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Article

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Eurostat Review on National Accounts and Macroeconomic Indicators (EURONA)

Provided in Cooperation with:

Eurostat, Luxembourg

Suggested Citation: Silver, Mick S. (2018) : How to measure hedonic property price indexes better, Eurostat Review on National Accounts and Macroeconomic Indicators (EURONA), ISSN 1977-978X, Publications Office of the European Union, Luxembourg, Iss. 1, pp. 35-66

This Version is available at:

<https://hdl.handle.net/10419/309823>

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2

How to measure hedonic property price indexes better ⁽¹⁾

MICK SILVER ⁽²⁾

Abstract: Hedonic regressions are used for residential property price index (RPPI) measurement to control for changes in the quality-mix of properties transacted. This paper consolidates the confusing array of existing approaches and methods of implementation. It further develops an innovative form of weighting at the (elementary) level of the individual property and, therefrom, quasi-superlative and superlative formulations that improve on those in the literature. Well-grounded, practical, quasi-superlative RPPIs with dual imputations are devised that are suitable for thin markets and sparse data and not subject to the vagaries of the periodic estimation of hedonic regressions. All of this is with no additional data requirements and suitable for real time production.

JEL codes: C43, E30, E31, R31

Keywords: hedonic regressions, residential property price index, commercial property price index, house price index, superlative index number formula

⁽¹⁾ An earlier version of this paper has been presented at a meeting of the Ottawa Group on 10-12 May 2017 in Eltville am Rhein (Germany).

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1. Introduction

This paper consolidates existing methods and provides improved practical methods for the timely measurement of hedonic residential property price indexes (RPPIs), though the proposed methods apply equally to hedonic commercial property price indexes (CPPIs). Hedonic regressions are the main mechanism recommended for and used by countries for a crucial aspect of RPPI estimation — preventing changes in the quality-mix of properties transacted translating to price changes ⁽³⁾.

RPPIs and CPPIs are hard to measure. Houses, never mind commercial properties, are infrequently traded and heterogeneous. Average house prices may increase over time, but this may, in part, be due to a change in the quality-mix of the houses transacted. For example, more four-bedroom houses in a better (more expensive) postcode transacted in the current quarter compared with the previous or some distant reference quarter would bias upwards a measure of the change in average house prices. There is a need to measure constant-quality property price changes and while there are alternative approaches the concern of this paper is with the hedonic approach as a recommended *method of choice* (Hill (2013), pp. 906) ⁽⁴⁾.

The aim of this paper is to further develop a best practice methodology grounded in both the practical considerations and methodological rigor required for such an important statistic. The methodology proposed is consistent with, but extends the provisions in, the *2013 Handbook on RPPIs* (Eurostat et al. (2013)) that form the international standards in this area.

There are three main hedonic approaches to RPPI measurement: the hedonic time dummy approach, the characteristics/repricing approach, and the imputation approach. This follows previous literature in this area including Berndt (1991), Triplett (2006), Silver and Heravi (2007a), Hill (2013), and De Haan and Diewert (2013a). These approaches are outlined in Section 2. A problem is that there are many alternative forms for each approach depending on (i) the functional form of the hedonic regression and aggregation; (ii) the choice of reference, current or some average of the two, period(s) to estimate hedonic coefficients or hold characteristics/weights constant; (iii) whether dual or single imputation is used for prices and/or weights; (iv) whether a direct or indirect formulation is used; (v) the periodicity of the estimation, say monthly/quarterly/annually; (vi) use of chained, rolling window or fixed baskets of characteristics; and more.

⁽³⁾ The methodology is based on a more detailed working paper, Silver (2016).

⁽⁴⁾ Hill (2013) concludes his survey paper: 'Hedonic indexes seem to be gradually replacing repeat sales as the method of choice for constructing quality-adjusted house price indexes. This trend can be attributed to the inherent weaknesses of the repeat sales method (especially its deletion of single-sales data and potential lemons bias) and a combination of the increasing availability of detailed data sets of house prices and characteristics, including geospatial data, increases in computing power, and the development of more sophisticated hedonic models that in particular take account of spatial dependence in the data'. Alternative methods are the repeat sales method, mainly used in the United States, and the sales price appraisal method (SPAR), outlined and surveyed in Eurostat et al. (2013). A survey and evaluation of the impact of methods is provided in Silver (2015).

The variety of approaches and myriad forms without a clear path of preferences is unhelpful to compilers. In Section 3 this paper consolidates the approaches to help narrow down the choice of methods compilers face. Quite reasonable specifications of the hedonic regression and aggregation procedure are given that enable an equivalence of results from the characteristics and imputation approach. The paper continues with a focus on the imputation approach. At the end of the paper the choice of methods is considered with a case argued for the use of the weighted hedonic imputation approach against a weighted time dummy approach.

The countrywide practice of hedonic RPPIs suffers from a major defect. Although hedonic regressions are estimated over strata of quite broad locations and types of properties, for example detached houses in a capital city, there is usually no weighting attached to a price change of an individual house. Price changes of more expensive properties are given the same (expenditure) weight as those of cheaper houses. This is an abrogation of a basic principle of price index measurement. In Section 4 we show how weights can be readily attached to individual property price changes. Having done so, a natural next step is to define a superlative hedonic RPPI that makes symmetric use of reference period and current period weights. This is undertaken in two steps by (i) defining hedonic 'quasi-superlative' and (ii) re-defining 'hedonic superlative' RPPIs, to advance on existing formulations in the literature of these target measures. The quasi-superlative formulation is tightly phrased as a component of a hedonic superlative index and its implicit assumptions are readily testable.

In Section 5 we turn to and successfully address the practical problem of measuring weighted (quasi-) superlative RPPIs in real-time without additional data demands. Moreover, we show how the methodology can be best-formulated for sparse data in thin housing markets. RPPI estimation is formulated in a manner that first grounds the hedonic price comparisons in a reference period that is relatively exhaustive of the property mix that arises in subsequent periods. Second, it is developed in a manner that avoids sparse data in thin markets as well as the vagaries and economic cost of regular periodic estimation of hedonic regressions. The issue of estimating a weighted hedonic regression is addressed and returned to in Section 7.

The intention of the paper is to provide a methodology that makes a marked improvement on existing methods. Again, all of this is without additional data and in real time. In achieving all of this, a glitch is found, that is the need for double imputation. A (testable) workaround is provided in Section 6.

In the final part of the analysis, Section 7, we return to look at the weighted time dummy approach and how it fares as a weighted (quasi-) superlative RPPI estimated in thin markets, in comparison with the weighted quasi-superlative imputation RPPI developed in the previous sections.

Throughout the paper the development of RPPI hedonic methods is undertaken for log-linear hedonic specification, although Silver (2016) develops similar results for a linear specification. A clear path of preference in index number choice is provided at the end of the paper.

2. Measures of hedonic constant quality property price change

A. Hedonic regressions

The starting point is an estimated hedonic regression for (a stratum of) properties in a country. The principles governing the specification and estimation of hedonic regressions are not the subject of this paper ⁽⁵⁾. The concern of this paper is with how hedonic regressions are used to derive RPPIs.

Throughout the paper, hedonic RPPIs are based on a log(arithmetic)-linear—semi-log—hedonic functional form, though similar principles apply to linear, log-log, and more flexible forms. The log-linear form: first, allows for curvature in the relationships say between square footage and price; second, for a multiplicative association between quality characteristics, for example, that possession of a garage and additional bathroom may be worth more than the (linear) sum of the two; and third, is more practical than a log-log form since many characteristics take a zero or one (possession or not of a characteristic) and logarithms cannot be taken of zero values. Silver (2016) provides a detailed exposition of the issues and methods for a linear functional form. Consider a relationship between the price of property i , p_i , and $k=1, \dots, K$ price-determining characteristics, $z_{k,i}$, along with a constant $z_{k=0,i} = 1$, given by:

$$(1) \quad p_i = \prod_{k=0}^K \beta_k^{z_{k,i}} \varepsilon_i$$

A log-linear hedonic regression equation for (the logarithm of) prices on $z_{k,i}^t$ characteristics for period t data is given by:

$$(2.1) \quad \ln p_i^t = \ln \beta_0^t + \sum_{k=1}^K z_{k,i}^t \ln \beta_k^t + \ln \varepsilon_i^t$$

An estimated ordinary least squares (OLS) regression equation for equation (1) is given as:

$$(2.2) \quad \ln \hat{p}_i^t = \ln \hat{\beta}_0^t + \sum_{k=1}^K z_{k,i}^t \ln \hat{\beta}_k^t$$

where \hat{p}_i^t (and p_i^t) are the predicted (and actual) price of property i in period t ; $z_{k,i}^t$ are the values of each $k=1, \dots, K$ price-determining characteristics for property i in period t ; $\hat{\beta}_0^t$ and $\hat{\beta}_k^t$ are the estimated (and actual) coefficients for each characteristic $z_{k,i}^t$; ε_i^t are independent identically distributed (i.i.d.) errors, using period t data and characteristics ⁽⁶⁾.

Hedonic RPPIs can be based on: (i) the hedonic time dummy variable, (ii) hedonic characteristics/repricing, or (iii) hedonic imputation approaches. We outline each in turn:

⁽⁵⁾ Readers are referred to Berndt (1991) and Triplett (2006) for a clear overview of hedonic regression methods, albeit not in the context of house prices, for the real estate sector to Sirmans et al. (2006), on explanatory variables for the hedonic regression to de Haan and Diewert (2013a), Hill and Scholz (2018), and for a land structure decomposition to Diewert and Shimizu (2015).

⁽⁶⁾ The log-linear regression output from estimating equation (2.2), that is $\ln \hat{p}_i^t$ on $z_{k,i}^t$, provides us with the logarithms of the coefficients from the original log-linear formulation in equation (1). Exponents of the estimated coefficients from the output of the software have to be taken if the parameters of the original function in equation (1) are to be recovered, that is: $\exp(\ln \hat{\beta}_k^t) = \hat{\beta}_k^t$.

B. The hedonic time dummy variable approach

A single hedonic regression equation is estimated with observations across properties transacted over several time periods, including the reference period 0 and successive subsequent periods t . (The logarithm of) prices of individual properties are regressed on their characteristics and dummy variables for time, taking the values of $D_1^1 = 1$ if the house is sold in period 1 and zero otherwise; $D_1^2 = 1$ if the house is sold in period 2 and zero otherwise ... , and $D_1^T = 1$ if the house is sold in period T and zero otherwise. We exclude in this case a period 0 dummy time variable. A log-linear specification for a time dummy variable hedonic regression over periods $t=0,1,2,\dots,T$ is given by:

$$(3) \quad \ln \hat{p}_i = \hat{\beta}_0 + \sum_{k=1}^K z_{k,i} \ln \hat{\beta}_k + \sum_{t=1}^T \hat{\delta}^t D_i^t$$

The $\hat{\delta}^t$ are estimates of the *proportionate* change in price arising from a change between the reference period $t=0$ — the period not specified as a dummy time variable — and successive periods $t=1,\dots,T$ having controlled for changes in the quality characteristics via the term

$$\sum_{k=1}^K z_{k,i} \ln \hat{\beta}_k.$$

In principle, the index, $100 \times \exp(\hat{\delta}^t)$ requires an adjustment for it to be a consistent (and almost unbiased) approximation of the proportionate impact of the time dummy (⁷). In practice, the adjustment usually has little effect.

The method implicitly restricts the coefficients on the quality characteristics to be constant over time: for example, for an adjacent period 0 and 1 time dummy hedonic regression, for $k=1,\dots,K$, $t=0,1$: $\beta_k = \beta_k^0 = \beta_k^1$ and $100 \times \exp(\hat{\delta}^1)$ is an estimate of the RPPI for period 1 (period 0=100). The extent of this restriction depends on the length of the time period over which the regression is run. If, for example, the regressions are run over quarterly data for a 10-year window, a property price comparison between say the first quarter of 2007 and the first quarter of 2017 with valuations of characteristics held constant may stretch credibility, though this can be alleviated by chained shorter and/or moving windows or adjacent period regressions (Silver (2016)).

C. The hedonic characteristics/repricing approach

A hedonic regression is run to determine the price-determining characteristics of properties in a say reference period 0. The average property in period 0 can then be defined as a tied bundle of the averages of each price-determining characteristic, for example, 2.8 bathrooms, 3.3 bedrooms, 0.8 garages, 0.2 transactions in an up-market location, and so forth — our starting point.

These average characteristics are held constant in each period but valued in turn using a period 0 and a period t hedonic regression. The (average) *characteristics* approach answers the question: what would be the price change of a set of average period t characteristics valued first, at period t hedonic valuations, and second, at period 0 hedonic valuations? A ratio of the results is a constant (period t) quality property price index.

(⁷) See Kennedy (1981), Van Garderen and Shah (2002), and the note at the end of Hill (2013) for the approximation, shown by Giles (2011) to be accurate, even for quite small samples.

Hill et al. (2018) in a survey of methodologies used by European national statistical institutes (NSIs) found the characteristics approach to be used by the NSIs of two countries in Europe, though a further eight countries used a variant of it, the repricing (of average characteristics) approach. It is shown in Hill et al. (2018) that the repricing approach can be represented as a fixed base average characteristics approach, that in turn in Section 3 is shown to be equivalent to the imputation approach. Rather than distinguish between the repricing and characteristics approach, we outline the latter since it encompasses properties of the former and our subsequent focus is, in any event, on the imputation approach.

A constant-quality *hedonic geometric mean characteristics (HGMC)* price index from a log-linear hedonic regression equation is a ratio of geometric means with characteristics held constant in the *current* period t , \bar{z}_k^t :

$$(4) \quad P_{HGMC\bar{z}^t}^{0 \rightarrow t} = \frac{\prod_{k=0}^K (\hat{\beta}_k^t)^{\bar{z}_k^t}}{\prod_{k=0}^K (\hat{\beta}_k^0)^{\bar{z}_k^t}} = \frac{\exp\left(\sum_{k=0}^K \bar{z}_k^t \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^K \bar{z}_k^t \ln \hat{\beta}_k^0\right)} \quad \text{where} \quad \bar{z}_k^t = \frac{1}{N^t} \sum_{i \in N^t} z_{i,k}^t$$

Equation (4) holds the (quality) characteristics constant in period t , though a similar index could be equally justified by valuing in each period a constant period 0 average quality set:

$$(5) \quad P_{HGMB\bar{z}^0}^{0 \rightarrow t} = \frac{\prod_{k=0}^K (\hat{\beta}_k^t)^{\bar{z}_k^0}}{\prod_{k=0}^K (\hat{\beta}_k^0)^{\bar{z}_k^0}} = \frac{\exp\left(\sum_{k=0}^K \bar{z}_k^0 \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^K \bar{z}_k^0 \ln \hat{\beta}_k^0\right)} \quad \text{where} \quad \bar{z}_k^0 = \frac{1}{N^0} \sum_{i \in N^0} z_{i,k}^0$$

Neither a period 0 constant-characteristics index nor a period t constant-characteristic quantity basket can be considered to be superior, both acting as bounds for their theoretical counterparts. Some average or compromise solution is required. Symmetric use of period 0 and period t characteristics values make sense. We do not draw on economic theory here since we have no weights.

$$(6) \quad P_{HGMC\sqrt{\bar{z}^0\bar{z}^t}}^{0 \rightarrow t} = \frac{\prod_{k=0}^K (\hat{\beta}_k^t)^{\bar{z}_k^r}}{\prod_{k=0}^K (\hat{\beta}_k^0)^{\bar{z}_k^r}} = \frac{\exp\left(\sum_{k=0}^K \bar{z}_k^r \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^K \bar{z}_k^r \ln \hat{\beta}_k^0\right)} \quad \text{where} \quad \bar{z}_k^r = (\bar{z}_k^0 + \bar{z}_k^t) / 2$$

We introduce weights in Section 5 and develop there a new formulation for a superlative hedonic RPPI.

D. The hedonic imputation approach

In contrast to the characteristics approach, the *imputation* approach works at the level of individual properties, rather than the average values of their characteristics. The rationale for the imputation approach lies in the matched model method. Consider a set of properties transacted in period t . We want to compare their period t prices with the prices of the same matched properties in period 0. In this way there is no contamination of the measure of price change by changes in the quality-mix of properties transacted. However, the period t properties were not sold in period 0 — there is no corresponding period 0 price. The solution — in the numerator of equation (7) — is to predict the period 0 price of each period

t property. We use a period 0 regression to predict prices of properties sold in period t to answer the counterfactual question: what would a property with period t characteristics have sold for in period 0?

A constant-quality *hedonic geometric mean imputation (HGMI)* price index is a ratio of the geometric means of prices of individual properties in period t compared with period 0 of properties transacted in the *current* period t . The value in the numerator of equation (7) is the geometric mean of the period t price of period t price-determining characteristics, $z_{i,k}^t$. This is compared, in the denominator, with the geometric mean of the period 0 predicted price of the self-same period t price-determining characteristics, $z_{i,k}^t$. For each property, the quantities of characteristics are held constant in period t , $z_{i,k}^t$; only the characteristic prices change. Where N^t is the number of properties transacted in period t :

$$(7) \quad P_{HGMI|z_i^t}^{0 \rightarrow t} = \frac{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^t)^{\frac{1}{N^t}} \exp\left(\frac{1}{N^t} \sum_{i \in N^t} \ln \hat{p}_{i|z_i^t}^t\right)}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\frac{1}{N^t}} \exp\left(\frac{1}{N^t} \sum_{i \in N^t} \ln \hat{p}_{i|z_i^t}^0\right)}$$

And a constant period 0 characteristics, z_i^0 , hedonic imputation HGMI where N^0 is the number of properties transacted in period 0 is given by:

$$(8) \quad P_{HGMI|z_i^0}^{0 \rightarrow t} = \frac{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^t)^{\frac{1}{N^0}} \exp\left(\frac{1}{N^0} \sum_{i \in N^0} \ln \hat{p}_{i|z_i^0}^t\right)}{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^0)^{\frac{1}{N^0}} \exp\left(\frac{1}{N^0} \sum_{i \in N^0} \ln \hat{p}_{i|z_i^0}^0\right)}$$

DUAL IMPUTATIONS

A natural question arises as to the phrasing of the predicted prices in equations (4) to (8) as **dual imputations**, that is they use predicted (imputed) prices in both the denominator and numerator — Silver (2001) and de Haan (2004a).

Dual imputation requires a predicted (imputed) price in both the denominator and numerator of equations (7) and (8), and for that matter equations (4), (5) and (6). For example, in equation (7) the single imputation index could be defined to use the actual price in the numerator and predicted price in the denominator. The denominator is a counterfactual price that a transacted property in period t would have sold for in period 0; a hedonic regression in period 0 is required. The logic for the need for dual imputations arises from the possibility of substantial omitted variable bias in the hedonic specification. For example, some cheaper terraced (row) houses may have no front garden, as they open directly onto the street. This poorer feature would be reflected in the actual price (numerator) of a constant period t index, but may be excluded or not properly represented in the hedonic specification and thus predicted price (denominator), unless a separate dummy variable: ‘no front garden’ is included in the hedonic regression. Without the new dummy variable the denominator would be biased upwards and the index downwards. The dual imputation hedonic index may to some extent offset any such upward bias by using predicted prices in both the numerator and denominator. Dual imputations are generally advised for hedonic price indexes, see: Silver and Heravi (2001); Silver (2004); de Haan (2004a); Hill and Melser (2008); Diewert, Heravi and Silver (2009); associated comments, de Haan (2009) and response, Hill (2013); and de Haan and Diewert (2013).

Yet, a feature of the OLS estimator is that the mean of actual prices is equal to the mean of predicted prices: $\frac{1}{N^0} \sum_{i \in N^0} \hat{p}_{ilz_i^0}^0 = \frac{1}{N^0} \sum_{i \in N^0} p_i^0$ and $\frac{1}{N^t} \sum_{i \in N^t} \hat{p}_{ilz_i^t}^t = \frac{1}{N^t} \sum_{i \in N^t} p_i^t$, and similarly for the logarithms of prices. Thus while the denominator of equation (7) must be counterfactual and use predicted prices, the numerator of equation (7) can use actual prices — see also de Haan and Diewert (2013), paragraph 5.38. Thus, when using unweighted hedonic imputation indexes or, as we will see, characteristics hedonic indexes, there is no need to estimate hedonic regressions in each period for (7), actual prices can be used in the numerator: equation (7) becomes:

$$(9) \quad P_{HGM|z_i^t}^{0 \rightarrow t} = \frac{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^t)^{\frac{1}{N^t}}}{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^0)^{\frac{1}{N^t}}} = \frac{\prod_{i \in N^t} (p_{ilz_i^t}^t)^{\frac{1}{N^t}}}{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^0)^{\frac{1}{N^t}}} = \frac{\exp\left(\frac{1}{N^t} \sum_{i \in N^t} \ln p_{ilz_i^t}^0\right)}{\exp\left(\frac{1}{N^t} \sum_{i \in N^t} \ln \hat{p}_{ilz_i^t}^0\right)}$$

This is an important result since, using the principles and practice extolled in Section 5, it aids the practical work of compilers, especially in thin housing markets not to have to estimate a hedonic regression equation in each period, but maybe once a year, or every two years and chain the resulting RPPIs together. We return to this issue in Sections 4 and 5 where weighting is considered and double imputation is more problematic.

E. An indirect approach to hedonic price indexes

The indirect approach is not new, as outlined in Triplett (2006). In log-linear form a constant period t hedonic imputation RPPI ⁽⁸⁾ is given by:

$$(10) \quad P_{InGM|z_i^t}^{0 \rightarrow t} = \left(\frac{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^t)^{\frac{1}{N^t}}}{\prod_{i \in N^0} (\hat{p}_{ilz_i^0}^0)^{\frac{1}{N^0}}} \right) / \left(\frac{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^0)^{\frac{1}{N^t}}}{\prod_{i \in N^0} (\hat{p}_{ilz_i^0}^0)^{\frac{1}{N^0}}} \right) = \frac{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^t)^{\frac{1}{N^t}}}{\prod_{i \in N^t} (\hat{p}_{ilz_i^t}^0)^{\frac{1}{N^t}}}$$

In calculating equation (10) we take the change in average prices in the numerator and divide it by the characteristics volume change between periods 0 and t , holding the marginal valuations of these characteristics constant in period 0. This yields a price index with quality characteristics held constant at current period values. A price index with quality characteristics held constant at reference period values can be similarly defined. The time dummy method is an implicit indirect approach measuring the change in average prices (the intercept shift) having controlled for the change in characteristics. De Haan (2004b) and Diewert, Heravi, and Silver (2009) show the equivalence of this indirect hedonic characteristics index to a hedonic time dummy one.

⁽⁸⁾ An indirect hedonic characteristics RPPI would take the form of a re-pricing index, see Hill et al. (2018).

3. Some equivalences

The three approaches have different, yet valid, intuitions. Yet, as long as the functional form of the aggregator is aligned to the hedonic regression in the manner shown in Table 2.1 below, the imputation and characteristics approaches yield the same result. This consolidation not only markedly narrows down the choice between approaches but validates the measure as one resulting from quite different intuitions.

Table 2.1: Equivalences of hedonic approaches

Hedonic regression: functional form	Characteristics approach: form of average of characteristics	Imputation approach: form of average of predicted prices
Linear	Arithmetic mean	Arithmetic mean
Log-linear	Arithmetic mean	Geometric mean
Log-log	Geometric mean	Geometric mean

For a log-linear functional form of a hedonic regression, the requirements are that (i) for the characteristics approach, \bar{z}_k^0 and \bar{z}_k^t are arithmetic means of characteristic's values, the right-hand-side (RHS) of the hedonic regression, and (ii) for the imputation approach, the ratio of average predicted prices is a ratio of geometric means, the left-hand-side (LHS). Similar equivalences shown in Table 2.1 apply to linear and log-log forms. While Hill and Melser (2008) confine the equivalences to the log-linear hedonic model, they identify the same property:

T3 [a geometric mean of geometric Laspeyres and geometric Paasche hedonic indexes] ... has attractive properties when the hedonic takes the log-linear form. The fact that it can be defined in either goods or characteristics space adds flexibility to the way the results can be interpreted. For example, T3 can be interpreted either as measuring the average of the ratios over the two region-periods of the imputed price of each house or as the ratio of the imputed price of the average house. Which perspective is most useful may depend on the context. Hill and Melser (2008, pp. 602).

A log-linear hedonic characteristics price index with constant reference-period average characteristics, $\bar{z}_k^0 = \frac{1}{N^0} \sum_{i \in N^0} z_{i,k}^0$, equals an imputation index for reference period properties:

$$\begin{aligned}
 (11) \quad p_{HGM|z_i^0}^{0 \rightarrow t} &= \frac{\prod_{k=0}^K (\hat{\beta}_k^t)^{\bar{z}_k^0}}{\prod_{k=0}^K (\hat{\beta}_k^0)^{\bar{z}_k^0}} = \frac{\exp\left(\sum_{k=0}^K \bar{z}_k^0 \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^K \bar{z}_k^0 \ln \hat{\beta}_k^0\right)} = \frac{\exp\left(\frac{1}{N^0} \sum_{k=0}^K \sum_{i \in N^0} z_{i,k}^0 \ln \hat{\beta}_k^t\right)}{\exp\left(\frac{1}{N^0} \sum_{k=0}^K \sum_{i \in N^0} z_{i,k}^0 \ln \hat{\beta}_k^0\right)} \\
 &= \frac{\exp\left(\frac{1}{N^0} \sum_{i \in N^0} \sum_{k=0}^K z_{i,k}^0 \ln \hat{\beta}_k^t\right)}{\exp\left(\frac{1}{N^0} \sum_{i \in N^0} \sum_{k=0}^K z_{i,k}^0 \ln \hat{\beta}_k^0\right)} = \frac{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^t)^{\frac{1}{N^0}}}{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^0)^{\frac{1}{N^0}}}
 \end{aligned}$$

and similarly, average characteristics held constant in the current period t , $\bar{z}_k^t = \frac{1}{N^t} \sum_{i \in N^t} z_{i,k}^t$ is equal to an imputation index for current period t properties:

$$(12) \quad p_{HGM|z_i^t}^{0 \rightarrow t} = \frac{\prod_{k=0}^K (\hat{\beta}_k^t)^{\bar{z}_k^t}}{\prod_{k=0}^K (\hat{\beta}_k^0)^{\bar{z}_k^t}} = \frac{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^t)^{\frac{1}{N^t}}}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\frac{1}{N^t}}}$$

4. Weights and superlative hedonic price indexes

A. Weights in a hedonic RPPI

So far we have made no mention of an essential element of index number construction: the weighting of price changes. If one index number formula has a superior weighting, other things being equal, it is preferred. As noted by Griliches (1971, pp. 326): *There is no good argument except simplicity for the one-vote-per-model approach to regression analysis* ⁽⁹⁾.

We distinguish between two levels of aggregation: the lower and higher levels. Property price indexes are often stratified by type and location to form more homogeneous strata of properties, say apartments in the capital city. The national or some higher-level index is compiled as a weighted average of the constant-quality price changes of the individual strata indexes. These higher-level strata are very broad, designed to ensure a large sample size is available for the estimation of hedonic regressions within them. At the lower or elementary level constant-quality price indexes are estimated for each stratum, generally as unweighted, that is equally-weighted, indexes. That say a price change of a three-bedroom apartment in an up-market area of a capital city should have the same weight as that of a studio apartment in a down-market area goes against the well-accepted principles, as embodied in international measurement standards (Eurostat et al. (2013), of expenditure-weighted price index numbers. Given the heterogeneity of price changes within these broad strata the absence of weighting systems at the lower level, within strata — at the level of the price change of the individual property — is a major shortcoming. To the author's knowledge no statistical office currently successfully uses weights except at the crudest higher level.

Weights at the higher and lower levels, as described in Silver (2016), can be the relative values of transactions or stocks of properties for each stratum ⁽¹⁰⁾. This choice between the use of 'transactions' or 'stocks' as weights depends on the purpose of the property price index and availability of adequate data on the stock of properties. Fenwick (2013) and Mehrhoff and Triebkorn (2016) outline issues relevant to such a choice, though the concern here is with the methodology for incorporating weights into the lower level within stratum RPPI measurement.

There is literature on elementary price index number formulas based on the needs of consumer, producer and trade price indexes. While some of these results have a bearing on the analysis here, the context differs in important respects. First, the matched prices are predicted constant-quality prices for *individual* properties. Second, an individual property sold in the reference (current) period has as its matched price in the current (reference) period a counterfactual predicted price. Third, the weight to be attached to each property's price

⁽⁹⁾ Griliches (1961, 1964) revived the hedonic approach to the construction of price indexes. Griliches (1971) raised methodological issues that foreshadowed many of the issues of concern in this paper including the need for weighting in regression estimates and the empirical form of the relationship, commenting on the preferred use of the semi-logarithmic form.

⁽¹⁰⁾ Rambaldi and Rao (2013) provide details on hedonic price indexes using democratic (equal) weights as opposed to plutocratic (stock or expenditure-share) weights.

change is its relative expenditure, that is, its price. Fourth, the elementary property price indexes are constant-quality indexes that make use of hedonic (or repeat sales) regressions. The weights given to the property price observations, for a time dummy method, are implicit in the way observations of prices enter into the regression or aggregation formula. We provide an improved mechanism for weighting at this lower elementary level.

In this section we consider three issues which allow us to develop a hedonic superlative price index number; in Section 4B we develop a proposed method for weighting hedonic property price indexes to form what we term as ‘quasi-superlative indexes’. Superlative price index number formulas are less likely to suffer from substitution bias, a bias that results when a single-period fixed basket index is used to estimate a cost of living index. The bias arises because a fixed basket index cannot take account of the effects on the cost of living of the substitutions made by consumers in response to changes in relative prices. In general, the earlier the period of which the basket is used, the greater the upward bias in the index.

Section 4C provides a definition of hedonic superlative price indexes and shows how they differ from the ‘quasi’ formulations in terms of an absence of sample selectivity bias. The quasi-superlative and superlative RPPIs defined in Sections 4B and 4C are derived from a hedonic imputation approach. Section 4D provides equivalent derivations from a hedonic characteristics approach. The formulations derived in Sections 4B and 4C differ from accepted wisdom and in Section 4E we use the, in many ways, seminal paper by Hill and Melser (2008) to show how this formulation improves on the one they advocate, which has been used by others in much subsequent work. In Section 4F we turn to a problem in using weights for the time dummy approach.

B. Quasi-superlative hedonic RPPIs

Consider again equation (8); the index is a measure of price change for constant-period 0 characteristics property price indexes:

$$(13) \quad p_{HGMI; z^0}^{0 \rightarrow t} = \frac{\prod_{i \in N^0} (\hat{p}_{i|z^0}^t)^{\frac{1}{N^0}}}{\prod_{i \in N^0} (p_i^0)^{\frac{1}{N^0}}} = \prod_{i \in N^0} \left[\frac{\hat{p}_{i|z^0}^t}{\hat{p}_{i|z^0}^0} \right]^{\frac{1}{N^0}}$$

There are three problems with this measure: (i) property price changes are equally weighted; (ii) the index is based on only the sample of properties transacted in period 0; and (iii) the introduction of explicit weights precludes our previous use of equating average predicted prices to average actual prices, as a means by which dual imputations are introduced. We consider each in turn.

The first task is to apply weights to these price changes. A useful opportunity exists using the imputation approach to explicitly introduce weights at this very lowest level. This approach, to the author’s knowledge, was first proposed in Feenstra (1995) and used by Ioannidis and Silver (1999) in an application, using scanner data, of hedonic methods to quality adjust price indexes for television sets, in Silver and Heravi (2007a), further developed in Diewert, Heravi, and Silver (2009), and in the context of RPPIs, in Hill and Melser (2008).

As outlined in Section 2D, the *imputation* approach works at the level of individual properties, rather than the average values of their characteristics. It provides for properties transacted in a reference (current) period an imputed matched price in the current (reference) period. This allows us to explicitly attach a weight to each property's matched price change. Period

0 weights would be $\frac{\hat{p}_{i|z_i^0}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i^0}^0}$ given to each price change, $\left(\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right)$ in equation (13). In this

unusual context, a property's relative transaction price is its expenditure weight. We *explicitly* weight price changes by their relative (predicted) price/transaction value in period 0. The price changes of more expensive properties are given a higher (period 0) proportionate weight:

$$(14) \quad P_{LaHGMI; z_i^0}^{0 \rightarrow t} = \prod_{i \in N^0} \left[\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right]^{\left(\frac{\hat{p}_{i|z_i^0}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i^0}^0} \right)} = \exp \left(\sum_{i \in N^0} \left(\frac{\hat{p}_{i|z_i^0}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i^0}^0} \right) \ln \left[\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right] \right)$$

There is then the question of why only period 0 transactions and weights are used for this measure of constant-quality price change. Equally justified is the use of period t transactions and weights:

$$(15) \quad P_{PaHGMI; z_i^t}^{0 \rightarrow t} = \prod_{i \in N^t} \left[\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right]^{\left(\frac{\hat{p}_{i|z_i^t}^t}{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^t} \right)} = \exp \left(\sum_{i \in N^t} \left(\frac{\hat{p}_{i|z_i^t}^t}{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^t} \right) \ln \left[\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right] \right)$$

Neither equations (14) nor (15) are superior to the other. However, we can use a symmetric average of period 0 and period t weights: a hedonic quasi-Törnqvist price index, but based on a period 0 sample selection is given by:

$$(16) \quad P_{QToHGMI; z_i^0}^{0 \rightarrow t} = \prod_{i \in N^0} \left(\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right)^{\hat{w}_i^t} = \exp \left(\sum_{i \in N^0} \hat{w}_i^t \ln \left(\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right) \right) = \exp \left(\sum_{i \in N^0} \hat{w}_i^t \ln \left(\hat{p}_{i|z_i^0}^t - \hat{p}_{i|z_i^0}^0 \right) \right)$$

$$\text{where } \hat{w}_i^t = \frac{1}{2} \left(\frac{\hat{p}_{i|z_i^t}^t}{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^t} + \frac{\hat{p}_{i|z_i^t}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i^t}^0} \right)$$

which is a quasi-hedonic formulation of a Törnqvist index (Feenstra (1995), Ioannidis and Silver (1999) and Balk (2008)), an index that has excellent properties in economic theory as a superlative index (Diewert (2004)). It is 'quasi' in the sense that it does not make use of the sample of period t transactions. It is 'superlative' in the sense that the index of price changes of transactions undertaken in period 0 makes symmetric use of reference and current period price information.

Equation (16) uses a period 0 sample of transactions. A similar quasi-hedonic Törnqvist index based on period t transactions is given by:

$$(17) \quad P_{QToHGM:z_i^t}^{0 \rightarrow t} = \prod_{i \in N^t} \left(\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right)^{\hat{w}_i^t} = \exp \left(\sum_{i \in N^t} \hat{w}_i^t \ln \left(\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right) \right) = \exp \left(\sum_{i \in N^t} \hat{w}_i^t \ln \left(\hat{p}_{i|z_i^t}^t - \hat{p}_{i|z_i^t}^0 \right) \right)$$

These innovative quasi-hedonic superlative formulas depart from conventional hedonic formulations — Diewert (2003), de Haan (2004a), Silver and Heravi (2005), de Haan and Krsinich (2014, Appendix A) — in which the weights attached to each price change for transactions in period 0 are the relative expenditures in period 0 (for $i \in N^0$) and for period t are the relative expenditures in t , (for $i \in N^t$), as opposed to an average of period 0 and t , as in equations (15) and (16).

We note that in using equations (14) to (17), we have a comparison between predicted prices in period 0 and counterfactual predicted prices in period t . We can no longer rely on the OLS property of average predicted prices equalling average actual prices to achieve double imputation. This need to undertake a hedonic regression each current period for equation (16) is revisited in Section 7 with a workaround. We also note that given these predicted prices act as corresponding weights in period t for the price change, it would be wasteful to abandon the thought experiment for the weights but not for the price change. Indeed, abandoning \hat{w}_i^t in favour of \hat{w}_i^0 would remove the analytical power of taking some account of substitution bias.

C. Hedonic superlative indexes and sample selection bias

The quasi-Törnqvist indexes in equations (16) and (17) were each based on samples of period 0 and t transactions respectively. In both cases, the distinction is not one of substitution bias; it is a sample selection bias. Substitution bias arises, in this context, from using period 0 or period t weights, rather than a symmetric mean of the two period's expenditure weights, as in a Törnqvist index. The quasi-superlative formulas outlined above make symmetric use of both periods' weights, but limits the sample to transactions in either period 0 or period t . Our hedonic Törnqvist price index should be based on samples of period 0 *and* period t transactions.

Some additional notation may help clarify the formulas. Let $S(0 \cap t)$ be the set of properties that are transacted in both periods 0 and t , $S(0 - t)$ is the set of properties transacted in period 0 but not period t , and $S(t - 0)$ is the set of properties that are transacted in period t but not period 0. The weights for each term are the relative transaction values of these sets of data, that is, where V is the total value of transaction prices (or stocks) for $S(0 \cap t)$, $S(0 - t)$ and $S(t - 0)$, $V = \sum_{i \in S(t-0) \cup S(0-t) \cup S(0 \cap t)} v_i$; $V_{0-t} = \sum_{i \in 0-t} v_i$; $V_{t-0} = \sum_{i \in t-0} v_i$; and $V_{0 \cap t} = \sum_{i \in 0 \cap t} v_i$ and w_i^t is an arithmetic mean of the weight (relative stock value or transaction (price) value) given to each property in periods 0 and t , that is $\hat{w}_i^t = 1/2(\hat{w}_i^0 + \hat{w}_i^t)$. Bear in mind that we are weighting the price change of each individual property and the weight is the relative

expenditure that equates to the price of the property. In this unusual situation we can use

predicted prices for weights, as argued above: $\hat{w}_i^t = \frac{1}{2} \left(\frac{\hat{p}_{i|z_i^t}^t}{\sum_{i \in S(0 \rightarrow t)} \hat{p}_{i|z_i^t}^t} + \frac{\hat{p}_{i|z_i^t}^0}{\sum_{i \in S(0 \rightarrow t)} \hat{p}_{i|z_i^t}^0} \right) = \frac{1}{2} (\hat{w}_i^0 + \hat{w}_i^t)$.

The hedonic Törnqvist price index is:

$$(18) \quad P_{ToHGMl:z_i^t}^{0 \rightarrow t} = \prod_{i \in S(0 \rightarrow t)} \left(\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right)^{\hat{w}_i^t \times \frac{V_{t-0}}{V}} \times \prod_{i \in S(t \rightarrow 0)} \left(\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right)^{\hat{w}_i^t \times \frac{V_{0-t}}{V}} \times \prod_{i \in S(0 \cap t)} \left(\frac{\hat{p}_{i|z_i^t}^t}{\hat{p}_{i|z_i^t}^0} \right)^{\hat{w}_i^t \times \frac{V_{0 \cap t}}{V}}$$

The superlative Törnqvist hedonic price index follows Triplett and McDonald (1977), Diewert (2003), Triplett (2006), de Haan (2004a), and Silver and Heravi (2005) ⁽¹⁾. We note that for repeat sales, $S(0 \cap t)$, we have used a double imputation, that is predicted prices, when actual prices are available. At first sight this goes against the principles of matched models measurement whereby actual prices are compared, say for the price change of a single standard can of Coca-Cola for a consumer price index, the price is compared over time like with like. However, as Hill and Melser (2008) explain:

As far as we are aware, the possibility of always imputing for a repeat observation ... has not previously been considered in the literature. For the case of computers, this would be hard to justify since a particular model is the same irrespective of when it is sold. Housing, however, is another matter. There is no guarantee even for a repeat sale that we are comparing like with like. This is because the characteristics of a house may change over time due to renovations or the building of a new shopping center nearby, etc. The only way to be sure that like is compared with like is to double impute all houses (even with repeat sales). Hill and Melser (2008, pp. 600).

Equation (18) has the following features:

- Its general form is a Törnqvist index, a superlative price index — an index number formula with good approximation to a price index without substitution bias.
- It has no sample selectivity bias in that it includes estimates of constant-quality price change using three sets of price observations: (i) transacted in period 0 (but not in period t); (ii) price observations transacted in period t (not in period 0); and (iii) repeat price transactions available in both periods 0 and t .
- The aggregate of each term, that is each set of transactions, is weighted by the expenditure share of that set, for example, if there are few repeat transactions in periods 0 and t , these price changes have a commensurately less weight. This is appropriate for a sample selection issue ⁽²⁾.

⁽¹⁾ This paper acknowledges the contribution from Erwin Diewert (University of British Columbia) who helpfully provided rigorous derivations of these results in a previous working version of Silver and Heravi (2005).

⁽²⁾ This inclusion of transactions confined to periods 0, t and both periods is akin to issues faced in productivity measurement, with entering, exiting and continuing firms, and cost-of-living measurement with new, obsolete and continuing products as is particularly apparent with the emergence of the digital economy, see Diewert and Feenstra (2017) and Reinsdorf and Schreyer (2017).

- A dual imputation is used for the constant-quality price change measurement for the weights and relative predicted values.
- We outline later some practical advantages of using a form of equation (16). What is apparent here is that equation (16) has a sample selectivity bias, but one that can be retrospectively tested by comparison with equation (18), in which it is identified here as a component.

D. And what about a weighted characteristics hedonic index?

As long as we adopt appropriate aggregators and functional forms as outlined in Table 2.1, the imputation and characteristics methods give the same result. This holds for weighted and unweighted versions. Were a weighted characteristics approach taken the weights would be introduced, for each transaction, in the measure of the arithmetic mean of the characteristics.

$$(19) \quad \bar{z}_k^0 = \frac{\sum_{i \in N^0} \hat{p}_{i|z_i^0}^0 z_{i,k}}{\sum_{i \in N^0} \hat{p}_{i|z_i^0}^0} \quad \text{and} \quad \bar{z}_k^t = \frac{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^t z_{i,k}}{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^t}$$

The view taken here is that the RPPIs should be weighted and especially so given there is no lack of information for weights at the elementary level. It is more intuitive to compile price indexes as weighted averages of price changes, rather than characteristic values. Thus, the weighted imputation approach is recommended.

E. Alternative hedonic superlative price index number formulas

Our formulation of a hedonic superlative index, equation (18), differs from Hill and Melser (2008) — hereafter HM — reiterated in Hill (2013) and used by Rambaldi and Rao (2013) ⁽¹³⁾. HM (2008, pp. 601-602) derive hedonic Fisher and Törnqvist hedonic price indexes from the imputation and characteristics approach for a semi-logarithmic functional form of a hedonic regression. In an important contribution, they first show how the derivations from the two approaches provide the same results. Second, they solve the absence of matched models (infrequent transactions) by separately considering geometric Laspeyres (for constant period 0 characteristics) and geometric Paasche indexes (for constant period t characteristic), and then taking a geometric mean of the two to derive a superlative hedonic price index. We show both of these below but take issue with their formulation of a hedonic superlative price index compared with our equation (18).

HM (2008, pp. 601) define a geometric, period 0 sample hedonic price index as:

$$(20) \quad \prod_{i \in N^0} \left[\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right]^{\left(\frac{p_{i|z_i^0}^0}{\sum_{i \in N^0} p_{i|z_i^0}^0} \right)} = \prod_{i \in N^0} \left[\frac{\hat{p}_{i|z_i^0}^t}{\hat{p}_{i|z_i^0}^0} \right]^{w_i^0} \quad \text{where } w_i^0 = p_{i|z_i^0}^0 / \sum_{i \in N^0} p_{i|z_i^0}^0$$

⁽¹³⁾ De Haan and Diewert (2013) in the *RPII handbook*, Eurostat et al. (2013) have a similar formulation to Hill and Melser (2008) except that it is unweighted.

A geometric, period t index is similarly defined and a superlative formulation is a geometric mean of the period 0 and period t hedonic indexes:

$$(21) \quad \sqrt{\prod_{i \in S(0 \rightarrow t)} \left[\frac{\hat{p}_{ilz_i^0}^t}{\hat{p}_{ilz_i^0}^0} \right]^{w_i^0} \times \prod_{i \in S(t \rightarrow 0)} \left[\frac{\hat{p}_{ilz_i^t}^0}{\hat{p}_{ilz_i^t}^t} \right]^{w_i^t}}$$

This formulation differs from the one proposed in equation (18) in some important respects, including — further points and detail are in Silver (2016): (i) the HM formulation captures the samples of transactions in periods 0 and t , but it does not include the symmetric weights of each transaction, and thus cannot take account of substitution effects; (ii) price changes of period 0 transactions are weighted by w_i^0 and price changes of period t transactions by w_i^t , as opposed to \hat{w}_i^t and (iii) the sets of the price changes, $S(0 \rightarrow t)$ and $S(t \rightarrow 0)$, are not weighted according to their sample sizes. A symmetric mean is taken akin to a superlative index.

F. Use of a weighted least squares (WLS) estimator for the hedonic regression if weights are to be applied in aggregation

Finally, a neglected issue for the imputation (and characteristic) approach is the use of a weighted least squares (WLS) estimator for the hedonic regression. Diewert (2005a) in a seminal paper on weighted aggregation in hedonic regression indexes argued for a WLS estimator using expenditure shares as weights. Diewert (2005a) showed that for a bilateral two-period aggregate price comparison with average expenditure shares $(w_{i,0} + w_{i,t})/2$ used as weights in a WLS estimator, the estimated price change is equivalent to the superlative Törnqvist index ⁽¹⁴⁾. There are two main reasons why this may not work.

LEVERAGE, INFLUENCE, AND ROBUST ESTIMATORS

Silver (2005, Appendix 1) and Silver (2016, Annex 2) ⁽¹⁵⁾ raised a concern that observations may have undue influence in a regression for reasons unrelated to their weighting. In a time dummy hedonic regression a property price observation whose characteristics differ markedly from their means — have a relatively high leverage — and whose price is not well predicted by the regression — has a relatively large residual — can have a weight/influence in determining the constant-quality price change that is markedly greater than merited by its singular transaction in OLS or expenditure (price) share in WLS. For example, an atypical six-bedroom (larger) house with high leverage may also have a high residual from the regression, and thus influence in determining the regression coefficients, in spite of expenditure shares being possibly minimal. This undue influence applies even when expenditure-share WLS is used as an estimator.

⁽¹⁴⁾ Further contributions on developing (value-share) weighting systems in regression-based estimates of aggregate price change include Feenstra (1995), Ioannidis and Silver (1999), de Haan (2004 and 2009), Diewert, Heravi and Silver (2009), Ivancic, Diewert, and Fox (2011) and de Haan and Krsinich (2014), and for the cross country-product dummy approach, see Rao (2005).

⁽¹⁵⁾ Much of this is drawn from a 2002 unpublished mimeo by the author, Cardiff University.

The problem is not just one of observations with high influence having a disproportionate effect on the estimated coefficients and predicted values. High-end properties with low residuals lying on the estimated hedonic surface will have little to no influence on the estimated coefficients, in spite of what may be relatively high expenditures.

Influence statistics, such as Cook's distance, are a method of discovering influential observations in a multivariate framework. Measures of leverage and residuals are readily available in econometric software ⁽¹⁶⁾ and may be used to investigate and remove observations with unduly high influence, but such a process may not be regarded as 'arms-length'.

An alternative approach to the treatment of observations with undue influence (weight) is to use a heteroscedastic-consistent covariance matrix estimator (HCCME). For example, the HC2 estimator replaces the squared OLS residuals $\hat{\mu}_i^2$ by a term that includes the leverage — see also the HC4 estimator proposed by Cribari-Neto (2004) ⁽¹⁷⁾. The i th residual is inflated more (less) when its leverage, h_i , is large (small) relative to the average of the h_i , which is k/n , see MacKinnon (2013). Such influence effects are particularly problematic with the use of WLS for the time dummy approach because both the estimation of the regression coefficients and the aggregation of the RPPI are part and parcel of the same process.

More generally, observations with undue influence — in relation to their expenditure (price) — should be detected, examined, and modified/deleted and/or a robust estimator applied with the results contrasted with those estimated by WLS.

WLS is typically used in econometrics to correct for a heteroscedastic error term to achieve more precise parameter estimates. If the error term was homoscedastic prior to weighting, the weighting will induce heteroscedasticity and imprecise estimation, Solon et al. (2015).

⁽¹⁶⁾ For example, *EViews 9 User's guide* (2015) provides least squares diagnostics for outlier detection, described in 'Leverage plots', pp 218 along with six diagnostic statistics/tests of the 'Influence of an observation', pp 220.

⁽¹⁷⁾ EViews also has routines for 'Robust least squares' and details of three robust estimators one of which has as its focus outliers with high leverage. HC2 replaces the squared OLS residuals with $\frac{\hat{\mu}_i^2}{(1-h_i)}$ and HC4 with $\frac{\hat{\mu}_i^2}{(1-h_i)\delta_i}$ where, $\delta_i = \min(4, nh_i k)$ and n is the number of observations and k the number of explanatory variables, $\hat{\mu}_i$ the residuals. MacKinnon (2013) notes that a few papers have taken different approaches: Furno (1996) uses residuals based on robust regression instead of OLS residuals in order to minimise the impact of data points with high leverage, see *EViews 9 User's Guide* (2015), pp. 387.

5. Practical problem of appropriate hedonic formulas for thin markets

Having defined a hedonic superlative index, our concern is with the development of a best practice, well-grounded practical formula for measuring hedonic property price indexes that are suitable for property markets where properties are heterogeneous and transactions sparse — thin markets ⁽¹⁸⁾. It is a concern that would more generally apply to regular hedonic estimation and its vagaries of estimation and specification as would be required for real time compilation, that is, for every current period t . These proposals are grounded in the theoretical framework in the previous sections. Section 5A below reminds us of the results on equivalences and points to a preference for the hedonic imputation approach. Section 5B outlines methods that only require a hedonic regression to be estimated in the reference period, yet still manages to include approximations to a superlative index, and Section 5C outlines the use of an extended reference period in this context. There are three caveats to this: first, in Section 5D, the need for frequent re-estimation of the reference period hedonic regression is outlined and a mechanism for testing the desired frequency of the re-estimation. Second, the methods outlined in Section 5B suffer from having a single imputation; Section 5E provides a workaround and Section 5F provides guidance on estimators for the hedonic regression to be consistent with the weighting system applied.

A. Equivalences

We have shown that for reasonable hedonic specifications and the use of appropriate aggregators outlined in Table 2.1 above, the hedonic characteristics and imputation approaches, and indirect approaches all yield the same result. Similar results hold for weighted variants of the measures. There is an axiomatic sense that gives credence to a measure that gives the same results when derived from different, but valid, intuitions; this helps consolidate choice. An imputation approach is proposed since it has a natural formulation when weights are applied. The application of weights requires no new data and can be readily undertaken, as outlined below. A weighted RPPI is preferable to an unweighted one.

B. A hedonic RPPI based only on an estimated regression in the reference period

The proposed measures below are imputation RPPIs based on a current period sample of period t transactions, as outlined above. They only require a hedonic regression for period 0. Limiting the regression estimation to the reference period is a major advantage. Hedonic regression estimates are subject to the vagaries of specification and estimation procedures, particularly in thin markets. A measure based on a well-grounded regression, especially one

⁽¹⁸⁾ There are other approaches to the problem of thin markets including (i) estimating a temporally aggregated price index for example, moving from a quarterly to a semiannual or annual index, Geltner (1993) and Bokhari and Geltner (2012); (ii) use of a time-series methodology, such as the Kalman Filter, including Goetzmann (1992), Francke (2008), and Rambaldi and Fletcher (2014); (iii) the inclusion of other related series as explanatory variables in thin markets, Baroni et al. (2007); and (iv) an improvement to the efficiency of the estimator using data on sample sizes, Silver and Graf (2014).

based on an extended reference period as outlined below, better grounds the index. An unweighted version is equation (22) — taken from equation (7) above:

$$(22) \quad p_{HGM|z_i^t}^{0 \rightarrow t} = \frac{\prod_{i \in N^t} (p_i^t)^{\frac{1}{N^t}} \exp\left(\frac{1}{N^t} \sum_{i \in N^t} \ln \hat{p}_{i|z_i^t}^t\right)}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\frac{1}{N^t}} \exp\left(\frac{1}{N^t} \sum_{i \in N^t} \ln \hat{p}_{i|z_i^t}^0\right)}$$

We note that an adjustment for double imputation is not required for the unweighted version in equation (22). This is because for an OLS regression, $\frac{1}{N^t} \sum_{i \in N^t} \hat{p}_{i|z_i^t}^t = \frac{1}{N^t} \sum_{i \in N^t} p_i^t$, and similarly for the logarithms of prices, a feature that does not carry over to weighted counterparts. In using only a reference period, regression equation (22) is akin to the characteristics-based repricing method used by some European countries, as outlined in Hill et al. (2018). However, these repricing indexes are unweighted. Given the simplicity and efficacy of using weights, equation (22) cannot be recommended ⁽¹⁹⁾. Weighted versions are preferred.

A period t weighted version is equation (23) — taken from equation (15) above:

$$(23) \quad p_{PaHGM|z_i^t}^{0 \rightarrow t} = \prod_{i \in N^t} \left[\frac{p_i^t}{\hat{p}_{i|z_i^t}^0} \right]^{\left(\sum_{i \in N^t} \frac{p_i^t}{\hat{p}_{i|z_i^t}^0} \right)} = \exp \left(\sum_{i \in N^t} \left(\frac{p_i^t}{\sum_{i \in N^t} p_i^t} \right) \ln \left[\frac{p_i^t}{\hat{p}_{i|z_i^t}^0} \right] \right)$$

A quasi-superlative version is equation (24), clearly superior to equation (23) — taken from equation (17) above:

$$(24) \quad p_{HLL}^J = \prod_{i \in N^t} \left[\frac{p_i^t}{\hat{p}_{i|z_i^t}^0} \right]^{\left(\frac{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^0 + \sum_{i \in N^t} p_i^t}{2} \right)^{1/2}} = \exp \left(\sum_{i \in N^t} \frac{\hat{w}_i^t}{2} \left[\ln p_i^t - \ln \hat{p}_{i|z_i^t}^0 \right] \right)$$

where $\hat{w}_i^t = \frac{1}{2} \left(\frac{p_i^t}{\sum_{i \in N^t} p_i^t} + \frac{\hat{p}_{i|z_i^t}^0}{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^0} \right)$

Equation (24) while only requiring the estimation of a hedonic regression in the reference period clearly provides an estimate that includes substitution effects for the sample of period t transactions. The thought experiment is of a price change of an individual house: its transaction price in period t compared to what its transaction price would have been in period 0 had it been sold then — a counterfactual price relevant to the needs of RPPI

⁽¹⁹⁾ The characteristics hedonic RPPI requiring a hedonic regression only in the reference period 0 is given by the first two terms of:

$$\frac{\prod_{k=0}^K (\bar{z}_k^t)^{\beta_k^t} \exp\left(\sum_{k=0}^K \bar{z}_k^t \ln \beta_k^t\right) \exp\left(\sum_{i \in N^0} p_i^t\right) \exp\left(\sum_{i \in N^0} \hat{p}_i^t\right)}{\prod_{k=0}^K (\bar{z}_k^0)^{\beta_k^0} \exp\left(\sum_{k=0}^K \bar{z}_k^0 \ln \beta_k^0\right) \exp\left(\sum_{k=0}^K \bar{z}_k^t \ln \hat{\beta}_k^0\right) \exp\left(\sum_{k=0}^K \bar{z}_k^t \ln \hat{\beta}_k^0\right)}$$

Making use of equations (9) and (11), a constant period t , double-imputation, hedonic characteristics RPPI can be measured by simply taking the geometric mean of the actual prices in the numerator which is equal for an OLS regression to that of the predicted prices.

measurement. The phrasing of the weights captures the (approximate) substitution effect being relevant to the price change measured. If the prices of houses in an up-market area rise faster than other houses, the weights will reflect the shift in expenditures since they are tied to the definitions of the price change. Identical results can be derived from a characteristics approach ⁽²⁰⁾.

However, while the weights are appropriate, the price change for the weighted version is a single imputation and for reasons outlined in Section 2D, a workaround is required to develop an approximation to a double imputation. We return to this in Section 5E below.

C. That an extended-current period formulation be used since sparse data is less problematic

A major problem in RPPI and CPPI estimation is that of sparse data on heterogeneous properties. However, this can be alleviated by the use of an extended reference period, noted as a useful feature of property price index construction by de Haan and Diewert (2013) ⁽²¹⁾. Nonetheless, extended periods may not be used for the current period hedonic regression estimation without being to the detriment of the periodicity of the series, for example, a quarterly series becoming bi-annual. This gives further support to the case for reference-period only hedonic regression estimation such as in equations (22) to (24).

There may not be an adequate number of observations and/or variation in the characteristics of the sample of properties transacted in period 0 to enable reliable and pertinent estimates to be made of the coefficients of price-determining characteristics that define properties sold in period t . For example, there may be a relatively small number of four-bedroom houses in a prime location sold in period t , but none sold in period 0. The problem of sparse data prevents reliable estimates of the predicted price from a period 0 regression of the period t characteristics ⁽²²⁾. The current period formulation can go some way to solving the problem of sparse data simply by defining the reference period 0, for example, for a quarterly series first quarter 2018, second quarter 2018, etc., to be an extended period of say a year with the index referenced as 2017 = 100.0 and centred at mid-2017. As such, the period 0 regression will be more likely to better encompass the characteristics of period t properties.

The advantage of not having to re-estimate a hedonic regression on a periodic basis is well recognised by NSIs in Europe. The repricing variant of the characteristics approach used by eight countries has an extended reference period of a year to establish the average values of the characteristics and the commensurate estimated marginal values from the hedonic regression. The repricing approach allows for this due to its correspondence to the characteristics approach and equivalence to the imputation approach when crafted following the principles in Table 2.1. We continue with the imputation approach.

⁽²⁰⁾ The interpretation of the characteristics approach is problematic, thus the focus on the imputation approach.

⁽²¹⁾ Though de Haan and Diewert (2013) refer to it in the context of an advantage of the indirect method, similar such formulations and advantages apply to the direct imputation and characteristics approach.

⁽²²⁾ More formally, the width (standard error) of a prediction interval from a regression of y on x , for a given value of say $x = x'$, depends not only on the fit of the regression — the larger the sample size and dispersion of the explanatory variables, the smaller the interval — but also on the distance the given value of x' is from the sample mean \bar{x} . The prediction will be better for values of x' closer to \bar{x} , see Maddala and Lahiri (2009).

D. Sample selectivity bias

Since the sample of period t transactions is only used, there may be a sample selectivity bias as explained in Section 4C. Yet equation (18) is a measure of a superlative Törnqvist RPPI for the complete period t sample of transactions; it is quasi-superlative. It would be a relatively trivial matter for a retrospective study to be conducted prior to the adoption of the methodology that compares the results of equation (24) with (18) to ascertain the extent and direction of any such bias. Sample selection bias can be mitigated by frequent re-estimation of the hedonic regression, say every year or two years, and chain-linking the results. This would be akin to rebasing a consumer price index to introduce new weights.

E. Dual imputations of price relatives: a workaround

Equations (23) and (24) differ from their counterpart equations (15) and (17) in that the measure of price change in the latter use dual imputations while the former uses a single imputation. This deficiency in equations (23) and (24) arise from the simple fact that our intention is to avoid estimation of a hedonic regression in the current period. The single imputations in equations (23) and (24) require workarounds so that approximations to predicted prices are used instead of actual prices. Define weights as:

$$(25) \quad \hat{w}_{i|z_i^t}^0 = \frac{\hat{p}_{i|z_i^t}^0}{\sum_{i \in N^t} \hat{p}_{i|z_i^t}^0}; \quad \hat{w}_{i|z_i^0}^0 = \frac{\hat{p}_{i|z_i^0}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i^0}^0}; \quad w_i^0 = \frac{p_i^0}{\sum_{i \in N^0} p_i^0}; \quad w_i^t = \frac{p_i^t}{\sum_{i \in N^t} p_i^t}$$

A workaround for the predicted value of period t prices for a dual imputation can be seen from equation (26):

$$(26) \quad P_{HIGL}^J = \frac{\prod_{i \in N^t} (p_i^t)^{w_i^t}}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\hat{w}_{i|z_i^t}^0}} \times \frac{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^0)^{\hat{w}_{i|z_i^0}^0}}{\prod_{i \in N^0} (p_i^0)^{w_i^0}} \neq \frac{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^t)^{\hat{w}_{i|z_i^t}^t}}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\hat{w}_{i|z_i^t}^0}}$$

The first term is the single imputation, 'adjusted' by the second term which is the ratio of the geometric mean of predicted values in period 0 to that of actual values in period 0, a term readily compiled from the real time data since we have estimated a regression in period 0. This is not equal to our desired measure, the third term in equation (26), but should be a close approximation. The desired expression is the period t ratio of predicted to actual values, that is we are assuming:

$$(27) \quad \frac{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^t)^{\hat{w}_{i|z_i^t}^t}}{\prod_{i \in N^t} (p_i^t)^{w_i^t}} = \frac{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^0)^{\hat{w}_{i|z_i^0}^0}}{\prod_{i \in N^0} (p_i^0)^{w_i^0}}$$

The validity of the assumption can be examined over time as the hedonic regression is updated; the more frequent the updates, the more likely the double-imputation workaround is likely to hold. The workaround in equation (26) can also be justified using the indirect method; that is we divide the change in actual average prices by the change in the characteristic mix ⁽²³⁾:

$$(28) \quad P_{HIGL}^J = \frac{\left(\frac{\prod_{i \in N^t} (p_i^t)^{w_i^t}}{\prod_{i \in N^0} (p_i^0)^{w_i^0}} \right)}{\left(\frac{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\hat{w}_{i|z_i^t}^0}}{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^0)^{\hat{w}_{i|z_i^0}^0}} \right)} = \frac{\prod_{i \in N^t} (p_i^t)^{w_i^t}}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\hat{w}_{i|z_i^t}^0}} \times \frac{\prod_{i \in N^0} (\hat{p}_{i|z_i^0}^0)^{\hat{w}_{i|z_i^0}^0}}{\prod_{i \in N^0} (p_i^0)^{w_i^0}} \neq \frac{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^t)^{\hat{w}_{i|z_i^t}^t}}{\prod_{i \in N^t} (\hat{p}_{i|z_i^t}^0)^{\hat{w}_{i|z_i^t}^0}}$$

The first term of equation (28) has integrity in the sense that the ratio of average actual prices between periods 0 and t in the numerator is of actual values, while the ratio in the denominator is a dual imputation of predicted prices.

In Section 2 three hedonic approaches were outlined: the imputation, characteristics, and time dummy approaches. The focus so far has been on the imputation approach as a natural vehicle to introduce weights supported in turn by its equivalence to the characteristics approach. We have neglected the time dummy approach outlined in Section 2A, to which we now turn.

6. What about the time dummy approach (TDA)?

A. Introduction

The focus on the hedonic imputation approach arose in this paper from: (i) an equivalence between the intuitive hedonic characteristics and imputation approaches to compiling RPPs. This consolidation strengthened the case for either measure against the time dummy approach (TDA); (ii) that hedonic imputation RPPs can be readily weighted using current information in real time production — weighting using the TDA is more problematic and less transparent as outlined below; (iii) the weights used in a hedonic imputation approach can take a quasi-superlative form not being prone to substitution bias; (iv) that a quasi-superlative form can be usefully derived for real time compilation that only requires estimation of a hedonic regression in the reference period; and (v) the potential exists to make use of an extended reference period for thin markets, though a TDA can be similarly constructed. The focus on the imputation method is due to its having a more natural intuition for weighting, an innovation of this paper.

⁽²³⁾ More generally recommended in Silver (2016).

This is neither to negate the advantages of a time dummy hedonic RPPI nor to rule it out as a feasible method. The TDA estimates the change in average prices while controlling for changes in the quality-mix of the characteristics. The TDA has a direct conceptual correspondence to the indirect method. Further, the TDA implicitly uses a dual imputation being concerned with the difference between predicted prices, controlled in the regression for quality-mix change. For thin markets, the estimation period for the hedonic regression can be readily extended by using a larger reference period or a moving window. The TDA also has a natural computational ease integrated into the estimation of a hedonic regression using panel/adjacent period data. Once estimated, the simple addition of time dummy variables provides, via the exponent of the parameter estimates, the RPPI (Section 2B).

B. Weights

The hedonic imputation (and characteristics) approaches can, unlike the time dummy method, have explicit weights readily and reliably applied in an easy-to-compute manner that can be interpreted in index number theory as a ‘quasi’ hedonic superlative index. Its difference from a full hedonic superlative index — equation (18) minus equation (17) — can be readily computed, identified and understood. Weighting for the TDA is problematic.

Weighting for the TDA can be undertaken using WLS as outlined in Diewert (2005a). The TDA estimates the parameters of the price-determining explanatory variables alongside the estimate of the time dummy parameters, as the basis for the estimated RPPI, as part and parcel of the same process. The use of WLS in the TDA benefits both. However, for reasons of influence and heteroscedasticity, as outlined in Section 4F, WLS may assign the wrong weights and OLS would be the preferred estimator. Solon et al. (2015) shows how OLS may be superior to WLS in determining the estimated parameters.

The weighting in the imputation approach is decoupled: that used for the aggregation is clear, sound and desirable, as outlined in the preceding sections. The weighting for the parameter estimates for the hedonic regression is less so. But we have the flexibility here to explore the efficacy of alternative estimators, of OLS against WLS. This is in sharp contrast to the TDA outlined above.

C. Ratio of averages versus average of ratios

The TDA has as its implicit measure of price change a ratio of the (geometric) mean of prices for properties whose characteristics are valued at constant period 0 (hedonic) prices and again at constant period t (hedonic) prices. For a WLS, the weights are attached to the individual prices and characteristics in each period. Thus, *the weighted hedonic time dummy estimate of the change in log prices is equal to a period t expenditure share weighted average of the quality-adjusted log prices ... less a period 0 expenditure share weighted average of the quality-adjusted log prices ...* (Diewert, Heravi, and Silver (2009), pp. 174). There is no such reasonable price index number formula of this form. The imputation approach calculates weighted average price changes.

D. Transparency

While the hedonic imputation (and characteristics) approaches are based on reasonable intuitions, the TDA can only be explained within the context of a regression equation.

A natural question is the extent of the difference between TDA and hedonic imputation indexes. Were this difference simply explained, the use of the TDA could be justified, at least for particular purposes. Diewert, Heravi, and Silver (2009), improving on Silver and Heravi (2007b), have formally determined the factors distinguishing between the results of (adjacent period) time-dummy and hedonic imputation hedonic indexes. It is not straightforward:

If either the weighted average amounts of each characteristic are much the same in the two periods being considered ..., or if the expenditure share weighted model characteristics variance covariance matrices are similar across periods, or if the separate weighted hedonic regression quality adjustment factors do not change much across the two periods, then it will not matter much which method is used, which is the new result that is demonstrated in this paper. Diewert, Heravi, and Silver (2009), pp. 180.

E. Estimation of hedonic regression in successive periods: adjacent period and moving window

Unlike the imputation approach, the TDA requires a hedonic regression to be estimated in successive periods. This may be problematic on resource and/or data grounds. The TDA can be based on chained adjacent successive periods, Diewert (2005b), or some moving window of data, O'Hanlon (2011). The adjacent-period hedonic TDA method is reliable in the sense that individual quarter-on-quarter price changes are only determined by the up-to-date data for these periods. It is a version of the rolling window approach that restricts the size of the window to two successive periods. Rolling windows of larger sizes, such as the four quarters, are advantageous when data are sparse and concern exists as to the robustness of regression estimates based on a series of hedonic regressions either due to specification or estimation, including sparse data, issues. However, the longer the window, the smoother will be the series and the longer the lag in tracking turns in the series. The adjacent-period rolling window if faithfully based on a sufficient sample size and well-specified hedonic regression should give timely information about changes in property price inflation that, while seemingly more volatile, are rightly so having not been subjected to what may be undue smoothing ⁽²⁴⁾. There is however, a caveat to this: the use of up-to-date weights, while desirable, can induce an unwarranted chain drift in the RPPI. This is in part an empirical matter dependent on the extent to which prices and expenditures 'bounce', a covariance term between short-term price changes and weights. Multilateral formulas are a solution to this problem, Ivancic et al. (2011).

⁽²⁴⁾ There is a case for using a Kalman Filter Smoother (Rambaldi and Fletcher (2014)). The Kalman Filter Smoother has been shown in some empirical work to produce relatively stable estimates that need only be estimated sporadically, not each period. It is argued that the indexes based on the Kalman Filter optimally weight current and past information while the rolling window constrains the estimation to the period of the window, two-periods in the case of the adjacent period window, used in the study.

F. Restriction of coefficients

We note that the coefficients on the price-determining characteristics for a TDA are restricted to be the same over time: an RPPI for all T time periods between periods 0 and t would restrict $\beta_k = \beta_k^0 = \beta_k^1, \dots, = \beta_k^T$, or $\beta_k = \beta_k^0 = \beta_k^1$ in an adjacent period context; this holding of coefficients constant is often used as a criticism of the TDA. The imputation approach holds quantities of characteristics constant either at period 0 characteristic values, or at period t characteristic values, equations (8) and (9). However, price indexes can be defined as changes in aggregate nominal values divided by changes in volume — the factor reversal test. In this context, it would be the change in average prices between periods 0 and t divided by the change in the volume of characteristics as given by the indirect approach in equation (10). The driving force behind the indirect measure is the holding \hat{p}_i^0 constant — the β_k^0 — when valuing z_i^0 and z_i^t ; both the imputation and TDA approaches are built on similar foundations, as shown by Diewert, Heravi, Silver (2009). Where the imputation approach has an advantage in this regard is its ability to decouple the restriction of $\beta_k = \beta_k^0 = \beta_k^1$ enabling separate RPPI estimates holding β_k^0 constant, and β_k^t constant, as in equations (8) and (9), and thus giving more insight by creating bounds on an averaged restriction.

G. Thin markets

A TDA does not allow for hedonic regressions in thin markets to be only estimated in the reference period, or for that matter, an extended reference period, that excludes the current period. An adjacent period hedonic for say the second quarter against the first quarter in 2017 (Q2-2017/Q1-2017) would require a time dummy hedonic regression estimated using both Q2-2017 and Q1-2017 data, or a rolling monthly index over three months, a regression including January, February and March, 2017, and for the April index, a regression including data for February, March and April, 2017. For thin markets there is the opportunity to extend the price reference period, but only insofar as data in the current period are also included. The Paasche-type quasi-imputation index does not require a regression that includes period t observations. Further supporting arguments for a hedonic imputation index against a TDI are given in Diewert, Heravi, and Silver (2009) and in Silver and Heravi (2007b).

H. Double imputation for the TDA and avoiding asymmetric parameter estimates

The TDA has an implicit double imputation. Consider this simple illustration of an unweighted regression of price $p^{0,t}$ on a single explanatory variable, $Z_1^{0,t}$, over two periods of data, period 0 and period t . A dummy variable for time is included, $D=1$ for period t observations and zero otherwise. This allows the intercepts for period 0 and period t to differ. The parameter estimate for $Z_1^{0,t}$ is constrained to be the same for each period, that is: $\beta_1^0 = \beta_1^t = \beta_1$ for the estimated regression:

$$(29) \quad \hat{p}_i = \hat{\beta}_0 + \hat{\beta}_1 Z_1 + \hat{\beta}_2 D$$

where $\hat{\beta}_2 = \hat{\beta}_0^t - \hat{\beta}_0^0$; that is, equation (29) estimates two regressions, albeit with a common error term, where the coefficient on the dummy variable is an estimate of the difference between period 0 and period t average prices having adjusted for changes in the quality characteristics; this is the TDA. The equations for period t and 0 are:

$$(30a) \quad \hat{p}_i^t = \hat{\beta}_0^t + \hat{\beta}_1 Z_1^t$$

$$(30b) \quad \hat{p}_i^0 = \hat{\beta}_0^0 + \hat{\beta}_1 Z_1^0$$

Subtract equation (30b) from (30a) and rearrange:

$$(31a) \quad (\hat{p}_i^t - \hat{p}_i^0) = (\hat{\beta}_0^t - \hat{\beta}_0^0) + \hat{\beta}_1 (Z_1^t - Z_1^0)$$

$$(31b) \quad (\hat{\beta}_0^t - \hat{\beta}_0^0) = (\hat{p}_i^t - \hat{p}_i^0) - \hat{\beta}_1 (Z_1^t - Z_1^0)$$

The difference between the intercepts is the TDA's estimate of the change in price adjusted for the change in the characteristics. Note first, that the price change is between predicted prices, a dual imputation, and second, that the estimated marginal value of the price-determining characteristic is $\hat{\beta}_1$ which is estimated using both period 0 and period t data. We are neither holding $\hat{\beta}_1$ constant as a period 0 estimate and deriving an index nor likewise for period t estimates, but cutting to the chase and using some average derived from the two period's data, as is right and proper.

7. Summary

There are serious problems linked to properly measuring RPPIs: transactions of properties are infrequent and properties are heterogeneous. Measures of average property price change can be confounded by changes in the quality-mix of properties transacted between the two periods compared. Hedonic regressions have been advocated as the primary method for adjusting measured price changes for the change in the quality-mix of transactions. De Haan and Diewert (2013) outline the three main approaches to using hedonic regressions for this purpose: the time dummy; characteristics/repricing; and imputation approaches. For each of these approaches there are myriad forms, including different forms of weights, sample selection, imputations, aggregators, direct and indirect methods and no straightforward guidelines. We demonstrate equivalencies between the approaches for quite straightforward formulations to narrow down the choice among formula. Real time RPPIs are currently unweighted, which cannot be justified. Of importance is that a methodological framework is established by which weighted hedonic RPPIs are best compiled. We devise an innovative form of weighting for property price indexes and, therefrom, derive quasi-superlative and superlative formulations of these hedonic indexes that improve on those in the literature. Arising from these definitions we develop well-grounded practical measures of hedonic property price inflation suitable for thin markets and sparse data. A formulation is provided that is not subject to the vagaries of the periodic estimation of hedonic regressions. It benefits from an innovative weighting system along with a 'quasi' superlative formulation that should take account of much of any substitution bias at this level. The 'quasi' superlative hedonic formulation is tightly phrased as a component of a hedonic superlative index and its implicit

assumptions easily testable and not, *prima facie*, problematic. All of this is without additional data currently used and practically applicable in real time.

Some readers may wonder what the fuss is about. Monetary authorities rightfully give a high priority to monitoring the irrational exuberances of property price inflation. Trends and turning points in property price inflation — bubbles — cannot be relied upon to be explained by the structural underpinnings of the economy. RPPIs need to be internally methodologically sound and reliable. Hedonic regressions are widely used in Europe for RPPI estimation. This paper provides readily applicable methods that can be applied in real time using currently available datasets. Hill et al. (2018) have found a variety of hedonic approaches to be used in Europe. This paper allows the different methods to be identified under a common framework and their pros and cons established. It also pays attention to the practical data and estimation needs that may be problematic for some countries, especially for commercial property price indexes, to which this self-same methodology applies. Yet, more particularly, for the large part, European RPPIs employ unweighted hedonic methods. Unweighted RPPIs are hard to justify.

The methodology outlined above has been rigorously defined as is appropriate for an important economic statistic. While the formula in the paper may appear untoward, the code for their implementation is quite straightforward. The preferred unweighted hedonic imputation index requires three lines of code in STATA and a quasi-superlative one, four lines of code (see Annex 1).

Acknowledgements

The author would like to thank the following for helpful comments: Erwin Diewert (University of British Columbia), Robert Hill (University of Graz), Claude Lamboray (Statistics Luxembourg) and Fernando Raul Mundaca Hurtado (Central Reserve Bank of Peru).

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Annex 1: Illustrative code in STATA

Consider a semi-log hedonic regression with two variables stacked quarterly data. The regression is estimated over the (extended) first four quarters of data for this simplified case of logarithm of price, **lnp**, on say **postcode** (a single variable with 1-100 outcomes), **size** (in square metres) stacked by **quarter**. The **i.postcode##c.size** will include 99 dummy variables for **postcodes**, a single variable for **size**, and interaction effects on **size** for each **postcode**. Simpler formulations are of course possible and the expression can be easily extended to further variables. STATA has extensive routines for chart/diagnostic tests and measures of heteroscedasticity; multicollinearity; normality of residuals; outliers, leverage, and influence; omitted variables; alternative estimators and more. All of this would serve a compiler in producing a companion paper to the release on the hedonic methodology, to help improve/justify the hedonic model.

The second line predicts **lnp** of each property transacted in the fifth quarter using the reference period hedonic regression. The third line would provide a mean of the log of the predicted price from which the exponent can be taken to give the denominator of equation (22). The numerator can be readily determined by summarising **lnp if quarter==5**. Weighted versions require an additional line of code to multiply the fitted/predicted values in the second line by their respective weights as in equation (24).

*Using data lnp postcode size — stacked by quarter

```
regress lnp i.postcode##c.size if quarter>0 & quarter<5
predict lnp5 if quarter==5
summarize lnp5
```