FTPL and within orthodox models of the economy, which implies that fiscal activism can much improve the economy's behaviour by supplementing the stabilising efforts of monetary policy.

From this bare summary of our paper we now proceed to review earlier work and describe our procedures and findings in detail.

Cochrane's suggestion comes after a long and heated debate in which a variety of tests have been put forward, though with little success. The FTPL has been set out and developed in Leeper (1991), Sims (1994), Woodford (1998a, 2001) and Cochrane (2001, 2005) — see also comments by McCallum (2001, 2003) and Buiter (1999, 2002), and surveys by Kocherlakota and Phelan (1999), Carlstrom and Fuerst (2000) and Christiano and Fitzgerald (2000). Empirical tests have been proposed by Bohn (1998), Canzoneri et al. (2001) and Bajo-Rubio et al. (2014). Loyo (2000), for example, argues that Brazilian policy in the late 1970s and early 1980s was non-Ricardian and that the FTPL provides a persuasive explanation for Brazil's high inflation during that time. The work of Tanner and Ramos (2003) also finds evidence of fiscal dominance for the case of Brazil for some important periods. Cochrane (1999, 2005) argues that the FTPL with a statistically exogenous surplus process explains the dynamics of U.S. inflation in the 1970s. This appears to be similar to what we see in the UK during the 1970s. In addition, there has been extensive work on FTPL in monetary unions<sup>2</sup> and specifically on European economies<sup>3</sup>.

Nonetheless, a number of problems beset empirical tests of the FTPL. Cochrane (1999, 2001, 2005, 2022 and 2023) has noted that there is a basic identification problem affecting tests of present value relations such as the intertemporal budget constraint (IGBC). This is a point also noted by Bajo-Rubio et al. (2014). The IGBC is a logical implication of (and is derived from) the government budget constraint which is an identity. Whether policy is non-Ricardian, as in the FTPL, or Ricardian as in orthodox theory, the IGBC must be satisfied. As a result, conventional tests of the long-run implications of the IGBC cannot distinguish between Ricardian and non-Ricardian policies. In the FTPL fiscal policy is exogenous which forces inflation to deliver fiscal solvency and hence satisfy the IGBC. The IGBC can also be consistent with an exogenous monetary policy that determines inflation in the 'orthodox' way, with Ricardian fiscal policy endogenously responding to the government budget constraint to ensure solvency given that inflation path; we will call this the Orthodox model. A further identification problem is that equations that appear to reflect the FTPL and are used to 'test' it, could also be implied by the Orthodox set-up. To put it more formally, the reduced form, or solved representation of an FTPL model, may in form be indistinguishable from that of an orthodox model; this is true of both single-equation implications of the model and complete solutions of it.

Bohn (1998) has proposed a 'backward-looking' feedback rule where the government is Ricardian with the government primary surplus reacting positively to lagged debt; this can be tested by checking the cointegration of revenue and spending with a unit coefficient. In the 'forward-looking' version due to Canzoneri et al. (2001), the future level of debt should react negatively to the current primary surplus. Here the test is of the impulse response function of debt to the surplus. This version too requires cointegration to hold.

The cointegration test needs in principle to include inflation-tax revenues. But these revenues include the reduction in the value of debt due to inflation. This inflation is precisely what is generated by FTPL to ensure solvency. As solvency is always assured in equilibrium by either Ricardian or non-Ricardian FTPL conditions, cointegration must hold for either condition; and so, while interpretation is possible, there is, strictly speaking, no way of distinguishing which condition is causing this to happen.

As a result of this critique, some authors — for example Bianchi (2012), Bianchi and Melosi (2013) — have abandoned the idea of testing whether the FTPL was, or was not, prevailing in an episode. Instead they have assumed that various possible combinations of fiscal and monetary policy were operating at different times, with switches between them occurring according to some Markov process. They then estimated, usually by Bayesian methods, which combinations were operating and when. In the context of the US, to which most of this work is devoted and where the constitution divides power between three branches of government, this idea, that at any one time there is not necessarily a definite 'regime' operating, but rather

<sup>&</sup>lt;sup>2</sup>See for example Sims (1997), Woodford (1998b), Bergin (2000), Canzoneri et al. (2001), and Bajo-Rubio et al. (2009).

<sup>&</sup>lt;sup>3</sup>See Mèlitz (2000), Afonso (2000) and Ballabriga and Martínez-Mongay (2003) and Bajo-Rubio et al. (2014).

a constant process of flux between transitory regimes, may well seem plausible.

After a long period in which the FTPL has been given little credence Cochrane (2022 and 2023) has returned to its defence. On p500 of Cochrane (2023) he says:

"Observational equivalence goes both ways. Any rejection of fiscal theory from equilibrium time series also rejects other theories of equilibrium formation. Observational equivalence opens the door to understanding any sample equally via fiscal theory as via new-Keynesian models. It guides us to find and examine the identifying assumptions of any proposed test. It guides us to look to institutions, regimes, commitments, and statements by fiscal and monetary authorities about how they operate, commentary on how people expect them to operate, narrative approaches to historical events, and times of regime change or construction."

Hence Cochrane (2022) argues that the models can best be tested by whether they give plausible accounts of historical episodes. He does this from the FTPL perspective, where the government is fiscally dominant, and accounts for inflation in each episode in terms of its policies towards primary surpluses. This 'narrative approach' seems attractive but has the drawback that 'plausibility' is difficult to achieve in an objective way. As an illustration, consider his account of why the introduction of low inflation targets introduced in the early 1980s for independent central banks led to the sharp fall in inflation. According to Cochrane, this was due to governments agreeing to back these new targets with higher primary surpluses, so that expected inflation melted away without the need for any monetary tightening via higher interest rates or induced recession. However, the Orthodox model can also provide a narrative rationale of these events. This is that the new monetary targets gave a credible commitment to prevent above target inflation by raising interest rates as necessary; by implication the government would have to produce primary surpluses to meet solvency in a Ricardian way. Inflation fell in response to the new rules, and fiscal policy had to adapt too.

A second episode, the recent aftermath of the 2008 banking crisis, provides a further example. According to the FTPL, the collapse of interest rates to the zero lower bound effectively made primary surpluses unnecessary as the solvency constraint was non-binding and hence no inflation was needed even though there was a massive printing of base money via 'QE'. An Orthodox account dwells on the role of a new regulatory assault on the banks that made it expensive to lend, with the result that broad money and credit hardly rose, demand weakened and there was no support from fiscal policy.

These rival accounts illustrate the limitations of the narrative approach. In the absence of quantitative support from models that specify the exact connection between surpluses and expected future surpluses, it is difficult to see any objective means for judging between them. We therefore propose an alternative approach, namely, to formulate the rival accounts of these and other episodes into quantitative model predictions whose accuracy can be properly measured. We aim to do this via tests based on indirect inference in which estimates of an auxiliary model using observed, or actual, data are compared with estimates using data simulated from each model, where the auxiliary model is chosen to capture observed quarterly U.S. data 1959-2017. This requires simulated data of the whole economy from each model which should, in principle, permit the full identification of the two rival models, thereby enabling a clear and powerful test of each theory. We distinguish between the two models through their implied terminal conditions. To carry out the test we need to specify a full dynamic model for each theory. We base this on the full model of Le et al. (2021) which has been estimated on US data across the whole postwar period from 1959. This model embodies Orthodox theory and fits the data behaviour well. Our task in this paper is to repurpose it as an FTPL model and test whether this too can pass our indirect inference test. In order to respond to Cochrane's charge that this is a futile task since the rival theories are observationally equivalent and so both will match the data equally well, we need to find alternative identifying restrictions which will clearly distinguish between the rival models. These restrictions may be provided through the contrasting and distinct terminal conditions imposed by each theory; it is these that create identification and different reduced forms. This is because Orthodox theory forces inflation and interest rates to be consistent with the Taylor Rule, whereas FTPL forces them to be consistent with terminal debt and the primary surplus.

An earlier effort by Fan et al. (2016) to apply these methods to the UK for the 1970s and 1980s tended to support Cochrane's assertion. They found evidence that the FTPL model did have influence during both periods in a weighted joint model of both while the FTPL model on its own could not be rejected, and nor could the Orthodox model. The weight on FTPL in the best-fitting joint model fell from about a half in the 1970s to around 15% in the 1980s, as fiscal policy was sharply tightened. These UK results suggest that it is hard to tell the models apart, even using the full powers of indirect inference on complete models; the need to use a weighted combination of them suggests that each regime may have been operating during different parts of each decade. We wish to discover whether using US data covering the majority of the postwar period can provide a clearer distinction between the two models. In effect we are using each framework to create a quantitative, model-based, account of the historical episodes encountered in the postwar period, in place of the qualitative accounts illustrated above. By using such identifying restrictions we hope to be able to determine which model provides the best explanation for these episodes and thereby move the debate forward.

In principle, the use of indirect inference to distinguish between the rival theories is straightforward. In practice we were confronted by an unexpected issue which was difficult to resolve and took us on a long and complicated journey. The reason for this was that while there was no difficulty in obtaining simulated data for the Orthodox model based on a conventional New Keynesian model with Ricardian fiscal policy, it proved difficult to obtain simulated data for the non-Ricardian FTPL version.

The problem was that the FTPL triggers a 'doom loop' in which inflation pushes up interest rates, deficits and debt which then raise inflation thereby starting the sequence again. Without some sort of endogenous feedback response this instability prevents the model from solving in finite space. We found that embedding FTPL in a classical RBC model gave the same result. This led us to examine alternative versions of the FTPL model in which the primary surplus was constrained in some way and which might then render the FTPL model sufficiently stable to be testable. We first considered an inflation cap ('tax reform') such that if inflation exceeds some high rate it overrides the FTPL terminal condition by inserting whatever terminal surplus will cap inflation at this rate. We impose FTPL with the inflation cap in both the New Keynesian model and then again in the RBC model. Both versions exhibit massive output and inflation volatility, regularly exceeding output and inflation ranges in the data, and implying highly frequent 'tax reform' — i.e. invocation of the cap — rendering them too incredible as models to be worth testing. Secondly, we looked, again in both model versions, for a Fiscal Rule accompanying FTPL with endogenous fiscal responses in which government spending stabilised the output gap while tax responded to inflation. The New Keynesian model version could not be stabilised sufficiently to be credible, again requiring highly frequent 'tax reform' and still creating massive volatility. However, although still generating high and volatile inflation, the RBC version required no tax reform at all and so might be considered potentially believable. When tested, it fell only slightly short of the acceptance threshold, though still leaving it far less probable than the orthodox model. The need to include such constraints on the FTPL model could, of course, in any case be regarded as a rejection of the strict non-Ricardian version of it in which the primary surplus is strongly exogenous.

The paper is set out as follows. In section 2 we describe the the rival Ricardian (Orthodox) and non-Ricardian (FTPL) versions of the benchmark New Keynesian model and of the alternative classical (RBC) model. We also describe in detail the price/wage setting structure of the benchmark New Keynesian model. In section 3 we discuss the required solvency conditions for each theory and the processes generating primary surpluses. In section 4 we set out the method of indirect inference and its application to testing the FTPL. Our initial results are given in section 5. As explained, these reveal a different issue with the FTPL from what we had anticipated. In section 6 we try to resolve this problem by using weaker versions of the FTPL involving the additional fiscal rules. Our conclusions are presented in section 7.

#### 2 The economic models

#### 2.1 An overview of their Ricardian and non-Ricardian representations

#### 2.1.1 The New Keynesian model

The benchmark model we use to represent both the FTPL and the Orthodox theories is the New Keynesian model. When we come to looking for alternative frameworks as part of our search for viable FTPL formulations, we also consider using a classical RBC-style model. Before giving details of the versions of the New Keynesian and classical models that we use, we reflect on their features that affect the FTPL.

The choice of New Keynesian model reflects its success in numerous studies in representing the U.S. economy. Its key features as far as FTPL is concerned are its price setting via a Phillips curve and its monetary policy framework via a Taylor Rule. Given the demand from households, firms and government, prices and wages adjust to any excess demand over equilibrium supply at equilibrium marginal costs, causing inflation via the Phillips Curve. Necessary control of demand is carried out by the central bank setting interest

rates through a Taylor Rule in order to push the economy to an inflation target and to equilibrium supply. With a coefficient on inflation in the Taylor Rule greater than unity, demand and hence inflation is controlled by raising interest rates until inflation returns to target and demand falls back to equilibrium.

As Cochrane has noted, behind this framework there is an implicit fiscal solvency issue that must prevent rising inflation and hence rising interest rates from causing government debt to also rise. This is provided by a Ricardian fiscal policy in which the government commits to raising future taxes to pay for its spending and pay interest on its debt, such that at the terminal date the present value of future taxes equals the real value of the debt. This then leads households to practice Ricardian equivalence. For example, as lump-sum taxes rise, households know their permanent income is unchanged as future taxes will be lowered to give the same present value of taxation. Hence they consume the same and their savings fall by the amount of the tax rise, and vice versa when taxes fall. Alternatively, solvency can be achieved under FTPL via inflation — in a non-Ricardian manner. For example, instead of assuming that lump-sum taxes are set to reduce government debt, pay for spending and ensure that any terminal real debt is paid for by the present value of future taxes, it could be assumed that taxes are set independently of debt and spending, with terminal prices intended to devalue terminal debt to a real value that is then payable out of the present value of future taxes which produce a primary surplus. Notice that there must still be a terminal surplus as otherwise the price level (and implied inflation up to then) would have to go to infinity, which is impossible, and not therefore a valid solution. Effectively, therefore, FTPL trades off terminal prices against terminal future surpluses in order to enjoy unconstrained borrowing up to that point.

We may note that with FTPL, households no longer have Ricardian equivalence. FTPL implies that expected inflation must deliver the terminal price level required for the solvency trade-off. This in turn determines the expected inflation path in the model. This expected inflation enters the Phillips Curve to determine current inflation each period. Household consumption does not react to tax changes directly. When tax rates change, causing changed borrowing, households understand that inflation will change too, thereby changing real interest rates and hence demand. Another way of putting this is that as the current tax rate falls, for example, the future 'inflation tax' must rise to give an offsetting present value; this higher inflation tax reduces the real interest rate, causing a rise in consumption demand, cet. par. This in turn raises inflation and excess demand, inducing reactions in interest rates.

#### 2.1.2 The classical (RBC) model

It would also be possible to embed the FTPL in a classical model with full price/wage flexibility. An example of such a model would be an RBC model in which output is set by supply via a production function dependent on labour supply, capital and productivity. Labour market clearing determines labour supply; productivity is an exogenous nonstationary process. Demand comes from consumption, investment and (exogenous) government spending; consumption and investment respond to productivity via permanent income and the expected marginal return on capital. Demand is brought into equality with supply by the real interest rate. This clears the market in bonds, thereby clearing the goods market via Walras' Law. FTPL then determines inflation and nominal demand growth. In an RBC model inflation is directly created by money supply policy via a Quantity Theory set-up, in which money is demanded as cash-in-advance. The central bank is expected to create money as needed for the model equilibrium, including the nominal interest needed to deliver the equilibrium real interest rate; it enforces the zero lower bound on interest rates by ensuring that inflation delivers negative real rates when required. Another way of putting this is that nominal demand is set by the money supply, and money demand is set via the Quantity Theory. In this framework the FTPL determines the price level by setting the growth rate of money to deliver the necessary inflation tax. The market-clearing nominal interest rate in this model is the market-clearing real rate,  $r^*$ , plus expected inflation:  $R_t = r_t^* + E_t \pi_{t+1}$ , where  $r_t^*$  is solved from the identity  $y_t = c_t + i_t + g_t$ , which involves the three elements in aggregate demand in the model. The essential point is that the nominal interest rate must respond to this solution for  $r^*$  and for inflation.

#### 2.2 The specification of the benchmark New Keynesian Model

The Orthodox model that forms the basis for testing the FTPL for the U.S. is a standard New Keynesian model. It is taken from Le et al. (2021) which was designed to match the full U.S. postwar data. Previous

models such as Le et al. (2011, 2016b) matched well the behaviour of periods within this full sample but failed to match the full sample period behaviour. The Le et al. (2021) model that we use here allows for state-dependent price and wage setting in the general equilibrium framework proposed by Le et al. (2016b) which was developed from Le et al. (2011) to deal with the zero-bound-interest rate era. This in turn had extended Smets and Wouters (2007) by allowing for 'hybrid' pricing in which a fraction of goods markets are flexprice while the rest set prices for longer durations; similarly with labour markets. These fractions or weights are included in the model used here. In addition to these frictions in labour and goods markets, the model incorporates financial frictions as proposed by Bernanke et al. (1999) and allows for cheap money collateral as in Le et al. (2016b) in order to make monetary policy effective via unconventional monetary measures (base money printing via Quantitative Easing, QE) even at the zero lower interest rate bound. This is the way that QE works: by providing more cheap collateral in the form of money, which lowers the cost of credit, so expanding the supply of money and credit used by entrepreneurs. The model also includes a variant monetary policy based on QE when the ZLB is triggered (a full model listing can be found in Appendix A, reproduced from Le et al., 2016b).

We require a state-dependent formulation of this model. The focus in obtaining this formulation is the Calvo structure of price/wage determination to which we now turn in the rest of this section. In the previous studies by Le et al. (2011, 2016b) it was assumed that imperfectly competitive firms and labour unions decide on changing their prices/wages based on Calvo fixed probabilities, but there were fixed weights on the fractions of goods and labour markets where there is 'long' duration of more than one quarter, and those in a 'short duration'/flexprice sector where prices and wages change continuously each quarter. That is, we assumed the structure of price/wage durations is fixed. Here we relax this assumption and assume this structure changes with the state of the economy, i.e. these durations vary as more firms/labour unions decide, in the face of aggregate shocks, to change their prices and wages continuously; and so shift from the long to the short duration sector. The short duration sector we describe as 'flexprice' (FP) since, in a quarterly context, it is continuously keeping prices equal to marginal costs plus the same constant mark-up as in the long-duration sector. The long duration sector we call 'New Keynesian' (NK) since it conforms to the Calvo sticky-price model.

For an imperfectly competitive firm, or for a labour union setting wages under imperfect competition, we interpret the probability of changing the price or wage as coming from the distribution of idiosyncratic shocks to the equilibrium relative price for the product or labour service. We assume these agents will only change prices/wages if the shock is larger than some particular value, representing the menu cost of changing prices: below this point, as Calvo (1983) puts it, the signal to change prices 'lights up'; they would rather stabilise the price in order to insure their customers against uncertainty, which is how we may interpret the menu cost. However, above this point the cost of providing this insurance is too great compared with the benefit it gives. We assume that this idiosyncratic distribution's variance is related to the size of recent inflation shocks to the economy, denoted by  $\Pi$  and measured by a moving average of inflation. These shocks to other prices set off price shocks to particular markets because they are shocks to the product's relative price. Thus, if prices in general, i.e. other prices, have moved substantially then demand and supply for the particular product must also be affected; hence, as recent inflation rises, so does the variance of the idiosyncratic distributions being used by price setters. This implies that the critical shock size is at a lower percentile of this more volatile distribution. This is illustrated in Figure 1. This percentile is then the Calvo percentage of firms not changing their price. This Calvo parameter is therefore a reduced form function of the idiosyncratic distribution, which in turn depends on  $\Pi$ . We do not derive this parameter but rather we estimate a general form from the macro data.

Hence the probability of not changing price is reduced by  $\Pi$  and so too is the Calvo parameter. As a result more sectors will become flexprice (i.e. have an overall duration of 1 quarter) and in the remaining sectors the Calvo parameter may fall. We should note, however, that the Calvo parameter for the sticky-price sector may actually rise as the sectors closest to the short duration sector migrate to it, leaving behind the sectors that have higher Calvo parameters. This 'abandonment effect' may more than offset the reduction effect on these remaining sectors' Calvo parameters, C, which we estimate in the usual way with the other model parameters.

Notice that in all this we are not changing our basic assumption that the macro shocks are drawn from constant distributions and are known to all agents. We assume that the idiosyncratic distributions, known only to the agents concerned, change over time as the draws from these macro distributions become

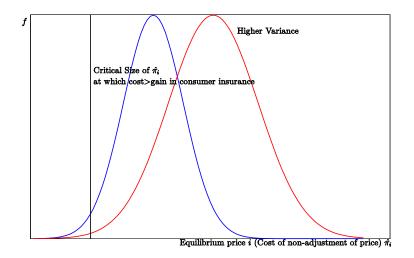


Figure 1: Distribution of Idiosyncratic Shocks

by chance larger or smaller for a substantial period of time and so affect inflation. These draws disturb the micro distributions because a succession of large macro shocks disturbing inflation create uncertainty about micro conditions. For a simple example one may think of the labour market in conditions where unemployment has been high for some time and wages have been falling: plainly the union's members will in some cases have lost jobs and in others fear they came close to it, while generally the union will face high member uncertainty about likely job offers. Also, while macro shock distributions are constant, the model wage/price parameters are changing so that the model is now nonlinear — its behaviour is changing in response to the history of shocks. This nonlinearity will feed back into macro variables' volatility which in turn will react on the wage/price parameters.

In making our assumptions about the parameters driving these shifts we look for a function relating wage/price parameters to the past history of inflation. A natural candidate is the square of a moving average of inflation over the recent past, say four years; this is our II. It allows for offsetting effects where inflation increases have been later reversed by inflation falls; but it will strongly register a sustained rise in inflation or a sustained fall into deflation. The response of the short-duration sector weights to this are determined empirically, by indirect inference estimation. The weights on the NK sectors are calculated according to the function  $\omega^i = \exp(-\theta_i \Pi)$ , where  $i = \pi$ , w. We add this price/wage setting state-dependence to the model of Le et al. (2016b). The resulting nonlinear, shifting-weights, model is then estimated and evaluated using the method of Indirect Inference on unfiltered US quarterly data from 1959–2017<sup>4</sup>.

This model was estimated and tested in Le et al. (2021). It passed the test comfortably with a pvalue of 21%. As explained more fully below, we obtain a FTPL version of the model by altering the terminal conditions on the Orthodox model to reflect the non-Ricardian assumptions about fiscal policy. Effectively government spending and taxation decisions which are chosen by exogenous political processes without regard for solvency and accumulate through the government budget constraint to create a terminal nominal public debt. As already explained, this has to be rendered sustainable by a terminal price level that produces solvency which requires that the debt's real value must equal the terminal present value of the future primary surpluses that the government has committed to. The terminal price level translates into an average inflation rate from the initial period to the terminal date; this acts as the expected inflation rate from the initial period, determining actual inflation via the Phillips curve.

<sup>&</sup>lt;sup>4</sup>For a description of the Indirect Inference method see Le et al. (2016b).

#### 3 Solvency conditions under Ricardian and FTPL assumptions

#### 3.1 Solvency under Ricardian assumptions

We now explain how the Ricardian and the non-Ricardian FTPL models differ in their treatment of the government budget solvency conditions. The nominal government budget constraint (GBC) may written as

$$B_t = -P_t s_t + (1 + R_t) B_{t-1}$$

where  $B_t$  is nominal debt,  $s_t$  is the real primary surplus,  $P_t$  is the price level and  $R_t$  is a nominal interest rate. The real GBC is then

$$b_t = -s_t + (1+r_t)b_{t-1}$$

where  $b_t$  is real debt and  $r_t = \frac{1+R_t}{1+\pi_t} - 1$  is the real interest rate ( $\pi_t = \text{inflation}$ , which for simplicity we treat here as expected to remain constant). This can be rewritten for constant r as

$$b_t = E_t \sum_{i=1}^N \frac{s_{t+i}}{(1+r)^i} + \frac{b_{t+N}}{(1+r)^N},$$

The crucial condition for solvency to hold is that the transversality condition for the inter-temporal GBC is satisfied, namely  $\lim_{N\to\infty} E_t \frac{b_{t+N}}{(1+r)^N} \rightarrow 0$ . This implies solvency with debt growing by less than the real interest  $\infty$ 

rate. It also implies that  $b_t = E_t \sum_{i=1}^{\infty} \frac{s_{t+i}}{(1+r)^i}$  equals the present value of (expected) future primary surpluses.

If  $g_{b,t}$ , is the growth rate of  $b_t$ , then

$$g_{b,t} = \frac{\Delta b_t}{b_{t-1}} = -\frac{s_t}{b_{t-1}} + r_t$$

As the solvency condition implies that  $E_t \frac{b_t}{(1+r)^t} \to 0$  as  $t \to \infty$ , we have  $\frac{b_t}{(1+r)^t} = \frac{b_0(1+g_b)^t}{(1+r)^t} \to 0$  as  $t \to \infty$ . This requires that  $\frac{\Delta b_t}{b_{t-1}} = g_{b,t} = -s_t/b_{t-1} + R_t - \pi_t < r$ . Hence the solvency condition is equivalent to enforcing that in the long run expected real debt growth is less than the real rate, i.e.  $E_t g_b < r$ .

Notice that the solvency condition must be met for a government that is expected to be solvent. A government may, of course, fail to be solvent, in which case the market will lower its debt price until its value equals the present value of future primary surpluses, whatever they are expected to be. For a developed economy that has never defaulted, we assume that it will not default and will remain solvent. Hence we treat the solvency condition as binding on policy.

With Ricardian fiscal policy, the Treasury sets s to satisfy the solvency condition at the terminal condition, t = T, which implies that  $-s_T/b_T + R_T - \pi_T < r_T$ ; and since  $R_T - \pi_T = r_T$  at T, this implies that  $-s_T/b_T < 0$ ; in other words  $s_T/b_T > 0$  and there must be a primary surplus at T which is expected to prevail indefinitely thereafter. In the Orthodox model, we set a terminal condition in which tax revenue exceeds government spending excluding debt interest, by an amount sufficient to pay the debt interest. In this model fiscal policy is constrained by the situation created by the monetary policy rule and households are indifferent between buying real debt and tax-financing government spending (we assume lump-sum tax to abstract from distortionary tax and supply-side issues), as they know the debt will be backed by future surpluses, with taxes paying for the spending stream.

We solve for debt by using the equation for the government budget constraint that is set out in detail below. We complement this with a Ricardian setting of the lump-sum tax,  $\tau$ , in which it responds negatively to the ratio of debt to GDP and reaches at the terminal date  $\tau_T - g_T = rb_T$ . This then defines each period's government budget deficit.

Because in the model both the goods and labour markets clear, by Walras's Law the bond market also clears. Given that in the goods market  $y_t = c_t + i_t + g_t$  it follows that  $y_t - \tau_t + RDI_t - c_t = g_t - \tau_t - RDI_t + i_t$  (where  $RDI_t$  is real debt interest paid to households) or, in other words there is bond market equilibrium with *Private Savings* = *Government deficit* + *Investment*. It follows that whatever lump-sum tax and debt interest occurs, raising the deficit and debt, creates an equal upward adjustment in private saving, leaving consumption and the rest of the solution the same. This is Ricardian equivalence. Thus the solutions for interest rates, inflation and output are unaffected by the evolution of taxes and debt which can be chosen freely by the government, provided it obeys the terminal solvency condition.

#### **3.2** Solvency under FTPL assumptions

We now move to an FTPL set-up. Solvency under FTPL also requires that the present value condition holds, namely:  $b_t = E_t \sum_{i=1}^{\infty} \frac{s_{t+i}}{(1+r)^i}$ , and that  $b_T = \frac{B_T}{P_T}$  where  $B_T$  results from the accumulated fiscal deficits up to T. It follows that

$$E_t b_T = E_t \sum_{i=1}^{\infty} \frac{s_{T+i}}{(1+r_T)^i}$$

Let  $s_{T+i} = \sigma_T b_0$  where  $\sigma$  has the dimension of a fraction and  $b_0$  is the baseline reference value of real debt that is used to convert this fraction into a real value term, then

$$b_T = \sum_{i=1}^{\infty} \frac{\sigma_T}{(1+r_T)^i} b_0 = \frac{\sigma_T}{r_T} b_0,$$

We allow  $\sigma_t(t=0,1,...,T-1)$  to be projected by exogenous government spending and tax processes that must be constrained to terminate in a real surplus,  $\sigma_T$ . At this point FTPL does not differ from the Ricardian regime. Where the difference occurs is that these exogenous fiscal trends determine nominal debt,  $B_T = P_T b_T$  and under FTPL,  $P_T$  reconciles  $B_T$  with the present value of the terminal surplus. In effect, these exogenous choices of fiscal outcomes before T are paid for by inflation tax revenue, determined at T. As  $P_T = e^{\pi T} P_0$ , for given  $P_0$  this determines long-run inflation,  $\pi$ , and the price level  $P_T$  as

$$P_T = e^{\pi T} P_0 = \frac{B_T r_T}{\sigma_T b_0}$$

 $s_T = \sigma_T b_0$ .  $\sigma_T$  can be chosen as required to maximise the model's fit; it will typically be positive because in the present value condition debt will typically be positive owing to past primary deficits. In practice we look for a  $\sigma_T$  that lies in a range close to zero, from 0.001 to 0.02. Then  $B_T$ , as well as  $\sigma_T$ , is found from the time series for g and tax,  $\tau$  which are assumed to reflect the exogenous fiscal choices of the government, ignoring Ricardian principles.

In this FTPL set-up, inflation emerges as the rate that will reduce the terminal value of debt to equate with the terminal present value of future surpluses. A country can fail to be solvent if no inflation rate is high enough to achieve this. In effect such a situation represents a failure to find a solution to the model. In this case the conditions for the FTPL to hold are not satisfied. It should be noted that under the FTPL the government can in principle freely choose its terminal primary surplus; however, to be credible, given the lack of any Ricardian constraint before the terminal condition, some limit to this must be set. Thus, in principle, the inflationary consequences of terminal debt can be largely avoided simply by promising a large terminal surplus; this would reduce inflationary behaviour generated by the FTPL model. In practice, however, in the US context, it would not be believable as there has only been a federal surplus in a few years around the late 1940s and 1990s — see Figure 2 which shows the primary surplus/deficit which we have calculated (sources:OMB; St. Louis Fed) as the gross federal surplus with debt interest added back, both as % of GDP. This reveals a general absence of primary surpluses over the postwar period.

#### 3.3 Modelling the government debt process under the FTPL

In order to implement the FTPL we need to add to the benchmark model the specification of the exogenous processes that determine government debt. The price level is calculated from this debt. We combine nominal government expenditures  $G_t$  and nominal tax revenues  $T_t$  into the surplus  $S_t$  that is represented by a nonstationary process:

$$\Delta S_t = c_0 + c_1 \Delta S_{t-1} + \eta_t$$

We then take the following steps:

1. From the U.S. data we estimate the  $S_t$  process. The value of  $c_1$  estimated on the data was 0.707.

2. Using the GBC we accumulate the resulting nominal debt, assuming nominal debt of 5-year maturity is issued (the average maturity of US public debt). We approximate the GBC in the following way. Each

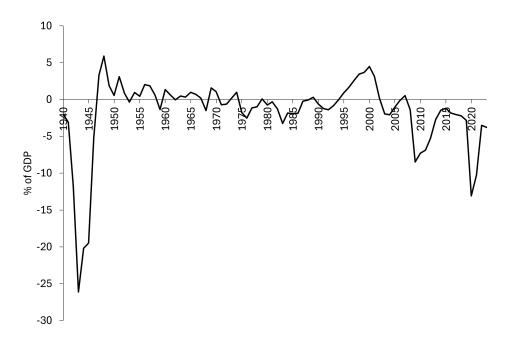


Figure 2: Primary Surplus/Deficit as a Percent of GDP

period all debt is assumed to be reissued at the new market value, allowing for the revaluation of debt. (We assume that the debt maturity share is constant and equal across the previous five years, i.e. 20 quarters.) In principle this is the most accurate approximation as each period we recalculate the market value of the debt as rates change; this revaluation then means that the interest rate on the revalued debt is the current market rate. The interest payments are then recalculated using current rates and revalued debt. We also allow for the effect of inflation via indexed debt which is about 7.5% of the total. There is also a proportion of the debt which is non-marketable, held by various statutory bodies such as state pension funds on behalf of private households. We assume this debt behaves like marketable debt. It turns out that this formula for debt evolution gives the closest approximation to actual debt over the sample, with an  $R^2$  of 0.99.

$$\Delta B_t = S_t + R_{5,t-1}(B_{t-1} + REVAL_t) - 0.075\pi_{t-1}B_{t-1}$$

where  $REVAL_t = -(20 \times \Delta R_{5,t})B_{t-1}$ ,  $R_{5,t}$  is the 5-year interest rate in period t derived from the model's solution — the rational expectation of the future expected interest rates at t.

3. The terminal condition is calculated from these processes as  $B_T$ . 4. The terminal price level is calculated as:  $P_T = e^{\pi T} P_0 = \frac{r^* B_T}{\sigma_T b_0}$ . From this we take  $\pi$  as the implied long run inflation rate enforced by FTPL.

5. This inflation rate is calculated by the model solution at each period t, using the t-based projections made above. This  $\pi$  enters the Phillips Curve as the expectation of future inflation,  $E_t \pi_{t+1}$ . Using all the model equations, including determining monetary policy from the Taylor Rule, the model solves for all time t variables, including inflation. As in the original New Keynesian model, a determinate solution for real interest rates, and hence demand, is obtained from the Taylor Rule.

#### 4 The method of Indirect Inference

The method of Indirect Inference is used to evaluate whether the model can fit the data. This method was proposed in Meenagh et al. (2009) and refined by Le et al. (2011) who used Monte Carlo experiments to evaluate the method. Indirect Inference uses an auxiliary model to describe the observed data. The auxiliary model is also estimated using data simulated from the model to be tested. A test of the model is obtained by comparing the two sets of estimates. The test statistic can be constructed in several ways, for example, using the scores from two sets estimates, from a measure of their respective fits such as a Likelihood Ratio test, and by testing for differences in the coefficient estimates of the auxiliary model using a Wald test. Our previous investigations favour the latter.

The auxiliary model is chosen to account for U.S. business cycle dynamics over the sample period 1959-2017. We use as the auxiliary model a VARX of the three macro variables, output, inflation and interest rates:

$$Y_t = AY_{t-1} + BX_{t-1} + e_t (1)$$

where  $Y_t \equiv (y_t, \pi_t, R_t)'$ ,  $X_t \equiv (ea_t, t)'$  where  $ea_t$  is productivity, t is the deterministic trend,  $e_t$  is the error vector, and A and B are the coefficient matrices. The Wald test statistic is:

$$Wald = (\Phi_T - \overline{\Phi})' \sum_{(\Phi\Phi)}^{-1} (\Phi_T - \overline{\Phi})$$
<sup>(2)</sup>

where  $\Phi_T$  is the vector of VARX estimates of the autoregressive coefficients and variances of the VARX residuals using the actual data,  $\overline{\Phi}$  are the corresponding mean estimates using the simulated data and  $\sum_{(\Phi\Phi)}$  is their variance-covariance matrix. Our null hypothesis  $H_0$  is that the model being tested is "true". The *p*-value of the test is calculated by:

$$p = (100 - WP)/100 \tag{3}$$

where WP is the percentile of the Wald statistic found with the actual data in the distribution of it generated by the simulated samples. The models would pass/fail the Wald test if their *p*-value is above/below a 5% threshold.

In order to estimate the model and then find its minimum-value Wald statistic we use a Simulated Annealing algorithm. This gives us a set of parameters that produces simulations that are closest to the data. These estimates have been shown to be consistent and asymptotically Normally distributed (see Smith, 1993; Gregory and Smith, 1991, 1993; Gourieroux et al., 1993; Gourieroux and Monfort, 1995; Canova, 2005).

We use estimates based on Indirect Inference rather than the now widely-used Bayesian approach to estimating our model because we wish to test the model as a whole against the data and avoid assuming prior distributions of the structural parameters which may unduly influence the posterior estimates. Evidence of the dangers of using Bayesian estimation was found by Le et al. (2011) for even such a major model as the Smets-Wouters (2007) model of the U.S. as their model was rejected by our indirect inference test. In order that we can rely on it for discussions of policy, our aim is to find a model that is not rejected by the data.

As noted, we could also use a Likelihood Ratio (LR) test instead of the Wald test. However, Le et al. (2016a) find that the two test statistics have quite different properties when used to check for model misspecification: the LR test is based on a model's in-sample current forecasting ability whereas the Wald is based on the ability of the model, including its implied errors, to replicate the behaviour of the data, as represented by the VAR coefficients and the data variances. In effect, the Wald test asks in a parsimonious way whether the model can replicate the impulse response functions found in the data.

It transpires that, in the context of a macro model like that used here, the Wald test has far greater power than the LR test. This property is demonstrated for the original Smets-Wouters model over the sample period 1947Q1 - 2004Q4 by Le et al. (2016b) whose comparative table we reproduce next. Table 1 shows a Monte Carlo experiment in which the Smets-Wouters model is treated as true and in which the alternative hypothesis is formulated generated by mis-specifying the parameters alternately by +/-x%. The rejection rate for the Wald statistic rises sharply with x. The table shows these results using both stationary and non-stationary data generated from the model. The the power of the Wald test remains much the same for both whereas the LR test loses power on non-stationary data, for reasons Le et al. (2016a) discuss.

Percent Mis-specified	Wald	LR	Wald LH	3
	Station	nary data	Non-statio	onary data
True	5.0	5.0	5.0	5.0
1	19.8	6.3	7.9	5.2
3	52.1	8.8	49.2	5.8
5	87.3	13.1	97.8	6.2
7	99.4	21.6	100.0	7.4
10	100.0	53.4	100.0	9.6
15	100.0	99.3	100.0	15.6
20	100.0	99.7	100.0	26.5

Table 1: Rejection Rates for Wald and Likelihood Ratio for 3 Variable VAR(1)

#### 5 The results of testing FTPL

#### 5.1 Results for the benchmark model of FTPL

We begin our discussion of the results of testing FTPL for its implementation using the Orthodox New Keynesian model of Le et al. (2021). This model performs well for the U.S. postwar period (1959–2017) in which there is Ricardian fiscal policy and orthodox monetary policy following a Taylor Rule. FTPL is embodied in the model by incorporating exogenous processes for government expenditures and tax. Later we implement the test on our classical model.

Our first discovery was that in the New Keynesian model FTPL does not coexist easily with orthodox monetary policy. This form of monetary policy can literally blow up the FTPL for some fiscal policy simulations because as the fiscal policy raises equilibrium inflation the consequent rise in interest rates causes instability due to escalating final debt causing an explosive loop between rising inflation and rising rates. As a result there is no solvent equilibrium for such cases. Hence it is not possible to generate the simulated data required for a Wald test.

If we interpret the FTPL as saying that the price level will adjust to deliver debt solvency no matter what exogenous fiscal policy is then, insofar as the benchmark model is a suitable representation of the economy, something supported by the evidence, this failure to find a solution to the benchmark model suggests that the FTPL is incorrect.

Nonetheless, it is of interest to check whether alternative exogenous processes for nominal spending, tax revenues and the primary surplus might produce a solution to the model. According to the data nominal spending, tax revenues and the primary surplus are I(1) processes. With this in mind, we carefully explored combining the spending and tax processes and looking for a specification of the implied surplus that would be both broadly consistent with the data and also give the closest match of the model to the auxiliary model's data behaviour. We found from the data that this process could either be trend-stationary — i.e. an AR(2) with a trend — or non-stationary — i.e. an AR(1) in first differences. We found the trend-stationary processes failed to solve for a large proportion of the bootstrap simulations. We found the same for the nonstationary versions. In short we found no time-series representation of the surplus that could consistently solve the model, let alone match the data-based auxiliary model.

We note that a non-stationary nominal surplus process is entirely possible; it results in an unbounded nominal value of debt at T which, if T were allowed to go to infinity, could generate infinite inflation, thereby preventing the model from solving in finite space. This is avoided by imposing a terminal finite date for Tat which a finite inflation must occur. This defines the FTPL inflation equilibrium. Hence it is not nonstationarity of the nominal surplus process that is the cause of the model's failure to solve. Rather, it is the loop connecting interest rates and so debt to inflation, with the debt feeding back into inflation. In Appendix B.1 we provide more details, showing how the model solves normally with a nonstationary nominal surplus process when nominal interest rates follow an exogenous process (so avoiding a doom loop from any reaction to inflation); we then show how the same model fails to solve when the interest rate responds to inflation as in a Taylor Rule in Appendix B.2. However, as we will go on to discuss, nominal interest rates cannot be permitted theoretically to follow an exogenous process.

#### 5.2 FTPL in the classical model

Our findings for the benchmark New Keynesian model show that virtually any response of interest rates to prices causes frequent instability in the FTPL model, with inflation pushing up interest rates and these blowing up inflation via terminal debt. In effect, for the FTPL to solve universally, we require a zero response of interest rates to inflation. This is also what Cochrane (2023) suggests for his 'simplified' macro model where the interest rate follows an exogenous ARIMA process — thus with no response to inflation. This model does indeed solve — see Appendix B.3. But the difficulty with this set-up is that there is no rationale for this equation. In a New Keynesian environment, where prices and wages are set by firms and unions for several periods ahead, goods and labour markets clear by supply meeting whatever is demanded; because by Walras' Law goods market clearing implies bond market clearing, real interest rates must be set by the central bank so that demand-determined output converges on equilibrium output supply otherwise long-run real interest rates and output will not solve for the steady-state equilibrium in the long run. Nominal interest rates must therefore equal this required real interest rate plus expected inflation for the model to solve with the necessary convergence. In effect, this implies a nominal interest rate rule that responds to inflation with a unit coefficient and to output with a positive coefficient. This is what we use in our FTPL model with a modified New Keynesian environment; notice that unlike the Taylor Rule it responds to inflation with the minimum coefficient (unity) that ensures delivery of convergence via the real interest rste response to output.

We also examine a version of FTPL where we abandon New Keynesian price/wage-setting in favour of market clearing prices and wages and, as discussed above in our theoretical overview, embed the FTPL in a purely classical RBC model. The real interest rate would then clear the bond market, and so also the goods market, while real wages would clear the labour market and so, via the production function, determine output supply. Inflation would then be determined by the FTPL directly, eliminating the Phillips Curve. Here we impose the zero lower bound by assuming that when a negative nominal rate is required to generate the equilibrium real rate, the central bank injects sufficient money into the economy to generate this real rate at a zero nominal rate through inflation; otherwise the central bank injects enough money growth to achieve the inflation implied by FTPL. We note that this alternative classical model implies that, when there is no ZLB, the nominal interest rate must respond to expected inflation with a unit coefficient in order to deliver the market-clearing real interest rate.

Again, with this RBC model, we find that there is general instability and the model cannot solve (see Appendix B.4). The problem is still the 'doom loop' that is unleashed by inflation interacting with interest rates and via debt interest, the cumulated debt creating the inflation. The model also fails to solve for a range of simulated surplus processes; debt accumulates enough to trigger this doom loop.

For robustness we also investigated for both models whether the choice of  $\sigma$  (the ratio of the terminal surplus to real debt) affects the results. For credibility reasons we set a boundary of 0.02 for  $\sigma$ . However, choosing a higher value could not prevent this instability and the resulting indeterminacy. Essentially, they were still affected by the strong feedback between inflation, interest rates and debt. While the higher  $\sigma$ reduced the direct impact of debt on inflation, it could not prevent this feedback from generating instability across numbers of simulations.

#### 5.3 Solving The FTPL model — discussion

It could be thought that the failure of the FTPL regime basic models to solve is due to some limitation in our model software. However, we are using the standard algorithm widely and frequently used to date — as set out by Fair and Taylor (1983) — which is designed to find a saddle path solution for rational expectations models. In this algorithm, the model set-up is assumed to eliminate unstable paths, such as sunspot equilibria, by restrictions on the model from transversality or other conditions. Thus, in the standard Ricardian model a sunspot path of exploding inflation would be eliminated by the central bank raising interest rates sufficiently to reduce output substantially and so bring down inflation back towards the inflation target. The problem we face in the FTPL model is that as fast as the monetary rule raises interest rates, fiscal policy raises inflation via rising debt, so preventing the path from being eliminated. This exploding path then becomes the only solution because lower inflation paths are inconsistent with the rise in debt dictated by the primary deficit path. Of course an exploding solution is not a defined finite solution of the model. Hence the algorithm can find no valid solution.

We can see these workings usefully in a much-simplified model.

Let the debt equation be:

$$D_t = (1 + R_{t-1})D_{t-1} + DEF_t$$

where D = debt; DEF = primary deficit; R = nominal interest rate.

Now let nominal interest rates follow a simple rule maintain a real interest rate,  $r_t$ , that controls output gap (ygap) and so the Phillips curve:

$$\pi_t = \gamma(ygap_t) + E_t \pi_{t+1} = \gamma ygap_t / (1 - B^{-1}); r_t = \psi ygap_{tr}$$

$$R_t = \pi_t + r_t$$

hence

$$R_t = (1 + \frac{\psi}{\gamma} [1 - B^{-1}]) \pi_t$$

where  $\pi_t = \text{inflation}$ .

Finally let inflation depend on Debt:

$$\pi_t = kD_t$$

It follows that

$$\pi_t/k = \left(1 + \left\{1 + \frac{\psi}{\gamma}[1 - B^{-1}]\right\}\pi_{t-1}\right)\pi_{t-1}/k + DEF_t$$

hence

$$\pi_t = (\pi_{t-1} + [1 + \frac{\psi}{\gamma}]\pi_{t-1} - \frac{\psi}{\gamma}\pi_t)\pi_{t-1} + kDEF_{tt}$$

The reduced form of this simple model for inflation is thus:

$$\pi_t (1 + \frac{\psi}{\gamma} \pi_{t-1}) = [2 + \frac{\psi}{\gamma}] \pi_{t-1}^2 + kDEF_t$$

Hence

$$\pi_t = \{ [2 + \frac{\psi}{\gamma}] \pi_{t-1}^2 + kDEF_t ] \} / 1 + \frac{\psi}{\gamma} \pi_{t-1} )$$

Let us evaluate this in log terms for  $DEF_t = 0$  approx. Then

$$\ln \pi_t = \{2 - [\frac{\frac{\psi}{\gamma}\pi_{t-1}}{1 + \frac{\psi}{\gamma}\pi_{t-1}}]\}(\ln \pi_{t-1}) + c_0$$

where  $c_0$  is a constant.

Plainly the log of inflation has an above-unit root (noting that the term  $\left[\frac{\frac{\psi}{\gamma}\pi_{t-1}}{1+\frac{\psi}{\gamma}\pi_{t-1}}\right]$  must lie between zero and one). This cannot be solved for any stable solution. The extra terms in the log of the deficit, if included, would change the above to

$$\ln \pi_t = (1-s)\{2 - [\frac{\frac{\psi}{\gamma}\pi_{t-1}}{1 + \frac{\psi}{\gamma}\pi_{t-1}}]\}(\ln \pi_{t-1}) + c_0 + s\ln DEF_t$$

where s is the ratio of the deficit to debt, typically small. When the deficit, debt and inflation are low, s will be a small value close to zero and  $\left\{2 - \left[\frac{\frac{\psi}{\gamma}\pi_{t-1}}{1+\frac{\psi}{\gamma}\pi_{t-1}}\right]\right\}$  close to 2; when debt and inflation are high, whereas the deficit remains within normal low bounds, s will be virtually zero and  $\left\{2 - \left[\frac{\frac{\psi}{\gamma}\pi_{t-1}}{1+\frac{\psi}{\gamma}\pi_{t-1}}\right]\right\}$  will be close to but above unity. Hence at all stages of the inflationary process the root,  $(1-s)\left\{2 - \left[\frac{\frac{\psi}{\gamma}\pi_{t-1}}{1+\frac{\psi}{\gamma}\pi_{t-1}}\right]\right\}$ , will be unstable, though it tends to fall as inflation rises.

Notice that as inflation rises, the output gap falls since future inflation exceeds current. Hence also the real interest rate falls.

We noted that in solving our standard model we ruled out unstable paths via a transversality condition on monetary policy. We also noted that any such actions by monetary policy would be overruled by fiscal policy as the determinant of inflation; only fiscal policy controls inflation under FTPL. But we can rule out the unstable solutions via a transversality condition on fiscal policy, implying that fiscal policy would prevent them by 'whatever it takes' in the form of fiscal restriction; under this assumption we need to find the only stable solution that remains. As there are no backward dynamics in this simple model, this is simply the equilibrium solution, as follows, where we assume DEF is constant:

$$\pi^*(1 + \frac{\psi}{\gamma}\pi^*) = [2 + \frac{\psi}{\gamma}]\pi^{*2} + kDEF$$

$$\left(1 + \frac{\psi}{\gamma}\pi^*\right) = \left[2 + \frac{\psi}{\gamma}\right]\pi^* + kDEF/\pi^*$$

or

or

$$1 = 2\pi^* + kDEF_t/\pi^*$$

or finally

$$0 = (\pi^*)^2 - 0.5\pi^* + 0.5kDEF$$

which produces the twin solutions:

$$\pi^* = \frac{0.5 \pm \sqrt{0.5^2 - 4 \cdot 0.5 \cdot kDEF}}{2}$$

The lower solution can be assumed to be chosen, since the higher will typically imply an interest rate above a politically feasible level.

We applied this approach to the FTPL model in its New Keynesian version; in this model the steady state solution for inflation is the same as in the simple model above. However, no stable solution path could be found because this steady state solution frequently did not exist. This was due to DEF being too large; essentially primary deficits require inflation above this steady state region. Applying the same approach to the RBC version of FTPL, the same occurred; the steady state solution for inflation did not exist. Our results for both models therefore imply that the FTPL in its basic form cannot replicate the data behaviour of the US postwar period. Since these models do not systematically solve for finite values, these FTPL regimes could not survive — as such, they cannot be candidates for testing. We therefore find that in the FTPL model there is no way of eliminating the unstable paths, without modifying the model in some way. We now turn to possible modifications, all of which involve endogenising fiscal policy in some way.

The above looks for a steady state solution, in effect letting T go however far ahead will generate a steady state equilibrium. It might be that the model would solve for a stable path up to a normal T (which we set at 50 quarters) over the sample period; we also investigate this possibility. For a stable path we need to find an inflation rate such that when expected as the terminal equilibrium the model-projected debt also implies it as the equilbrium value needed for solvency. We find that for a large range of simulated primary surpluses no such stable path can be found — as shown in Table 2. In it we see the distribution of simulated inflation rates corresponding to an assumed equilibrium expected inflation. For much of the simulated distribution the model generates inflation rates well above this, indicating that for many bootstrapped surplus simulations there is no stable path. Thus as with the absence of steady state solutions for many possible surplus trajectories, we find the same for a stable path over the sample period: many surplus trajectories require inflation well above what would be the stable path for a low debt trajectory.

Thus the difficulty we face with the FTPL model is that in its bootstrapped simulation behaviour it very frequently generates instability, with no stable path possible. The bottom line of Table 2 shows that simulations without a stable solution are between 91% and 100% of the total.

	E	quilibrium	Expected	Inflation	L
Debt Implied Inflation	1%	5%	10%	15%	20%
Median	6.5019	6.3742	6.1280	6.1171	5.5071
Lower C.I.	0.9879	0.9879	0.8906	0.8907	0.8906
Upper C.I.	30394.9	18701.7	11709.5	7752.4	6738.4
Unstable Sim.	100%	100%	98%	94%	91%

Table 2: Debt Implied Inflation Comparison with Stable Path (NK Model)

We repeated this analysis with the RBC version of the FTPL model. Here again we found a general absence of a steady state positive inflation solution<sup>5</sup>; and a widespread absence of a stable simulated solution over the sample period. We show the equivalent Table 3 for the RBC model over the sample period below. Here the proportion of simulations without a stable solution come to between 61% and 86%, the vast majority.

	Stable $\pi^*$					
Debt Implied Inflation	1%	5%	10%	15%	20%	
Median	0.5728	0.5107	0.4904	0.4722	0.4556	
Lower C.I.	-0.0014	-0.0014	-0.0014	-0.0014	-0.0014	
Upper C.I.	7822.8	7604.8	7340.7	7085.5	6839.1	
Unstable Sim.	86%	73%	66%	62%	61%	

Table 3: Debt Implied Inflation	Comparison with Stable	Path (RBC Model)
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Since we cannot find any stable solution for the basic FTPL model in which fiscal policy chooses a primary surplus exogenously according to its postwar practice, we turn next to explore versions of FTPL in which fiscal policy responds endogenously to events in various possible ways. In these models inflation is always set by fiscal policy but the latter responds to the economy so as both to ensure a viable solution and to prevent obvious economic crises. Our aim is to find an FTPL version that could be tested as a plausible model of the US economy in the postwar period.

#### 6 Fiscal reforms to the FTPL

#### 6.1 Emergency fiscal responses to FTPL

We consider two types of modification to the FTPL framework: an emergency fiscal response and fiscal rules. An emergency fiscal response could be implemented if the doom loop is triggered. Cochrane (2023, p. 536, opening para) suggests as much:

In retrospect, 1980 looks a lot like a classic inflation stabilization combined with fiscal and pro-growth reform, such as inflation targeting countries introduced. The fiscal and pro-growth reform came after monetary policy changes, and may have been partly induced by the interest expense provoked by higher interest rates. The interest expense channel can provoke fiscal reform rather than spark a doom loop. Or, the fiscal reform may have been the clean-up effort that made the monetary tightening stick. Many attempted monetary tightenings have failed when promised fiscal reforms did not materialize.

<sup>&</sup>lt;sup>5</sup>Note on RBC model steady state. Let the debt equation be:  $D_t = (1 + R_{t-1})D_{t-1} + DEF_t$  where D = debt; DEF = primary deficit; R = nominal interest rate.

Now let nominal interest rates follow a simple rule maintain a real interest rate,  $r_t$ , that keeps ygap = 0 (there is now no Phillips curve):

 $r_t - r^* = \psi ygap_{tr} = 0; R_t = \pi_t + r_t;$  where  $\pi_t =$  inflation.

Finally let inflation depend on Debt:  $\pi_t = kD_t$ 

It follows that in steady state  $\pi^* = [1 + r^* + \pi^*]\pi^* + kDEF$  or  $0 = \pi^{*2} + r^*\pi^* + kDEF$  which produces the twin solutions:  $\pi^* = -0.5r^* + / -0.5\sqrt{(r^{*2} - 4kDEF)}$ 

A solution can be found with the positive branch of the square root expression, provided DEF is small enough. But there is no positive solution at all.

This extract suggests that 'fiscal reform' (for example, a sharp increase of tax and the surplus) could indeed avoid a crisis in inflation triggering the interest doom loop. Our bootstrap simulations of debt that trigger the loop could be constrained by such an emergency fiscal response. We can think of this as a statecontingent response of  $\sigma$  to such simulations, designed to prevent inflation from spiralling out of control. In the U.S. context such a crisis would be identified by quarterly inflation exceeding some trigger rate — which U.S. history suggests would be around 3% (an annualised 13%), as this has never been exceeded in postwar history — see Figure 3.

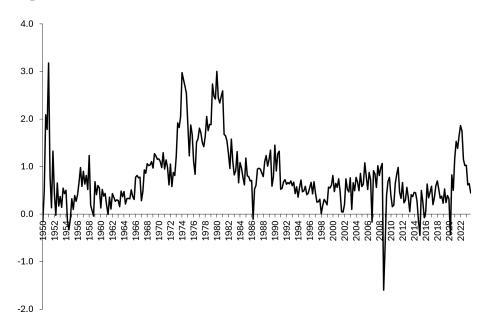


Figure 3: US inflation: percent per quarter, chained PCE index (Source: FRED)

If we integrate this emergency response into our two FTPL models, we can obtain solutions for all our bootstrap simulations (see Appendices B.5 and B.6 for more details). We found that this extreme reform of taxes on its own would allow both models to solve but it would result in substantial instability both of the output gap (which often exceeded the 10% maximum ever experienced in the postwar period — see Figure 4) and inflation, as illustrated in the Table 4 below for the New Keynesian model. Furthermore, the tax reform needed to be invoked for about 75% of the simulations, and several times in some simulations. Similar results are obtained for the RBC model. These upheavals suggest that such reforms would not make this FTPL policy regime sustainable, any more than the one with total instability that it attempts to remedy. It too we treat as untestable because not viable.

	Actual		Sim	ilations
	mean	std. dev.	$\mathrm{mean}$	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	17.2486
PI	0.8065	0.5795	1.5699	1.1666
R	1.2751	0.9101	2.0833	2.0900

Table 4: Data and Simulations Statistics: NK Model with Zero Lower Bound and Fiscal Reform

#### 6.2 Fiscal Rules: Between Scylla and Charybdis?

Like Odysseus on his perilous return to Ithaca after the Trojan war, FTPL must navigate between the perils of excessive output instability, its Scylla, creating political regime death and of excessive inflation triggering the doom loop, its Charybdis. If fiscal policy reacts to both inflation (e.g. by raising tax) and the output

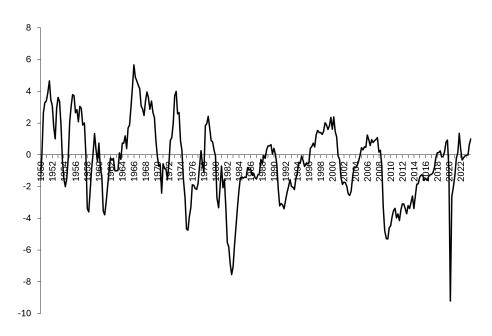


Figure 4: US Output Gap (Source: FRED)

gap (e.g. by raising spending), it may become a viable policy regime and one that could also fit the facts. By adding fiscal rules we are switching from exogenous to endogenous fiscal policy. This entails a rejection of the FTPL formulation according to which the price level will adjust to produce debt solvency no matter what fiscal policy is pursued.

We found that a fiscal rule in which spending responds to the output gap while tax responds as strongly as necessary to head off above 5% inflation would stabilise both output and inflation and allow the New Keynesian model to solve (see Appendix B.7). However, the rule triggered extreme tax reforms in 40% of the simulations, about five times in each, which would prove too regularly disruptive for voters' tastes to be sustainable. In terms of the FTPL, even with this fiscal rule, the doom loop sets up intolerable pressures causing large-scale inflation instability. While we can control output instability, the Scylla side of the channel, we need extreme tax reform to eliminate the extreme inflation arising from the interaction of inflation shocks and the interest rates in order to prevent the doom loop — the Charybdis whirlpool, into which the economy is sucked for 40% of its simulations. Hence, this FTPL regime too we regard as unsustainable and not worth testing.

	Actual		Simu	ilations
	$\mathrm{mean}$	std. dev.	$\mathrm{mean}$	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	0.9621
PI	0.8065	0.5795	2.9817	2.4372
R	1.2751	0.9101	3.5850	2.5556

Table 5: Data and Simulations Statistics: NK Model with Zero Lower Bound and Fiscal Rule

	Inflation Cap $(3\%)$
Percent of Periods	0.9080
Percent of Simulations	42.2535

Table 6: Percentage of Simulations/Periods Hitting Expected Inflation Cap: NK Model with Zero Lower Bound and Fiscal Rule

We had more success using the RBC version of the model (see Appendix B.8). The fiscal rule did not

trigger the 'tax reform' cap on inflation and it completely avoided extreme output gaps. This makes it potentially politically viable, though inflation and interest rates are still highly volatile. The reason for it not triggering the cap in the RBC model is that the doom loop is not so threatening due to the lower (unit) interest rate response to inflation. A further concern in this RBC model is that the Fed is following a policy of monetary injection that is maximally cooperative with the fiscal authority; it is injecting money growth to satisfy the inflation required by the FTPL terminal condition, and also as needed to satisfy the zero lower bound (i.e. to implement any necessary negative real interest rates without pushing nominal interest rates below zero). Such a monetary policy does not much resemble what we know of Fed policy; it also produces much higher and variable inflation, which could undermine its viability. In terms of fit, this RBC version is on the margins of rejection and acceptance, with a *p*-value of 4.2%, which is well below the *p*-value of the benchmark Orthodox New Keynesian model. The estimated coefficients, Wald statistic, and *p*-values can be found in Table 7.

Models' Coefficients						
		Orthodox Model	RBC with Fiscal Rule			
Elasticity of capital adjustment	$\varphi$	6.881	5.124			
Elasticity of consumption	$\sigma_c$	1.283	1.490			
External habit formation	$\lambda$	0.767	0.560			
Probability of not changing wages	$\xi_w$	0.635	_			
Elasticity of labour supply	$\sigma_L$	2.865	_			
Probability of not changing prices	$\xi_p$	0.746	_			
Wage indexation	$\iota_w$	0.376	_			
Price indexation	$\iota_p$	0.107	_			
Elasticity of capital utilisation	$\hat{\psi}$	0.128	0.274			
Share of fixed costs in production $(+1)$	$\Phi$	1.083	1.588			
Taylor Rule response to inflation	$r_p$	2.913	_			
Interest rate smoothing	$\hat{\rho}$	0.732	_			
Taylor Rule response to output	$r_y$	0.019	_			
Taylor Rule response to change in output	$r_{\Delta y}$	0.019	_			
Share of capital in production	$\alpha$	0.222	0.341			
Elasticity of the premium with respect to leverage	$\chi$	0.032	_			
Money response to premium	$\psi_2$	0.059	_			
Elasticity of the premium to M0	$\bar{\psi}$	0.058	_			
Money response to credit growth	$\psi_1$	0.052	_			
Parameter response of NK weight — prices	$\theta_{\pi}$	0.052	_			
Parameter response of NK weight — wages	$\theta_w$	0.071	_			
Wald $(Y, \pi, R)^*$		15.525	32.2414			
<i>p</i> -value		0.21	0.04			

Thus, even the most successful version of our FTPL models that include fiscal rules cannot be considered to be the best available match to the data. This is provided by the Orthodox model which gives the closest account of postwar U.S. behaviour.<sup>6</sup>

Table 7: Coefficient Estimates

 $<sup>^{6}</sup>$ In the course of testing a wide variety of DSGE models, we have found that when faced with a model with large outliers the test loses its power. It does so because these outliers create extreme values in the variance-covariance matrix and hence very large standard deviations on the auxiliary model coefficients. This may also apply with our models here. When outliers are a problem, the test's power is restored by excluding possible outliers — see Minford, Xu and Dong (2023) and Minford and Xu(2024) in the case of trade models. In the models here this exclusion would be on the grounds that these outliers represent 'crises' which will be restored to normality by suitable policy measures such as regulations or fiscal measures like bailouts. In ongoing work, we are investigating other ways in which the test's power can be restored in the presence of outliers.

If we exclude the top 5% of the variation in these two models, in case this problem applies here, we find our conclusions are unaltered: that the Orthodox model matches the data behaviour much better than the modified FTPL model. Their respective p-values are 8% and 0.04%

### 7 Does the empirical failure of the FTPL condemn all fiscal activism? The fiscal spectrum

We have found that the empirical evidence over the whole postwar period favours the Ricardian Orthodox New Keynesian model. Nonetheless, it is possible that in particular sub-episodes the FTPL model could match the data, transitionally as it were, as assumed in the work of Leeper (1991); plainly that lies well outside the scope of this paper. FTPL is at the extreme end of a spectrum of fiscal activism which stretches from active counter-cyclical fiscal policy and 'tax-smoothing' all the way to entirely independent fiscal policy largely dependent on inflationary finance as exemplified by the FTPL. We have shown that it is possible to modify the strict FTPL framework to include a fiscal response that controls this inflation to produce more stability; but that even so the behaviour of these modified-FTPL regimes still exhibits intolerable instability. 'Fiscal rules' designed to prevent destabilising threats to solvency, are widespread among developed countries, including the German 'debt brake', the EU's Stability and Growth Pact, the current UK government's debt/GDP reduction rule and the US government's 'debt ceiling'. While these rules vary in their rigour of application, they all exemplify attempts to put limits around fiscal independence, due to fears of destabilisation that this paper shows are entirely reasonable.

At the same time there is considerable evidence from DSGE models that monetary policy designed to stabilise the economy can too easily trigger a zero lower bound for interest rates, implying a beneficial role for fiscal policy to control output and take the pressure off monetary policy. Evidence for this is found for the U.S. in Le et al. (2021). A similar finding is made for the UK in Le et al. (2023a). In relying solely on monetary policy, the eurozone has a particular problem due to the disparities of the economies in the zone. Minford et al. (2022) found that fiscal policy can play a major stabilising role in the eurozone. Japan too can benefit from an active fiscal policy as found in Le et al. (2023b).

The evidence, therefore, indicates that fiscal activism in the form of countercyclical fiscal policy can improve welfare. Such policy, however, stops well short of extreme FTPL fiscal independence. In fact, its optimising contribution is to complement monetary policy in its primary role of controlling inflation by supplying another source of output stabilisation. In this way, it does not undermine the inflationary discipline supplied by monetary policy, while the FTPL in all its suggested forms, including that with the Cap brake, seriously undermines it.

While fiscal policy of this sort is beneficial, we have not investigated whether or not it was actually operating in the U.S. during our sample postwar period. In our version of the Orthodox New Keynesian model no fiscal responses were assumed, only a Ricardian commitment to providing future surpluses to pay off debt. Our work leaves open the question of whether fiscal activism, well short of the FTPL, was operating in this period.

#### 8 Conclusions

In this paper we have implemented a quantitative empirical test of FTPL defined as a non-Ricardian model of fiscal policy (the 'FTPL model'). We used indirect inference to check whether it can match postwar U.S. data as well as a standard New Keynesian model with Ricardian fiscal policy. The distinguishing feature of this FTPL version of the Orthodox New Keynesian model is that government spending and taxation are allowed to evolve according to data-based exogenous non-stationary processes. Nominal debt accumulates according to the government's budget constraint; at a terminal date, this must be paid off by a future real primary surplus whose present value must be equal to accumulated nominal debt, deflated to real terms by a terminal price level, which in turn defines an average inflation rate to that point. This is assumed to equal expected inflation which then enters the model up to the terminal date. Thus inflation is caused by expected fiscal policy. This FTPL model is estimated, with all its coefficients and the terminal surplus rate chosen to match the data behaviour as closely as possible. We have repeated this with an FTPL version of the classical RBC model. In order to carry out these tests using indirect inference we need the models to provide a credible explanation of the data and we require data simulated from the FTPL version of the models. But we have found that the New Keynesian version fails to solve owing to serious instability generated by the links between interest rates, debt and inflation; and that the RBC version generates similar instability. Hence, by default, the FTPL is rejected as an explanation of postwar U.S. data for both models.

We went on to investigate whether these models could avoid this instability through the inclusion of endogenous fiscal responses to output and inflation. While this might render them testable, it also undermines the strict version of FTPL: if FTPL is interpreted as always determining a price level that makes debt solvent for exogenous fiscal policy then the need to introduce fiscal rules amounts to a rejection of it. If our models are regarded as an accurate theoretical representation of the economy then their failure to solve is both an empirical and a theoretical rejection of strict FTPL. Even including fiscal rules, the FTPL model in its New Keynesian version exhibits extreme variability, rendering it unviable; in its RBC version, it exhibits excessive inflation volatility but if one accepts it as testable, it can almost reach the test threshold, though it falls well short of the Orthodox New Keynesian model in the closeness of its match to postwar U.S. behaviour. It also implies an unlikely degree of monetary cooperation by the Fed.

Despite its failure to account for postwar data, however, the FTPL is at the extreme end of a spectrum of fiscal activism and the evidence suggests that some such activism in the form of counter-cyclical stabilisation can improve welfare in a number of major developed economies within an otherwise New Keynesian policy regime. In future work on the U.S. it would be useful to establish how far such policies, well short of FTPL, have actually been pursued and what precise form they should ideally take.

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## Appendices

## A Listing of the Orthodox New Keynesian model

Consumption Euler equation

$$c_t = \frac{\frac{\lambda}{\gamma}}{1+\frac{\lambda}{\gamma}}c_{t-1} + \frac{1}{1+\frac{\lambda}{\gamma}}E_t c_{t+1} + \frac{(\sigma_c - 1)\frac{W_*L_*}{C_*}}{\left(1+\frac{\lambda}{\gamma}\right)\sigma_c}\left(l_t - E_t l_{t+1}\right)$$
(4)

$$-\left(\frac{1-\frac{\lambda}{\gamma}}{\left(1+\frac{\lambda}{\gamma}\right)\sigma_c}\right)\left(r_t - E_t\pi_{t+1}\right) + eb_t\tag{5}$$

Investment Euler equation

$$inn_t = \frac{1}{1 + \beta \gamma^{(1-\sigma_c)}} inn_{t-1} + \frac{\beta \gamma^{(1-\sigma_c)}}{1 + \beta \gamma^{(1-\sigma_c)}} E_t inn_{t+1}$$

$$\tag{6}$$

$$+\frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi}qq_t + einn_t\tag{7}$$

Tobin Q equation

$$qq_t = \frac{1-\delta}{1-\delta + R_*^K} E_t qq_{t+1} + \frac{R_*^K}{1-\delta + R_*^K} E_t rk_{t+1} - E_t cy_{t+1}$$
(8)

Capital Accumulation equation

$$k_{t} = \left(\frac{1-\delta}{\gamma}\right)k_{t-1} + \left(1-\frac{1-\delta}{\gamma}\right)inn_{t} + \left(1-\frac{1-\delta}{\gamma}\right)\left(1+\beta\gamma^{(1-\sigma_{c})}\right)\gamma^{2}\varphi\left(einn_{t}\right)$$
(9)

Labour demand

$$l_t = -w_t + \left(1 + \frac{1 - \psi}{\psi}\right) rk_t + k_{t-1} \tag{10}$$

NK Price Setting equation  $(\pi_t^{NK})$ 

$$\pi_{t} = \frac{\frac{\beta\gamma^{(1-\sigma_{c})}{\iota_{P}}}{1+\beta\gamma^{(1-\sigma_{c})}{\iota_{P}}} E_{t}\pi_{t+1} + \frac{\iota_{P}}{1+\beta\gamma^{(1-\sigma_{c})}{\iota_{P}}}\pi_{t-1} - \left(\frac{1}{1+\beta\gamma^{(1-\sigma_{c})}{\iota_{P}}}\right)}{\left(\frac{(1-\beta\gamma^{(1-\sigma_{c})}\xi_{p})(1-\xi_{p})}{\xi_{p}((\phi_{p}-1)\epsilon_{p}+1)}\right)(\alpha rk_{t} + (1-\alpha)w_{t} - ea_{t}) + ep_{t}}$$
(11)

NK Wage Setting equation  $(w_t^{NK})$ 

$$w_{t} = \frac{\frac{\beta\gamma^{(1-\sigma_{c})}}{1+\beta\gamma^{(1-\sigma_{c})}}E_{t}w_{t+1} + \frac{1}{1+\beta\gamma^{(1-\sigma_{c})}}w_{t-1} + \frac{\beta\gamma^{(1-\sigma_{c})}}{1+\beta\gamma^{(1-\sigma_{c})}}E_{t}\pi_{t+1} - \frac{1+\beta\gamma^{(1-\sigma_{c})}\iota_{w}}{1+\beta\gamma^{(1-\sigma_{c})}}\pi_{t} \\ w_{t} = +\frac{\iota_{w}}{1+\beta\gamma^{(1-\sigma_{c})}}\pi_{t-1} - \frac{1}{1+\beta\gamma^{(1-\sigma_{c})}}\left(\frac{\left(1-\beta\gamma^{(1-\sigma_{c})}\xi_{w}\right)(1-\xi_{w})}{(1+\epsilon_{w}(\phi_{w}-1))\xi_{w}}\right) \\ \left(w_{t} - \sigma_{l}l_{t} - \left(\frac{1}{1-\frac{\lambda}{\gamma}}\right)\left(c_{t} - \frac{\lambda}{\gamma}c_{t-1}\right)\right) + ew_{t}$$
(12)

FP Marginal Product of Labour  $(w_t^{FP})$ 

$$rk_t = \frac{1}{\alpha} \left[ (1 - \alpha) w_t + ea_t \right]$$
(13)

FP Labour Supply  $(\pi_t^{FP})$ 

$$w_t = \sigma_l l_t + \left(\frac{1}{1 - \frac{\lambda}{\gamma}}\right) \left(c_t - \frac{\lambda}{\gamma}c_{t-1}\right) - (\pi_t - E_{t-1}\pi_t) + ew_t^S$$

Weighted Inflation

$$\pi_t = \theta_\pi \pi_t^{NK} + (1 - \theta_\pi) \pi_t^{FP} \tag{14}$$

Weighted Wage

$$w_t = \theta_w w_t^{NK} + (1 - \theta_w) w_t^{FP} \tag{15}$$

Market Clearing condition in goods market

$$y_{t} = \frac{C}{Y}c_{t} + \frac{I}{Y}inn_{t} + R_{*}^{K}k_{y}\frac{1-\psi}{\psi}rk_{t} + c_{y}^{e}c_{t}^{e} + eg_{t}$$
(16)

Aggregate Production equation

$$y_t = \phi \left[ \alpha \frac{1 - \psi}{\psi} rk_t + \alpha k_{t-1} + (1 - \alpha) l_t + ea_t \right]$$
(17)

Taylor Rule

$$r_t = \rho r_{t-1} + (1-\rho) \left( r_p \pi_t + r_y y_t \right) + r_{\Delta y} \left( y_t - y_{t-1} \right) + er_t \text{ for } r_t > 0.0625$$
(18)

Premium

$$E_t cy_{t+1} - (r_t - E_t \pi_{t+1}) = pm_t = \chi \left( qq_t + k_t - n_t \right) - \psi m_t + \xi_t + epr_t$$
(19)

Net worth

$$n_{t} = \frac{K}{N} \left( cy_{t} - E_{t-1} cy_{t} \right) + E_{t-1} cy_{t} + \theta n_{t-1} + enw_{t}$$
(20)

Entrepreneurial consumption

$$c_t^e = n_t \tag{21}$$

M0

$$\Delta m_t = \psi_1 \Delta M_t + errm_{2t} \text{ for } r_t > 0.0625 \text{ and}$$
(22)

$$\Delta m_t = \psi_2(s_t - c^*) + errm_{2t} \text{ for } r_t \le 0.0625$$
(23)

M2

$$M_t = (1 + \nu - \mu)k_t + \mu m_t - \nu n_t \tag{24}$$

## **B** Analysis of various FTPL regime model versions

#### B.1 NK Model with exogenous interest rate

We implement the FTPL in the NK model whilst setting interest rates exogenous. The model fails to converge on the steady state equilibrium for output and real interest rates and so the model is not admissible theoretically.

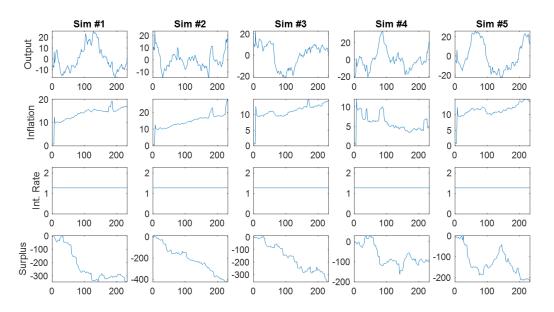


Figure 5: Example of Simulations: NK Model with exogenous interest rate

	Actual		Simu	ilations
	$\mathrm{mean}$	std. dev.	$\mathrm{mean}$	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	9.0758
PI	0.8065	0.5795	9.0161	3.1127
R	1.2751	0.9101	1.2751	0.0000

Table 8: Data and Simulations Statistics: NK Model with exogenous interest rate

#### B.2 NK Model with Zero Lower Bound

We implement the FTPL in the NK model with a zero lower bound. None of the simulations solved, and so the FTPL regime plainly could not survive and is not sustainable.

#### B.3 NK Model with AR(1) Interest Rate

We implement the FTPL in the NK model whilst setting interest rates to follow an AR(1) process with a persistence parameter of 0.9. This model is not theoretically admissible as it does not converge to a steady state equilibrium, so not viable.

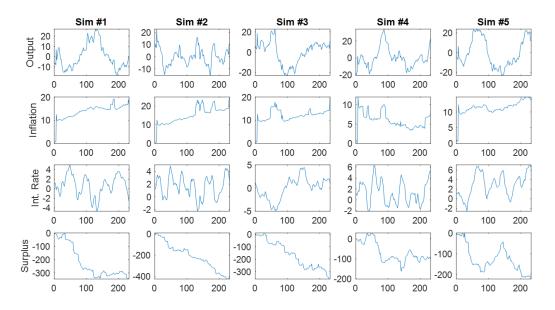


Figure 6: Example of Simulations: NK Model with AR(1) Interest Rate

	Actual		Simu	ilations
	$\mathrm{mean}$	std. dev.	$\operatorname{mean}$	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	9.0257
PI	0.8065	0.5795	9.0824	3.3716
R	1.2751	0.9101	1.7259	1.9248

Table 9: Data and Simulations Statistics: NK Model with AR(1) Interest Rate

#### B.4 RBC Model with Zero Lower Bound

Here we implement the FTPL model in the RBC model including a zero lower bound. Only 28% of the simulations solved; so again the FTPL policy regime is unsustainable, and so the model is automatically discounted.

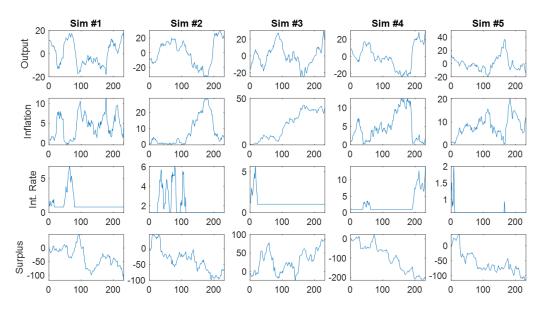


Figure 7: Example of Simulations: RBC Model with Zero Lower Bound

	Actual		Simulations	
	mean	std. dev.	mean	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	1.27E + 14
PI	0.8065	0.5795	1.75E + 15	3.60E + 15
R	1.2751	0.9101	1.77E + 15	3.51E + 15

Table 10: Data and Simulations Statistics: RBC Model with Zero Lower Bound

#### B.5 NK Model with Zero Lower Bound and Fiscal Reform

We implement the FTPL in the NK model with a zero lower bound whilst imposing an inflation cap ('tax reform'). The simulations show large volatility, with the inflation cap being hit in 75% of the simulations; this renders it unsustainable, like the totally unstable regime without the cap.

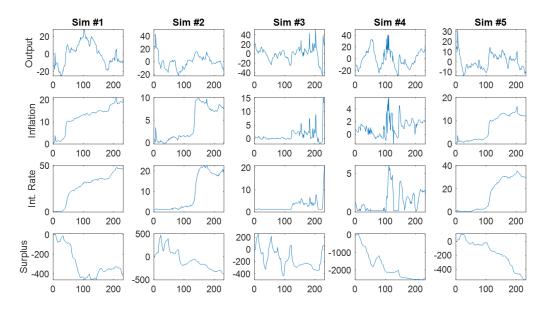


Figure 8: Example of Simulations: NK Model with Zero Lower Bound and Fiscal Reform

	Actual		Simu	ilations
	$\mathrm{mean}$	std. dev.	$\mathrm{mean}$	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	17.2486
PI	0.8065	0.5795	2.6712	2.2000
R	1.2751	0.9101	4.4809	3.9579

Table 11: Data and Simulations Statistics: NK Model with Zero Lower Bound and Fiscal Reform

	Inflation Cap $(3\%)$
Percent of Periods	14.3776
Percent of Simulations	74.6479

Table 12: Percentage of Simulations/Periods Hitting Expected Inflation Cap: NK Model with Zero Lower Bound and Fiscal Reform

#### B.6 RBC Model with Zero Lower Bound and Fiscal Reform

We implement the FTPL in the RBC model with a zero lower bound whilst imposing an inflation cap ('tax reform'). The simulations show large volatility, with the inflation cap being hit in 75% of the simulations; this regime too is unsustainable.

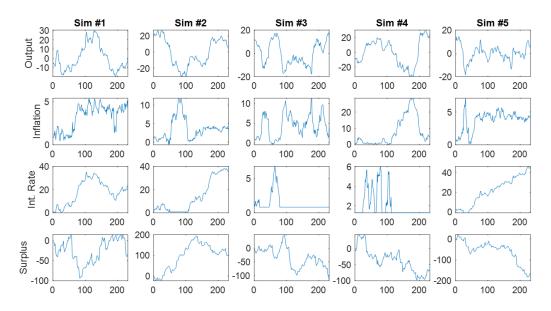


Figure 9: Example of Simulations: RBC Model with Zero Lower Bound and Fiscal Reform

	Actual		Simulations	
	$\mathrm{mean}$	std. dev.	mean	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	11.2287
PI	0.8065	0.5795	4.3105	2.6282
R	1.2751	0.9101	11.5410	7.3825

Table 13: Data and Simulations Statistics: RBC Model with Zero Lower Bound and Fiscal Reform

	Inflation Cap $(3\%)$
Percent of Periods	33.5116
Percent of Simulations	74.4980

Table 14: Percentage of Simulations/Periods Hitting Expected Inflation Cap: RBC Model with Zero Lower Bound and Fiscal Reform

#### B.7 NK Model with Zero Lower Bound and Fiscal Rule

We implement the FTPL in the NK model with a zero lower bound whilst imposing an inflation cap ('tax reform') and a fiscal rule to avoid output and inflation instability. The simulations show much less volatility than without the fiscal rule but the inflation cap is hit in 42% of the simulations, showing that the model suffers from extreme volatility. This makes this regime too unsustainable.

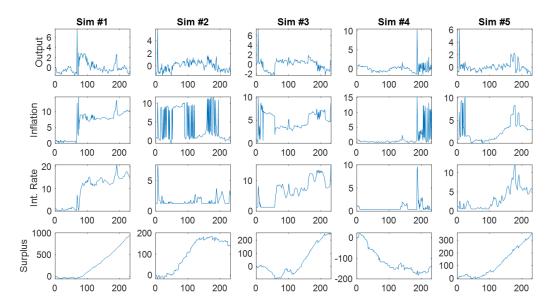


Figure 10: Example of Simulations: NK Model with Zero Lower Bound and Fiscal Rule

	Actual		Simulations	
	mean std. dev.		$\mathrm{mean}$	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	0.9621
PI	0.8065	0.5795	2.5882	2.4957
R	1.2751	0.9101	2.5024	2.1845

Table 15: Data and Simulations Statistics: NK Model with Zero Lower Bound and Fiscal Rule

	Inflation Cap $(3\%)$
Percent of Periods	0.9080
Percent of Simulations	42.2535

Table 16: Percentage of Simulations/Periods Hitting Expected Inflation Cap: NK Model with Zero Lower Bound and Fiscal Rule

#### B.8 RBC Model with Zero Lower Bound and Fiscal Rule

We implement the FTPL in the RBC model with a zero lower bound whilst imposing an inflation cap ('tax reform') and a fiscal rule to avoid output and inflation instability. The simulations show much less volatility and the inflation cap is not hit in any of the simulations. This is therefore a model that is potentially sustainable, the only one of the FTPL regimes that can be so regarded. We therefore proceed to test it, with the result that it nearly reaches the non-rejection theshold.

Wald	32.2414
Transformed Wald (t-stat)	1.8922
P-Value	0.0420
Percent of Simulations Solving	100

Table 17: Model Estimation Statistics: RBC Model with Zero Lower Bound and Fiscal Reform

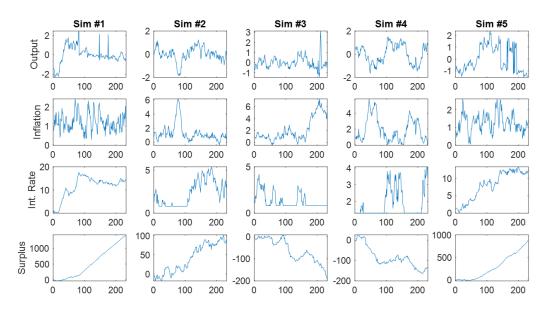


Figure 11: Example of Simulations: RBC Model with Zero Lower Bound and Fiscal Rule

	Actual		Simulations	
	$\mathrm{mean}$	std. dev.	mean	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	0.8309
PI	0.8065	0.5795	2.4534	1.5831
R	1.2751	0.9101	2.7349	1.6984

Table 18: Data and Simulations Statistics: RBC Model with Zero Lower Bound and Fiscal Rule

	Inflation Cap $(3\%)$
Percent of Periods	0.0000
Percent of Simulations	0.0000

Table 19: Percentage of Simulations/Periods Hitting Expected Inflation Cap: RBC Model with Zero Lower Bound and Fiscal Reform

#### B.9 Orthodox Model

From the orthodox model that passed the Wald test, we can see that some of the parameters of the auxiliary model are outside the bounds, but overall the test is passed. The simulations are largely in line with the data, with the occasional high inflation and interest rate.

	Actual	Lower	Upper	Mean	IN/OUT
Y_Y	0.9852	0.8698	0.9788	0.9299	OUT
Y_PI	-0.1735	-0.6829	0.8176	0.0367	IN
$Y_R$	-0.1808	-0.1169	0.2219	0.0364	OUT
PI_Y	0.012	-0.0163	0.0420	0.0056	IN
PI_PI	0.7173	0.4391	1.0658	0.7184	IN
PI_R	0.1110	-0.0632	0.0593	0.0110	OUT
R_Y	0.0052	-0.0814	0.0256	-0.0176	IN
R_PI	0.1084	-0.7293	0.4592	-0.1377	IN
$R_R$	0.9058	0.6906	1.0204	0.9281	IN
$\operatorname{var}(\mathbf{Y})$	0.6175	0.6456	1.3710	0.9030	OUT
$\operatorname{var}(\operatorname{PI})$	0.0642	0.0238	0.4273	0.0766	IN
$\operatorname{var}(\mathbf{R})$	0.0476	0.0217	1.0261	0.4815	IN

Table 20: Auxiliary Model Parameter Bounds: Orthodox Model

	Actual		Simulations	
	$\mathrm{mean}$	std. dev.	mean	std. dev.
Y (detrended)	0.0000	4.5265	0.0000	5.0475
PI	0.8065	0.5795	-0.1229	2.9202
R	1.2751	0.9101	5.8011	4.4545

Table 21: Data and Simulations Statistics: RBC Model with Zero Lower Bound and Fiscal Rule

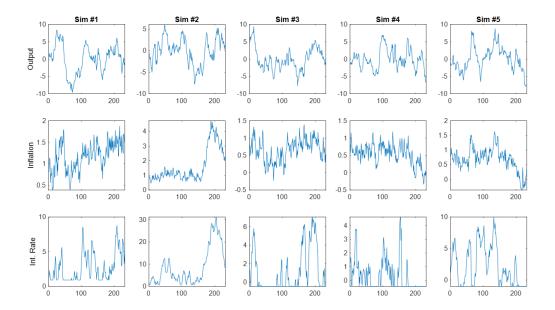


Figure 12: Example of Simulations: Orthodox Model